Aditya Das

axd5763

1001675762 Assignment - 4

10.31 $X=4$ accidents occured on month 1.
The no. of accidents is a poisson distribution.

To find

Bayes estimator

given
$X = 4$
$n = 1$ (first month)

The poisson distribution with θ number of accidents θ is gamma $(5, 1)$.

Prior distribution of θ is gamma (α, λ).

Posterior distribution of θ is gamma $(\alpha + n\bar{x}, \lambda + n)$

$\Rightarrow$ gamma $(5 + 1 \times 4, 1 + 1)$
$=$ gamma $(9, 2)$

Bayes estimator of $\theta = E(\theta|x)$
$$= \frac{\alpha + n\bar{x}}{(\lambda + n)^2} = \frac{9}{2} = 4.5$$

10.36 Probability mean between 5.0 and 6.0
And the given probability $= 0.95$

To find

a) Conjugate prior distribution

Using the formula

$[\mu - 2.025z, \mu + 2.025z] = [\mu - 1.96z, \mu + 1.96z]$

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$\Rightarrow$ Solving

$$\begin{aligned} \mu - 1.47 &= 5.0 \\ \mu + 1.47 &= 6.0 \\ \hline 2\mu &= 11.0 \\ \mu &= 5.5 \end{aligned}$$

Using $\mu - 1.96\tau = 5.0$ and substituting $\mu = 5.5$

$\Rightarrow 5.5 - 1.96\tau = 5.0$

$\Rightarrow \tau = 0.5/1.96$

$= 0.255$

Hence $(\mu, \tau) = (5.5, 0.255)$.

b). Derive the posterior distribution and find the Bayes estimator of μ.

$x = 6$, $\bar{x} = 6.5$ $\sigma = 2.2$

Posterior distribution

$$\pi(\mu|\bar{x}) \sim N\left(\frac{n\bar{x}/\sigma^2 + \mu/\tau^2}{n/\sigma^2 + 1/\tau^2}, \frac{1}{\sqrt{n/\sigma^2 + 1/\tau^2}}\right)$$

$$\sim N\left(\frac{6 \times 6.5/2.2^2 + 5.5/(0.255)^2}{6/2.2^2 + 1/0.255^2}, \frac{1}{\sqrt{6/(2.2)^2 + 1/(0.255)^2}}\right)$$

$$= N\left(\frac{\frac{39}{4.84} + \frac{5.5}{0.065}}{6 \times 4.84 + \frac{1}{0.065}}, \frac{1}{\sqrt{\frac{6}{4.84} + \frac{1}{0.065}}}\right)$$

$$= N\left(\frac{8.051 + 84.61}{29.04 + 15.38}, \frac{1}{\sqrt{16.618}}\right)$$

$$\sim N\left(\frac{92.641}{16.618}, \frac{1}{\sqrt{16.618}}\right)$$

$$\sim N(5.575, 0.245)$$

Bayes estimator, $\mu = E(\mu|x)$
$= 5.575$

Posterior Risk $= Var(M|x)$
$\Rightarrow \frac{1}{16.618} = 0.060$

c) Compute 95% HPD credible set for μ.

HPD $=$ ~~[...]~~ $\mu_x \pm z_{\frac{\alpha}{2}} \tau_x$
$= 5.575 \pm (1.96)(0.245)$
$= 5.575 \pm 0.480$
$= [5.095, 6.055]$

10.57 Given prior probability

$$\pi(\theta) = \begin{cases} \text{fair} & 0.99 \\ \text{biased} & 0.01 \end{cases}$$

$f(x/\theta = \text{fair})$ represent as Binomial distribution
$n=10$, $p=0.5$

$f(x/\theta = \text{biased}) = \text{Uniform}(0,1)$

$$\pi(\theta = \text{fair}|x) = \frac{f(x/\theta = \cancel{\text{fair}})\,\pi(\text{fair})}{m(x)}$$

$$= \frac{0.5^{10} \times 0.99}{0.5^{10} \times (0.99) + \int_0^1 \theta^{10}\, d\theta (0.01)}$$

$$= \frac{0.5^{16} \times 0.99}{0.5^{10}(0.99) + \frac{0.01}{11}}$$

$$= 0.5154$$

Hence $\pi(\theta = \text{biased}|x) = 1 - \pi(\theta = \text{fair}|x)$

$$\Rightarrow 1 - 0.5154$$

$$\Rightarrow 0.4846.$$