ECE 558

Lecture 1 August 25th

Focus:

1. Centralised Stochastic Control
   - Stochastic dynamical System (!)
   - One Controller (!)
   - Controller has perfect recall (!)

2. Structural and Foundational aspects

Why?

Algorithmic ideas will be developed but explored only to some extent.

Algorithms Focus of Reinforcement Learning (RL)

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ECE 602 RL LEI YING WN 2026

Motivation: lots of applications Controls, Communications, Networks, Signal Processing, OR, Math, Finance, Economics, Statistics, Manufacturing, Robotics, AI, . . . , Quantum

Examples:

1. Scheduling in Cloud Systems/ Data Centers

Servers

Dispatcher

Jobs

Many servers millions

Service times are stochastic/ random

When to send jobs?

Server

queue

Criteria:

Minimize delay of processing.

Perspective A: Dispatcher sees full state, knows ability of servers

Based on job type, send the job to a specific server!

$\boxed{\text{MARKOV DECISION PROCESS (MDP)}}$

Perspective B: SENDING INFORMATION IS COSTLY

Dispatcher polls for information

⇒ Poll a fixed number of servers

Partial observability - imperfect information

$\boxed{\text{PARTIALLY OBSERVED MDPs (POMDPs)}}$

Poll a fixed number of servers

(i) Soft constraint - On the average meet this

(ii) Hard constraint - Never allowed to exceed this.

$\boxed{\text{CONSTRAINED MDPs + POMDPs}}$ (More advanced, Could be a Project topic)

2. Controlling the trajectory of a spacecraft, autonomous car, robot

System state known - MDP (State "feedback")

If sensors used - POMDP (Output "feedback")

3. Portfolio management

Have some money on day t - x_t

Have K instruments to choose from.

$K+1$ options - Choosing distribution π_t on $K+1$ options

Option k gets $\pi_t^k x_t$ amount of money

Y_{t+1}^k - a stochastic outcome

Function of $\pi_t^k x_t$ & Market behaviour (Stochastic)

$$x_{t+1} = \sum_{k=0}^{K} Y_{t+1}$$

Goal: T periods, π_t's each day to maximize wealth at the end of the period, but minimizing the chance of going bankrupt.

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4. Active Hypothesis Testing

Compare 2 images to find differences (small differences)

C	A
B	D

A	C
B	D

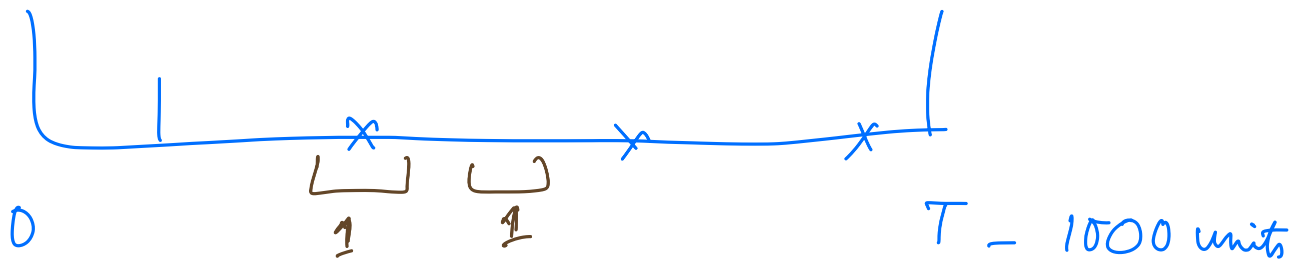
(KNOWN MODEL)

Goal: Tell apart the images (if different) & identify differences. FAST

How? Take different views at different granularities

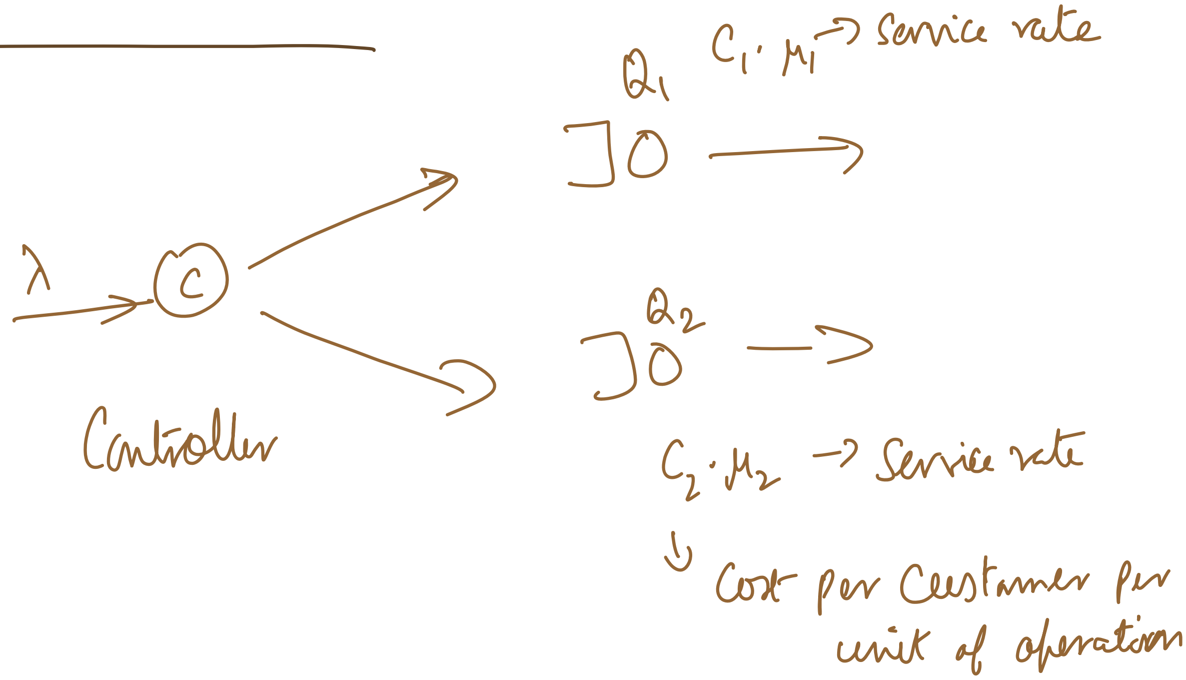
Choice {
- High resolution over a small region
- low resolution over a bigger region

Choosing next resolution & region based on past choices & results.



1 unit at a time

(1)



Network operates until time T

Occupancy of each queue at time $t \in \{1, 2, \ldots, T\} \triangleq [T]$

X_t^1 in Q_1

X_t^2 in Q_2

$$A_t = \begin{cases} \text{Route to } Q_1 & Q_1 \\ \text{Route to } Q_2 & Q_2 \end{cases} \qquad A_t \in \{Q_1, Q_2\}$$

Goal: Find a routing policy

$$g = (g_1, g_2, g_3, \dots, g_T)$$

that minimizes

$$\mathbb{E}^g \left[\sum_{t=1}^{T} \left(c_1 x_t^1 + c_2 x_t^2 \right) \right]$$

g_t — $(x_1^1, x_1^2), (x_2^1, x_2^2), \dots, (x_t^1, x_t^2)$

(Past states values including the present)

$A_1, A_2, \dots, A_{t-1}$

(Past action values)

Controller using all the past information

PERFECT RECALL

Assmpt: Queues are finite — max B

$$x_t^1 \in \{0, 1, 2, \dots, B\} \quad (B+1)$$

$g_t : H_t \to A_t$ $\qquad$ $A_t = \{Q_1, Q_2\}$

H_t takes values in a space that has size

$$((B+1)^2)^t \times 2^{t-1}$$

Number of functions $= 2^{((B+1)^2)^t \times 2^{t-1}}$

- Paring down to simpler functions to choose from while ensuring good performance.

- DP - breaks the time dependence

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② Spacecraft - Vector state

$t \in \{0, 1, \dots\}$

$$x_{t+1} = \underset{\text{State}}{x_t} + \underset{\text{Control}}{u_t} + \underset{\text{noise}}{w_t}$$

Observation $\quad y_t = C x_t + v_t$ → noise in sensors.

Define $\quad y_{0:t} = (y_0, y_1, \dots, y_t)$

$\quad\quad\quad\; u_{0:t} = (u_0, u_1, \dots, u_t)$

$u_t = g_t(y_{0:t}, u_{0:t-1})$ and at time t

"Cost" is $\quad x_t^* Q x_t + u_t^* R u_t \quad t < T$

$\quad\quad\quad\quad x_T^* Q x_T \quad$ & if $t = T$

x^* - Transpose of x (Hermitian transpose)

$$x = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \text{ then } x^* = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}, \quad A^* = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$$

Goal: Choose g to minimize

$$\mathbb{E}^g \left[\sum_{t=0}^{T-1} \left(x_t^* Q x_t + u_t^* R u_t \right) + x_T^* Q x_T \right]$$

$Q = I \qquad R = I$

$$x_t^* Q x_t = \sum_{k=1}^{K} (x_t^k)^2$$

Covered - Single controller with perfect recall.

Discrete-time, but state discrete or continuous.

Not covered (1) Continuous time problems (Technically more demanding, conceptually same ideas apply)

(2) Multi-controller problems (Conceptual leap)

(a) Team

(b) Games (c) Games of teams.