Lab Practice sheet on Interpolation — October 15, 2015

Topics:- Lagrange, Newton's forward, backward and divided difference methods.

1. **Lagrange interpolating polynomials**
   (i) **Pseudocode**

   - INPUT: numbers $x_0, x_1, \ldots, x_n$; $f(x_0), f(x_1), \ldots, f(x_n)$
   - for $i, k = 0, 1, \ldots, n$. Here, n is the degree of the polynomial.
     $L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$. Here, value of x is given.
   - $P = \sum_{k=0}^{n} f(x_k) L_k(x)$
     OUTPUT P, STOP

   (ii) (a) Use appropriate Lagrange interpolating polynomials of degree one, two and three to approximate $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$.

   (b) For the function $f(x) = \sin \pi x$, $x \in (0, 2)$. Using Lagrange method, construct interpolation polynomials (i) of degree two and (ii) of degree five to approximate $f(1/4)$. Find the absolute error in each case.

2. **Newton's forward, backward and divided difference formula:**
   (i) **Pseudocode**
   To obtain the divided difference coefficients of the interpolating polynomial P on the $n + 1$ distinct numbers $x_0, x_1, \ldots, x_n$ for the function f:

   - INPUT: numbers $x_0, x_1, \ldots, x_n$; $f(x_0), f(x_1), \ldots, f(x_n)$ as $F_{0,0}, F_{1,0}, \ldots, F_{n,0}$.
   - for $i = 1 : n$
     for $j = 1 : i$
     $F_{i,j} = \frac{F_{i,j-1} - F_{i-1,j-1}}{x_i - x_{i-j}}, \qquad F_{i,j} = f[x_{i-j}, \ldots, x_i]$
   - OUTPUT: The numbers $F_{0,0}, F_{1,1}, \ldots, F_{n,n}$ where

   Newton's divided difference formula:

$$P_n(x) = F_{0,0} + \sum_{i=1}^{n} F_{i,i} \prod_{j=0}^{i-1} (x - x_j). \qquad \text{where} \quad F_{i,i} = f[x_0, x_1, \ldots, x_i].$$

Newton's forward difference formula:

$$P_n(x) = f(x_0) + \sum_{k=1}^{n} \binom{s}{k} \triangle^k f(x_0)$$

where $\binom{s}{k} = \dfrac{s(s-1)\dots(s-k+1)}{k!}$, is the binomial coefficient and $\triangle$ is forward difference.

Newton's backward difference formula:

$$P_n(x) = f(x_n) + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n)$$

with ∇ is backward difference.

(ii)

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
$f(x)$	1.0000	1.2214	1.4918	1.8221	2.2255	2.7177	3.3166	4.0426	4.9185	5.9694

(a) Use the Newton forward-difference formula to approximate $f(0.05)$.

(b) Use the Newton backward-difference formula to approximate $f(1.75)$.

(iii) Using the following table for $\tan x$, approximate its value at 0.71. Also, find an error estimate (Note $\tan(0.71) = 0.85953$).

x_i	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88
$\tan x_i$	0.8423	0.8771	0.9131	0.9505	0.9893	1.0296	1.0717	1.1156	1.1616	1.2097

(iv) Using Newton divided difference formula to construct the interpolating polynomial of degree four for the unequally spaced points given in the following table and approximate its value at 0.85:

x	$f(x)$
0	-6.00000
0.1	-5.89483
0.3	-5.65014
0.6	-5.17788
1.0	-4.28172