APS -1

Aditya
231057015
AI & ML

1) a) <u>No restriction</u>:

$n = 10$
$r = 7$

$$\therefore {}^{10}C_7 = \frac{10!}{7!(10-7)!}$$

$$= \frac{10!}{7!\,3!}$$

$$= \underline{\underline{120}}$$

b) answer exactly 2 of last 4

→ 6 question answer 2
→ last 4 question answer 2

$\therefore {}^4C_2 = 6$

${}^6C_5 = 6$

$\text{Total} = {}^4C_2 \times {}^6C_2$

$= \underline{\underline{36}}$

c) Exactly 2 of first 6

$\rightarrow 6C_2 \times 4C_5$

$= 15 \times 1$

$= 15$

d) at least 3 of first 5

$\rightarrow 5C_3 + 5C_4 + 5C_5 + 5C_6 + 5C_7$

$= 10 + 5 + 1$

$= 16$

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2) $HLA\text{-}A_n \rightarrow 18$ $\quad$ $HLA\text{-}A_r = 2$

$HLA\text{-}B_n \rightarrow 40$ $\quad$ $HLA\text{-}B_r = 2$

$HLA\text{-}DR_n \rightarrow 14$ $\quad$ $HLA\text{-}C_r = 2$

HLA-A chosen as $(n+r-1)$

$= (18+2-1)$

$= 19$

$C(HLA\text{-}A) = 19C_2 = 171$

$HLA-B = (40+2-1)$
$= 41$

$C(HLA B) = {}^{41}C_2$

$= \frac{41 \times \cancel{40}^{20}}{\cancel{2} \times 1}$

$= \underline{820}$

$HLA D_R = (14+2-1)$
$= 15$

$C(HLA-D_R) = {}^{15}C_2$

$= \frac{15 \times 14}{2 \times 1}$

$= \underline{105}$

$\therefore$ total strings $= C(HLA-A) \cdot C(HLA-B) \cdot C(HLA D_R)$

$= 820 \times 15 \times 105$

$= 14407500$

3) □ □ / □ □ / □ □ / □ □

a) $n = 8$

$r = 4$

$$8C_r = \frac{8!}{4!\,4!}$$

$= 70$

b) atleast 1

$$4C_4 = 1$$

4 3 PC , 4Mac , 2 linux

a) distinguishable ways of service

$$\rightarrow \frac{9!}{4!\,3!\,2!}$$

$= 1260$

b) first 5 malli has 4 mac

$$\frac{9!}{1! \times 8!}$$

$$\underline{\underline{9}}$$

c) 2 PC first 3
1 PC in last 3 } of 9

2 PC in first 3 $= \frac{3!}{2!\,1!} \times \frac{6!}{4!\,2!}$

$= 90$

1 PC in last 3 $= \frac{6!}{4!\,2!} \times \frac{3!}{2!\,1!}$

$= 90$

$\therefore$ 180 ways

5 100 on 5
V1 → atleast 10 , V2 = V3 = atleast 12
$V_6 + V_7$ = atleast 4

$\rightarrow {}^{100}C_{10} \cdot {}^{90}C_{12} \cdot {}^{78}C_{12} \cdot {}^{66}C_{4} \cdot {}^{62}C_{5}$

$= \frac{100}{10!(100-10)} \times \frac{90}{12!(90-12)!} \cdot \frac{78}{12!(78-12)!} \cdot \frac{66}{4(66-4)!} \cdot \frac{62}{5(62-5)!}$

$= \frac{100!}{10! \cdot 12! \cdot 12! \cdot 4! \cdot 57!}$

6 11 Players 4 groups
3 F , 3 M , ~~4~~ 4 D, 1 G

a) 2 goal
b) Can play any position
c) of 9, 4 → F or M, 5 → D or M

$$2C_1 \left(6C_1 \cdot 5C_3 \cdot 6C_3 \cdot 3C_3 \right) + \left(10C_1 \cdot 9C_2 \cdot 4C_4 \cdot 3C_3 \right) + 3C_1 \cdot 4C_2 \cdot 7C_3 \cdot 4C_4 \Big)$$

$$= 2(1200 + 360 + 1050)$$
$$= \underline{5220}$$

7 No of ways to distribute r among n so that i receives at least m_i.

$$C(r - m_1 - m_2 - m_3 \dots - m_n + n - 1, n - 1)$$

Eg: $r = 10$
$n = 3$
$m_1 = 2$
$m_2 = 3$
$m_3 = 3$

$$C(10 - 2 - 3 - 3 + 3 - 1, 3 - 1)$$

$$6C_2 = \frac{6!}{4! \cdot 2!}$$

$$= \underline{15}$$

9) Court

	Correct	Wrong
A	0.95	0.05
B	0.95	0.05
C	0.9	0.1
D	0.9	0.1
E	0.8	0.2

a) $S = \{0, 1\}$

where 0 → correct

1 → incorrect

outcome 1 → 10111

↳ Judge A voted 1

B voted 0

C voted 1

D voted 1

E voted 0

3/5 voted to wrong, so prisinor was not released

outcome → 00001

Judge A, B, C, D voted 0

Judge E voted 1

4/5 voted to correct, released

b) no of outcomes $= 2^n$

$= 2^5$

$= 32$

c) $n(E) = 5C_3 + 5C_4 + 5C_5$

$= 10 + 10 + 10$

$= 30$

d) probability of event is ratio of favourable outcome to total possible outcome. Here, judge have diff probabilities of voting correctly, so not all outcomes are equally likely.
$\therefore$ directly using ratio of no. of outcomes to total outcomes $n(E)$ might not give correct probability

10 type A → 1. type B → 1.
10 of type A 10 type B
20 tablets

a) take 2 from 20

$P(Pill_A) = \frac{10}{18} = 0.55$

$P(Pill_B) = \frac{8}{18} = 0.44$

$P(\text{each tab from Pile is } A \& B) = \frac{10}{18} + \frac{8}{18}$

$= 1$

b) P(Random)
↳ Possibilities

$P(A) + P(A) = \frac{10}{20} \times \frac{9}{19} = \frac{90}{380} = P(B) + P(B)$

$P(A) + P(B) = P(B) + P(A) = \frac{10}{20} \times \frac{10}{19} = \frac{100}{380}$

$\therefore P = \frac{90}{380} + \frac{100}{380} + \frac{100}{380} + \frac{90}{380}$

$= 1$

11 $A \rightarrow 28\%$ $B \rightarrow 7\%$ $A \cap B \Rightarrow 5\%$

i) $(A \cup B)^c$
$= 100\% - P(A \cup B)$
$= 100 - [P(A) + P(B) - P(A \cap B))$

$= 100 - (28 + 7 - 5)$

$= 60$

11) $B \cap A^c$

$= P(B) - P(A \cap B)$

$= 7 - 5$

$= 2\%$

12 quantitative support

a) atleast one 6 in 4 rolls of single die

$$P = 1 - \left(1 - \frac{1}{6}\right)^4$$

$= 0.517$

b) one 12 in 24 rolls of pair

$$P = 1 - \left(1 - \frac{1}{36}\right)^{24}$$

≈ 0.491

after comparing the higher probabality of obtaining at least one 6 in 4 rolls is more likely toocur.

13 a) age < 25 years

Total $= 952 + 1050 + 53 + 456 + 2055 + 1570 + 54 + 952 + 1008$

$= 8150$

age $< 25 = 952 + 1050 + 53$
$= 2055$

% of people < 25 years $= \frac{2055}{8150} = 25.2\%$

b) age > 25 years

$P(8150 - 2055)$
$P(6095) = \frac{6095}{8150}$
$= 0.7478$

c) Salaore < 70000
$= \frac{952 + 1050 + 456 + 2055 + 54 + 952}{8130}$

$= 0.6771 = 67.7\% < \$70k$

d) < 25y & > 70 k
$\frac{53}{8130} = 0.0065 = 0.65\%$

e) < 25 K & 25-45 years

$$\frac{456}{8150} = 5.6\%$$

f) > 45 & < 70 K

$$\frac{54+952}{8150} = 12.34\%$$