Math 501: Intro to Real Analysis
Homework 7

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Problem

Definition 1. Suppose $\{x_n\}_{n=1}^{\infty} \subseteq \mathbb{R}$. We say $\{x_n\}$ diverges to infinity if for every $M \in \mathbb{R}$, there exists a $N \in \mathbb{N}$ such that for all $n \geq N$, $x_n \geq M$. We write $\lim_n x_n = +\infty$.
1. State the analogous definition for diverges to negative infinity. 2. Suppose $\{x_n\}_{n=1}^{\infty} \subseteq \mathbb{R}$ is not bounded above. Prove that there exists a subsequence $\{x_{n_k}\}$ such that $\lim_k x_{n_k} = +\infty$. 3. Suppose $\{x_n\}_{n=1}^{\infty} \subseteq \mathbb{R}$ is not bounded below. Prove that there exists a subsequence $\{x_{n_k}\}$ such that $\lim_k x_{n_k} = -\infty$

Solution