Canny Edge Detection

CSE 6367 – Computer Vision
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What Is an Edge?

- An edge pixel is a pixel at a “boundary”.
- There is no single definition for what is a “boundary”.

output1

input

output2

Image Derivatives

- We have all learned how to calculate derivatives of functions from the real line to the real line.
  - Input: a real number
  - Output: a real number
- In taking image derivatives, we must have in mind two important differences from the “typical” derivative scenario:
  - Input: two numbers (x, y), not one number.
  - Input: discrete (integer pixel coordinates).
  - Output: Integer between 0 and 255.

Directional Derivative

- Let f(x, y) be a function mapping two *real* numbers to a real number.
- Let theta be a direction (specified as an angle from the x axis).
- Let (x_1, y_1) be a specific point on the plane.
- Define $g(x) = f(x_1 + x \cos(\text{theta}), y_1 + x \sin(\text{theta}))$.
- Then, g(x) is a function from the real line to the real line.
- The directional derivative of f at x_1, y_1 is defined to be g'(0).

dx, dy Directional Derivatives

- For the directional derivative of f along the x axis, we use notation df/dx.
- For the directional derivative of f along the y axis, we use notation df/dy.

Vertical and Horizontal Edges

- Consider the image as a function f(i,j) mapping pixels to intensity values.
  - Function f(i,j) can be seen as a discretized version of a more general function g(y,x), mapping pairs of real numbers to intensity values.
  - Vertical edges correspond to points in g with high dg/dx.
  - Horizontal edges correspond to points in g with high dg/dy.

Approximating dg/dx via Filtering

- In the discrete domain of f(i,j), dg/dx is approximated by filtering with the right kernel:

```
dx = [-1 0 1;
      -2 0 2;
      -1 0 1];
dx = dx / (sum(abs(dx(:))));
dxgray = abs(imfilter2(gray, dx));
```

- Interpreting **`imfilter(gray, dx)`**:
  – Results far from zero (positive and negative) correspond to strong vertical edges.
    - These are mapped to high positive values by **`abs`**.
  – Results close to zero correspond to weak vertical edges, or no edges whatsoever.

Result: Vertical/Horizontal Edges

```
gray = read_gray('data/hand20.bmp');
dx = [-1 0 1;
      -2 0 2;
      -1 0 1];
dx = dx / (sum(abs(dx(:))));
dy = dx'; % dy is the transpose of dx
dxgray = abs(imfilter(gray, dx, 'symmetric', 'same'));
dygray = abs(imfilter(gray, dy, 'symmetric', 'same'));
```

gray

dxgray
(vertical edges)

dygray
(horizontal edges)

Blurring and Filtering

- To suppress edges corresponding to small-scale objects/textures, we should first blur.

```
% generate two blurred versions of the image, see how it
% looks when we apply dx to those blurred versions.
filename = 'data/hand20.bmp';
gray = read_gray(filename);
dx = [-1 0 1; -2 0 2; -1 0 1] / 8;
dy = dx';

blur_window1 = fspecial('gaussian', 19, 3.0); % std = 3
blur_window2 = fspecial('gaussian', 37, 6.0); % std = 6
blurred_gray1 = imfilter(gray, blur_window1, 'symmetric');
blurred_gray2 = imfilter(gray, blur_window2 , 'symmetric');
dxgray = abs(imfilter(gray, dx, 'symmetric'));
dxb1gray = abs(imfilter(blurred_gray1, dx , 'symmetric'));
dxb2gray = abs(imfilter(blurred_gray2, dx , 'symmetric'));
```

Blurring and Filtering: Results

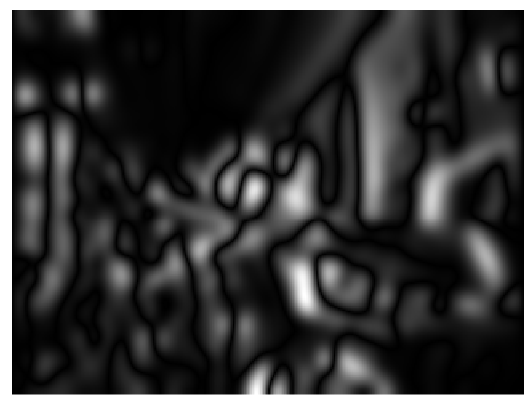
`gray`

- Smaller details are suppressed, but the edges are too thick.
  - Will be remedied in a few slides, with *non-maxima suppression.*

`dxgray`
No blurring

`dxb1gray`
Blurring, std = 3

`dxb1gray`
Blurring, std = 6



Finding Edges at Other Angles

- Extracting edges at angle theta:
  - Rotate dx by theta, or
  - Rotate image by –theta.
  - Rotating filter is typically more efficient.

```
% detecting edges with orientation 45 or 135 degrees:

fcircle = read_gray('data/blurred_fcircle.bmp');
dx = [-1 0 1; -2 0 2; -1 0 1] / 8;
rot45 = imrotate(dx, 45, 'bilinear', 'loose');
rot135 = imrotate(dx, 135, 'bilinear', 'loose');

edges45 = abs(imfilter(fcircle, rot45, 'symmetric'));
edges135 = abs(imfilter(fcircle, rot135, 'symmetric'));
```

Results: Edges at Degrees 45/135

fcircle

edges45

edges135

More Edges at 45/135 Degrees

original image

edges at 45 degrees

edges at 135 degrees

Computing Gradient Norms

- Let:
  - dxA = imfilter(A, dx);
  - dyA = imfilter(A, dy);
- Gradient norm at pixel (i,j):
  - The norm of vector (dxA(i,j), dyA(i,j)).
  - sqrt(dxA(i,j)^2 + dyA(i,j)^2).
- The gradient norm operation identifies pixels at all orientations.
- Also useful for identifying smooth/rough textures.

Computing Gradient Norms: Code

```
gray = read_gray('data/hand20.bmp');
dx = [-1 0 1; -2 0 2; -1 0 1] / 8;
dy = dx';

blurred_gray = blur_image(gray, 1.4 1.4);
dxgray = imfilter(blurred_gray, dx, 'symmetric');
dygray = imfilter(blurred_gray, dy, 'symmetric');

% computing gradient norms
grad_norms = (dxb1gray.^2 + dyb1gray.^2).^0.5;
```

- See following functions online:
  - gradient_norms
  - blur_image

Gradient Norms: Results

gray

grad_norms

dxgray

dygray

Notes on Gradient Norms

- Gradient norms detect edges at all orientations.
- However, gradient norms in themselves are not a good output for an edge detector:
  - We need thinner edges.
  - We need to decide which pixels are edge pixels.

Non-Maxima Suppression

- Goal: produce thinner edges.
- Idea: for every pixel, decide if it is maximum along the direction of fastest change.
  - Preview of results:

gradient norms

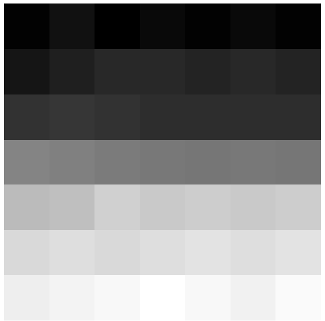
result of nonmaxima suppression

Nonmaxima Suppression Example

- Example:

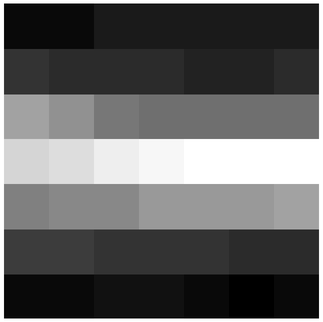
```
img = [112  118  111  115  112  115  112
       120  124  128  128  126  128  126
       132  134  132  130  130  130  130
       167  165  163  162  161  162  161
       190  192  199  196  198  196  198
       203  205  203  205  207  205  207
       212  214  216  219  216  213  217];
```

img



```
grad_norms = round(gradient_norms(img));
grad_norms = [ 5   5   7   7   7   7   7
              10   9   9   9   8   8   9
              23  21  18  17  17  17  17
              29  30  32  33  34  34  34
              19  20  20  22  22  22  23
              11  11  10  10  10   9   9
               5   5   6   6   5   4   5];
```

grad_norms

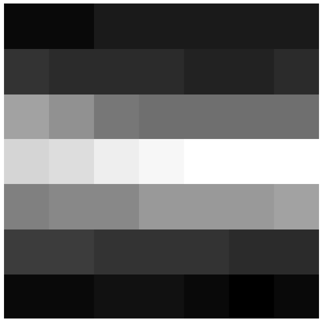


Nonmaxima Suppression Example

- Should we keep pixel (3,3)?
- result of dx filter [-0.5 0 0.5]
  - (img(3,4) – img(3, 2)) / 2 = -2.
- result of dy filter [-0.5; 0; 0.5]
  - (img(4,3) – img(2, 3)) / 2 = 17.5.
- Gradient = (-2, 17.5).
- Gradient direction:
  - atan2(17.5, -2) = 1.68 rad = 96.5 deg.
  - Unit vector at gradient direction:
    - [0.9935, -0.1135]   (y direction, x direction)

img

grad_norms

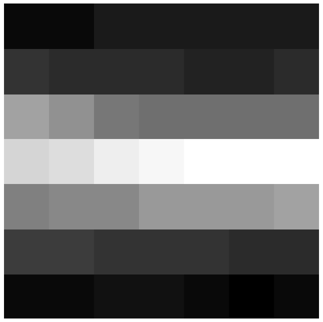


Nonmaxima Suppression Example

- Should we keep pixel (3,3)?
- Gradient direction: 96.5 degrees
  - Unit vector: disp = [0.9935, -0.1135].
  - disp defines the direction along which pixel(3,3) must be a local maximum.
  - Positions of interest:
    - [3,3] + disp, [3,3] – disp.
  - We compare grad_norms(3,3) with:
    - grad_norms(3.9935, 2.8865), and
    - grad_norms(2.0065, 3.1135)

img

grad_norms



Nonmaxima Suppression Example

- We compare grad_norms(3,3) with:
  - grad_norms(3.9935, 2.8865), and
  - grad_norms(2.0065, 3.1135)
- grad_norms(3.9935, 2.8865) = ?
  - Use bilinear interpolation.
  - (3.9935, 2.8865) is surrounded by:
    - (3,2) at the top and left.
    - (3,3) at the top and right.
    - (4,2) at the bottom and left.
    - (4,3) at the bottom and right.

img

grad_norms

Nonmaxima Suppression Example

- grad_norms(3.9935, 2.8865) = ?
  - Weighted average of surrounding pixels.

```
top_left = image(top, left);
top_right = image(top, right);
bottom_left = image(bottom, left);
bottom_right = image(bottom, right);
wy = 3.9935 – 3 = 0.9935;
wx = 2.8865 – 2 = 0.8865;
result = (1 – wx) * (1 – wy) * top_left +
         wx * (1 - wy) * top_right +
         (1 – wx) * y * bottom_left +
          x * y * bottom_right;
```

  - See function bilinear_interpolation online.

img

grad_norms

Nonmaxima Suppression Example

- grad_norms(3.9935, 2.8865) = 33.3
- grad_norms(2.0065, 3.1135) = 10.7
- grad_norms(3,3) = 18
  - Position 3,3 is not a local maximum in the direction of the gradient.
  - Position 3,3 is set to zero in the result of non-maxima suppression
  - Same test applied to all pixels.

img

grad_norms

Nonmaxima Suppression Result

```
grad_norms =
[ 5   5   7   7   7   7   7
 10   9   9   9   8   8   9
 23  21  18  17  17  17  17
 29  30  32  33  34  34  34
 19  20  20  22  22  22  23
 11  11  10  10  10   9   9
  5   5   6   6   5   4   5];
```

img

grad_norms

```
nonmaxima suppression(grand norms, thetas, 1) =
[ 0   0   0   0   0   0   0
  0   0   0   0   0   0   0
  0   0   0   0   0   0   0
  0  29  34  33  34  33   0
  0   0   0   0   0   0   0
  0   0   0   0   0   0   0
  0   0   0   0   0   0   0
```

result of non-maxima suppression

Nonmaxima Suppression Result

gradient norms

result of nonmaxima suppression

Side Note: Bilinear Interpolation

- grad_norms(3.9935, 2.8865) = ?
  - Weighted average of surrounding pixels.

```
top_left = image(top, left);
top_right = image(top, right);
bottom_left = image(bottom, left);
bottom_right = image(bottom, right);
wy = 3.9935 – 3 = 0.9935;
wx = 2.8865 – 2 = 0.8865;
result = (1 – wx) * (1 – wy) * top_left +
         wx * (1 - wy) * top_right +
         (1 – wx) * y * bottom_left +
         wx * wy * bottom_right;
```

- Interpolation is a very common operation.
  - Images are discrete, sometimes it is convenient to treat them as continuous values.

bilinear_interpolation.m

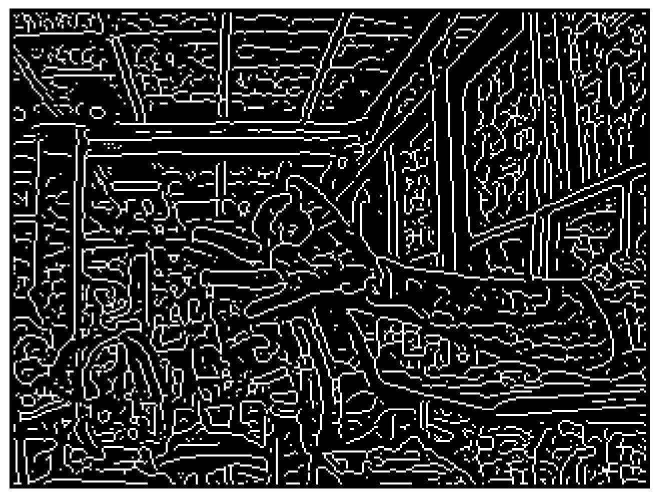
```
function result = bilinear_interpolation(image, row, col)

% row and col are non-integer coordinates, and this function
% computes the value at those coordinates using bilinear interpolation.

% Get the bounding square.
top = floor(row);
left = floor(col);
bottom = top + 1;
right = left + 1;

% Get values at the corners of the square
top_left = image(top, left);
top_right = image(top, right);
bottom_left = image(bottom, left);
bottom_right = image(bottom, right);

x = col - left;
y = row - top;

result = (1 - x) * (1 - y) * top_left;
result = result + x * (1 - y) * top_right;
result = result + x * y * bottom_right;
result = result + (1 - x) * y * bottom_left;
```

The Need for Thresholding

`gray`

- Many non-zero pixels in the result of nonmaxima suppression represent very weak edges.

`nonmaxima`



`nonmaxima > 0`

The Need for Thresholding

gray

- Decide which are the edge pixels:
  - Reject maxima with very small values.
  - Hysteresis thresholding.

gradient norms

result of nonmaxima suppression

Hysteresis Thresholding

- Use two thresholds, t1 and t2.
- Pixels above t2 survive.
- Pixels below t1 do not survive.
- Pixels >= t1 and < t2 survive if:
  - They are connected to a pixel >= t2 via an 8-connected path of other pixels >= t1.

Hysteresis Thresholding Example

`A = nonmaxima >= 4`

`B = nonmaxima >= 8`

`C = hysthresh(nonmaxima, 4, 8)`

- A pixel is white in C if:
  - It is white in A, and
  - It is connected to a white pixel of B via an 8-connected path of white pixels in A.

Canny Edge Detection

- Blur input image.
- Compute dx, dy, gradient norms.
- Do non-maxima suppression on gradient norms.
- Apply hysteresis thresholding to the result of non-maxima suppression.
  - Check out these functions online:
    - blur_image
    - gradient_norms
    - gradient_orientations
    - nonmaxsup
    - hysthresh
    - canny
    - canny4

Side Note: Angles/Directions/Orientations

- To avoid confusion, you must specify:
  - Unit (degrees, or radians).
  - Do you use undirected or directed orientation?
    - Undirected: 180 degrees = 0 degrees.
    - Directed: 180 degres != 0 degrees.
  - Which axis is direction 0? Pointing which way?
    - Class convention: direction 0 is x axis, pointing right.
  - Do angles increase clockwise or counterclockwise?
    - Class convention: clockwise.
  - Does the y axis point down? (in this class: yes)
  - What range of values do you allow/expect?
    - [-180, 180]? Any real number?

Side Note: Angles/Directions/Orientations

- Confusion and bugs stemming from different conventions are extremely common.
- When combining different code, **<u>make sure that you account for different conventions</u>**.

```
thetas =  atan2(dyb1gray, dxb1gray);

% atan2 convention:
% values: [-pi, pi]
% 0 degrees: x axis, pointing left
% y axis points down
% values increase clockwise
```

Side Note: Edge Orientation

- How is the orientation of an edge pixel defined?

Side Note: Edge Orientation

- How is the orientation of an edge pixel defined?
  - It is the direction PERPENDICULAR to the gradient, i.e., the (dx, dy) vector for that pixel.
  - Typically (not always) 0 degrees = 180 degrees.
    - In other words, typically we do not care about the direction of the orientation.