# Simultaneous **batching** and **scheduling** in multi-stage multi-product process


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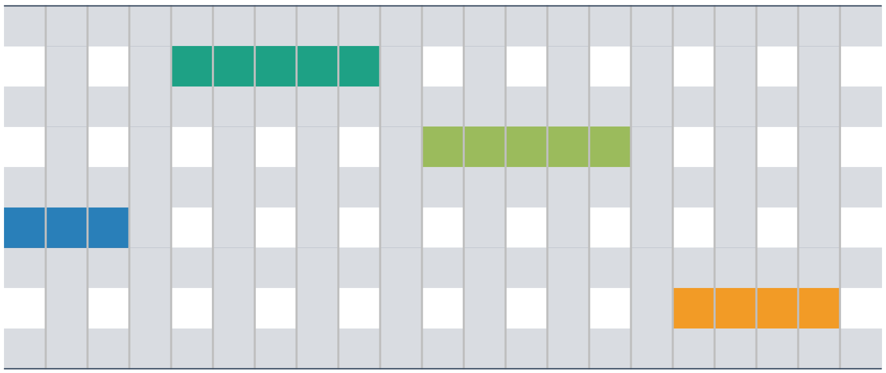
# Motivation

Limitations in traditional discrete time & continuous time approach

## Discrete Time



*State-Task Network*

Kondili et al., (1993)

$$S_{st-1} + \sum_{i\in \bar{T}_s} \bar{\rho}_{is} \sum_{j\in K_i} B_{ijt-pi} + R_{st} = S_{st} + \sum_{i\in T_s} \rho_{is} \sum_{j\in K_i} B_{ijt} + D_{st} \quad \forall s,t$$

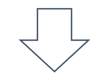
$$\sum_{i\in I_j} \sum_{\hat{t}=t}^{t-p_i+1} W_{ij\hat{t}} \le 1 \quad \forall j,t$$

Require Constant Processing Time

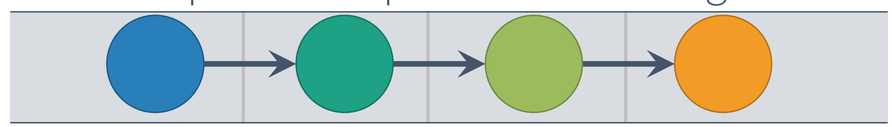
Handles Batching using summation

## Continuous Time

Use of continuous time variable



Sequence Dependent Ordering



*Mixed Product Campaign*

Birewar and Grossmann, 1989

$$s.t. \quad V_j \ge S_{ij} B_i \qquad i = 1,\ldots,NP \qquad j = 1,\ldots,M$$

$$n_i B_i = Q_i \qquad i = 1,\ldots,NP$$

$$\sum_{k=1}^{NP} NPRS_{ik} = n_j \qquad i = 1,\ldots,NP$$

$$\sum_{i=1}^{NP} NPRS_{ik} = n_k \qquad k = 1,\ldots,NP$$

Require Constant Batch Size / Number

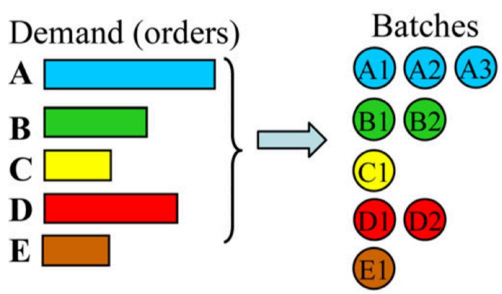
Handles Variable Processing Time

# Conventional Approach

Two-step approach for continuous time produces sub-optimal solution

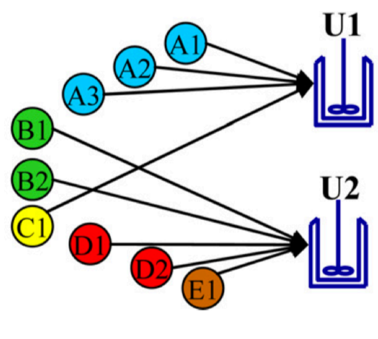
1 Batching Decisions:

$\boldsymbol{n_i}, \boldsymbol{B_i}$



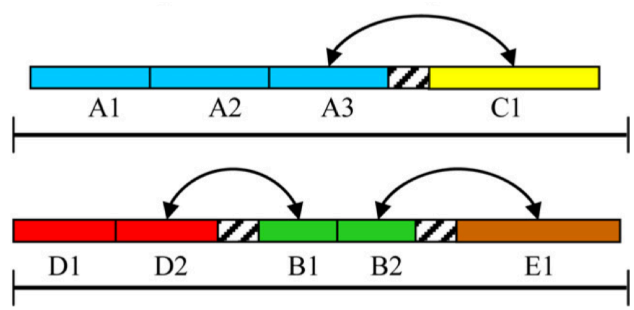

2 Unit Assignment:

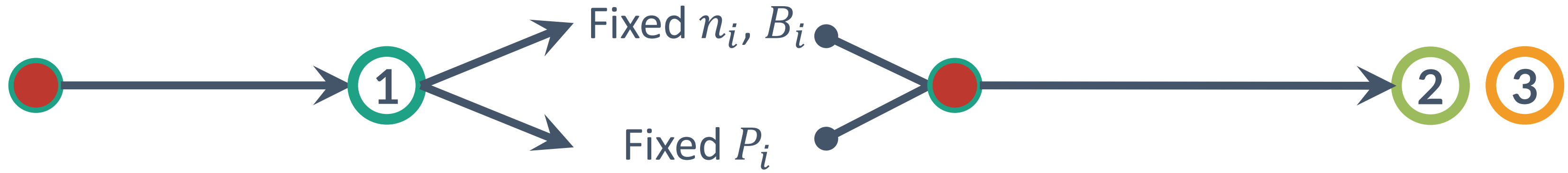
$\boldsymbol{B_{i,j}}$




3 Sequencing & Timing:

$\boldsymbol{Y_{i,i',j}}, \boldsymbol{T_{i,j}}$




Solve a separate batching problem

Solve Assignment and sequencing simultaneously

Simultaneous approach: Further Disaggregation

# GDP with Convex Hull Formulation

Generalized disjunctive programming with 2 levels of decomposition

Decompose

$n_i \cdot B_i = Q_i$

| | |
|---|---|
| 1 | ✔ |
| 2 | ✘ |
| 3 | ✔ |
| ⋮ | |
| L − 1 | ✘ |
| L | ✔ |

**1**

$$\sum_{1}^{L} B_{il} = Q_i$$

$$\hat{b}_i^{min} Z_{il} \le B_{il} \le \hat{b}_i^{max} Z_{il}$$

✘ ✘ ✔

**2**

$$Z_{il} = \sum_{j \in JAik} X_{ilj}$$

$$B_{il} = \sum_{j \in JAik} \bar{B}_{ilj}$$

$$b_j^{min} X_{ilj} \le \bar{B}_{ilj} \le b_j^{max} X_{ilj}$$

$n_i = \max[n_i] = \mathrm{L}$ ⇨ Batch Decisions ⇨ Unit Assignments

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$$\left[\begin{array}{c} Z_{il} = 1 \\ \cdots \\ \begin{pmatrix} X_{ilj} = 1 \\ \hat{b}_i^{min} \le B_{il} \le \hat{b}_i^{\max} \end{pmatrix} \bigvee \begin{pmatrix} X_{ilj} = 0 \\ \bar{B}_{ilj} = 0 \end{pmatrix} \end{array}\right] \bigvee \left[\begin{array}{c} Z_{il} = 0 \\ \cdots \\ \begin{pmatrix} X_{ilj} = 0 \\ \bar{B}_{ilj} = 0 \end{pmatrix} \end{array}\right] \qquad \forall i, l, j$$

# Solution Methods

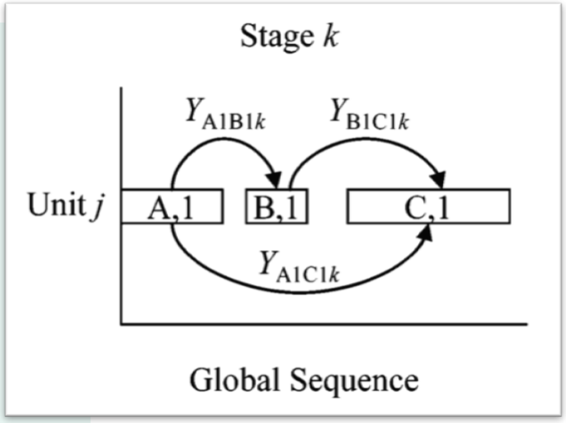
Sequencing and scheduling: global vs local

## 1 Global Sequence: $Y_{ili'l'k}, T_{ilk}$

$$X_{ilj} + X_{i'l'j} - 1 \leq Y_{ili'l'k} + Y_{i'l'ilk}$$

$$T_{il(k+1)} \geq T_{ilk} + \sum_{j \in JAi(k+1)} (\tau^F_{ij} X_{ilj} + \tau^P_{ij} \bar{B}_{ilj})$$

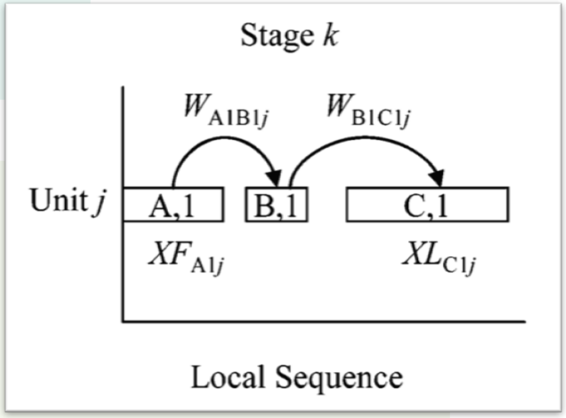
$$T_{i'l'k} \geq T_{ilk} + \sum_{j \in JA_{ik}} \left(\tau^F_{i'j} X_{i'l'j} + \tau^P_{i'j} \bar{B}_{i'l'j}\right) - H(1 - Y_{ili'l'k})$$




## 2 Local Sequence: $W_{ili'l'k}, XF_{ilj}, XL_{ilj}, T_{ilk}$

$$\sum_{i'l' \in L_{i'}} W_{i'l'ilj} + XF_{ilj} = X_{ilj} \qquad \sum_{i'l' \in L_{i'}} W_{ili'l'j} + XL_{ilj} = X_{ilj}$$

$$T_{i'l'k} \geq T_{ilk} + \sum_{j \in JAik \cap JAi'k} \left(\tau^F_{i'j} X_{i'l'j} + \tau^P_{i'j} \bar{B}_{i'l'j} + \tau^{ch}_{il'j} W_{ii'l'j}\right) - H\left(1 - \sum_{j \in JAik \cap JAi'k} W_{ili'l'j}\right)$$

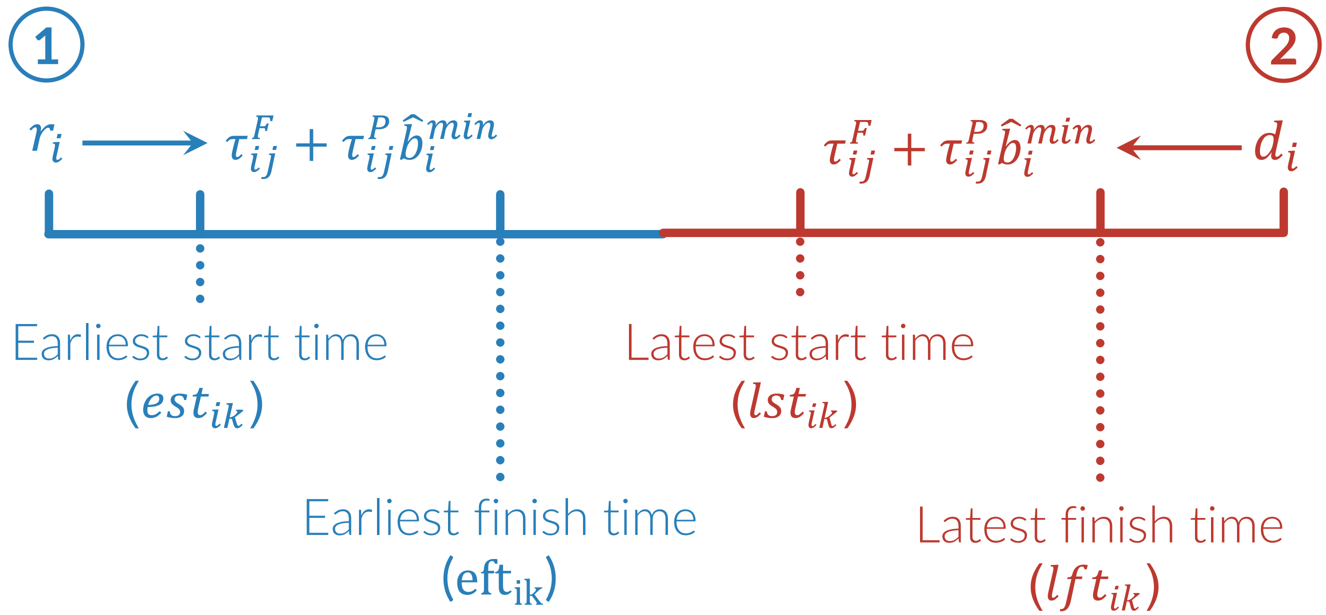


# Tighting Constraints

How to enhance the performance of a generic model

### ❖ Sample Problem Size

| | |
|---|---|
| Product | 10 |
| Stages | 2 |
| Units | 6 |
| Continuous | 532 |
| Binary | 7257 |
| Equality | 2974 |
| Inequality | 19260 |




3

$$\min\begin{pmatrix} d_i - \sum_{k'>K, j\in JA_ik'} \min({\tau_{ij'}}^{min}) - {\tau_{ij}}^{min}, \\ d_{i'} - \sum_{k'>K, j'\in JA_{i'k'}} \min({\tau_{i'j'}}^{min}) - {\tau_{i'j}}^{min} - {\tau_{ij}}^{min} \end{pmatrix} = ub_{ii'j} \geq est_{ik}$$

: **Feasible Changeovers**

# Tightening Constraints

How to apply time window concept to further tighten the model

for $k \in \{1,2,...|\mathbf{K}|\}$,<br>
  for $j \in \mathbf{J}_k$,<br>
    Initialize auxiliary set: $\mathbf{IC}' = \mathbf{I} \in \mathbf{IA}_j$<br>
    Reset cut number: $n = 0$<br>
    Do while $\mathbf{IC}' \neq \varnothing$,<br>
      Initialize active set: $\mathbf{IC}_{jn} = \mathbf{IC}'$<br>
      Do while $\mathbf{IC}_{jn} \neq \varnothing$ and $\max_{i \in \mathbf{IC}_{jn}} (\mathrm{lft}_{ik}) = \max_{i \in \mathbf{IC}'_{jn}} (\mathrm{lft}_{ik})$,<br>
        Update cut number: $n = n + 1$<br>
        Calculate parameters:<br>
          $\mathrm{lbw}_{jn} = \min_{i \in \mathbf{IC}_{jn}} (\mathrm{est}_{ik})$<br>
          $\mathrm{ubw}_{jn} = \max_{i \in \mathbf{IC}_{jn}} (\mathrm{lft}_{ik})$<br>
          $\mathrm{tail}_{jn} = \min_{i \in \mathbf{IC}_{jn}} \sum_{k' > k} \min_{j' \in \mathbf{JA}_{ik'}} (\tau_{ij'}^{min})$<br>
        Update $\mathbf{IC}_{jn}$ by removing the order(s) with the earliest start time:<br>
          $\mathbf{IC}_{jn} = \mathbf{IC}_{jn}\{i \in \mathbf{I}: \mathrm{est}_{i'k} = \mathrm{lbw}_{jn}\}$<br>
      Update $\mathbf{IC}'$ by removing the order(s) with the latest finish time:<br>
        $\mathbf{IC}' = \mathbf{IC}' \backslash \{i \in \mathbf{I}: \mathrm{lft}_{ik} = \mathrm{ubw}_{jn}\}$<br>
Calculate the number of inequalities for unit $j$: $\mathbf{N}_j = n$


"All possible combinations"

⇕

$\mathrm{ubw}_{jn} - \mathrm{lbw}_{jn}$ **> Sum $\tau_{ij}$?**

Eliminate impossible combinations

| Cuts | Upper Bound | Lower Bound | $\mathrm{IC}_{\mathrm{jn}}$ (All Possible Combinations) |
|---|---|---|---|
| 0 | 80 | 0 | B, C, D, E, F, G, H, I, J. K, L |
| 1 | 80 | 5 | B, D, G, I, J, K, L |
| 2 | 80 | 25 | D, G, I. J. K, L |
| 3 | 80 | 30 | G, J, K, L |
| 4 | 80 | 40 | J, K |
| 5 | 76.8 | 0 | B, C, D, E, F, G, H, K, L |
| 6 | | 5 | B, D, G, K, L |
| … | … | … | … |
| 21 | 16.8 | 0 | C, H |
| 22 | 17.7 | 0 | C |

# Case Studies

We tested 4 formulations on 5 cases

| | | Sequential | Global Sequence | Tightening A | Tightening A + B | Local Sequence |
|---|---|---|---|---|---|---|
| 1 | min | Make Span | Make Span | | | |
| | s.t. | 2-step | Eqn (3) – (17) | | | |
| 2 | min | Earliness | Earliness | | | |
| | s.t. | 2-step | Eqn (3) – (17) | | | |
| 3 | min | | Cost | Cost | Cost | |
| | s.t. | | Eqn (3) – (17) | Eqn (3) – (17)<br>Eqn (8) – (21) | Eqn (3) – (17)<br>Eqn (8) – (21) + (22) | |
| 4 | max | | Profit | Profit | Profit | |
| | s.t. | | Eqn (3a) – (17) | Eqn (3a) – (17)<br>Eqn (8) – (21) | Eqn (3a) – (17)<br>Eqn (8) – (21) + (22) | |
| 5 | min | | | | | Cost + Changeover |
| | s.t. | | | | | Eqn (3) – (17)<br>(9abcd) (10b)<br>Eqn (8) – (21) + (22)<br>(18a) + (21a) +(22a) |

**Comparison** between 2-step approach and simultaneous approach

**Evaluation** of different tightening constraints

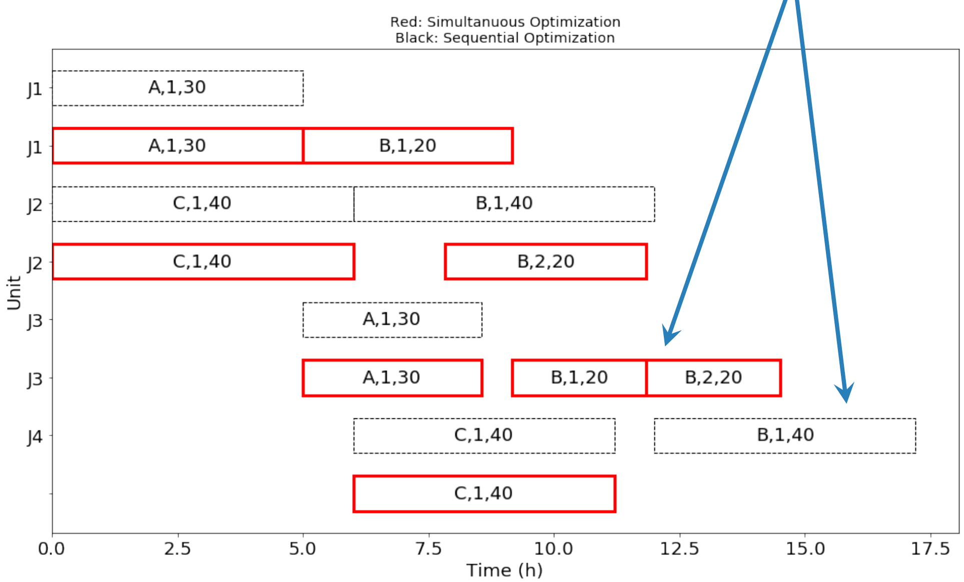
**Demonstration** of local sequence

# Simultaneous Optimization

Conventional approach gives sub-optimal solutions

Objective:
min Make Span

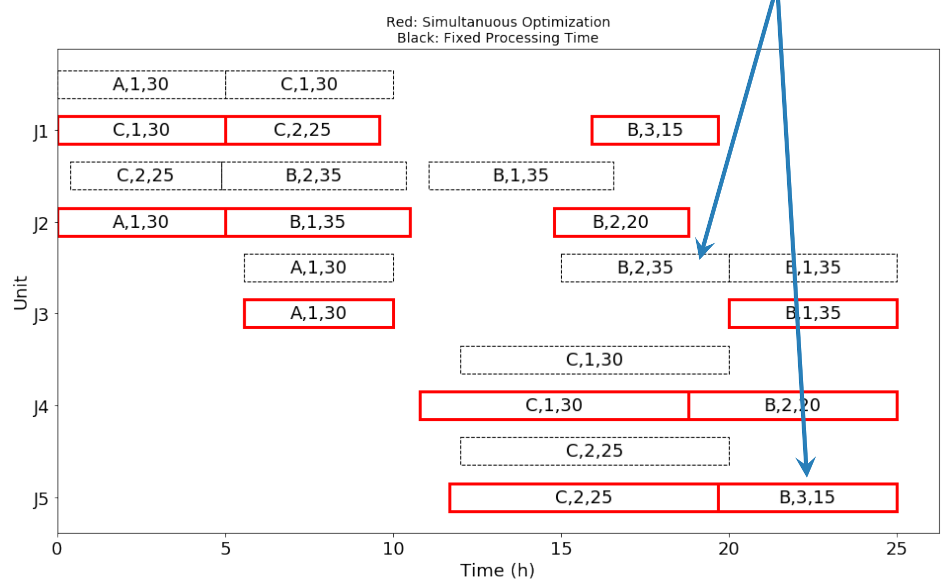
Separate B to 2 batches




| | Sequential | Simultaneous |
|---|---|---|
| Obj | 17.2h | 14.5h |

Objective:
min Earliness

Utilize small slacks

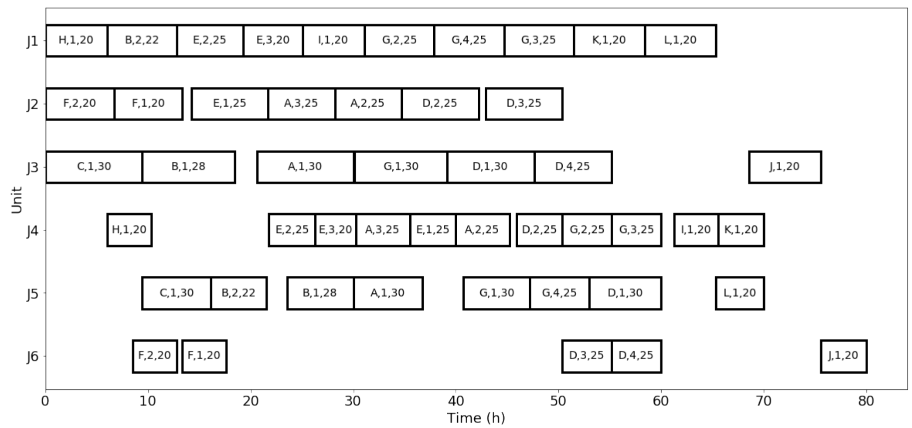



| | Sequential | Simultaneous |
|---|---|---|
| Obj | 5h | 1.56h |

# Computational Enhancement

Both tightening A and B offers noticeable improvements

## Objective: min Cost = $3057




❖ **Tightening A:**
- Fix binary variables
- Increase equality constraints

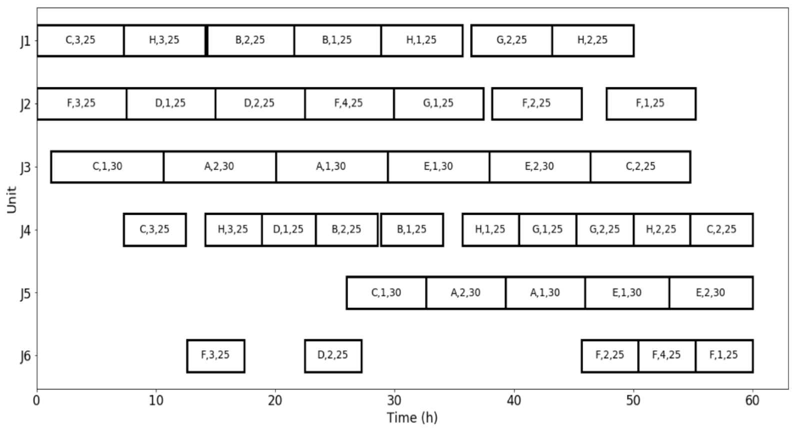
❖ **Tightening B:**
- Cut binary variables
- Increase inequality constraints

| | | | Global Sequence | Tightening A | Tightening A + B |
|---|---|---|---|---|---|
| Variable | Before Pre-solve | Continuous | 532 | | |
| | | Binary | 7257 | | |
| | After Pre-solve | Continuous | 390 | 390 | 387 |
| | | Binary | **6161** | **5327** | **5136** |
| Constraints | Equality | | **256** | **2974** | 2974 |
| | Inequality | | 19077 | **19077** | **19260** |
| CPLEX Solution Time | | | 267s | 136s | 52s |

# Limitations

Tightening A can sometimes be ineffective

**Objective: max Profit = $2056**




- Tightening A:
  - Data doesn't offer any cuts
  - Ineffective
- Tightening B:
  - Deeper processing of data
  - Still effective

| | | | Global Sequence | Tightening A | Tightening A + B |
|---|---|---|---|---|---|
| Variable | Before Pre-solve | Continuous | 487 | | |
| | | Binary | 6102 | | |
| | After Pre-solve | Continuous | 394 | 394 | 394 |
| | | Binary | 6032 | 6032 | 6032 |
| Constraints | Equality | | **223** | **223** | **223** |
| | Inequality | | **16183** | **16183** | **16226** |
| CPLEX Solution Time | | | 10000+ s | 10000+ s | 520 s |

# Summary

Simultaneous batching and scheduling of plant

## ❖ Problem Formation

1. Motivation
   - MINLP formulation
2. GDP Formation
   - Disaggregation
   - Sequencing & Timing
   - Tightening

## ❖ Case Studies

1. Advantages
   - Flexible Batching
   - Variable Processing Time
2. Enhancements
   - Effectiveness
   - Limitations
3. Local Sequence