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1. Apakah sistem dinamik berikut bersifat causal, time invariant, dan/atau zero state linear? (beri alasan)

a) $y(t) = \int_{t}^{t+2} t^2 u(\tau)\, d\tau$

↳ bersifat non-causal, karena bergantung pada waktu mendatang $(t+2)$.

↳ Time invariant or Time Variying ?

* Actual Output

$$\int_{t}^{t+2} t^2 u(\tau)\, d\tau$$

$$= \int_{t-t_0}^{t-t_0+2} (t-t_0)^2 u(\tau - t_0)\, d\tau$$

$$= \int_{t-t_0}^{t-t_0+2} (t-t_0)^2 u(\tau - t_0)\, d\tau$$

$$= \int_{t-t_0}^{t-t_0+2} (t-t_0)^2 u(\alpha)\, d\alpha$$

* Desired Output

$$y(t-t_0) = \int_{t-t_0}^{t-t_0+2} (t-t_0)^2 u(\tau)\, d\tau$$

Actual Output $\neq$ Desired Output $\Rightarrow$ Time Variying.

2. Cari Transformasi Laplace untuk fungsi berikut (asumsi $f(t) = 0$, untuk $t<0$)

a. $F(t) = t^2 e^{-at}$ for $t \geq 0$

Penyelesaian :

$$F(t) = t^2 e^{-at} = \int_0^\infty e^{-st} \cdot t^2 \cdot e^{-at}\, dt$$

$$= \int_0^\infty t^2 e^{-st-at}\, dt$$

$$= \int_0^\infty t^2 e^{-(s+a)t}\, dt \longrightarrow u = t^2,\ du = 2t\, dt$$

$$dv = e^{-(s+a)t},\quad v = \frac{e^{-(s+a)t}}{-(s+a)}$$

$$= \int u\,dv = uv - \int v\,du$$

$$= \frac{t^2 \cdot e^{-(s+a)t}}{-(s+a)} - \int_0^\infty \frac{e^{-(s+a)t}}{-(s+a)} \cdot 2t\, dt$$

$$= \frac{-t^2 e^{-(s+a)t}}{s+a} + \int_0^\infty \frac{2t \cdot e^{-(s+a)t}}{s+a}\, dt \qquad u = 2t,\ du = 2\,dt$$

$$dv = \frac{e^{-(s+a)t}}{s+a},\quad v = \frac{e^{-(s+a)t}}{-(s+a)^2}$$

$$= \frac{-t^2 e^{-(s+a)t}}{s+a} - \frac{2te^{-(s+a)t}}{(s+a)^2} + \int \frac{2 \cdot e^{-(s+a)t}}{(s+a)^2}\, dt$$

$$= \left. \frac{-t^2 \cdot e^{-(s+a)t}}{s+a} - \frac{2te^{-(s+a)t}}{(s+a)^2} - \frac{2e^{-(s+a)t}}{(s+a)^3} \right]_0^\infty$$

$$= \frac{-\infty^2 \cdot e^{-(s+a)\infty}}{s+a} - \frac{2\infty e^{-(s+a)\infty}}{(s+a)^2} - \frac{2e^{-(s+a)\infty}}{(s+a)^3} - \left(0 - 0 - \frac{2}{(s+a)^3} \right)$$

$$= \frac{2}{(s+a)^3}$$

b. $f(t) = e^{-0.4t} \cos 12t$ for $t \geq 0$

Penyelesaian :

$$\int_0^\infty e^{-st} \cdot e^{-0.4t} \cdot \cos 12\, dt$$

$$= \int_0^\infty e^{(-s-0,4)t} \cdot \cos 12t \Rightarrow u = \cos 12t \quad du = -12 \sin 12t$$

$$dv = e^{(-s-0,4)t}, \quad v = -\frac{1}{s+0,4} e^{(-s-0,4)t}$$

$uv - \int v\,du$

$$= \left. \frac{-\cos 12t}{s+0,4} e^{(-s-0,4)t} \right]_0^\infty - \int \frac{12}{s+0,4} e^{(-s-0,4)t} \sin 12t\, dt$$

$$= \left(\frac{-\cos 12(\infty) \cdot e^{(-s-0,4)\infty}}{s+0,4} + \frac{\cos 12(0)}{s+0,4} \cdot e^{(-s-0,4)0} \right) - \frac{12}{s+0,4} \int e^{(-s-0,4)t} \sin 12t$$

$$u = \sin 12t, \quad du = 12 \cos 12t$$

$$dv = e^{(-s-a)t}, \quad v = -\frac{1}{s+0,4} e^{(-s-0,4)}$$

$$= \frac{1}{s+0,4} - \frac{12}{s+0,4} \left(\left. \frac{-\sin 12t}{s+0,4} e^{(-s-0,4)t} \right]_0^\infty - \int -\frac{12}{s+0,4} e^{(-s-0,4)t} \cos 12t\, dt \right)$$

$$= \frac{1}{s+0,4} - \frac{12}{s+0,4} \left(\frac{-\sin 12(\infty)}{s+0,4} + \frac{\sin(2(0)) \cdot e^{(-s-0,4)0}}{s+0,4} \right) + \frac{12}{s+0,4} \int e^{(-s-0,4)t} \cos 12t\, dt$$

$$\mathcal{L}\{e^{-0,4t} \cos 12t\} = y$$

$$y = \frac{1}{s+0,4} - \frac{12^2}{(s+0,4)^2} y$$

$$y + \frac{12^2}{(s+0,4)^2} y = \frac{1}{s+0,4}$$

$$\frac{(s+0,4)^2 + 12^2}{(s+0,4)^2} = \frac{1}{s+0,4}$$

$$y = \frac{\frac{1}{s+0,4}}{\frac{(s+0,4)^2 + 12^2}{(s+0,4)^2}}$$

$$y = \frac{1}{\cancel{(s+0,4)}} \cdot \frac{(s+0,4)^{\cancel{2}}}{(s+0,4)^2 + 12^2}$$

$$= \frac{(s+0,4)}{(s+0,4)^2 + \cancel{12^2}\, 144}$$

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