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MT2022161

TASK 2

Polynomial classifier with one feature. Generate 200 points $x^{(1)}, \ldots, x^{(200)}$, uniformly spaced in the interval $[-1, 1]$, and take

$$y^{(i)} = \begin{cases} +1 & -0.5 \leq x^{(i)} < 0.1 \text{ or } 0.5 \leq x^{(i)} \\ -1 & \text{otherwise} \end{cases}$$

for $i = 1, \ldots, 200$. Fit polynomial least squares classifiers of degrees $0, \ldots, 8$ to this training data set.

(a) Evaluate the error rate on the training data set. Does the error rate decrease when you increase the degree?

(b) For each degree, plot the polynomial $\tilde{f}(x)$ and the classifier $\hat{f}(x) = \mathbf{sign}(\tilde{f}(x))$.

(c) It is possible to classify this data set perfectly using a classifier $\hat{f}(x) = \mathbf{sign}(\tilde{f}(x))$ and a cubic polynomial

$$\tilde{f}(x) = c(x + 0.5)(x - 0.1)(x - 0.5),$$

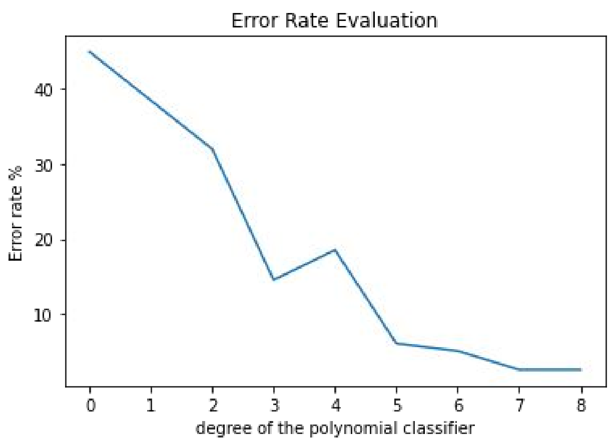
for any positive c. Compare this classifier with the least squares classifier of degree 3 that you found and explain why there is a difference.

a.)Evaluate the error rate on the training data set. Does the error rate decrease when you increase the degree.

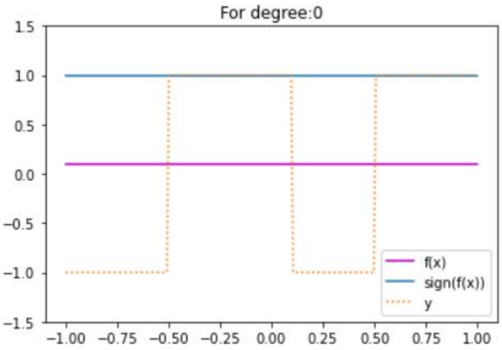
Solution:

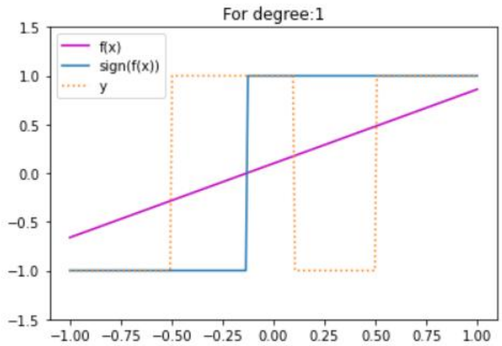
From the below plot we can observe that error rate increases as the degree of the of the polynomial classifier increases from 3 to 4.

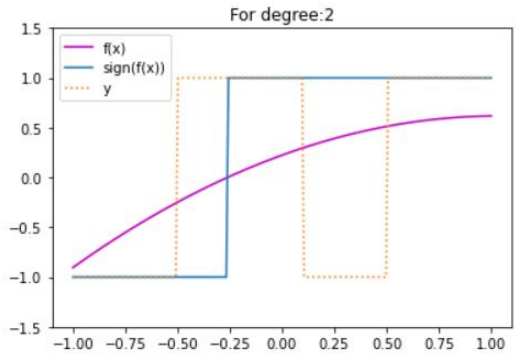
Even though we might think that increasing the degree of the polynomial should decrease the error rate at least with respect to the training data(because the model will over fit) but here error rate increases when degree of the classifier is increased from 3 to 4. This is because we are trying to fit our model by reducing RMSE between f(x) and y and not sign(f(x)) and y. Hence polynomial classifier of degree 4 might predicts f(x) better than polynomial classifier of degree 3, but when it comes to predicting the sign(f(x)) it performs worst.

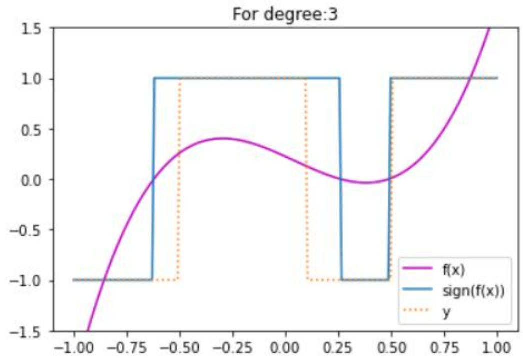


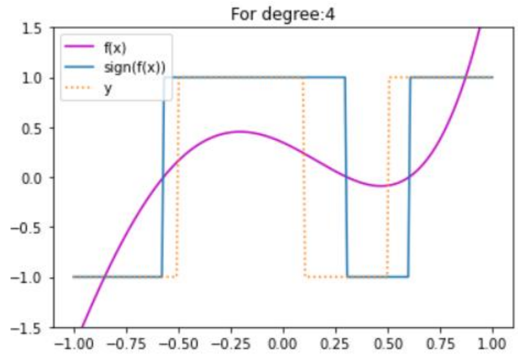
b.) Plotting f(x) and sign(f(x)) for least squares classifier of all degrees from 0 till 8

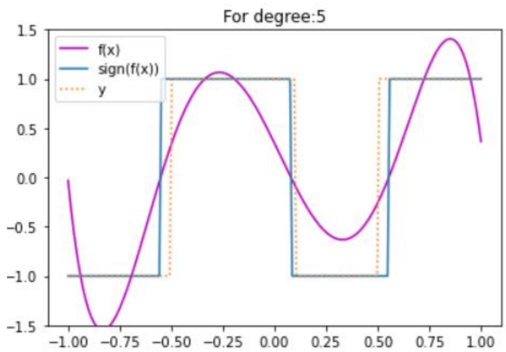

For degree:0
f(x)
sign(f(x))
y

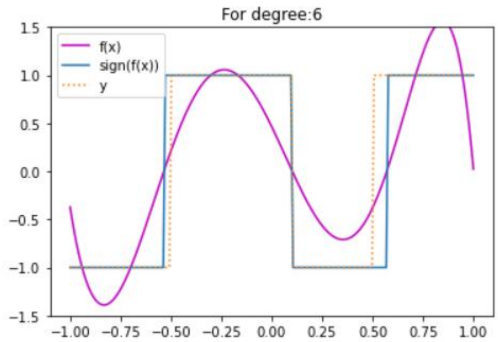

For degree:1
f(x)
sign(f(x))
y

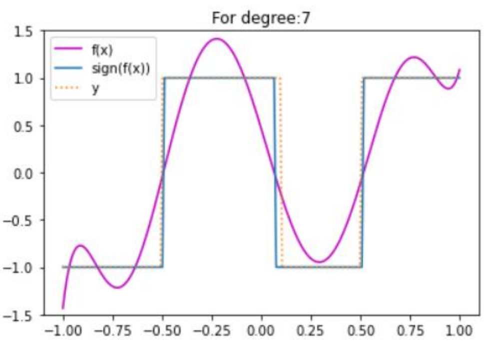

For degree:2
f(x)
sign(f(x))
y

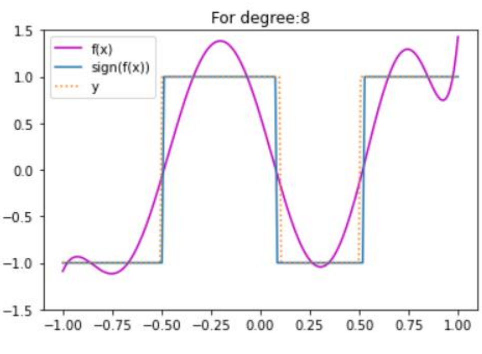

For degree:3
f(x)
sign(f(x))
y


For degree:4
f(x)
sign(f(x))
y


For degree:5
f(x)
sign(f(x))
y


For degree:6
f(x)
sign(f(x))
y


For degree:7
f(x)
sign(f(x))
y

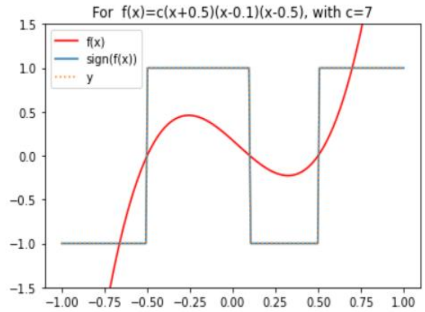

For degree:8
f(x)
sign(f(x))
y

c.) Is it possible to classify this data set perfectly using a classifier f(x)=sign(f(x)) ,where f(x)=c(x+0.5)(x-0.1)(x-0.5)

Solution:

The equation f(x)=c(x+0.5)(x-0.1)(x-0.5) perfectly classifies the training data when compared to the polynomial classifier of 3rd degree.

The RMSE is trying to fit by minimize the error between f(x) and y which is not same as minimizing the error between sign(f(x)) and y. Hence our 3rd degree polynomial classifier will not perfectly fit the training data.

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RMSE of training set is:0.0
error rate is : 0.0
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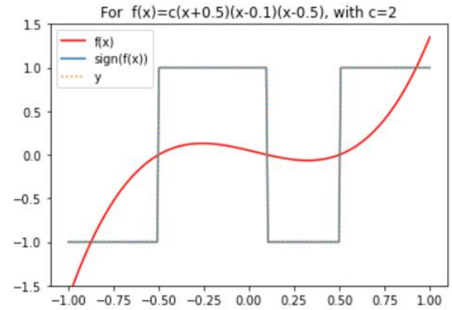
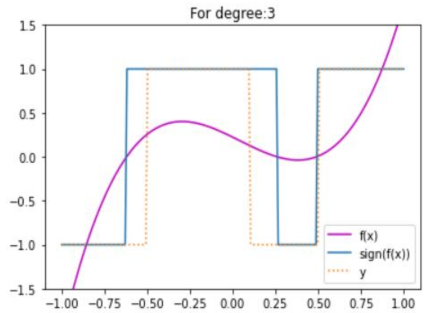
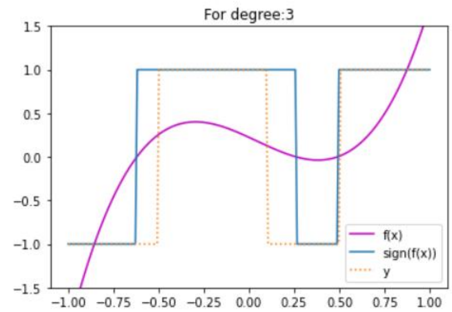



```
RMSE of training set is:0.0
error rate is : 0.0
```




```
RMSE of training set is:0.58
error rate is : 14.499999999999998
```




```
RMSE of training set is:0.58
error rate is : 14.499999999999998
```