An Improved Maze Solving Algorithm Based on An Amoeboid Organism

Ya Juan Zhang, Zi Li Zhang and Yong Deng

***Abstract*— Maze solving algorithm is used to find the shortest path between the source and target point in a given labyrinth. In this paper, an improved algorithm based on existing mathematical model inspired by an amoeboid organism, *Physarum polycephalum*, is proposed to solve maze solving problems. The positive feedback mechanism in the mathematical model is adopted in our algorithm. Meanwhile, some fuzzy rules generated from experiments are integrated to reduce convergence time and improve the performance of our algorithm. An illustrative example is given to prove the efficiency of the proposed algorithm in maze solving problems.**

***Index Terms*— Maze solving algorithm, Fuzzy rule, Physarum polycephalum**

I. INTRODUCTION

Maze solving problem is one of the classical problems in the fields of graphics, graph theory, data structure, etc. Strategic decision-making, intelligent robot path planning problems can be transformed into the problem of find the shortest path in a maze. Maze solving algorithm is used to find the shortest path between the source and target point in a given labyrinth. Usually, breadth-first search(BFS) or depth-first search(DFS) algorithm is used by most traditional maze solving algorithms. However, the computational complexity and time complexity of those search algorithms will increase exponentially as the increase in the size of the maze. With further study in intelligent algorithms, many biologically inspired algorithms have been applied to find the optimal path in labyrinths, such as particle swarm algorithm [1], genetic algorithm[2], et al.[3].

In the study of [4], it is shown that an amoeboid organism, called *Physarum polycephalum*, has the ability to track the shortest path between two selected points in a labyrinth. The plasmodium of the slime mold *Physarum polycephalum* is a large amoeba-like organism. The body of the plasmodium is made up of a network of tubes, by means of which nutrients, chemical signals and body mass are transported throughout the organism. When placed in a maze with food sources at two exits, the plasmodium will develop a network of tubes within its body which connect the food sources along the shortest path so that the rates of nutrient absorption and intracellular communication are maximized. After extracting the underlying physiological mechanism that a tube thickens as the flux through it increases, Tero et al. [5] proposed a mathematical model to solve maze problems in a manner similar to that used by the plasmodium of *Physarum*.

In this paper, based on their mathematical model, we proposed an improved algorithm by adopting some fuzzy rules to reduce the complexity of calculation and accelerate the convergence speed. The rest of this paper is organized as follows. Section II begins with a brief introduction to the basic theory used in this paper. Section III describes the improved algorithm in details. In Section IV, an illustrative example is given to show the performance of the algorithm. Finally, the conclusion is given in Section V.

{Y. J. Zhang, Z. L. Zhang and Y. Deng} are with School of Computer and Information Sciences, Southwest University, Chongqing, 400715, China {zyjuan, zhangzl, ydeng}@swu.edu.cn

Corresponding author: Y. Deng, School of Computer and Information Sciences, Southwest University, Chongqing, 400715, China, E-mail address: ydeng@swu.edu.cn

II. BASIC THEORY

A. Existing Mathematical Model

Through observation and experiments on the plasmodium of true slime mold *Physarum polycephalum* as described in [4], the physiological mechanism of tube formation has been established: tubes thicken in a given direction when shuttle streaming of the protoplasm persists in that direction for a certain time. This implies positive feedback between flux and tube thickness, as the conductance of the sol is greater in a thicker channel.

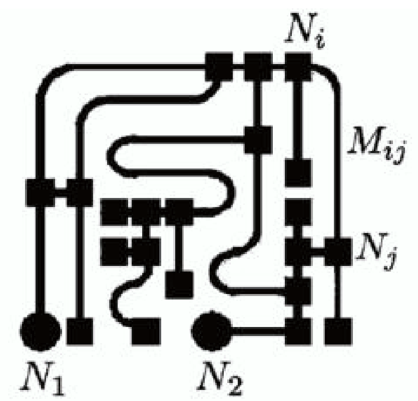

Fig. 1: Graphical maze: the source node N1 and the sink node N2 are indicated by solid circles and other nodes are shown by solid squares

According to the mechanism, two rules describing the changes in the tubular structure of the plasmodium are: first, open-ended tubes, which are not connected between the two food sources, are likely to disappear; and second, when two or more tubes connect the same two food sources, the longer tube is likely to disappear[6]. With these two rules, a mathematical model for maze solving problems has been constructed.

Using the maze illustrated in Fig. 1, the model can be described as follows. Each segment in the diagram represents a section of tube. Two special nodes, which are also called food source nodes, are named N_1 and N_2, and the other nodes are denoted as N_3, N_4, N_5, and so on. The section of tube between N_i and N_j is denoted as M_{ij}. If several tubes connect the same pair of nodes, intermediate nodes will be placed in the center of the tubes to guarantee the uniqueness of the connecting segments.

The variable Q_{ij} is used to expressed the flux through tube M_{ij} from N_i to N_j. Assuming the flow along the tube as an approximately Poiseuille flow, the flux Q_{ij} can be expressed as:

$$Q_{ij} = \frac{D_{ij}}{L_{ij}} (p_i - p_j) \tag{1}$$

where p_i is a pressure at the node N_i, L_{ij} is a length of the tube M_{ij} and D_{ij} is the conductivity of the tube. By considering Kirchhoff's law at each node, we have

$$\sum_i Q_{ij} = 0 \quad (j \neq 1, 2) \tag{2}$$

For the source node N_1 and the sink node N_2, the following two equations hold:

$$\sum_i Q_{i1} + I_0 = 0, \quad \sum_i Q_{i2} - I_0 = 0 \tag{3}$$

where I_0 is the flux from the source node which is set as a constant in the model. In order to describe the adaptation of tubular thickness, it is assumed that the conductivity D_{ij} changes in time according to the flux Q_{ij}. The adaptation equation for conductivity is expressed as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t} D_{ij} = f\left(|Q_{ij}|\right) - r D_{ij} \tag{4}$$

where $f(Q)$ is a increasing function with $f(0) = 0$. This equation implies that conductivity tends to decline exponentially, but is enhanced by the flux through the tube. It should be noted that the edge length L_{ij} is constant throughout the adaptation process.

The network Poisson equation for the pressure is derived from Eqs. (1)-(3) as follows:

$$\sum_i \frac{D_{ij}}{L_{ij}} (p_i - p_j) = \begin{cases} -1 & \text{for } j = 1, \\ +1 & \text{for } j = 2, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

By setting $p_2 = 0$ as a basic pressure level, all p_i's can be determined by solving equation Eqs. 5, and each $Q_{ij} = D_{ij}/L_{ij}\,(p_i - p_j)$ is also obtained.

It should be noted that the variable D_{ij} evolves according to the adaptation equation Eqs. 4 while variables such as p_i and Q_{ij} are determined by solving the network Poisson Eqs. 5 defined by the D_{ij}'s (and L_{ij}'s) *at each instant*. As conductivity is closely related to tube thickness, the disappearance of tubes is expressed by zero conductivity of the corresponding tube.

B. Fuzzy Logic

Since proposed in 1965 by Zadeh [7], fuzzy set theory has been very successful in solving problems dealing with vague expert knowledge, uncertainty, or imprecise/insufficient data[8], [9], [10], [11]. Fuzzy logic is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than accurate.

Fuzzy set theory defines fuzzy operators on fuzzy sets. The problem in applying this is that the appropriate fuzzy operator may not be known. For this reason, fuzzy logic usually uses IF-THEN rules of the form "*If* antecedent proposition *Then* consequence proposition". The antecedent proposition is always a fuzzy proposition of the type "*x* is *A*" where *x* is a linguistic variable and *A* is a linguistic constant term. The proposition's truth-value (a real number between 0 and 1) depends on degree of similarity between *x* and *A*.

III. MAZE SOLVING ALGORITHM

A. Extraction of Fuzzy Rules

With simulations of maze solving using the mathematical model described in II-A, the model can automatically eliminates some tubes by reducing their conductivities and reinforces other tubes, to arrive at a solution of the maze. However, it usually takes a large number of iterations for the model to arrive at a final solution.

After observing the iteration process of the model solving a maze, we find some phenomena:

First, at the beginning of the iteration, the changes of the conductivities of each tube is ruleless;

Second, after a certain number of iterations, the open-ended tubes, also called dead ends, which are not connected between two food sources, always vanish first;

Third, the changes of the conductivities in some other tubes maintain stable trend, that is, some conductivities keep increasing as the tubes thicken, while some keep decreasing as the tubes thinner until the conductivities converge to 0 and the tubes tend to disappear.

Our experiments reveal that it is the process of conductivity reduction that results in the large number of iteration. Therefore, some empirical rules can be adopted from those phenomena to reduce the complexity of calculation and accelerate the convergence speed.

Rule 1: if the tube is open-ended, then the tube will be cut.

Rule 2: if the change of the conductivity of the tube maintains a stable trend and keep reducing, then the tube will be cut.

Here, "stable trend" as a linguistic constant term, has been defined as that the conductivity has continuously increased or reduced for several iterations and will keep the change tendency until the final solution is derived.

B. Formulation of Algorithm

With these two fuzzy rules, the mathematical model can be improved by integrating the rules. Our improved algorithm for finding the shortest path in a maze is expressed in the following steps.

//N: the total number of nodes in the maze;

//L: a matrix of length, in which L_{ij} is the length of the tube connecting node N_i and N_j;

//Q: a matrix to record current moment flux in each tube, in which Q_{ij} is the flux through the tube from node N_i to N_j;

//Q_pre: a matrix to record previous moment flux in each tube;

//Q_cha: a matrix to record the change of flux in each tube;

//D: a matrix to record conductivities, in which D_{ij} is the conductivity of tube L_{ij};

//P: a matrix to record pressure at each node; $P_i = 0$ if N_i is the exit node.

1) Initialize the matrix L and the number of nodes N, according to the given maze.
2) While (if there is a node connected only by one tube) remove the node and the tube and update the matrix L and N;
3) Remove the node connected only by two tubes, combine the two tube into one tube whose length is the sum of the two tubes' length and update L and N;
4) Set iterate number $count = 0$, the flux in each tube $Q_{ij} = 0, (\forall i, j = 1, 2, \ldots, N)$ and $Q\_pre_{ij} = Q_{ij}$, the flux change in each tube $Q\_cha_{ij} = 0, (\forall i, j = 1, 2, \ldots, N)$, the conductivities of each tube in the maze $D_{ji} = D_{ij} = C_{ij}$(where C_{ij} ranges from 0 to 1), and the pressure of each node $P_i = 0$;
5) Set $count = count + 1$ and calculate the pressure P at each node according to Eqs. 5;
6) Calculate the flux Q in each tube according to Eqs. 1;
7) If count $\leq C_1$, go to Step 10; else go to Step 8;
8) Update the flux change Q_cha_{ij} in each tube using the following equation:

$$Q\_cha_{ij} = \begin{cases} Q\_cha_{ij} + 1, & |Q_{ij}| - |Q\_pre_{ij}| \geq 0 \\ & and\ Q_{ij} \geq 0 \\ 1, & |Q_{ij}| - |Q\_pre_{ij}| \geq 0 \\ & and\ Q_{ij} < 0 \\ Q\_cha_{ij} - 1, & |Q_{ij}| - |Q\_pre_{ij}| < 0 \\ & and\ Q_{ij} \leq 0 \\ -1, & |Q_{ij}| - |Q\_pre_{ij}| < 0 \\ & and\ Q_{ij} > 0 \end{cases}$$

9) If the flux change $Q\_cha_{ij} \leq C_2$(C_2 is negative), then set $Q_{ij} = 0$;
10) Calculate the conductivity D of each tube according to Eqs. 4;
11) Repeat Step 5 until the termination condition of the iteration is satisfied.

IV. ILLUSTRATIVE EXAMPLE

To verify the theoretical result and the effectiveness of our improved algorithm compared to the existing mathematical model, experiment has been carried out.

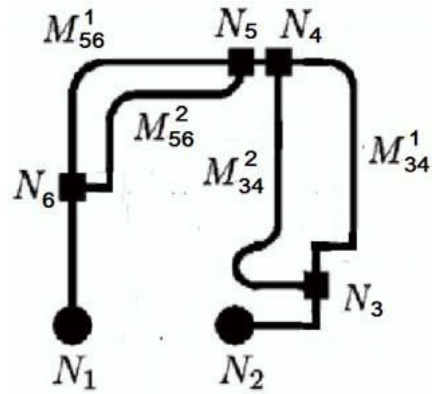

Fig. 2: Intermediate state of the maze

Our experiment is based on the maze with 23 nodes and 24 tubes, illustrated in Fig. 1. At first, Step 1~3 will be implemented to reduce the complexity of the calculation. Then an intermediate state will be derived which is similar to the intermediate state of plasmodial maze-solving process, as depicted in Fig. 2. With this simplified maze, some initial values shall be set to relevant nodes and tubes as described in Step 4. Without loss of generality, the conductivity of each tube is supposed to be equal, which is set $C_{ij} = 0.5$. After that, the pressure at each node can be calculated in Step 5. In our example, Eqs. 5 can be expressed as follows:

With the pressure at each node, the flux in each tube will be derived by Step 6. In Step 7, constant C_1, which is set $C_1 = 6$, is used to avoid the ruleless changes in flux and conductivity at the beginning of the iteration. If iterate number $count$ is lager than C_1, then rule 2 described in Section III can be applied in Step 9. If the flux in the tube keeps reducing for more than $|C_2|$ times, the tube is considered as the flux in it will keep reducing until it arrives at 0, when the tube will vanish. In this situation, the flux in the tube will directly be set 0. With the flux calculated, the conductivity can be derived in Step 10, where we use Eqn. 6 instead of Eqn.4, adopting the functional form $f(Q) = |Q|$.

$$\frac{D_{ij}^{n+1} - D_{ij}^{n}}{\delta t} = |Q| - D_{ij}^{n+1} \tag{6}$$

Up to now, one iteration is finished. The next is to test whether the termination condition is satisfied or not, that is, if the flux in all tubes remain unchanged, the final solution has already been found out. Otherwise, Step 5 will be repeat to continue the iteration.

Assume the length of each tube in Fig. 2 are given in TABLE I. The final solution derived from our improved algorithm is the shortest path connecting N_1 and N_2 in the given maze, as depicted in Fig. 3. We also compare the performance between our algorithm and the mathematical

TABLE I: Length of Tubes

M_{16}	M_{56}^1	M_{56}^2	M_{54}	M_{34}^1	M_{34}^2	M_{32}
7.0	16.0	15.0	2.0	19.0	18.0	5.0

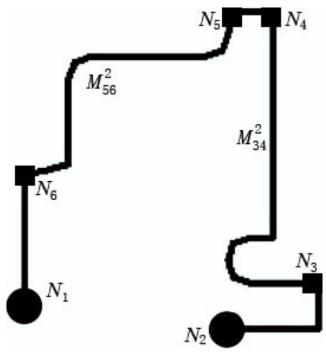

Fig. 3: Final Solution of the Maze

model. The comparison result is shown in TABLE II. Solving the same maze, both our algorithm and the model can find the shortest path between two selected points. However, it is only 11 times that our algorithm iterates to derive the right solution, while the model uses 295 times. And the time we spends is much shorter than the model spends. Therefore, our improved algorithm acts better in maze solving with higher performance.

V. CONCLUSIONS

In this paper, we have addressed the problem of finding a path through a maze of a given size. An improved algorithm based on existing mathematical model inspired by an amoeboid organism is proposed to find the shortest path between two selected points in a maze. The positive feedback mechanism in the mathematical model that greater conductivity results in greater flux and this in turn increases conductivity is adopted in our algorithm. Further, some fuzzy rules generated from experiments is applied to improve the performance by reducing the complexity of calculation and accelerating the convergence speed. Simulation result shows the efficiency of our algorithm with less iterative times and shorter time compared to the existing model.

VI. ACKNOWLEDGMENTS

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TABLE II: Performance Comparison

Method	Iterate Number	Time(second)
Existing Mathematical Model	295	0.0150000
Our Improved Algorithm	11	0.0000001

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