MML - Assignment - 2 :-

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Task 1 : Network Topology

Given

$\Rightarrow$ n links l_1 l_2 l_3 . . . l_n

$\Rightarrow$ $\vec{P}$ is a $N \times n$ matrix where,

$$P_{ij} = \begin{cases} 1 & \text{if link } j \text{ is on path } i \\ 0 & \text{otherwise} \end{cases}$$

$\Rightarrow$ number of paths is N.

$\Rightarrow$ number of links is n.

$$\therefore \vec{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & & & \\ p_{N1} & p_{N2} & \cdots & p_{Nn} \end{bmatrix}_{N \times n}$$

Assumption 1: $N > n$

→ $\vec{t}$ is a N vector whose entries are noisy travel times along N paths.

$$\vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ \vdots \\ t_N \end{bmatrix}$$

Assumption 2: $\vec{t}$ contains noisy data.

Goal: Predict link delays $\vec{d}$

$$\vec{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$$

Procedure: For the given delays $\vec{d}$ travel times can be predicted by,

$$\vec{t}_{pred} = P_{N \times n} \cdot d_{n \times 1}$$

Root Mean Square deviation is given by : $\sqrt{\frac{\Sigma(y_{pred.} - y)^2}{N}}$

for better estimation of $\vec{d}$, RMS deviation b/w $\vec{t}$ & $\vec{t}_{pred}$ should be minimized.

$$\therefore d = \frac{\left((\vec{t}_{pred} - t)(t_{pred} - t)^T \right)^{1/2}}{(N)^{1/2}}$$

{ignoring constants}

$$d = \left((t_{pred} - t)(t_{pred} - t)^T \right)^{1/2}$$

$$d = \left((p.d - t)(p.d - t)^T \right)^{1/2}$$

$$d = \left(pd\, d^T p^T - t d^T p^T - pd\, t^T - t t^T \right)^{1/2}$$

$$d = \left(p d d^T p^T - 2 t d^T p^T - t t^T \right)^{1/2}$$

Applying derivative of first order with respect to d, to find the minima by equating to 0.

$$\frac{\partial L}{\partial d} = \frac{1}{2}\left(2P^TPd - 2t^TP + 0\right)$$

$$\frac{\partial L}{\partial d} = P^TPd - t^TP$$

$$\frac{\partial L}{\partial d} = 0 \Rightarrow P^TPd - t^TP = 0$$

$$P^TPd = t^TP$$

$$P^TPd = P^Tt$$

$$\vec{d} = \underbrace{(P^TP)^{-1}P^T}_{\text{Moore - Penrose Pseudo Inverse}} \cdot t$$

$$\vec{d} = P^+ \cdot \vec{t}$$

where P^+ is the Pseudo Inverse of $\vec{P}$.