

CSL603 - Machine Learning

Lab 2

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1 Linear Ridge Regression

Given X and Y we will find W that minimizes $J(W)$, the error function and are defined as:

$$f(X) = \underbrace{\begin{pmatrix} 1 & x_{11} & \cdots & x_{1D} \\ 1 & x_{21} & \cdots & x_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{N1} & \cdots & x_{ND} \end{pmatrix}}_X \underbrace{\begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_D \end{pmatrix}}_W = \underbrace{\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{pmatrix}}_Y$$

$$\min_W J(W) \equiv \min_W \frac{1}{2} (XW - Y)^T (XW - Y) + \frac{1}{2} \lambda \|W\|^2$$

Which when solved gives us:

$$W = (X^T X + \lambda I)^{-1} X^T Y$$

Observations

The following observations were obtained:

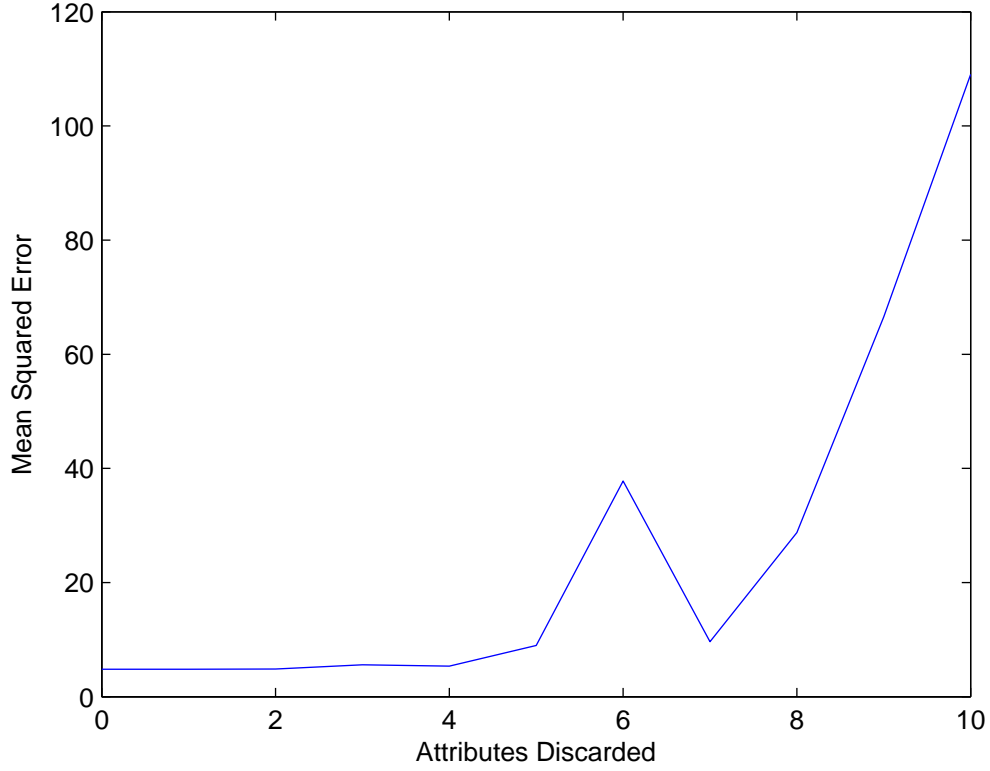


Figure 1: Mean Squared Error for after discarding increasing number of least significant weights in W .

- A particular value of λ (say 0) was chosen and then the magnitude of entries in the weights W was compared and one by one the least significant ones were discarded and the mean squared error changes can be seen in Figure 1. We can see that discarding 2-4 least significant attributes does not make any major change to the mean squared error, hence we can conclude that the input data contained some attributes that were irrelevant in estimating the output values.

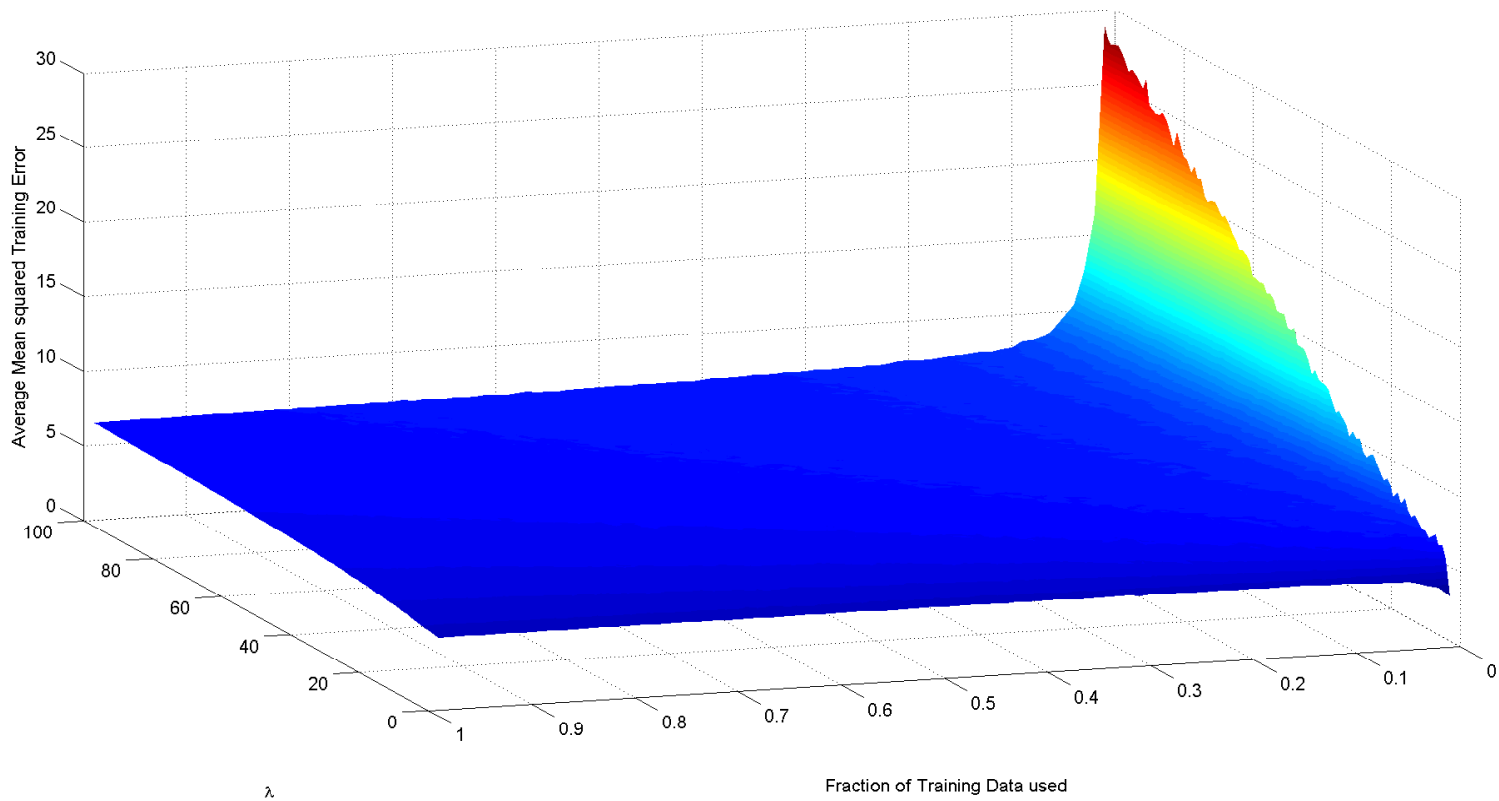


Figure 2: Average Mean Squared Error for various values of training set fraction and λ values used in Ridge Regression for Training Data.

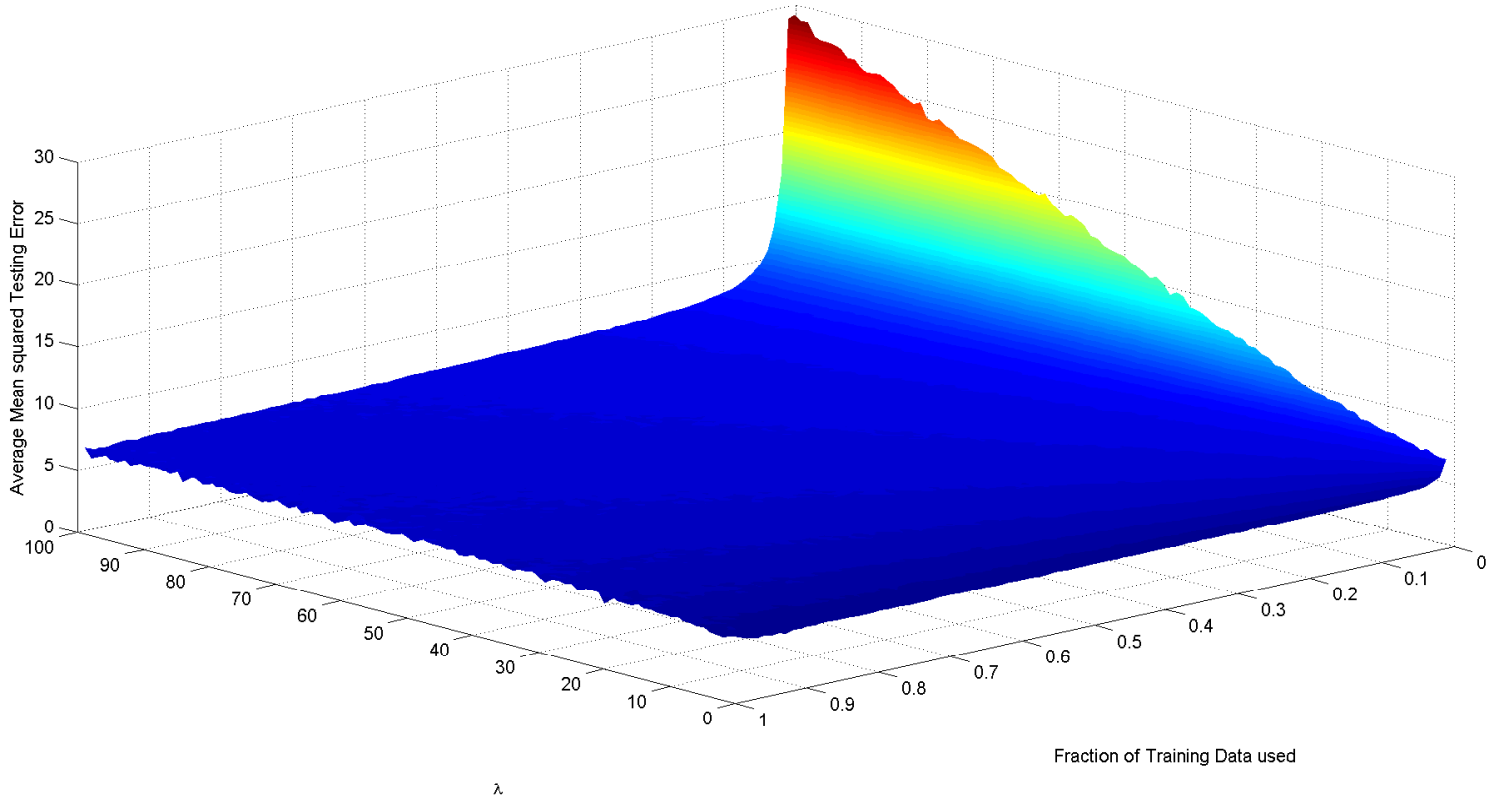


Figure 3: Average Mean Squared Error for various values of training set fraction and λ values used in Ridge Regression for Testing Data.

- The effect of λ on error was observed for different partitions of the data into training and testing sets. The average mean squared error for 100 repetitions for splitting-fractions varying from 1% to 99% and lambda values from 0 to 100 was observed. The surface corresponding to the average mean absolute error can be seen in Figure 2 and 3. We can see that for low values of training set fraction or high λ values the average mean squared error increased quite a bit.

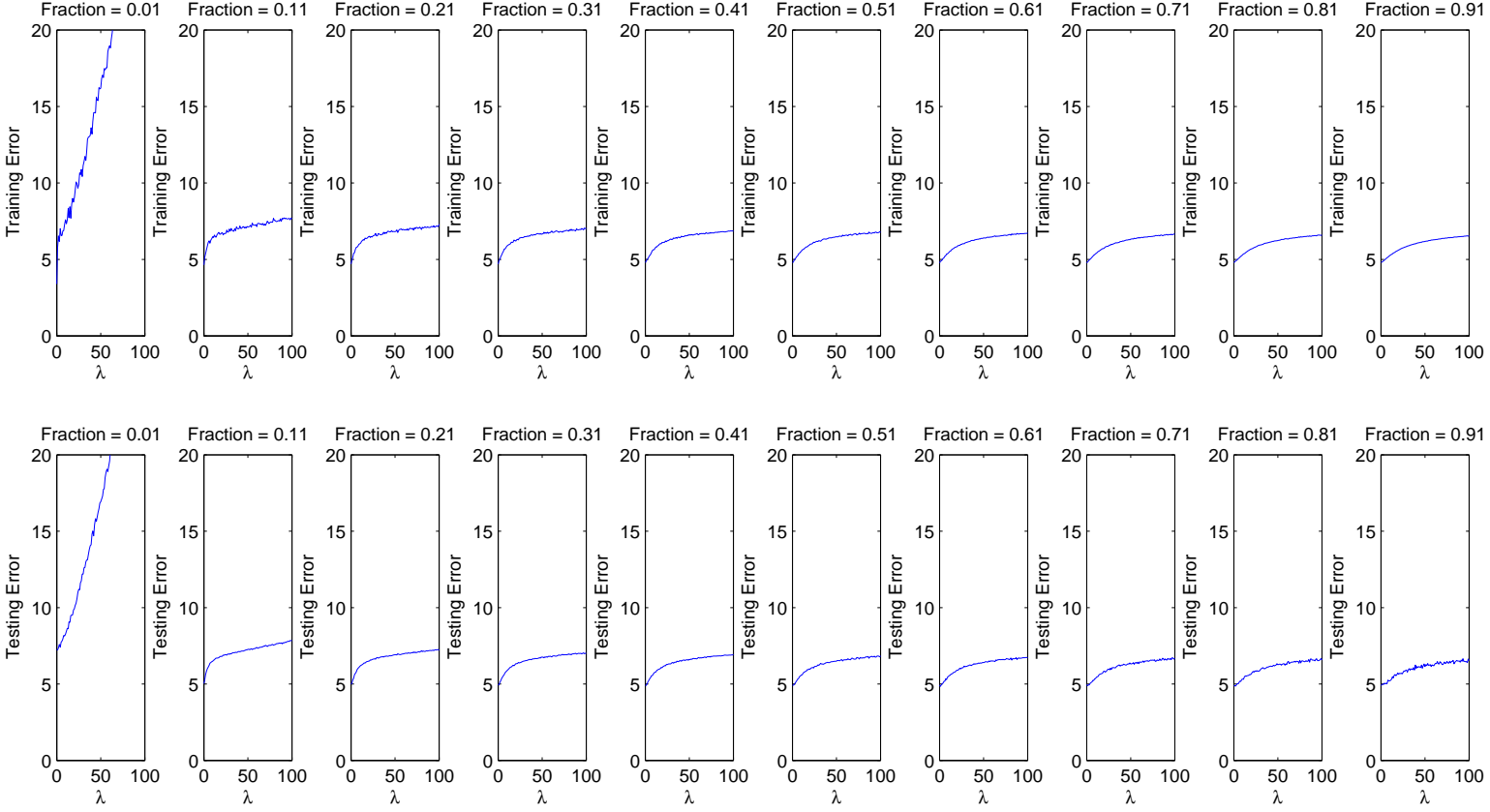


Figure 4: Average Mean Squared Error for various training set fractions varying against the λ values

- Figure 2 and 3's surfaces can be plotted into different graphs for few particular values of splitting-fractions and varying lambda and observing the change in average mean absolute error. Figure 4 shows that for high λ values the average mean squared error increases and is shaped like a convex function and the increase is more apparent in low training set fraction values.

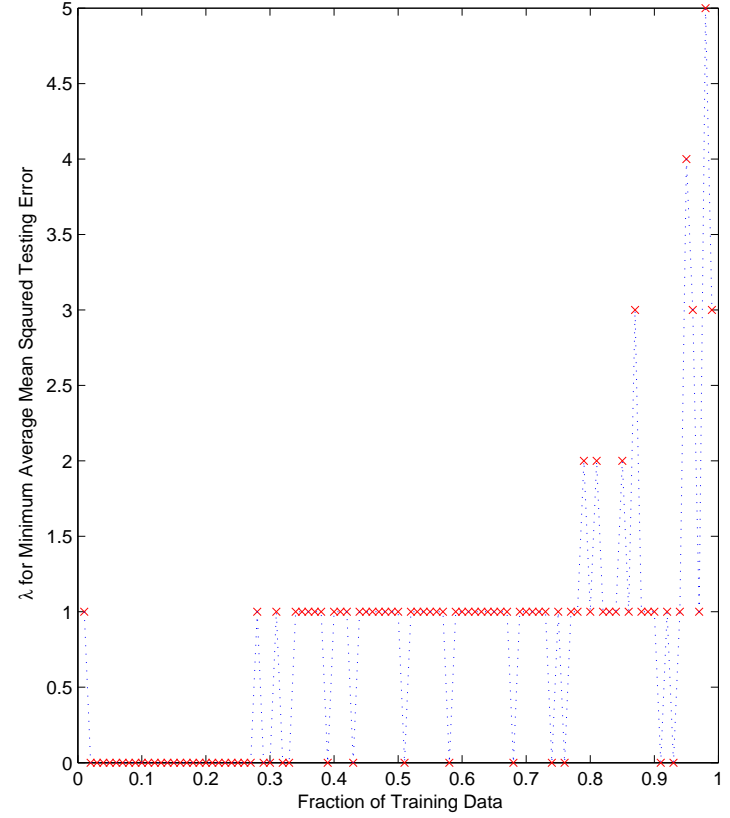
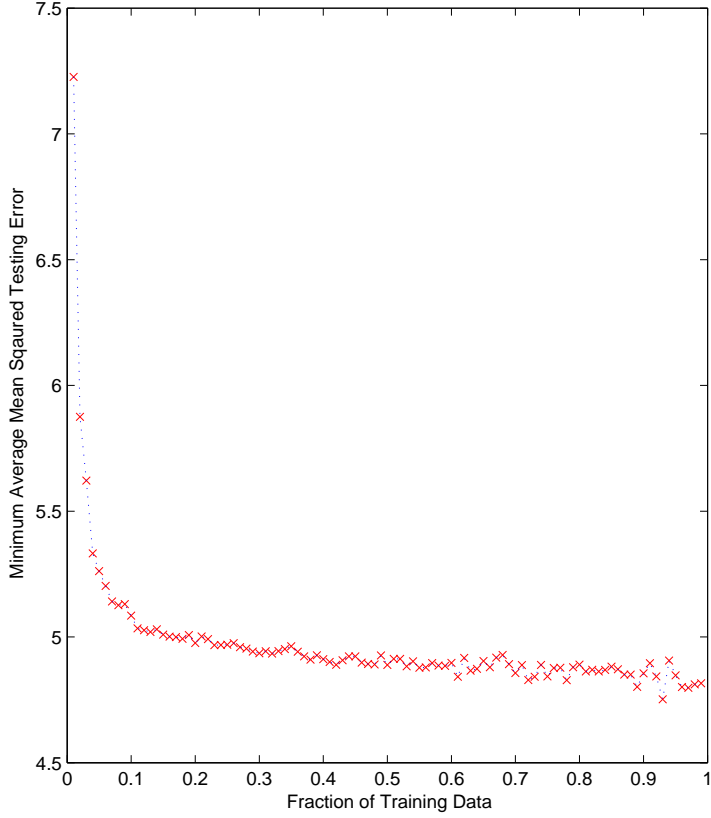


Figure 5: Minimum Average Mean Squared Error for various values of training set fraction and the corresponding λ values.

- Now we noted the minimum average mean squared testing error for each training set fraction values. Also the corresponding λ value was observed. We can see from Figure 5 that with high training set fraction the minimum average mean squared error decreases and though the λ values at which these values are obtained in general have higher magnitude (λ) as we increase the training set fraction, we can say that probably the model is overfitting and to penalize it we need higher magnitude of λ .

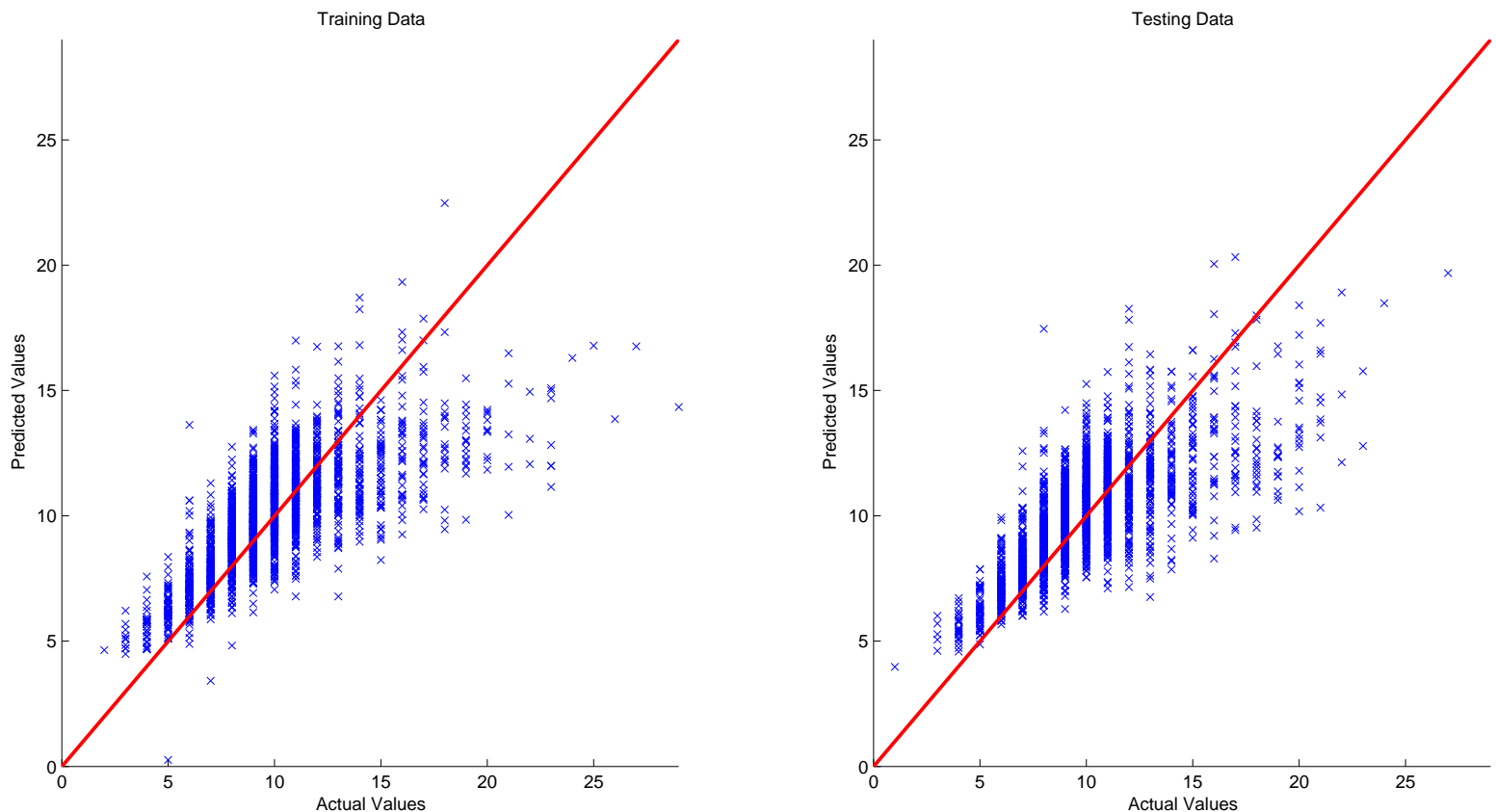


Figure 6: Relation between the actual data set values and predicted values for training and testing data. The reference line $y = x$ is shown in red.

- The correspondence between the actual and predicted values was also observed. For perfect prediction, this should correspond to a straight line through origin at 45° degrees. Figure 6 shows that the actual values and the predicted values lie close to the line $y = x$ thus ensuring that there is a significant correlation and accuracy to the predicted values.

Summary: Conclusions

- **Underfitting:** With high λ values the average mean squared error increases.
- **Decrease in Variance:** With high training set fraction the average mean squared error decreases.

- **Correctness of the Model:** The actual and predicted values lie quite close to the line $y = x$.
- **Redundant Attributes:** Discarding few attributes doesn't make quite a difference hence their irrelevancy to the use in prediction of output values.

2 Regularized Logistic Regression