Assignment - 9 :

1. Given
$A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$
$P = [[b_1]_A \ [b_2]_A \ [b_3]_A]$.
$\therefore$ matrix from A to B
$= b_j = P_{1j} a_1 + P_{2j} a_2 + P_{3j} a_3 \ , 1 \le j \le 3$ — ①

$\Rightarrow [x_B] = [x_1^B, x_2^B, x_3^B]$ which is in ordered basis B and coordinate of vector x.
$\therefore \ x = x_1^b b_1 + x_2^b b_2 + x_3^b b_3$

$$= \sum_{i=1}^{3} x_i^B b_i$$

$$= \sum_{i=1}^{3} x_i^B (P_{1j} a_1 + P_{2i} a_2 + P_{3i} a_3) \ \ldots \text{in ①}$$

$$\therefore [x]_A = \left[\sum_{i=1}^{3} P_{1j} x_i^B, \sum_{j=1}^{3} P_{2i} x_i^B, \sum_{i=1}^{3} P_{3i} x_i^B \right]$$

$\Rightarrow [x]_B = P^{-1} [x]_A$ as P has a change in coordinate matrix i.e from A to B.

2. Given.

$$B = \left\{ \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ 0 \end{bmatrix} \right\}$$

$$C = \left\{ \begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ -7 \\ -7 \end{bmatrix}, \begin{bmatrix} -5 \\ 6 \\ -5 \end{bmatrix} \right\}, \quad [x]_B \begin{Bmatrix} 3 \\ \ldots \end{Bmatrix}$$

$\therefore [x]_C =$

$$\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} = x \begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix} + y \begin{bmatrix} 1 \\ -7 \\ -7 \end{bmatrix} + z \begin{bmatrix} -5 \\ 6 \\ -5 \end{bmatrix}$$

$$4x + y - 5z = 0$$
$$7x - 7y + 6z = -1$$
$$9x - 7y - 5z = 3$$

$$\begin{bmatrix} 4 & 1 & -5 & 0 \\ 7 & -7 & 6 & -1 \\ 9 & -7 & -5 & 3 \end{bmatrix} \xrightarrow{R_1/4} \begin{bmatrix} 1 & \frac{1}{4} & -\frac{5}{4} & 0 \\ 7 & -7 & 6 & -1 \\ 9 & -7 & -5 & 3 \end{bmatrix} R_2 - 7R_1$$

$$\begin{bmatrix} 1 & 1/4 & -5/4 & 0 \\ 0 & -35/4 & 59/4 & -1 \\ 9 & -7 & -5 & 3 \end{bmatrix} R_3 - 9R_1 \sim \begin{bmatrix} 1 & 1/4 & -5/4 & 0 \\ 0 & -35/4 & 59/4 & -1 \\ 0 & -37/4 & -\frac{25}{4} & 3 \end{bmatrix} -\frac{4}{35}R_2$$

$$\sim \begin{bmatrix} 1 & 1/4 & -5/4 & 0 \\ 0 & 1 & -59/35 & 4/35 \\ 0 & -37/4 & 25/4 & 3 \end{bmatrix} R_1 - R_2/4 \sim \begin{bmatrix} 1 & 0 & -29/35 & -1/35 \\ 0 & 1 & -59/35 & 4/35 \\ 0 & -37/4 & \frac{25}{4} & 3 \end{bmatrix} R_3 + 37R_2/4$$

$$\left(\begin{matrix} 1 & 0 & -29/35 & -1/35 \\ 0 & 1 & -59/35 & 4/35 \\ 0 & 0 & -327/35 & 142/35 \end{matrix}\right) \frac{-35R_3}{327} \sim \left(\begin{matrix} 1 & 0 & \frac{-29}{35} & \frac{-1}{35} \\ 0 & 1 & \frac{-59}{35} & 4/35 \\ 0 & 0 & 1 & \frac{-142}{327} \end{matrix}\right)$$

$$x - z\left(\frac{29}{35}\right) = \frac{-1}{35}$$

$$y - \frac{59}{35} z = 4/35$$

$$z = \frac{-142}{327}$$

$$y = \frac{4}{35} + \frac{59}{35}\left(\frac{-142}{327}\right) = -202/327$$

$$x = \frac{-127}{327}$$

∴ P being the change of basis matrix from C to B will be $\left(\begin{matrix} \frac{-127}{327} \\ \frac{-202}{327} \\ \frac{-142}{327} \end{matrix}\right)$

then for 2nd column

$$\begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix} = x\begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix} + y\begin{bmatrix} 1 \\ -7 \\ -7 \end{bmatrix} + z\begin{bmatrix} -5 \\ 6 \\ -5 \end{bmatrix}$$

$$\left(\begin{matrix} 4 & 1 & -5 & \Big| & -3 \\ 7 & -7 & 6 & \Big| & 2 \\ 9 & -7 & -5 & \Big| & -5 \end{matrix}\right) \cdot \frac{1}{4}$$

$$\sim \begin{pmatrix} 1 & \frac{1}{4} & -5/4 & -3/4 \\ 7 & -7 & 6 & 2 \\ 9 & -7 & -5 & -5 \end{pmatrix} -7R_1 \sim \begin{bmatrix} 1 & 1/4 & -5/4 & -3/4 \\ 0 & -35/4 & 59/4 & 29/4 \\ 9 & -7 & -5 & -5 \end{bmatrix}$$

$-9R_1$ to 3^{rd} row

$$\begin{bmatrix} 1 & 1/4 & -5/4 & -3/4 \\ 0 & -35/4 & 59/4 & 29/4 \\ 0 & \frac{-37}{4} & 25/4 & 7/4 \end{bmatrix} -4/35 \sim \begin{bmatrix} 1 & 1/4 & \frac{-5}{4} & -3/4 \\ 0 & 1 & \frac{-59}{35} & -29/35 \\ 0 & \frac{-37}{4} & 25/4 & \frac{7}{4} \end{bmatrix}$$

$\frac{37}{4} R_2$ to 3^{rd} row

$$\sim \begin{pmatrix} 1 & 1/4 & -5/4 & -3/4 \\ 0 & 1 & -59/35 & -29/35 \\ 0 & 0 & \frac{-327}{35} & -207/35 \end{pmatrix} \cdot \left(\frac{-35}{327}\right)$$

$$\sim \begin{pmatrix} 1 & 1/4 & -5/4 & -3/4 \\ 0 & 1 & -59/35 & -29/35 \\ 0 & 0 & 1 & 69/109 \end{pmatrix} + 59/35\, R_3$$

$$\sim \begin{pmatrix} 1 & 1/4 & -5/4 & -3/4 \\ 0 & 1 & 0 & 26/109 \\ 0 & 0 & 1 & 69/109 \end{pmatrix} - 5/4\, R_3$$

$$\sim \begin{pmatrix} 1 & 1/4 & 0 & 9/218 \\ 0 & 1 & 0 & 26/109 \\ 0 & 0 & 1 & 69/109 \end{pmatrix} + \left(-\frac{1}{4} R_2\right)$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -2/109 \\ 0 & 1 & 0 & 26/109 \\ 0 & 0 & 1 & 69/109 \end{bmatrix} \Rightarrow 2^{nd} \text{ column} \begin{pmatrix} -\frac{296}{327} \\ -512/327 \\ -143/327 \end{pmatrix}$$

for 3rd column

$$\begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix} = x\begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix} + y\begin{pmatrix} 1 \\ -7 \\ -7 \end{pmatrix} + z\begin{pmatrix} -5 \\ 6 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 & -5 & 5 \\ 7 & -7 & 6 & -5 \\ 9 & -7 & -5 & 0 \end{pmatrix} \cdot \tfrac{1}{4} \sim \begin{pmatrix} 1 & 1/4 & -5/4 & 5/4 \\ 7 & -7 & 6 & -5 \\ 9 & -7 & -5 & 0 \end{pmatrix} -7R_1$$

$$\sim \begin{pmatrix} 1 & 1/4 & -5/4 & 5/4 \\ 0 & -35/4 & 59/4 & -55/4 \\ 9 & -7 & -5 & 0 \end{pmatrix} -9R_1 \sim \begin{pmatrix} 1 & 1/4 & -5/4 & 5/4 \\ 0 & -35/4 & 59/4 & -55/4 \\ 0 & -37/4 & 25/4 & -45/4 \end{pmatrix}$$

multiply 2nd row by $-4/35$

$$\sim \begin{pmatrix} 1 & 1/4 & -5/4 & 5/4 \\ 0 & 1 & -59/35 & 11/7 \\ 0 & -37/4 & 25/4 & -45/4 \end{pmatrix} +\tfrac{37}{4}R_2$$

$$\sim \begin{pmatrix} 1 & 1/4 & -5/4 & 5/4 \\ 0 & 1 & -59/35 & 11/7 \\ 0 & 0 & -\frac{327}{35} & 23/7 \end{pmatrix} \left(-\tfrac{35}{327}\right) \sim \begin{pmatrix} 1 & 1/4 & -5/4 & 5/4 \\ 0 & 1 & -\frac{59}{35} & 11/7 \\ 0 & 0 & 1 & -115/327 \end{pmatrix}$$

$+\frac{59}{35}R_3$ to 2nd row

$$\sim \begin{pmatrix} 1 & 1/4 & -5/4 & 5/4 \\ 0 & 1 & 0 & 320/327 \\ 0 & 0 & 1 & -115/327 \end{pmatrix} +\tfrac{5}{4}R_3 \sim \begin{pmatrix} 1 & 1/4 & 0 & \frac{265}{327} \\ 0 & 1 & 0 & 320/327 \\ 0 & 0 & 1 & -\frac{115}{327} \end{pmatrix}$$

in 1st row $+\left(-\frac{1}{4}\right)R_3$

$$\sim \begin{pmatrix} 1 & 0 & 0 & \frac{185}{327} \\ 0 & 1 & 0 & \frac{320}{327} \\ 0 & 0 & 1 & -\frac{115}{327} \end{pmatrix}$$

$\therefore z = -115/327$

$\therefore y = \frac{320}{327}$

$x = \frac{185}{327}$

cont: 2

$$\therefore P = \begin{pmatrix} -127/327 & -296/327 & 185/327 \\ -202/327 & -512/327 & 320/327 \\ -142/327 & -143/327 & -115/327 \end{pmatrix}$$

$$P[x]_B = [x]_C$$

$$\Rightarrow [x]_C = P\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{-127 + 3(-296) + 5(185)}{327} \\ \dfrac{-202 + 3(-512) + 5(320)}{327} \\ \dfrac{-142 + 3(-143) - 115(5)}{327} \end{pmatrix}$$

$$\therefore [x]_C = \begin{pmatrix} -90/327 \\ -138/327 \\ -1146/327 \end{pmatrix}$$

Q3.

Given

$$B = \left\{ \begin{bmatrix} 6 \\ -5 \end{bmatrix}, \begin{bmatrix} 8 \\ -3 \end{bmatrix} \right\}, C = \left\{ \begin{bmatrix} -6 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$$

$$1 \mid a \mid B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

$\therefore |x|_C =$

$$\begin{bmatrix} 6 \\ -5 \end{bmatrix} = x \begin{bmatrix} -6 \\ -5 \end{bmatrix} + y \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$-6x + 2y = 6$$
$$-5x - y = -5$$

$$\begin{bmatrix} -6 & 2 & 6 \\ -5 & -1 & -5 \end{bmatrix} \overset{R1/6}{\sim} \begin{bmatrix} 1 & -1/3 & -1 \\ -5 & -1 & -5 \end{bmatrix} R_2 + 5R_1$$

$$\begin{bmatrix} 1 & -1/3 & -1 \\ 0 & -8/3 & -10 \end{bmatrix} \frac{-3R2}{8} \sim \begin{bmatrix} 1 & -1/3 & -1 \\ 0 & 1 & 15/4 \end{bmatrix} R_1 + \frac{R_2}{3}$$

$$\begin{bmatrix} 1 & 0 & 1/4 \\ 0 & 1 & 15/4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{matrix} x = 1/4 \\ y = 15/4 \end{matrix}$$

$$1^{st} \text{ colum} = \begin{pmatrix} 1/4 \\ 15/4 \end{pmatrix}$$

$$2^{nd} \text{ column} = \begin{bmatrix} -6 & 2 & 8 \\ -5 & 1 & -3 \end{bmatrix} \overset{R1/-6}{\sim} \begin{bmatrix} 1 & -1/3 & -4/3 \\ -5 & 1 & -3 \end{bmatrix} +5R_1$$

$$\begin{bmatrix} 1 & -1/3 & -4/3 \\ 0 & -8/3 & -29/3 \end{bmatrix} -\tfrac{3}{8}R_2 \sim \begin{bmatrix} 1 & -1/3 & -4/3 \\ 0 & 1 & 29/8 \end{bmatrix} +\tfrac{R_2}{3}$$

$$\sim \begin{bmatrix} 1 & 0 & -1/8 \\ 0 & 1 & 29/8 \end{bmatrix} \quad \therefore x = -1/8,\ y = 29/8$$

$$P = \begin{bmatrix} 1/4 & -1/8 \\ 15/4 & 29/8 \end{bmatrix}$$

$$\Rightarrow P[x]_B = [x]_C$$

$$[x]_C = \begin{bmatrix} 1/4 & -1/8 \\ 15/4 & 29/8 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$[x]_B = \begin{bmatrix} 1/4 - 1/4 \\ 15/4 + 29/4 \end{bmatrix}$$

$$[x]_C = \begin{bmatrix} 0 \\ \frac{44}{4} \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$$

2.4. Assume P be the change of coordinate matrix from B to C.

$$[B] = \begin{bmatrix} -3 & 1 & 5 \\ 6 & 8 & 5 \\ 2 & -4 & -2 \end{bmatrix} \ \& \ [C] = \begin{bmatrix} 1 & 1 & -7 \\ -8 & 5 & 1 \\ -8 & 8 & -1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} -3 & 1 & 5 & 1 & 1 & -7 \\ 6 & 8 & 5 & -8 & 5 & 1 \\ 2 & -4 & -2 & -8 & 8 & -1 \end{array}\right] R_1/-3$$

$$\sim \begin{pmatrix} & -1/3 & -5/3 & -1/3 & -1/3 & 7/3 \\ 6 & 8 & 5 & -8 & 5 & 1 \\ 2 & -4 & -2 & -8 & 8 & -1 \end{pmatrix} R_2 - 6R_1$$

$$\begin{pmatrix} 1 & -1/3 & -5/3 & -1/3 & -1/3 & 7/3 \\ 0 & 10 & 15 & -6 & 7 & -13 \\ 2 & -4 & -2 & -8 & 8 & -1 \end{pmatrix} R_3 - 2R_1$$

$$\begin{pmatrix} 1 & -1/3 & -5/3 & -1/3 & -1/3 & 7/3 \\ 0 & 10 & 15 & -6 & 7 & -13 \\ 0 & -10/3 & 4/3 & -22/3 & 26/3 & -17/3 \end{pmatrix} \frac{R_2}{10}$$

$$\begin{pmatrix} 1 & -1/3 & -5/3 & -1/3 & -1/3 & 7/3 \\ 0 & 1 & 3/2 & -3/5 & 7/10 & -13/10 \\ 0 & -10/3 & 4/3 & -22/3 & 26/3 & -17/3 \end{pmatrix} R_1 + R_2/3$$

$$\begin{pmatrix} 1 & 0 & -7/6 & -8/15 & -1/10 & 19/10 \\ 0 & 1 & 3/2 & -3/5 & 7/10 & -13/10 \\ 0 & -10/3 & 4/3 & -22/3 & 26/3 & -17/3 \end{pmatrix} R_3 + \frac{10R_2}{3}$$

$$\begin{pmatrix} 1 & 0 & -7/6 & -8/15 & -1/10 & 19/10 \\ 0 & 1 & 3/2 & -3/5 & 7/10 & -13/10 \\ 0 & 0 & 19/3 & -28/5 & 11 & -30 \end{pmatrix} 3R_3/19$$

$$\begin{pmatrix} 1 & 0 & -7/6 & -8/15 & -1/10 & 19/10 \\ 0 & 1 & 3/2 & -3/5 & 7/10 & -13/10 \\ 0 & 0 & 1 & -28/19 & 33/19 & -30/19 \end{pmatrix} \begin{matrix} +7R_3/6 \\ -3\frac{R_3}{2} \end{matrix}$$

$$\left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \;\middle|\; \begin{matrix} \frac{-214}{95} & 183/95 & 11/190 \\ \frac{153}{95} & -181/95 & 203/190 \\ \frac{-28}{19} & 33/19 & -30/19 \end{matrix}\right)$$

$$\therefore P = \begin{bmatrix} \frac{-214}{95} & \frac{183}{95} & \frac{11}{190} \\ \frac{153}{95} & \frac{-181}{95} & \frac{203}{190} \\ \frac{-28}{19} & \frac{33}{19} & \frac{-30}{19} \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -7 & -3 & 1 & 5 \\ -8 & 5 & 1 & 6 & 8 & 5 \\ -8 & 8 & -1 & 2 & -4 & -2 \end{array}\right] R_2 + 8R_1$$

$$\sim \begin{bmatrix} 1 & 1 & -7 & -3 & 1 & 5 \\ 0 & 13 & -55 & -18 & 16 & 45 \\ -8 & 8 & -1 & 2 & -4 & -2 \end{bmatrix} +8R_1$$

$$\sim \begin{bmatrix} 1 & 1 & -7 & -3 & 1 & 5 \\ 0 & 13 & -55 & -18 & 16 & 45 \\ 0 & 16 & -57 & -22 & 4 & 38 \end{bmatrix} R_2/13$$

$$\sim \begin{bmatrix} 1 & 1 & -7 & -3 & 1 & 5 \\ 0 & 1 & -55/13 & -18/13 & 16/13 & 45/13 \\ 0 & 16 & -57 & -22 & 4 & 38 \end{bmatrix} R_1 - R_2$$

$$\sim \left(\begin{array}{cccccc} 1 & 0 & -36/13 & -21/13 & -3/13 & 20/13 \\ 0 & 1 & -55/13 & -18/13 & 16/13 & 45/13 \\ 0 & 16 & -57 & -22 & -\frac{204}{13} & -\frac{226}{13} \end{array}\right) \frac{13}{139}R_3$$

$$\sim \left(\begin{array}{cccccc} 1 & 0 & -36/13 & -21/13 & -3/13 & 20/13 \\ 0 & 1 & -55/13 & -18/13 & 16/13 & 95/13 \\ 0 & 0 & 1 & 2/139 & -204/139 & \frac{-226}{139} \end{array}\right) R_1 + \frac{36}{13}R_3$$

$$\sim \left(\begin{array}{cccccc} 1 & 0 & 0 & -219/139 & -597/139 & -412/139 \\ 0 & 1 & -55/13 & -18/13 & 16/13 & 45/13 \\ 0 & 0 & 1 & \frac{2}{139} & \frac{-204}{139} & \frac{-226}{139} \end{array}\right) \frac{55}{13}R_3$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-219}{139} & \frac{-597}{139} & \frac{-412}{139} \\ 0 & 1 & 0 & -184/139 & \frac{-692}{139} & \frac{-475}{139} \\ 0 & 0 & 1 & 2/139 & \frac{-204}{139} & \frac{-226}{139} \end{array}\right)$$

$$\Rightarrow Q = -\begin{pmatrix} \frac{219}{139} & -\frac{597}{139} & -\frac{412}{139} \\ -\frac{186}{139} & -\frac{692}{139} & -\frac{475}{139} \\ \frac{2}{139} & -\frac{204}{139} & -\frac{226}{139} \end{pmatrix}$$

$$[T]_C^B = \left([T]_B^C\right)^{-1}$$

$$c_1 = ab_1 + bb_2 + cc_2$$

Cont. 4

$\Rightarrow a = \frac{-214}{95} \mid b = \frac{153}{95},\ c = \frac{-28}{19}$

$c_2 = ab_1 + bb_2 + cc_2$

$\Rightarrow a = \frac{183}{95} \mid b = \frac{-181}{95},\ c = \frac{33}{19}$

$c_3 = ab_1 + bb_2 + cc_2$

$\Rightarrow a = \frac{11}{190},\ b = \frac{203}{190},\ c = \frac{-30}{19}$

$$[T]_C^B = \begin{pmatrix} -214/95 & 183/95 & 11/190 \\ 153/95 & -181/95 & 203/190 \\ -28/19 & 33/19 & -30/19 \end{pmatrix}$$

Q-5 $P = \begin{bmatrix} 5 & 0 & -2 \\ 6 & -2 & -5 \\ 0 & 5 & 4 \end{bmatrix}$, $a_1 = \begin{bmatrix} -2 \\ -5 \\ -2 \end{bmatrix}$, $a_2 = \begin{bmatrix} -8 \\ -7 \\ 8 \end{bmatrix}$,

$a_3 = \begin{bmatrix} 9 \\ 1 \\ -8 \end{bmatrix}$

$\therefore$ As P being change of coordinate matrix from $\{a_1, a_2, a_3\}$ to $\{b_1, b_2, b_3\}$

$\Rightarrow b_1 = 5a_1 + 6a_2 + 0a_3$

$= 5\begin{bmatrix} -2 \\ -5 \\ -2 \end{bmatrix} + 6\begin{bmatrix} -8 \\ -7 \\ 8 \end{bmatrix}$

$= \begin{bmatrix} -10-48 \\ -25-42 \\ -10+48 \end{bmatrix} = \begin{bmatrix} -58 \\ -67 \\ 38 \end{bmatrix}$

$b_2 = 0 \cdot a_1 + (-2)a_2 + 5 \cdot a_3$

$= (-2)\begin{bmatrix} -8 \\ -7 \\ 8 \end{bmatrix} + 5\begin{bmatrix} 9 \\ 1 \\ -8 \end{bmatrix}$

$= \begin{bmatrix} 16+45 \\ 14+5 \\ -16-40 \end{bmatrix} = \begin{bmatrix} 61 \\ 19 \\ -56 \end{bmatrix}$

$b_3 = (-2)\begin{bmatrix} -2 \\ -5 \\ -2 \end{bmatrix} - 5\begin{bmatrix} -8 \\ -7 \\ 8 \end{bmatrix} + 4\begin{bmatrix} 9 \\ 1 \\ -8 \end{bmatrix}$

$$= \begin{bmatrix} 4+40+36 \\ 10+35+4 \\ 4-40-32 \end{bmatrix} = \begin{bmatrix} 80 \\ 49 \\ -68 \end{bmatrix}$$

$\therefore b_1 = \begin{bmatrix} -58 \\ -67 \\ 38 \end{bmatrix}, b_2 = \begin{bmatrix} 61 \\ 19 \\ -56 \end{bmatrix},$

$b3 = \begin{bmatrix} 80 \\ 49 \\ -68 \end{bmatrix}$

Q.6.a) $x:y$ aspect ratio, the image is x units wide and y units high.

So for 4:3 basis will be

$B = \left\{ \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right\}$

$= \{ 4e_1, 3e_2 \}$

As $\{e_1, e_2\}$ are independent so is $\{4e_1, 3e_2\}$

& for 16:9 basis will be $C = \left\{ \begin{bmatrix} 16 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 9 \end{bmatrix} \right\}$

$= \{ 16 \cdot e_1, 9e_2 \}$

b) Assume P the change of co-ordinate matrix from B to C for 1st column $\begin{bmatrix} 16 \\ 0 \end{bmatrix} = a \begin{bmatrix} 4 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$\Rightarrow a = 4, \ b = 0$

$\therefore$ first column of P is $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

for 2nd column ~~of P~~ is =

$\begin{bmatrix} 0 \\ 9 \end{bmatrix} = a \begin{bmatrix} 4 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$\Rightarrow a = 0, \ b = 3$

$\Rightarrow P = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

Assume Q the change in coordinate matrix from C to B

then $Q = P^{-1}$

$= \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix}^{-1}$

$Q = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

c) $[x]_C = (854, 1001)$

Since $P^{-1}[x]_B = [x]_C$

$\Rightarrow [x]_B = P[x]_C$

$= \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 854 \\ 1001 \end{bmatrix}$

$= \begin{bmatrix} 3916 \\ 3003 \end{bmatrix}$