MANIPAL SCHOOL OF INFORMATION SCIENCES
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AML5101 | Applied Linear Algebra | In-class Problem Set-1

Suppose we have an ECG signal with several missing samples that may have been lost due to transmission errors, interference, or noise. One such dataset (`ecg_missing.txt`) is available on Teams via *Discussions ALA ¿ ALA Bootcamp ¿ Data* folder where the missing values are represented as `NaN`. We will later formulate the problem of recovering the missing ECG samples as a so-called *linear least squares problem*, the foundations of which will be discussed in future lectures. In this assignment we will practice the basics of vector operations that will be extended to a code. To that end, for simplicity, assume that the full ECG signal is a 6-vector x with samples 3, 5, and 6 missing as shown below:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ {\color{red}x_3} \\ x_4 \\ {\color{red}x_5} \\ {\color{red}x_6} \end{bmatrix}.$$

For computational purposes that we will discuss later, we will rename the missing components of the vector x as follows:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ {\color{red}v_1} \\ x_4 \\ {\color{red}v_2} \\ {\color{red}v_3} \end{bmatrix}.$$

1. Fill in the entries of the matrix below that will fetch the non-missing components from the vector x:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ {\color{red}v_1} \\ x_4 \\ {\color{red}v_2} \\ {\color{red}v_3} \end{bmatrix}.$$

   How do the rows of the matrix correspond to the positions of the non-missing components of the vector x?

2. Now that you have identified the matrix above, explain how it is related to the matrix below:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

How do the columns of the matrix correspond to the positions of the non-missing components of the vector x?

3. Explain in plain English (in terms of the signal vector x) what the multiplication below achieves:

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\\0&0&1\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_4\end{bmatrix}.$$

4. Fill in the entries in the two matrices below:

$$\underbrace{\begin{bmatrix}?&?&?\\?&?&?\\?&?&?\\?&?&?\\?&?&?\\?&?&?\end{bmatrix}}_{S_1}\begin{bmatrix}x_1\\x_2\\x_4\end{bmatrix}+\underbrace{\begin{bmatrix}?&?&?\\?&?&?\\?&?&?\\?&?&?\\?&?&?\\?&?&?\end{bmatrix}}_{S_2}\begin{bmatrix}\color{red}{v_1}\\\color{red}{v_2}\\\color{red}{v_3}\end{bmatrix}=\begin{bmatrix}x_1\\x_2\\\color{red}{v_1}\\x_4\\\color{red}{v_2}\\\color{red}{v_3}\end{bmatrix}.$$

How do the columns of the matrices correspond to the positions of the non-missing and missing components of the vector x?

5. Now conisder a generic vector 6-vector y. Write the result of the following multiplication and explain in plain English what the result tells us about the vector y itself (for example, what if all the components of the vector y are the same?):

$$\underbrace{\begin{bmatrix}1&-2&1&0&0&0\\0&1&-2&1&0&0\\0&0&1&-2&1&0\\0&0&0&1&-2&1\end{bmatrix}}_{D}\begin{bmatrix}y_1\\y_2\\y_3\\y_4\\y_5\\y_6\end{bmatrix}.$$