#### Classification: Decision Tree

CSE 4334 / 5334 Data Mining Spring 2019

#### **Won Hwa Kim**

(Slides courtesy of Pang-Ning Tan, Michael Steinbach and Vioin Kumar, and Jiawei Han, Michaeline Kamber and Jian Pei)



## Classification: Definition



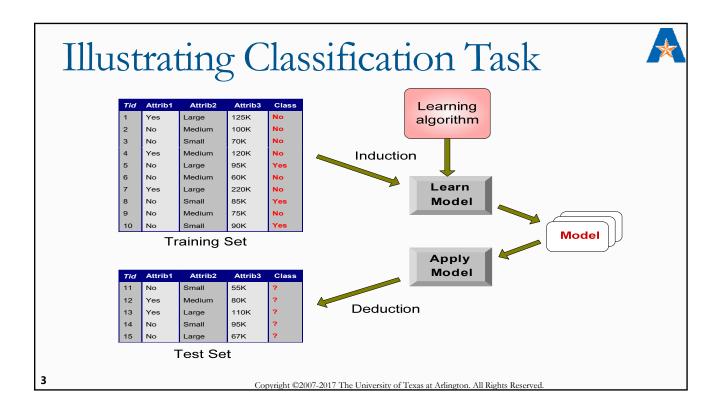
Given a collection of records (training set)

o Each record contains a set of *attributes*, one of the attributes is the *class*.

Find a *model* for class attribute as a function of the values of other attributes.

Goal: <u>previously unseen</u> records should be assigned a class as accurately as possible.

o A *test set* is used to determine the accuracy of the model. Usually, the given data set is divided into training and test sets, with training set used to build the model and test set used to validate it.

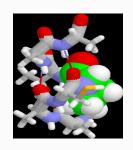


# Examples of Classification Task



- o Predicting tumor cells as benign or malignant
- o Classifying credit card transactions as legitimate or fraudulent
- o Classifying secondary structures of protein as alpha-helix, beta-sheet, or random coil
- o Categorizing news stories as finance, weather, entertainment, sports, etc







## Classification vs. Prediction

#### Classification

- o Predicts categorical class labels
- o Most suited for nominal attributes
- o Less effective for ordinal attributes

#### Prediction

- o models continuous-valued functions or ordinal attributes, i.e., predicts unknown or missing values
- o E.g., Linear regression

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.



# Supervised vs. Unsupervised Learning

### Supervised learning (e.g., classification)

- o Supervision: The training data (observations, measurements, etc.) are accompanied by labels indicating the class of the observations
- o New data is classified based on the training set

### Unsupervised learning (e.g., clustering)

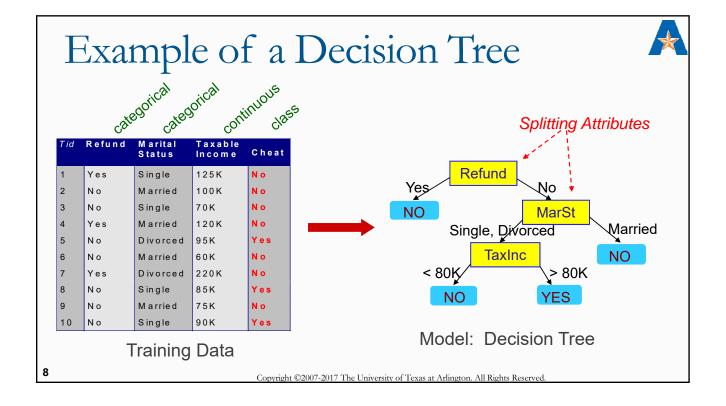
- o The class labels of training data is unknown
- o Given a set of measurements, observations, etc. with the aim of establishing the existence of classes or clusters in the data

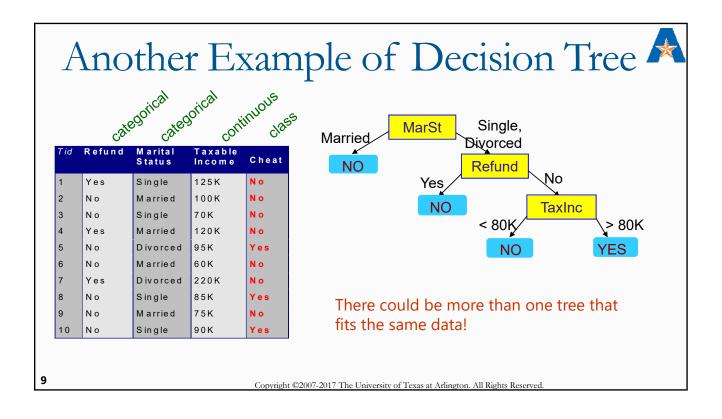
# Classification Techniques

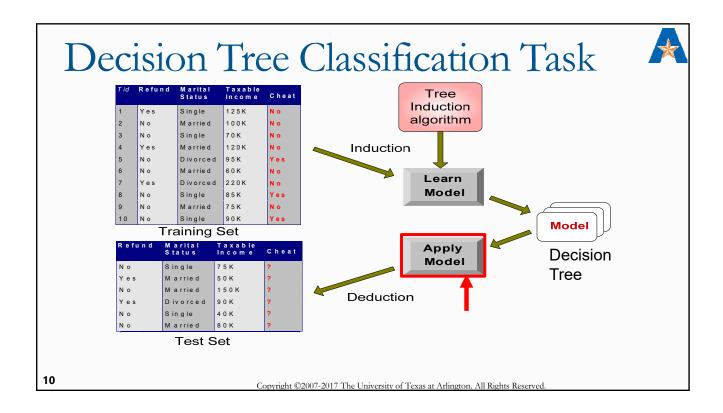


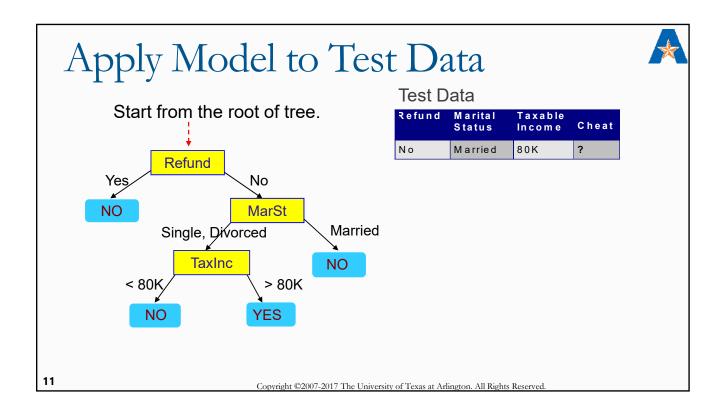
- o Decision Tree based Methods
- o Rule-based Methods
- o Memory based reasoning
- o Neural Networks
- o Naïve Bayes and Bayesian Belief Networks
- o Support Vector Machines

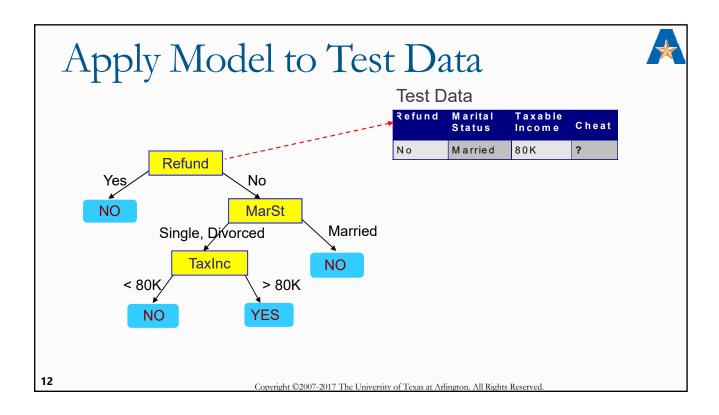
7

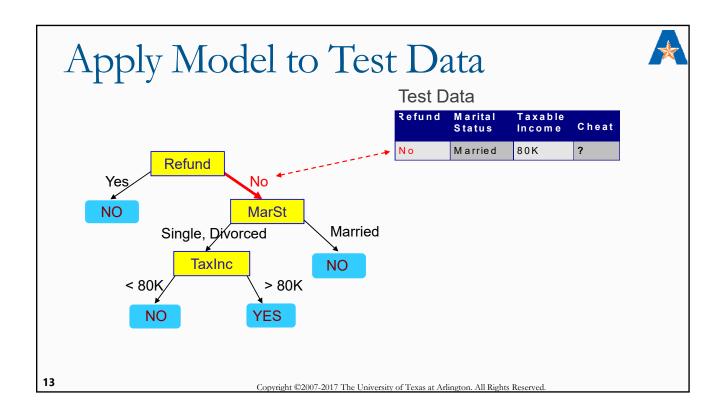


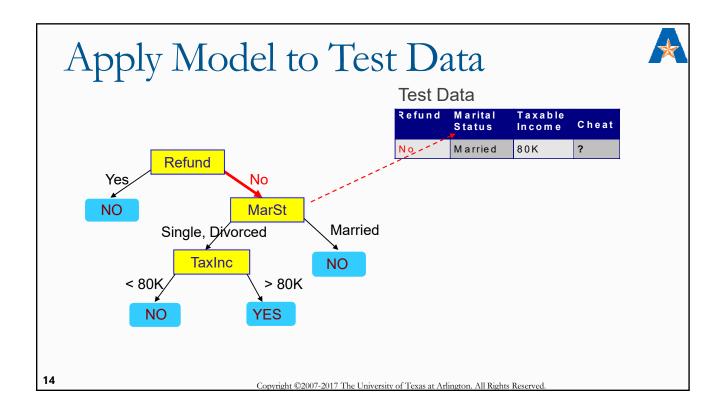


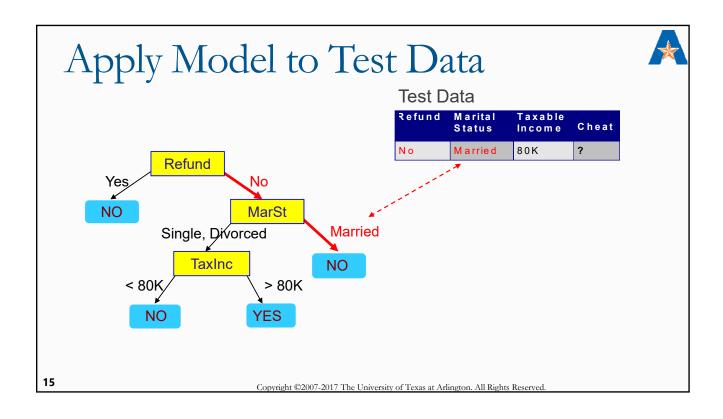


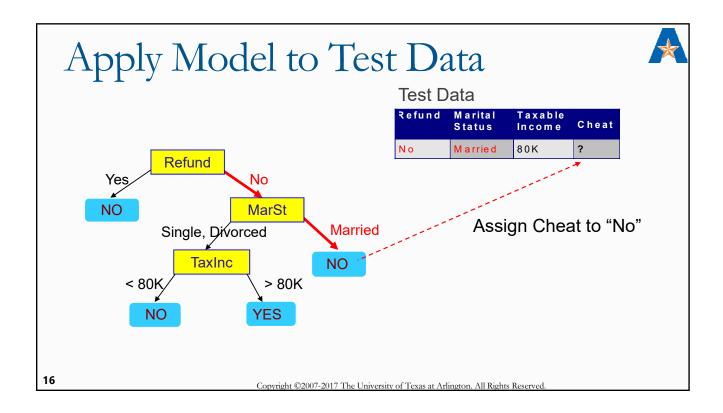


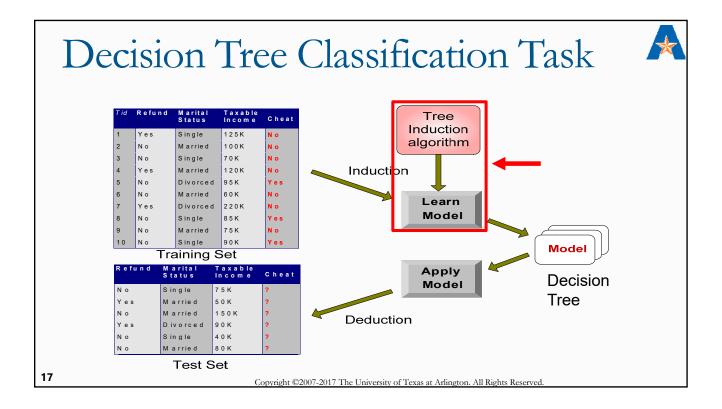












## Decision Tree Induction



#### Large search space

- o Exponential size, with respect to the set of attributes
- o Finding the optimal decision tree is computationally infeasible

# Efficient algorithm for accurate suboptimal decision tree

o Greedy strategy

18

o Grow the tree by making locally optimally decisions in selecting the attributes

## Decision Tree Induction



#### Many Algorithms:

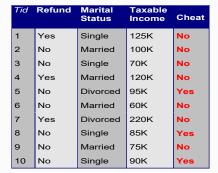
- o Hunt's Algorithm (one of the earliest)
- o CART
- o ID3, C4.5
- o SLIQ, SPRINT

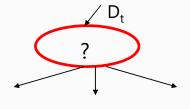
19

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved

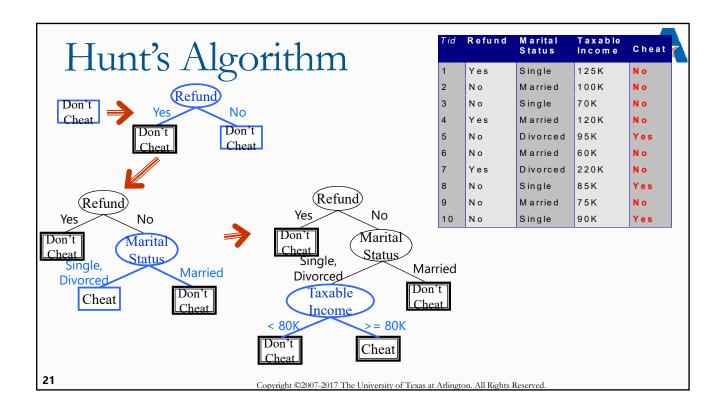
# General Structure of Hunt's Algorithm

- o Let D<sub>t</sub> be the set of training records that reach a node t
- o General Procedure:
  - o If D<sub>t</sub> contains records that belong the same class y<sub>t</sub>, then t is a leaf node labeled as y<sub>t</sub>.
    o If D<sub>t</sub> is an empty set, then t is a leaf node labeled
  - o If D<sub>t</sub> is an empty set, then t is a leaf node labeled by the majority class among the records of Dt's parent node.
  - If D<sub>t</sub> contains records that have identical values on all attributes but the class attribute, then t is a leaf node labeled by the majority class among D<sub>t</sub>'s records.
  - o If none of the above conditions is satisfied, use an attribute test to split the data into smaller subsets. Recursively apply the procedure to each subset.





20



### Tree Induction



#### Greedy strategy.

o Split the records based on an attribute test that optimizes certain criterion.

#### Issues

- o Determine how to split the records
  - How to specify the attribute test condition?
  - How to determine the best split?
- o Determine when to stop splitting

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

22

### Tree Induction



#### Greedy strategy.

o Split the records based on an attribute test that optimizes certain criterion.

#### **Issues**

- o Determine how to split the records
  - How to specify the attribute test condition?
  - How to determine the best split?
- o Determine when to stop splitting

23

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

# How to Specify Test Condition?



### Depends on attribute types

- o Categorical vs. Numeric
  - Categorical attributes: Nominal, Ordinal
  - Numeric attributes: Interval, Ratio
- o Discrete vs. Continuous

### Depends on number of ways to split

- o 2-way split
- o Multi-way split

24

# Splitting Based on Nominal Attributes



Multi-way split: Use as many partitions as distinct values.

Family CarType Luxury
Sports

Binary split: Divides values into two subsets.

Need to find optimal partitioning.



OR



25

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

# Splitting Based on Ordinal Attributes



Multi-way split: Use as many partitions as distinct values.

Size Large Medium

Binary split: Divides values into two subsets.

Need to find optimal partitioning.

{Small, Size {Large}

OR

{Medium, Size Large} {Small}

What about this split?

{Small, Size {Medium}

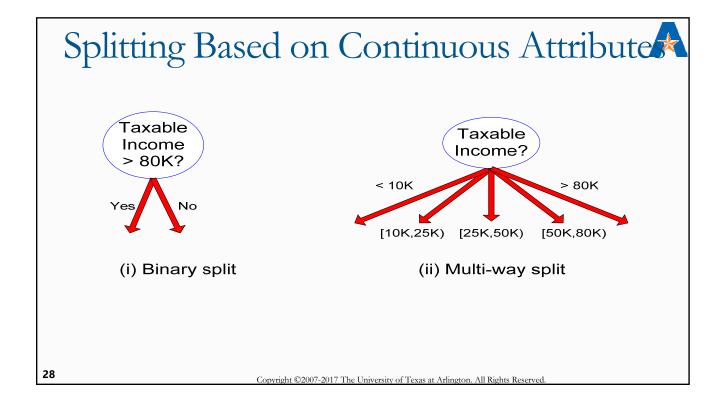
26



#### Different ways of handling

- o Discretization to form an ordinal categorical attribute
  - Static discretize once at the beginning
  - Dynamic ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
- o Binary Decision: (A < v) or  $(A \ge v)$ 
  - consider all possible splits and finds the best cut
  - can be more compute intensive

27



### Tree Induction



#### Greedy strategy.

o Split the records based on an attribute test that optimizes certain criterion.

#### **Issues**

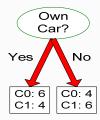
- o Determine how to split the records
  - How to specify the attribute test condition?
  - How to determine the best split?
- o Determine when to stop splitting

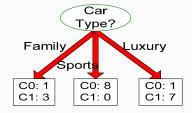
ppyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved

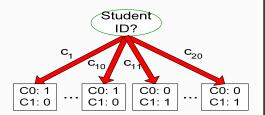
# How to determine the Best Split



Before Splitting: 10 records of class 0, 10 records of class 1







Which test condition is the best?

30

# How to determine the Best Split



- o Greedy approach:
  - Nodes with homogeneous class distribution are preferred
- o Need a measure of node impurity:

C0: 5 C1: 5

C0: 9 C1: 1

Non-homogeneous,

Homogeneous,

High degree of impurity

Low degree of impurity

31

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

# Measures of Node Impurity

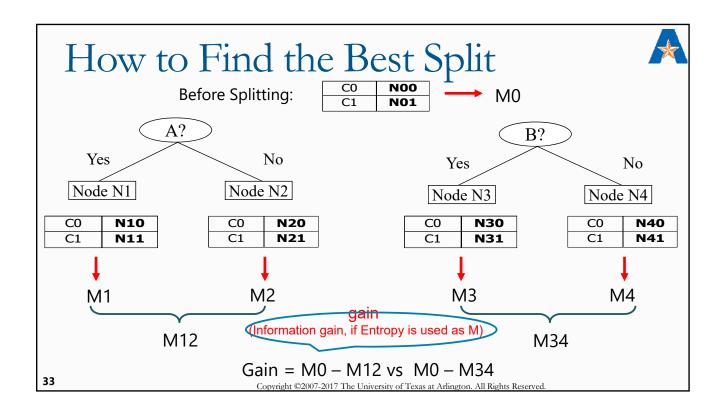


Gini Index

Entropy

Misclassification error

32



# Measure of Impurity: GINI



Gini Index for a given node t:

GINI 
$$(t) = 1 - \sum_{j} [p(j | t)]^{2}$$

(NOTE:  $p(j \mid t)$  is the relative frequency of class j at node t).

- o Maximum  $(1 1/n_c)$  when records are equally distributed among all classes, implying least interesting information
- o Minimum (0.0) when all records belong to one class, implying most interesting information

C1	0
C2	6
Gini= 0.000	

C1 1 C2 5 Gini= 0.278 C1 2 C2 4 Gini= 0.444

C1 3 C2 3 Gini= 0.500

34

# Examples for computing GINI



GINI 
$$(t) = 1 - \sum_{j} [p(j | t)]^{2}$$

C 1	0
C 2	6

$$P(C1) = 0/6 = 0$$
  $P(C2) = 6/6 = 1$ 

Gini = 
$$1 - P(C1)^2 - P(C2)^2 = 1 - 0 - 1 = 0$$

$$P(C1) = 1/6$$
  $P(C2) = 5/6$ 

Gini = 
$$1 - (1/6)^2 - (5/6)^2 = 0.278$$

$$P(C1) = 2/6$$
  $P(C2) = 4/6$ 

Gini = 
$$1 - (2/6)^2 - (4/6)^2 = 0.444$$

35

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

# Splitting Based on GINI



- o Used in CART, SLIQ, SPRINT.
- o When a node p is split into k partitions (children), the quality of split is computed as,

$$GINI _{split} = \sum_{i=1}^{k} \frac{n_i}{n} GINI (i)$$

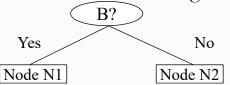
where,  $n_i$  = number of records at child i, n = number of records at node p.

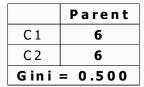
36

## Binary Attributes: Computing GINI Index



- Splits into two partitions
- o Effect of Weighing partitions:
  - Larger and Purer Partitions are sought for.





Gini(N1)

$$= 1 - (5/7)^2 - (2/7)^2$$

$$= 0.408$$

Gini(N2)

$$= 1 - (1/5)^2 - (4/5)^2$$

 $= 1 - (1/5)^2 - (1/5)^2 - (1/5)^2 = 0.32$ 

	N1	N2
C1	5	1
C2	2	4
Gini=0.371		

Gini(Children)

$$= 0.371$$

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

### Categorical Attributes: Computing Gini Index



- o For each distinct value, gather counts for each class in the dataset
- o Use the count matrix to make decisions

Multi-way split

	CarType		
	Family	Sports	Luxury
C 1	1	2	1
C 2	4	1	1
Gini		0.393	

Two-way split (find best partition of values)

	CarType	
	{Sports, Luxury} {Family}	
C 1	3	1
C 2	2	4
Gini	0.4	0 0

	CarType	
	{Sports}	{Family, Luxury}
C 1	2	2
C 2	1	5
Gini	0.4	19

38

### Continuous Attributes: Computing Gini Index



- o Use Binary Decisions based on one value
- o Several Choices for the splitting value
  - Number of possible splitting valuesNumber of distinct values
- o Each splitting value has a count matrix associated with it
  - o Class counts in each of the partitions,  $A \le v$  and  $A \ge v$
- o Simple method to choose best v
  - o For each v, scan the database to gather count matrix and compute its Gini index
  - o Computationally Inefficient! Repetition of work.

Cheat Status Income 125K No Yes No 100K Married No No 70K No Yes 120K No No 95K No 60K Yes 220K No No 85K Yes No Married No 75K Single 90K



39

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

### Continuous Attributes: Computing Gini Index...



For efficient computation: for each attribute,

- o Sort the attribute on values
  - o Linearly scan these values, each time updating the count matrix and computing gini index
  - o Choose the split position that has the least gini index



### Alternative Splitting Criteria based on INFO



Entropy at a given node t:

Entropy 
$$(t) = -\sum_{j} p(j \mid t) \log p(j \mid t)$$

(NOTE:  $p(j \mid t)$  is the relative frequency of class j at node t).

- o Measures homogeneity of a node.
  - Maximum (log n<sub>c</sub>) when records are equally distributed among all classes implying least information
  - Minimum (0.0) when all records belong to one class, implying most information
- o Entropy based computations are similar to the GINI index computations

  Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

41

Examples for computing Entropy



Entropy 
$$(t) = -\sum_{j} p(j \mid t) \log_{2} p(j \mid t)$$

C 1	0
C 2	6

$$P(C1) = 0/6 = 0$$
  $P(C2) = 6/6 = 1$ 

Entropy = 
$$-0 \log 0 - 1 \log 1 = -0 - 0 = 0$$

$$P(C1) = 1/6$$
  $P(C2) = 5/6$ 

Entropy = 
$$-(1/6) \log_2 (1/6) - (5/6) \log_2 (5/6) = 0.65$$

$$P(C1) = 2/6$$
  $P(C2) = 4/6$ 

Entropy = 
$$-(2/6) \log_2(2/6) - (4/6) \log_2(4/6) = 0.92$$

42



# Why is that $0 \log 0 = 0$ ?

$$\lim_{x \to 0} x \log_2(x) = \lim_{x \to 0} \frac{\frac{\ln(x)}{\ln(2)}}{x^{-1}} = \lim_{x \to 0} \frac{\frac{x^{-1}}{\ln(2)}}{-x^{-2}} = \lim_{x \to 0} \frac{-x}{\ln(2)} = 0$$

L'Hospital's Rule (Wikipedia)

 $\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0 \text{ or } \pm \infty, \text{ and}$   $\lim_{x \to c} \frac{f'(x)}{g'(x)} \text{ exists, and}$   $g'(x) \neq 0 \text{ for all } x \text{ in } I \text{ with } x \neq c,$ 

then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

43

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved

### Splitting Based on INFO...



#### Information Gain:

GAIN <sub>split</sub> = Entropy 
$$(p) - \left(\sum_{i=1}^{k} \frac{n_i}{n} Entropy (i)\right)$$

Parent Node, p is split into k partitions;

n<sub>i</sub> is number of records in partition i

- o Measures Reduction in Entropy achieved because of the split. Choose the split that achieves most reduction (maximizes GAIN)
- o Used in ID3 and C4.5
- o Disadvantage: Tends to prefer splits that result in large number of partitions, each being small but pure.

1

### Splitting Based on INFO...



#### Gain Ratio:

$$GainRATIO \qquad _{split} = \frac{GAIN _{Split}}{SplitINFO}$$

$$SplitINFO = -\sum_{i=1}^{k} \frac{n_{i}}{n} \log \frac{n_{i}}{n}$$

Parent Node, p is split into k partitions n; is the number of records in partition i

- o Adjusts Information Gain by the entropy of the partitioning (SplitINFO). Higher entropy partitioning (large number of small partitions) is penalized!
- o Used in C4.5
- o Designed to overcome the disadvantage of Information Gain

45

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

### Splitting Criteria based on Classification Error



Classification error at a node t:

Error 
$$(t) = 1 - \max_{i} P(i \mid t)$$

Measures misclassification error made by a node.

- o Maximum  $(1 1/n_c)$  when records are equally distributed among all classes, implying least interesting information
- o Minimum (0.0) when all records belong to one class, implying most interesting information

46

# **Examples for Computing Error**



Error 
$$(t) = 1 - \max_{i} P(i \mid t)$$

C 1	0
C 2	6

$$P(C1) = 0/6 = 0$$
  $P(C2) = 6/6 = 1$ 

Error = 
$$1 - \max(0, 1) = 1 - 1 = 0$$

$$P(C1) = 1/6$$
  $P(C2) = 5/6$ 

Error = 
$$1 - \max(1/6, 5/6) = 1 - 5/6 = 1/6$$

$$P(C1) = 2/6$$
  $P(C2) = 4/6$ 

Error = 
$$1 - \max(2/6, 4/6) = 1 - 4/6 = 1/3$$

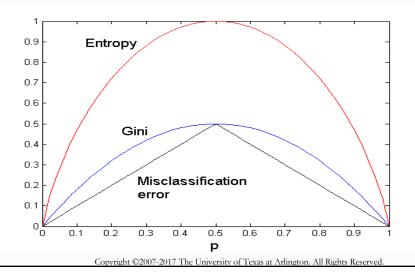
47

48

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

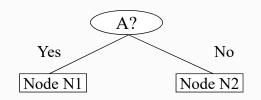
# Comparison among Splitting Criteria

#### For a 2-class problem:



## Misclassification Error vs Gini





	Parent
C 1	7
C 2	3
Gini = 0.42	

Gini(N1)  
= 
$$1 - (3/3)^2 - (0/3)^2$$
  
= 0

Gini(Children) = 3/10 \* 0 + 7/10 \* 0.489 = 0.342

Gini improves !!

Gini(N2) =  $1 - (4/7)^2 - (3/7)^2$ = 0.489

49

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

## Tree Induction



#### Greedy strategy.

o Split the records based on an attribute test that optimizes certain criterion.

#### Issues

- o Determine how to split the records
  - o How to specify the attribute test condition?
  - o How to determine the best split?
- o Determine when to stop splitting

50

# Stopping Criteria for Tree Induction



- o Stop expanding a node when all the records belong to the same class
- o Stop expanding a node when all the records have similar attribute values
  - o What to do? majority voting
- o Early termination, e.g., when the information gain is below a threshold.

51

Copyright ©2007-2017 The University of Texas at Arlington. All Rights Reserved.

## Decision Tree Based Classification



#### Advantages:

- o Inexpensive to construct
- o Extremely fast at classifying unknown records
- o Easy to interpret for small-sized trees
- o Accuracy is comparable to other classification techniques for many simple data sets

52

# Example: C4.5



Simple depth-first construction.

Uses Information Gain

Sorts Continuous Attributes at each node.

Needs entire data to fit in memory.

Unsuitable for Large Datasets.

o Needs out-of-core sorting.

You can download the software from:

http://www.cse.unsw.edu.au/~quinlan/c4.5r8.tar.gz

53