Report: BDD Assignment (EE709)

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Analysis of a 4-bit adder

Consider a 4-bit adder without an incoming carry.

It has 8 inputs

x3 x2 x1 x0

y3 y2 y1 y0

and produces five output bits cout s3 s2 s1 s0

Q1. Consider the following subset A of the domain: those combinations of x and y such that an odd number of bits of x and an odd number of bits of y are 1. Using the BDD package, find the image of the set A in the range.

Solution : [C code - 213076003_Q1.c]

```
if var.8
  if var.9
    if var.10
      !var.12
    else if !var.10
      1
    endif var.10
  else if !var.9
    1
  endif var.9
else if !var.8
  if var.9
    1
  else if !var.9
    if var.10
      1
    else if !var.10
      if var.11
        1
      else if !var.11
        var.12
      endif var.11
    endif var.10
  endif var.9
endif var.8
```

Q2. Consider the following subset B of the range: the set of all 5-bit numbers c y3 y2 y1 y0 such that the number of bits in the number is odd. Using the BDD package, find the pre-image of the set B in the domain.

Solution : [C code - 213076003_Q2.c]

```
if var.0
  if var.1
    if var.2
      if var.3
        if var.4
          0: if var.5
            1: if var.6
              0
            else if !var.6
              var.7
            endif var.6
          else if !var.5
            var.6
          endif var.5
        else if !var.4
          2: if var.5
            !subformula 1
          else if !var.5
            subformula 1
          endif var.5
        endif var.4
      else if !var.3
        if var.4
          3: if var.5
            !var.7
          else if !var.5
            4: if var.6
              var.7
            else if !var.6
              !var.7
            endif var.6
          endif var.5
        else if !var.4
          5: if var.5
            var.7
          else if !var.5
            !var.7
          endif var.5
        endif var.4
      endif var.3
    else if !var.2
      if var.3
```

```
      if var.4
        6: if var.5
          7: if var.6
            var.7
          else if !var.6
            1
          endif var.6
        else if !var.5
          !var.6
        endif var.5
      else if !var.4
        8: if var.5
          !subformula 7
        else if !var.5
          subformula 7
        endif var.5
      endif var.4
    else if !var.3
      if var.4
        !subformula 4
      else if !var.4
        9: if var.5
          subformula 4
        else if !var.5
          !subformula 4
        endif var.5
      endif var.4
    endif var.3
  endif var.2
else if !var.1
  if var.2
    if var.3
      if var.4
        10: if var.5
          var.6
        else if !var.5
          !subformula 1
        endif var.5
      else if !var.4
        !subformula 2
      endif var.4
    else if !var.3
      if var.4
        11: if var.5
          subformula 4
        else if !var.5
          var.7
        endif var.5
```

```
      else if !var.4
        !subformula 5
      endif var.4
    endif var.3
  else if !var.2
    if var.3
      if var.4
        12: if var.5
          !var.6
        else if !var.5
          !subformula 7
        endif var.5
      else if !var.4
        !subformula 8
      endif var.4
    else if !var.3
      !subformula 9
    endif var.3
  endif var.2
endif var.1
else if !var.0
  if var.1
    if var.2
      if var.3
        if var.4
          subformula 2
        else if !var.4
          !subformula 0
        endif var.4
      else if !var.3
        if var.4
          subformula 5
        else if !var.4
          !subformula 3
        endif var.4
      endif var.3
    else if !var.2
      if var.3
        if var.4
          subformula 8
        else if !var.4
          !subformula 6
        endif var.4
      else if !var.3
        if var.4
          subformula 9
        else if !var.4
          subformula 4
```

```
      endif var.4
    endif var.3
  endif var.2
 else if !var.1
  if var.2
    if var.3
      if var.4
        !subformula 2
      else if !var.4
        !subformula 10
      endif var.4
    else if !var.3
      if var.4
        !subformula 5
      else if !var.4
        !subformula 11
      endif var.4
    endif var.3
  else if !var.2
    if var.3
      if var.4
        !subformula 8
      else if !var.4
        !subformula 12
      endif var.4
    else if !var.3
      if var.4
        !subformula 9
      else if !var.4
        subformula 9
      endif var.4
    endif var.3
  endif var.2
 endif var.1
endif var.0
```

Q3. Lets prove a property about a four bit adder: show (using BDD's) that every even 4-bit number can be expressed as a sum of two prime numbers.

Solution : [C code - 213076003_Q2.c]

```
if var.8
 if var.9
  if var.10
   0
  else if !var.10
   !var.12
```

```
    endif var.10
  else if !var.9
    !var.12
  endif var.9
else if !var.8
  if var.9
    if var.10
      1
    else if !var.10
      if var.11
        !var.12
      else if !var.11
        1
      endif var.11
    endif var.10
  else if !var.9
    if var.10
      1
    else if !var.10
      var.11
    endif var.10
  endif var.9
endif var.8
```

----Subset of subset A(even outputs when inputs are prime numbers)----

----------BDD name : e----------

```
if var.8
  if var.9
    if var.10
      0
    else if !var.10
      !var.12
    endif var.10
  else if !var.9
    !var.12
  endif var.9
else if !var.8
  if var.9
    !var.12
  else if !var.9
    if var.10
      !var.12
    else if !var.10
      if var.11
```

```
      !var.12
    else if !var.11
      0
    endif var.11
  endif var.10
 endif var.9
endif var.8
```


----------All possible nonzero even output numbers----------
----------BDD name : i----------

```
if var.8
 !var.12
else if !var.8
 if var.9
  !var.12
 else if !var.9
  if var.10
   !var.12
  else if !var.10
   if var.11
    !var.12
   else if !var.11
    0
   endif var.11
  endif var.10
 endif var.9
endif var.8
```


To prove: Every even 4-bit number can be expressed as a sum of two prime numbers
We need to prove: bdd 'e' is a subset of bdd 'i' i.e. ~e+i = 1 OR ~i.e = 0
----------Proof 1 : ~i.e = 0----------

0
Result is a zero BDD hence ~i.e = 0

----------Proof 2 : ~e+i = 1----------

1
Result is a one BDD hence ~e+i = 1

Hence the property is proved