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Indian Institute of Technology Hyderabad

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A Comprehensive Analysis of Motorcycle Dynamics

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ABSTRACT

An attempt to create a realistic simulation of the motion of a motorcycle taking into consideration its multiple rigid bodies and constraints, and forces acting on it by utilising Lagrangian mechanics. Later using the simulation to compare different riding styles and maneuvers and see how the motorcycle reacts to various inputs.

Contents

1 Geometry of Motorcycles

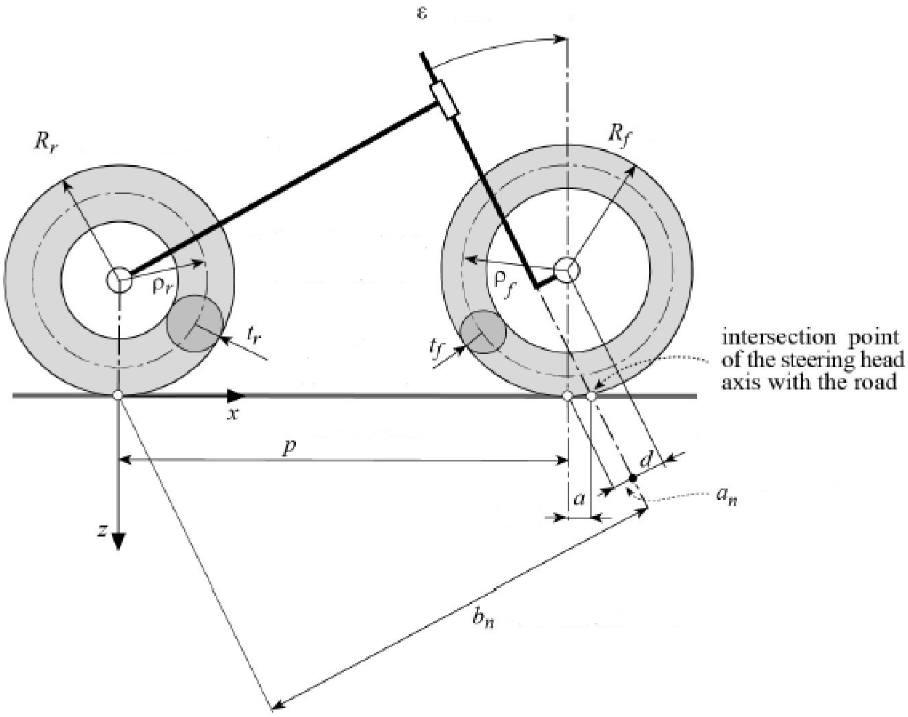

Figure 1: Geometry of a motorcycle [1]

Fig.1 shows us the various geometrical parameters of a motorcycle to be taken into consideration, which include:

- p wheelbase : distance between the contact points of tires on the road
- d fork offset : perpendicular distance between the axis of the steering head and the center of the front wheel
- ε caster angle/rake : the angle between the vertical axis and the rotation axis of the steering head
- R_r and R_f : radius of the rear and front wheel
- t_r and t_f : radius of the rear and front tire cross section
- a trail : the distance between the contact point of the front wheel and the intersection point of the steering head axis with the road measured in the ground plane

1.1 Importance of wheelbase, rake and trail

Trail (a), rake (ε) and wheelbase (p) are important in defining the maneuverability of the motorcycle as perceived by the rider. Each of these values are intertwined and combinations of them determine riding styles of motorcycles.
Wheelbase ranges from 1200 *mm* in the case of small scooters to 1300 *mm* for light motorcycles (125 *cc* displacement) to 1350 *mm* for medium displacement motorcycles (250 *cc*) up to 1600 *mm*,

and beyond, for touring motorcycles with greater displacement.
In general an increase in wheelbase leads to a :

- unfavourable increase in deformability of frame
- unfavorable increase in the minimum curvature radius
- favourable decrease in pitching motion (due to road conditions or due to throttle)
- favorable increase in directional stability

Rake and trail directly influence the wheelbase. An increase in rake or trail results to an increase in wheelbase and as such the above mentioned points are true for these two parameters as well.

1.1.1 Stabilizing effect of positive trail

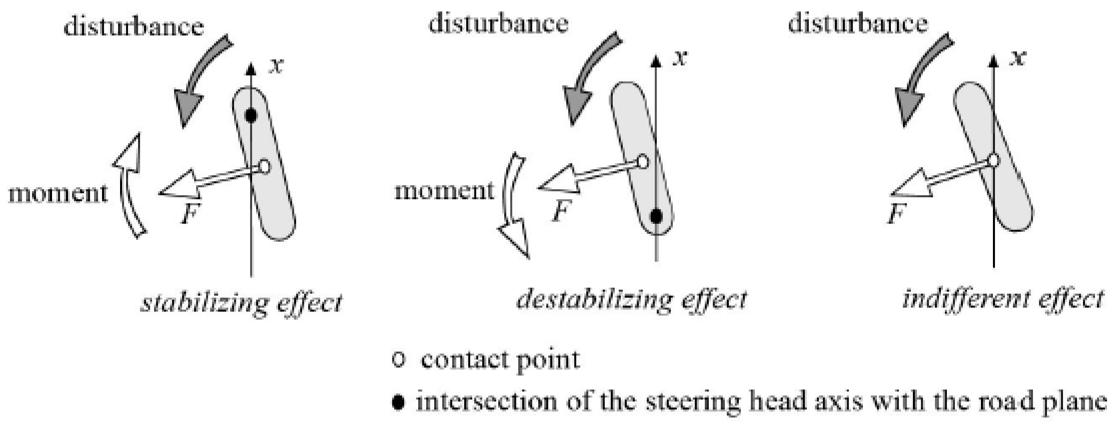

Figure 2: Trail and stability [1]

In the three figures displayed F is the lateral force acting on the front tire produced due to the steering angle. It acts at the contact point of the tire on the road. The wheel rotates about the vertical axis (in this case the axis is coming out of the paper) about the intersection of the steering head axis with the road plane.The motorcycle is moving in the positive x direction.
Let us consider a scenario where an external disturbance forced the tire to rotate as shown in the images. In the right-most image, both the points of interest are coincidental and hence the lateral force causes no moment about the tire's rotating axis. In the middle image the trail is negative (black dot lies before white dot) and it can be seen how the moment due to the lateral force aligns with the disturbance causing a destabilizing effect. The opposite happens when the trail is positive as seen in the left-most image causing a stabilizing effect.

1.2 Kinematic steering angle (Δ) and Front wheel camber (β)

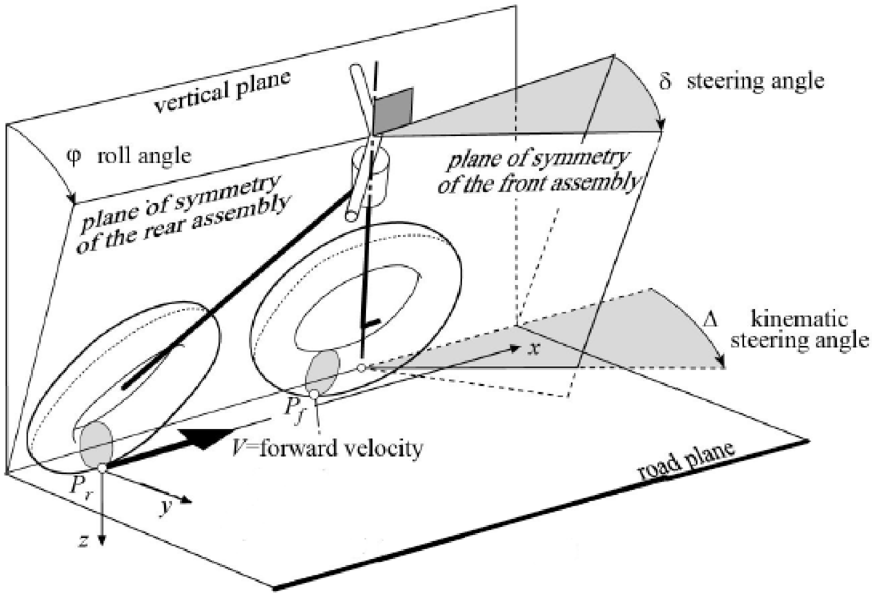

Figure 3: Kinematic steering angle [1]

The steering angle δ is the angle between the rear and front wheel planes, while the kinematic steering angle Δ represents the intersection of this actual angle with the road plane. The kinematic steering angle can be expressed in terms of the caster angle (ε), the roll angle (ϕ) and the steering angle (δ), as [1] :

$$\Delta = \tan^{-1}\left(\frac{\cos\varepsilon}{\cos\phi}\tan\delta\right) \tag{1}$$

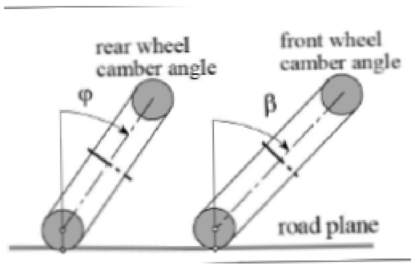

Figure 4: Camber angles of both tires [1]

The camber angle β of the front wheel is different from the roll angle ϕ of the rear frame when the steering angle δ is non-zero. The front frame is always more titled than the rear frame when δ is non zero and the same sign as ϕ. It can be expressed as [1] :

$$\beta = \sin^{-1}(\cos\delta\sin\phi + \cos\phi\sin\delta\sin\varepsilon) \tag{2}$$

2 Forces and Moments

Most of the forces and moments experienced by a motorcycle are due to the tire and road interactions. A major factor in these forces is the deformability and elasticity of the material tires are made out of.

2.1 Longitudinal forces

During the tire's rolling, the contact patch of the tire on the ground, which is not a simple line as the tire is squished from the weight above it, undergoes deflections. Stresses are generated in the contact area which are both normal, due to load, and shear, due to the presence of both adhesion and slip zones. Due to material properties of tires, some portion of the contact patch 'sticks' to the road, but this bond is temporary due to the rolling motion of the rest of the tire. The contact patch that does not stick to the track is called the sliding zone.
This longitudinal slip can be quantified as :

$$\kappa = \frac{V - \omega R}{V} \tag{3}$$

where V is the forward velocity, ω is the angular velocity of the wheel and R is the rolling radius.
the longitudinal force due to thrust or braking at nominal load N can be described by the means of Pacejka's Magic Formula (1993) :

$$F(braking)/S(thrust) = ND_\kappa \sin\left[C_\kappa \tan^{-1}\left[B_\kappa\kappa - E_\kappa\left(B_\kappa\kappa - \tan^{-1}\left(B_\kappa\kappa\right)\right)\right]\right] \tag{4}$$

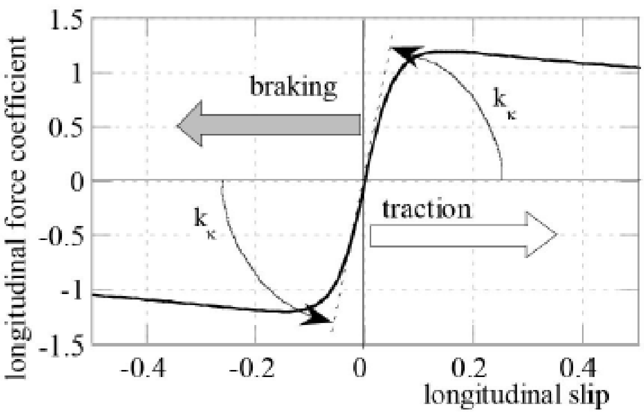

Figure 5: Qualitative variation of the braking/driving force coefficient versus slip [1]

The fitting parameters B_κ, C_κ, D_κ, E_κ are unique to different tires. $D_\kappa C_\kappa B_\kappa$ is a measure of the longitudinal slip stiffness.
Comparing the amount of slip necessary to produce forces at the tire between tires can also tell us about their wear. A tire that would require more slip to produce a certain amount of traction would experience more rapid wear than a tire that requires lesser slip.

2.2 Rolling Resistance

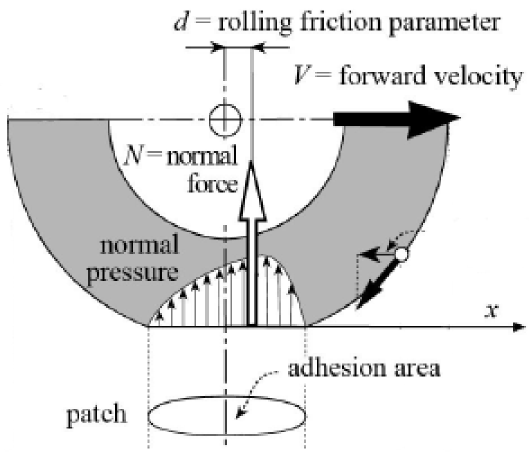

Figure 6: Description of contact pressures on a rolling wheel [1]

Assuming that the adhesion area forms in the front part ($+x$ direction here) of the contact patch, the normal force is displaced forward with respect to the center of the wheel by a distance d.

$$d = f_w R \tag{5}$$

where f_w is called the rolling resistance coefficient. And to move the wheel forward a rolling resistance moment (M_w) has to be overcome,

$$M_w = dN \tag{6}$$

2.3 Lateral Force

The lateral force is generated mainly by camber (β or ϕ) and the sideslip angle (λ). The sideslip angle is defined as the angle measured in the road plane between the direction of travel and the intersection of the wheel plane with the road plane as show in Fig7.

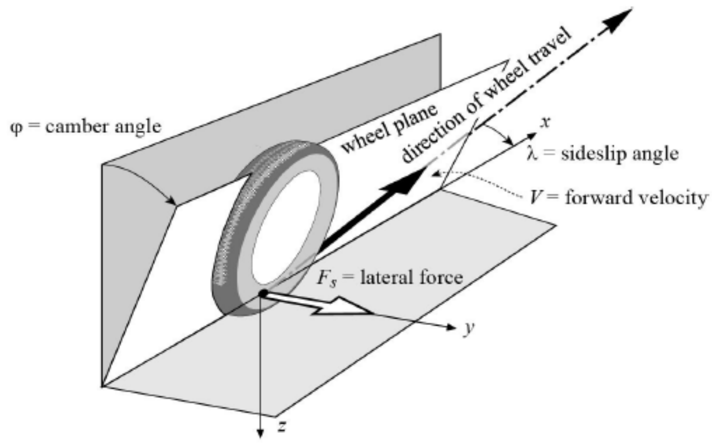

Figure 7: Depiction of sideslip angle [1]

Pacejka's Magic Formula can be used once again to describe the dependency of lateral force on camber, sideslip and normal force :

$$F_s = ND_s\left[\sin\left[C_\lambda \tan^{-1}\left[B_\lambda\lambda - E_\lambda\left(B_\lambda\lambda - \tan^{-1}(B_\lambda\lambda)\right)\right]\right] + \sin\left[C_\phi \tan^{-1}\left[B_\phi\phi - E_\phi\left(B_\phi\phi - \tan^{-1}\left(B_\phi\phi\right)\right)\right]\right]\right] \tag{7}$$

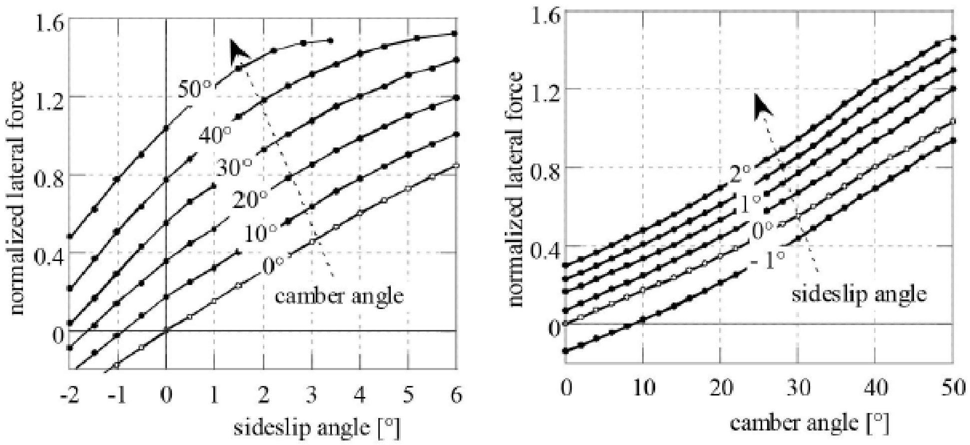

Figure 8: Measured values of lateral force with varying camber and sideslip [1]

$D_\lambda C_\lambda B_\lambda$ and $D_\phi C_\phi B_\phi$ are a measure of the cornering and camber stiffness coefficients respectively.

2.3.1 Contribution of Camber

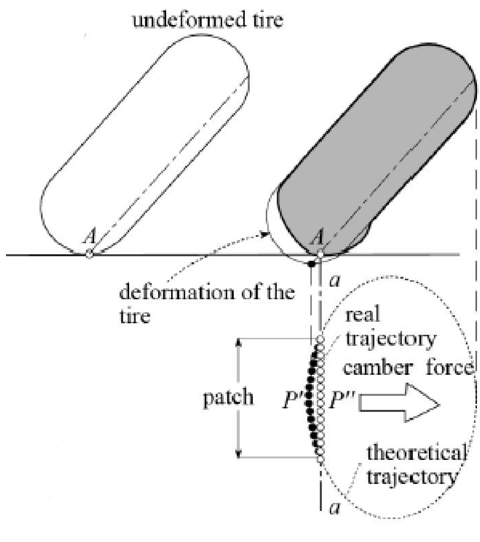

Figure 9: Camber thrust origin [1]

The deformation of the tire due to vertical load initially and camber thrust itself a little later on cause the patch's trajectory to change from P' to P'' as shown in Fig9.

2.3.2 Contribution of Sideslip

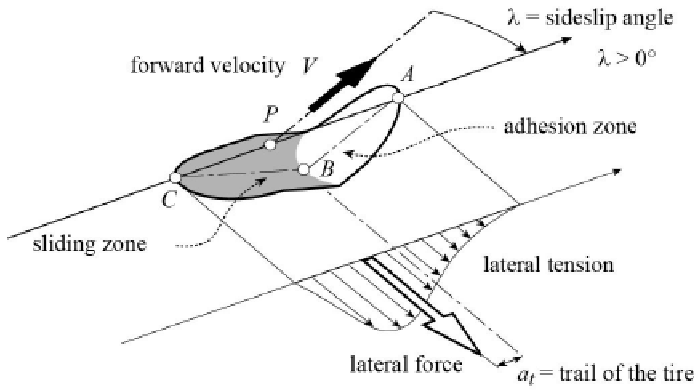

Figure 10: Deformation of a patch due to sideslip [1]

The tire whose magnified patch is shown above in Fig10 is rotating and slipping laterally at the same time. Consider a point A which while rotation travels to B and takes a path parallel to V. once it reaches B and is outside the adhesion zone, the restoring shear stress of the tire rubber takes over and pulls B towards C. The lateral shear stresses and resultant lateral force are also shown in Fig10.

2.3.3 Lateral force and Equilibrium

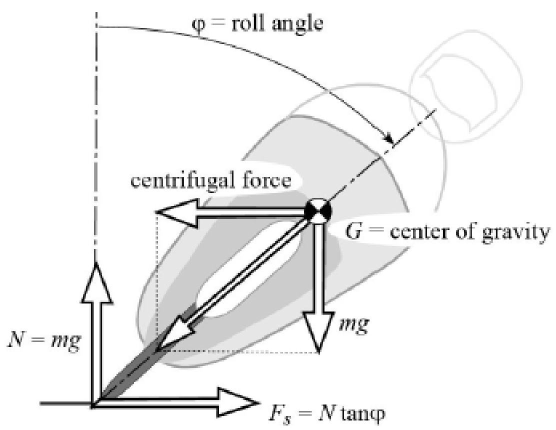

Figure 11: Equilibrium of the motorcycle in a curve [1]

From the diagram above it is obvious that the normalised lateral force should be equal to the tangent of the roll angle for equilibrium of the motorcycle,

$$\frac{F_s}{N} = \tan\phi \tag{8}$$

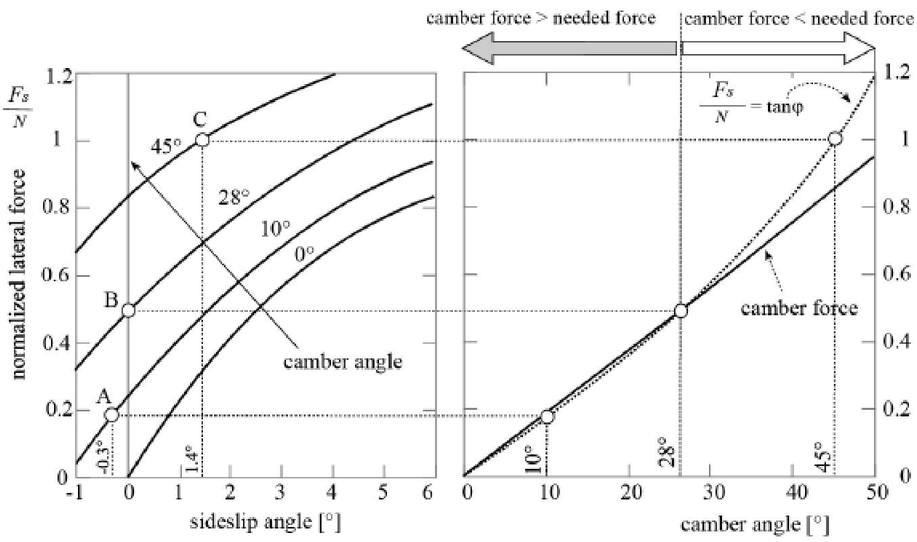

Figure 12: Lateral force due to slip and camber [1]

In the right graph the dotted line represents the necessary camber force to maintain equilibrium and the solid line is the lateral force produced by camber alone. For this given tire at a roll angle of 28°, camber force alone is sufficient for equilibrium. For any roll angle below 28°, equilibrium is achieved by a negative sideslip angle. And for any roll angle above 28°, equilibrium is achieved by a positive sideslip angle.

2.4 Self Aligning Moment

The distribution of the lateral shear stress generated by the lateral slip of the tire is not symmetric as seen in Fig10. The resulting lateral force is thus applied a point displaced by a certain distance from the theoretical contact point. This distance a_t is called the pneumatic trail of the tire. The self aligning moment can be calculated as

$$M_z = a_t F_s \tag{9}$$

The pneumatic trail can be approximated with the help of sideslip angle (λ) as follows :

$$a_t = a_{t_o}\left(1 - \left|\frac{\lambda}{\lambda_{max}}\right|\right) \tag{10}$$

where a_{t_o} is the maximum pneumatic trail (ranges from 1.5 to 5 cm) and λ_{max} is the maximum sideslip angle (usually around 15°).

2.5 Torque from Driving/Braking Force

$$M_z = (s + s_p)S = \left(t \tan\phi + \frac{F_s}{k_s}\right)S \tag{11}$$

s - lateral displacement of contact point of tire
s_p - lateral deformation of tire
k_s - lateral stiffness of tire
t - radius of cross section of tire

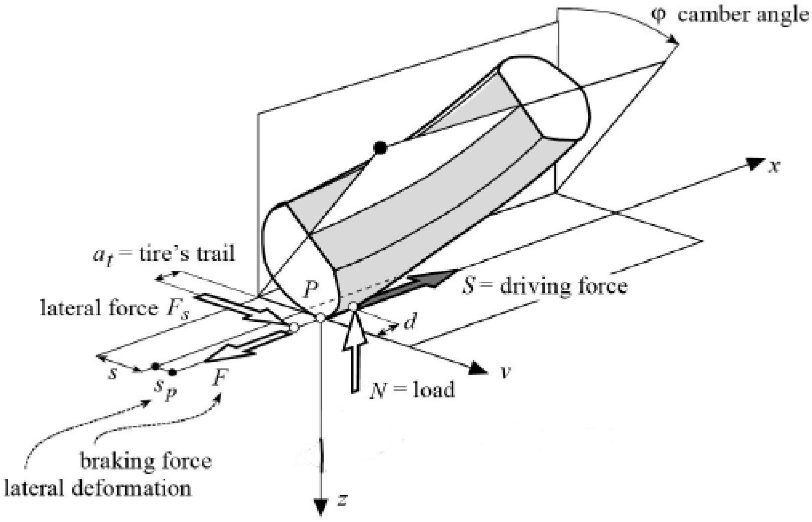

Figure 13: Forces and Moments on the tire [1]

Torque due to thrust would be aligning and due to braking force would be disaligning.

2.6 Overturning Moment

The overturning moment is generated by the vertical load N whose arm is the lateral deformation s_p, refer to Fig13.

$$M_x = s_p N \tag{12}$$

This moment tends to align the motorcycle back to the vertical position.

2.7 Load transfer

The position of a motorcycle's center of gravity has a significant influence on the motorcycle's handling. It's height or position with respect to the frame of the body has a great effect on cornering. Apart from that the distance between the rear contact point and the center of mass determines how the load of the bike is split between both the wheels.

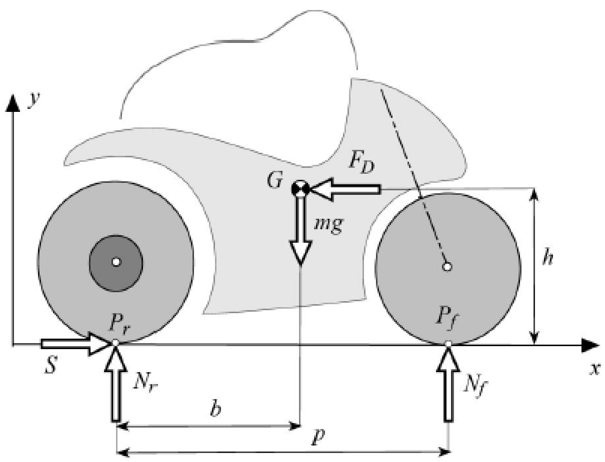

Figure 14: Distribution of load [1]

The dynamic load is on both the tires is give by (variables are as shown in Fig14) :

$$N_f = mg\frac{b}{p} - S\,\frac{h}{p} \tag{13}$$

$$N_r = mg\frac{p-b}{b} + S\,\frac{h}{p} \tag{14}$$

2.8 Aerodynamic drag

The drag force F_D is assumed to act on the center of mass and is given by :

$$F_D = \frac{1}{2}\rho C_D A V^2 \tag{15}$$

ρ - density of air
C_D - drag coefficient
A - frontal area of the motorcycle

3 Lagrangian Mechanics Approach

The approach I will follow is to define the Lagrangian of the motorcycle system along with the external and dissipating forces using generalised coordinates (in terms of the degrees of freedom of the bike, thereby removing the need for Lagrange multipliers) and solve the Euler-Lagrange equations which are given by ;

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} + \frac{\partial \mathcal{D}}{\partial \dot{q}_i} = \mathcal{Q}_i \tag{16}$$

Here, q_i and $\dot{q}_i$ are the generalised coordinates and their derivatives with time respectively.
$\mathcal{Q}_\rangle$ is the generalised external force corresponding to the generalised coordinate q_i. $\mathcal{Q}_i$ can be calculated from the Work Function $\mathcal{W}$ of the system.

$$\mathcal{Q}_i = \frac{\partial \mathcal{W}}{\partial q_i} \tag{17}$$

The Work Function is the sum of all the dot products of the forces acting on the body and the position at which they act on. This definition can be treated analogously for moments and angles.
$\mathcal{D}$ is the Rayleigh's Dissipation Function. Used to introduce frictional forces into the system where d_i are the dissipation constants.

$$\mathcal{D}(\dot{q}_i) = \frac{1}{2}\sum d_i \dot{q}_i{}^2 \tag{18}$$

4 Testing Code Structure

```
syms alpha1(t) alpha2(t) alpha3(t) Y
syms q [3 1]
syms Lag [3 1]
syms eqns [3 1]

q(1) = alpha1;
q(2) = alpha2;
q(3) = alpha3;
dq = diff(q,t);

m1 = 0.5;
m2 = 0.15;
m3 = 0.05;
l1 = 0.1;
l2 = 0.1;
l3 = 0.1;
g = 9.8;

% REMEBER alpha is the angle with the vertical
x1 = l1*sin(alpha1);
y1 = -l1*cos(alpha1);
x2 = l1*sin(alpha1) + l2*sin(alpha2);
y2 = -l1*cos(alpha1) - l2*cos(alpha2);
x3 = l1*sin(alpha1) + l2*sin(alpha2) + l3*sin(alpha3);
y3 = -l1*cos(alpha1) - l2*cos(alpha2) - l3*cos(alpha3);

T = 0.5*(m1*((diff(x1,t))^2 + (diff(y1,t))^2) + m2*((diff(x2,t))^2 + (diff(y2,t))^2) + m3*((diff(x3,t))^2 + (diff(y3,t))^2));
V = m1*g*y1 + m2*g*y2 + m3*g*y3;
L = T-V;

for i=1:3
    Lag(i) = diff(diff(L,dq(i)),t) - diff(L,q(i)) == 0;
end
for i=1:3
    eqns(i) = simplify(lhs(Lag(i)) - rhs(Lag(i)));
end
eqns == 0 %#ok<EQEFF>

[VF,Subs] = odeToVectorField(Lag)
odefcn = matlabFunction(VF, 'Vars',{t,Y})
[X,Y] = ode45(odefcn, [0 5], [pi/2 0 pi/2 0 pi 0]);

plot(X, Y)
grid
legend(string(Subs))
```

Before jumping straight into writing code for a motorcycle, which has multiple degrees of freedom and multiple rigid bodies to keep track of, I wanted to test a certain structure of code on a simpler example such as a triple pendulum. Here is a brief description of how the code is built :

- Lines 1-4 : defining my generalised coordinates as functions of time and creating symbolic arrays to store said coordinates and the later derived Euler-Lagrange equations (this is where MATLAB's symbolic math toolbox came in very handy)
- Lines 6-9 : storing the generalised coordinates in the previously made arrays and defining $\dot{q}_i$'s
- Lines 11-17 : physical parameters
- Lines 20-25 : defining the relations between the generalised coordinates and 2-dimensional Cartesian coordinates (alpha is the angle each string makes with the vertical)
- Lines 27-29 : defining the Kinetic and Potential energies as well as the Lagrangian
- Lines 31-33 : for loop to calculate the Euler-Lagrange equations
- Lines 39-41 : converting the system of higher order ODEs into multiple single order equations, making a matlabFunction out of them and injecting them into an ODE solver. ode45 was used here.
- Lines 43-45 : viewing results

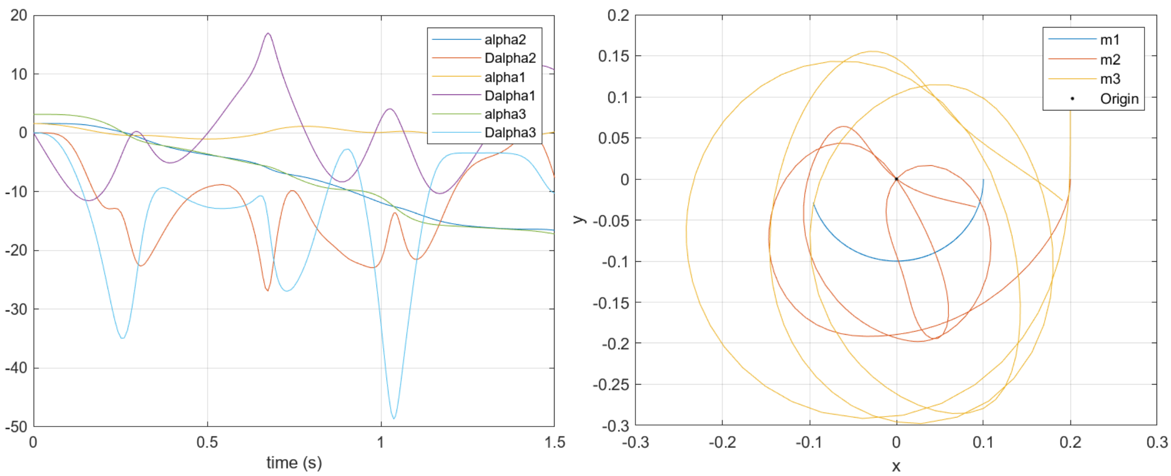

Figure 15: Generalised coordinates and their velocities as a function of time (left), path of each mass traced in xy plane

An animation of this simulation can be found at
https://github.com/adityagangula/Motorcycle_Simulation

5 The Motorcycle Model

I chose to work with a 6 body - 11 degree of freedom motorcycle model based on [2].

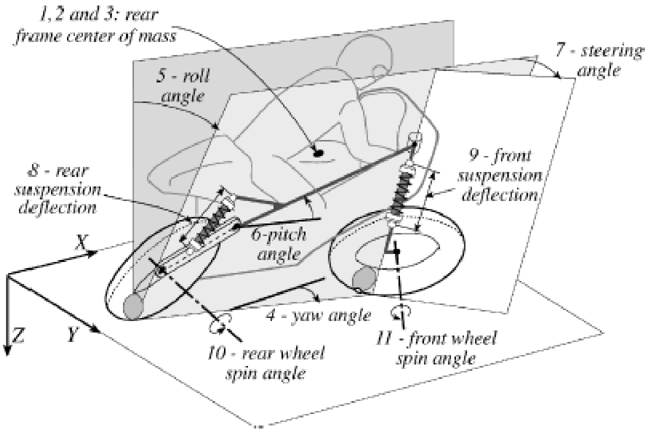

Figure 16: 11 degrees of freedom described [2]

The six bodies are :

- Front Wheel
- Rear Wheel
- Rear Assembly (Frame, Rider, Engine, Fuel Tank)
- Front Assembly (Steering column, Handlebar, Inverted fork)
- Rear Swingarm
- Front unsprung mass (Fork, Brake pliers)

The 11 degrees of freedom :

- x, y, z translational motion of Center of Mass
- Roll ϕ
- Yaw ψ
- Pitch μ
- Steering angle δ
- Front and Rear suspension travel
- Spin of Front and Rear wheels

The eleven degrees of freedom are enough to completely describe the system and are hence, directly used as the generalised coordinates. The next step (following the flow of Section 4) is to determine and track the positions of all six bodies with respect to the center of mass. To do this we define a coordinate basis attached to the center of mass of each body and a few intermediate basis.Then we define how all these variables are related to each other using the relations mentioned in Section 1 and 2.

My aim is to predict how the motorcycle reacts to a person riding it. This would mean out of the eleven degrees of freedom, I set three of them to be functions of time which I initially define. Those three being the coordinates of the Centre of Mass. This way I can use the rider's riding style as an input parameter and try and predict how the bike reacts to it. From the trajectory of the center of mass, the amount of throttle/brakes and steering angle applied I should be able to see how the motorcycle responds to all this.

Frames and their labels :

- A - rear frame
- B - rear swingarm
- C - rear wheel
- D - front frame
- E - front wheel
- F - unsprung mass
- G - ground
- H, M, N - intermediate

Angle of rotation between frames about either the vertical, longitudinal or lateral axis:

- G $\leftrightarrow$ M : ψ
- M $\leftrightarrow$ C : ϕ
- A $\leftrightarrow$ C : μ
- B $\leftrightarrow$ A : swingarm angle
- A $\leftrightarrow$ N : ε
- D $\leftrightarrow$ N : δ
- D $\leftrightarrow$ E : $\varepsilon + \mu$
- H $\leftrightarrow$ E : β
- M $\leftrightarrow$ H : Δ

After using the geometrical parameters and other physical properties of the suspension and tires from [2], all that is left to do is to calculate and solve the Euler-Lagrange equations.

6 Trouble with the ODE Solver

```
Error using mupadengine/feval_internal
System contains a nonlinear equation in 'diff(y(t), t)'. The system must be
quasi-linear: highest derivatives must enter the differential equations
linearly.

Error in odeToVectorField>mupadOdeToVectorField (line 171)
T = feval_internal(symengine,'symobj::odeToVectorField',sys,x,stringInput);

Error in odeToVectorField (line 119)
sol = mupadOdeToVectorField(varargin);
```

Figure 17: Error message when solving

As the error message above reads - the highest derivatives must enter the differential equations linearly. This is not true just for $y(t)$ but the same error pops up when checking for other variables. The **odeToVectorField** function in MATLAB usually does a decent job simplifying the equations to make it work with it's in-built algorithm. This does not seem to be the case here. The equations to be solved are extremely complicated especially because I used dependent coordinates instead of independent coordinates. Try to imagine all 15 of the mentioned equations in Section 1 and 2 intertwined along with the additional rotation operators mixed in.

7 Workarounds for the error

Clearly, the complexity of my system had to be reduced as I had no knowledge of how exactly each equation was non-linear with which variable. I started off by using the less accurate linear approximations of the tire forces. Then I started reducing the number of degrees of freedom by removing the suspension and neglecting pitch and yaw. these reductions were not enough to solve the problem.
Eventually I reduced the degrees of freedom till my aim mentioned in Section 5 was flipped.
Initial aim :

$$\begin{array}{c} x \\ y \\ z \\ \delta \\ \text{thrust/brake} \end{array} \longrightarrow \begin{array}{c} \phi \\ \psi \\ \mu \\ \text{suspension lengths} \\ \lambda_r \text{ and } \lambda_f \end{array}$$

New aim :

$$\begin{array}{c} \delta \\ \text{thrust/brake} \\ \phi \\ \lambda_r \text{ and } \lambda_f \end{array} \longrightarrow \begin{array}{c} x \\ y \\ z \end{array}$$

8 Results and Conclusions

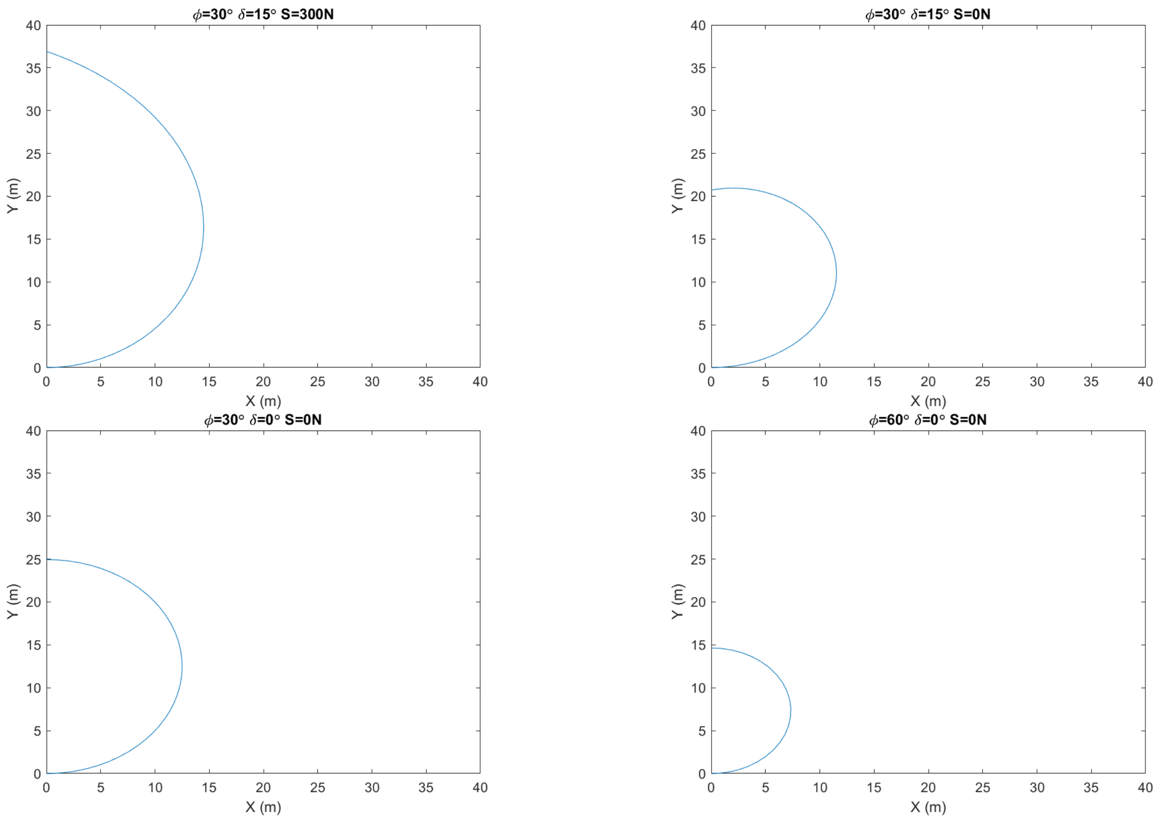

Figure 18: Resultant trajectories from various combinations of roll, steering angle and thrust where the initial velocity was 30kmph along X axis

When both steering angle and thrust are zero, the motorcycle follows perfectly circular paths as the camber thrust always acts laterally. An increase in thrust shows how the motorcycle tends to exit the curve. An increase in roll angle is followed by a increase in path curvature.

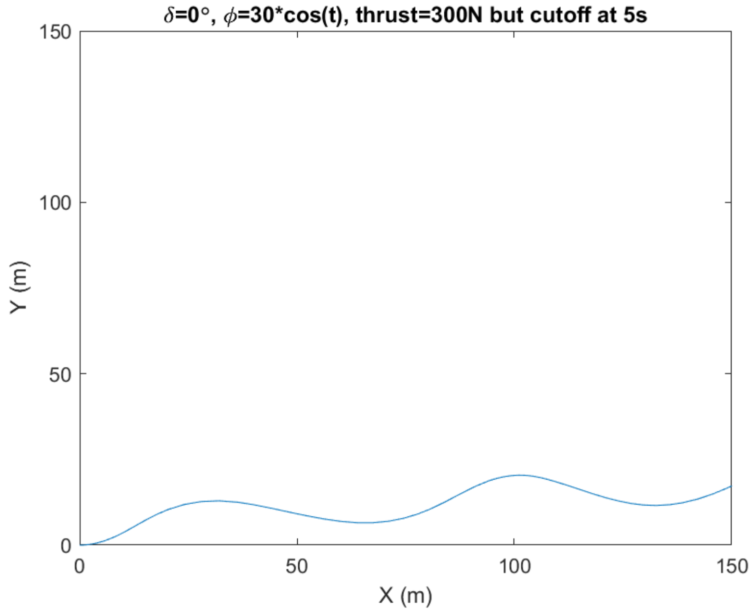

Figure 19: Roll of the motorcycle follows a sinusoidal trend and the initial thrust is cutoff at 5s

Due to the thrust being cutoff somewhere in between $x = (50 - 80)$ the second curve of the path of the bike is more squished and it's peak is raised.

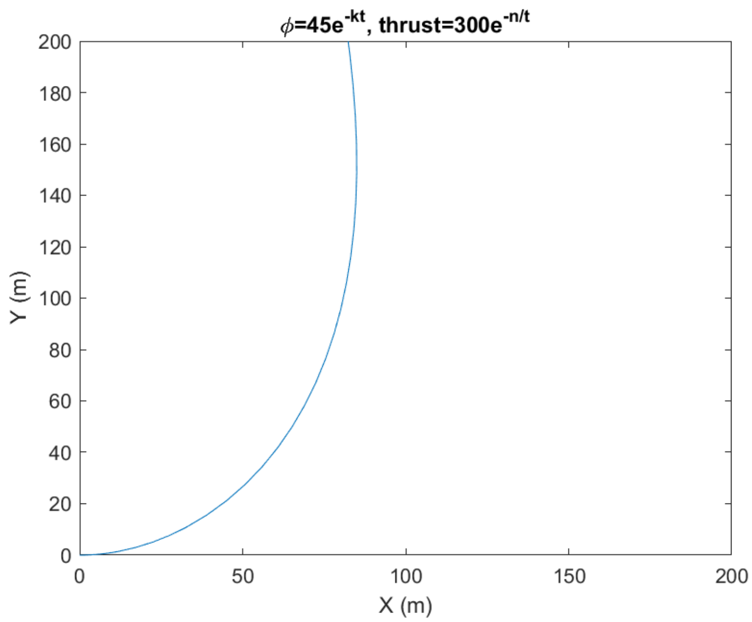

Figure 20: Roll and thrust follow an exponential decay and growth trend respectively

These functions were chosen to recreate a rider's behaviour while exiting a corner. They would raise the throttle and lean the motorcycle back up vertical gradually as they pass the apex of the curve.

For all project related resources :
https://github.com/adityagangula/Motorcycle_Simulation

References

[1] Vittore Cossalter, Motorcycle Dynamics

[2] Vittore Cossalter and Roberto Lota, Motorcycle Multi-Body Model for Real Time Simulations Based on the Natural Coordinates Approach, 2002