AML5103 | Applied Probability and Statistics | In-class Problem Set-2

1. A salesman has scheduled two appointments to sell software, one in the morning and another one in the afternoon. There are two software editions available: the base edition costing Rs. 5000 and the premium edition costing Rs. 10000. His morning appointments typically lead to a sale with a 30% chance while the afternoon ones typically lead to a sale with a 60% chance independent of what happened in the morning. If the morning appointment ends up in sale, the salesman has a 70% chance of selling the premium edition and if the afternoon appointment ends up in a sale, he is equally likely to sell either of the editions. Let X be the random variable representing the total Rupee value of sales. What are the different values that X can take? Calculate the probability that X takes the value 5000? Use the preliminary steps below to calculate that probability.

Solution: The event $X = 5000$ is equivalent to the following:

morning appointment ends up in sale and sell standard model and afternoon appointment no sale

or

morning appointment no sale and afternoon appointment ends up in sale and sell standard model

We can now calculate the probability $P(X = 5000)$ as follows:

$$P(X=5000) = \begin{cases} P\,(\text{morning appointment ends up in sale and sell standard model and afternoon appointment no sale}) \\ \qquad\qquad \underbrace{+}_{\text{why?}} \\ P\,(\text{morning appointment no sale and afternoon appointment ends up in sale and sell standard model}) \end{cases}$$

$$= \begin{cases} P\,(\text{morning appointment ends up in sale and sell standard model}) \underbrace{\times}_{\text{why?}} P\,(\text{afternoon appointment no sale}) \\ \qquad\qquad \underbrace{+}_{\text{why?}} \\ P\,(\text{morning appointment no sale}) \underbrace{\times}_{\text{why?}} P\,(\text{afternoon appointment ends up in sale and sell standard model})\,. \end{cases}$$

Finish the solution using pen & paper using the approach above and then try the coding approach using the code snippet hints below:

```
# Sampling space for appointment success (0 corresponds to no sale, 1 corresponds to a sale)
s_appointment = c(0, 1)

# Appointment success and failure probabilities
p_morning = ? # Success probability of sales in morning
p_afternoon = ? # Success probability of sales in afternoon
p_appointment = matrix(nrow  = 2, ncol = 2, c(1-p_morning, ?, ?, p_afternoon), byrow = TRUE)

# Sampling space for software type
s_software = c(?, ?)

# Software type probabilities
p_software = c(?, ?)
```

```
# Function that simulates one trial of the random experiment which is
# what the salesman earns on a random day
salesResult = function(){
  result = numeric(2)
  # Simulate whether sales happen in morning and afternoon appointments
  for (j in c(1:2)){
    result[j] = sample(?, size = 1, prob = p_appointment[?, ?])
  }
  earnings = ifelse(result[1] == 1, sample(?, size = 1, prob = p_software), 0) +
    ifelse(result[?] == ?, sample(s_software, size = 1, prob = p_software), 0)
  return(?)
}

# Number of simulations
nsimulations = 1e5
simulatedData = replicate(nsimulations, salesResult())
```