2019

Engineering Mathematics-I

Q.1. Answer the following:

(i) A.M between (x - y) and (x + y) is equal to:

(a) 2x (b) 2y (c) x (d) y

Ans. x

(ii) The value of $\log_a 1$ is equal to

(a) 1 (b) 0 (c) a (d) None

Ans.

(iii) ${}^nC_r$ is equal to

(a) ${}^nC_{n-r}$ (b) ${}^nC_{n-1}$ (c) ${}^nC_{r-n}$ (d) None

Ans.

(iv) The value of cosec 270° is equal to

(a) 1 (b) 0 (c) -1 (d) None

Ans.

(v) The value of $\begin{vmatrix} 3 & 5 & 8 \\ 1 & 1 & 2 \\ 2 & 3 & 5 \end{vmatrix}$ *is equal to*

(a) 5 (b) 0 (c) 8 (d) None

Ans.

(vi) The principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ *is equal to*

(a) $\frac{3\lambda}{4}$ (b) $\frac{\lambda}{4}$ (c) $\frac{-\lambda}{4}$ (d) None

Ans.

(vii) Two non vertical lines with slopes m_1 and m_2 are perpendicular if and only if

(a) $m_1 = m_2$ (b) $m_1.m_2 = 1$

(c) $m_1.m_2 = -1$ (d) None of these

Ans.

(viii) If the equation $x^2 + y^2 + 4x + 6y + 7 = 0$ represents a circle, then its centre will be

(a) (2, 3) (b) (–2, –3)

(c) (2, 7) (d) None of these

Ans.

Q2.(a) If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$. Prove that

$$\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=1.$$

Ans. LHS $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$

Putting the value of x, y, z , we get

$$\text{LHS} = \frac{1}{\log_a(bc)+1}+\frac{1}{\log_b(ca)+1}+\frac{1}{\log_c(ab)+1}$$

$$=\frac{1}{\log_a(bc)+\log_a(a)}+\frac{1}{\log_b(ca)+\log_b(b)}+\frac{1}{\log_c(ab)+\log_a(b)}$$

$$=\frac{1}{\log_a(abc)}+\frac{1}{\log_b(abc)}+\frac{1}{\log_c(abc)}$$

$$=\log_{abc}(a)+\log_{abc}(b)+\log_{abc}(c)$$

$$=\log_{abc}(abc)=1=\text{RHS}. \quad \text{Proved.}$$

Q2.(b) If a, b, c are in G.P. prove that $\log_a x, \log_b x, \log_c x$ are in H.P.

Ans. Given: a, b, c are in G.P. Then, $\frac{b}{a}=\frac{c}{b}$.

If $\log_a x$, $\log_b x$ and $\log_c x$ are in HP

Then, $\frac{1}{\log_a x}, \frac{1}{\log_b x}$ and $\frac{1}{\log_c x}$ are in AP.

Now,

$$\frac{1}{\log_b x}-\frac{1}{\log_a x}=\frac{1}{\log_c x}-\frac{1}{\log_b x}$$

$$\log_x b-\log_x a=\log_x c-\log_x b$$

$$\log_x \frac{b}{a}=\log_x \frac{c}{b}$$

$$\therefore \frac{b}{a}=\frac{c}{b}=1 \quad \text{Hence Proved.}$$

Q3.(a) Resolve into partial fraction $\frac{x-1}{(x+1)(x-2)}$.

Ans. $\frac{x-1}{(x+1)(x-2)}=\frac{A}{x+1}+\frac{B}{x-2}$

Multiplying both sides by (x + 1) (x – 2), we get

$x-1=A(x-2)+B(x+1)$

$\Rightarrow x-1=Ax-2A+Bx+B$

$\Rightarrow x-1=(A+B)x+B-2A$

$A + B = 1$ .... (i) and $B - 2A = -1$ ..... (ii)

Putting the value of A in eqn. (ii)

$B - 2(1 - B) = -1$

$B - 2 + 2B = -1$

$3B = 1$

$B=\frac{1}{3}$ and $A=1-\frac{1}{3}=\frac{2}{3}$

$$\therefore \frac{x-1}{(x+1)(x-2)}=\frac{2}{3(x+1)}+\frac{1}{3(x-2)} \quad \text{Ans.}$$

Q3.(b) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Ans.

Q4.(a) Find the coefficient of x7 in the expansion of $\left(x^2 + \frac{1}{x}\right)$.

Ans. Binomial expansion of $\left(x^2 + \frac{1}{x}\right)^{11}$

$$= {}^{11}C_r (x^2)^{11-r}\left(\frac{1}{x}\right)^r$$

$$= {}^{11}C_r\, x^{22-2r}.x^{-r}$$

$$= {}^{11}C_r\, x^{22-3r}$$

For the term containing x^7, we have

$22 - 3r = 7$

$3r = 15$

$r = \frac{15}{3}$ 5

So, the term containing x^7 in binomial expansion of given expression is

$$= {}^{11}C_5\, x^{22-3\times 5}$$

$$= {}^{11}C_5\, x^7$$

$$= \frac{11!}{5!(11-5)!}x^7$$

$$= \frac{11\times10\times9\times3\times7}{5\times4\times3\times2}x^7$$

$$= 11\times3\times2\times7\, x^7$$

$$= 462x^7$$

Hence, the coefficient of x^7 is 462.

Q4.(b) Find the middle term in the expansion of $\left(1 - \frac{x^2}{2}\right)$.

Ans. Binomial expension of $\left(1 - \frac{x^2}{2}\right)^{14}$

$$= {}^{14}C_r (1)^{14-r}\left(\frac{-x^2}{2}\right)^r$$

Hence, n = 14, which is even

So, the middle term of the expansion is $\left(\frac{n+2}{2}\right)^{th}$ term,

i.e., 8^{th} terms

For 8^{th} term, we have

$r + 1 = 8$

$r = 7$

Hence, the middle term in binomial expansion of given expression is

$$T_8 = {}^{14}C_7 (1)^{14-7}\left(\frac{-x^2}{2}\right)^7$$

$$= \left(\frac{-1}{2}\right)^7 {}^{14}C_7\, x^{14}$$

$$= -\frac{1}{128}\times\frac{14\times13\times12\times11\times10\times9\times8}{7\times6\times5\times4\times3\times2\times1}x^{14}$$

$$= \frac{-429}{16}x^{14} \quad \text{Ans.}$$

Q5.(a) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, ***show that $A^2 = 5A + 7 = 0$.***

Ans. Given:

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$-5A = -5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Now,

$$LHS = A^2 - 5A + 7 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = 0 = RHS$$

Hence Proved.

Q5.(b) Solve the following by matrix inversion method.

$$\left.\begin{aligned} x-y+z&=1 \\ 2x+y-z&=2 \\ x-2y-z&=4 \end{aligned}\right\}$$

Ans. Given:
$x - y + z = 1$
$2x + y - z = 2$
$x - 2y - z = 4$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & -1 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$\text{adj } A = \begin{bmatrix} (-1-2) & (-2+1) & (-4-1) \\ (1+2) & (-1-1) & (-2+1) \\ (1-1) & (-1-2) & (1+2) \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -1 & -5 \\ 3 & -2 & -1 \\ 0 & -3 & 3 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} -3 & 3 & 0 \\ -1 & -2 & -3 \\ -5 & -1 & 3 \end{bmatrix}$$

$$|A| = 1(-1-2) + 1(-2+1) + 1(-4-1)$$

$$= -3 - 1 - 5 = -9$$

$\therefore |A| = 9$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{9}\begin{bmatrix} -3 & 3 & 0 \\ -1 & -2 & -3 \\ -5 & -1 & 3 \end{bmatrix}$$

$$X = \frac{1}{9}\begin{bmatrix} -3 & 3 & 0 \\ -1 & -2 & -3 \\ -5 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -3+6+0 \\ -1-4-12 \\ -5-2+12 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 3 \\ -17 \\ 5 \end{bmatrix}$$

$$X = \begin{bmatrix} 1/3 \\ -17/9 \\ 5/9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/3 \\ -17/9 \\ 5/9 \end{bmatrix}$$

$$x = \frac{1}{3};\ y = \frac{-17}{9};\ z = \frac{5}{9} \quad \text{Ans.}$$

Q6.(a) Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

Ans. Same as 2016, Q.no. 6(b).

Q6.(b) If $\tan A = \frac{5}{6}$ ***and*** $\tan B = \frac{1}{11}$, ***prove that*** $A + B = 45^0$.

Ans. Given:

$$\tan A = \frac{5}{6} \text{ and } \tan B = \frac{1}{11}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}}$$

$$= \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6}\times\frac{1}{11}}$$

$$= \frac{\frac{55+6}{66}}{\frac{66-5}{66}} = \frac{\frac{61}{66}}{\frac{61}{66}}$$

$\tan(A+B) = 1$

$(A+B) = \tan^{-1}(1)$

$= \frac{\pi}{4} = RHS$ Hence proved.

Q7.(a) In any ΔABC, if a^2, b^2, c^2 are in A.P. prove that CotA, CotB, CotC are in A.P.

Ans. Given: a^2, b^2, c^2 are in A.P

$$\Rightarrow b^2 - a^2 = c^2 - b^2$$

$$\Rightarrow K^2\sin^2 B - K^2\sin^2 A = K^2\sin^2 C - K^2\sin^2 B$$

$$\Rightarrow \sin(B+A).\sin(B-A) = \sin(C+B).\sin(C-B)$$

$$\Rightarrow \sin C.\sin(B-A) = \sin A.\sin(C-B)$$

$$(\because A + B + C = \pi)$$

$$\Rightarrow \frac{\sin(B-A)}{\sin A} = \frac{\sin(C-B)}{\sin C}$$

$$\Rightarrow \frac{\sin B.\cos A - \cos B.\sin A}{\sin A.\sin B} = \frac{\sin C.\cos B - \cos C.\sin B}{\sin C.\sin B}$$

$$\Rightarrow \cot A - \cot B = \cot B - \cot C$$

Hence, cotA, cotB, cotC are in A.P.

Q7.(b) Prove that $\sin 75^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$.

Ans. $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$$= \sin 45^\circ.\cos 30^\circ + \cos 45^\circ.\sin 30^\circ$$

$$(\sin(A+B) = \sin A.\cos B + \cos A.\sin B)$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Multiplying $\sqrt{2}$ in numerator and denominator

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4} = \text{RHS}$$ Hence Proved.

Q8.(a) Find the equation of a line passing through the points. (–1, 1) and (2, –4).

Ans. Points are (–1, 1) and (2, –4) equation to the straight line passing the points (–1, 1) and (2, –4) is

$$y-(1) = \frac{-4-1}{2+1}(x+1)$$

$$y-1 = \frac{-5}{3}(x+1)$$

$$3y-3 = -5x-5$$

$$5x-3y+2 = 0$$

Q8.(b) Find the equation of line passing through the point (3, –2) and perpendicular to the line x – 3y + 720.

Ans. The equation of any line perpendicular to

$x-3y+7=0$ is

$$-3x+y+\lambda = 0$$

If it passes through the point (3, –2), then

$$-3\times 3+(-2)+\lambda = 0$$

$$-9-2+\lambda = 0$$

$$\lambda = 11$$

Thus equation of required line is $-3x+y+11=0$.

Q9.(a) Find the equation of the circle whose centre is (2, –5) and which passes through the point (3, 2).

Ans. The general equation for a circle with center (a, b) and radius r is

$$(x-a)^2+(y-b)^2 = r^2$$

we are given,

$$a = 2 \text{ and } b = -5$$

Given the center (2, –5) and a point on the circumference radius (3, 2), we can evaluate the radius by pythagorean theorem

$$r^2 = (2-3)^2 + (-5-2)^2$$

$$r^2 = 1+49$$

$$r^2 = 50$$

Therefore the equation of the circle is

$$(x-2)^2 + (y-(-5))^2 = 50$$

$$(x-2)^2 + (y+5)^2 = 50$$

Q9.(b) Show that the equation $x^2+y^2-6x+4y-36=0$ represents a circle. Also find its centre and radius.

Ans. Given: The equation is

$$x^2+y^2-6x+4y-36=0$$

Here $2g = -6$

$$g = -3$$

$$2f = 4$$

$$f = 2 \text{ and } c = -36$$

Thus radius $r = \sqrt{g^2+f^2-c}$

$$= \sqrt{(-3)^2+(2)^2-(-36)}$$

$$= \sqrt{9+4+36}$$

$$= \sqrt{49} = 7 > 0$$

As the radius is > 0, so the given equation represent a real circle

Centre is $(-g, -f)$

$= (3, -2)$ and radius = 7.

Q10.(a) Prove that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{d})+\vec{c}\times(\vec{a}+\vec{b}) = 0$

Ans. Same as 2014, Q.12(b)

Q10.(b) A force $\vec{F} = 3\vec{i}+2\vec{j}-4\vec{k}$ acting at a point (1, –1, 2). Find the moment of the force about the point (2, –1, 3).

Ans. $\vec{F} = 3\hat{i}+2\hat{j}-4\hat{k}$

P(1, -1, 2) Q(2, -1, 3)

$$\vec{r} = \vec{PQ} = (2-1)\hat{i}+(-1+1)\hat{j}+(3-2)\hat{k}$$

$$= \hat{i}+\hat{k}$$

∴ Moment of force $= \vec{r}\times\vec{F}$

$$= (\hat{i}+\hat{k})\times(3\hat{i}+2\hat{j}-4\hat{k})$$

$$\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 3 & 2 & -4 \end{vmatrix}$$

$$= \{(0\times -4) - (2\times 1)\}\ \hat{i} + (3\times 1+4)\ \hat{j} + (2-0)\ \hat{k}$$

$$\vec{r}\times\vec{F} = -2\hat{i}+7\hat{j}+2\hat{k}$$

2018

Engineering Mathematics-I

Q1. Answer the following :

(i) The value of $\log_2 \log_2 \log_2 16$ is equal to

(a) 1 (b) 2 (c) -1 (d) None of the above

Sol. $\log_2 \log_2 \log_2 16 = \log_2 \log_2 \{\log_2 (2)^4\}$

$$= \log_2 \log_2 \{4 \log_2 2\}$$

$$= \log_2 \log_2 \{4\times 1\}$$

$$= \log_2 \log_2 4$$

$$= \log_2 \{\log_2 (2)^2\}$$

$$= \log_2 \{2\times \log_2 2\}$$

$$= \log_2 2 = 1$$

(ii) **The number of terms in the expansion of $x^6(1+3x^4)^{15}$ is**

(a) 21 (b) 15 (c) 16 (d) 19

Sol. (c)

(iii) The value of sin 18° is equal to

(a) $\frac{\sqrt{5}+1}{4}$ (b) $\frac{1-\sqrt{5}}{4}$

(c) $\frac{\sqrt{5}-1}{4}$ (d) None of the above

Sol. Let $\theta = 18^0$

$$5\theta = 5\times 18^0 = 90^0$$

$$3\theta + 2\theta = 90^0$$

$$2\theta = 90^0 - 3\theta$$

$\sin 2\theta = \sin(90^0 - 3\theta)$ {Applying sin on both side}

$$\Rightarrow \sin 2\theta = \cos 3\theta$$

$$\Rightarrow 2\sin\theta\ \cos\theta = 4\cos^3\theta - 3\cos\theta$$

$$\Rightarrow 2\sin\theta\ \cos\theta = \cos\theta(4\cos^2\theta - 3)$$

$$\Rightarrow 2\sin\theta - 4\cos^2\theta + 3 = 0$$

$$\Rightarrow 2\sin\theta - 4(1-\sin^2\theta) + 3 = 0$$

$$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$$

Using form

$$ax^2+bx+c = 0$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

So, $\sin\theta = \frac{-2\pm\sqrt{4-4\times 4\times(-1)}}{2\times 4}$

$$= \frac{-2\pm\sqrt{20}}{8}$$

$$= \frac{-1\pm\sqrt{5}}{4}$$

$$= \frac{-1+\sqrt{5}}{4}, \frac{-1-\sqrt{5}}{4}$$

$$1 \le \sin\theta \le 1$$

$$\sin\theta = \frac{-1+\sqrt{5}}{4}$$

$$\sin 18^0 = \frac{\sqrt{5}-1}{4}$$

(iv) The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ ***is equal to***

(a) x + y + z (b) 0 (c) 1 (d) None of the above

Sol. $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$$R_1 \to R_1 + R_2$$

$$\therefore \Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Taking x+y+z common from R_1

$$\therefore \Delta = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (x+y+z)\{1(x-y)+1(y-z)+1(z-x)\}$$

$$= (x+y+z)\ (x-y+y-z+z-x)$$

$$= 0$$

(v) The Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ ***is equal to***

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{-\pi}{6}$ (d) None of the above

Sol. Let $\sin^{-1}\left(\frac{1}{2}\right) = y$

$$\Rightarrow \sin y = -\frac{1}{2}$$

$$\Rightarrow \sin y = \sin\left(\frac{-\pi}{6}\right)$$

Range of principal value of $\sin^{-1}$ is $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

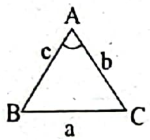
∴ Principal value is $\left(-\frac{\pi}{6}\right)$

(vi) In a ΔABC, if a, b and c are the sides of the corresponding angles respectively, then cos A is equal to

(a) $\frac{b^2+c^2-a^2}{2bc}$ (b) $\frac{a^2+b^2-c^2}{2ab}$

(c) $\frac{c^2+a^2-b^2}{2ca}$ (d) None of the above

Sol. (a) Using cosine formula -



$$\cos = \frac{b^2+c^2-a^2}{2bc}$$

(vii) If the points (-2, -5), (2,-2) and (8,k) are collinear then the value of k is equal to

(a) 5 (b) 3 (c) $\frac{5}{2}$ (d) None of the above

Sol. (c) A(-2, -5) B(2, -2) C(8, K)

If Point A, B, C are collinear

Then, Slope of line AB = Slope of line AC

$$\therefore \frac{-2+5}{2+2} = \frac{K+5}{8+2}$$

$$\Rightarrow \frac{3}{4} = \frac{K+5}{10}$$

$$\Rightarrow 10 \times 3 = 4K + 20$$

$$\Rightarrow 4K = 30 - 20 = 10$$

$$\Rightarrow K = \frac{10}{4} = 2.5 = \frac{5}{2}$$

(viii) The equation of straight line parallel to the line 4x + 7y + 5 = 0 and passing through the point (1, -2) is given by

(a) 4x + 7y + k = 0 (b) 4x - 7y + 10 = 0

(c) 4x + 7y + 10 = 0 (d) None of the above

Sol. (c) Equation of line parallel to 4x+7y+5 = 0 and passing through (1, -2) is

$4x + 7y + \lambda = 0$ ......(i)

Since, (i) passes through (1, -2)

$$\therefore 4 \times 1 + 7 \times (-2) + \lambda = 0$$

$$\Rightarrow 4 - 14 + \lambda = 0$$

$$\Rightarrow \lambda = 10$$

$$\therefore 4x + 7y + 10 = 0$$

Q2. (a) Prove that $\log_l m = \log_b m \times \log_a b$

Sol. (a) $LHS = \log_a m$

$RHS = \log_b m \times \log_a b$

$$= \frac{\log m}{\log b} \times \frac{\log b}{\log a} \qquad \left\{\because \log_a b = \frac{\log b}{\log a}\right\}$$

$$= \frac{\log m}{\log a}$$

$$= \log_a m$$

Q2.(b) *If* $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$ *then prove that* $a^a.b^b.c^c = 1$.

Sol. Given : $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = K$

loga = K (b - c) logb = K (c - a) logc = k(a -b)

aloga = aK (b - c) blogb= bK(c-a) clogc= CK(a-b)

$$\log a^a = K(ab - ac) \quad \log b^b = K(bc - ab) \quad \log c^c = K(ac - bc)$$

$$\log a^a + \log b^b + \log c^c = K[ab - ac + bc - ab + ac - bc]$$

$$= K \times 0$$

$$\log a^a + \log b^b + \log c^c = 0$$

$$\log(a^a b^b c^c) = \log 1$$

$$a^a b^b c^c = 1$$

Q3. (a) Resolve into partial fractions : $\frac{x^2+1}{x(x^2-1)}$

Sol. $\frac{x^2+1}{x(x^2-1)} = \frac{A}{x} + \frac{Bx+C}{(x^2-1)}$

$$\Rightarrow \frac{x^2+1}{x(x^2-1)} = \frac{A(x^2-1)+x(Bx+C)}{x(x^2-1)}$$

$$\Rightarrow x^2 + 1 = x^2(A+B) + (x-A)$$

Comparing coefficients on both side -

A + B = 1; C = 0; A = -1; B = 2

$$\therefore \frac{x^2+1}{x(x^2-1)} = \frac{-1}{x} + \frac{2x}{(x^2-1)}$$

$$= \frac{2x}{(x^2-1)} - \frac{1}{x}$$

Q3.(b) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Sol. $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 = R_1 + R_2 + R_3$

$$\begin{vmatrix} a-b-c+2b+2c & 2a+b-c-a+2c & 2a+2b+c-a-b \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking (a+b+c) common from R1

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$c_2 \to c_2 - c_1 \qquad c_3 \to c_3 - c_1$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -a-b-c \end{vmatrix}$$

Taking (a+b+c) common from c_2 and c_3

$$\Delta = (a+b+c)^3\begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$

$$= (a+b+c)^3 \times 1\{1-0\}$$

$$= (a+b+c)^3$$

Q4. (a) Solve the following by matrix inversion method:

$x + y + z = 5$

$x + 2y + 3z = 12$

$2x - y + z = 4$

Sol. x + y + z = 5 x + 2y + 3z = 12

2x - y + z = 4

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 12 \\ 4 \end{bmatrix}$$

AX = B

$X = A^{-1}B$

$$adj\ A = \begin{bmatrix} (2\times1+3) & (2\times3-1) & (-1-4) \\ (-1-1) & (1-2) & (2+1) \\ (3-2) & (1-3) & (2-1) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 & -5 \\ -2 & -1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 5 & -2 & 1 \\ 5 & -1 & -2 \\ -5 & 3 & 1 \end{bmatrix}$$

|A| = 1 × 5 + 1 × 5 + 1 × (-5) = 5

$$\therefore X = \frac{1}{|A|} adj\ A\,B \qquad \left[\because A^{-1} = \frac{1}{|A|} adj\ A\right]$$

$$X = \frac{1}{5}\begin{bmatrix} 5 & -2 & 1 \\ 5 & -1 & -2 \\ -5 & 3 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 12 \\ 4 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{5}\begin{bmatrix} 5 \\ 5 \\ 15 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore x = 1; y = 1; z = 3$

Q4.(b) Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ ***satisfies the equation*** $x^2 - 3x - 7 = 0$.

Sol. $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$\therefore x^2 - 3x - 7$

$= A^2 - 3A - 7I$

$$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} -15 & -9 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9+0 \\ -3+3+0 & 1+6-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore A^2 - 3A + 7I = 0$

Q5. (a) Find the middle term in the expansion of $\left(\frac{2x}{3} - \frac{3}{2x}\right)^8$.

Sol. Binomial Expansion of $\left(\frac{2x}{3} - \frac{3}{2x}\right)^8$

$$= {}^8C_r\left(\frac{2x}{3}\right)^{8-r}\left(\frac{-3}{2x}\right)^r$$

$\because$ n = 8 is even

$\therefore$ 4th term will be middle term

$$= {}^8C_4\left(\frac{2x}{3}\right)^4\left(\frac{-3}{2x}\right)^4$$

$$= {}^8C_4(-1)^4 \times \left(\frac{2x}{3}\right)^4 \times \left(\frac{3}{2x}\right)^4$$

$$= {}^8C_4$$

$= \frac{8}{4} \times \frac{7}{3} \times \frac{6}{2} \times 5$

$= 70$

Q5.(b) Find the coefficient of x^2 in the expansion of $\left(3x - \frac{1}{x}\right)^{12}$

Sol. Binomial Expansion of $\left(3x - \frac{1}{x}\right)^{12}$

$$= {}^{12}C_r (3x)^{12-r} \left(\frac{-1}{x}\right)^r$$

$$= (-1)^r \cdot {}^{12}C_r (3)^{12-r} (x)^{12-r-r}$$

$$= (-1)^r (3)^{12-r} \, {}^{12}C_r (x)^{12-2r}$$

We have to coefficient of x^2

$\therefore$ $12 - 2r = 2$

$\Rightarrow 2r = 10 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^2 = (-1)^5 (3)^7 \times {}^{12}C_5$

$$= = -3^7 \times {}^{12}C_5$$

Q6. (a) Prove that $\tan^{-1}\frac{2}{5} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{12} = \frac{\pi}{4}$.

Sol. $= \tan^{-1}\left(\frac{2}{5}\right) + \tan^{-1}\left[\frac{\frac{1}{3} + \frac{1}{12}}{1 - \left(\frac{1}{3} \times \frac{1}{12}\right)}\right]$

$\therefore$ $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$$= \tan^{-1}\left(\frac{2}{5}\right) + \tan^{-1}\left(\frac{15}{35}\right)$$

$$= \tan^{-1}\left(\frac{2}{5}\right) + \tan^{-1}\left(\frac{3}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{2}{5} + \frac{3}{7}}{1 - \left(\frac{2}{5} \times \frac{3}{7}\right)}\right) = \tan^{-1}\left(\frac{\frac{29}{35}}{\frac{29}{35}}\right)$$

$$= \tan^{-1} 1 = \frac{\pi}{4}$$

Q6.(b) If $A + B = 45^\circ$ then prove that $(\cot A - 1)(\cot B - 1) = 2$.

Sol. $A + B = 45^0$

$(\cot A - 1)(\cot B - 1)$

$= \cot A \cot B - \cot A - \cot B + 1$

$= (\cot A \cot B - \cot A - \cot B) + 1$ ....(i)

$\cot(A + B) = \cot 45^0$

$\frac{\cot A \cot B - 1}{\cot A + \cot B} = 1$

$\Rightarrow$ $\cot A \cot B - 1 = \cot A + \cot B$

$\Rightarrow$ $\cot A \cot B - \cot A - \cot B = 1$ ....(ii)

From (i) & (ii)

$(\cot A - 1)(\cot B - 1) = 1 + 1 = 2$

Q7. (a) Prove that in a triangle ABC, $\tan\frac{A-B}{2} = \frac{a-b}{a+b} \cot\frac{C}{2}$.

Sol. In ABC Triangle

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\frac{a-b}{a+b} = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}$$

$$= \frac{\sin A - \sin B}{\sin A + \sin B}$$

$$= \frac{2\cos\frac{A+B}{2} . \sin\frac{A-B}{2}}{2\sin\frac{A+B}{2} . \cos\frac{A-B}{2}}$$

$$= \cot\frac{A+B}{2} . \tan\frac{A-B}{2}$$

$$= \cot\left(\frac{\pi - c}{2}\right) \tan\frac{A-B}{2} \qquad [as\ A + B + C = \pi]$$

$$= \tan\left(\frac{C}{2}\right) \tan\frac{A-B}{2}$$

$\therefore$ $\tan\frac{A-B}{2} = \frac{a-b}{a+b} \cot\frac{C}{2}$

Q7.(b) In a triangle ABC, Prove that $a(b\cos C - c\cos B) = b^2 - c^2$.

Sol. Using cosine formula

$$a^2 + b^2 - 2ab\cos C = c^2$$

$$a^2 + c^2 - 2ac\cos B = b^2$$

Substracting (i) and (ii)

$\therefore$ $b^2 - c^2 = (a^2 + c^2 - 2ac\cos B) - (a^2 + b^2 - 2ab(\cos C)$

$$= c^2 - 2ac\cos B - b^2 + 2ab\cos C$$

$$= -(b^2 - c^2) - 2ac\cos B + 2ab\cos C$$

$\Rightarrow$ $2(b^2 - c^2) = 2a[-c\cos B + b\cos C]$

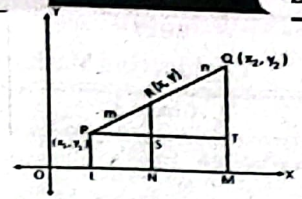
$\Rightarrow$ $(b^2 - c^2) = a[-c\cos B + b\cos C]$

$\therefore$ a [bcosc - acosb] = $b^2 - c^2$

Q8. (a) Find the co-ordinates of points which divide the given line segments joining the point (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$

Sol. (a) Internal Division of line segment :

Let (x_1, y_1) and (x_2, y_2) be the cartesian co-ordinates of the points P and Q respectively referred to rectangular co-ordinate axes $\overline{OX}$ and $\overline{OY}$ and the point R divides the line-segment $\overline{PQ}$ internally in a given ratio m : n (say, i.e. $\overline{PR} : \overline{RQ}$ = m : n. We are to find the co-ordinates of R.



Let, (x, y) be the required co-ordinate of R. From P, Q and R, draw $\overline{PL}, \overline{QM}$ and $\overline{RN}$ perpendiculars on $\overline{OX}$. Again, draw $\overline{PT}$ parallel to $\overline{OX}$ to cut $\overline{RN}$ at S and $\overline{QM}$ at T.

Then,

$\overline{PS} = \overline{LN} = \overline{ON} - \overline{OL} = x - \overline{x_1}$

$\overline{PT} = \overline{LM} = \overline{OM} - \overline{OL} = x_2 - x_1$

$\overline{RS} = \overline{RN} - \overline{SN} = \overline{RN} - \overline{PL} = y - y_1$

and $\overline{QT} = \overline{QM} - \overline{TM} = \overline{QM} - \overline{PL} = y_2 - y_1$

Again, $\overline{PR} / \overline{RQ} = m / n$

or, $\overline{RQ} / \overline{PR} + 1 = n / m + 1$

or, $(\overline{RQ} + \overline{PR} / \overline{PR}) = (m + n) / m$

or, $\overline{PQ} / \overline{PR} = (m + n) / m$

Now, by construction, the triangles PRS and PQT are similar; hence,

$\overline{PS} / \overline{PT} = \overline{RS} / \overline{QT} = \overline{PR} / \overline{PQ}$

Taking, $\overline{PS} / \overline{PT} = \overline{PR} / \overline{PQ}$ we get,

$(x - x_1) / (x_2 - x_1) = m / (m + n)$

or, $x(m + n) - x_1(m + n) = mx_2 - mx_1$

or, $x(m + n) = mx_2 - mx_1 + mx_1 + nx_1 = mx_2 + nx_1$

Therefore, $x = (mx_2 + nx_1) / (m + n)$

Again, taking $\overline{RS} / \overline{QT} = \overline{PR} / \overline{PQ}$ we get,

$(y - y_1) / (y_2 - y_1) = m / (m + n)$

or, $(m + n)y - (m + n)y_1 = my_2 - my_1$

or, $(m + n) = my_2 - my_1 + my_1 + ny_1 = my_2 + ny_1$

Therefore, $y = my_2 + ny_1 / (m + n)$

Therefore, the required co-ordinates of the point R are

$((mx_2 + nx_1) / (m + n), (my_2 + ny_1) / (m + n))$

Q8.(b) Find the angle between the straight lines $2x - 8y = 7$ and $6x - y = 12$.

Sol. $2x - 8y = 7 \quad 6x - y = 12$

$8y = 2x - 7 \qquad y = 6x - 12$

$\Rightarrow$ $y = \frac{2x}{8} - \frac{7}{8} \qquad m_2 = 6$

$\Rightarrow$ $y = \frac{x}{4} - \frac{7}{8}$

$m_1 = \frac{1}{4}$

Comparing above equations with y = mx + c

$\therefore$ $m_1 = \frac{1}{4} \qquad m_2 = 6$

Angle between lines

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \pm\frac{\left(\frac{1}{4} - 6\right)}{1 + \left(\frac{1}{4} \times 6\right)}$$

$$= \pm\frac{\left(\frac{-23}{4}\right)}{\frac{10}{4}}$$

$$= \pm\left(\frac{-23}{10}\right)$$

$$\tan\theta = \mp\frac{-23}{10}$$

$\therefore$ $\theta = \tan^{-1}\left(\mp\frac{-23}{10}\right)$

Q9. (a) If one end of a diameter of the circle $x^2 + y^2 - 2x - 4y + 1 = 0$ be (1,0). Find other end.

Sol. $x^2 + y^2 - 2x - 4y + 1 = 0$

P (1, 0)

Comparing above equation with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$2g = -2 \quad \Rightarrow g = -1$

$2f = -4 \quad \Rightarrow f = -2$

$\therefore$ Centre = (-g, -f) = (1, 2)

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{1 + 4 - 1} = \sqrt{4} = 2$

Let other end of dia be (x, y)

$\therefore$ $\frac{(x+1)}{2} = 1$

$\Rightarrow x = 1$

$\frac{y+0}{2} = 2 \quad \Rightarrow y = 4$

$\therefore$ Point = (1, 4)

Q9.(b) Find the equation of the circle passing through the points (2,3), (-1,6) and having its centre on the line $2x + 5y + 1 = 0$.

Sol. Let equation of circle be -

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It posses through (2, 3) and (-1, 6)

$2^2+3^2+2g\times2+2f\times3+c=0$

$4g + 6f + c = -13$ .....(i)

$1 + 36 + 2g \times(-1) + 2f \times 6 + c = 0$

$12f - 2g + c = -37$ ......(ii)

Substracting (i) and (ii)

$6g - 6t = 24$

$\therefore$ $g - f = 4$ ....(iii)

Since centre (-g, -f) lie on $2x + 5y + 1 = 0$

$\therefore$ $-2g - 5f + 1 = 0$

$2g + 5f + 1$ ....(iv)

On solving (iii) and (iv) we get

$g = 3; f = (-1)$

$\therefore$ $4 \times 3 + 6 \times (-1) + c = -13$

$\therefore$ $c = -19$

$\therefore$ $x^2 + y^2 + 6x - 2y - 19 = 0$

Q10.(a) *$\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$. Find $|\vec{a}-\vec{b}|$.*

Sol. $|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$

$$= \vec{a}.\vec{a}-\vec{a}.\vec{b}-\vec{b}.\vec{a}+\vec{b}.\vec{b}$$

$$= |\vec{a}|^2 - 2\vec{a}.\vec{b} + |\vec{b}|^2$$

$$= 2^2 - 2\times4 + 3^2$$

$$= 4 - 8 + 9 = 5$$

$\therefore$ $|\vec{a}-\vec{b}| = \pm\sqrt{5}$

Since, magnitude is not negative

$\therefore$ $|\vec{a}-\vec{b}| = \sqrt{5}$

Q10.(b) *A force $\vec{F} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ is applied at the point (1, -1,2). Find the moment of the force about the point (2, -1, 3).*

Sol. $\vec{F} = 2\hat{i} + 3\hat{j} - 5\hat{k}$

P(1, -1, 2) Q(2, -1, 3)

$\vec{r} = \vec{PQ} = (2-1)\hat{i} + (-1+1)\hat{j} + (3-2)\hat{k}$

$= \hat{i} + \hat{k}$

$\therefore$ Moment of force $= \vec{r}\times\vec{F}$

$$= (\hat{i}+\hat{k})\times(2\hat{i}+3\hat{j}-5\hat{k})$$

$$\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 3 & -5 \end{vmatrix}$$

$$= \{(0\times-5) - (3\times1)\}\,\hat{i} + (2\times1+5)\,\hat{j} + (3-0)\,\hat{k}$$

$\vec{r}\times\vec{F} = -3\hat{i} + 7\hat{j} + 3\hat{k}$