

SUPPLEMENTARY FILE OF “AN INERTIAL DERIVATIVE-FREE  
ITERATIVE METHOD FOR CONSTRAINTS NONLINEAR  
GENERALIZED MONOTONE EQUATIONS WITHOUT LIPSCHITZ  
ASSUMPTIONS”

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Table 1: Results for Problem (5.1), Problem (5.2), Problem (5.3), and Problem (5.4)

$n \times 10^4$	STTCGPM (1)	FITTCGPM-PRP [1]	FITTCGPM-DY [1]	IMSMNNE [2]
<b>Problem 5.1</b>				
	Itr/NF/Tcpu/ $\ \Upsilon\ $			
1	25.4/145.6/0.188/6.38e-07	59.2/187.8/0.259/7.34e-07	58.4/185.6/0.253/7.14e-07	121.8/1197.8/1.646/1.16e-06
3	25.8/148.8/0.580/6.89e-07	59.6/188.8/0.752/8.95e-07	59.2/188.0/0.745/6.88e-07	125.0/1230.0/4.861/1.17e-06
5	24.0/139.6/0.888/7.77e-07	62.0/195.8/1.254/9.24e-07	60.8/192.6/1.234/7.94e-07	125.8/1238.2/7.930/1.33e-06
8	24.2/140.8/1.431/7.12e-07	60.2/189.2/1.929/7.34e-07	60.4/189.8/1.962/8.75e-07	127.0/1251.0/12.760/1.24e-06
10	25.0/144.2/1.826/7.48e-07	59.8/186.8/2.355/7.51e-07	60.0/187.6/2.389/7.49e-07	127.0/1251.0/15.869/1.35e-06
<b>Problem 5.2</b>				
	Itr/NF/Tcpu/ $\ \Upsilon\ $			
1	20.8/84.2/0.019/5.14e-07	40.0/121.0/0.028/7.60e-07	40.0/121.0/0.027/7.41e-07	180.6/1791.2/0.378/9.16e-07
3	20.0/81.0/0.036/9.28e-07	41.0/124.0/0.059/9.37e-07	41.0/124.0/0.057/9.13e-07	187.0/1855.2/0.880/9.37e-07
5	21.0/85.0/0.050/4.63e-07	42.0/127.0/0.086/8.05e-07	42.0/127.0/0.081/7.84e-07	190.8/1891.4/1.084/9.46e-07
8	21.0/85.0/0.066/5.59e-07	43.0/130.0/0.114/6.81e-07	43.0/128.0/0.109/9.83e-07	193.6/1918.6/1.495/9.71e-07
10	21.0/85.0/0.077/6.13e-07	43.0/130.0/0.127/7.76e-07	43.0/130.0/0.135/7.55e-07	194.8/1931.0/1.711/9.61e-07
<b>Problem 5.3</b>				
	Itr/NF/Tcpu/ $\ \Upsilon\ $			
1	17.0/101.0/0.038/3.71e-07	32.6/317.0/0.110/2.37e-07	27.2/252.4/0.085/8.73e-08	24.4/233.0/0.086/3.05e-07
3	17.0/101.0/0.092/8.17e-07	26.8/273.4/0.239/7.00e-08	31.2/358.4/0.304/1.19e-09	18.8/185.4/0.174/0.00e+00
5	19.0/115.0/0.176/6.96e-07	12.8/47.2/0.063/2.07e-07	11.8/43.6/0.061/0.00e+00	20.2/199.6/0.301/3.30e-05
8	19.0/115.0/0.278/9.32e-07	108.6/2558.4/5.690/5.07e-08	35.6/450.0/0.967/7.33e-08	19.6/195.0/0.460/2.27e-05
10	20.0/116.0/0.341/9.45e-07	110.2/1467.4/4.075/0.00e+00	70.4/847.8/2.318/2.26e-07	19.0/186.0/0.533/0.00e+00
<b>Problem 5.4</b>				
	Itr/NF/Tcpu/ $\ \Upsilon\ $			
1	48.0/311.0/0.093/5.18e-07	102.2/522.0/0.154/6.10e-07	104.2/532.0/0.153/5.90e-07	98.4/1016.8/0.280/9.66e-07
3	38.0/253.0/0.199/4.95e-07	108.4/553.0/0.431/7.15e-07	108.4/553.0/0.428/7.05e-07	101.6/1050.4/0.825/9.39e-07
5	38.0/253.0/0.310/6.61e-07	109.4/558.0/0.682/8.37e-07	108.6/554.0/0.666/8.14e-07	103.0/1063.8/1.305/8.97e-07
8	38.0/253.0/0.495/8.61e-07	113.6/579.2/1.106/8.23e-07	110.0/561.0/1.057/9.62e-07	104.4/1080.4/2.082/9.09e-07
10	38.0/253.0/0.594/9.71e-07	127.8/651.0/1.501/1.85e-07	127.4/649.0/1.491/1.83e-07	105.2/1084.4/2.555/9.70e-07

Table 2: Results for Problem (5.5), Problem (5.6), Problem (5.7), and Problem (5.8)

$n \times 10^4$	STTCGPM (1)	FITTCGPM-PRP [1]	FITTCGPM-DY [1]	IMSMINE [2]
<b>Problem 5.5</b>				
	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $
1	17.0/101.0/0.036/9.51e-07	20.0/81.0/0.027/6.35e-07	20.0/81.0/0.027/6.35e-07	56.0/547.0/0.162/9.22e-07
3	18.0/107.0/0.091/7.11e-07	21.0/85.0/0.076/4.07e-07	21.0/85.0/0.075/4.08e-07	57.0/566.0/0.479/8.92e-07
5	18.0/107.0/0.142/9.87e-07	21.0/85.0/0.117/5.16e-07	21.0/85.0/0.116/5.16e-07	58.2/570.4/0.770/1.01e-06
8	19.0/113.0/0.236/4.80e-07	21.0/85.0/0.181/6.49e-07	21.0/85.0/0.181/6.49e-07	59.0/586.0/1.244/7.23e-07
10	19.0/113.0/0.290/5.52e-07	21.0/85.0/0.222/7.19e-07	21.0/85.0/0.224/7.20e-07	59.0/586.0/1.519/8.05e-07
<b>Problem 5.6</b>				
	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $
1	16.0/95.0/0.195/4.23e-07	461.2/1845.8/3.592/9.86e-07	661.8/2648.2/4.931/9.90e-07	50.0/487.2/0.913/1.09e-06
3	16.2/95.2/0.441/8.89e-07	485.4/1942.6/8.948/9.87e-07	698.6/2795.4/12.843/9.92e-07	49.8/486.8/2.248/1.11e-06
5	17.0/101.0/0.922/3.17e-07	493.8/1976.2/17.086/9.87e-07	710.4/2842.6/24.271/9.93e-07	NaN/NaN/NaN/NaN
8	17.0/101.0/1.338/4.30e-07	504.8/2020.2/25.774/9.82e-07	727.2/2909.8/37.394/9.87e-07	NaN/NaN/NaN/NaN
10	17.0/101.0/1.686/5.24e-07	509.0/2037.0/33.512/9.83e-07	733.2/2933.8/45.974/9.87e-07	50.4/498.2/7.978/8.42e-07
<b>Problem 5.7</b>				
	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $
1	8.0/39.0/0.036/2.44e-07	12.0/37.0/0.021/3.67e-07	12.0/37.0/0.021/3.63e-07	64.0/626.0/0.321/8.34e-07
3	9.0/46.0/0.063/4.23e-07	12.0/37.0/0.052/9.02e-07	12.0/37.0/0.051/8.95e-07	67.0/665.0/0.897/6.89e-07
5	10.0/52.0/0.111/2.78e-07	13.0/38.0/0.084/7.42e-07	13.0/38.0/0.083/7.35e-07	68.0/666.0/1.441/9.35e-07
8	11.0/58.0/0.193/2.19e-07	13.0/38.0/0.131/9.83e-07	13.0/38.0/0.128/9.74e-07	69.0/685.0/2.357/9.11e-07
10	11.0/58.0/0.240/2.48e-07	13.0/40.0/0.170/2.69e-07	13.0/40.0/0.173/2.66e-07	70.0/686.0/2.924/8.61e-07
<b>Problem 5.8</b>				
	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $
1	3.0/15.0/0.006/0.00e+00	10.0/32.0/0.010/9.60e-07	10.0/32.0/0.010/9.86e-07	59.0/585.0/0.169/8.13e-07
3	2.0/10.0/0.010/0.00e+00	11.0/35.0/0.029/9.16e-07	11.0/35.0/0.028/9.22e-07	62.0/606.0/0.480/9.41e-07
5	3.0/16.0/0.024/0.00e+00	11.0/35.0/0.043/8.16e-07	11.0/35.0/0.042/8.28e-07	63.2/625.2/0.769/9.21e-07
8	3.0/16.0/0.037/0.00e+00	11.0/35.0/0.065/7.72e-07	11.0/35.0/0.064/7.89e-07	65.0/645.0/1.197/6.96e-07
10	3.0/16.0/0.047/0.00e+00	11.0/35.0/0.084/7.66e-07	11.0/35.0/0.081/7.86e-07	65.0/645.0/1.483/9.06e-07

Table 3: Comparison of STTCGPM, FITTCGPM-PRP, FITTCGPM-DY, and IMSMNE methods.

Init(n)	STTCGPM (1)		FITTCGPM-PRP [1]		FITTCGPM-DY [1]		IMSMNE [2]	
	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $	Itr/NF/Tcpu/ $\ \Upsilon\ $				
1	60.8/270.6/2.302/7.60e-07	157.6/473.8/4.159/9.65e-07	157.6/473.4/4.062/9.66e-07	157.6/473.4/4.062/9.66e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
2	66.0/296.6/2.464/6.89e-07	157.8/474.4/4.077/9.56e-07	157.8/474.0/3.932/9.57e-07	157.8/474.0/3.932/9.57e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
3	65.4/292.2/2.266/6.89e-07	157.8/473.6/3.678/9.83e-07	157.4/473.2/3.640/9.72e-07	157.4/473.2/3.640/9.72e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
4	64.0/285.8/2.091/7.01e-07	157.6/473.8/3.517/9.52e-07	157.6/473.0/3.486/9.68e-07	157.6/473.0/3.486/9.68e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
5	67.2/301.6/2.052/8.72e-07	157.6/473.4/3.203/9.77e-07	157.4/473.2/3.249/9.65e-07	157.4/473.2/3.249/9.65e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
6	68.8/306.6/1.645/7.85e-07	156.8/471.4/2.719/9.66e-07	156.8/471.0/2.569/9.67e-07	156.8/471.0/2.569/9.67e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
7	63.0/279.4/1.153/7.85e-07	158.2/475.2/1.926/9.81e-07	158.0/475.0/1.988/9.68e-07	158.0/475.0/1.988/9.68e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
8	62.4/279.4/0.616/6.86e-07	157.8/474.0/1.012/9.61e-07	157.4/473.2/1.009/9.66e-07	157.4/473.2/1.009/9.66e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
9	61.2/273.8/1.034/7.00e-07	157.4/472.8/1.801/9.85e-07	157.2/472.6/1.779/9.72e-07	157.2/472.6/1.779/9.72e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
10	48.6/215.8/0.013/5.37e-07	137.8/413.6/0.020/9.68e-07	137.2/412.6/0.019/9.77e-07	137.2/412.6/0.019/9.77e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
11	17.8/66.2/0.780/6.84e-07	164.8/495.0/6.033/9.61e-07	164.6/494.8/6.176/9.47e-07	164.6/494.8/6.176/9.47e-07	830.4/8295.8/70.166/9.92e-07	830.4/8295.8/70.166/9.92e-07	830.4/8295.8/70.166/9.92e-07	830.4/8295.8/70.166/9.92e-07
12	17.0/63.4/0.754/4.87e-07	164.8/495.4/5.889/9.59e-07	164.8/495.0/5.801/9.59e-07	164.8/495.0/5.801/9.59e-07	833.8/8324.8/68.490/9.96e-07	833.8/8324.8/68.490/9.96e-07	833.8/8324.8/68.490/9.96e-07	833.8/8324.8/68.490/9.96e-07
13	18.6/70.2/0.682/4.38e-07	164.6/494.8/4.384/9.57e-07	164.6/494.8/4.214/9.42e-07	164.6/494.8/4.214/9.42e-07	877.2/8760.6/70.030/9.91e-07	877.2/8760.6/70.030/9.91e-07	877.2/8760.6/70.030/9.91e-07	877.2/8760.6/70.030/9.91e-07
14	17.4/64.4/0.564/7.54e-07	164.6/494.8/4.088/9.71e-07	164.6/494.8/4.154/9.56e-07	164.6/494.8/4.154/9.56e-07	842.4/8416.2/62.867/9.88e-07	842.4/8416.2/62.867/9.88e-07	842.4/8416.2/62.867/9.88e-07	842.4/8416.2/62.867/9.88e-07
15	17.8/66.8/0.508/6.74e-07	164.6/494.4/3.768/9.76e-07	164.4/494.2/3.837/9.63e-07	164.4/494.2/3.837/9.63e-07	871.2/8704.2/61.415/9.87e-07	871.2/8704.2/61.415/9.87e-07	871.2/8704.2/61.415/9.87e-07	871.2/8704.2/61.415/9.87e-07
16	16.6/62.4/0.373/8.15e-07	164.2/493.6/3.193/9.65e-07	164.2/493.2/3.287/9.65e-07	164.2/493.2/3.287/9.65e-07	803.2/8020.6/43.210/9.90e-07	803.2/8020.6/43.210/9.90e-07	803.2/8020.6/43.210/9.90e-07	803.2/8020.6/43.210/9.90e-07
17	17.0/63.2/0.362/4.01e-07	165.0/495.6/2.289/9.75e-07	164.8/495.4/2.357/9.61e-07	164.8/495.4/2.357/9.61e-07	814.8/8136.6/32.773/9.93e-07	814.8/8136.6/32.773/9.93e-07	814.8/8136.6/32.773/9.93e-07	814.8/8136.6/32.773/9.93e-07
18	15.6/56.8/0.157/6.40e-07	164.6/494.8/1.270/9.61e-07	164.4/494.2/1.391/9.63e-07	164.4/494.2/1.391/9.63e-07	950.0/9491.8/19.498/9.95e-07	950.0/9491.8/19.498/9.95e-07	950.0/9491.8/19.498/9.95e-07	950.0/9491.8/19.498/9.95e-07

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