1 Week 2 - Gradient Descent

We want to find θ_0, θ_1 that will minimise our cost function

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

We will use gradient descent to do that. Here is the idea: Imagine that you are in a hilly landscape blindfolded, and you want to get to the lowest elevation around. What would you do? You'd probably tap around your immediate area, and then move to the lowest elevation that you found. Then you'd repeat until you tapped around your immediate area and found that the ground around you to all be higher or on the same level. That is what gradient descent does.

Our plan:

- Pick some θ_0, θ_1 (Perhaps $\theta_0 = 1, \theta_1 = 1$, but it doesn't really matter).
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$, until we have found a minimum

More formally:

Data: A set of points to which we are fitting a line
$(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)})$
Result: θ_0, θ_1, parameters for the line that fits the input data
set some initial values for θ_0, θ_1
while *we have not reached convergence* **do**

$$\theta_0 := \theta_0 - \alpha \frac{\delta}{\delta \theta_0} J(\theta_0, \theta_1)$$

$$\theta_1 := \theta_1 - \alpha \frac{\delta}{\delta \theta_1} J(\theta_0, \theta_1)$$

end
return θ_0, θ_1

Algorithm 1: algorithm for **gradient descent**

We will update θ_0, θ_1 simultaneously.

α is the learning rate, and it represents the size of the step you take in each iteration. More on that later. $\frac{\delta}{\delta \theta_0} J(\theta_0, \theta_1)$ and $\frac{\delta}{\delta \theta_1} J(\theta_0, \theta_1)$ are partial

derivatives for the cost function, and they represent the direction we need to go if we want $J(\theta_0, \theta_1)$ to decrease in the next iteration. Here is the algorithm with the partial derivatives for our cost function subbed in:

Data: A set of points to which we are fitting a line
$(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)})$
Result: θ_0, θ_1, parameters for the line that fits the input data
set some initial values for θ_0, θ_1
while *we have not reached convergence* **do**

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) \times x^{(i)}$$

end
return θ_0, θ_1

Algorithm 2: algorithm for **gradient descent**

1.1 Gradient Descent - how do we detect convergence?

The number of iterations we need will vary, but note that:

- For sufficiently small α, $J(\theta)$ should decrease on every iteration
- But if α is too small, gradient descent is *slow*. It can also encourage getting stuck in a local minima.

The best way to know when we can stop iterating is to keep track of $J(\theta)$ on each iteration, and then plot it like the following:

We can tell gradient descent is working if it decreases on each iteration, and at the point where the cost seems to level off is when we can stop iterating.

You can also use these plots to spot if something has gone wrong.

This usually means that our α is too big. We are probably overshooting the minimum. To go back to the analogy of getting to the lowest elevation, imagine that your steps are so large that it's like your landscape is actually a puddle, and you just keep stepping over it.

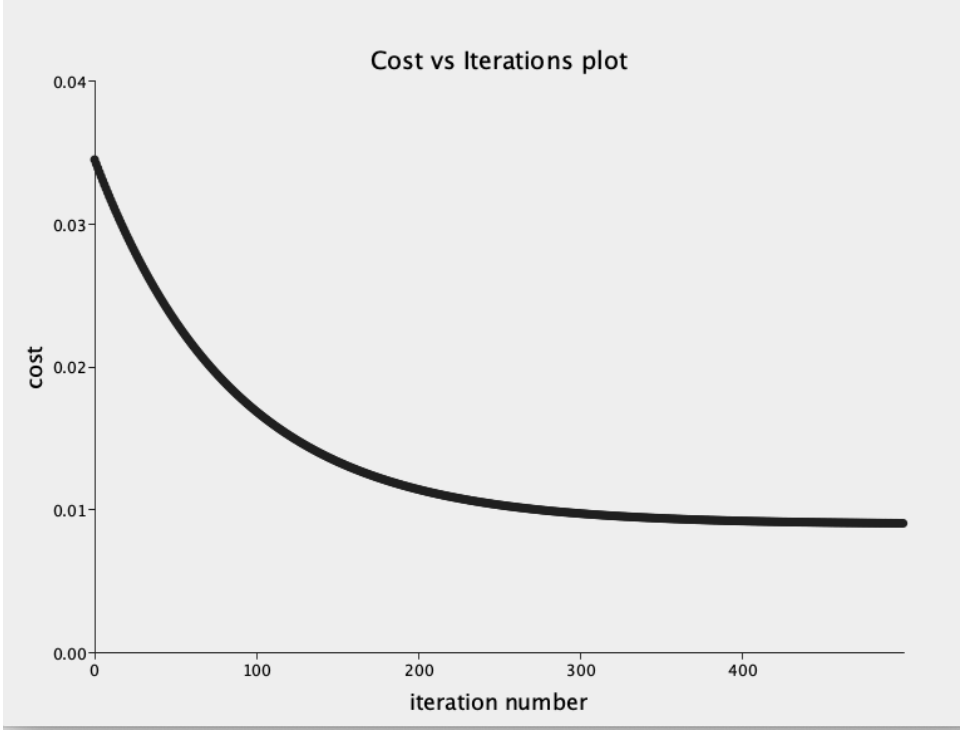


Figure 1: A cost vs iterations plot where gradient descent has converged.

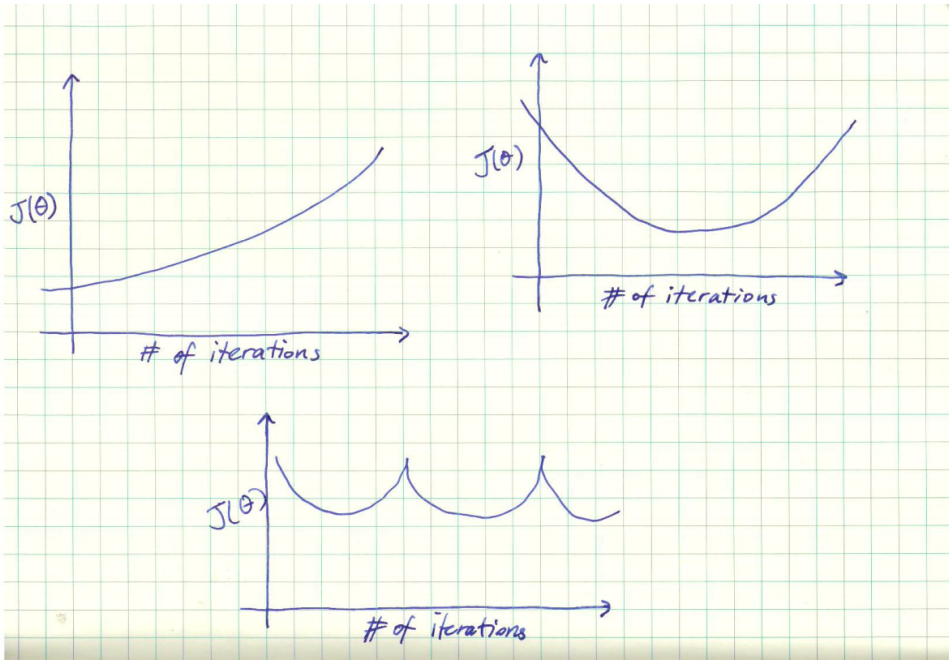


Figure 2: Some cost vs iterations plots where gradient descent has diverged.

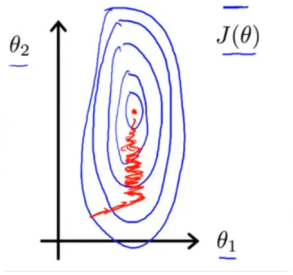
1.1.1 Feature scaling

Imagine we are building a model where we and we have a feature x_1 which corresponds to age, and another x_2 which is annual income in pounds. In our dataset, the range of x_1 might be 18-65, and x_2 could be £10K-£500K. These features have very different ranges! If we tried to run gradient descent with this dataset as is, we'd end up trying to traverse a warped landscape. This is going to:

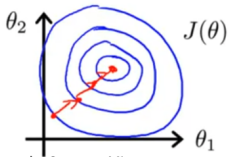
- Make our run of gradient descent *slow*
- Make it harder to choose α and the number of iterations needed

We can speed up gradient descent by ensuring that our data falls into similar ranges, while maintaining the shape of the data. In our case with one feature, LotArea, LotArea and selling price seem to have a linear relationship. After scaling LotArea, it will still have a linear relationship with selling price.

One common technique is *mean normalisation*. We can apply mean normalisation to our features in the following way:



(a) Gradient descent traversal when we our features vary greatly in range



(b) Gradient descent traversal when we've scaled our features

Figure 3: Feature scaling for gradient descent. Figures from Andrew Ng's ML course on Coursera.

Each feature x_i has a range s_i which is {the max value in x_i} - {the min value in x_i}. Each feature also has a mean u_i.

Then for each point j in x_i:

$$x_i^{(j)} \leftarrow \frac{x_i^{(j)} - \mu_i^{(j)}}{s_i}$$

and then for each $x_i^{(j)}$, $-0.5 \leq x_i^{(j)} \leq 0.5$

Mean normalisation isn't the only approach to take for feature scaling, but it will work well enough for our purposes here. *Min-max scaling* and *unit vector scaling* are also common, but we won't cover them here.

We already know that feature scaling is a good idea if we need to run gradient descent. To do so, we apply a technique such as mean normalisation just to our xs. So, when we run gradient descent, it is on data with scaled features. This means that when we want to make predictions, we must apply the same technique we used to scale our features when building the model, to the features in the new data we want to make predictions for. See Figure 4.

1.2 How good is our model?

Recall from week 1 that we do not use all of our labelled data to build our model. We want to set aside a set of *test data* so that we can evaluate how

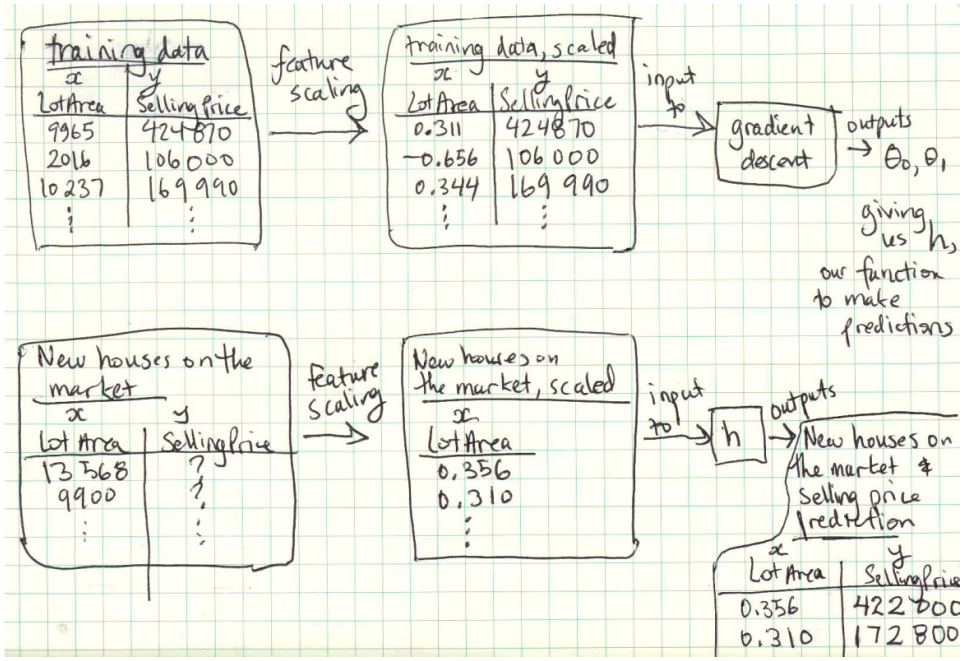


Figure 4: Feature scaling when building a model and when making predictions

accurate our model is. That way, when we make changes to our model, we'll be able to quantify if that has caused the model to perform better or not.

Recall our cost function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

This was based on Mean Squared Error which squared the error for each data point, and then summed over all of the squared errors.

Now that (for the moment) we have finished training our model, we will sum the absolute value of the error for each data point:

$$MeanAbsoluteError = \frac{1}{m} \sum_{i=1}^{m} |h_\theta(x^{(i)}) - y^{(i)}|$$

Then, later on, if we were to make some improvements to our model, we could calculate the absolute again and see which model is more accurate.