CSL603 - Machine Learning
Lab 2

Aditya Gupta
2015CSB1003

September 9, 2017

Linear Ridge Regression

Given X and Y we will find W that minimizes $J(W)$, the error function and are defined as:

$$f(X) = \underbrace{\begin{pmatrix} 1 & x_{11} & \cdots & x_{1D} \\ 1 & x_{21} & \cdots & x_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{N1} & \cdots & x_{ND} \end{pmatrix}}_{X} \underbrace{\begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_D \end{pmatrix}}_{W} = \underbrace{\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{pmatrix}}_{Y}$$

$$\min_W J(W) \equiv \min_W \frac{1}{2}(XW - Y)^T(XW - Y) + \lambda||W||^2$$

Which when solved gives us:

$$W = (X^tX + \lambda I)^{-1}X^TY$$

Observations

The following observations were obtained:

- A particular value of λ(say 0) was choosen and then the magnitude of entries in the weights W was compared and one by one the least significant ones were discarded and the mean squared error changed as can be seen in Figure 1. We can see that discarding 2-4 least significant attributes does not make any major change to the mean squared error, hence we can conclude that the input data contains some attributes that are irrelevant in estimating the output values.

- The effect of λ on error was observed for different partitions of the data into training and testing sets. The average mean sqaured error for 100 repetitions for splitting-fractions varying from 1% to 99% and lambda

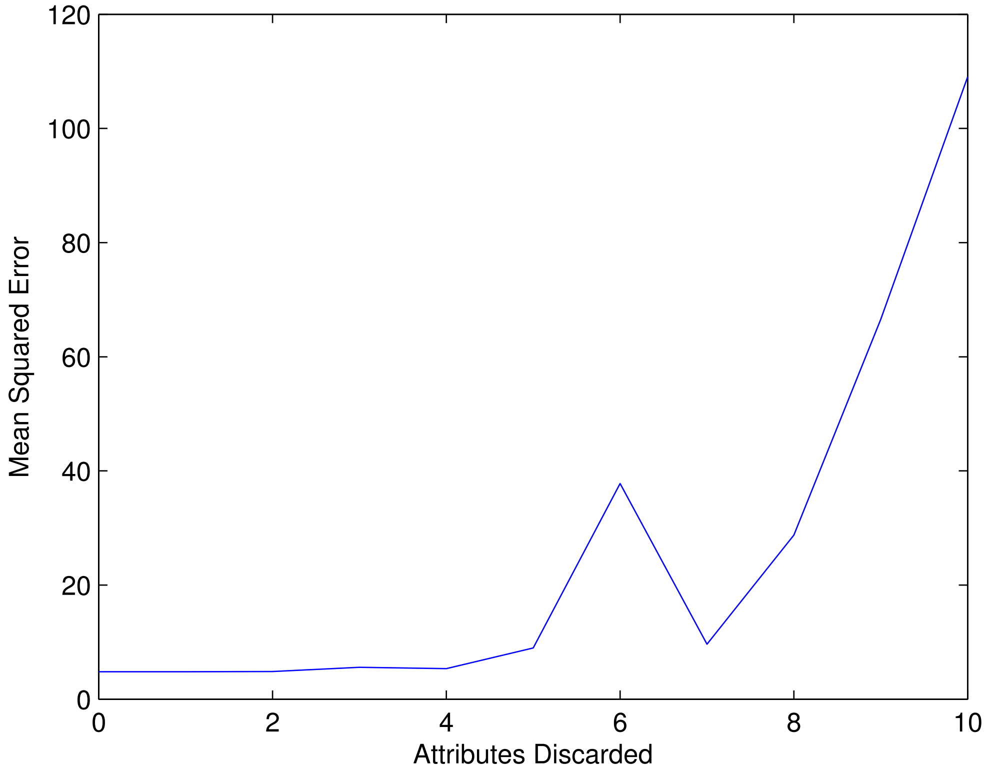

Figure 1: Mean Sqaured Error for after discarding increasing number of least significant weights in W.

values from 0 to 100 was observed. The surface corresponding to the average mean absolute error can be seen in Figure 2 and 3. We can see that for low values of training set fraction or high λ values the average mean squared error increased quite a bit.

- Figure 2 and 3's surfaces can be plotted into different graphs for few particular values of splitting-fractions and varying lambda and obsreving the change in average mean absolute error. Figure 4 shows that for high λ values the average mean squared error increases and is shaped like a convex function and the increase is more apparent in low training set fraction values.

- Now we noted the minimum average mean squared testing error for each training set fraction values. Also the corresponding λ value was observed. We can see from Figure 5 that with high taining set fraction the minimum

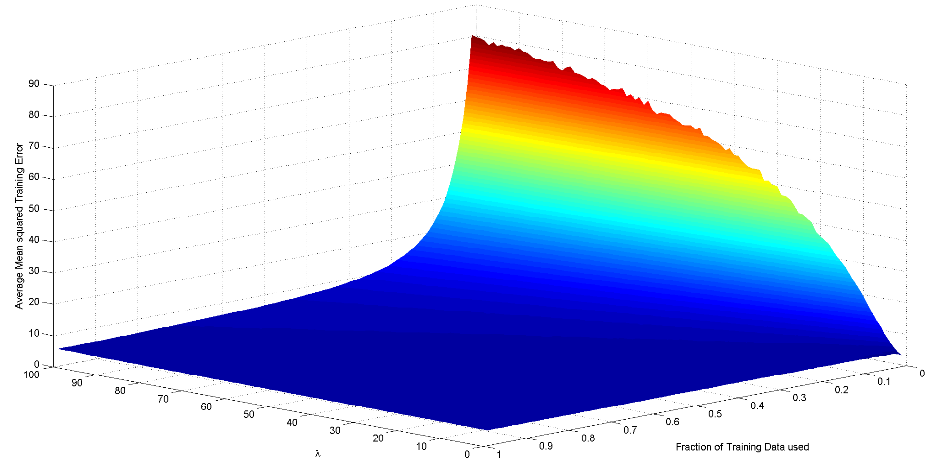

Figure 2: Average Mean Sqaured Error for various values of training set fraction and λ values used in Ridge Regression for Training Data.

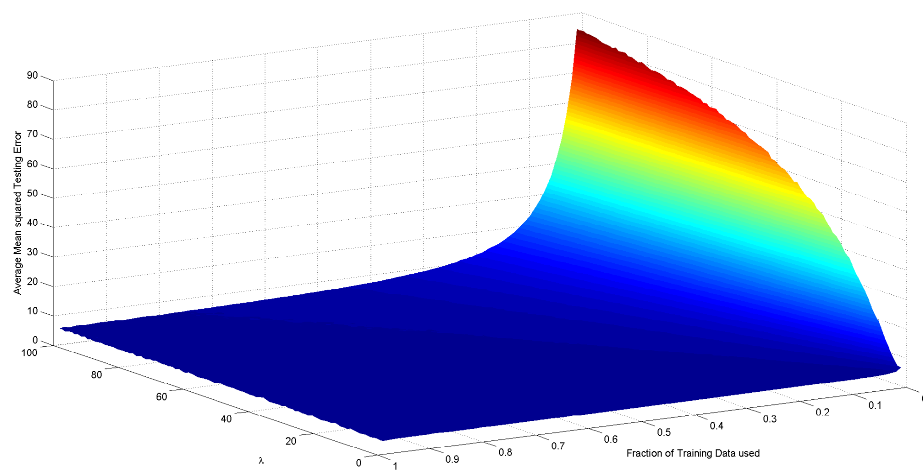

Figure 3: Average Mean Sqaured Error for various values of training set fraction and λ values used in Ridge Regression for Testing Data.

average mean sqaured error decreases and though the λ values at which these values are obtained appear chaotic, probably due to the noise in the data, they do in general have higher magnitude (λ) as we increase the

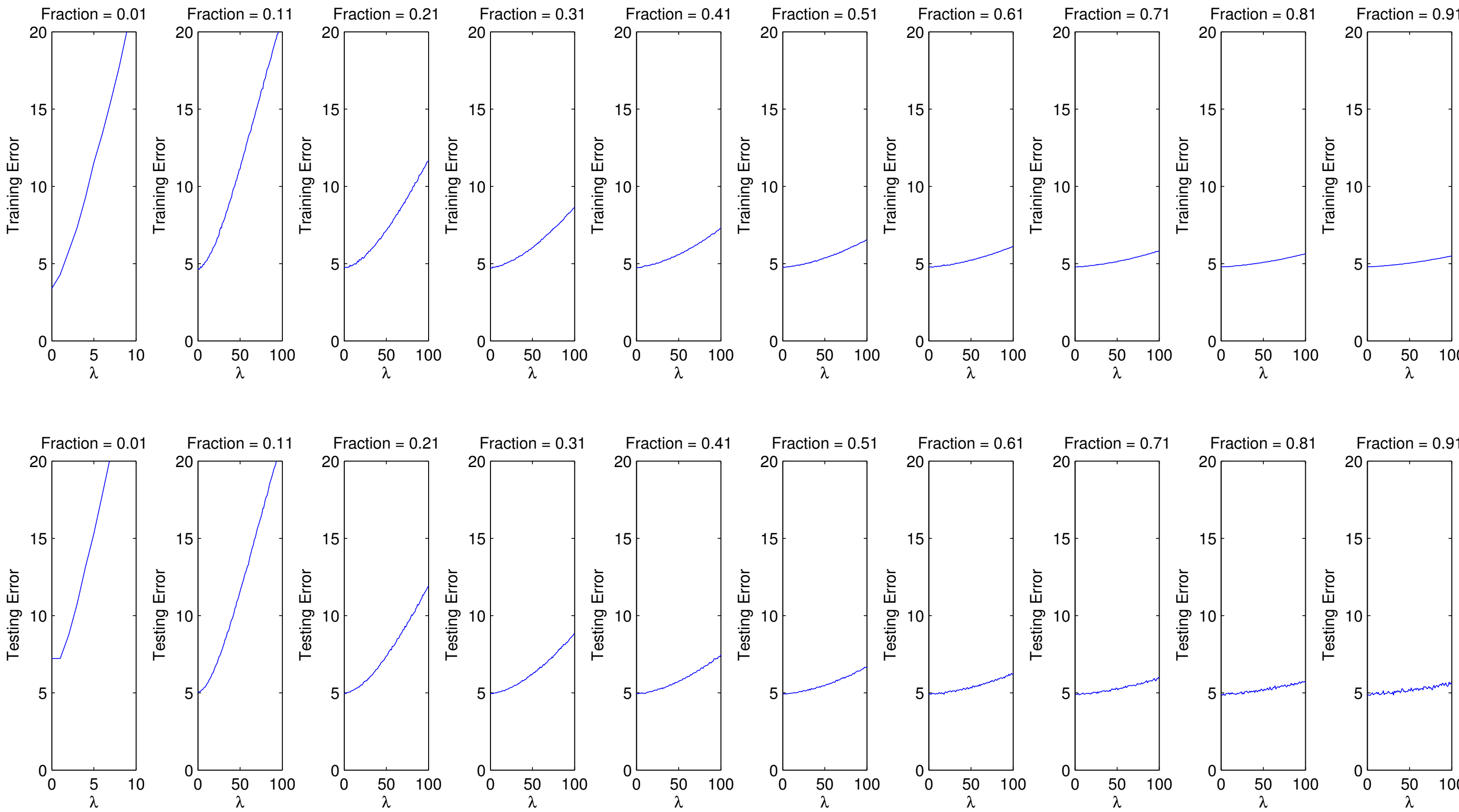

Figure 4: Average Mean Squared Error for various training set fractions varying against the λ values

training set fraction.

- The correspondence between the actual and predicted values was also observed. For perfect prediction, this should correspond to a straight line through origin at 45° degrees. Figure 6 shows that the actual values and the predicted values lie close to the line $y = x$ thus ensuring that there is a significant correlation and accuracy to the predicted values.

Summary: Conclusions

- With high λ values the average mean sqaured error increases.
- With high training set fraction the average mean squared error decreases.

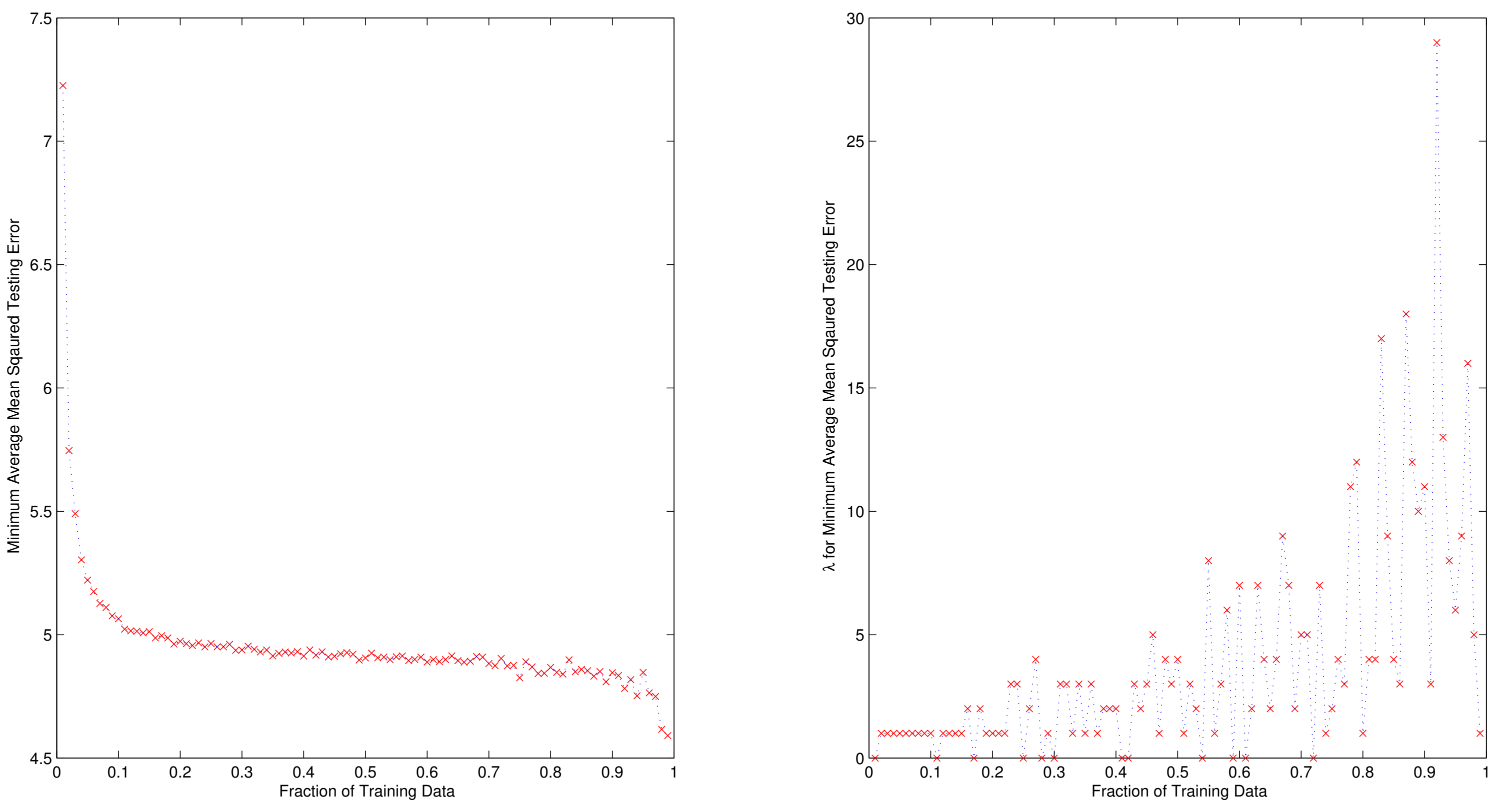

Figure 5: Minimum Average Mean Squared Error for various values of training set fraction and the corresponding λ values.

- The actual and predicted values lie quite close to the line $y = x$.
- Discarding few attributes doesn't make quite a difference hence their irrelevancy to the use in prediction of output values.

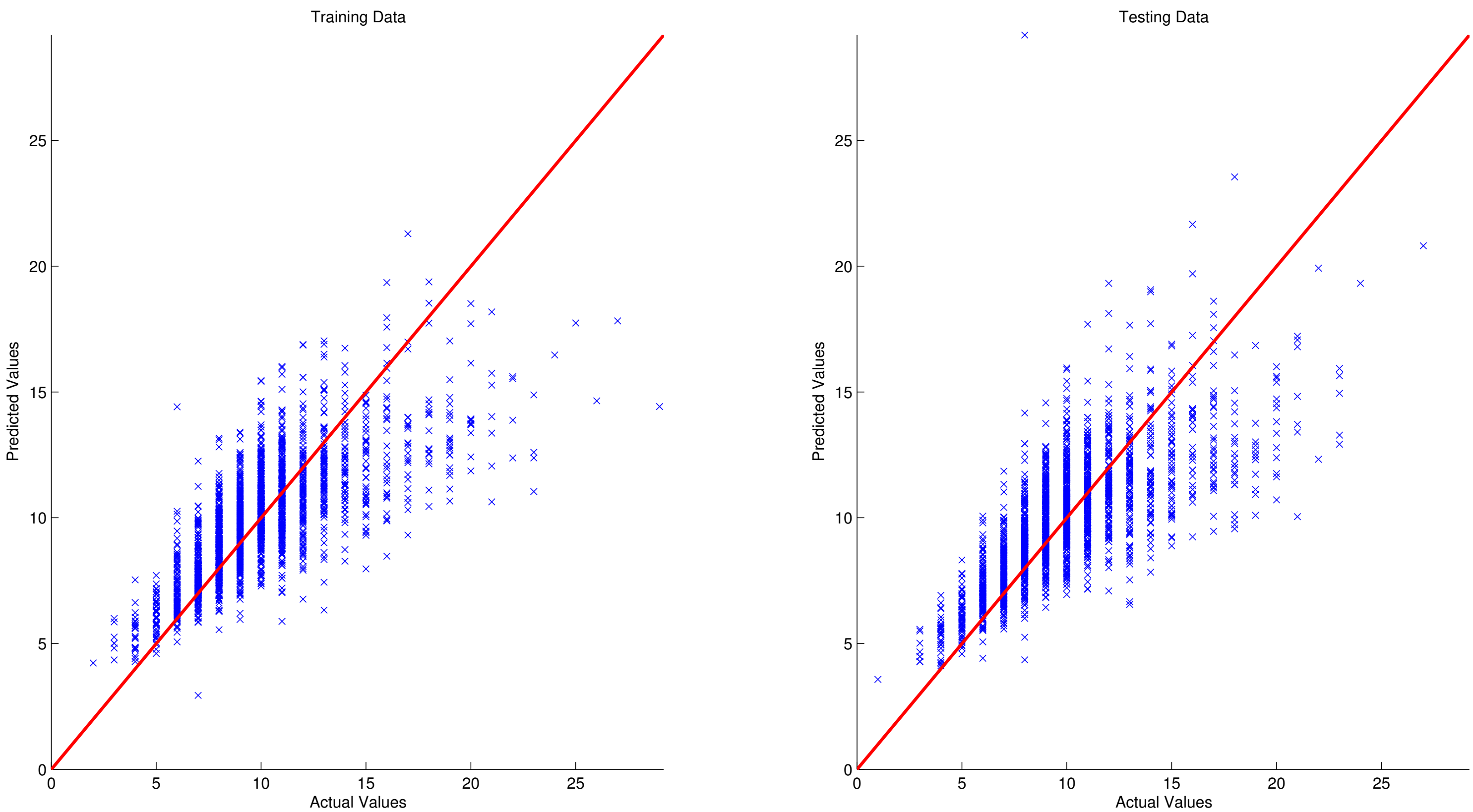

Figure 6: Relation between the actual data set values and predicted values for training and testing data. The reference line $y = x$ is shown in red.