Math 501: Intro to Real Analysis
Homework 3

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Problem

Let $A = \{x \in Q : x^2 \leq 2\}$. Prove that if $\alpha \in A$, then there exists a $\beta \in A$ such that $\alpha < \beta$. Note: you may use the following fact without proof: there does not exist any rational number c with $c^2 = 2$.

Solution

Let us assume that $\forall \alpha, \beta \in A \qquad \beta \leq \alpha$.

Hence, our assumption was wrong, Thus, using the trichotomy property of the Ordered Fields, if $\beta \not\leq \alpha$, then $\beta > \alpha$.