

SUPPLEMENTARY FILE OF “AN INERTIAL DERIVATIVE-FREE
ITERATIVE METHOD FOR CONSTRAINTS NONLINEAR
GENERALIZED MONOTONE EQUATIONS WITHOUT LIPSCHITZ
ASSUMPTIONS”

Aditya Sharma, V Antony Vijesh

Table 1: Results for Problem (5.1), Problem (5.2), Problem (5.3), and Problem (5.4)

$n \times 10^4$	STTCGPM (1)	FITTCGPM-PRP [1]	FITTCGPM-DY [1]	IMSMNE [2]
Problem 5.1				
	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y
1	23.2/132.8/0.154/5.91e-07	58.4/184.0/0.219/7.30e-07	57.4/181.4/0.219/7.97e-07	122.8/1212.8/1.438/9.70e-07
3	23.2/133.0/0.558/7.24e-07	59.6/188.6/0.795/8.54e-07	58.4/184.4/0.772/7.88e-07	126.0/1248.0/5.167/9.41e-07
5	23.4/135.2/0.891/6.21e-07	62.0/195.2/1.312/7.93e-07	60.2/189.8/1.271/7.56e-07	127.6/1260.8/8.282/9.52e-07
8	23.2/134.4/1.353/6.24e-07	59.8/187.2/1.896/7.54e-07	60.4/189.2/1.916/7.43e-07	128.0/1269.6/12.905/9.37e-07
10	23.2/133.8/1.690/6.90e-07	58.8/182.8/2.326/8.49e-07	59.0/183.4/2.339/8.22e-07	128.0/1270.0/16.165/9.76e-07
Problem 5.2				
	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y
1	20.0/80.0/0.024/8.38e-07	40.0/120.0/0.031/9.23e-07	40.0/120.0/0.028/9.00e-07	180.6/1790.6/0.421/9.58e-07
3	20.0/80.0/0.040/7.12e-07	41.0/124.0/0.066/9.37e-07	41.0/124.0/0.067/9.13e-07	187.0/1854.8/1.217/9.66e-07
5	20.0/80.0/0.057/8.75e-07	42.0/126.0/0.099/9.77e-07	42.0/126.0/0.094/9.52e-07	190.8/1891.2/1.386/9.61e-07
8	21.0/84.0/0.083/4.29e-07	43.0/129.0/0.143/8.27e-07	43.0/128.0/0.139/9.83e-07	193.6/1918.6/1.993/9.71e-07
10	21.0/84.0/0.097/4.70e-07	43.0/129.0/0.169/9.42e-07	43.0/129.0/0.172/9.17e-07	194.8/1931.0/2.317/9.61e-07
Problem 5.3				
	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y
1	17.0/100.0/0.039/7.07e-07	32.4/315.8/0.104/2.54e-07	27.2/252.4/0.078/8.73e-08	24.4/233.0/0.077/3.05e-07
3	17.0/101.0/0.092/8.17e-07	26.8/273.2/0.236/1.07e-07	31.2/358.2/0.305/1.27e-07	18.8/185.4/0.173/0.00e+00
5	19.0/115.0/0.173/6.96e-07	12.4/45.6/0.061/2.98e-07	11.8/43.6/0.059/0.00e+00	NaN/NaN/NaN/NaN
8	19.0/115.0/0.274/9.32e-07	108.6/2558.4/5.535/5.07e-08	35.6/450.0/0.939/7.33e-08	NaN/NaN/NaN/NaN
10	20.0/116.0/0.327/9.45e-07	110.2/1467.4/3.978/0.00e+00	70.4/847.6/2.276/1.87e-07	19.0/186.0/0.523/0.00e+00
Problem 5.4				
	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y
1	45.0/290.0/0.084/8.58e-07	102.2/522.0/0.151/6.10e-07	102.2/521.6/0.144/6.83e-07	98.4/1016.4/0.265/9.46e-07
3	36.0/238.0/0.183/9.89e-07	108.4/553.0/0.423/7.15e-07	108.4/553.0/0.417/7.05e-07	101.6/1050.0/0.814/9.21e-07
5	38.0/252.0/0.302/9.91e-07	109.4/558.0/0.659/8.37e-07	108.6/554.0/0.648/8.14e-07	102.6/1062.6/1.276/9.15e-07
8	38.0/253.0/0.484/8.61e-07	110.2/561.8/1.058/8.65e-07	110.0/561.0/1.046/9.62e-07	104.0/1079.0/2.056/9.15e-07
10	38.0/253.0/0.591/9.71e-07	111.0/565.0/1.289/4.16e-07	111.0/565.0/1.286/4.13e-07	105.2/1084.4/2.590/9.70e-07

Table 2: Results for Problem (5.5), Problem (5.6), Problem (5.7), and Problem (5.8)

$n \times 10^4$	STTCGPM (1)	FITTCGPM-PRP [1]	FITTCGPM-DY [1]	IMSMNE [2]
Problem 5.5				
	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y
1	17.0/101.0/0.035/9.51e-07	20.0/80.0/0.027/7.49e-07	20.0/80.0/0.026/7.49e-07	56.0/547.0/0.164/9.22e-07
3	18.0/107.0/0.089/7.11e-07	21.0/84.0/0.074/4.80e-07	21.0/84.0/0.073/4.81e-07	57.0/565.0/0.473/9.68e-07
5	18.0/107.0/0.143/9.87e-07	21.0/84.0/0.113/6.08e-07	21.0/84.0/0.113/6.08e-07	59.0/584.0/0.773/9.60e-07
8	19.0/112.0/0.232/8.14e-07	21.0/84.0/0.178/7.65e-07	21.0/84.0/0.179/7.65e-07	59.0/585.0/1.223/7.85e-07
10	19.0/112.0/0.289/9.37e-07	21.0/84.0/0.219/8.48e-07	21.0/84.0/0.215/8.49e-07	59.0/585.0/1.504/8.74e-07
Problem 5.6				
	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y
1	16.0/94.0/0.195/8.29e-07	375.4/1501.6/2.529/9.80e-07	527.2/2108.8/3.522/9.88e-07	50.6/497.8/0.839/9.14e-07
3	16.2/95.2/0.409/8.89e-07	398.4/1593.6/6.851/9.80e-07	562.2/2248.8/9.458/9.90e-07	50.4/497.4/2.093/8.71e-07
5	17.0/100.0/0.790/6.27e-07	406.8/1627.2/12.774/9.83e-07	574.2/2296.8/18.765/9.88e-07	NaN/NaN/NaN/NaN/NaN
8	17.0/100.0/1.110/8.49e-07	417.2/1668.8/18.838/9.83e-07	590.0/2360.0/26.759/9.90e-07	NaN/NaN/NaN/NaN/NaN
10	17.0/100.8/1.403/6.20e-07	421.4/1685.6/23.537/9.84e-07	596.0/2384.0/33.150/9.91e-07	50.4/497.8/6.926/8.90e-07
Problem 5.7				
	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y
1	8.0/39.0/0.026/2.44e-07	12.0/36.0/0.021/8.83e-07	12.0/36.0/0.020/8.73e-07	64.0/626.0/0.313/8.34e-07
3	9.0/46.0/0.060/4.23e-07	12.0/37.0/0.051/9.02e-07	12.0/37.0/0.051/8.95e-07	67.0/664.0/0.880/8.63e-07
5	10.0/52.0/0.112/2.78e-07	13.0/38.0/0.084/7.42e-07	13.0/38.0/0.084/7.35e-07	68.0/666.0/1.456/9.35e-07
8	11.0/57.0/0.187/9.83e-07	13.0/38.0/0.127/9.83e-07	13.0/38.0/0.128/9.74e-07	69.0/685.0/2.259/9.11e-07
10	11.0/58.0/0.233/2.48e-07	13.0/39.0/0.159/6.46e-07	13.0/39.0/0.165/6.40e-07	70.0/686.0/2.772/8.61e-07
Problem 5.8				
	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y	Itr/NF/Tcpu/ Y
1	3.0/15.0/0.008/0.00e+00	10.0/32.0/0.010/9.60e-07	10.0/32.0/0.008/9.86e-07	59.0/584.8/0.135/8.51e-07
3	2.0/10.0/0.010/0.00e+00	11.0/35.0/0.027/9.16e-07	11.0/35.0/0.026/9.22e-07	62.0/606.0/0.448/9.41e-07
5	3.0/16.0/0.024/0.00e+00	11.0/35.0/0.042/8.16e-07	11.0/35.0/0.041/8.28e-07	63.2/625.2/0.719/9.21e-07
8	3.0/16.0/0.037/0.00e+00	11.0/35.0/0.064/7.72e-07	11.0/35.0/0.063/7.89e-07	65.0/644.0/1.171/8.71e-07
10	3.0/16.0/0.045/0.00e+00	11.0/35.0/0.077/7.66e-07	11.0/35.0/0.077/7.86e-07	65.0/645.0/1.427/9.06e-07

Table 3: Comparison of STTCGPM, FITTCGPM-PRP, FITTCGPM-DY, and IMSMNE methods.

Init(n)	STTCGPM (1)		FITTCGPM-PRP [1]		FITTCGPM-DY [1]		IMSMNE [2]	
	Itr/NF/Tcpu/ Y		Itr/NF/Tcpu/ Y		Itr/NF/Tcpu/ Y		Itr/NF/Tcpu/ Y	
1	60.8/270.6/2.302/7.60e-07	157.6/473.8/4.159/9.65e-07	157.6/473.4/4.062/9.66e-07	157.6/473.4/4.062/9.66e-07	157.6/473.4/4.062/9.66e-07	157.6/473.4/4.062/9.66e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
2	66.0/296.6/2.464/6.89e-07	157.8/474.4/4.077/9.56e-07	157.8/474.0/3.932/9.57e-07	157.8/474.0/3.932/9.57e-07	157.8/474.0/3.932/9.57e-07	157.8/474.0/3.932/9.57e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
3	65.4/292.2/2.266/6.89e-07	157.8/473.6/3.678/9.83e-07	157.4/473.2/3.640/9.72e-07	157.4/473.2/3.640/9.72e-07	157.4/473.2/3.640/9.72e-07	157.4/473.2/3.640/9.72e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
4	64.0/285.8/2.091/7.01e-07	157.6/473.8/3.517/9.52e-07	157.6/473.0/3.486/9.68e-07	157.6/473.0/3.486/9.68e-07	157.6/473.0/3.486/9.68e-07	157.6/473.0/3.486/9.68e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
5	67.2/301.6/2.052/8.72e-07	157.6/473.4/3.203/9.77e-07	157.4/473.2/3.249/9.65e-07	157.4/473.2/3.249/9.65e-07	157.4/473.2/3.249/9.65e-07	157.4/473.2/3.249/9.65e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
6	68.8/306.6/1.645/7.85e-07	156.8/471.4/2.719/9.66e-07	156.8/471.0/2.569/9.67e-07	156.8/471.0/2.569/9.67e-07	156.8/471.0/2.569/9.67e-07	156.8/471.0/2.569/9.67e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
7	63.0/279.4/1.153/7.85e-07	158.2/475.2/1.926/9.81e-07	158.0/475.0/1.988/9.68e-07	158.0/475.0/1.988/9.68e-07	158.0/475.0/1.988/9.68e-07	158.0/475.0/1.988/9.68e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
8	62.4/279.4/0.616/6.86e-07	157.8/474.0/1.012/9.61e-07	157.4/473.2/1.009/9.66e-07	157.4/473.2/1.009/9.66e-07	157.4/473.2/1.009/9.66e-07	157.4/473.2/1.009/9.66e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
9	61.2/273.8/1.034/7.00e-07	157.4/472.8/1.801/9.85e-07	157.2/472.6/1.779/9.72e-07	157.2/472.6/1.779/9.72e-07	157.2/472.6/1.779/9.72e-07	157.2/472.6/1.779/9.72e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
10	48.6/215.8/0.013/5.37e-07	137.8/413.6/0.020/9.68e-07	137.2/412.6/0.019/9.77e-07	137.2/412.6/0.019/9.77e-07	137.2/412.6/0.019/9.77e-07	137.2/412.6/0.019/9.77e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
11	17.8/66.2/0.780/6.84e-07	164.8/495.0/6.033/9.61e-07	164.6/494.8/6.176/9.47e-07	164.6/494.8/6.176/9.47e-07	164.6/494.8/6.176/9.47e-07	164.6/494.8/6.176/9.47e-07	830.4/8295.8/70.166/9.92e-07	830.4/8295.8/70.166/9.92e-07
12	17.0/63.4/0.754/4.87e-07	164.8/495.4/5.889/9.59e-07	164.8/495.0/5.801/9.59e-07	164.8/495.0/5.801/9.59e-07	164.8/495.0/5.801/9.59e-07	164.8/495.0/5.801/9.59e-07	833.8/8324.8/68.490/9.96e-07	833.8/8324.8/68.490/9.96e-07
13	18.6/70.2/0.682/4.38e-07	164.6/494.8/4.384/9.57e-07	164.6/494.8/4.214/9.42e-07	164.6/494.8/4.214/9.42e-07	164.6/494.8/4.214/9.42e-07	164.6/494.8/4.214/9.42e-07	877.2/8760.6/70.030/9.91e-07	877.2/8760.6/70.030/9.91e-07
14	17.4/64.4/0.564/7.54e-07	164.6/494.8/4.088/9.71e-07	164.6/494.8/4.154/9.56e-07	164.6/494.8/4.154/9.56e-07	164.6/494.8/4.154/9.56e-07	164.6/494.8/4.154/9.56e-07	842.4/8416.2/62.867/9.88e-07	842.4/8416.2/62.867/9.88e-07
15	17.8/66.8/0.508/6.74e-07	164.6/494.4/3.768/9.76e-07	164.4/494.2/3.837/9.63e-07	164.4/494.2/3.837/9.63e-07	164.4/494.2/3.837/9.63e-07	164.4/494.2/3.837/9.63e-07	871.2/8704.2/61.415/9.87e-07	871.2/8704.2/61.415/9.87e-07
16	16.6/62.4/0.373/8.15e-07	164.2/493.6/3.193/9.65e-07	164.2/493.2/3.287/9.65e-07	164.2/493.2/3.287/9.65e-07	164.2/493.2/3.287/9.65e-07	164.2/493.2/3.287/9.65e-07	803.2/8020.6/43.210/9.90e-07	803.2/8020.6/43.210/9.90e-07
17	17.0/63.2/0.362/4.01e-07	165.0/495.6/2.289/9.75e-07	164.8/495.4/2.357/9.61e-07	164.8/495.4/2.357/9.61e-07	164.8/495.4/2.357/9.61e-07	164.8/495.4/2.357/9.61e-07	814.8/8136.6/32.773/9.93e-07	814.8/8136.6/32.773/9.93e-07
18	15.6/56.8/0.157/6.40e-07	164.6/494.8/1.270/9.61e-07	164.4/494.2/1.391/9.63e-07	164.4/494.2/1.391/9.63e-07	164.4/494.2/1.391/9.63e-07	164.4/494.2/1.391/9.63e-07	950.0/9491.8/19.498/9.95e-07	950.0/9491.8/19.498/9.95e-07

References

- [1] Jinbao Jian, Qiongquan Huang, Jianghua Yin, and Guodong Ma. A family of inertial three-term cgpms for large-scale nonlinear pseudo-monotone equations with convex constraints. *Numerical Linear Algebra with Applications*, 32(1):e2589, 2025.
- [2] Muhammad Abdullahi, Kejia Pan, Auwal Bala Abubakar, and Abubakar Sani Halilu. An inertial spectral conjugate gradient method for monotone nonlinear equations with applications. *Mathematical Methods in the Applied Sciences*, 48:10623–10638, 2025.