9.7) a) X, Given p-X4)

$n = 100$

$\bar{X} = 37.7$ $\alpha = 0.1$

$s = 9.2$

90% confidence interval for the expectation of the no. of concurrent users.

Using $\bar{X} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$ where $\alpha = 0.1$

$\Rightarrow \bar{X} \pm Z_{0.05} \frac{s}{\sqrt{100}}$ $Z_{0.05} = 1.645$
From the normal table

$\Rightarrow 37.7 \pm 1.645 \frac{9.2}{10} = 37.7 \pm 1.5 = [36.2, 39.2]$.

b). Given $\alpha = 0.01$.

Our null hypothesis of mean $H_0 : \mu = 35$.

The alternative hypothesis of $H_1 : \mu > 35$ with $\alpha = 0.01$ needs to be calculated.

The test statistics here is $Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$.

The distribution of the Test statistics under the null region

$$z = \frac{\bar{X} - 35}{\frac{\sigma}{\sqrt{n}}} \underset{\sim}{\text{under}} H_0 \text{ Normal}(0,1)$$

$$= \frac{37.7 - 35}{\frac{9.2}{\sqrt{100}}} = 2.9348.$$

And we know that the rejection region $Z > Z_\alpha = 2.33$ from the normal table.

Thus H_0 is rejected as $Z = 2.93 > 2.33$.

Hence, significant evidence at 1% significance level that the mean no. of concurrent users is greater than 35.

9.8 a) Given.

$\bar{X} = 42\text{min}$ $\quad n = 64$.

$\alpha = 0.05$

$\sigma = 5\text{ min}$.

Using the formula $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$= 42 \pm z_{0.025} \frac{5}{\sqrt{64}} = 42 \pm 1.225$

$z_{0.025} = 1.96$ from the normal table

$= ~~[40.8, 43.2]~~$

$= [40.775, 43.225]$

b). Mean installation time $= 40$ mins.

$P(40.775 \leq X \leq 43.225) = P\left(\frac{40.775 - \mu}{\sigma} \leq Z \leq \frac{43.225 - \mu}{\sigma}\right)$

$= P\left(\frac{40.775 - 40}{5} \leq Z \leq \frac{43.225 - 40}{5}\right)$

$= P(0.645 \geq Z \geq 0.155)$

$= 0.7406 - 0.5616$

$= 0.1790$.

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9.10 Given $n = 200$
Find 24 defective items.

a) The defect sample proportion $\frac{24}{200} = 0.12 \; (\hat{p})$

$\alpha = 1 -$ conf. interval (0.96).
$= 0.04$.

$z_{0.02} = 2.054$

~~Using~~ $\cancel{\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}}$. $\cancel{q} \cancel{z}(1+\alpha)/2$.

$\Rightarrow \cancel{0.12 \pm 2.054}$

Using $\hat{p} \pm z_{(1+\alpha)/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\Rightarrow 0.12 \pm 2.054 \sqrt{\frac{0.12(1-0.12)}{200}}$

$= 0.12 \pm 0.047$
$= [0.073, 0.167]$.

b) . Null hypothesis. $H_0: p = 0.1$ vs $H_A: p > 0.1$.

Given. 4% level of significance.

$H_0 : p = 0.1$
$H_1 = p > 0.1$

$$z = \frac{0.12 - 0.10}{\sqrt{\frac{0.1(1-0.1)}{200}}} \approx 0.94.$$

$$p\text{-value} = 1 - \phi(0.94) = 1 - 0.8264 = 0.1736$$

For $\alpha = 0.04$, ~~the p value is 15% the significance level.~~
we have more than 15% level.

q.11 Given old suppliers $\hat{p}_1 = 0.12$
New suppliers $\hat{p}_2 = \frac{13}{150} = 0.0867$

Therefore pooling proportions $\hat{p} = \frac{24+13}{200+150} = \frac{37}{350} = 0.1057$

Let,

$H_0 : p_1 \leq p_2$

$H_A : p_1 > p_2$

$\alpha = 0.05$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.12 - 0.0867}{\sqrt{0.1057(1-0.1057)\left(\frac{1}{200} + \frac{1}{150}\right)}}$$

$$= \frac{0.033}{\sqrt{0.0011}} = \frac{0.033}{0.033} = 1$$

$$p\text{-value } (P(Z > Z_a)) = 1 - P(Z \leq Z_a) = 1 - 0.8413 = 0.1587$$

9.17. Given

A leads B by 10%

$n = 900$

$N_s A - 45\%$

$N_s B = 35\%$

$CI = 95\%$

$\alpha = 0.05$

$Z_{\alpha/2} = 1.96$

~~For~~ Using formula

Error Margin for A $= Z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}$

$= 1.96 \sqrt{\frac{0.45(1-0.45)}{900}}$

$= 0.03250$

$= 3.25\%$

Error Margin for B $= Z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}$

$= 1.96 \sqrt{\frac{0.35(0.65)}{900}}$

$= 0.0312$

$= 3.12\%$

Error margin for A and B

$= Z_{\alpha/2} \sqrt{\frac{p_A(1-p_A)}{n} + \frac{p_B(1-p_B)}{n}}$

$= 1.96 \sqrt{\frac{0.45(0.55) + 0.35(0.65)}{900}}$

$= 1.96(0.0229734)$
$= 0.0456$
$= 4.5\%$ (margin)

9.20 $\alpha = 0.02$ $\sigma_1 = 6.2$ min
$n = 40$ $\sigma_0 = 5$ min

Let
$H_0: \sigma = 5$
$H_A: \sigma \neq 5$

$\therefore$
$$\chi^2 = \frac{(n-1) \times s^2}{\sigma^2} = \frac{(40-1) \times 6.2^2}{5^2} = 59.9664$$

Degrees of freedom (df) = $n-1 = 39$.

p-value = CHIDIST(59.9664, 39) = 0.017043

$t_{critical} = t_{0.02}(39) = 2.426$

The p index is less than the given significance value of 0.02 thus we reject the null hypothesis and accept the alt hypothesis.

9.18
$$\bar{X} = \frac{\sum x_i}{n} \qquad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}.$$

From the attached excel file.

	N	Mean	Std dev
Before change of firewall	14	50	7.62
After change of firewall	20	40.2	7.96

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~~Diff~~ Degrees of freedom $= n_1 + n_2 - 2 = 14 + 20 - 2 = 32$.

a. $\alpha = 1 - 0.95 = 0.05$.

Confidence interval of T-tabulation of $(\alpha, df): \bar{X}_1 - \bar{X}_2 \pm t_{critical}$

$\times \left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} \right)$

$= 50 - 40.2 \pm \cancel{t_{0.025}} * \frac{(7.62^2}{14} + \frac{7.96^2)}{20} = 4.8 \pm (2.037 + 2.7047)$

$= 4.8 \pm 5.5095 = [4.2965, 15.3095]$.

b. H_0 : no signifant diff between avg intrusions.
H_A : Significant diff between the avg ~~test~~ intrusion.

$\alpha = 1 - 0.95 = 0.05$
$z_{\alpha/2} = 2.307$.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{50 - 40.2}{\sqrt{\frac{7.62^2}{14} + \frac{7.96^2}{20}}} = \frac{4.8}{2.7047} = 3.6233$$

p-index = Tdist(3.6233, 32, 2) = 0.000978

As the p-index value is less than the significance value of 0.05, H_0 is rejected and H_A is accepted.

Equal Variances

$$S_p^2 = \Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2 = 7.8206$$

$$t = (\bar{X}_1 - \bar{X}_2)^2 / \left(S_p^2 * \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right) = 3.5962$$

$V = n_1 + n_2 - 2 = 32$

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p-index = Tdist (3.6233, 29, 2) = 0.00978

In both cases we reject H_0.

9.23 From the attached excel sheet.

	Mean	Std. dev
Anthony	85	12.7593
Eric	80	3.2249.

Also given

$\alpha = 0.05$

$n = 6$.

$\therefore \quad z_\alpha = 1.943$

a. Let

$H_0 : \mu_A \leq \mu_E$

$H_A : \mu_A > \mu_E$

Pooled Variance

$$S_p^2 = \frac{(n_A - 1) S_A^2 + (n_B - 1) S_B^2}{n_A + n_B - 2}$$

$$= \frac{(5-1)(12.76)^2 + (5-1)(3.23)^2}{5+5-2}$$

$= 86.593$.

$$t = \frac{X_A - X_B}{\sqrt{S_p^2 * \left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$$

$$= \frac{85 - 80}{(86.593)^2 * \left(\frac{1}{6} + \frac{1}{6}\right)} = 0.9297$$

n

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Degrees of freedom = $df = n_A + n_B - 2 = 6+6-2 = 10$

p-index = Tdist (0.9297, 10, 1) = 0.187138

As p-index is greater than significance ~~te~~ value, so we fail to reject the null hypothesis, and accept that there is no evidence ~~of~~ to Anthony's statement.

b) $H_0 : \sigma_A^2 \leq \sigma_B^2$
$H_A : \sigma_A^2 > \sigma_E$

$$F = \frac{S_A^2}{S_B^2} = \frac{12.76^2}{3.23^2} = 15.70325.$$

Degrees of Freedom $V_A = V_B = n-1 = 5.$

p-index = Fdist (15.70325, 5, 5) = 0.004464

As p-index is lesser than the significance value, hence H_0 is rejected.

Q.9

i. Average amt of information $H = -\Sigma\, p(x) * \log_2 (p(x))$

$$H(\text{Dice } 1) = \left(\frac{-1}{6}\log_2\frac{1}{6}\right) \times 6 = 0.1667 * 2.5849 * 6$$

$$H(\text{Dice } 1) = 2.5849 \approx 2.59.$$

$$H(\text{Dice } 2 = \left(\left(\frac{-1}{9}\log_2\frac{1}{9}\right) * 3\right) + \left(\left(-\frac{2}{9}\log_2\frac{2}{9}\right) * 3\right)$$

$= (0.1111 * 3.1669 * 3) + (0.2222 * 2.1699 * 3)$
$= 2.5032$

$H(Dice\ 3) = \left(\left(-\frac{1}{9}\log_2\frac{1}{9}\right) * 3\right) + \left(\left(\frac{-2}{9}\log_2\frac{2}{9} * 3\right)\right)$

$H(Dice\ 3) = 1.0566 + 1.4466$
$= 2.5032$

(iii) Calculating the entropy : $d = \Sigma\, p(x) * \log_2 \left(\frac{p(x)}{q(x)}\right)$.

a). 2nd dice w.r.t to 1st dice.

$d = \frac{1}{9}\log_2\frac{1/9}{1/6} + \frac{1}{9}\log_2\frac{1/9}{1/6} + 1/9 \log_2\frac{1/9}{1/6} + 2/9\log_2\frac{2/9}{1/6}$

$+ \frac{2}{9}\log_2\frac{2/9}{1/6} + 2/9\log_2\frac{2/9}{1/6}$.

$d = \left(3 * \left(\frac{1}{9}\log_2\frac{6}{9}\right)\right) + \left(3 * \left(\frac{2}{9}\log_2\frac{12}{9}\right)\right)$

$d = (3 * 0.1111 * (-0.5849)) + (3 * 0.2222 * 0.4150)$
$= -0.1950 + 0.2767$
$= 0.0817$

b) 3rd dice w.r.t to 1st dice

As the second and third dice as have the same values.

$d = \frac{1}{9}\log_2\frac{6}{9} + \frac{1}{9}\log_2\frac{6}{9} + \frac{1}{9}\log_2\frac{6}{9} + \frac{2}{9}\log_2\frac{12}{9}$

$+ \frac{2}{9} \log_2 \frac{9/2}{9}$

$= 0.0817$

c) <u>Second dice w.r.t third dice</u>.

$d = \frac{1}{9}\log_2 \frac{1}{} + \frac{1}{9}\log_2 1 + \frac{1}{9}\log_2 \frac{1/9}{2/9} + \frac{2}{9}\log_2 \frac{2/9}{2/9} + \frac{2}{9}\log_2 \frac{2/9}{2/9}$

$+ \frac{2}{9}\log_2 \frac{2/9}{1/9}$.

$d = \left(\frac{6}{9}\log_2 1\right) + \left(\frac{2}{9}\log_2 2\right) + \left(\frac{1}{9}\log_2 \frac{1}{2}\right)$

$d = (0.6667 \times 0) + (0.2222 \times 1) + (0.1111 * -1)$

$d = 0.1111$