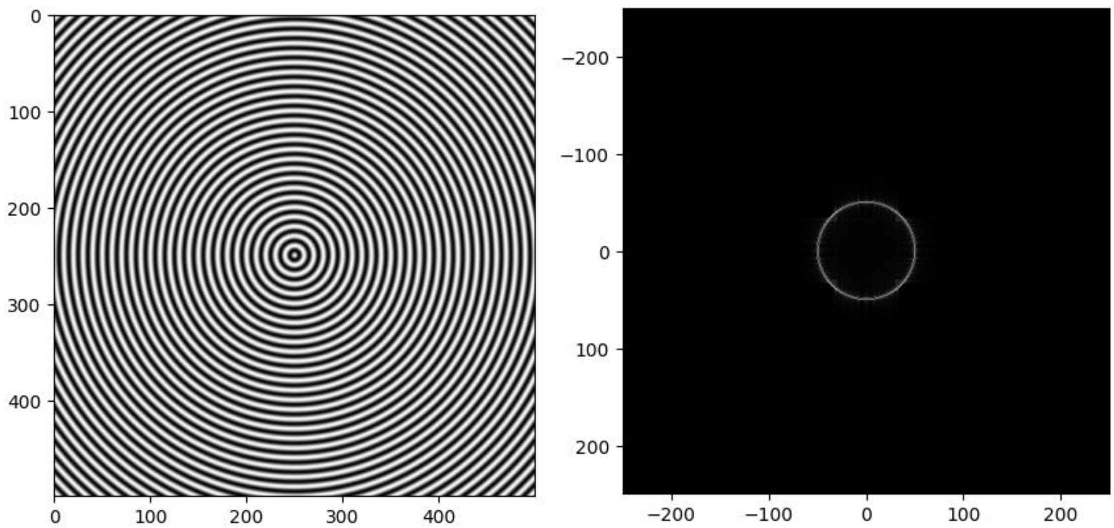
Report- Assignment 3

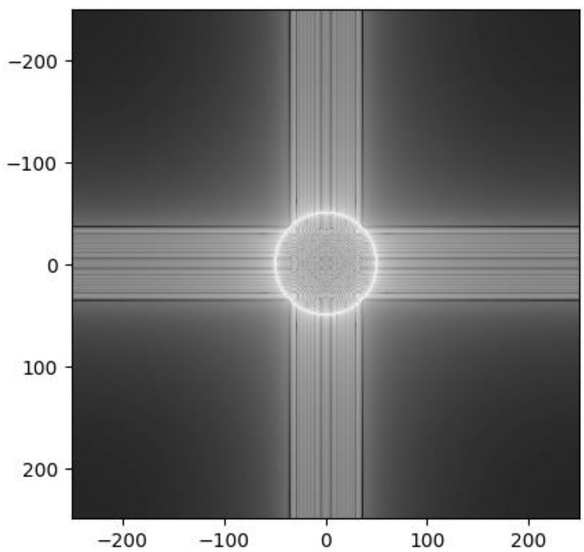
1. **Radial Sinusoid and its Frequency Response:**
   (a) Generate a radial sinusoid image of size M ×M using $x(u, v) = \cos(2\pi f0\ M \times D(u, v))$, where the distance from centre $D(u, v) = p\ (u - M/2)^2 + (v - M/2)^2$ with the side length M = 500 and frequency f0 = 50.
   (b) Compute the DFT of this image. The DFT is computed with origin at (0, 0). To visualize the DFT response, cyclically shift this to the centre (using the library function fftshift). Visualize the magnitude of DFT.
   (c) Compute IDFT of the DFT response and compare the reconstructed image with the input.

Results:

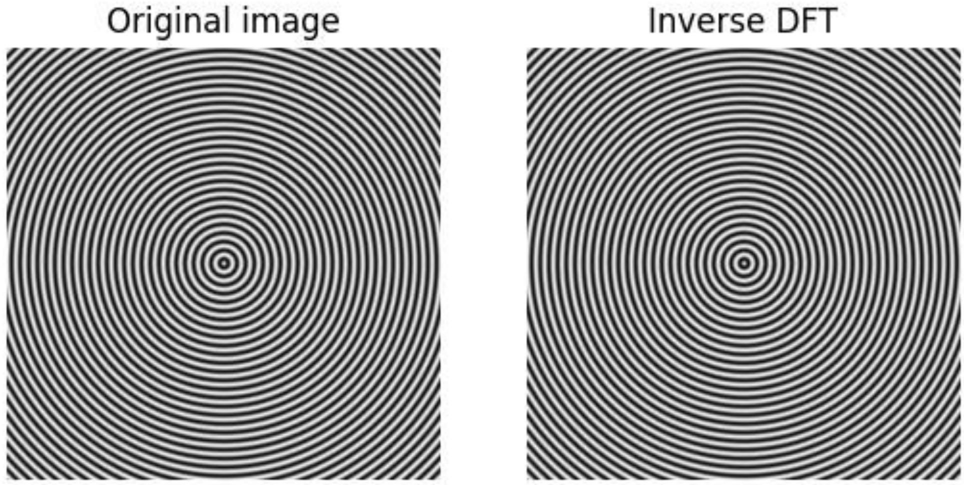
Radial Sinusoidal Image, DFT of radially sinusoidal image,



Log_10 of the DFT,



Results of Inverse DFT



Inferences:

- This question helped to understand how, DFT behaves, with the frequency of 50 Hz, it is intuitively correct that a ring appear In an image, when we know that for 1D case, two impulses appear at equal distance from center.
- Vectorization and broadcasting also helped in the question.
- Comparing the original image and the DFT, we can see that they are identical.

2. **Frequency Domain Filtering:**
   (a) Filter the image characters.tif in the frequency domain using an ideal low pass filter (ILPF). The expression for the ILPF is

$$H(u, v; D0) = 1 \quad D(u, v) \leq D0$$

$$= 0 \quad D(u, v) > D0$$

where D0 is a positive constant referred to as the cut-off frequency and D(u, v) is the distance between a point (u, v) in the frequency domain and the center of the frequency rectangle, i.e., $D(u, v) = p\ (u - P/2)2 + (v - Q/2)2$, where P and Q are the number of rows and columns in the image. What artefacts do you notice in the image obtained by computing the inverse DFT of the filtered image?

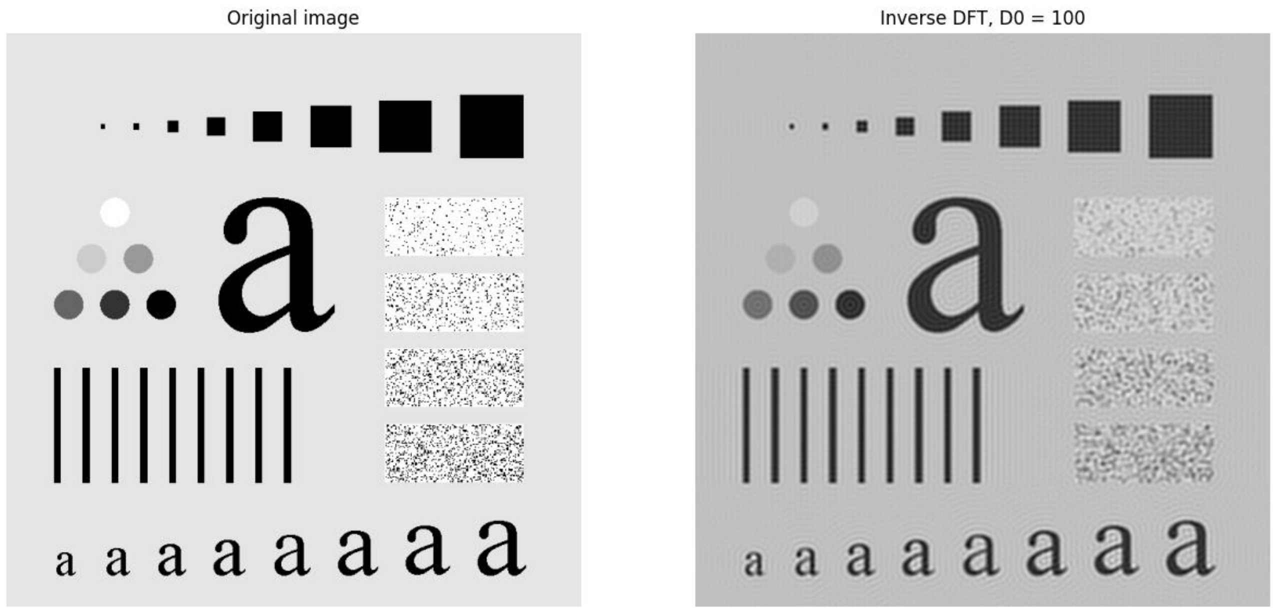
(b) Filter the image characters.tif in the frequency domain using the Gaussian low pass filter given by

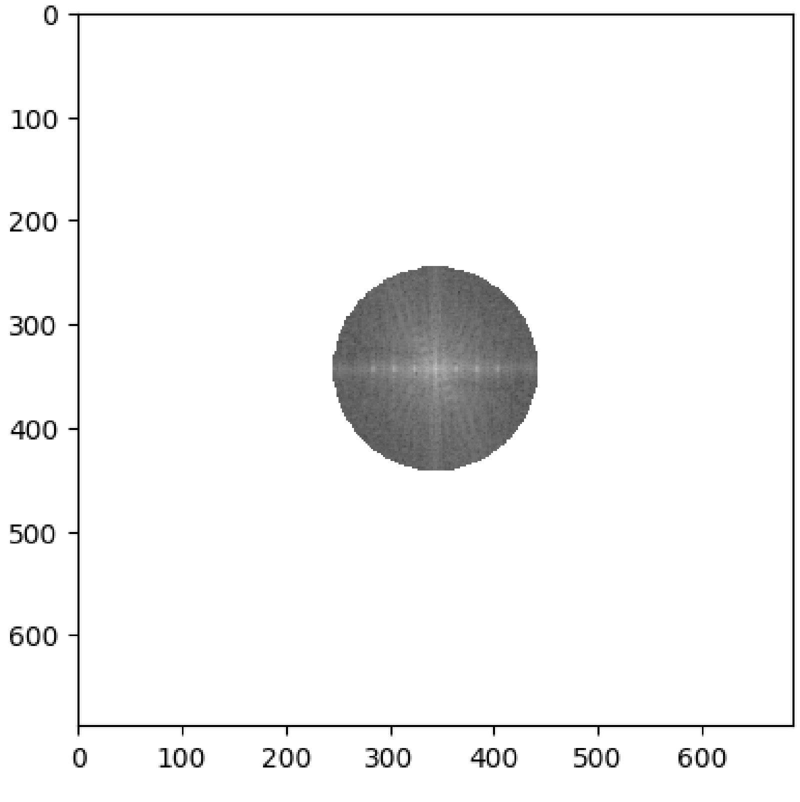
$$H(u, v; D0) = \exp(-D\ (u, v)^2/2(D0)^2\ )$$

where all the terms are as explained in part (b). For D0 = 100, compare the result with that of the ILPF.

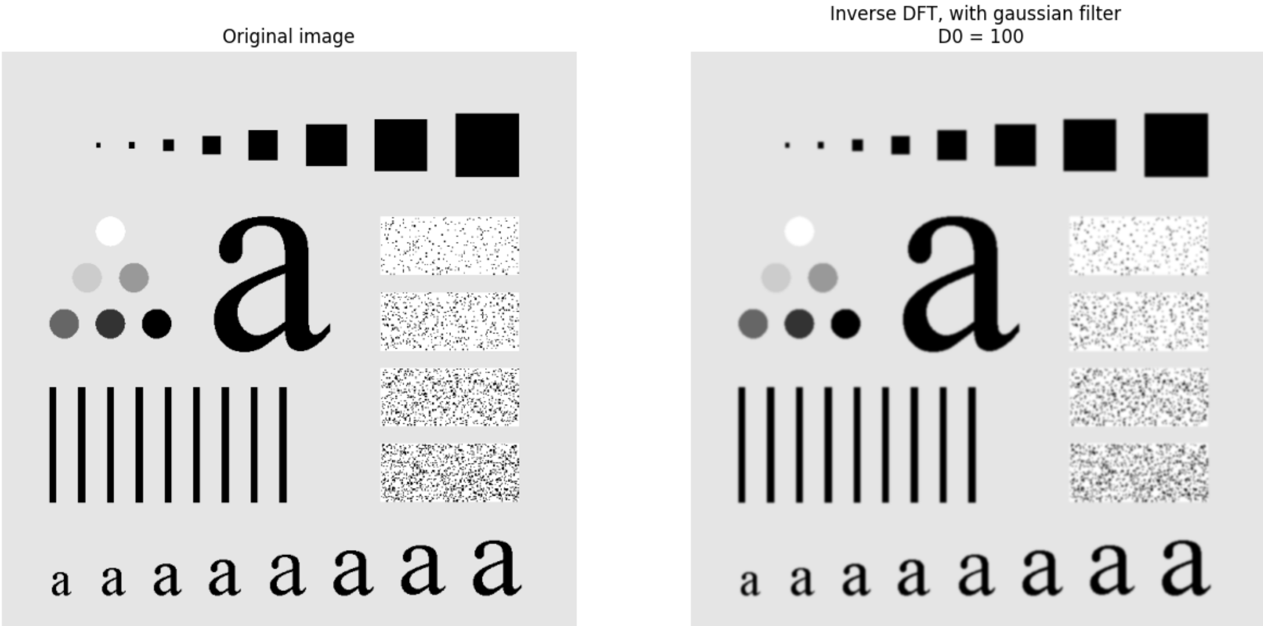
Results:

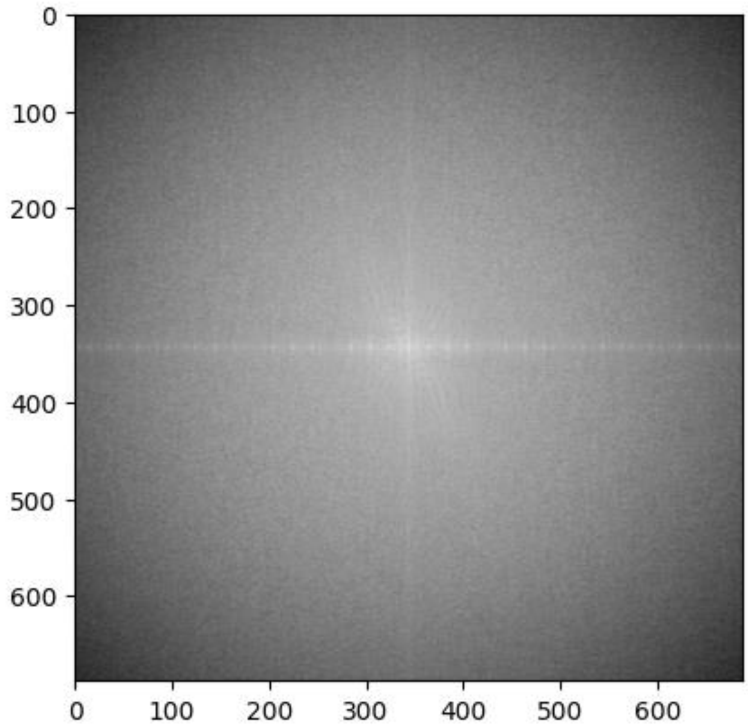
Inverse DFT of image applied with ideal LPF and D0=100





Inverse filter with Gaussian LPF and D0=100





Inferences:

- For the ideal LPF case clear ringing artifacts are visible in the image, where we can see that the edges have pulses like appearance around it.
- The inverse DFT with the Gaussian filter does not have the ringing artifacts in it as compared to the ideal LPF.
- The image from the Gaussian is sharper as compared to the Ideal filter. This is because more information is retrieved in the case off guassian filter.

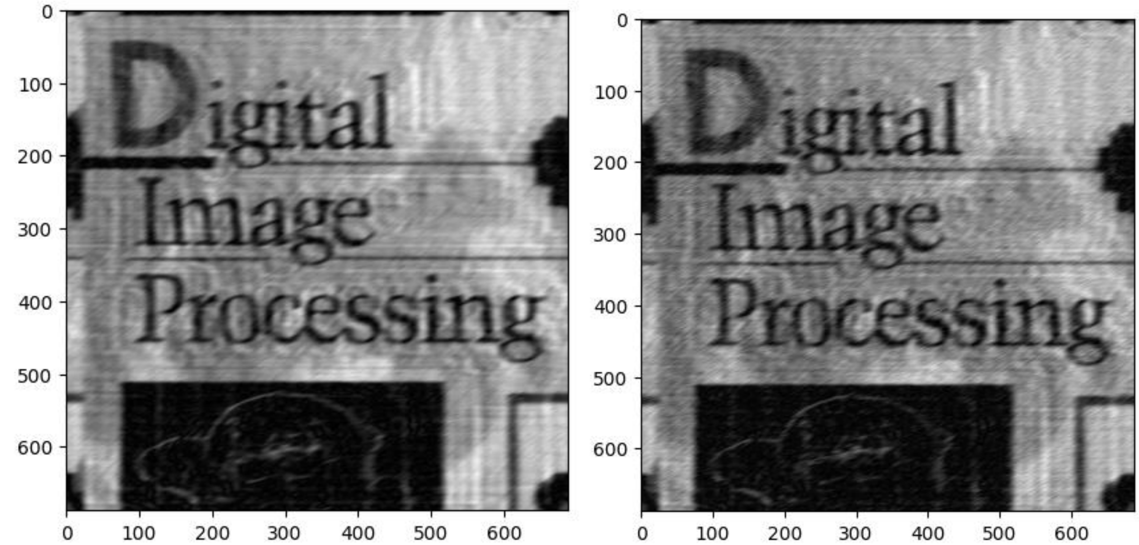
3. **Image Deblurring: Deblur the images Blurred LowNoise.png (White Noise Standard Deviation (σ)=1) and Blurred HighNoise.png (σ = 10) which have been blurred by the kernel BlurKernel.mat using**
   (a) Inverse filtering: Simple inverse filtering may lead to amplification of noise (why?). To mitigate amplification of high frequency noise, set the inverse filter fft values to 0 wherever blur filter fft values are below a threshold t. Set t = 0.1.
   (b) Wiener filter: You can assume the PSD of the white noise is equal to σ specified. For signal PSD, use power law,

   Sf (u, v) = k*root(u^2 + v^2) .

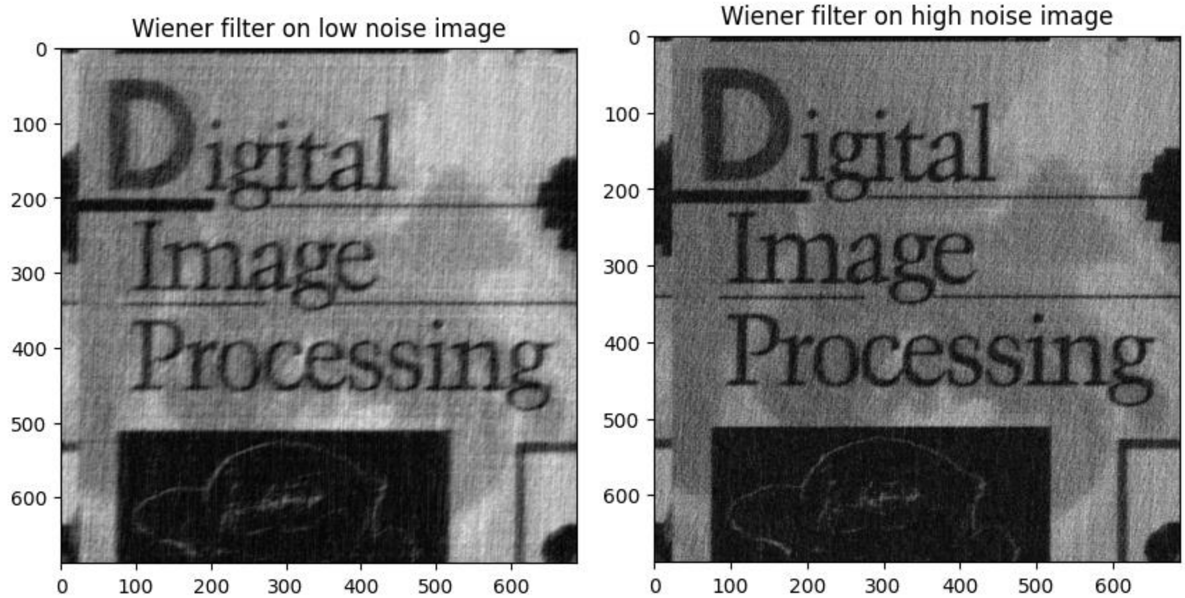
   Use k = 10^5.

Results:

Low noise image(Left) and High noise(Right) image restoration using Inverse Filter



Low noise image(Left) and High noise(Right) image restoration using wiener Filter



Inferences:

- A simple debluring filter results may result in a noisy image because when the H(u,v) filter becomes gets close to zero then the inverse filter gets becomes very large, which may result in the amplification of the noise in the image.
- The wiener filter being derived from the minimization of the error of the noise free image and the filtered image by varying the noise free image. This results in the formula

$$H_{wien}(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + SNR^{-1}}$$

$$SNR = \frac{S_f(u,v)}{S_n(u,v)}$$

- Here the Sf is referred to the power spectral density used as the prior for the image. In this case it is given in the question as the inverse distance from the origin. Sn is referred to the power spectral density of the noise, it is assumed to be constant and is given in this question equal to the variance.
- Where as in case of the inverse filter the value is made 0 in the case that H(u,v) becomes close to zero this results in loss of information, and does not account for removing noise from the image.
- The results clearly make the wiener filter the more superior.