Pairs Trading: Different Weights, Methods and Markets

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Resumo

Este artigo analisa a lucratividade de portfólios de pairs trading auto-financiados para os mercados acionários Brasileiro, Europeu e Americano utilizando duas metodologias diferentes de seleção de pares: os métodos da distância e cointegração. Uma comparação ampla das metodologias de pairs trading com uma base de dados grande de diferentes mercados é capaz de elucidar os principais benefícios e fragilidades de cada método. De modo geral, os resultados mostram que diferentes estruturas de mercado favorecem diferentes estratégias de pairs trading. Mais especificamente, a seleção via cointegração desempenha melhor nos mercados Brasileiro e Europeu, enquanto que o método da distância gera resultados melhores para os Estados Unidos. A melhor estratégia em cada mercado possui um alpha significante com um beta negligenciável.

Palavras-chave: Arbitragem Estatística, Pairs Trading, Cointegração, Soma dos Quadrados dos Desvios.

Classificação JEL: C58, G11, G14, G15.

Abstract

This paper analyzes the profitability of self-financing portfolios using pairs trading for the Brazilian, European, and American stock markets using two different pairs selection methodologies: the distance and cointegration methods. A comprehensive comparison of pairs trading methodologies using large datasets from different markets uncovers the main benefits and drawbacks of each approach. Overall, the results show that different market structures favor different pairs trading strategies. More specifically, the cointegration approach performed better on the Brazilian and European markets, while the distance method delivered better results for the US. The best strategy in each market yields significant alpha with negligible beta.

Keywords: Statistical Arbitrage; Pairs Trading; Cointegration; Sum Square Deviation

JEL Classification: C58, E44, E47, G11, G14, G15.

1 Introduction

The computational advances of the past decades have stimulated the development of trading via computer programs and the rise of algorithmic trading. These systems are designed to search for patterns in financial markets, detect deviation of market prices from these patterns, and profit from detected anomalies. Algorithmic trading is now responsible for more than 70 percent of the trading volume in the the US markets (HENDERSHOTT et al, 2011). On the other hand, events like the Flash Crash of May 6 2010, when the Dow Jones Industrial Average dropped 600 points in less than 5 minutes, revealed the lack of knowledge about the consequences and robustness of algorithms used in practice (NUTI et al, 2011).

This paper proposes the use of large datasets from stock markets in the USA, Europe and Brazil in order to compare two popular pairs trading algorithms out-of-sample, namely: the sum of squared deviations approach of Gatev et al. (2006), and the cointegration approach suggested by Alexander and Dimitriu (2002). The markets selected allow for a detailed comparative study, and the analysis of both strategies under different market conditions allows us to uncover differences between trading algorithms. Although the literature on pairs trading strategies is growing fast, it still lacks a comprehensive study of the performance of different methodologies across developed and emerging markets. Moreover, most studies use different trading periods, different criteria to select assets to be included in the sample, and different formation period, rendering a cross study comparison impossible. Since pairs trading performance is influenced by the methodology chosen, it is important to compare them under different circumstances to understand if there is an overall winner, or if some strategies are better suited to specific market conditions. To this end, we use the database of an important world financial center, the USA, a developed monetary union, the Euro Area, and an emerging market, Brazil, along with equal parameters for each methodology in order to compare them and test to see which is more suitable in a given environment. Using financial data from emerging markets is a particularly difficult task. There are specific challenges that distort the data and the transaction prices (unexpected information, insider trading, lack of liquidity), rendering classical statistical inference biased. We use data for an emerging market because they tend to have a more volatile stock market and also because of its dependence with the main developed markets. Seeing how both strategies perform in such an environment is particularly helpfull from an asset allocation perspective. Also, Brazilian listed companies tend to have preferred and common stocks listed, which is an uncommon characteristic allowing closer examination of causes of potential price deviation between assets that have rights to the same cash flow source.

In a nutshell, pairs trading strategies speculate on future convergence of spread between similar securities. Similarity concerns industry, sector, market capitalization, and other common exposures that might imply a comovement between stocks. However a profitable strategy might also be constructed with stocks covering different sectors based purely on statistical properties of the time series. Gatev et al. (2006) test a simple non-parametric pairs trading algorithm on the US market between 1963 and 2002, finding average annualized returns of up to 11.00% for portfolios of pairs. They suggested that the abnormal returns to pairs strategies were a compensation to arbitrageurs for enforcing the law of one price. Another popular algorithm to select pairs is based on the presence of a cointegration relation between stock prices. For a textbook treatment of the subject see Vidyamurthy (2004). The use of the cointegration technique to asset allocation was pioneered by Alexander (1999) and in the previous decade it was increasingly applied in financial econometrics. [1]. For example, Caldeira and Moura, (2013), using the cointegration method for Brazil, found that for the period between 2005 and 2012 the strategy exhibits annualized excess returns of up to 16,38%.

It's clear that many studies attempted to test the profitability of pairs trading strategies. However, all of them focus on either a single market studies or on a single methodology to select pairs. Do et al (2012) examined the impact of trading costs on pairs trading profitability in the U.S. equity market and documented that, after 2002, pairs trading strategies were largely unprofitable. Bowen et al. (2010) back-test a pairs trading algorithm using intraday data over a 12 month period in 2007, and conclude that returns are highly sensitive to the speed of execution. Moreover, accounting for transaction costs and enforcing a "wait one period" restriction, excess returns are completely eliminated. Broussard and Vaihekoski (2012) tested the profitability of pairs trading under different weighting structures and trade initiation conditions using data from the Finnish stock market. Although the proposed strategy is profitable, the authors note that returns have declined in recent years possible due to increased competition among hedge funds, and/or a reduction in the importance of an underlying common factor that drives the returns in a pairs trading strategy.

The datasets used in these analysis can be divided into two groups: First, the data comes from developed countries which have plenty of historical financial information available, as is the case of the United States. The articles by Gatev et al. (2006), Engelberg et al. (2009), Huck (2010), Do and Faff (2010), Bowen et al. (2010) and Do and Faff (2012) are examples that use data from the United States. The second group includes datasets from developing countries. These studies analyse shorter time periods and a smaller number of assets in the database. Yuksel et al. (2010) analyses pairs trading in Turkey, Broussard and Vaihekoski (2012) in Finland, Perlin (2009), and Caldeira and Moura (2013) in Brazil.

Summarizing, the contributions of this paper are three fold: (i) we test different strategies in the same market using the same database and parameters in order to mantain comparability between strategies and with Gatev et al. (2006) and Caldeira and Moura (2013); (ii) we are able to analyze how different strategies perform differently in a developed country and a developed monetary union; (iii) finally, we chose Brazil, one of the most important and open emerging markets as the database of interest in the developing world. The results show that the cointegration method has statistically superior Sharpe Ratio against the distance method for the Euro Area and Brazilian markets for all 3 portfolios formed, while the distance method had a superior performance for the American market, signalling that different market structures affect the performance of each methodology.

The article is organized as follows. In the next section we describe both pairs trading methodologies and some. Section 3 presents some implementation details common to both methods, as well as the evaluation strategy. Section 4 describes the three large data sets used and discusses the results of our comparison. Section 5 evaluates the performance of both methodologies and the last section concludes with some final remarks.

2 Pairs Trading Methodologies

Pairs trading is an algorithmic trading strategy designed to exploit short-term deviations from an existing long run equilibrium between two stocks. However, different methods have been

[1] See among others, Alexander and Dimitriu (2002), Bessler and Yang (2003), Yang et al. (2004) Galenko et al. (2012) Caldeira and Moura (2013) and Gatarek et al. (2014)

proposed in the literature to identify pairs to be traded (VIDYAMURTHY, 2004; ALEXANDER; DIMITRIU, 2005; ELLIOTT et al, 2005; GATEV et al, 2006; CALDEIRA; MOURA, 2013). The motivation for trading pairs has its roots in works that preach the existence of long term relation between stocks. If there exists indeed a long term equilibrium, deviation from this relation are expected to revert. Since future observations of a mean-reverting time series can potentially be forecasted using historical data, this literature challenges the notion that stock prices cannot be predicted (LO; MACKINLAY, 1997; GUIDOLIN et al, 2009). Active asset allocation strategies based on mean-reverting portfolios, which generally fall under the umbrella of statistical arbitrage, have been used by investment banks and hedge funds for several years (GATEV et al, 2006). The word statistical in context of an investment approach is an indication of the speculative character of investment strategy. It is based on the assumption that the patterns observed in the past are going to be repeated in the future. This is in opposition to the fundamental investment strategy that both explores and predicts the behaviour of economic forces that influence the share prices. Pairs trading is possibly one of the simplest statistical arbitrage strategy, since it consists of a portfolio of only two assets. In this approach, we are not interested about trends for particular assets but with a common trend among a pair of stocks, which defines a long-run equilibrium between them. The idea behind pairs trading is that when prices of two shares move together there could be short term deviations to be arbitraged. Thus, this trading strategy consists in detecting pairs of stocks that historically move together, waiting for the spread between them to widen, longing the underpriced stock and shorting the overpriced one to profit when prices revert back to their long-run equilibrium.

Thus, pairs trading is a purely statistical approach designed to exploit equity market inefficiencies defined as the deviation from a long-term equilibrium across stock prices observed in the past. As argued by Do and Faff (2010), pairs trading falls under the big umbrella of the long-short investing approach. According to Avellaneda and Lee (2010) the term statistical arbitrage includes investment strategies that have certain characteristics in common: (i) trading signals follow a systematic rule, in opposition to fundamentals based strategies; (ii) strategies seek to be market-neutral, in the sense that they are not exposed to broad market risk, i.e, they have a zero beta; (iii) the mechanism used to obtain abnormal returns is based on statistical analysis. The success of pairs trading, especially statistical arbitrage strategies, depends heavily on the modeling and forecasting of the spread time series although fundamental insights can aid in the pre-selection step. Pairs trading needs not be market neutral although some say it is a particular implementation of market neutral investing (JACOBS et al, 1999).

Broadly defined, there are three different approaches to pairs trading: the distance approach, the stochastic approach and the cointegration approach. These methods all vary with regard to how the spread of the stock pairs is defined. This paper compares two most popular methods of selecting pairs of stocks between practitioners and researchers: the distance method proposed by Gatev et al. (2006) and the cointegration approach used in Lucas (1997), Alexander and Dimitriu (2005), Do et al. (2006), and Caldeira and Moura (2013).

In order to understand the economic source of potential mispricing of assets and the profits that may arise, it's important to explain how asset pricing can be viewed. Asset pricing may be done on absolute or on relative terms. The source of value on absolute pricing arises from fundamentals such as discounted future cash flow. This is considered difficult (GATEV et al, 2006), with relative pricing being slightly easier. On relative terms, two securities that are close substitutes for each other should, on theory, sell for the same price, without defining which price it would be. Hence, it's possible for relative pricing to allow bubbles in the economy, without permiting arbitrage or profitable speculation. Relative pricing is similar to the Law of One Price

(LOP), as in its ability to generate asset prices that are related to each other, even if they are wrong. Chen and Knez (1995) explain that markets that are closely integrated should have similar prices for investments with similar playoffs. The relative pricing is a weaker condition and subject to bounds on prices for unusual states, howeverm this allows for the examination of quasi-efficient economies, or near integrated markets as is the European Union and the less developed Brazilian market. The Brazilian market includes different assets for the same company, which creates the possibility of evaluating the price fluctuations of assets that are related to the same cash flow. Also, the European Union has a near-integrated financial market

It's important to mention that this theory corresponds to the objective of finding two stocks whose prices move closely together as long as we can identify states of nature as the time series of observed historical trading days. We use two pairs methodologies to select the pairs portfolios. We then trade pairs whose prices closely match in historical state-space, because the LOP suggests that in an efficient market their prices should be nearly identical. In this framework, the current study can be viewed as a test of the LOP and near-LOP in the U.S., EU, and Brazilian equity markets, under certain stationarity conditions. We are effectively testing the integration of very local markets—the markets for specific assets. This is similar in spirit to Bossaerts' (1988) test of co-integration of asset prices at the portfolio level. It's also possible that the marginal profits to be had from risk arbitrage of these temporary deviations is crucial to the maintenance of first-order efficiency.

According to Gatev et al (2006), pairs trading can be justified within an equilibrium asset-pricing framework with nonstationary common factors as in Bossaerts and Green (1989) and Jagannathan and Viswanathan (1988). If the long and short components fluctuate with common nonstationary factors, then the prices of the assets in the portfolios would be co-integrated and the pairs trading strategy would be expected to work. Evidence of exposures to common nonstationary factors would support a nonstationary factor pricing framework. The space of cumulative total returns with dividends reinvested is the basic space for the pairs trading strategies in this article. Since we want to keep the notion of the empirically observed co-movement of asset prices, without unnecessarily restrictive assumptions, we proceed in the same way as the co-integrated prices literature. More specifically, our matching in price space can be interpreted as follows. Suppose that prices obey a statistical model of the form,

$$p_{it} = \sum \beta_{il} p_{lt} + \varepsilon_{it} \tag{1}$$

where ε_{it} denotes a weakly dependent error in the sense of Bossaerts (1988). Assume also that p_{it} is weakly dependent after differencing once. Under these assumptions, the price vector $\mathbf{p_t}$ is co-integrated of order 1 with cointegrating rank $r = n–k$, in the sense of Engle and Granger (1987) and Bossaerts (1988). Thus, there exist r linearly independent vectors $\alpha_{q_q} = 1, ..., r$ such that $z_q = \alpha_q^{'} \mathbf{p}_t$ are weakly dependent. In other words, r linear combinations of prices will not be driven by the k common nonstationary components p_l. Note that this interpretation does not imply that the market is inefficient, rather it says that certain assets are weakly redundant, so that any deviation of their price from a linear combination of the prices of other assets is expected to be temporary and reverting. To interpret the pairs as co-integrated prices, we need to assume that for $n >> k$, there are co-integrating vectors that have only two nonzero coordinates. In that case, the sum or difference of scaled prices will be reverting to zero and a trading rule could be constructed to exploit the expected temporary deviations. Our strategy relies on exactly this conclusion. In principle one could construct trading strategies with trios, quadruples, and so on of stocks, which would presumably capture more co-integrated prices and would yield better profits. The assumption that a linear combination of two stocks can be weakly dependent may be interpreted as saying that a co-integrating vector can be partitioned in two parts, such that the two corresponding portfolios are priced within a weakly dependent error of another stock. Given the large universe of stocks, this statement is always empirically valid and provides the basis of our formation procedure. However, it is important to recognize the possibility of spuriously correlated prices, which are not de facto co-integrated. The two approaches used in this paper build on the above premise.

Also, according to Andrade, Pietro and Seasholes (2005), uninformed trading shocks help explain the returns of relative value trading strategies. As in traditional asset pricing models, assume that stock returns are determined by loadings on risk factors plus an idiosyncratic component. Given a long enough observation period, two stocks that have historically moved together can be thought of as having similar factor loadings. Assuming factor loadings remain constant in the future, the two stock returns should continue to move together. Now, in a market with limited risk bearing capacity, uninformed traders place buy and sell orders that are uncorrelated with asset fundamentals. Optimizing investors, who are risk averse, accommodate the demands but require compensation. Thus, uninformed buying is accompanied by a contemporaneous rise in prices. Following a demand shock, prices mean revert back to pre-shock (fundamental) levels. Likewise, uninformed selling is accompanied by a contemporaneous fall in prices and similar mean-reversion. In a world with limited risk-bearing capacity, a pairs trading strategy effectively matches stocks with similar (historical) factor loadings. When prices diverge, the strategy takes a risky position by "betting" the divergence stems from different uninformed trading shocks and not different informational shocks. In models such as Greenwood (2004), prices mean revert back towards fundamentals in a linear fashion.

Economically speaking, the distance approach identifies assets that move closely together, but do not impose any kind of long term relationship or convergence between stock prices. Hence, there is nothing in the modelling of the pairs through the distance method that requires both stocks to have a close long run relationship relying purely on a statistical relationship, and the stocks selected may have no economic relationship with each other. Nonetheless, the strategy has had a positive performance in the United States as Gatev et al (2006) has shown. On the other hand, the cointegration approach depends on identifying a long run relationship with an error correction model between assets implying that the stocks will converge towards an equilibrium. The reason as for why there is more than one methodology steams mainly because the source of positive returns in pairs trading is not very well understood. Hence all strategies may be capturing different sources of profitability from pairs trading. Since those may have different relative importance in different markets, none of the strategies are being consistently superior across all markets.

2.1 The Distance Approach

The distance approach is proposed by Gatev et al. (2006) and is used among others by Andrade et al. (2005), Engelberg et al. (2009), Do and Faff (2010), Bowen et al. (2010), and Broussard and Vaihekoski (2012). By this approach the co-movement in the pair is measured by the *distance*, which is defined as the sum of squared deviations (SSD) between the two normalized price series. Normalized price series are defined to start from one, and then evolve using the return series. The normalized price series for a stock is given by its cumulative total returns index over the moving formation period of 252 days. Formally, we compute

$$\tilde{P}_{it} = \prod_{\tau=1}^{t}(1 + r_{i\tau}) \tag{2}$$

where $\tilde{P}_{ii}$ is the normalized price of stock i at time t, $r_{i\tau}$ is the dividend-adjusted return of stock i at time τ, and τ is the index for all trading days between $t - 252$ and t. The normalized series begin the observation period with a value equal to one, and increases or decreases each day given its return. For each stock i, we find the stock j that minimizes the sum of square deviations between the two normalized price series. The distance is thus defined as

$$\triangle_t^{ij} = \sum_{t=1}^{252}(\tilde{P}_{it} - \tilde{P}_{tj})^2 \tag{3}$$

where $\triangle_t^{ij}$ is the distance between the normalized prices of stock i and j over the formation period. This means that pairs are formed by exhaustive matching in normalized price space, where price is the daily closing price adjusted for dividends and splits. We rank all possible pairs by distance, identify the combinations with the highest measure of co-movement and monitor these pairs for the duration of the trading period. Similar to Gatev et al. (2006), we set the periodicity of pair updates to 6 months.

In order to select a pair for a given stock, we search on the database for an asset whose normalized price has the smallest squared distance to the normalized price of the chosen stock up to time t . A long-short position is opened when the distance exceeds a pre-specified threshold[2]:

$$q = \delta\sigma_{spread}$$

Gatev et al. (2006), Andrade et al. (2005) and Do and Fa (2010) set $\delta = 2$, whereas Bowen et al. (2010) and Broussard and Vaihekoski (2012) experiment with a range of values. It is also possible to let q be a variable by defining as a rolling parameter with window size n; this may allow us to better capture the profit potential of periods with higher volatility in the spread. Following Gatev et al. (2006), the signal to start trading occurs when the distance between the normalized price diverges by more than two standard deviations. An open long-short position is closed either upon convergence in normalized prices or at the end of the trading period. The latter imposes a restriction on the investment horizon and works as an automatic risk control mechanism.

The distance approach is a model free approach and non-parametrically exploits a statistical relationship among two stocks prices. From a practical point of view, the distance method is easy to implement and independent of economic models, which avoids misspecification problems. On the other hand, non-parametric strategies have lower predictive ability compared to well-specified parametric models. The fundamental assumption of this approach is that pair spreads exhibit mean-reversion. Accordingly, a price-level divergence is an indication of disequilibrium and price distance is the measure of mispricing.

2.2 The Cointegration Approach

The use of the cointegration technique to asset allocation was pioneered by Lucas (1997) and Alexander (1999) and in the previous decade it was increasingly applied in financial econometrics (see, among others, Alexander and Dimitriu (2002); Bessler and Yang (2003); Yang et

[2] The threshold can be constructed in a variety of ways, but the most common method is to select some proportion of the historical standard deviation of the spread.

al. (2004). Cointegration is an extremely powerful technique, which allows dynamic modelling of non-stationary time-series sharing a common stochastic trend. The fundamental observation that justifies the application of the concept of cointegration to the analysis of stock prices is that a system involving non-stationary stock prices in levels can display a common stochastic trend (Gatev et al., 2006). When compared to the concept of correlation, the main advantage of cointegration is that it enables the use of the information contained in the levels of financial variables.

Similar to the previous trading strategy, the main concern of the cointegration approach is the mean reversion of the spread. However, instead of defining the spread as the distance between standardized prices of a pair of stocks, the spread is defined with respect to the long-run equilibrium of a cointegrated system; that is, the long-run mean of the linear combination of two time series, Vidyamurthy (2004). Deviations from the equilibrium should revert to the long-run mean, implying that one or both time series should adjust in order to restore the equilibrium.

Using cointegration as a theoretical basis, the spread is generated based on the actual error term of the long-run relation:

$$log(P_{it}) - \gamma log(P_{jt}) = \mu + \varepsilon_t \tag{4}$$

where γ is the cointegration coeficient, the constant term μ captures a possible premium in stock i versus stock j, and ε_t is the estimated error term. Thus, it is not needed to predict P_t^i and P_t^j but only their dierence $log(P_{it}) - log(P_{jt})$. If we assume that $\{log(P_{it}), log(P_{jt})\}$in 4 is a nonstationary VAR(p) process, and there exists a value γ such that $log(P_{it}) - \gamma log(P_{jt})$ is stationary, we will have a cointegrated pair.

For detected cointegrating relations, the algorithm creates trading signals based on predefined investment decision rules. In order to implement the strategy we need to determine when to open and when to close a position. First, we calculate the spread between the shares. The spread is calculated as

$$\varepsilon_t = log(P_{it}) - \gamma log(P_{jt}) - \mu \tag{5}$$

where ε_t is the value of the spread at time t. Accordingly, we compute the dimensionless z-score defined as

$$z_t = \frac{\varepsilon_t - \mu_\varepsilon}{\sigma_\varepsilon} \tag{6}$$

the z-score measures the distance to the long-term mean in units of long-term standard deviation. After selecting the most appropriate pairs, the same trading strategy used under the distance approach is executed using the z-score series instead. This method is based on Vidyamurthy (2004); Avellaneda and Lee (2010) and Caldeira and Moura (2013). It is an attempt to parametrize the long-term relationship between two assets and explore price-deviations from their historical relationship using cointegration. Even if two time series are non-stationary, cointegration implies the possibility that a linear combination of both series could be stationary. If this is indeed the case, both series move closely together as if they were connected to each other.

The quality of estimation of the correction error model depends on the econometric technique applied. The first method for testing cointegration by Engle and Granger (1987) is a two step procedure in which the first step, stationarity test of the residuals errors, renders results sensitive to the ordering of the variables, and such mispecification error is carried to the second step, the error correction model estimation. The way found to reduce this error is to use two cointegration tests. Besides the Engle and Granger (1987) we also used the Johansen (1988) test, and use only the pairs that are considered cointegrated by both tests. Nonetheless Engle and Granger (1991) well-known limitations (small sample problems, maximum of one cointegrating vector, treating the variables assymetricaly) are not an issue in this work, due to our samples having 252 observations, only two variables are included in the estimation procedure, and it is only possible to find one cointegrating vector.

3 Implementation Details

In this work we follow the methodology by Gatev et al. (2006) and Broussard and Vaihekoski (2012) to implement the distance method and the methodology used by Caldeira and Moura (2013) and Vidyamurthy (2004) in the implementation of cointegration methodology. The formation period for the pairs is 12 months long, and the trading period comprises the following 6 months. The pairs of assets are selected by minimizing the sum of squared deviations in the portfolios formed from the distance method and ranked beginning from the smallest sum of squared deviations. In portfolios formed from the cointegration method, the pairs are selected if they are found cointegrated with both tests, Engle and Granger (1987) and Johansen (1988), and later ranked by their Sharpe index within the sample as in Caldeira and Moura (2013).

Next, portfolios are formed with 5, 10, and 20 pairs for each methodology, and are used in the trading period in the 6 months following the formation of pairs. At the end of each period of trading all positions are closed. A new 12 month period for the pairs formation is created and ends on the last observation of the previous trading period, when all cointegration tests and pairing are redone. The assets to be used must be traded in the 12 month formation period, but not necessarily they will be listed during the 6 month trading period.

In order to generate trading signals, it is necessary to calculate the distance between the asset prices in the pair, measured by the spread $\varepsilon_t = P_t^l - \gamma P_t^s$, where ε_t is the spread value at time t. From the spread, the distance measure is given by the formula $z_t = \frac{\varepsilon_t - \mu_\varepsilon}{\sigma_\varepsilon}$. The goal is to identify when z_t departs from the long term average, given by the error correction model, measured in terms of standard deviation. Initially, the position opens when $|z_t| > 2$ and closes when $z_t = 0$.

Let P_t^l be the long asset price and P_t^s t the price of the asset sold short, then the net return in t of pair i is given by:

$$r_{it}^{raw} = log\left[\frac{P_t^l}{P_{t-1}^l}\right] - \gamma\left[\frac{P_t^s}{P_{t-1}^s}\right] + 2log\left(\frac{1-C}{1+C}\right) \tag{7}$$

This equation already includes transaction costs in its second term[3]To calculate the net return of a portfolio with N pairs, we do the weighted average net returns of each pair, with the weight defined by the percentage of the amount invested in each pair with respect to the value of the portfolio in time t. Let p be a portfolio with N pairs, where ω_i is the weight for each pair i. Thus, the net return of the portfolio in t is $R_t^p = \sum_{i=1}^N \omega_{it} R_{it}$. As explained in the Caldeira and

[3]This formula can be explained intuitively. Suppose we buy stock ξ at price $P_{t-1}^\xi(1+C)$ and the profit of selling is $P_{t-1}^\xi(1-C)$. This corresponds to the decomposed net return: $log\left[\frac{P_t^\xi(1+C)}{P_{t-1}^l(1+C)}\right] = log\left[\frac{P_t^\xi}{P_{t-1}^\xi}\right] + log\left[\frac{1-C}{1+C}\right] = r_t^\xi + log\left[\frac{1-C}{1+C}\right]$

Moura (2013), the calculation of compound return (log returns) of a portfolio of assets is, for small values, close to the weighted average of the continuously compounded returns for each asset i.e, $R_t^p \approx \sum_{i=1}^{N} \omega_{it} r_{it}$. However, to calculate the return accurately, log-returns are transformed back to simple return, with the monthly compound rate of return, r_{it} given by:

$$r_{it} = log(1 + R_{it}) = log\left(\frac{P_t}{P_{t-1}}\right) \tag{8}$$

to transform back we just multiply by e to remove the logarithm and obtain the net return R_{it}. $e^{r_{it}} = 1 + R_{it} \Rightarrow R_{it} = e^{r_{it}} - 1$.

From this net return of the portfolio equation, we used two weighting scheme of returns as in Gatev et al. (2006) and Broussard and Vaihekoski (2012). The first scheme used is the weighting of the returns to the capital previously committed (committed capital scheme), in which an amount of capital is distributed evenly across the entire universe of pairs for the period. Even if the pair does not open or if it closes before the trading period finishes, capital remains committed to that pair. This scheme divides the payoff in pairs for all pairs that were selected for the period of trading. This method considers the opportunity cost of hedge funds when they commit resources on a pair that ends up not being used during trading. The second scheme used divides the returns by the number of pairs that are open at the moment the return was obtained (fully invested scheme). In other words, this is a less conservative scheme where the portfolios resources are are all invested in the pairs that are currently open. We are conservative and assume a rate of return of zero for capital in pairs that are not open, as in Broussard and Vaihekoski (2012), and unlike Gatev et al. (2006), which assumes a risk-free rate of return.

The change in the weights of the pairs within the portfolio follows the method of equal weights (Equally weighted approach), defined as in Broussard and Vaihekoski (2012), although we do use the value weighted method and find similar results, hence, we do not report them, but are readily available upon request. The sum of returns of each pair is divided by the number of pairs that were selected for the period of trading, in the committed capital scheme. In practice, the use of stop-loss is critical to minimize losses. However, most academic works on pairs trading don't use them. Exceptions are Nath (2006), and Caldeira and Moura (2013), and in this work we follow the method of Caldeira and Moura (2013) and the stop-loss is triggered and the position in the pair is closed when losses reach 10% and we also include a stop gain of 20% with other values being tested.[4]

Transaction costs considered follow Dunis et al. (2010) and Caldeira and Moura (2013) and total 0.4% for each change of position in the pair (opening and closing): 0.1% brokerage in total for each action (buying and selling), totaling 0.2% for each pair in brokerage costs. Slippage of 0.05% for each stock in the pair, and 0.2% for the lease of the asset to be sold short (divided in 0.1% for opening and 0.1% when closing the position). The performance of the pairs portfolios is measured from 4 statistics:

$$Cumulative\ Returns:\ R^A = 252 \times \left(\frac{1}{T}\sum_{t=1}^{T} R_t\right) \tag{9}$$

[4]We considered values of 5%, 7%, 10%, 15%, 20% and no trigger for the stop-loss or stop-gain, nding that the lower/higher the stop-loss/gain better the strategy performance being the the highest sharpe ratio when no trigger was used either for stop-loss or stop-gain.

$$Variance\,of\;Returns:\;\hat{\sigma}^A = \sqrt{252} \times \left(\frac{1}{T} \sum_{t=1}^{T} (R_t - \hat{\mu})^2 \right) \tag{10}$$

$$Sharpe\;Index:\;SR = \frac{\hat{\mu}}{\hat{\sigma}},\;where\;\hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} w_{it} R_{wit} \tag{11}$$

$$Maximun\;Drawdown:\;MDD = \sup_{t\epsilon[0,T]} \left[\sup_{t\epsilon[0,t]} R_s - R_t \right] \tag{12}$$

4 Data Sets

Previous pairs trading studies aimed at testing a specic methodology for a given stock market. However, the availability of big nancial datasets across the globe allows the researcher to expand the analysis to markets with dierent characteristics, allowing a more robust evaluation of the strategies. Our data comprises three highly liquid nancial markets described below: United States Stock Market, Euro Area Stock Markets and the Brazilian Stock Market.

The US dataset was obtained at CRSP and contains the 1000 most liquid stocks for every year, totalling 4,471 stocks. The period analyzed goes from 1962 to 2012 comprising a total of 12,586 observations and the market index used is the S&P 500, that is available for the whole sample period. We did not limit the universe with which each stock could pair up. The US database is the most homogenous one with stocks being traded in stock exchanges close to each other. Also, all stocks considered for trade are ordinary stocks and consequently, unlike for the brazilian and the euro area markets, where pairs can be from the same company due to diferent class shares, in the US all stock pairs will necessarily be from dierent companies, allowing for the comparison of each methodology on diferent institutional environments and types of stocks selected. Finally, from an academic point of view, despite considerable theory about market eciency, not enough empirical information is known regarding how eciency arises in practice.

The European Union dataset has stocks from companies based in Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain and data from the main stock exchanges based on all those countries, hence we use the MSCI Europe excluding UK and Switzerland Index. This dataset is relevant because the monetary union through the Euro currency eliminates the exchange rate variation. Also, it allows direct arbitrage between different countries and different stock exchanges within countries. The EU dataset contains daily data from the 1,000 most liquid stocks from 1973 to 2012. All the data is quoted in Euro converted by the conversion rates of the moment the country entered the monetary Union, directly done by the data provider Datastream at it's source. The data was obtained from Datastream comprising 10,435 observations. All countries in the euro area were considered for the sample, however not all countries have stocks in the 1000 most liquid for the sample period.

There are country specific reasons to analyze pairs trading in the brazilian financial markets. A considerable amount of international crisis caught Brazil off guard during the 90's and the beginning of the 2000's decade. We use database between 1995 and 2012, which had an amount of 4 international crisis that hit Brazil,[5] and also 2 internal crisis[6] that directly affected the brazilian

[5]1Asian financial crisis in 1997. Russian moratorium and rublo devaluation in 1998. Argentinian moratorium and peso devaluation in 2001 and the U.S. financial crisis in 2008.

[6]Brazilian real devaluation in 1999 and 2002 election of former president Lula.

Table 1 – Descriptive statistics of the data sets

Datasets	US data	European Data	Brazilian Data
Start date	Jan-1962	Jan-1973	Jan-1995
End date	Dec-2012	Dec-2012	Dec-2012
Number of stocks	4,471	1,000	450
Number of observations in the sample	12,586	10,435	4,087
Number of formation periods	100	80	34
Average number of days in formation period	252	260	247
Average number of days in trading period	126	130	123

financial markets. To evaluate pairs trading in such stressful financial conditions provides valuable insight to how this strategy can act as a risk management alternative to other trading strategies, not to mention in a reduced liquidity environment compared to the U.S. Also, brazilian listed companies tend to have preferred and common stocks listed, which is an uncommon characteristic allowing closer examination of causes of potential price deviation between assets that have rights to the same cash flow source. The database used in this study consists of all daily closing prices for stocks that have daily trading during the 12 month formation period and are listed in the Bolsa de Valores de São Paulo (Bovespa). The data were obtained from Economática for the period between 1995 and 2012, and are adjusted for dividends and splits, in order to avoid false trading signals. The market index considered is the Ibovespa for the whole sample period. Table 1 summarizes the descriptive statistics of the database.

4.1 Empirical Findings

In this section we report the main results of the empirical analysis. It is found that the performance of pairs trading strategies could vary wildly across individual markets and approaches (see Tables 3, 9, and 15).

4.1.1 US Data Set

Table 2 shows the results found for the American data set for the whole sample period between January 1962 and December 2012. The average number of pairs opened increases with the number of pairs in the portfolio due to the fact that there are more available pairs that can open at any given moment. However, the distance methodology is much more "trigger happy" than the cointegration method, opening almost double the number of pairs for each different size of portfolio. Nonetheless, it's very interesting to note that the average time the pairs are open is very similar for both strategies, which indicates that they react similarly, but the distance method identifies more trading opportunities. That also reflects on the share of negative excess returns, which is higher for the distance method on the 5 pairs portfolios. Since the distance method opens more pairs, the situation where the returns are 0, due to the lack of any open pairs, occur less frequent. On the other hand, the cointegration approach opens pairs less frequently, and has more zero return days that does not count as negative returns, which reduces the share of negative excess returns. Hence, even though the distance method performs better, as seen on the next section, it has a tendency of having a higher share of returns below zero.

The average number of round-trip trades per pairs, i.e., the average of pairs that manage to open and close before the closing of the 6 month period is higher for the distance method, for all portfolios sizes. Each pair opens and closes on average between 4.20 and 3.70 times for the

Table 2 – Summary statistics of unrestricted pairs trading for the USA data set

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1962 and December 2012 for the Committed Capital weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence sinal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window.

Methodology	**Distance Approach**			**Cointegration Approach**		
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Total number of pairs opened	2110	3,944	7,385	1,184	2,263	4,670
Total number of 6 month trading periods	100	100	100	100	100	100
Mean price deviation for opening pairs	0.028	0.030	0.032	0.406	0.409	0.409
Mean Nº of pairs opened each 6 month period	21.1	39.4	73.85	11.84	22.63	46.7
Mean Nº of pairs traded when at least one pair opened	4.228	3.948	3.694	2.368	2.265	2.339
Average Nº of round-trip trades per pair	4.22	3.94	3.693	2.368	2.263	2.335
Standard deviation of round-trips per pair	2.539	2.379	2.185	1.5458	1.479	1.549
Average time pairs are open in days	17.488	19.363	21.515	16.945	17.283	17.24
Median time pairs are open in days	7	8	10	11	11	11
Average time pairs are open in months	0.833	0.922	1.025	0.806	0.823	0.82
Standard deviation of time open per pair in days	25.396	26.701	28.228	18.110	18.331	18.469
Standard deviation of time open per pair in months	1.209	1.271	1.344	0.862	0.872	0.879
Share of negative excess returns	0.432	0.441	0.430	0.379	0.442	0.469

distance portfolios while the cointegration method has an average of round trips per pair of 2.30. It's interesting to note that the average time pairs are open is very similar, but the standard deviation is higher for the distance approach, suggesting that this method is less selective on when to open the pairs and consequently they may take a longer time to converge or they close too rapidly generating a higher S.D..

4.1.2 Committed Capital Scheme

This section uses the Committed Capital weighting scheme, where the returns of the pairs in the portfolio are weighted against all pairs, not only the open pairs at the moment the return was realized. It's clearly a more conservative measure and it takes into account the opportunity cost of committing capital to a given strategy even if there are no trades. Table 3 summarizes the excess return of the pairs portfolios with opening positions at the end of the day that occurs the price divergence and closing positions at the end of the day that prices converge.

The average annualized return for all portfolios created through the distance method is over 10.00% yearly return, similar to what Gatev et al (2006) found. The cointegration method performance is lower, with a Sharpe ratio ranging between 0.20 and 0.60, while the distance method has a Sharpe ratio of up to 2.90. The superior performance of the latter is a direct consequence of not only its higher average return, but also a lower volatility. Both strategies correlation with the market, as seen through the CAPM beta and the Spearman Rho, are not significantly different from zero, exactly what we want and would expect from a self-financed market neutral strategy. Although the beta and rho are important measures of risk, we also considered the maximum drawdown for all portfolios. This is a simple measure that indicates the largest cumulative loss after a given maximum of a cumulative positive rallying of the returns, signaling how fast the leverage can increase. In the 50 years of the data spam the biggest cumulative loss ranges between 6.90% and 11.17% for the distance method and between 14% to 37% for the cointegration approach. Both are considerably inferior to the S&P 500 maximum drawdown of 56%.

Table 3 – Comparison of strategies for unrestricted pairs trading for the USA data set

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1962 and December 2012 for the Committed Capital weighting scheme. Pairs are formed over a 12-month period according to a distance and a cointegration criterion and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted weighting scheme.

Methodology	**Distance Approach**			**Cointegration Approach**		
Weighting Scheme	**Committed Capital**					
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Average Annualized Return (in %)	15.061	13.969	13.169	3.956	1.911	3.708
Average Annualized Volatility (in %)	7.342	5.557	4.273	11.527	8.084	5.946
Total Sample Sharpe Ratio	1.911	2.354	2.896	0.336	0.234	0.612
Largest Daily Return (in %)	6.404	6.471	4.328	7.64	3.841	2.657
Lowest Daily Return (in %)	-4.639	-3.225	-2.781	-7.57	-4.323	-2.879
Spearman Rho	0.030	0.045	0.048	0.012	0.003	0.015
CAPM Beta	0.028	0.024	0.020	0.033	0.021	0.023
Jensen's Alpha	0.0005	0.0005	0.0005	0.00014	0.0006	0.0001
Jense's Alpha p.-value	0.00	0.000	0.000	0.0263	0.129	0.0003
Annual Skewness	1.498	0.677	0.497	0.680	0.298	0.459
Annual Kurtosis	5.978	3.104	2.530	3.388	2.822	2.756
Total Sample MDD (in %)	11.173	9.307	6.990	34.23	37.380	14.77

It is interesting to see the daily returns as measured by the Jensen's alpha. The method of measuring the excess returns is solely through the returns obtained by the pairs, without being necessary to account for the risk free return, since, in theory, pairs trading are not supposed to have any positive performance. However, in order to be more conservative, we measure Jensen's alpha using 3 months U.S. Treasury Bills as the risk free rate and the market beta using the S&P500. Even so, the alphas produced by this strategy are statistically significant for all portfolio sizes for the distance methods and for the portfolios with 5 and 20 pairs for the cointegration approach.

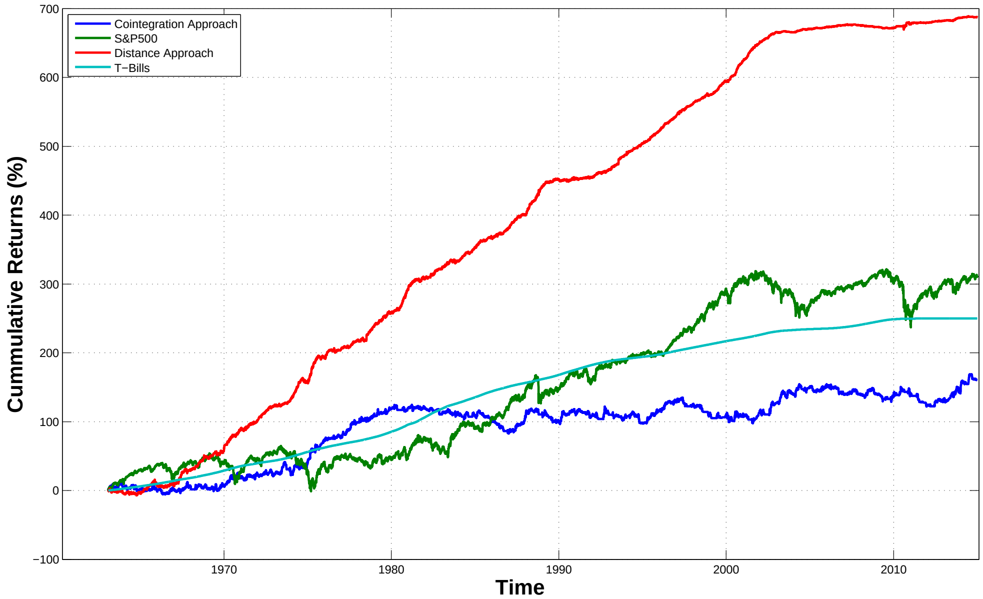
Figure 1 shows the cumulative excess returns of both methodologies using the 5 pairs portfolios. We can see that the performance of the distance method is consistente throughout the whole period, in contrast with the volatility of the S&P500 and the cointegration method but more volatile than T-bills. The distance method performs well during difficult times in the U.S and its performance is stable until 2003, when it starts performing more conservatively until the end of the sample in 2012. It also performs well as a whole beating all other 3 alternatives. On the other hand the cointegration approach has a relatively good period during the 70's but its performance is relatively flat for the rest of the period with small cycles of ups and downs that tend to cancel each other.

4.2 Fully Invested Scheme

The fully invested portfolios are presented on Table 4 and show a slightly different picture than the committed capital portfolios. These portfolios have necessarily an equal or higher average return than the committed capital for all portfolio sizes due to its construction methodology. This weighting scheme scales the payoffs by the number of pairs that actually open, implying that all the money is distributed between all open pairs. The distance method is still superior to the cointegration method, with average annualized returns between 23% and 28% while the cointegration method returns are between 9% and 18%. However, its volatility is proportionally higher for both methods and portfolio sizes, and in turn, the sharpe ratios are very similar to the ones found previously. Another consequence of the higher volatility is its impact on the maximun

Figure 1 – Cumulative Returns Index for the USA (starts at 0)

Note: This figure reports the cumulative excess returns for the USA in log for the 5 pairs portfolios between January 1963 and December 2012 using the distance method and the cointegration approach using the committed capital weighting scheme.



drawdown. All distance method portfolios MDD increased and reached up to 27%, while for the cointegration method the MDD is up to 75%, superior than the S&P500 56% MDD.

These portfolios have a higher average return and volatility, but its betas and rhos are still negligible, and these portfolios can be considered market neutral. This indicates that these portfolios volatility are not based on its market risk, but rather on its own decision on when to open and close pairs and consequently can be included with an index tracking strategy without increasing the portfolios risk. The alphas obtained by the distance approach are higher than the cointegration method generates, but they all are statistically significant, being possible to conclude that both strategies can generate positive performance.

The American market structure, being considered a very liquid market and relatively more efficient than the less developed Brazilian market or the more segmented European Union market, favors the distance method possibily due to its basic characteristics. All stocks in the American market are ordinary and none are preferential. Hence, every asset is tied to a different cash flow. Even though they can be similar due to both belonging to the same industry and sector, they are not the same. This implies that the forces of convergence through cointegration are weaker than if both assets were entitled to the same cash flow and that the distance method, by not being limited to the existence of a long run relationship may be capturing arbitrage opportunities than the cointegration method does not identify.

4.2.1 Robustness Checks: Returns on the long and short side, no transaction costs and one day waiting for the US

The inspection of the performance of both sides of the strategy in a separate manner is potentially interesting and is depicted in table 5. Since one of the aspects that makes pairs trading

Table 4 – Comparison of strategies for unrestricted pairs trading for the USA data set

Methodology	Distance Approach			Cointegration Approach		
Weighting Scheme	Fully invested					
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	27.730	24.922	23.358	14.138	9.602	18.12
Average Annualized Volatility (in %)	13.849	9.915	7.431	31.560	27.954	23.476
Total Sample Sharpe Ratio	1.768	2.245	2.826	0.419	0.328	0.709
Largest Daily Return (in %)	10.673	8.088	6.658	32.14	21.626	34.648
Lowest Daily Return (in %)	-23.195	-6.449	-4.039	-18.92	-17.381	-23.427
Spearman Rho	0.028	0.042	0.044	0.011	0.001	0.012
CAPM Beta	0.049	0.035	0.035	0.083	0.052	0.068
Jensen's Alpha	0.0009	0.0009	0.0008	0.0004	0.0003	0.0006
Jense's Alpha p-value	0.000	0.000	0.000	0.0048	0.026	0.00001
Annual Skewness	1.582	1.051	0.705	2.049	1.162	7.423
Annual Kurtosis	5.630	3.759	2.681	10.072	4.596	65.540
Total Sample MDD (in %)	27.055	13.569	11.625	74.21	75.071	42.81

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1962 and December 2012 for the Fully Invested weighting scheme. Pairs are formed over a 12-month period according to a distance and a cointegration criterion and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted weighting scheme.

profitable is its mean reversion characteristic, we should expect the excess returns of the long and short side to be equal. Another reason is that if the short side of the strategy is the one driving the results, the short selling barriers and market depth for such instrument should be considered in order to assess why there still exist a profit to be obtained. The results show that for both methods and weighting schemes the long side has a superior return, which are the stocks that have lost value relative to their pairs before the strategy is started. For the cointegration method this difference is even more pronounced, while for the distance approach both sides have more similar performances, although the short side still performs slightly worse. This provides evidence that the returns are not driven by non realized profit opportunities created by short selling barriers.

Table 5 – Comparison of long and short returns for unrestricted pairs trading for the USA data set

Methodology		Distance Approach				Cointegration Approach			
Weighting Scheme		Committed Capital		Fully Invested		Committed Capital		Fully Invested	
Pairs Portfolio		Long	Short	Long	Short	Long	Short	Long	Short
5 Pairs	Excess Return	8.561	5.990	14.121	11.932	5.796	0.143	16.491	2.759
	Std. Dev.	10.385	10.076	18.679	18.450	10.096	9.460	26.316	24.684
	S.R	0.791	0.577	0.707	0.611	0.558	0.015	0.58	0.11
10 Pairs	Excess Return	8.263	5.272	14.250	9.346	5.676	-1.799	18.357	-2.242
	Std. Dev.	9.154	8.929	14.685	14.360	7.666	7.300	24.576	22.966
	S.R	0.868	0.575	0.907	0.622	0.720	-0.248	0.686	-0.098
20 Pairs	Excess Return	7.733	5.047	12.656	9.504	5.948	-0.264	23.609	1.561
	Std. Dev.	8.007	7.770	12.454	11.972	6.348	5.954	22.455	20.784
	S.R	0.930	0.634	0.957	0.759	0.91	-0.044	0.944	0.074

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs for the long and the short side between January 1962 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully Invested weighting scheme.

The analysis of the performance without transaction costs can be very elucidating. If there are no costs to open or close any position, we can measure through the difference between the returns of the portfolios with and without transaction costs what is the size of the possible trading gains. The inefficiency generated by transaction costs can be measured in terms of excess return that could be obtained. Table 6 shows that the returns without transaction costs are almost double the ones calculated with a 0.8% transaction cost. This suggests that, since transaction costs account for a percentage of the price, the higher the return obtained in a given portfolio, the higher it could have been in absolute terms. Also, since the excess return is higher and the volatility remains the same, the sharpe ratio is superior for all portfolios types and sizes.

Table 6 – Excess return of unrestricted pairs trading for the USA data set without transaction costs

Methodology		**Distance Approach**		**Cointegration Approach**	
Pairs Portfolio		**Committed Capital**	**Fully Invested**	**Committed Capital**	**Fully Invested**
5 Pairs	Excess Return	22.992	43.269	8.218	25.563
	Std. Dev.	7.535	14.125	11.685	31.878
	S.R	2.748	2.547	0.676	0.714
10 Pairs	Excess Return	21.339	39.138	6.044	22.479
	Std. Dev.	5.700	10.085	8.236	27.916
	S.R	3.395	3.277	0.712	0.726
20 Pairs	Excess Return	20.035	36.320	7.902	31.619
	Std. Dev.	4.361	7.562	6.048	22.914
	S.R	4.189	4.100	1.257	1.199

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1962 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully Invested weighting scheme. 0% Transaction Costs.

We report the one day waiting period results on table 7. The reason for analyzing the one day waiting results is for the possibility of a delay with the implementation of the trade after the signal has been observed. Also, it allows for a more conservative view of the strategy since there may be a bid-ask bounce effect. The results indicate that both methodologies have a significant loss of performance when a one day lag is introduced but the distance approach still performs positively just as Gatev et al (2006) found. For the distance approach, all portfolios have a positive average annualized return between 6.6% and 7.4%, while the cointegration approach has negative average returns for its 5 and 10 pairs portfolios. This suggests that pairs trading profits can still survive in a competitive environment, when problems with executing a trade because of a thin market may happen or due to the bid-ask bounce vanishing when a one day waiting period is inserted. However, the profitability of pairs trading is tightly linked with the speed of execution, which although it raises questions on its profitability, the fact that we could still find considerable excess return with one day waiting basis suggests that pairs trading is still profitable. The tendency in terms of profitability regarding market efficiency is for the intra-day execution of pairs trading to take away the profits that could be obtained by a daily execution, with its limits on return being set by the transaction costs that they incur and consequently the necessity for the spread to widen enough in order to create a profitable opportunity.

Table 7 – Excess return of unrestricted pairs trading for the USA data set with one day waiting

Methodology	Distance Approach			Cointegration Approach		
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Average Annualized Return (in %)	7.393	6.894	6.608	-0.664	-0.598	0.326
Average Annualized Volatility (in %)	7.173	5.481	4.235	11.444	8.319	6.039
Total Sample Sharpe Ratio	0.994	1.217	1.511	-0.058	-0.072	0.054
Largest Daily Return (in %)	6.244	6.351	3.945	7.855	4.442	2.856
Lowest Daily Return (in %)	-4.719	-3.305	-2.841	-7.598	-4.27	-2.895
Spearman Rho	0.025	0.043	0.046	0.012	0.004	0.005
CAPM Beta	0.025	0.022	0.020	0.025	0.015	0.014
Jensen's Alpha	0.0003	0.0003	0.0002	-0.0003	-0.0002	0.00008
Jensen's Alpha p-value	0.000	0.000	0.00	0.596	0.541	0.79
Annual Skewness	1.618	0.769	0.499	0.792	0.795	0.345
Annual Kurtosis	7.112	3.390	2.437	3.454	3.966	3.37
Total Sample MDD (in %)	44.126	39.116	36.445	73.056	55.158	35.527
Share of negative excess returns (in %)	0.463	0.480	0.472	0.394	0.455	0.488

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1962 and December 2012 for the Committed Capital weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. One day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window.

4.3 European Union Data Set

Table 8 shows the summary statistics for the european data set for the whole sample period between January 1973 and December 2012. The average number of pairs opened, also increases with the number of pairs in the portfolio. However, when compared to the USA, the difference between both strategies is smaller. Similar to the USA results, the cointegration approach opens less pairs on average than the distance method. It is interesting to note that the median time pairs are open in days is very similar for both strategies. This suggests that both strategies may be closing too soon, and its performance could be improved by either identifiying an opening opportunity earlier, or by taking longer to close. The distance method has a higher average time pairs are open as well as a standard deviation, which is a consequence of this method identification procedure, not requiring a long term relationship between the stocks. Also, the average and median time pairs remain open increases with the portfolio size, specially for the cointegration method, indicating that in this approach the convergence takes longer as we increase the number of pairs, suggesting that the stocks may not move so close together as we increase the number of pairs in our portfolio, but end up converging after a while. As a matter of fact, the distance method does not considers any kind of convergence towards a long term relationship, unlike the cointegration methods. The distance method selects stocks that are very close to each other, but not based on convergence, hence explaining why the average time pairs remain open is higher for the distance method.

We can see that the share of negative excess returns is higher for the distance method, similar to the result obtained for the USA. As the share of negative returns shows, the distance method tends to have more open pairs at a given moment, due to its higher average time pairs are open, which consequently generates the opportunity for more non-zero returns. Hence, unlike the cointegration approach that tends to have pairs open for a shorter period, and less opportunities to have negative returns days, the distance method has a higher share of negative excess returns.

Table 8 – Summary statistics of unrestricted pairs trading for the European data set

Methodology	**Distance Approach**			**Cointegration Approach**		
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Total number of pairs opened	2,107	4,261	8,254	1,733	3,404	6,470
Total number of 6 month trading periods	78	78	78	78	78	78
Mean price deviation for opening pairs	0.017	0.019	0.021	0.225	0.25	0.286
Mean Nº of pairs opened each 6 month period	27.01	54.62	105.80	22.22	43.64	82.95
Mean Nº of pairs traded when at least one pair opened	5.472	5.533	5.359	4.524	4.438	4.264
Average Nº of round-trip trades per pair	5.402	5.462	5.291	4.443	4.364	4.147
Standard deviation of round-trips per pair	3.404	3.531	3.585	3.288	3.211	3.236
Average time pairs are open in days	10.12	9.80	10.21	7.40	8.32	8.87
Median time pairs are open in days	3	3	3	3	3	3
Average time pairs are open in months	0.481	0.466	0.486	0.352	0.396	0.422
Standard deviation of time open per pair in days	20.70	19.65	19.77	12.91	14.86	15.47
Standard deviation of time open per pair in months	0.985	0.936	0.941	0.614	0.707	0.736
Share of negative excess returns	0.366	0.404	0.411	0.284	0.365	0.394

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1973 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

4.3.1 Committed Capital Scheme

Table 9 shows the results for the committed capital weighting scheme for the European Union, for both investment strategies. Both strategies have a very good performance, and an average annualized return superior to 6% a year. Also, their volatility is fairly low, and falls with the size of the pairs portfolios. The result of such facts is that both methods have very high sharpe ratios. The distance method has a SR ranging between 1.2 and 2.1 while the cointegration approach varies between 1.5 and 2.4. Also, when we look at the Jensen's alpha, we see that they are statistically significant, which adds to the conclusion that both strategies have a positive performance.

In terms of neutrality, both strategies have a close to zero beta and rho, another relevant characteristic, given that both strategies perform well. This shows that the performance is not a consequence of a high beta combined with a good market performance in the period. When we look at the strategies maximum drawdown we see that they perform very well and have a small MDD. The distance method has an MDD that can be as low as 13% while the cointegration approach loses up to only 16.9% of its value, much smaller than the MSCI Europe's MDD of 48%. All results indicate that both strategies have a very positive average performance through the sample period, and a high return stability, as seen by the low MDD, as well as a negligible beta.

The graph 2 shows the cumulative returns for the 5 pairs portfolios for both strategies. Both strategies underperform until the beginning of the 90's when their annual return becomes superior to the market. The cointegration method performs better from the beginning of the 1990s all the way to the end of the sample in 2012. On the other hand, the distance method outperforms the market on the 90's but in 2003 its performance slowed down until the end of the sample, with an excess return similar to the market. Both strategies have a smaller volatility, as seen in the graph, and are not correlated with the market, a positive feature of pairs trading.

Table 9 – Comparison of strategies for unrestricted pairs trading for the European Union data set

Methodology	**Distance Approach**			**Cointegration Approach**		
Weighting Scheme			**Committed Capital**			
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Average Annualized Return (in %)	6.560	7.477	7.753	14.338	15.092	13.221
Average Annualized Volatility (in %)	5.231	4.208	3.417	8.77	6.623	5.00
Total Sample Sharpe Ratio	1.214	1.713	2.185	1.528	2.122	2.483
Largest Daily Return (in %)	4.173	2.227	1.964	7.078	4.392	3.714
Lowest Daily Return (in %)	-5.23	-1.97	-1.83	-7.309	-3.746	-3.306
Spearman Rho	0.015	0.003	0.015	0.009	0.012	0.029
CAPM Beta	0.001	0.001	0.002	0.003	0.006	0.008
Jensen's Alpha	0.00025	0.00028	0.00029	0.0005	0.0005	0.0005
Jense's Alpha p.value	0.00	0.00	0.00	0.00	0.00	0.00
Annual Skewness	1.513	1.386	1.346	0.507	0.751	0.627
Annual Kurtosis	7.161	5.671	5.095	2.622	3.116	2.419
Total Sample MDD (in %)	23.66	13.86	30.35	40.554	23.515	16.936

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1973 and December 2012 for the Commited Capital weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion or cointegration approach and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted.

4.3.2 Fully Invested in the European Union

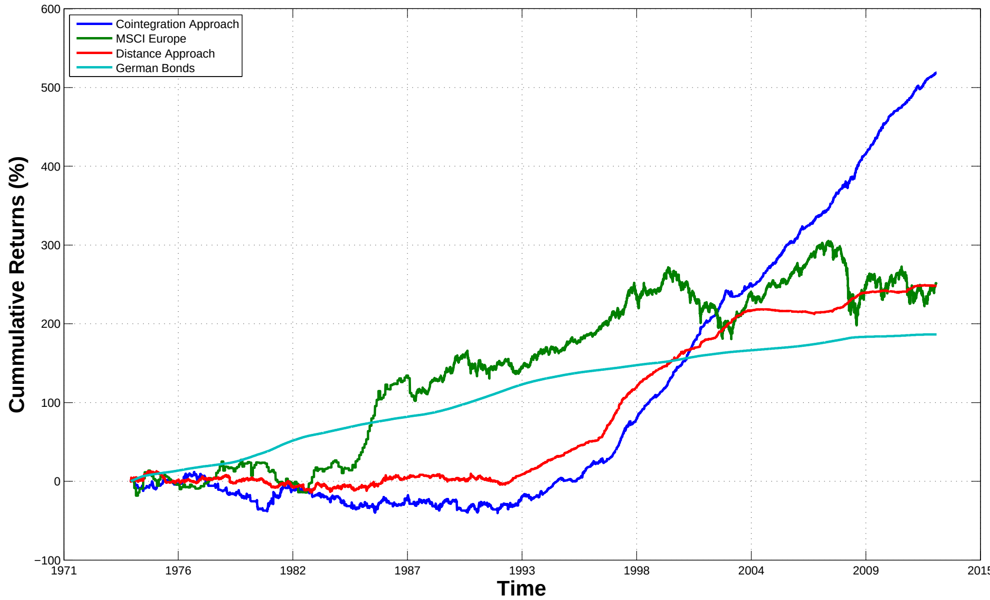
Table 10 shows that the results for the fully invested portfolios are a little different than the committed capital ones. The performances for the cointegration method are superior in terms of average excess return when compared with the distance approach but when we look at the sharpe ratios, the results are very similar. This emanates from the higher volatility the cointegration portfolios have, hence, not being able to generated a better portfolio.

The fully invested scheme has a higher sharpe ratio than the committed capital scheme possibly due to the short length of time pairs remain open at a single time. Hence, although the volatility is higher, the higher returns more than compensates the gains, resulting in a higher sharpe ratio. For the cointegration method, it has a higher excess return for all portfolio sizes as well as a higher sharpe ratio than when we use the committed capital scheme. This happens because the cointegration method has a median time of open pairs of only three days and a smaller standard deviation of the average time pairs remain open, causing the fully invested scheme to weight the returns by a much smaller number of pairs than the committed capital or even than the distance method that uses the fully invested scheme. Hence the returns increase much more than for the distance method. However, the Jensen's alpha is higher for the distance method, a result of the high risk that the cointegration approach has.[7].

Even though the volatility is higher for all portfolio sizes and both strategies, the beta is still not statistically different than zero, as well as the Spearman Rho. Finally, the MDD for both strategies is higher for all portfolio sizes, something that is expected, given the weighting methodology, but the MDD for the 5 pairs cointegration approach is higher than 80%, a result that signals that even though the strategy is market neutral, its downside risk is still very high. Nonetheless, the cointegration method underperforms before the beginning of the 1990s, which is when the high MDD occurs, having a very small MDD after and until the end of the sample.

[7] 0,002 for the cointegration method using fully invested scheme, compared to the 0,0005 using the committed capital weighting scheme.

Figure 2 – Cumulative Returns for the European Union



Note: This figure reports the cumulative excess returns for the European Union in log for the 5 pairs portfolios between July 1974 and December 2012 using the distance method and the cointegration approach using the committed capital weighting scheme. Data source: Eurostat (2014).

Table 10 – Comparison of strategies for unrestricted pairs trading for the European Union data set

Methodology	**Distance Approach**			**Cointegration Approach**		
Weighting Scheme			**Fully Invested**			
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Average Annualized Return (in %)	19.45	27.79	27.94	56.016	82.69	81.849
Average Annualized Volatility (in %)	10.37	9.785	8.198	25.567	23.092	19.025
Total Sample Sharpe Ratio	1.714	2.507	3.007	1.741	2.612	3.147
Largest Daily Return (in %)	6.956	6.115	5.245	15.20	13.54	10.697
Lowest Daily Return (in %)	-12.8	-4.82	-6.55	-20.725	-11.631	-14.371
Spearman Rho	0.0152	0.0145	0.0161	0.008	0.0105	0.028
CAPM Beta	0.015	0.005	0.017	0.015	0.028	0.039
Jensen's Alpha	0.009	0.005	0.009	0.001	0.002	0.002
Jensen's Alpha p-value	0.000	0.000	0.000	0.000	0.000	0.00
Annual Skewness	6.666	1.557	1.651	1.853	2.539	2.767
Annual Kurtosis	3.403	3.680	2.379	6.652	9.940	11.309
Total Sample MDD (in %)	37.05	39.24	48.67	80.593	49.179	37.579

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1973 and December 2012 for the Fully Invested weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion or cointegration approach and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted.

Also, the results have shown that share of pairs formed through the distance method that are composed of assets from the same corporations is higher than for the pairs formed through the cointegration method. Because some pairs are same company assets, they are tied to the same cash flow, and hence their prices move so closely together that the distance method, by only considering the historical price distance, ends up including a high amount of same company pairs. However, these pairs have a price distance so small that their trading isn't profitable, possibly due to their obvious connection, creating less arbitrage opportunites. On the other hand, a smaller share of same company pairs are selected through the cointegration method, and the assets tend to have a higher price divergence, allowing for more arbitrage opportunities. Since investor may be less aware of long run relationships between stocks that do not belong to the same corporation, this may be driving the superior results of the cointegration method.

4.3.3 Robustness Checks: Returns on the long and short side, no transaction costs and one day waiting for the EU

When we look at table 11 we see clearly see that both approaches have a tendency for the long side to perform better than the short side. The long side returns with committed capital for both strategies range between 7.8% and 11.8% while the short side is between 2.2% and 6.5%. For the fully invested scheme the difference is a bit bigger, but nonetheless the strategy still benefits strongly from both sides of the pair. Another important characteristic is that the sharpe ratio on both sides are smaller than the pairs sharpe ratio, which indicates that the strategy fullfills its objective of portfolio building and benefiting from the diversification, not only based on the selection of different stocks but also on symmetrically opposite investments.

The results for the cointegration approach are not very different but, nonetheless, have some interesting characteristics. The long side's return is more than double the short side for all pairs size and weighting schemes. This can be explained by the existance of a bull market through the most part of the analyzed period, and the fact that the strategy selects different stocks and pairs than the distance method. This also reflects on the sharpe ratio with the long side having a superior performance than the short side. On the other hand, the distance method has a long side performance that is between 3 to 4 times superior to the short side, hence, with a higher dependence on the long side.

The performances without transaction costs are presented on table 12. For both strategies the excess return increases when compared both weighting schemes with their counterparts with transaction costs. The main difference is that the cointegration method has a higher absolute and percentage change on its excess return when using the fully invested scheme. In general, the results indicate that the transaction costs reduces the profitability of pairs trading substantially.

When we include the one day waiting period we see a shift in performance for both strategies. Table 13 depicts the statistics for both strategies using a committed capital weighting scheme and including a one day waiting period for the opening and closing of pairs in order to account for unexpected situations, for example, the difficulty in executing the order as well as bid-ask bounce. The distance method has a negative average excess return for the sample period which translates in a negative sharpe and alpha. The MDD is high and the Jensen's Alpha is negative and not statistically significant, which indicates that pairs trading strategy, if the orders execution is delayed, can be very unprofitable. These results indicate that in a competitive environment, although daily pairs trading can still be profitable given a sufficient speed of execution, it is no longer profitable to trade with a one day delay, be it due to low processing capabilities, thin trading or any other reason that would impose a delay between the moment the price is discovered, and the

Table 11 – Comparison of long and short returns for unrestricted pairs trading for the European Union data set

Methodology		Distance Approach				Cointegration Approach			
Weighting Scheme		Committed Capital		Fully Invested		Committed Capital		Fully Invested	
Pairs Portfolio		Long	Short	Long	Short	Long	Short	Long	Short
5 Pairs	Excess Return	7.836	3.058	24.520	6.626	11.868	5.797	42.772	21.578
	Std. Dev.	8.012	7.908	19.950	19.730	8.377	7.996	25.142	23.565
	S.R	0.941	0.381	1.099	0.325	1.339	0.704	1,417	0.829
10 Pairs	Excess Return	9.395	2.514	3.640	11.380	11.699	6.599	51.832	37.172
	Std. Dev.	6.737	6.621	21.010	20.590	6.612	6.486	24.640	23.676
	S.R	1.333	0.375	1.272	0.523	1.673	0.985	1.696	1.335
20 Pairs	Excess Return	9.863	2.202	30.300	11.270	11.567	4.826	57.908	31.667
	Std. Dev.	5.973	5.903	18.250	17.860	5.601	5.435	22.200	21.630
	S.R	1.575	0.368	1.451	0.598	1.954	0.867	2.059	1.272

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs for the long side and the short side between January 1973 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully invested weighting scheme.

Table 12 – Excess return of pairs trading for the European Union data set without transaction costs

Methodology		Distance Approach		Cointegration Approach	
Pairs Portfolio		Committed Capital	Fully Invested	Committed Capital	Fully Invested
5 Pairs	Excess Return	16.12	48.20	22.28	92.94
	Std. Dev.	5.58	11.35	9.05	26.35
	S.R	2.67	3.46	2.22	2.49
10 Pairs	Excess Return	27.56	52.30	23.09	137.44
	Std. Dev.	6.24	11.23	6.87	24.00
	S.R	3.89	3.74	3.02	3.60
20 Pairs	Excess Return	24.95	46.1	20.73	135.66
	Std. Dev.	4.91	8.42	5.22	19.84
	S.R	4.53	4.50	3.60	4.32

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1973 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 0% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully Invested weighting scheme.

Table 13 – Excess return of unrestricted pairs trading for the European Union data set with one day waiting

Methodology	Distance Approach			Cointegration Approach		
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Average Annualized Return (in %)	-2.001	-1.279	-0.737	6.831	7.236	5.822
Average Annualized Volatility (in %)	5.126	4.087	3.277	8.498	6.365	4.823
Total Sample Sharpe Ratio	-0.394	-0.315	-0.226	0.778	1.098	1.173
Largest Daily Return (in %)	4.094	2.148	1.725	6.998	4.073	3.414
Lowest Daily Return (in %)	-5.390	-2.094	-1.913	-5.557	-3.827	-3.386
Spearman Rho	0.016	0.003	0.015	0.004	0.010	0.023
CAPM Beta	0.001	0.001	0.002	0.003	0.006	0.008
Jensen's Alpha	-0.00008	-0.00005	-0.00003	0.00026	0.00028	0.00022
Jensen's Alpha p-value	0.012	0.046	0.145	0.000	0.000	0.000
Annual Skewness	1.315	1.213	1.474	0.570	0.767	0.622
Annual Kurtosis	6.085	5.400	5.729	2.980	3.373	2.540
Total Sample MDD (in %)	61.579	54.303	62.927	63.340	46.521	52.744
Share of negative returns (in %)	0.427	0.498	0.512	0.304	0.391	0.424

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1973 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. One day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

order executed. Albeit at much more modest levels, the cointegration approach still mantains its profitability and Jensen's Alpha significance, suggesting such strategy not only remains profitable nowadays, but it also resists to the speed of execution . In general, as seen for the USA, and now for the european union, the speed of execution is of paramount importance in pairs trading.

4.4 Brazilian Data Set

The results for the brazilian dataset are shown on table 14 for the whole sample period between january 1996 and december 2012. The results are very similar to the american and european datasets, with the main difference being the higher median and average time pairs remain open. This reflects also on a higher standard deviation of time pairs remain open as well as in a lower average number of round-trip trades per pair. Since pairs remain open longer, they have less opportunites to have more round-trip trades in a given trading window. Another consequence of its longer average time open, is a higher share of negative excess return, which is superior to the ones found for the USA and european datasets.

4.4.1 Committed Capital Scheme

The performance of both strategies is presented on table 15 using the committed capital weighting scheme. Some periods exhibit less than 20 cointegrated pairs, specially the years between 1996 and 2002, consequently when applicable, we used the maximun number available of pairs to form a portfolio. Unlike for the previous databases, the performance is fairly low for both methods. The top performance in terms of annual excess returns and sharpe ratio is for the 5 pairs cointegration approach reaching, respectively, 8.30% and 0.64%. The performance of all other portfolios range between 0.30% and 3.60% annual excess return. The alphas are not statistically significant, except for the 5 pairs cointegration approach which are significant at the 5% level. Although both strategies do not exhibit great results, their MDD are small ranging between 15%

Table 14 – Summary statistics of unrestricted pairs trading for the Brazilian data set

Methodology	**Distance Approach**			**Cointegration Approach**		
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Total number of pairs opened	415	734	1407	389	638	1156
Total number of 6 month trading periods	34	34	34	34	34	34
Mean price deviation for opening pairs	0.049	0.063	0.078	0.745	0.775	0.759
Mean Nº of pairs opened each 6 month period	12.205	21.588	41.382	11.441	18.764	34
Mean Nº of pairs traded when at least one pair opened	2.515	2.224	2.131	2.542	2.294	2.298
Average Nº of round-trip trades per pair	2.441	2.158	2.069	2.288	1.876	1.7
Standard deviation of round-trips per pair	1.810	1.562	1.45	1.748	1.589	1.576
Average time pairs are open in days	30.828	30.209	27.064	17.051	18.155	18.135
Median time pairs are open in days	13	15	14	11	12	12
Average time pairs are open in months	1.468	1.438	1.288	0.811	0.864	0.863
Standard deviation of time open per pair in days	37.418	35.174	31.712	19.538	19.87	19.383
Standard deviation of time open per pair in months	1.781	1.674	1.510	0.930	0.946	0.923
Share of negative excess returns	0.472	0.492	0.493	0.383	0.426	0.443

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1996 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital Weighting Scheme, where the returns are pondered by all possible pairs chosen.

and 23%, which suggest that this strategies are very stable over time. This can be seen through its beta and rho, which are very close to zero, similar to these statistics obtained for the previous datasets.

4.4.2 Fully Invested Scheme

Table 16 presents the results for the whole trading out of sample, between July 1996 and December 2012 already discounted for transaction costs and slippage effects. The performance for the cointegration method are superior in terms of average excess return, as well as in sharpe ratio. However, this does not mean much, since its sharpe ratio is fairly low for the cointegration method, between 0.36 and 0.57, but still higher than the ones for the distance approach and superior to the market sharpe of 0.41 in two portfolios.

This low performance reflects on the Jensen's alpha, with it not being statistically significant for the distance method, and only the five pairs portfolio through the cointegration method is significant at the 5% level. Both strategies are market neutral, as reflected by its beta and rho, and as expected from this self-financing strategy. The MDD from both strategies is around 50%, indicating a quite risky strategy, but still lower than the 65% MDD of the market index (Ibovespa).

Figure 3 shows why the cointegration method does not perform better in terms of SR than the distance method for the whole sample. Its volatility is 31% annually, although inferior to the 34% annually for the Ibovespa, is higher than the 17% for the five pairs portfolio formed using the distance method. Until the year 2000 the cointegration method underperformed both the market and the distance method. Possibly due to the lack of pairs available, with the number of pairs that cointegrated being very often inferior to 10 and sometimes less than five pairs where cointegrated before the year 2002, which reduces the possibilities of choosing the best performing pairs according to the Sharpe Ratio. From then on, the cointegration method overperformed the distance method, and kept a similar pace to the market index Ibovespa. Before the 2008 crash, the market index soared, and the cointegration strategy kept its pace, with barely no correlation with

Table 15 – Comparison of strategies for unrestricted pairs trading for the Brazilian data set

Methodology	**Distance Approach**			**Cointegration Approach**		
Weighting Scheme	**Committed Capital**					
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Average Annualized Return (in %)	3.188	0.640	0.378	8.370	3.534	3.688
Average Annualized Volatility (in %)	8.252	6.676	5.297	12.53	10.37	9.314
Total Sample Sharpe Ratio	0.380	0.095	0.071	0.641	0.334	0.388
Largest Daily Return (in %)	2.934	3.571	2.036	5.512	5.512	5.512
Lowest Daily Return (in %)	-3.41	-3.94	-2.30	-7.68	-7.68	-7.68
Spearman Rho	0.064	0.060	0.071	0.069	0.050	0.042
CAPM Beta	0.017	0.013	0.013	0.014	0.008	0.005
Jensen's Alpha	0.00014	0.000017	0.000007	0.00031	0.00013	0.00014
Jense's Alpha p.value	0.157	0.785	0.890	0.011	0.192	0.125
Annual Skewness	0.663	-0.37	0.728	1.375	1.280	1.536
Annual Kurtosis	4.099	3.020	4.543	4.185	4.586	5.948
Total Sample MDD (in %)	15.06	23.17	18.68	19.55	18.56	18.56

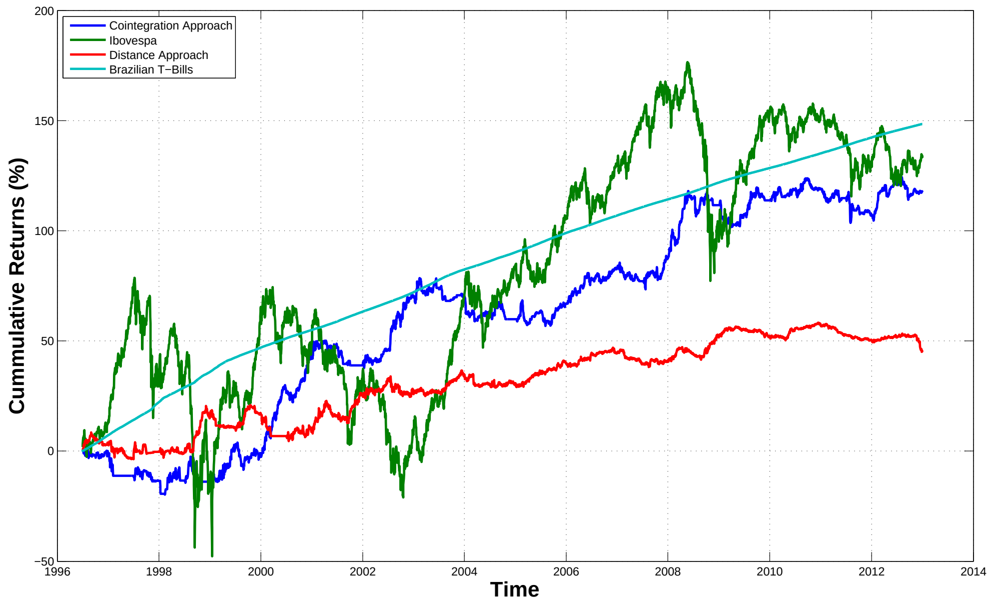
Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1996 and December 2012 for the Commited Capital weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion or cointegration approach and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted weighting scheme.

Table 16 – Comparison of strategies for unrestricted pairs trading for the Brazilian dataset

Methodology	**Distance Approach**			**Cointegration Approach**		
Weighting Scheme	**Fully invested**					
Pairs Portfolio	5 Pairs	10 Pairs	20 Pairs	5 Pairs	10 Pairs	20 Pairs
Average Annualized Return (in %)	6.507	0.091	-0.84	19.50	10.99	12.39
Average Annualized Volatility (in %)	17.65	17.89	16.38	31.01	28.26	24.25
Total Sample Sharpe Ratio	0.357	0.005	-0.05	0.574	0.369	0.481
Largest Daily Return (in %)	13.52	13.52	14.49	13.53	15.27	11.52
Lowest Daily Return (in %)	-15.1	-9.80	-8.01	-18.3	-16.0	-15.3
Spearman Rho	0.058	0.050	0.064	0.069	0.051	0.044
CAPM Beta	0.033	0.035	0.038	0.052	0.035	0.022
Jensen's Alpha	0.00023	-0.00006	-0.000052	0.00067	0.000039	0.000045
Jense's Alpha p.value	0.183	0.924	0.731	0.026	0.157	0.059
Annual Skewness	5.010	3.273	2.790	1.832	2.401	2.631
Annual Kurtosis	27.735	17.352	12.878	5.77	9.735	9.737
Total Sample MDD (in %)	46.66	58.73	59.83	46.98	57.73	41.68

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1996 and December 2012 for the Fully Invested weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion or cointegration approach and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving SD window of 20 days and no limit on the time pairs remain open inside the trading window. Equally weighted weighting scheme.

Figure 3 – Cumulative Returns for Brazil

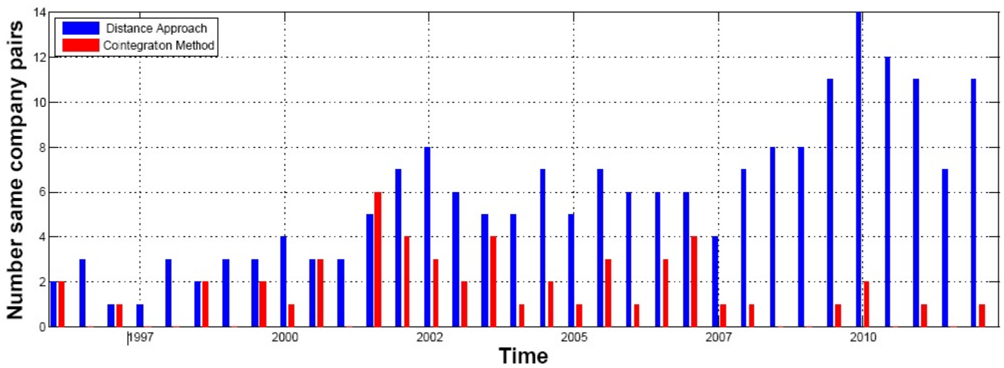


Note: This figure reports the cumulative excess returns for Brazil in log for the 5 pairs portfolios between January 1996 and December 2012 using the distance method and the cointegration approach using the fully invested weighting scheme. Data source: Economática (2014).

the market as shown by its negligible beta. The strategy has performed reasonably consistently throughout the whole period with returns ranging between 10% and 19%, in line with the Caldeira and Moura (2013) results, that used a smaller sample.

The brazilian market structure is very different than the other two databases we are investigating. It is not only less liquid than the american and the european ones, but it also has a lower number of assets, and some of those assets are ordinary and preferential stocks, meaning they are entitled to the same cash flow with the main difference being two: (i) the seniority of the preferential stock against the ordinary stock regarding mainly bankruptcy credits and (ii) the right to vote of the ordinary stock, that the preferential asset does not have. Therefore we have identified that most stocks traded in the brazilian market are same companies assets, with the same cash flow, which it implies that there is a reason of fundamentalist valuation for both assets to have similar prices. As can be seen on Figure 4 a larger portion of the pairs selected through the distance method have stocks from the same company, which eliminates firm-specific shocks as a source of deviation. Since these stocks obviously have a common component, their normalized price distance tends to be very small and hence the distance method selects a higher share. Also the arbitrage opportunities are smaller causing the distance method to underperform. The process may be better modelled by the cointegration method, that explicitly fits a mean reverting equation to the dynamic between both assets, than by the distance method, that requires only the distance between the asset prices to be small, thus, explaining the superior performance of the cointegration method. In summary, the presence of more than one share class guides the distance method into creating pairs that have low arbitrage opportunites, due to its inherent feature as a "closeness measure" in contrast to a "convergence/divergence" modelling.

Figure 4 – Number of pairs formed through the cointegration amd distance method that are composed of same company assets



Note: This figure reports the number of pairs that are composed of stocks from the same company for Brazil between January 1996 and December 2012 using the distance method and the cointegration approach. Data source: Economática (2014).

4.4.3 Robustness Checks: Returns on the long and short side, no transaction costs and one day waiting for Brazil

The results on the long and short side are quite different than the ones found for the american and european dabase as seen on Table 17. For example, the distance approach short side has a negative average annual excess return. This may be because the brazilian stock market rallied after 2002, and this method might have selected stocks that were both increasing in value together. Hence, the short side would lose money. Another interesting fact is that the cointegration approach using the fully invested weighting scheme also has a negative performance on its portfolios with 10 and 20 stocks. As a general rule, the long side is responsible for the results of the whole portfolio, different from the american and european datasets. This indicates that the hedging abilities of the short side could be improved in order to increase the portfolios returns. We also see that the short side is not the one driving the results due to short selling barriers or lack of liquidity.

For the brazilian database the results without transaction costs are presented on table 18. It's clear that all portfolios perform much better without transaction costs, and also have a higher sharpe ratio. However, the 5 and 10 pairs portfolios created through the distance method have an average annual excess return of slightly over 4% (4.30 and 4.08%, respectively). Since their performance with transaction costs is slightly over 0% as seen on table 15, we can estimate that transaction costs after including bid-ask bounce, slippage effects and liquidity effects, is around 4 percentage points of the final return on these portfolios.

As previously seen with the american and european databases, the returns with one day waiting period tend to be considerably smaller. As seen on table 19, the brazilian dataset is not very different. The exception is the positive returns obtained by the 5 and 10 pairs portfolios created using the cointegration method, which have an average annualized excess return of 2.40% and 0.80% respectively. Nonetheless, their Sharpe ratio are 0.191 and 0.081, and their Jensen's alpha are not statistically significant. The share of negative excess returns is not much higher than without the one day waiting period, indicating that the number of days the strategy loses money is not more frequent, but the loss in itself is higher. As for the MDD, both strategies have a worse maximum drawdown when compared to the committed capital scheme without one day waiting period. These results could be due to a relatively smaller amount of possible pairs at each trading period, consequence of a small universe of available stocks in a given moment. Hence, the pairs

Table 17 – Comparison of long and short returns for unrestricted pairs trading for the Brazilian data set

Methodology		Distance Approach				Cointegration Approach			
Weighting Scheme		Committed Capital		Fully Invested		Committed Capital		Fully Invested	
Pairs Portfolio		Long	Short	Long	Short	Long	Short	Long	Short
5 Pairs	Excess Return	6.84	-1.44	18.30	-7.05	8.33	2.11	22.35	2.49
	Std. Dev.	21.97	21.21	36.81	35.87	16.12	15.81	34.27	33.24
	S.R	0.30	-0.06	0.45	-0.20	0.50	0.13	0.59	0.07
10 Pairs	Excess Return	7.12	-4.34	14.72	-9.71	3.27	2.15	18.62	-1.62
	Std. Dev.	17.64	17.13	35.61	34.46	14.95	14.71	33.68	32.89
	S.R	0.39	-0.25	0.38	-0.29	0.21	0.14	0.51	-0.04
20 Pairs	Excess Return	5.42	-3.12	13.76	-9.49	4.46	1.14	22.60	-3.56
	Std. Dev.	14.34	13.78	33.34	32.25	14.52	14.27	33.63	32.94
	S.R	0.36	-0.23	0.39	-0.30	0.30	0.08	0.61	-0.11

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs for the long side and the short side between January 1996 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted. 4% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully invested weighting scheme.

Table 18 – Excess return of pairs trading for the Brazilian data set without transaction costs

Methodology		Distance Approach		Cointegration Approach	
Pairs Portfolio		Committed Capital	Fully Invested	Committed Capital	Fully Invested
5 Pairs	Excess Return	7.52	12.96	12.54	30.60
	Std. Dev.	8.49	17.54	12.66	31.60
	S.R	0.85	0.69	0.93	0.84
10 Pairs	Excess Return	4.34	7.19	7.93	23.13
	Std. Dev.	6.81	17.94	10.53	28.15
	S.R	0.62	0.39	0.72	0.74
20 Pairs	Excess Return	4.09	6.76	8.02	24.51
	Std. Dev.	5.47	16.83	9.45	24.30
	S.R	0.73	0.39	0.82	0.90

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1996 and December 2012. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 0% Transaction Costs. Without one day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window. Committed Capital and Fully Invested weighting schemes.

Tabela 19 – Excess return of unrestricted pairs trading for the Brazilian data set with one day waiting

Methodology	**Distance Approach**			**Cointegration Approach**		
Pairs Portfolio	**5 Pairs**	**10 Pairs**	**20 Pairs**	**5 Pairs**	**10 Pairs**	**20 Pairs**
Average Annualized Return (in %)	-4.24	-2.33	-1.81	2.449	0.861	-0.03
Average Annualized Volatility (in %)	10.34	9.705	8.537	12.66	10.51	9.371
Total Sample Sharpe Ratio	-0.41	-0.24	-0.21	0.191	0.081	-0.00
Largest Daily Return (in %)	5.919	4.946	4.089	5.205	4.927	4.927
Lowest Daily Return (in %)	-4.17	-3.15	-2.61	-6.97	-6.97	-6.97
Spearman Rho	0.038	0.063	0.091	0.061	0.050	0.043
CAPM Beta	0.012	0.019	0.023	0.018	0.012	0.011
Jensen's Alpha	-0.00	-0.00	-0.00008	-0.00008	0.00002	-0.00007
Jensen's Alpha p.value	0.078	0.271	0.305	0.492	0.795	0.934
Annual Skewness	0.449	0.622	0.848	0.175	0.920	0.800
Annual Kurtosis	2.950	3.756	3.735	2.010	3.800	3.235
Total Sample MDD (in %)	58.24	39.55	38.05	27.37	24.34	23.78
Share of negative excess returns (in %)	0.503	0.505	0.509	0.401	0.437	0.453

Note: This table reports a summary statistics of the excess returns on portfolios of 5, 10 and 20 pairs between January 1996 and December 2012 for the Committed Capital weighting scheme. Pairs are formed over a 12-month period according to a minimum-distance criterion and a cointegration method and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis. Equally weighted weighting scheme. 4% Transaction Costs. One day waiting. 2 SD signal to open a pair and full convergence signal to close a pair (0 SD). Stop loss of 10% and Stop gain of 20%. Moving average and SD window of 20 days and no limit on the time pairs remain open inside the trading window.

created, although the best available, are not very profitable and a one day waiting period is just enough to reduce the profitability overall.

5 Pairs Trading Performance Evaluation

In order to evaluate the performance of the strategies, we compare it to a naive strategy, i.e., we create bootstrapped return series in which the signal to start the strategy of pairs trading is inserted,and the performance of such a strategy is monitored and compared to the performance of the original series of returns. We follow the method used by Gatev et al. (2006) and Caldeira and Moura (2013), in which the bootstrap initiates at the time at which the signal is sent to begin trading pairs. In each bootstrap, the original series is replaced by two series of random assets similar to the assets earlier, similarity being defined as returns in the previous month belonging to the same decile. Thus, the difference in performance of the original assets and simulated give an indication of performance. The net return of the naive strategy is given by:

$$R_t^{naive} = \sum_{i=1}^{N} w_{it} r_{it} + 2N\, ln\left(\frac{1-C}{1+C}\right) \tag{13}$$

The results were calculated in every 6 month trading period and are withheld due to space constraints and can be obtained by contacting the author. We bootstrap each period 2500 for each of the pairs selection methodology and for each portfolio size, and found that both strategies obtain statisticallly significant positive performance when compared to a naive trader for both countries. In other words, the pairs trading strategies based on the selection of pairs through cointegration and through the distance method have a superior performance when compared to the random selection of pairs of stocks to be traded. The average returns on the random pairs is slightly negative for all databases, possibly due to the inclusion of transaction costs, and the standard deviations are large compared to the pairs trading portfolio's standard deviations. Beating a naive trader with random opening and closing pairs signals is not particularly difficult, but it does indicate that the use of information to choose pairs and decide when to trade does have a positive return, contrary to what a weakly efficient market would allow.

5.1 Hypothesis Testing for the Difference Between the Sharpe Ratios

Given that the objective of this paper is to compare the performance of two pairs selection methods, we must use a metric in order to assess if any of the strategies has a superior performance. In order to test the statistical significance of the difference between the Sharpe ratios of both strategies we use the methodology proposed in Ledoit and Wolf (2008) and obtain the p-values of the stationary bootstrap of Politis and Romano (1994) with B = 1000 bootstrap resamples and block length b= 5.

The whole sample result for the difference between Sharpe ratios through the methodology proposed by **?** indicates that for Brazil, the cointegration method is superior during the whole sample period to the distance method. However, for the sub-periods the results are not as robust, with most sub-periods results, available upon request, indicating that the cointegration method does not deliver a statistically significant higher Sharpe Ratio which hints at the fact that some subperiods might be driving the full sample results, or that the performance is slightly superior in each period, but not statistically higher due to sample limitations, since the size of most subperiods sample is 120 compared to the whole period that comprises 4087 observations. For europe the results also indicate that for the whole sample period the cointegration strategy is superior when using a portfolio consisting of 10 and 20 pairs. However for the 5 pairs portfolio the p-value of the statistic calculated is 0.107, and the cointegration strategy cannot be considered superior. Also, for the 6 month sub-periods the cointegration strategy in most subsample periods is not superior, with the results most likely being driving by some sub-periods. These findings hint that the cointegration strategy may be superior to the distance method in some periods, while the distance method may be superior in other sub-periods, but on average the cointegration method delivers a higher Sharpe Ratio. For the USA the results are the other way around. The distance method Sharpe Ratio is statistically superior than on the cointegration method, for all 3 portfolio sizes. However, again as in Brazil and for Europe, the results in the sub-periods are mixed, with the distance method not being superior in most 6 month subperiods due to high volatility and small period (tables available upon request). Since all returns are relatively small in each subperiod, the sample is small (120 observations) and the test is a very strict one, it is expected for there to not exist a statistical superiority in a given subsample. Nonetheless, some strategies are, when using the full sample, considered statistically superior by Ledoit and Wolf (2008).

6 Conclusion

In this paper we compared two methodologies for the strategy called pairs trading. The distance method presented in Gatev et al. (2006) and the cointegration method used by Caldeira and Moura (2013), for the american stock market between 1962 and 2012, for the Brazilian stock market between 1996 and 2012 and for the european market between 1973 and 2012. We create portfolios comprising 5, 10 and 20 pairs for each method, and bootstrap the results in order to compare the their performance. The pairs were ranked by their in sample sharpe in the cointegration method and by the smallest to the highest SSD for the distance method in order to form the portfolios. The signal to open the position out of sample was given whenever the distance between the stocks on a given pair crossed the 2 standard deviation threshold. Both methodologies had a good performance when compared to a naive trader that randomly selection pairs to trade on a given period. For Brazil, when compared to each other, the cointegration method had a clear, statistically significant higher average annualized return, with a superior Sharpe Ratio. Both strategies can be considered market neutral, with a close to zero spearman correlation with the market.

For Europe, while the results were not so clear cut, they also pointed towards the cointegration method being superior, delivering an average of 15% of excess returns for the committed capitals against 7% for the distance method. The Sharpe Ratio was also considered superior for the whole sample period, although in some subsamples both strategies were very similiar. Both strategies had an excess returns superior than a naive trader. For the United States, the results were very different, indicating that the distance method is superior, delivering up to 13.% of average annual excess return, more than the 3% of the cointegration method. Considering that this strategy is self-financed, since the cash obtained by shortening a stock is used to buy the long stock in the pair, these results are encouraging and indicate a clear path for more research regarding the drivers of such difference in performance, the optimality of the trading thresholds and the stability of the cointegration parameters.

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