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Analysis of Optimization Model for Carbon Market of Power Plants

I. Introduction

Climate change brought about by increased greenhouse gas (GHG) emission is a serious challenge that threatens society. In attempt to curtail GHG emissions, governments have implemented various policies that incentivizes industries to reduce carbon emissions. In the case of power plants, governments have implemented a limit in which plants can emit under the pain of heavy fines. In some countries where there is an emission cap and trade system (ETS), power plants can trade their limits of carbon emissions with other power plants. Furthermore, some industries, such as beverage or oil drilling industries have commercial use for carbon dioxide, so the captured carbon from plants can be sold to these industries. Thus, in order to meet carbon emission mandates, power plants have the option to either capture and storage carbon (CCS), capture and utilize carbon (CCU), or trade carbon in the ETS. As time progresses, governments have enforced stricter carbon emission limits. In other words, the limit in which a plant can emit carbon decreases gradually over time, hence there is a growing incentive to implement CCS or CCU eventually. Furthermore, if there is a profit that can be made, it would be financially wise to do so early on the time horizon since not doing so early would result in higher cost in terms of net present value and lost opportunity cost. Semra Agrali and colleagues modeled the carbon market of a power plant and applied the model to a case study for coal power plants in Turkey using GAMS (Agrali, Uctug and Turkmen). For this project, we plan to study the structure of the carbon market and replicate the results of optimizing the carbon market of Turkey in GAMS.

II. Modeling

Agrali modeled this problem as a type of MINLP investment type problem. From the perspective of the owner of power plants, the model considers whether it would be profitable to invest in either CCS or CCU facilities and support the operating cost along the time horizon of 20 years. Utilizing any of the CCS or CCU option would incur fixed and variable costs and since the objective of this model is to minimize cost, the sensitivity of this model is dependent on the price of carbon. Although an ETS does not exist in Turkey, Agrali points out the favorable prospect of an ETS being implemented in the near future, and thus takes ETS into account to the model. The model borrows values for price of carbon based on countries where ETS exists. Agrali assumes that all transport of carbon is facilitated by a network of pipelines; therefore, the investment and operating cost of installing pipelines need to be considered in the model. The sets and parameters used in this model can be summarized by the following table.

Table 1. Set and Parameters

Symbol	Type	Description
N	Set	All nodes
i	Set	Origin node
j	Set	Destination node
T	Set	Planning years
S	Set	Source (Power plant) nodes
R	Set	Reservoir (storage) nodes
D	Set	Possible diameters for pipeline
Q_i^s	Parameter	Annual carbon production of node i ∈ S (ton/year)
Q_i^r	Parameter	Total carbon storage capacity of node i ∈ R (ton)
Q^p_{ijd}	Parameter	Carbon capacity of a pipelines between nodes i and j with a diameter of d (ton/year)
cap_{it}	Parameter	Cap value on the emissions level of node i ∈ S at year t (ton/year)
$IC(a_i)$	Parameter	Investment cost of a carbon capture unit with capacity a_i (\$)
$OC(aa_{it})$	Parameter	Operating cost of a carbon capture unit with annual capacity of a_{it} (\$)
$SC(b_i)$	Parameter	Investment cost of building storage site with total capacity of b_i (\$)
$VC(bb_{it})$	Parameter	Operating cost of storing bb_{it} amount annually (\$/ton)
F^p_{ijdt}	Parameter	Investment cost of building a pipeline with diameter d between nodes i and j in year t (\$)
V^p_{ij}	Parameter	Operating cost of transporting carbon between nodes i and j (\$/ton)
p^c_t	Parameter	Price of carbon sold to utility (\$/ton)
p^e_t	Parameter	Price of carbon in the ETS market in year t (\$/ton)
$dist_{ij}$	Parameter	Distance between nodes i and j (km)

The carbon capacity of a pipeline, Q^p_{ijd} is a function of the length, diameter, and mass flow rate of the pipe. The mass flow rate of the pipe, q_m, can be calculated by the following equation, where D, is the diameter of the pipeline, f_F is the Darcy friction factor, ρ is the density of carbon dioxide, L the length of the pipeline, and Δp is the pressure drop.

$$D^5 = \frac{32 f_F q_m^2}{\rho\, \pi (\Delta p / L)} \qquad (1)$$

The Darcy friction factor is assumed to be constant for all diameters and is calculated using the Blasius correlation. The length of the pipe between nodes i and j are given by parameter $dist_{ij}$. The density of carbon dioxide is 200 kg/m^3. The pressure drop is considered to be constant for all diameter and lengths with a value of 10 MPa.

The model assumes that the annual carbon production amount of a source node is the same as the allowed carbon emission limit, cap_{it}, at year 0. Due to progressively stricter carbon emissions reduction policies, the carbon emission limit is reduced by 3% annually. Thus, cap_{it} is equal to 97% of $cap_{i\,t-1}$.

The investment cost of building a pipeline between nodes i and j with diameter d at year t, F_{ijdt}, is given by the following formula, where ρ_{steel} is 7850 kg/m^3.

$$F_{ijdt} = \pi * \rho_{steel} * 0.008(D + 0.008) * dist_{ij} * 1000 \qquad (2)$$

Agrali assumes that the net present value of the investment cost of a pipeline does not change over time.

For the other parameters, such as the investment cost and operating costs of a carbon capture unit, Agrali references average values across the world. The decision variables used for this model can be summarized in the following table.

Table 2. Decision Variables

Symbol	Type	Description
a_i	Positive variable	Capacity of a carbon capture unit build in node i ∈ S (ton)
aa_{it}	Positive variable	Amount of carbon captured in node i ∈ S in year t (ton)
b_i	Positive variable	Total amount of carbon stored in node i ∈ R (ton)
bb_{it}	Positive variable	Amount of carbon stored in node i ∈ R in year t(ton)
c_{it}	Positive variable	Amount of carbon sold to node i ∈ L in year t (ton)
e_{it}	Free variable	Amount of carbon traded from node i ∈ S to the ETS market at year t (ton) *Positive value corresponds to carbon purchased from ETS and visa versa.
rr_{it}	Boolean variable	1 = If a reservoir is built in node i ∈ R at year t 0 = else
ss_{it}	Boolean variable	1 = If carbon capture unit is installed at node i ∈ S at time t 0 = else
x_{ijt}	Positive variable	Amount of carbon sent from node i to j in year t
y_{ijdt}	Boolean variable	1 = if pipeline connecting nodes i and j with a diameter is built at year t 0 = else
z	Free variable	Capital expenditure in terms of net present value (objective value)

The model developed by Agrali seeks to minimize the capital expenditure cost to optimize the carbon market and its objective function is shown by the following equation.

$$z = \sum_{t\in T} \frac{1}{(1+r_{interest})^t} \left[\sum_{i\in S} \left(IC(a_i) * ss_{it} + OC(aa_{it}) \right) + \sum_{i\in R} \left(SC(b_i) * rr_{it} + VC(bb_{it}) \right) + \sum_{i\in N} \sum_{j\in N_i} \left(\sum_{d\in D} F^p_{ijdt} * y_{ijdt} + V^p_{ij} * x_{ijt} \right) + \sum_{i\in S} p^e_t * e_{it} - \sum_{i\in L} p^c_t * c_{it} \right] \qquad (3)$$

As previously mentioned, the model is based on an investment type problem, so there is a boolean variable associated with the fixed investment costs and a positive variable that determines the variable operating cost. Except for the investment cost of the ETS and CCU facility, the owner of the coal power plant incurs the investment cost. Due to the fact that the time horizon is 20 years, in order to accurately represent the capital expenditure cost, the cost is adjusted accordingly using annual interest rate so that the capital expenditure cost is given in terms of net present value.

The constraints given for this model can be generalized as carbon mass balances in the ETS, the source nodes, reservoir nodes, and utility nodes, and maximum annual carbon capacity constraints. There are also constraints that limit the construction of carbon capture units, carbon storage units and, pipelines between nodes i and j, to occur only once during the time horizon of 20 years. The constraints used in this model are as follows

$$aa_{it} = \sum_{j\in N_i} x_{ijt} \;\; \forall\, i \in S, t \in T \qquad (4)$$

Equation 4 is a carbon mass balance around source nodes and equates the total amount of carbon outputted by a source node equal to the amount of carbon captured for a given time period.

$$a_i \geq aa_{it} \;\forall\; i \in S, t \in T \qquad (5)$$

Equation 5 limits the annual carbon capture amount in source nodes to be less than the total carbon production amount in source nodes.

$$bb_{it} = \sum_{t\in T} x_{jit} \;\forall\, i \in R, t \in T \qquad (6)$$

Similar to equation 4, equation 6 is mass balance around the reservoir nodes and ensures the amount of inputted carbon for a reservoir node equal to the amount of carbon stored for a given time period

$$b_i = \sum_{t\in T} bb_{it} \;\forall\, i \in R, \qquad (7)$$

Equation 7 is a carbon mass balance ensuring that the sum over the time horizon of annual carbon stored in a reservoir node equal to the total amount of carbon stored in a reservoir node.

$$c_{it} = \sum_{j \in N_i} x_{jit} \; \forall \; i \in L, t \in T \qquad (8)$$

Similar to equation 4, equation 8 is a carbon mass balance around the utilization nodes, and ensures the amount of carbon inputted equal to the amount of carbon sold for a utilization node.

$$x_{ijt} \leq \sum_{d \in D} Q_{ijd}^{p,max} \left(\sum_{t' \leq t} y_{ijdt} \right) \; \forall \; i \in N, j \in N_{i,} \; t \in T \qquad (9)$$

$$x_{ijt} \geq \sum_{d \in D} Q_{ijd}^{p,min} \left(\sum_{t' \leq t} y_{ijdt} \right) \; \forall \; i \in N, j \in N_{i,} \; t \in T \qquad (10)$$

Equations 9 and 10 are similar to that of big M constraints and limits the amount carbon transported in a given pipeline between nodes i and j with diameter d to the minimum and maximum of the pipeline capacity.

$$\sum_{j \in N_i} x_{ijt} - \sum_{j \in N_{i \in R}} x_{ijt} - aa_{it} + bb_{it} + c_{it} = 0 \qquad \forall i \in N, t \in T \qquad (11)$$

Equation 11 is an overall carbon mass balance among all nodes for a given time period.

$$aa_{it} \leq Q_i^s \sum_{t' \leq t} s_{it} \; \forall \; i \in S, t \in T \qquad (12)$$

Equation 12 ensures that the amount of carbon captured at source nodes is less than the maximum annual carbon capture capacity of the given source node.

$$b_i \leq Q_i^r \sum_{t' \leq t} r_{it} \; \forall \; j \in R, t \in T \qquad (13)$$

Similar equation 12, equation 13 ensures the total amount of carbon stored at reservoir nodes does not exceed the maximum capacity of reservoir nodes.

$$c_{it} \leq Q_i^l \; \forall \; i \in L, t \in T \qquad (14)$$

Equation 14 ensures that the amount of carbon sold to utilization nodes does not exceed the maximum utilization capacity.

$$aa_{it} + e_{it} = Q_i^S - cap_{it} \; \forall \; i \in S, t \in T \qquad (15)$$

Equation 15 is a carbon mass balance on the captured carbon at source nodes.

$$\sum_{t \in T} ss_{it} \leq 1 \; \forall i \in S \qquad (16)$$

$$\sum_{t \in T} rr_{it} \leq 1 \;\; \forall i \in R \quad (17)$$

$$\sum_{d \in D} \sum_{t \in T} y_{ijdt} \leq 1 \;\; \forall i \in N, j \in N_i \quad (18)$$

Equations 16 through 18 ensures that carbon capture units in a source node, storage units in reservoir nodes, and pipeline connecting nodes i and j with diameter d are installed only once during the time horizon.

When Agrali applies this model to the case study of Turkey there are critical assumptions that are made in order to simplify the model. As previously mentioned, an ETS does not exist yet in Turkey; thus, the structure of ETS is assumed to be already set in place. Also, the price of carbon in ETS was taken as the average price of carbon in countries where exists. Furthermore, only 2 coal plants are considered although there are numerous more coal plants in Turkey. The storage units in reservoir nodes in Lake Tuz and Mediterranean Sea are assumed to be already constructed and funded by the Government of Turkey. Without accounting for other commercial use of carbon dioxide, only 1 mode of utilization of carbon dioxide, enhanced oil recovery in oil wells, is considered and only 1 oil well location in Mosul, Iraq is considered. The variable operating cost of carbon capture units, carbon storage facilities, and pipelines are either considered as a constant or a linear function of the investment cost. Furthermore, the annual interest rate is assumed to be a constant of 5%. The location of the nodes for this case study is shown in the below map.

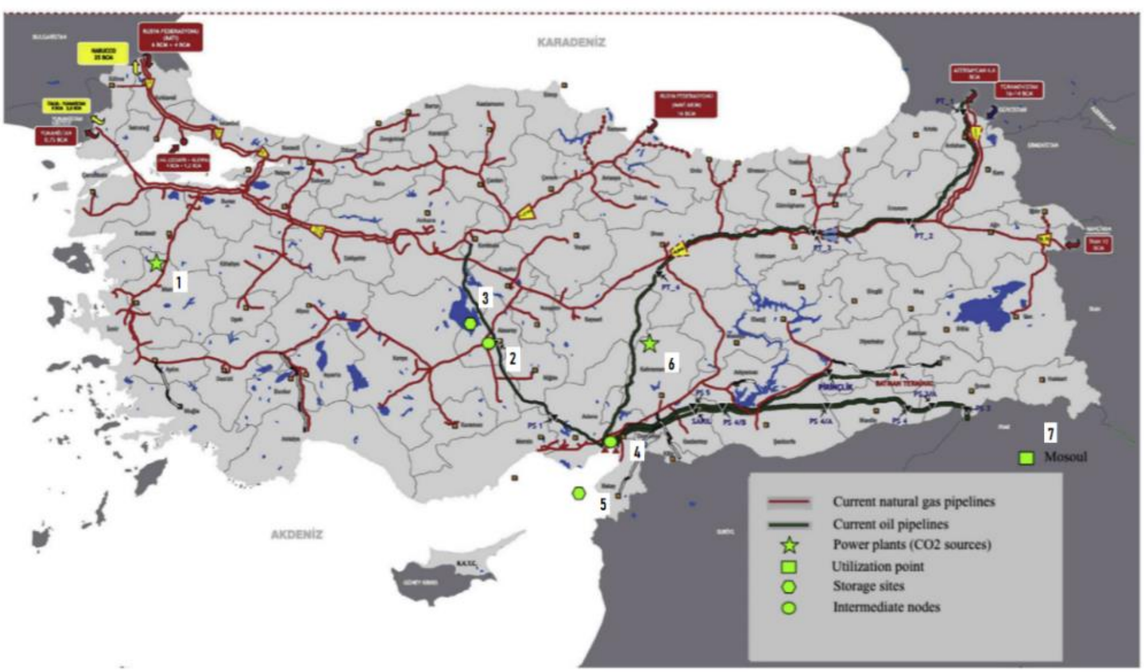

Figure 1. Location of Nodes on Map of Turkey

When attempting to replicate the results of this case study, we found that not all parameter data was given. For instances, the annual carbon production capacity of the source nodes was not given and as a result we resorted to other sources for values (Mapped: The World's Coal Power Plants). Few parameters were referenced from other journal articles and not explicitly given in this journal article by Agrali. Despite the fact data and parameters for the case study of Turkey was not presented in a clear manner, the model was straightforward, and we were able to reproduce the model in GAMS. Please refer to the appendix for the GAMS code.

III. Results and Discussion

In our attempt to reproduce the results of applying the case study of turkey to the carbon market model, the optimal value for capital expenditure was determined to be 512 million dollars. This value, however, could not be corroborated as the journal article did not provide the results. In the journal article, Agrali discusses how changes in various parameters changes the decision variables but fails to disclose the objective values of each parametric study. Our solution did not produce infeasibility errors but returned odd values for decision variables, which leads us to conclude that the solution is doubtful. We suspect that there are missing constraints not indicated in the journal article. Our optimal solution indicates that the it would be most profitable to purchase carbon emission limit from ETS and not utilize any of the CCS and CCU options. Since the objective is to minimize expenditure, it would make sense that the selling captured carbon for revenue would minimize expenditure; however, in our optimal solution no pipelines, or carbon capture units are installed. Since the model sensitivity is based on carbon prices, we hypothesize that a higher selling price for carbon for CCU and lower investment and operating cost for CCS would produce results favorable to utilizing carbon mitigating options.

The journal article by Agrali presents a holistic model of an interesting topic, the carbon market. When presenting the model, Agrali introduces all the parameters, variables, objective function and constraints in orderly and clear manner. However, when applying the case study of Turkey to the model, we often found ourselves questioning why and how Agrali used some values for parameters. The values for parameters for the case study of Turkey are not presented in a clear and concise manner and the objective values of the parametric studies are not given. As a result, we have some doubts over the results presented by Agrali.

IV. Conclusion

In order to reduce GHG emissions in power plants, governments have implemented policies mandating a limit on carbon emissions. Power plants have the option to trade carbon emissions limits with other power plants or capture carbon and then store or sell them to other industries. Agrali and colleagues modeled this carbon market as a MINLP investment type problem from the perspective of a power plant owner and sought to minimize the capital expenditure cost, while satisfying carbon emission policies. Agrali then applied the model to a case study of Turkey; however, in doing so Agrali made critical assumptions that may have skewed the model. We were able to reproduce the model in GAMS; however, we cannot corroborate the results due to the fact that the objective values are not given by Agrali. Judging from the results of decision variables, we suspect that the results are not optimal and that there are some constraints missing from the journal article.

V. References

Agrali, Semra, Fehmi Gorkem Uctug and Burcin Atilgan Turkmen. "An Optimization Model for Carbon Capture & Storage/Utilization Vs. Carbon Trading: A Case Study of Fossil-Fired Power Plants in Turkey." *Journal of Enviromental Management* (2018): 305-315.

Mapped: The World's Coal Power Plants. 30 March 2020. 12 April 2020. <https://www.carbonbrief.org/mapped-worlds-coal-power-plants>.

VI. Appendices

GAMS Code