ASSESSMENT 1

IE6200 Fall 2025

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ACADEMIC INTEGRITY

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OVERALL SUMMARY

The analysis first examined the performance of three different sensors: LiDAR, Camera, and Radar by modeling their reliability at 5000 hours, their expected lifespans (MTTF), and their accuracy under severe environmental conditions using Bayes' Theorem. The results demonstrated that the Radar sensor was the most robust, achieving a reliability of 99.62% and the longest MTTF of 9000 hours, while the Camera proved to be the weakest, with a reliability of 43.6% and an MTTF of 6000 hours. LiDAR fell in between, with a reliability of 73.45% and an MTTF of 7098 hours. Bayesian analysis further revealed that, given a correct detection, the probability of conditions actually being severe was 9.62%, slightly less than the prior probability of 10%, underscoring that correct detections are more likely in clear or moderate conditions.

Building on this, the system-level analysis applied a 2-out-of-3 redundancy rule to evaluate reliability across different sensor configurations. Among the configurations, the three Radar setup achieved near-perfect system reliability ($R_{sys} = 1$), while mixed configurations and the three LiDAR setups also performed strongly ($R_{sys} \approx 0.85$). By contrast, the three Camera system was far less reliable at 0.4024. The study also included a Venn diagram and derived formulas to illustrate how system reliability emerges from different combinations of wrong and failing sensors, highlighting the effectiveness of redundancy when components are independent.

Finally, sensitivity analysis was conducted to measure the impact of hardware upgrades on sensor performance. The upgrades involved extending LiDAR's characteristic life, reducing the Camera's failure rate, and lowering the Radar's variance. The results showed the greatest percentage improvement for the Camera (+21.45%), the largest absolute gain for LiDAR (+0.0906), and negligible improvement for Radar due to its already near-perfect reliability. Overall, the findings recommend the three Radar configuration as the most reliable choice for critical applications, while also emphasizing that upgrades offer the most value when applied to components with moderate reliability. Additionally, the report cautions that stochastic dependence and common cause failures could reduce true system reliability, meaning reported values should be treated as optimistic upper bounds.

STAGE 1 - Data Analysis and Modeling

The Stage 1 analysis comprehensively assesses three sensor types (LiDAR, Camera, Radar) based on their reliability at 5000 hours, their Mean Time to Failure (MTTF), and the probability of actually severe conditions given a correct detection (Bayes' Theorem).

The Radar sensor, modelled by a Normal Distribution, is by far the most reliable component with R(5000) = 0.9962 (99.62%). The LiDAR (Weibull Distribution) is moderately reliable with R(5000) = 0.7345 (73.45%). The Camera (Exponential Distribution) is the least reliable at the 5000-hour mark, with R(5000) = 0.4346 (43.46%).

The Radar also has the longest expected lifespan, with an MTTF = 9000 hours. The LiDAR is next with an MTTF ≈ of 7098 hours, and the Camera has the shortest expected lifespan at MTTF = 6000 hours. The probability of failing before the MTTF is exactly 50% for the radar but is greater than 50% for both the LiDAR (due to its Weibull shape parameter $\beta > 1$) and the Camera (≈ 63.21% for Exponential Distribution).

Using Bayes' Theorem, the overall chance of a correct detection (P(CD)) is calculated to be 0.9875 (98.75%). Given that a correct detection has occurred, the probability that the conditions were actually Severe (P(S|CD)) is only 0.0962 (9.62%). This slight decrease from the prior probability of Severe conditions (P(S) = 0.1) suggests that a correct detection is marginally more likely to occur in Clear or Moderate conditions than in Severe conditions.

Numerical Analysis Details

Reliability of Each Sensor at 5000 hours

Reliability (R_i) is the probability that a sensor's lifespan (T_i) is greater than 5000 hours
Therefore, the probability $P(T_i > 5000 \text{ hours})$
Using the formula R(t) = 1 - F(t); where F(t): Cumulative Distribution Function for lifetime T_i

Reliability Formula: R(t) = P(T>t)
For LiDAR: $R_L(t) = e^{-(t/n)^\beta}$ since it is Weibull Distribution
where t=5000, η=8000, β=2.5
After substituting the above values, we get the following:
$R_L(5000) = e^{-(5000/8000)^{2.5}}$
$R_L(5000) = e^{-(0.625)^{2.5}}$
$R_L(5000) = e^{-0.3085}$
$R_L(5000) = 0.7345$
Therefore, the reliability of the LiDAR sensor at 5000 hours is approximately 73.45%

For Camera: $R_C(t) = e^{-\lambda t}$ since it is Exponential Distribution
where t=5000, λ=1/6000
After substituting the above values, we get the following:
$R_C(5000) = e^{(-1/6000)\,5000}$
$R_C(5000) = e^{-(5000/6000)}$
$R_C(5000) = e^{-0.8333}$

R_C (5000) = $1/e^{0.8333}$; $e^{0.8333} \approx 2.3006$
R_C (5000) = 1/2.3006
R_C (5000) = 0.4346
Therefore, the reliability of the Camera sensor at 5000 hours is approximately 43.46%

For Radar: $R_R(t) = 1 - \Phi(t- \mu/\sigma)$ since it is Normal Distribution
where t=5000, μ=9000, σ=1500
After substituting the above values, we get the following:
R_R (5000) = 1 - Φ(5000-9000/1500)
R_R (5000) = 1 - Φ(-4000/1500)
R_R (5000) = 1 - Φ(-2.6667) ; Using symmetry of the standard normal CDF: 1 - Φ(-a) = Φ(a)
R_R (5000) = Φ(2.6667)
R_R (5000) = 0.9962
Therefore, the reliability of the Radar sensor at 5000 hours is approximately 99.62%

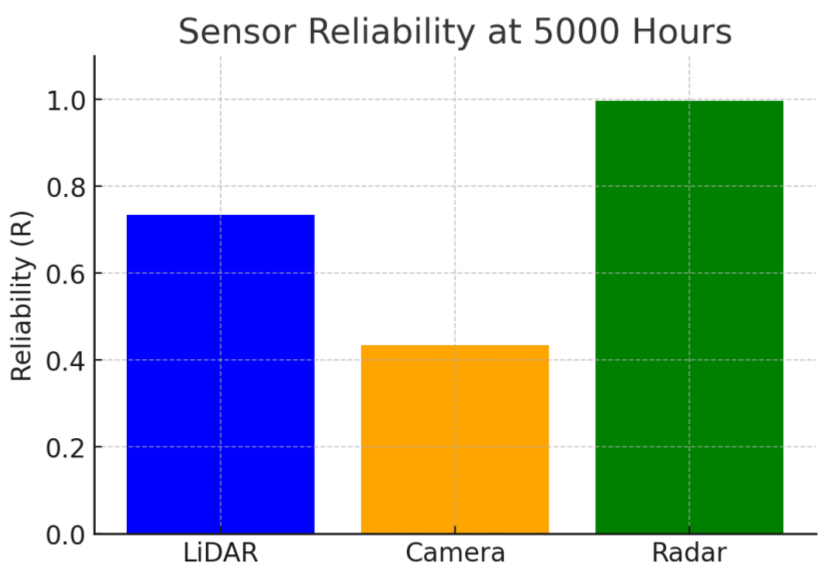

***Figure 1:** Sensor Reliability at 5000 hours*

The above graph indicates the reliability comparison between LiDAR, Camera and Radar sensors at the 5000-hour mark. It is a bar chart that highlights Radar as the most reliable and Camera as the least reliable component.

Sensor	Distribution Model	Parameters	R(5000 hours)
LiDAR	Weibull	t=5000, η=8000, β=2.5	0.7345
Camera	Exponential	t=5000, λ=1/6000	0.4346
Radar	Normal	t=5000, μ=9000, σ=1500	0.9962

***Table 1:** Sensor Reliability at 5000 hours*

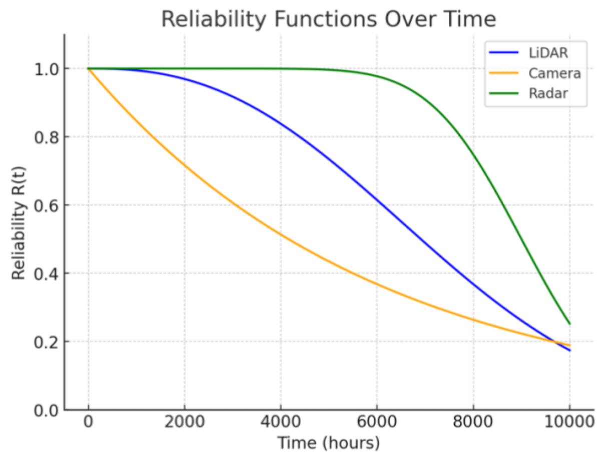

Figure 2: *Reliability Functions Over Time (0 to 10,000 hours)*

The above graph indicates how the reliability of each sensor decreases over time. It is a line graph showing that Camera reliability drops fastest, LiDAR shows moderate decay, and Radar remains highly reliable for most of the timeline.

Conclusion: The LiDAR achieves a reliability of 0.7343. This indicates a moderately high probability that it will remain functional after 5,000 hours. The Camera achieves a reliability of 0.4346, which makes it the weakest component in the analysis. The Radar achieves the highest reliability of 0.9962, making failure highly unlikely.

Chances of Actually Severe Conditions Given Correct Detection

If a sensor experienced correct detection in a severe condition, we need to find the probability that the conditions were actually severe. We need to find P(S|CD), where
S: Event of severe conditions
CD: Event of correct detection

Using Bayes' Theorem (since it is a conditional probability problem): P(S|CD) = P(CD|S) . P(S)/ P(CD)
Defining the events:
P(Clear) = P(C) = 0.6
P(Moderate) = P(M) = 0.3
P(Severe) = P(S) = 0.1
P(Correct Detection|Clear) = P(CD|C) = 0.995
P(Correct Detection|Moderate) = P(CD|M) = 0.985
P(Correct Detection|Severe) = P(CD|S) = 0.950

Now, using the **Law of Total Probability:**
P(CD) = P(CD|C).P(C) + P(CD|M).P(M) + P(CD|S).P(S)
After substituting the above value, we get the following:
P(CD) = (0.995).(0.6) + (0.985)(0.3) + (0.950)(0.1)
P(CD) = 0.597 + 0.2955 + 0.095

$P(CD) = 0.9875$
Therefore, the overall chance that a detection is correct is 98.75%

Now we substitute the found answers to the formula $P(S|CD) = P(CD|S). P(S)/ P(CD)$
$P(S|CD) = (0.950). (0.1)/0.9875$
$P(S|CD) = 0.095/0.9875$
$P(S|CD) = 0.0962$
Therefore, the chances of actually severe conditions given correct detection is 9.62%

Conclusion:
By applying Bayes' Theorem and the Law of Total Probability, we find that when a detection is correct, the probability that the underlying condition was Severe is only about 9.62%, which is slightly lower than the unconditional prior of 10%. This outcome is intuitive as the system is marginally more accurate in Clear and Moderate conditions than Severe, so observing a correct detection provides a small amount of evidence against the environment being Severe.

Mean Time to Failure for Each Sensor Type

The Mean Time is the expected value, or average lifespan, of a sensor. The calculation method depends entirely on the probability distribution that models the sensor's lifetime.

We calculate the expected lifetime $E[T_i]$ for each sensor type
MTTF Formula: $E[T_i]$
For LiDAR: $E[T_{LiDAR}] = MTTF_{LiDAR} = \eta \,.\, \Gamma(1 + 1/\beta)$ since it is Weibull Distribution
where $\eta=8000$, $\beta=2.5$
After substituting the above values, we get the following:
$MTTF_{LiDAR} = 8000.\ \Gamma(1 + 1/2.5)$
$MTTF_{LiDAR} = 8000.\ \Gamma(1.4)$; Using Gamma recurrence: $\Gamma(1+z) = z\,\Gamma(z)$: $\Gamma(1+0.4) = 0.4\,\Gamma(0.4)$
$MTTF_{LiDAR} = 8000 \times 0.4\,\Gamma(0.4)$; $\Gamma(0.4) \approx 2.2181$
$MTTF_{LiDAR} = 8000 \times 0.4 \times 2.2181$
$MTTF_{LiDAR} = 7097.92 \approx 7098$ hours
Therefore, the expected lifetime of the LiDAR sensor is 7097.92 ≈ 7098 hours.

For Camera: $E[T_{Camera}] = MTTF_{Camera} = 1/\lambda$ since it is an Exponential Distribution
where $\lambda = 1/6000$
After substituting the above values, we get the following:
$MTTF_{Camera} = 1/1/6000$
$MTTF_{Camera} = 6000$ hours
Therefore, the expected lifetime of the Camera sensor is 6000 hours.

For Radar: $E[T_{Radar}] = MTTF_{Radar} = \mu$ since it is a Normal Distribution (considered for non-negative time)
where $\mu = 9000$, $\sigma = 1500$
After substituting the above values, we get the following:
$MTTF_{Radar} = 9000$ hours
Therefore, the expected lifetime of the Radar sensor is 9000 hours.

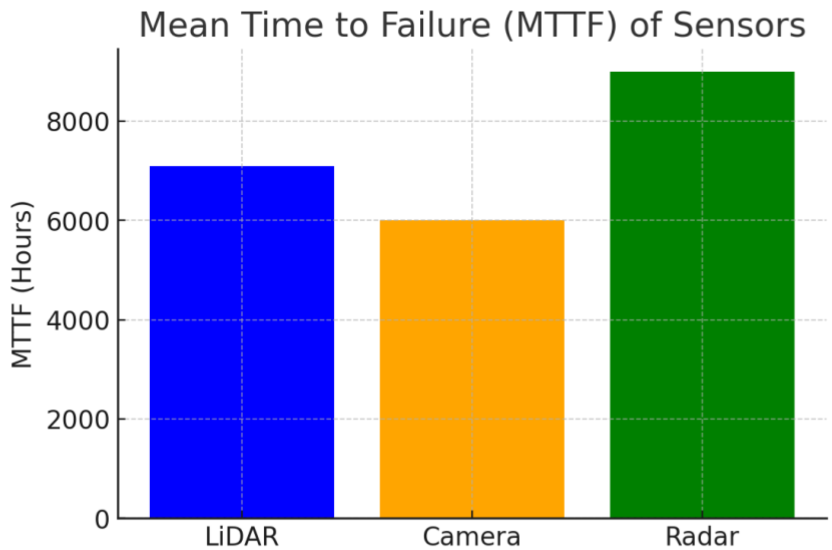

Figure 3: *Mean Time to Failure (MTTF) of Sensors*

The above graph indicates the expected average lifetime of each sensor. It is a bar chart showing that Radar lasts the longest, followed by LiDAR, while Camera has the shortest lifespan.

Sensor	**Distribution Model**	**Parameters**	**MTTF (hours)**
LiDAR	Weibull	η=8000, β=2.5	7098 hours
Camera	Exponential	λ = 1/6000	6000 hours
Radar	Normal	μ = 9000, σ = 1500	9000 hours

Table 2: *Mean Time to Failure (MTTF) of Sensors*

MTTF is the expected value or average time to failure. In terms of likelihood, it is the actual time to failure is less than the MTTF (Actual time to failure < MTTF). Therefore, the likelihood P (T_i < MTTF) depends on the shape of the probability distribution function

- For LiDAR (Weibull Distribution with β=2.5): since shape parameter β>1, the distribution is skewed to the left (positive skew). This means the mode (most likely failure time) and median are less than the mean (MTTF)

$P(T_{LiDAR} < MTTF) = P(T_{LiDAR} < 7098 \text{ hours}) > 0.50$
The probability of failing before the MTTF is greater than 50%

- For Camera (Exponential Distribution): This distribution is memoryless and always skewed to the right (decreasing failure rate). For this distribution, the median is less than the mean (MTTF).

  $P(T_{Camera} < MTTF) = P(T_{Camera} < 6000 \text{ hours}) = 1 - e^{-\lambda(1/\lambda)} = 1 - e^{-1} = 0.6321$
  The probability of failing before the MTTF is approximately 63.21%, which is greater than 50%

- For Radar (Normal Distribution): This distribution is symmetrical around the mean (μ). The mean (MTTF) is equal to the median.

$P(T_{Radar} < MTTF) = P(T_{Radar} < 9000 \text{ hours}) = 0.50$
The probability of failing before or after MTTF is exactly 50%

Stage 2 - Decision-Analysis

The Stage 2 analysis focuses on determining the system reliability (R_{sys}) for various 2-out-of-3 sensor configurations, evaluating the impact of hardware upgrades through sensitivity analysis, and providing final recommendations with constraints.

The system success criterion is defined by the 2-out-of-3 rule, meaning at least two sensors must be operational. The reliability is calculated using $R_{sys} = 3R^2 - 2R^3$ for identical sensors or an expanded formula for the mixed configuration. The Three Radar configuration achieved the highest system reliability ($R_{sys} = 1$), showing a near-perfectly reliable setup for the data shown. The Mixed (0.8478) and Three LiDAR (0.8258) configurations demonstrated strong performance, while the Three Camera system was the least reliable (0.4024).

The individual sensor reliabilities (R') were improved based on specified upgrades (20%7ncrease in LiDAR's characteristic life η, 15% decrease in Camera's failure rate λ, 25% reduction in Radar's variance σ^2). The resulting system reliabilities (R'_{sys}) were then compared to the initial values. The sensitivity analysis reveals that the Camera sensor gains the most in percentage change (21.45%), while the LiDAR sensor shows the largest absolute increase (+0.0906), reaching a strong $R'_{sys} = 0.9164$. The Radar sensor sees a negligible benefit ($\approx$ 0.00%) as its reliability was already near perfect, confirming diminishing returns for improving already high-reliability components.

Analysis Details

Schematic Diagram of System Reliability

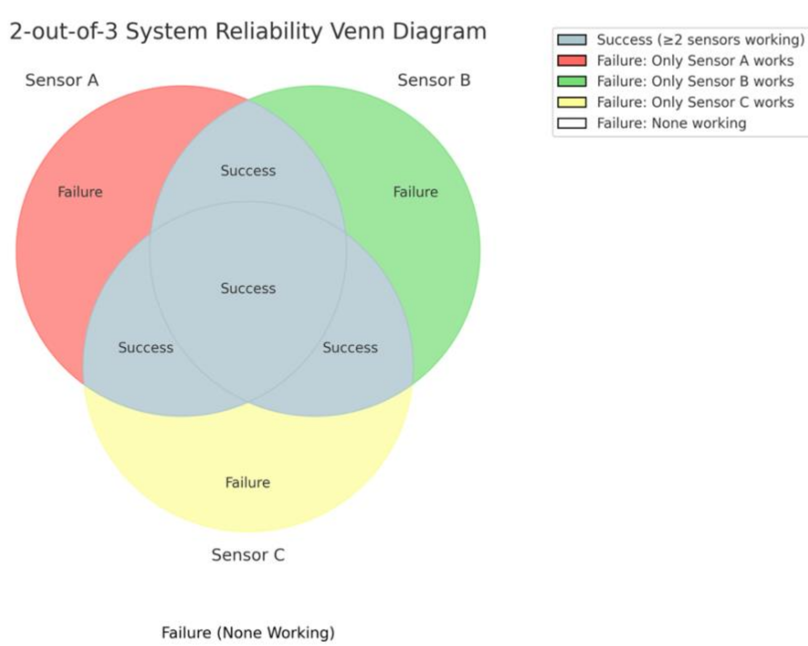

Figure 4: *Venn Diagram generated by ChatGPT 5 after providing calculated data*

We use a Venn Diagram to illustrate the successful outcomes. It has three overlapping circles representing LiDAR (Sensor A), Camera (Sensor B) and Radar (Sensor C) sensor operations, with the shaded area (grey) showing where system success occurs. Since there are three sensors, there are a total of eight mutually exclusive regions, i.e., each sensor either works or fails ($2^3 = 8$). Let S be the success of the system reliability, with at least two of the three sensors operating.

Interpretation of the Venn Diagram:
Event Definitions:
A: LiDAR sensor works
B: Camera sensor works
C: Radar sensor works
A': LiDAR sensor fails
B': Camera sensor fails
C': Radar sensor fails

Success Regions:
A'BC: Both the Camera and Radar sensors are operating, but the LiDAR sensor fails **($A' \cap B \cap C$)**
AB'C: Both LiDAR and Radar sensors are operating, but the Camera sensor fails **($A \cap B' \cap C$)**
ABC': Both LiDAR and Camera sensors are operating, but the Radar sensor fails **($A \cap B \cap C'$)**
ABC: All three sensors are operating **($A \cap B \cap C$)**
These are the grey regions in the above Venn Diagram.

Failure Regions:
AB'C': Both the Camera and Radar sensors fail, but the LiDAR sensor is operating **($A \cap B' \cap C'$)**
A'BC': Both the LiDAR and Radar sensors fail, but the Camera sensor is operating (**$A' \cap B \cap C'$)**
A'B'C: Both the LiDAR and Camera sensors fail, but the Radar sensor is operating **($A' \cap B' \cap C$)**
A'B'C': All three sensors fail **($A' \cap B' \cap C'$)**
These are the red, green and yellow regions in the above Venn Diagram. The white background is the region where all sensors fail.

Succession Criteria (2-out-of-3 rule):
The system is considered successful if at least two sensors are operational. Thus, the regions in the diagram where two or three circles overlap give system success.
A'BC: Both the Camera and Radar sensors are operating, but the LiDAR sensor fails **($A' \cap B \cap C$)**
AB'C: Both LiDAR and Radar sensors are operating, but the Camera sensor fails **($A \cap B' \cap C$)**
ABC': Both LiDAR and Camera sensors are operating, but the Radar sensor fails **($A \cap B \cap C'$)**

Therefore, we conclude that the system succeeds in the union of four regions:

$$\mathbf{S = (A \cap B \cap C) \cup (A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C)}$$

System Reliability Formula

The formula for system reliability ($R_{sys} = 3R^2 - 2R^3$) is derived from the first principles using the success regions identified in the Venn Diagram.
The system is successful if at least two of the three sensors are operating. Let R be the reliability of a single sensor, and Q be the probability of its failure, where Q = 1-R. Since all three sensors are of the same type, they have the same reliability R. The sensors are independent.

Derivation of $R_{sys} = 3R^2 - 2R^3$:
The probability of the union of these mutually exclusive events is the sum of their individual probabilities.
$R_{sys} = P(S) = (A \cap B \cap C) \cup (A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C)$
Since the sensors are independent, the probability of the intersection is the product of the individual probabilities.
P(A) = R(Success)
P(A') = Q = 1-R(Failure)

The system reliability (R_{sys}) is calculated as follows:

- Three sensors working: $P(A \cap B \cap C) = P(A)P(B)P(C) = R.R.R = R^3$
- Two sensors are working (and one failing):
  $P(A \cap B \cap C') = P(A)P(B)P(C') = R.R.Q = R^2Q = R^2(1-R) = R^2 - R^3$
  $P(A \cap B' \cap C) = P(A)P(B')P(C) = R.Q.R = R^2Q = R^2(1-R) = R^2 - R^3$
  $P(A' \cap B \cap C) = P(A')P(B)P(C) = Q.R.R = R^2Q = R^2(1-R) = R^2 - R^3$
  Therefore, summing the above probabilities gives $R_{sys} = S = P(S)$
  $R_{sys} = R^3 + (R^2 - R^3) + (R^2 - R^3) + (R^2 - R^3)$
  $R_{sys} = R^3 + 3(R^2 - R^3)$
  $R_{sys} = R^3 + 3R^2 - 3R^3$
  $R_{sys} = 3R^2 - 2R^3$
  This is the system reliability formula for a 2-out-of-3 system with identical and independent components.

System Reliability for Each Configuration

We need to calculate the system reliability R_{sys} at t=5000 hours for four different configurations. We use the derived formula: $R_{sys} = 3R^2 - 2R^3$

(A) **Three LiDAR Sensors**
Using the formula $R_{sys} = 3R^2_L - 2R^3_L$; Given that $R_L = 0.7345$
After substituting the above values, we get the following:
$R_{sys} = 3(0.7345)^2 - 2(0.7345)^3$
$R_{sys} = 3(0.5394) - 2(0.3962)$
$R_{sys} = 1.6182 - 0.7924 = 0.8258$

(B) **Three Radar Sensors**
Using the formula $R_{sys} = 3R^2_R - 2R^3_R$; Given that $R_R = 0.9962$
After substituting the above values, we get the following:
$R_{sys} = 3(0.9962)^2 - 2(0.9962)^3$

$R_{sys} = 3(0.9924) - 2(0.9886)$
$R_{sys} = 2.9772 - 1.9772 = 1$

(C) **Three Camera Sensors**

Using the formula $R_{sys} = 3R^2_C - 2R^3_C$; Given that $R_C = 0.4346$
After substituting the above values, we get the following:
$R_{sys} = 3(0.4346)^2 - 2(0.4346)^3$
$R_{sys} = 3(0.1888) - 2(0.0820)$
$R_{sys} = 0.5664 - 0.1640 = 0.4024$

(D) **Mixed Configuration Reliability Formula**

If the sensors are of different types (independent but not identical), then
$R_{sys} = P(ABC') + P(AB'C) + P(A'BC) + P(ABC)$
$R_{sys} = P(A)P(B)P(C') + P(A)P(B')P(C) + P(A')P(B)P(C) + P(A)P(B)P(C)$
$R_{sys} = R_AR_B(1-R_C) + R_A(1-R_B)R_C + (1-R_A)R_BR_C + R_AR_BR_C$
We already know that A: LiDAR sensor, B: Camera Sensor, C: Radar Sensor
$R_{sys} = R_LR_C(1-R_R) + R_L(1-R_C)R_R + (1-R_L)R_CR_R + R_LR_CR_R$
After substituting the above values, we get the following:
$R_{sys} = (0.7345)(0.4346)(1 - 0.9962) + (0.7345)(1 - 0.4346)(0.9962) + (1 - 0.7345)(0.4346)(0.9962) + (0.7345)(0.4346)(0.9962)$
$R_{sys} = 0.3192(0.0038) + 0.7317(0.5654) + 0.2655(0.4329) + 0.3180$
$R_{sys} = 0.0012 + 0.4137 + 0.1149 + 0.3180$
$R_{sys} = 0.8478$

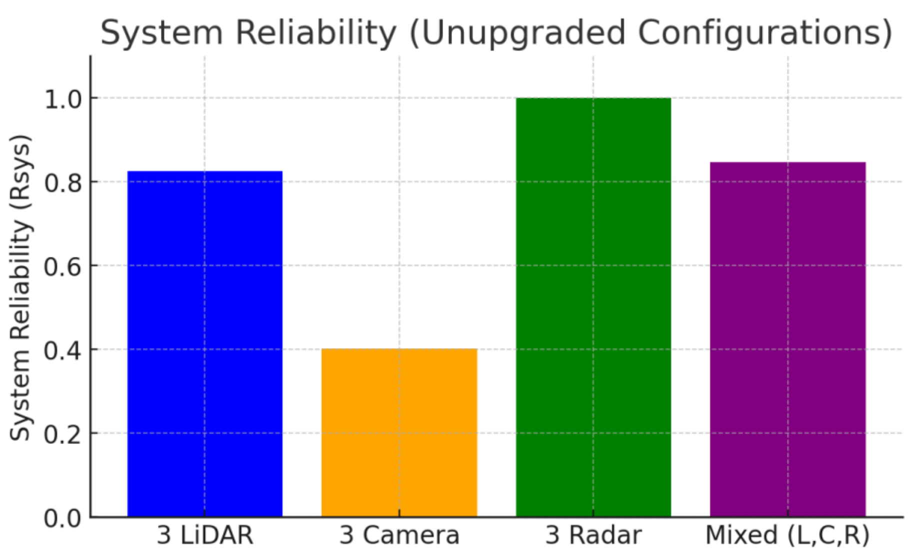

Figure 5: *System Reliability (Unupgraded Configurations)*

The above graph indicates the system reliability values for different 2-out-of-3 sensor setups before upgrades. It is a bar chart showing that the three Radar configuration achieve near-perfect reliability, while the Camera configuration is the weakest.

Configuration	Formula Used	R_{sys}
Three LiDAR	$R_{sys} = 3R^2_L - 2R^3_L$	0.8258
Three Camera	$R_{sys} = 3R^2_C - 2R^3_C$	0.4024
Three Radar	$R_{sys} = 3R^2_R - 2R^3_R$	1
Mixed (L, C, R)	$R_{sys} = R_LR_C(1-R_R) + R_L(1-R_C)R_R + (1-R_L)R_CR_R + R_LR_CR_R$	0.8478

Table 3: *System Reliability for Each Configuration (Unupgraded)*

Conclusion: The three Radar configuration achieved the highest system reliability ($R_{sys} = 1$), showing a perfectly reliable setup for the data shown. The three LiDAR and Mixed configurations also demonstrated strong performance, with R_{sys} values of 0.8258 and 0.8478, respectively, while the three Camera system was the least reliable ($R_{sys} = 0.4024$)

Sensitivity Analysis

The following is the impact of the upgrades on the individual sensors at reliability t=5000 hours

For LiDAR: $R_L(t) = e^{-(t/n)^\wedge\beta}$ since it is Weibull Distribution

where t=5000, η=8000 + 20% of 8000, β=2.5

new parameters are t=5000, η=9600, β=2.5

After substituting the above values, we get the following:

$R'_L\ (5000) = e^{-(5000/9600)^\wedge 2.5}$

$R'_L\ (5000) = e^{-(0.5208)^\wedge 2.5}$

$R'_L\ (5000) = e^{\ -0.1957}$

$R'_L\ (5000) = 0.8222$

For Camera: $R_C(t) = e^{-\ \lambda t}$ since it is Exponential Distribution

where t=5000, λ=1/6000 - 15% of 1/6000

new parameters are t=5000, λ= 1/7058.82

After substituting the above values, we get the following:

$R'_C\ (5000) = e^{(-1/7058.82)\ 5000}$

$R'_C\ (5000) = e^{-(5000/7058.82)}$

$R'_C\ (5000) = e^{-0.7083}$

$R'_C\ (5000) = 0.4925$

For Radar: $R_R(t) = 1 - \Phi(t- \mu/\sigma)$ since it is Normal Distribution

where t=5000, μ=9000, σ=1500, σ^2=2,250,000 - 25% of 2,250,000

new parameters are t=5000, μ=9000, σ^2 = 1,687,500, σ=1299.038

After substituting the above values, we get the following:

$R'_R\ (5000) = 1 - \Phi(5000\text{-}9000/1299.038)$

$R'_R\ (5000) = 1 - \Phi(-4000/1299.038)$

$R'_R\ (5000) = 1 - \Phi(-3.0792)$; Using symmetry of the standard normal CDF: $1 - \Phi(-a) = \Phi(a)$

$R'_R\ (5000) = \Phi(3.0792)$

$R'_R\ (5000) = 0.9990$

Sensor Type	New Parameters	R'(5000 hours)
LiDAR	t=5000, η=9600, β=2.5	0.8222
Camera	t=5000, λ= 1/7058.82	0.4925
Radar	t=5000, μ=9000, σ^2 = 1,687,500, σ=1299.038	0.9990

Table 4: *Upgraded Sensor Reliability at 5000 hours*

The new individual reliabilities R' are plugged into the "all three sensors are the same type" model.

(A) **Three LiDAR Sensors**

Using the formula $R'_{sys} = 3R'^2_L - 2R'^3_L$; Given that $R'_L = 0.8222$

After substituting the above values, we get the following:

$R'_{sys} = 3(0.8222)^2 - 2(0.8222)^3$

$R'_{sys} = 3(0.6760) - 2(0.5558)$

$R'_{sys} = 2.0280 - 1.1116 = 0.9164$

(B) **Three Radar Sensors**

Using the formula $R'_{sys} = 3R'^2_R - 2R'^3_R$; Given that $R'_R = 0.9990$

After substituting the above values, we get the following:

$R'_{sys} = 3(0.9990)^2 - 2(0.9990)^3$

$R'_{sys} = 3(0.9980) - 2(0.9970)$

$R'_{sys} = 2.9940 - 1.9940 = 1$

(C) **Three Camera Sensors**

Using the formula $R'_{sys} = 3R'^2_C - 2R'^3_C$; Given that $R'_C = 0.4925$

After substituting the above values, we get the following:

$R'_{sys} = 3(0.4925)^2 - 2(0.4925)^3$

$R'_{sys} = 3(0.2425) - 2(0.1194)$

$R'_{sys} = 0.7275 - 0.2388 = 0.4887$

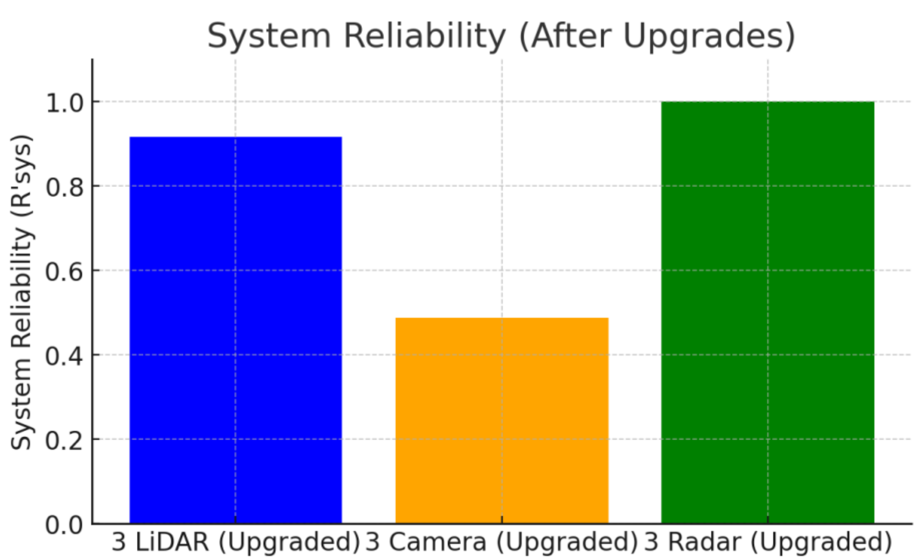

Figure 6: *System Reliability After Upgrades*

The above graph indicates the improvement in reliability after hardware upgrades. It is a bar chart showing significant gains for LiDAR and Camera, with Radar remaining at the maximum value.

Configuration	Formula Used	R'_{sys}
Three LiDAR	$R'_{sys} = 3R'^2_L - 2R'^3_L$	0.9164
Three Camera	$R'_{sys} = 3R'^2_C - 2R'^3_C$	0.4887
Three Radar	$R'_{sys} = 3R'^2_R - 2R'^3_R$	1

Table 5: **System Reliability After Upgrades**

The benefit of the upgrades is evaluated by comparing the system reliability before the upgrades (R_{sys}) to the system reliability after the upgrade (R'_{sys}). All three sensors are the same model, which is used for comparison.

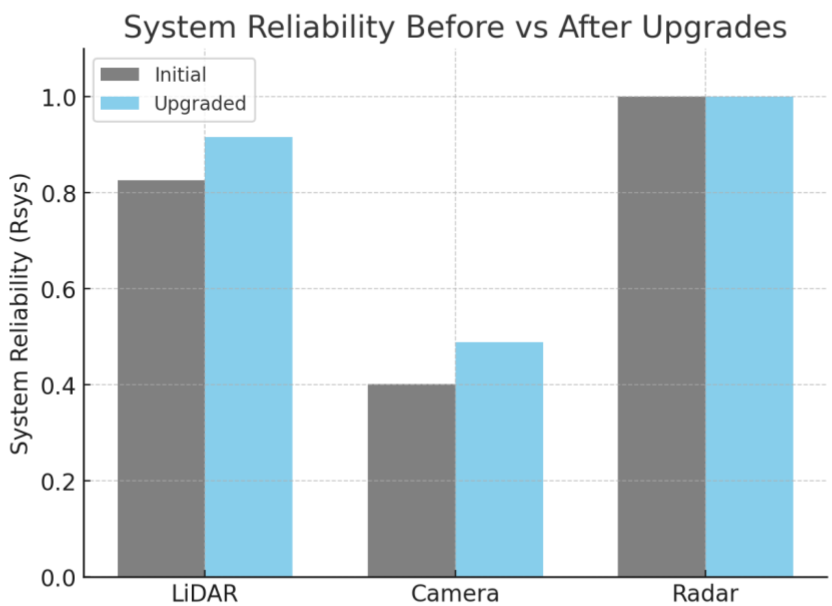

Figure 7: *Comparison of System Reliability Before vs After Upgrades*

The above graph indicates the side-by-side comparison of system reliability values before and after upgrades. It is a grouped bar chart clearly showing how LiDAR and Camera benefited, while Radar remains unchanged.

Sensor Type	Initial R	Initial R_{sys}	Upgraded R'	Upgraded R'_{sys}	ΔR_{sys} (Absolute Benefit)	% Change
LiDAR	0.7345	0.8258	0.8222	0.9164	+0.0906	10.97%
Camera	0.4346	0.4024	0.4925	0.4887	+0.0863	21.45%
Radar	0.9962	1	0.9990	1	Negligible ≈ 0	≈ 0.00%

Table 6: *Sensitivity Analysis - Before and After Upgrades*

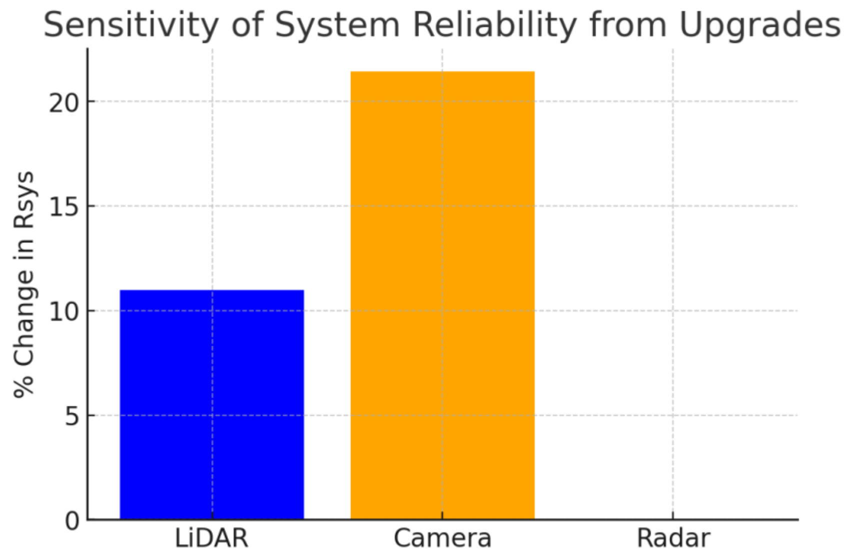

Figure 8: *Sensitivity of System Reliability from Upgrades*

The above graph indicates how sensitive each sensor type is to the proposed upgrades. It is a bar chart showing that Camera had the highest percentage improvement, LiDAR had the highest gain, and Radar showed a negligible change.

Conclusion: The sensitivity analysis reveals that the Camera sensor gains the most in percentage change (21.45%), while the LiDAR sensor shows the largest absolute increase (+0.0906), reaching a strong R'_{sys} = 0.9164. The Radar sensor sees a negligible benefit (0.00%) as its reliability was already near perfect. This confirms that the upgrades are most effective for components with moderate starting reliability, and the Radar configuration remains the most reliable overall.

Recommendations

Best

The best system configuration is the Three Radar configuration as it achieves the highest system reliability R_{sys} = 1 at t=5000 hours, under the 2-out-of-3 rule, outperforming 3 LiDAR (0.8258), Mixed L+C+R(0.8478), and Three Camera (0.4024). Even after the suggested upgrades, the Three Radar configuration remains at R_{sys} = 1. These results make it the most robust choice for critical applications in the analysis.

If an alternate needs to be picked, the next best is the Mixed configuration on baseline (R_{sys} = 0.8478), offering modality diversity that can help mitigate common-cause risks if engineered properly. Then comes the Three LiDAR configuration (R_{sys} = 0.8258 baseline; 0.9164 post-upgrade), giving the largest absolute improvement from upgrades among the non-Radar sets. Last is the Three Camera configuration, and it is not recommended for reliability-critical contexts (R_{sys} = 0.4024 baseline; 0.4887 post-upgrade)

Limits and Constraints of Recommendations

Influence of Stochastic Dependence on the System Reliability Formula

The 2-out-of-3 reliability formula $R_{sys} = 3R^2 - 2R^3$ assumes that sensors fail independently. In real-world settings, this is rarely the case. If positive dependence (eg: common-cause failures from shared power supplies, environmental stressors, or identical firmware bugs) exists, the likelihood of simultaneous failures increases, which means the actual system reliability will decrease compared to our calculated values. Thus, the reported values should be treated as optimistic upper bounds. On the other hand, if diverse redundancy is designed into the system (different sensing modalities, separate power domains, or vendor diversity), failures may become less correlated, and the system reliability can stay the same or even slightly improve relative to the independent assumption. Overall, the expected trend in practice is a decrease in system reliability unless steps are taken to mitigate common-cause dependencies.

Sensitivity of the Reliability from the suggested hardware upgrades

The sensitivity of system reliability improvements depends heavily on the initial reliability of each component. Mathematically, the derivative of the system reliability function, $dR_{sys}/dR = 6R(1 - R)$, shows that sensitivity is highest when a component's reliability is moderate (around 0.3 to 0.8) and lowest when reliability is already very high or very low. This explains our results: the Camera, with an initial reliability of ~0.43, showed the largest percentage improvement (+21.45%); the LiDAR, at ~0.73, had the largest absolute system gain (+0.0906); and the Radar, already near 1.0 (almost to near perfect), showed negligible improvement. Thus, hardware upgrades are most effective for components with moderate baseline reliability, while improving already near-perfect sensors yields diminishing returns.