Indian Institute of Technology Ropar
Department of Mathematics
MAL 111: Mathematics Laboratory

Lab Practice sheet on root finding methods — October 7, 2015

Topics:- Newton Raphson, Secant, Regula falsi, Fixed point methods

1. **Method of false positions (Regula falsi)**
   (i) **Pseudocode**

   - Choose $p_0 < p_1$ such that $f(p_0)f(p_1) < 0$
   - for $i = 2, 3, ..., N_{max}$. Here, N_{max} is maximum no. of iterations.
   - $p = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)}$
   - if $|p - p_1| < \epsilon$
     output p, STOP
   - $i = i + 1$
   - if $f(p_0)f(p) < 0, p_1 = p$, else $p_0 = p$

   (ii) **Use Regula falsi method to find the approximate roots of the following functions, accurate within 10^{-4}, in the specified intervals**

   (a) $x^3 + 3x^2 - 1 = 0$ in the interval $[-3, -2]$

   (b) $(x-2)^2 - lnx = 0$ in the interval $[e, 4]$

2. **Newton Raphson Method**
   (i) **Pseudocode**

   - Choose p_0 as initial approximation
   - for $i = 1, 2, 3, ..., N_{max}$
   - $p = p_0 - \frac{f(p_0)}{f'(p_0)}$
   - if $|p - p_0| < \epsilon$
     output p, STOP
   - $i = i + 1, p_0 = p$

   (ii) **Use Newton Raphson's method to find the approximate roots of the following functions in the specified intervals:**

   (a) $x^2 - 2xe^{-x} + e^{-2x} = 0, 0 \leq x \leq 1$

   (b) $(x-1)^3 + 0.512 = 0$. Try with number of iterations $n = 10, n = 20$ and compare the results. What inference do you draw from the results?

3. **Secant method**
   (i) **Pseudocode**

- Choose p_0 as initial approximation
- for $i = 2, 3, ..., N_{max}$
- $p = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)}$
- if $|p - p_1| < \epsilon$
  output p, STOP
- $i = i + 1, p_0 = p_1, p_1 = p$

(ii) **Use Secant method to find the approximate roots of the following functions with the given initial approximations and find which is converging faster.**

(a) $3e^x - 4\cos(x) = 0, p_0 = 0.1, p_1 = 0.5$

(b) $1 - 10^x + 25x^2 = 0, p_0 = 0.9, p_1 = 1.0$

(c) Also, solve 1(a), 1(b) using Regula falsi method and compare the results with those of with secant method.

4. **Fixed-Point Iteration (or functional iteration)**
   (i) **Pseudocode**

- Choose p_0 as initial approximation
- for $i = 1, 2, 3, ..., N_{max}$
- $p = g(p_0)$
- if $|p - p_0| < \epsilon$
  output p, STOP
- $i = i + 1$
- $p_0 = p$

(ii) **Use Fixed-point iterative method with different choices of $g(x)$ to find the roots of $x^3 + 4x^2 - 10 = 0$ in the interval $[1, 2]$. Display the output in tabular form and compare the results.**

(a) $x = \frac{1}{2}(10 - x^3)^{1/2}$

(b) $x = \left(\frac{10}{4+x}\right)^{1/2}$

(c) $x = x - \frac{x^3 + 4x^2 - 10}{3x^2 + 8x}$

5. **Find the roots of the following functions using Newton's, Regula falsi, Secant method and compare the outputs and convergence rates for each method.**

(a) $x + 1 - 2\sin(\pi x) = 0, 0 \leq x \leq 1/2$

(b) $3xe^x = 0, 1 \leq x \leq 2$