

# Left recursion & Left factoring

# Left recursion

- ▶ A grammar is said to be **left recursive** if it has a non terminal  $A$  such that there is a derivation  $A \rightarrow A\alpha$  for some string  $\alpha$ .

## Algorithm to eliminate left recursion

1. Arrange the non terminals in some order  $A_1, \dots, A_n$
2. For  $i := 1$  to  $n$  **do begin**  
    for  $j := 1$  to  $i - 1$  **do begin**  
        replace each production of the form  $A_i \rightarrow A_j\gamma$   
        by the productions  $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \dots \mid \delta_k\gamma$ ,  
        where  $A_j \rightarrow \delta_1 \mid \delta_2 \mid \dots \mid \delta_k$  are all the current  $A_j$   
        productions;  
    **end**  
    eliminate the immediate left recursion among the  $A_i$  - productions  
**end**

# Left recursion elimination

$$A \rightarrow A\alpha \mid \beta \quad \longrightarrow \quad \begin{array}{l} A \rightarrow A' \\ A' \rightarrow A' \mid \epsilon \end{array}$$

# Examples: Left recursion elimination

$E \rightarrow E+T \mid T$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \varepsilon$

$T \rightarrow T*F \mid F$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \varepsilon$

$X \rightarrow X\%Y \mid Z$

$X \rightarrow ZX'$

$X' \rightarrow \%YX' \mid \varepsilon$

# Exercise: Left recursion

1.  $A \rightarrow Abd \mid Aa \mid a$   
 $B \rightarrow Be \mid b$
2.  $A \rightarrow AB \mid AC \mid a \mid b$
3.  $S \rightarrow A \mid B$   
 $A \rightarrow ABC \mid Acd \mid a \mid aa$   
 $B \rightarrow Bee \mid b$
4.  $\text{Exp} \rightarrow \text{Exp} + \text{term} \mid \text{Exp} - \text{term} \mid \text{term}$

# Left factoring

Left factoring is a grammar transformation that is useful for producing a grammar suitable for predictive parsing.

## Algorithm to left factor a grammar

**Input:** Grammar G

**Output:** An equivalent left factored grammar.

**Method:**

For each non terminal A find the longest prefix  $\alpha$  common to two or more of its alternatives. If  $\alpha \neq \epsilon$ , i.e., there is a non trivial common prefix, replace all the A productions  $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \dots \mid \alpha\beta_n \mid \gamma$  where  $\gamma$  represents all alternatives that do not begin with  $\alpha$  by

$$\begin{aligned} A &\rightarrow \alpha A' \mid \gamma \\ A' &\rightarrow \beta_1 \mid \beta_2 \mid \dots \mid \beta_n \end{aligned}$$

Here A' is new non terminal. Repeatedly apply this transformation until no two alternatives for a non-terminal have a common prefix.

# Left factoring elimination

$$A \rightarrow \alpha \beta \mid \alpha \delta \longrightarrow \begin{array}{l} A \rightarrow A' \\ A' \rightarrow \mid \end{array}$$

# Example: Left factoring elimination

$S \rightarrow aAB \mid aCD$

$S \rightarrow aS'$

$S' \rightarrow AB \mid CD$

$A \rightarrow xByA \mid xByAzA \mid a$

$A \rightarrow xByAA' \mid a$

$A' \rightarrow \epsilon \mid zA$

$A \rightarrow aAB \mid aA \mid a$

$A \rightarrow aA'$

$A' \rightarrow AB \mid A \mid \epsilon$

$A' \rightarrow AA'' \mid \epsilon$

$A'' \rightarrow B \mid \epsilon$



# Exercise

1.  $S \rightarrow iEtS \mid iEtSeS \mid a$
2.  $A \rightarrow ad \mid a \mid ab \mid abc \mid x$



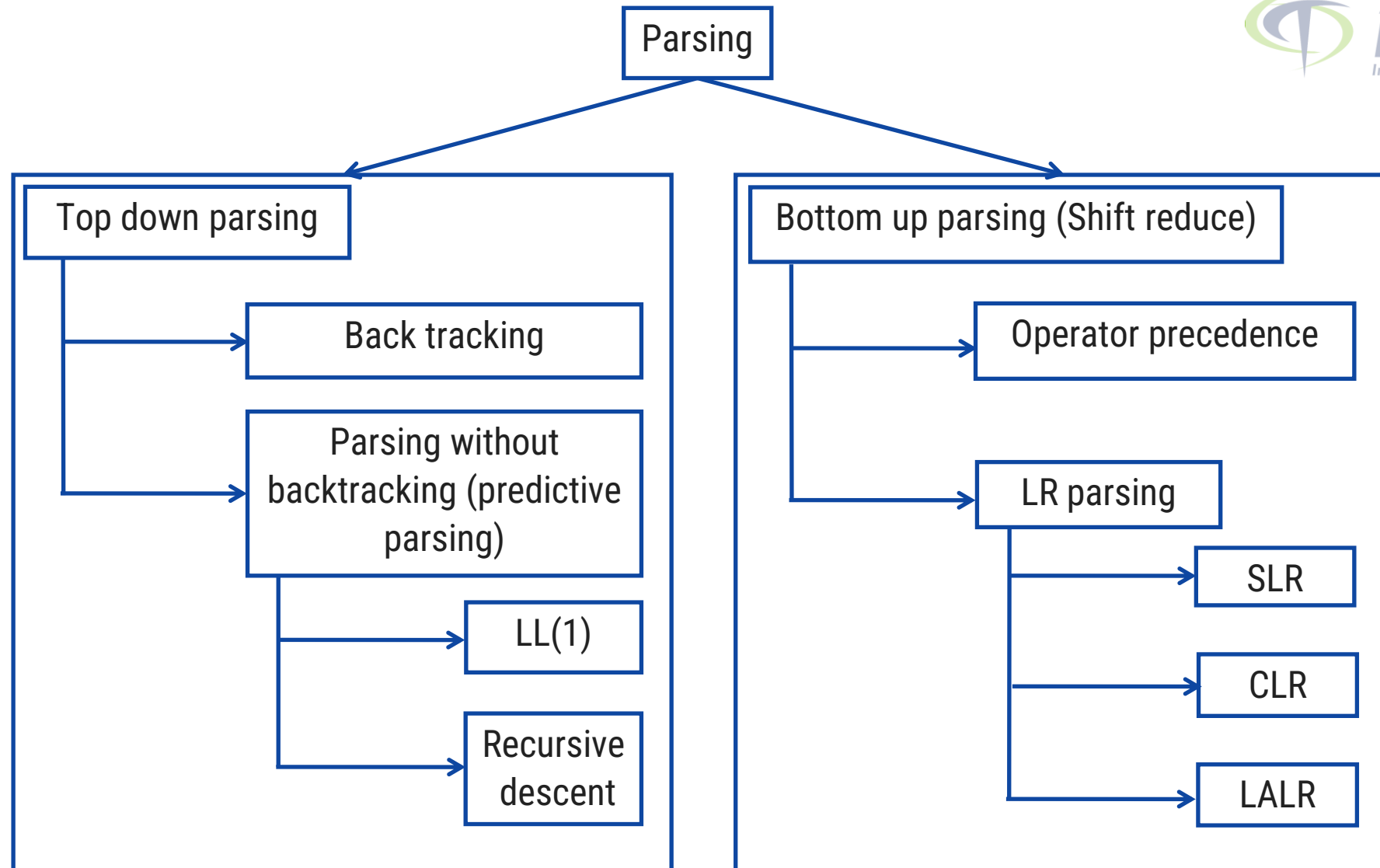
# Parsing



# Parsing

- ▶ Parsing is a technique that takes input string and produces output either a **parse tree** if string is valid sentence of grammar, or an **error message** indicating that string is not a valid.
- ▶ Types of parsing are:
  1. **Top down parsing:** In top down parsing parser build parse tree from top to bottom.
  2. **Bottom up parsing:** Bottom up parser starts from leaves and work up to the root.

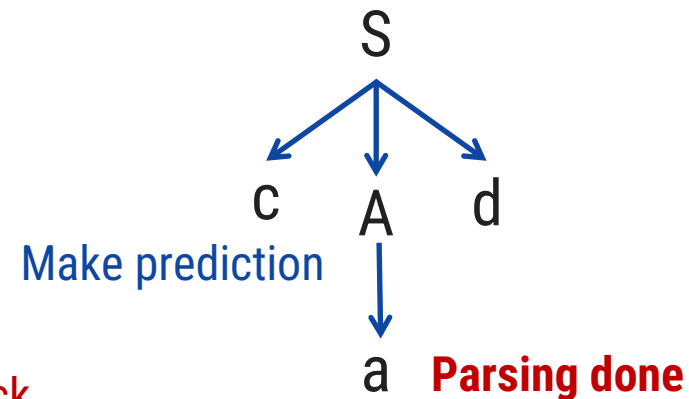
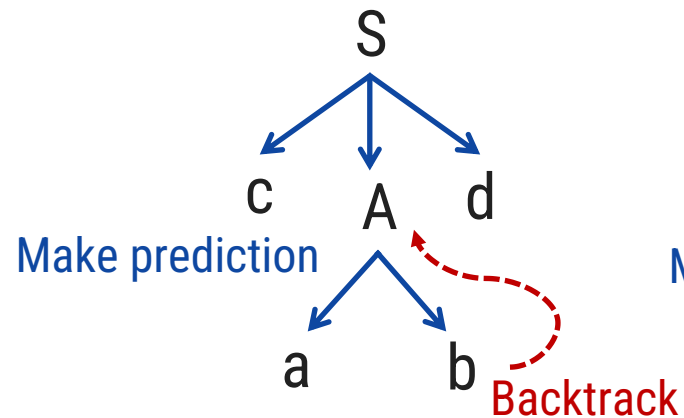
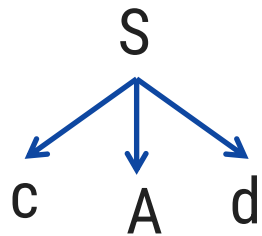
# Classification of parsing methods



# Backtracking

- ▶ In backtracking, expansion of nonterminal symbol we choose one alternative and if any mismatch occurs then we try another alternative.

▶ Grammar:  $S \rightarrow cAd$                       Input string: cad  
               $A \rightarrow ab \mid a$



# Exercise

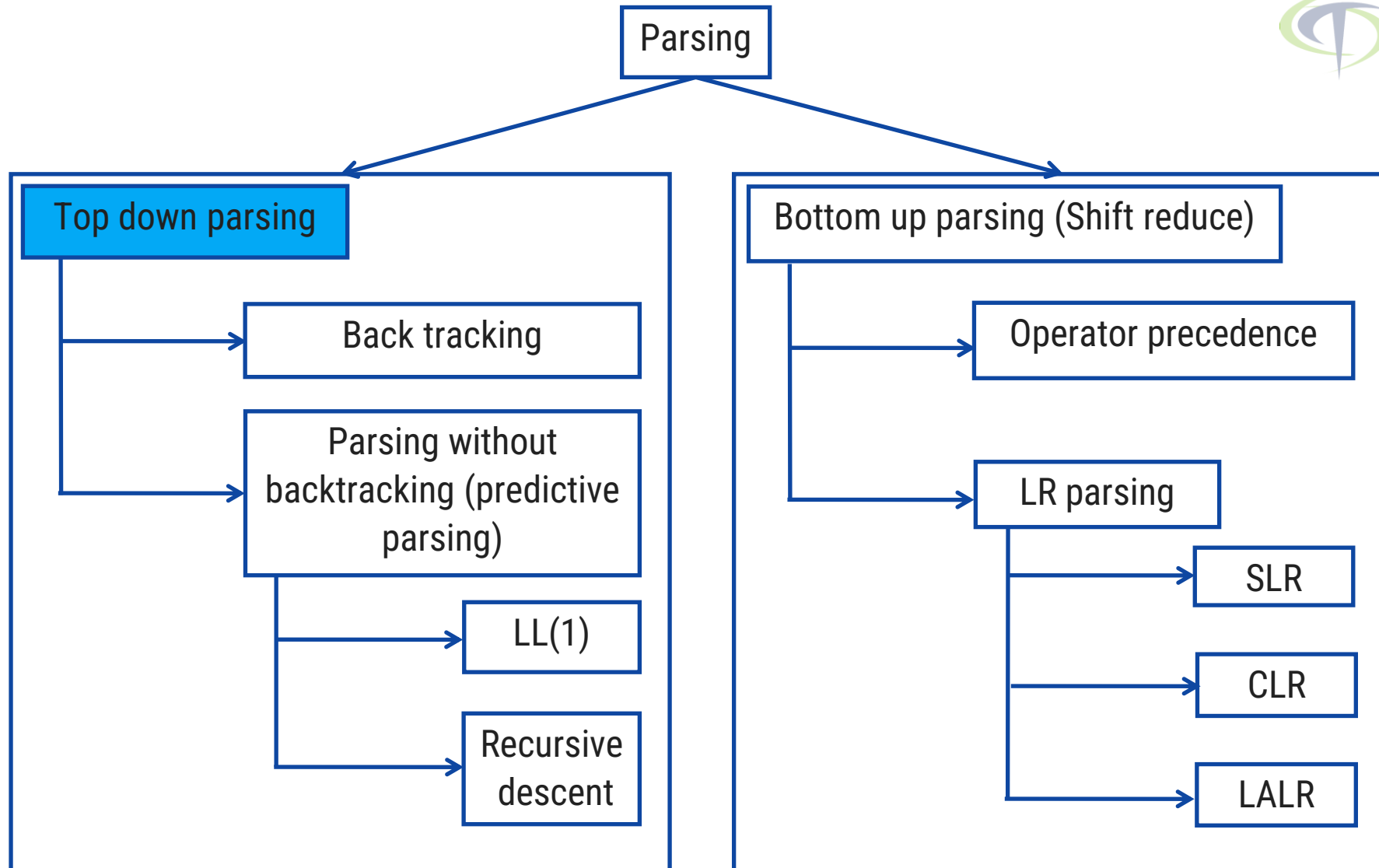
1.  $E \rightarrow 5+T \mid 3-T$

$$T \rightarrow V \mid V*V \mid V+V$$

$$V \rightarrow a \mid b$$

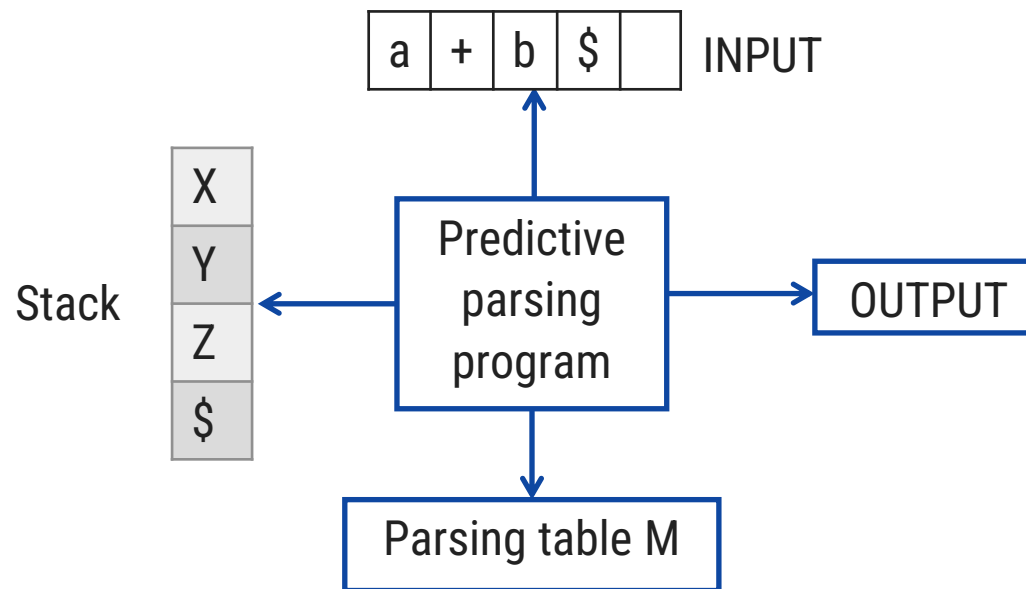
String: 3-a+b

# Parsing Methods



# LL(1) parser (predictive parser)

- ▶ LL(1) is non recursive top down parser.
  1. First **L** indicates input is scanned from left to right.
  2. The second **L** means it uses leftmost derivation for input string
  3. **1** means it uses only input symbol to predict the parsing process.





# LL(1) parsing (predictive parsing)

Steps to construct LL(1) parser

1. Remove left recursion / Perform left factoring (if any).
2. Compute FIRST and FOLLOW of non terminals.
3. Construct predictive parsing table.
4. Parse the input string using parsing table.

# Rules to compute first of non terminal

1. If  $A \rightarrow \alpha$  and  $\alpha$  is terminal, add  $\alpha$  to  $FIRST(A)$ .
2. If  $A \rightarrow \epsilon$ , add  $\epsilon$  to  $FIRST(A)$ .
3. If  $X$  is nonterminal and  $X \rightarrow Y_1 Y_2 \dots Y_k$  is a production, then place  $a$  in  $FIRST(X)$  if for some  $i$ ,  $a$  is in  $FIRST(Y_i)$ , and  $\epsilon$  is in all of  $FIRST(Y_1), \dots, FIRST(Y_{i-1})$ ; that is  $Y_1 \dots Y_{i-1} \Rightarrow \epsilon$ . If  $\epsilon$  is in  $FIRST(Y_j)$  for all  $j = 1, 2, \dots, k$  then add  $\epsilon$  to  $FIRST(X)$ .

Everything in  $FIRST(Y_1)$  is surely in  $FIRST(X)$  If  $Y_1$  does not derive  $\epsilon$ , then we do nothing more to  $FIRST(X)$ , but if  $Y_1 \Rightarrow \epsilon$ , then we add  $FIRST(Y_2)$  and so on.

# Rules to compute first of non terminal

## Simplification of Rule 3

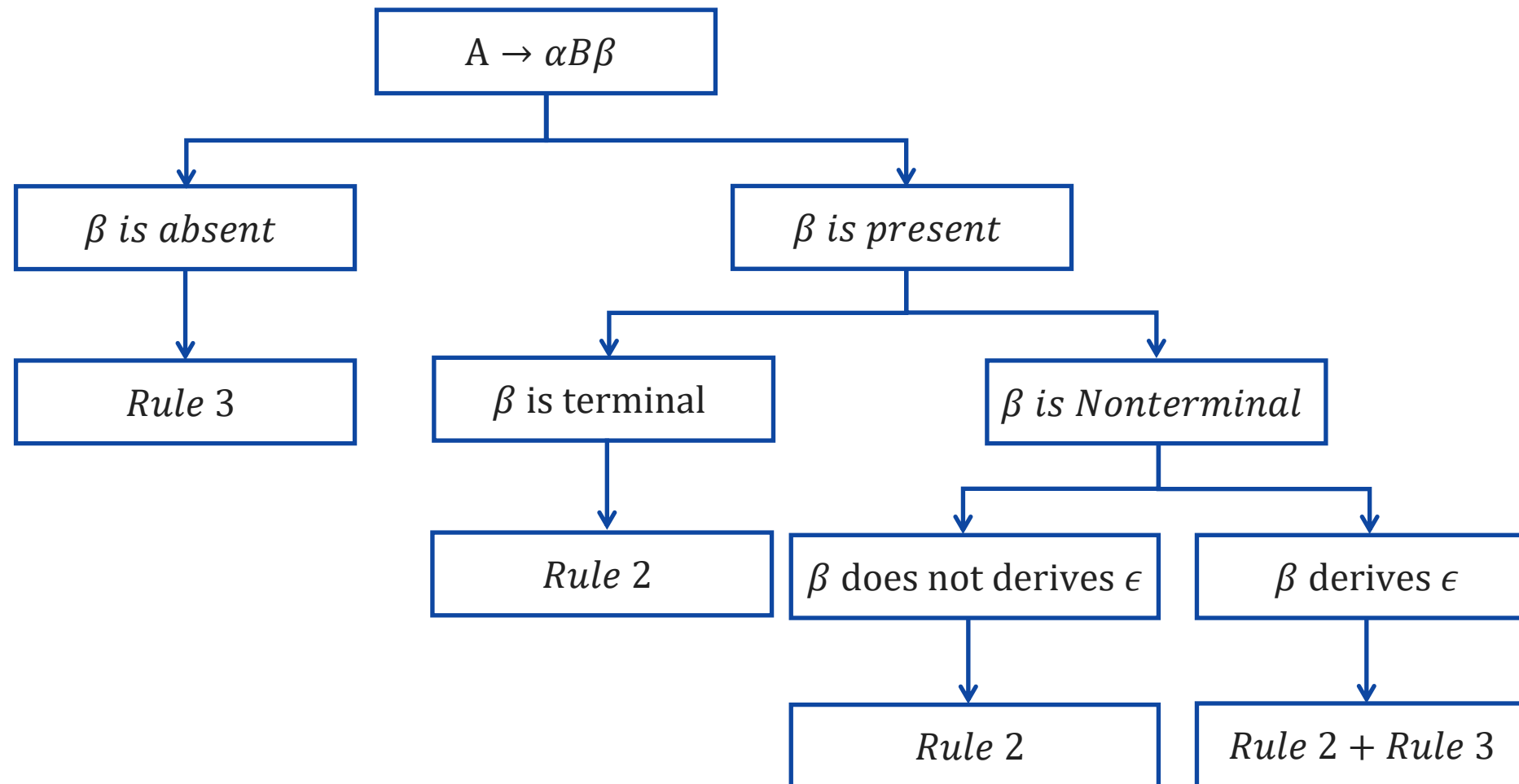
If  $A \rightarrow Y_1 Y_2 \dots \dots Y_K$ ,

- If  $Y_1$  **does not derives**  $\epsilon$  then,  $FIRST(A) = FIRST(Y_1)$
- If  $Y_1$  **derives**  $\epsilon$  then,  
$$FIRST(A) = FIRST(Y_1) - \epsilon \cup FIRST(Y_2)$$
- If  $Y_1$  &  $Y_2$  **derives**  $\epsilon$  then,  
$$FIRST(A) = FIRST(Y_1) - \epsilon \cup FIRST(Y_2) - \epsilon \cup FIRST(Y_3)$$
- If  $Y_1, Y_2$  &  $Y_3$  **derives**  $\epsilon$  then,  
$$FIRST(A) = FIRST(Y_1) - \epsilon \cup FIRST(Y_2) - \epsilon \cup FIRST(Y_3) - \epsilon \cup FIRST(Y_4)$$
- If  $Y_1, Y_2, Y_3 \dots Y_K$  all **derives**  $\epsilon$  then,  
$$FIRST(A) = FIRST(Y_1) - \epsilon \cup FIRST(Y_2) - \epsilon \cup FIRST(Y_3) - \epsilon \cup FIRST(Y_4) - \epsilon \cup \dots \dots \dots FIRST(Y_K)$$
 (note: if all non terminals derives  $\epsilon$  then add  $\epsilon$  to  $FIRST(A)$ )

# Rules to compute FOLLOW of non terminal

1. Place \$ in  $follow(S)$ . (S is start symbol)
2. If  $A \rightarrow \alpha B \beta$ , then everything in  $FIRST(\beta)$  except for  $\epsilon$  is placed in  $FOLLOW(B)$
3. If there is a production  $A \rightarrow \alpha B$  or a production  $A \rightarrow \alpha B \beta$  where  $FIRST(\beta)$  contains  $\epsilon$  then everything in  $FOLLOW(A) = FOLLOW(B)$

# How to apply rules to find FOLLOW of non terminal?



# Rules to construct predictive parsing table

1. For each production  $A \rightarrow \alpha$  of the grammar, do steps 2 and 3.
2. For each terminal  $a$  in  $first(\alpha)$ , Add  $A \rightarrow \alpha$  to  $M[A, a]$ .
3. If  $\epsilon$  is in  $first(\alpha)$ , Add  $A \rightarrow \alpha$  to  $M[A, b]$  for each terminal  $b$  in  $FOLLOW(B)$ . If  $\epsilon$  is in  $first(\alpha)$ , and  $\$$  is in  $FOLLOW(A)$ , add  $A \rightarrow \alpha$  to  $M[A, \$]$ .
4. Make each undefined entry of  $M$  be error.

# Example-1: LL(1) parsing

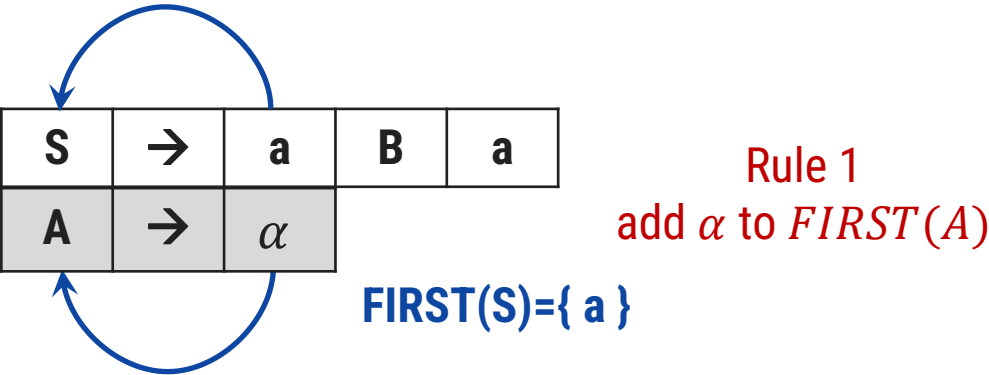
$S \rightarrow aBa$   
 $B \rightarrow bB \mid \epsilon$

Step 1: Not required

Step 2: Compute FIRST

First(S)

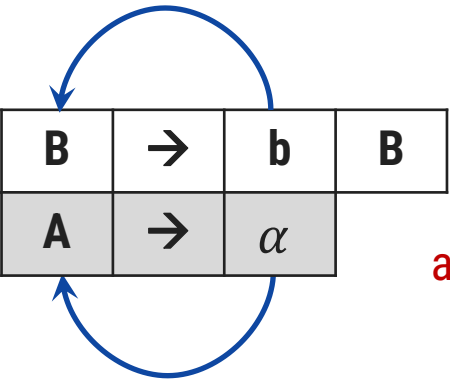
$S \rightarrow aBa$



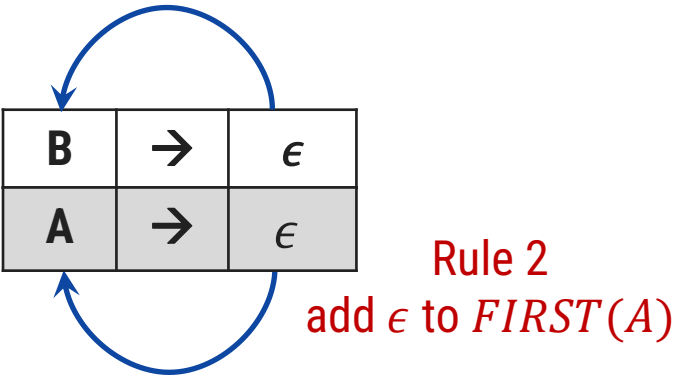
NT	First
S	
B	

First(B)

$B \rightarrow bB$



$B \rightarrow \epsilon$



$FIRST(B) = \{ b, \epsilon \}$

# Example-1: LL(1) parsing

$S \rightarrow aBa$

$B \rightarrow bB \mid \epsilon$

Step 2: Compute FOLLOW

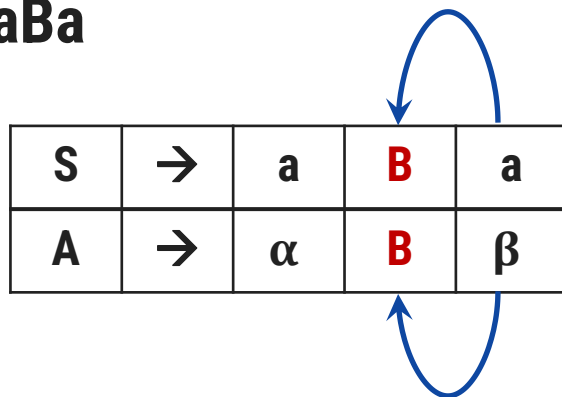
Follow(S)

Rule 1: Place \$ in FOLLOW(S)

$\text{Follow}(S) = \{ \$ \}$

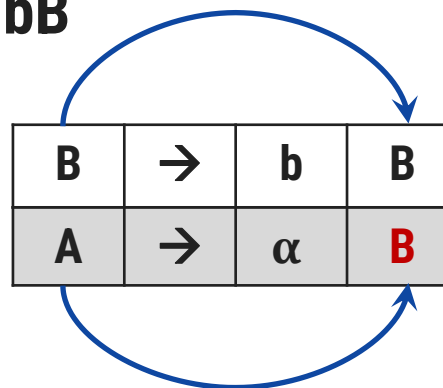
Follow(B)

$S \rightarrow aBa$



Rule 2  
 $\text{First}(\beta) = \epsilon$

$B \rightarrow bB$



Rule 3  
 $\text{Follow}(A) = \text{follow}(B)$

$\text{Follow}(B) = \{ a \}$

NT	First	Follow
S	{a}	
B	{b,ε}	



# Example-1: LL(1) parsing

$S \rightarrow aBa$

$B \rightarrow bB \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a}	{ \$ }
B	{b, $\epsilon$ }	{a}

NT	Input Symbol		
	a	b	\$
S			
B			

$S \rightarrow aBa$

$a = \text{FIRST}(aBa) = \{ a \}$

$M[S, a] = S \rightarrow aBa$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$

# Example-1: LL(1) parsing

$S \rightarrow aBa$

$B \rightarrow bB \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a}	{ \$ }
B	{b, $\epsilon$ }	{a}

NT	Input Symbol		
	a	b	\$
S	$S \rightarrow aBa$		
B			

$B \rightarrow bB$

$a = \text{FIRST}(bB) = \{ b \}$

$M[B, b] = B \rightarrow bB$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$

# Example-1: LL(1) parsing

$S \rightarrow aBa$

$B \rightarrow bB \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a}	{\$}
B	{b, $\epsilon$ }	{a}

NT	Input Symbol		
	a	b	\$
S	$S \rightarrow aBa$		
B		$B \rightarrow bB$	

$B \rightarrow \epsilon$

$b = \text{FOLLOW}(B) = \{ a \}$

$M[B, a] = B \rightarrow \epsilon$

Rule: 3

$A \rightarrow \alpha$

$b = \text{follow}(A)$

$M[A, b] = A \rightarrow \alpha$

# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

$C \rightarrow cS \mid \epsilon$

Step 1: Not required

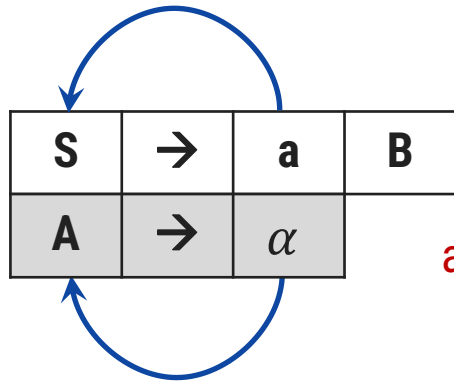
Step 2: Compute FIRST

First(S)

$S \rightarrow aB$

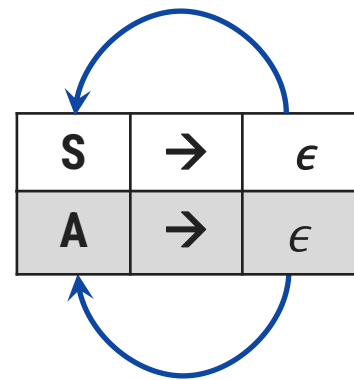
$S \rightarrow \epsilon$

NT	First
S	
B	
C	



Rule 1  
add  $\alpha$  to  $FIRST(A)$

$FIRST(S) = \{ a, \epsilon \}$



Rule 2  
add  $\epsilon$  to  $FIRST(A)$

# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

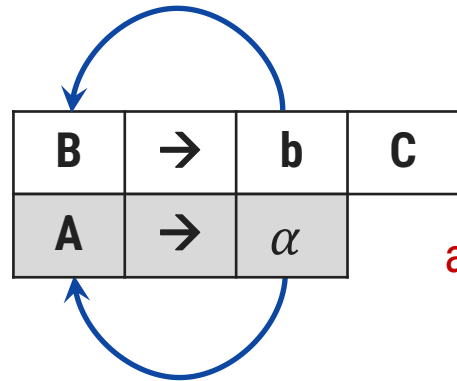
$C \rightarrow cS \mid \epsilon$

Step 1: Not required

Step 2: Compute FIRST

First(B)

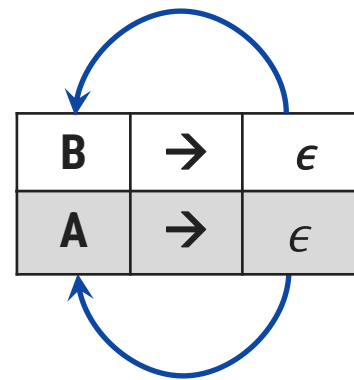
$B \rightarrow bC$



Rule 1  
add  $\alpha$  to  $FIRST(A)$

$FIRST(B) = \{ b, \epsilon \}$

$B \rightarrow \epsilon$



Rule 2  
add  $\epsilon$  to  $FIRST(A)$

NT	First
S	{ a, $\epsilon$ }
B	
C	

# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

$C \rightarrow cS \mid \epsilon$

Step 1: Not required

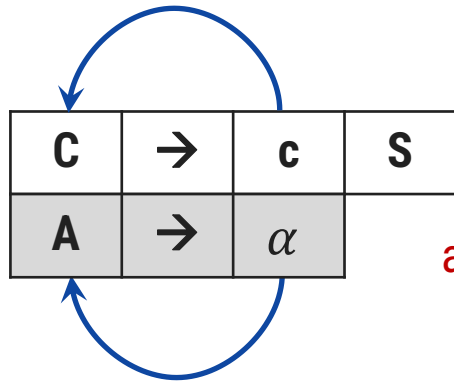
Step 2: Compute FIRST

First(C)

$C \rightarrow cS$

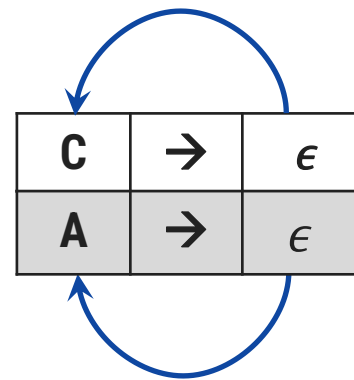
$C \rightarrow \epsilon$

NT	First
S	{ a, $\epsilon$ }
B	{ b, $\epsilon$ }
C	



Rule 1  
add  $\alpha$  to  $FIRST(A)$

$FIRST(B) = \{ c, \epsilon \}$



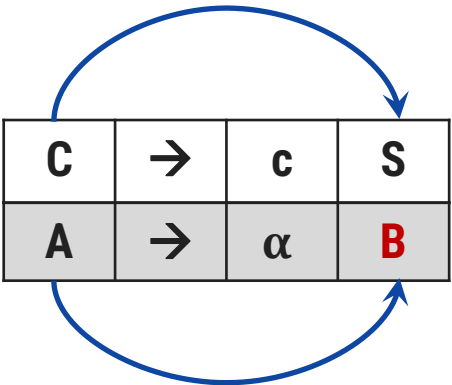
Rule 2  
add  $\epsilon$  to  $FIRST(A)$

# Example-2: LL(1) parsing

Step 2: Compute FOLLOW  
Follow(S)

Rule 1: Place \$ in FOLLOW(S)  
Follow(S)={ \$ }

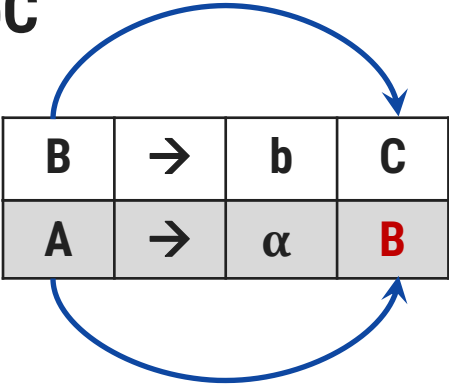
C→cS



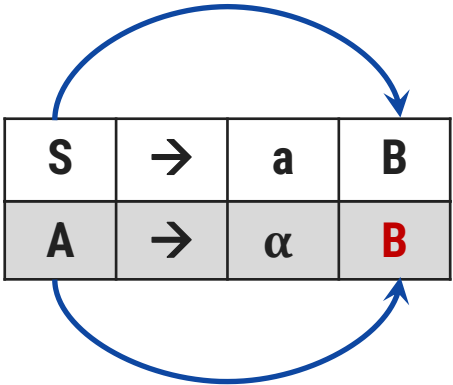
Rule 3  
Follow(A)=follow(B)  
Follow(S)=Follow(C)={\$}

S→aB | ε  
B→bC | ε  
C→cS | ε

B→bC



Rule 3  
Follow(A)=follow(B)  
Follow(C)=Follow(B)={\$}



Rule 3  
Follow(A)=follow(B)  
Follow(B)=Follow(S)={\$}

NT	First	Follow
S	{a,ε}	
B	{b,ε}	
C	{c,ε}	

# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

$C \rightarrow cS \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a, $\epsilon$ }	{ $\epsilon$ }
B	{b, $\epsilon$ }	{ $\epsilon$ }
C	{c, $\epsilon$ }	{ $\epsilon$ }

N T	Input Symbol			
	a	b	c	\$
S				
B				
C				

$S \rightarrow aB$

$a = \text{FIRST}(aB) = \{ a \}$

$M[S, a] = S \rightarrow aB$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$



# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

$C \rightarrow cS \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a}	{}
B	{b,}	{}
C	{c,}	{}

N T	Input Symbol			
	a	b	c	\$
S	$S \rightarrow aB$			
B				
C				

$S \rightarrow \epsilon$

$b = \text{FOLLOW}(S) = \{ \$ \}$

$M[S, \$] = S \rightarrow \epsilon$

Rule: 3

$A \rightarrow \alpha$

$b = \text{follow}(A)$

$M[A, b] = A \rightarrow \alpha$

# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

$C \rightarrow cS \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a}	{ \$ }
B	{b, $\epsilon$ }	{ \$ }
C	{c, $\epsilon$ }	{ \$ }

N T	Input Symbol			
	a	b	c	\$
S	$S \rightarrow aB$			$S \rightarrow \epsilon$
B				
C				

$B \rightarrow bC$

$a = \text{FIRST}(bC) = \{ b \}$

$M[B, b] = B \rightarrow bC$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$

# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

$C \rightarrow cS \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a}	{ \$ }
B	{b, $\epsilon$ }	{ \$ }
C	{c, $\epsilon$ }	{ \$ }

N T	Input Symbol			
	a	b	c	\$
S	$S \rightarrow aB$			$S \rightarrow \epsilon$
B		$B \rightarrow bC$		
C				

$B \rightarrow \epsilon$

$b = \text{FOLLOW}(B) = \{ \$ \}$

$M[B, \$] = B \rightarrow \epsilon$

Rule: 3

$A \rightarrow \alpha$

$b = \text{follow}(A)$

$M[A, b] = A \rightarrow \alpha$

# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

$C \rightarrow cS \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a}	{\$}
B	{b, $\epsilon$ }	{\$}
C	{c, $\epsilon$ }	{\$}

N T	Input Symbol			
	a	b	c	\$
S	$S \rightarrow aB$			$S \rightarrow \epsilon$
B		$B \rightarrow bC$		$B \rightarrow \epsilon$
C				

$C \rightarrow cS$

$a = \text{FIRST}(cS) = \{ c \}$

$M[C, c] = C \rightarrow cS$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$

# Example-2: LL(1) parsing

$S \rightarrow aB \mid \epsilon$

$B \rightarrow bC \mid \epsilon$

$C \rightarrow cS \mid \epsilon$

Step 3: Prepare predictive parsing table

NT	First	Follow
S	{a}	{\$}
B	{b, $\epsilon$ }	{\$}
C	{c, $\epsilon$ }	{\$}

N T	Input Symbol			
	a	b	c	\$
S	$S \rightarrow aB$			$S \rightarrow \epsilon$
B		$B \rightarrow bB$		$B \rightarrow \epsilon$
C			$C \rightarrow cS$	

$C \rightarrow \epsilon$

$b = \text{FOLLOW}(C) = \{ \$ \}$

$M[C, \$] = C \rightarrow \epsilon$

Rule: 3

$A \rightarrow \alpha$

$b = \text{follow}(A)$

$M[A, b] = A \rightarrow \alpha$

# Example-3: LL(1) parsing

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

Step 1: Remove left recursion

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

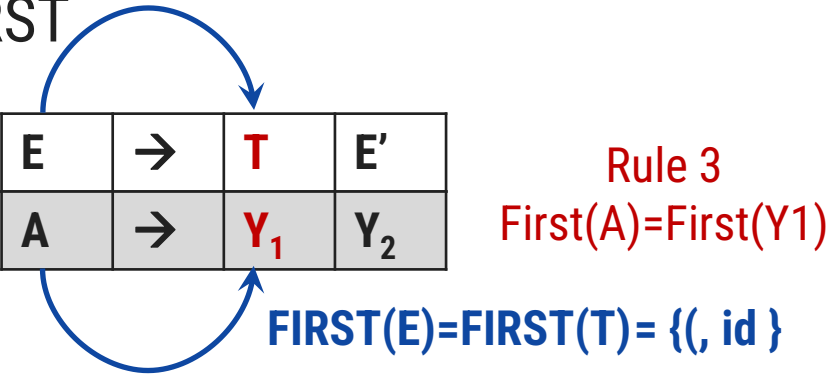
$F \rightarrow (E) \mid id$

# Example-3: LL(1) parsing

Step 2: Compute FIRST

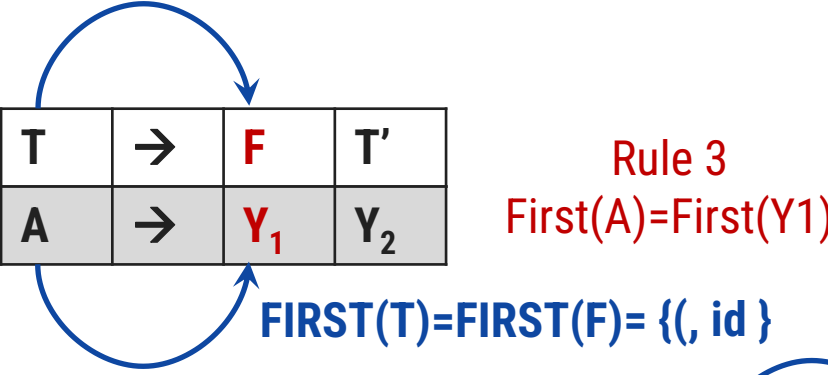
First(E)

$E \rightarrow TE'$



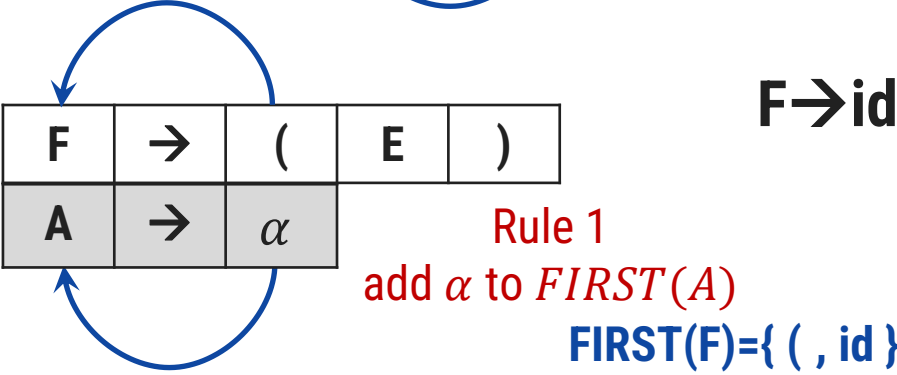
First(T)

$T \rightarrow FT'$

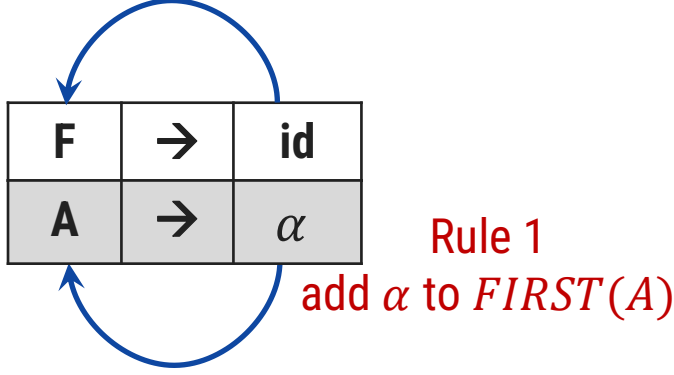


First(F)

$F \rightarrow (E)$



$F \rightarrow \text{id}$



$E \rightarrow TE'$   
 $E' \rightarrow +TE' \mid \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E) \mid \text{id}$

NT	First
E	
E'	
T	
T'	
F	

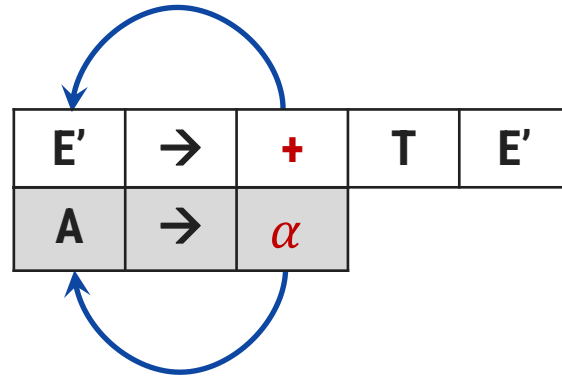
# Example-3: LL(1) parsing

Step 2: Compute FIRST

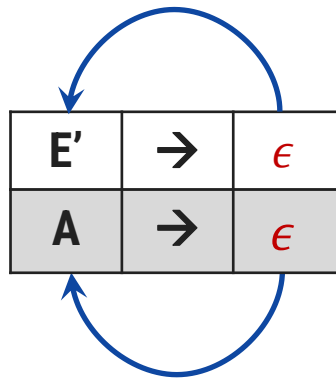
First( $E'$ )

$E' \rightarrow +TE'$

$E' \rightarrow \epsilon$



Rule 1  
add  $\alpha$  to  $FIRST(A)$



Rule 2  
add  $\epsilon$  to  $FIRST(A)$

$FIRST(E') = \{ +, \epsilon \}$

$E \rightarrow TE'$   
 $E' \rightarrow +TE' \mid \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E) \mid id$

NT	First
E	{ (,id }
E'	
T	{ (,id }
T'	
F	{ (,id }



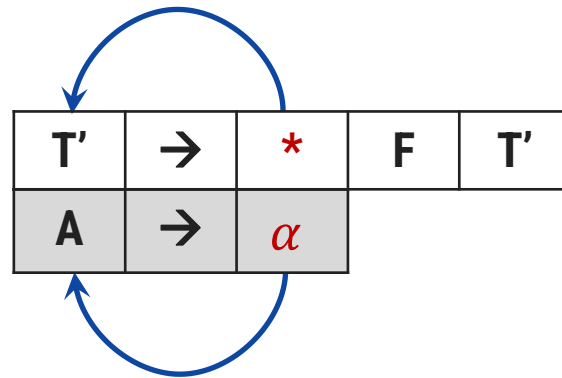
# Example-3: LL(1) parsing

Step 2: Compute FIRST

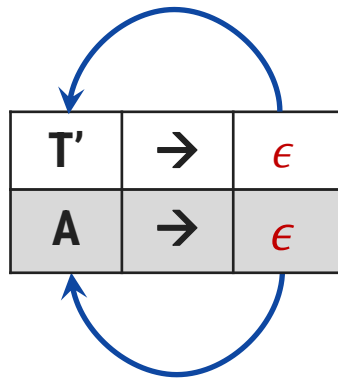
First( $T'$ )

$T' \rightarrow *FT'$

$T' \rightarrow \epsilon$



Rule 1  
add  $\alpha$  to  $FIRST(A)$



Rule 2  
add  $\epsilon$  to  $FIRST(A)$

$FIRST(T') = \{ *, \epsilon \}$

$E \rightarrow TE'$   
 $E' \rightarrow +TE' \mid \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E) \mid id$

NT	First
E	{ (, id }
E'	{ +, $\epsilon$ }
T	{ (, id }
T'	
F	{ (, id }

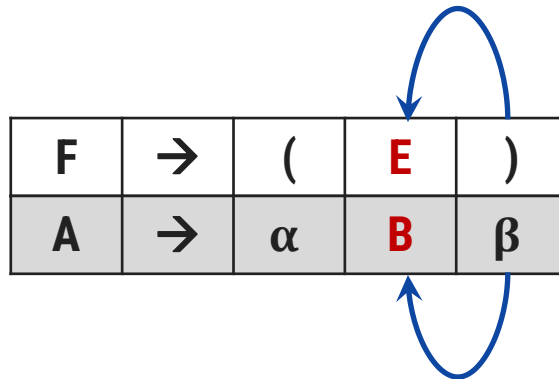
# Example-3: LL(1) parsing

Step 2: Compute FOLLOW

FOLLOW(E)

Rule 1: Place \$ in FOLLOW(E)

$F \rightarrow (E)$



Rule 2

$\text{FOLLOW}(E) = \{ \$, ) \}$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$

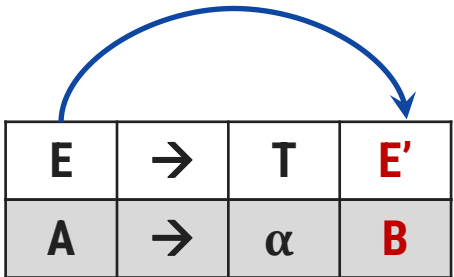
NT	First	Follow
E	{ (, id }	
E'	{ +, ε }	
T	{ (, id }	
T'	{ *, ε }	
F	{ (, id }	

# Example-3: LL(1) parsing

Step 2: Compute FOLLOW

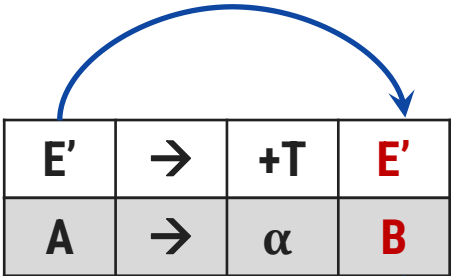
FOLLOW(E')

E → TE'



Rule 3

E' → +TE'



Rule 3

FOLLOW(E') = { \$, ) }

E → TE'  
E' → +TE' | ε  
T → FT'  
T' → \*FT' | ε  
F → (E) | id

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, ε }	
T	{ (, id }	
T'	{ *, ε }	
F	{ (, id }	

# Example-3: LL(1) parsing

Step 2: Compute FOLLOW

$\text{FOLLOW}(T)$

$E \rightarrow TE'$

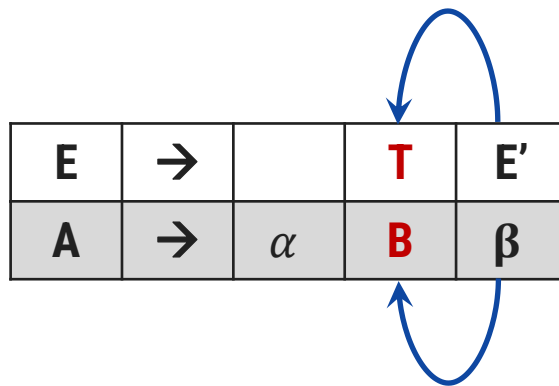
$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

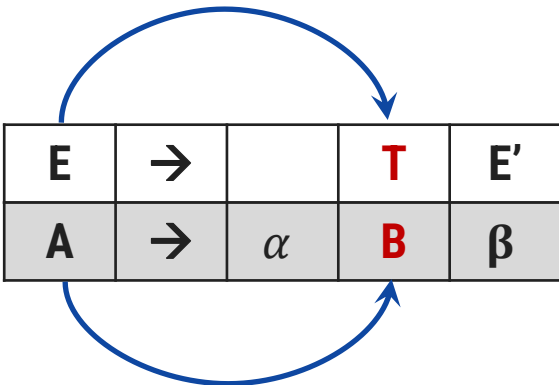
$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$



Rule 2



Rule 3

$\text{FOLLOW}(T) = \{ +, \$, ) \}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	
T'	{ *, $\epsilon$ }	
F	{ (, id }	

# Example-3: LL(1) parsing

Step 2: Compute FOLLOW

$\text{FOLLOW}(T)$

$E' \rightarrow +TE'$

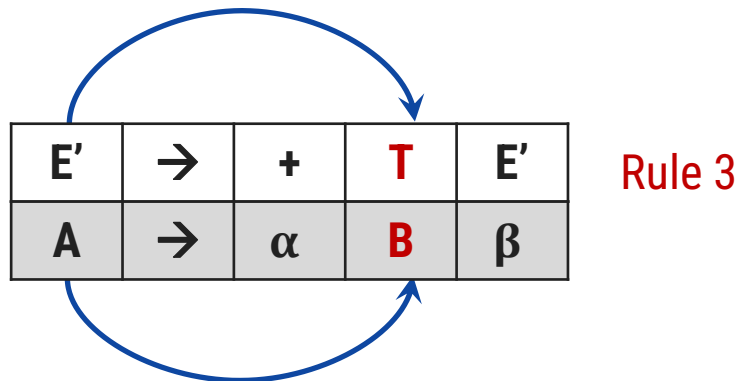
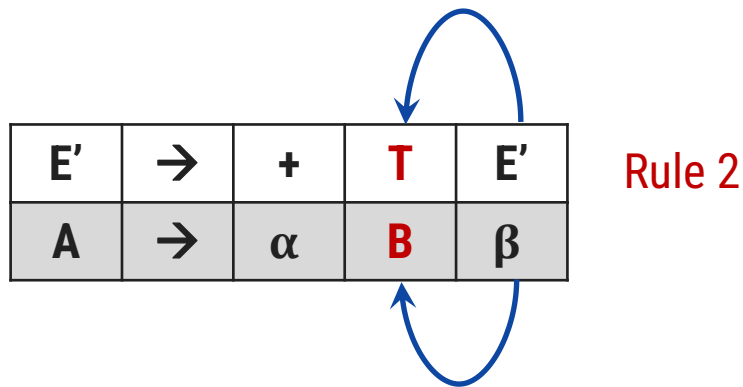
$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$



$\text{FOLLOW}(T) = \{ +, \$, ) \}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, ε }	{ \$, ) }
T	{ (, id }	
T'	{ *, ε }	
F	{ (, id }	

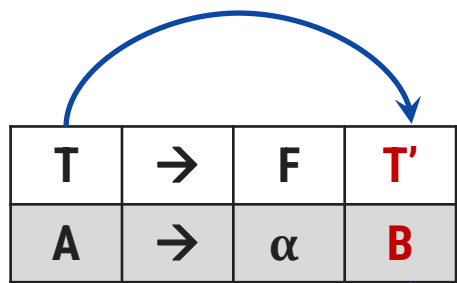
# Example-3: LL(1) parsing

Step 2: Compute FOLLOW

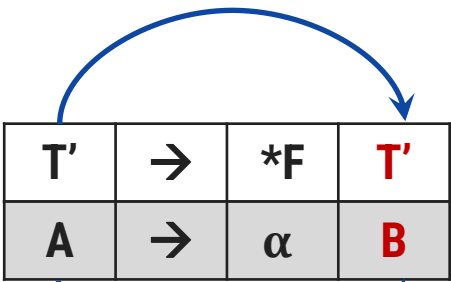
FOLLOW(T')

T → FT'

T' → \*FT'



Rule 3



Rule 3

FOLLOW(T')={+ \$,) }

E → TE'  
E' → +TE' | ε  
T → FT'  
T' → \*FT' | ε  
F → (E) | id

NT	First	Follow
E	{ (,id }	{ \$,) }
E'	{ +, ε }	{ \$,) }
T	{ (,id }	{ +,\$,) }
T'	{ *, ε }	
F	{ (,id }	

# Example-3: LL(1) parsing

Step 2: Compute FOLLOW

$\text{FOLLOW}(F)$

$T \rightarrow FT'$

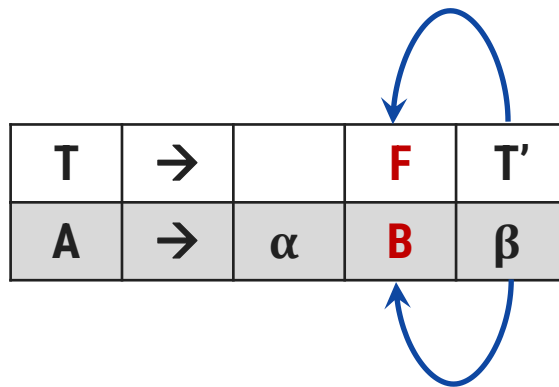
$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

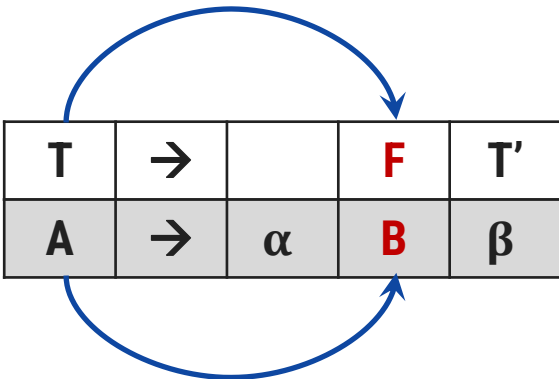
$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$



Rule 2



Rule 3

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, ε }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, ε }	{ +, \$, ) }
F	{ (, id }	

$\text{FOLLOW}(F) = \{ *, +, \$, ) \}$

# Example-3: LL(1) parsing

Step 2: Compute FOLLOW

$\text{FOLLOW}(F)$

$T' \rightarrow *FT'$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$

$T'$	$\rightarrow$	$*$	$F$	$T'$
$A$	$\rightarrow$	$\alpha$	<b>B</b>	$\beta$

Rule 2

$T'$	$\rightarrow$	$*$	$F$	$T'$
$A$	$\rightarrow$	$\alpha$	<b>B</b>	$\beta$

Rule 3

$\text{FOLLOW}(F) = \{ *, +, \$, ) \}$

NT	First	Follow
E	$\{ (, \text{id} \}$	$\{ \$, ) \}$
E'	$\{ +, \epsilon \}$	$\{ \$, ) \}$
T	$\{ (, \text{id} \}$	$\{ +, \$, ) \}$
T'	$\{ *, \epsilon \}$	$\{ +, \$, ) \}$
F	$\{ (, \text{id} \}$	



# Example-3: LL(1) parsing

Step 3: Construct predictive parsing table

NT	Input Symbol					
	id	+	*	(	)	\$
E						
E'						
T						
T'						
F						

$E \rightarrow TE'$

$a = \text{FIRST}(TE') = \{ (, \text{id} \}$

$M[E, (] = E \rightarrow TE'$

$M[E, \text{id}] = E \rightarrow TE'$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, $\epsilon$ }	{ +, \$, ) }
F	{ (, id }	{ *, +, \$, ) }

# Example-3: LL(1) parsing

Step 3: Construct predictive parsing table

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'						
T						
T'						
F						

$E' \rightarrow +TE'$

$a = \text{FIRST}(+TE') = \{ + \}$

$M[E', +] = E' \rightarrow +TE'$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, $\epsilon$ }	{ +, \$, ) }
F	{ (, id }	{ *, +, \$, ) }

# Example-3: LL(1) parsing

Step 3: Construct predictive parsing table

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$				
T						
T'						
F						

$E' \rightarrow \epsilon$

$b = \text{FOLLOW}(E') = \{ \$, ) \}$

$M[E', \$] = E' \rightarrow \epsilon$

$M[E', )] = E' \rightarrow \epsilon$

Rule: 3

$A \rightarrow \alpha$

$b = \text{follow}(A)$

$M[A, b] = A \rightarrow \alpha$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, $\epsilon$ }	{ +, \$, ) }
F	{ (, id }	{ *, +, \$, ) }

# Example-3: LL(1) parsing

Step 3: Construct predictive parsing table

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T						
T'						
F						

$T \rightarrow FT'$

$a = \text{FIRST}(FT') = \{ (, \text{id} \}$

$M[T, (] = T \rightarrow FT'$

$M[T, \text{id}] = T \rightarrow FT'$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \epsilon$

$F \rightarrow (E) \mid \text{id}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, $\epsilon$ }	{ +, \$, ) }
F	{ (, id }	{ *, +, \$, ) }

# Example-3: LL(1) parsing

Step 3: Construct predictive parsing table

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'						
F						

$T' \rightarrow *FT'$

$a = \text{FIRST}(*FT') = \{ * \}$

$M[T', *] = T' \rightarrow *FT'$

Rule: 2  
 $A \rightarrow \alpha$   
 $a = \text{first}(\alpha)$   
 $M[A, a] = A \rightarrow \alpha$

$E \rightarrow TE'$   
 $E' \rightarrow +TE' \mid \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E) \mid \text{id}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, $\epsilon$ }	{ +, \$, ) }
F	{ (, id }	{ *, +, \$, ) }

# Example-3: LL(1) parsing

Step 3: Construct predictive parsing table

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'			$T' \rightarrow *FT'$			
F						

$T' \rightarrow \epsilon$

$b = \text{FOLLOW}(T') = \{ +, \$, ) \}$

$M[T', +] = T' \rightarrow \epsilon$

$M[T', \$] = T' \rightarrow \epsilon$

$M[T', )] = T' \rightarrow \epsilon$

Rule: 3  
 $A \rightarrow \alpha$   
 $b = \text{follow}(A)$   
 $M[A, b] = A \rightarrow \alpha$

$E \rightarrow TE'$   
 $E' \rightarrow +TE' \mid \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E) \mid \text{id}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, $\epsilon$ }	{ +, \$, ) }
F	{ (, id }	{ *, +, \$, ) }

# Example-3: LL(1) parsing

## Step 3: Construct predictive parsing table

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F						

$F \rightarrow (E)$

$a = \text{FIRST}((E)) = \{ ( \}$

$M[F, (] = F \rightarrow (E)$

Rule: 2

$A \rightarrow \alpha$

$a = \text{first}(\alpha)$

$M[A, a] = A \rightarrow \alpha$

$E \rightarrow TE'$   
 $E' \rightarrow +TE' \mid \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E) \mid \text{id}$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, $\epsilon$ }	{ +, \$, ) }
F	{ (, id }	{ *, +, \$, ) }

# Example-3: LL(1) parsing

## Step 3: Construct predictive parsing table

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F				$F \rightarrow (E)$		

$E \rightarrow TE'$   
 $E' \rightarrow +TE' \mid \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E) \mid id$

NT	First	Follow
E	{ (, id }	{ \$, ) }
E'	{ +, $\epsilon$ }	{ \$, ) }
T	{ (, id }	{ +, \$, ) }
T'	{ *, $\epsilon$ }	{ +, \$, ) }
F	{ (, id }	{ *, +, \$, ) }

$F \rightarrow id$

$a = \text{FIRST}(id) = \{ id \}$

$M[F, id] = F \rightarrow id$

Rule: 2  
 $A \rightarrow \alpha$   
 $a = \text{first}(\alpha)$   
 $M[A, a] = A \rightarrow \alpha$



# Example-3: LL(1) parsing

- Step 4: Make each undefined entry of table be Error

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$	Error	Error	$E \rightarrow TE'$	Error	Error
E'	Error	$E' \rightarrow +TE'$	Error	Error	$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$	Error	Error	$T \rightarrow FT'$	Error	Error
T'	Error	$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$	Error	$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$	Error	Error	$F \rightarrow (E)$	Error	Error

# Example-3: LL(1) parsing

Step 4: Parse the string : **id + id \* id \$**

STACK	INPUT	OUTPUT
E\$	id+id*id\$	

NT	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$	Error	Error	$E \rightarrow TE'$	Error	Error
E'	Error	$E' \rightarrow +TE'$	Error	Error	$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$	Error	Error	$T \rightarrow FT'$	Error	Error
T'	Error	$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$	Error	$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$	Error	Error	$F \rightarrow (E)$	Error	Error
