4704

# Jharkhand University of Technology, Ranchi

## 1st Semester Diploma Examination, 2022

**Subject : Engg. Mathematics-I** **Subject Code : 102**

**Time Allowed : 3 Hours** **Full Marks : 80**

**Pass Marks : 26**

*Answer in your own words.*

*Answer* five *questions in which* Question No. 1 *is compulsory and answer* any four *from rest questions.*

*All questions carry equal marks.*

**1.** Choose the correct answer from given choices:

(i) The value of ${}^{n}C_{r} + {}^{n}C_{r+1}$ is equal to

(a) ${}^{n+1}C_{r}$ (b) ${}^{n+1}C_{r+1}$

(c) ${}^{n}C_{r+1}$ (d) None of these

(ii) The value of $\log_{2\sqrt{3}} 144$ is

(a) 3 (b) 5

(c) 4 (d) None of these

(iii) The points (2, 3), (5, $k$) and (6, 7) are collinear. Then the value of $k$ will be

(a) 5 (b) 7

(c) 6 (d) None of these

(iv) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$, then the transpose of ($AB$) is

(a) $\begin{bmatrix} 4 & 21 \\ 3 & 13 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 15 \\ 3 & 12 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 \\ 21 & 13 \end{bmatrix}$ (d) None of these

(v) For the circle $x^2 + y^2 - 6x - 2y - 6 = 0$, the radius and centre is

(a) 3; (0, 0) (b) 5; (3, 4)

(c) 4; (3, 1) (d) None of these

(vi) The value of $\sin^{-1} x + \cos^{-1} x$ is equal to

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) None of these

(vii) If $A = \begin{vmatrix} 4 & 7 & 3 \\ 6 & 8 & 9 \\ 8 & 14 & 6 \end{vmatrix}$, then the value of $A$ is

(a) 1 (b) 2

(c) 0 (d) –1

(viii) If $\vec{a} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$, then modulus of $|\vec{a}|$ will be

(a) $4\sqrt{5}$ (b) $5\sqrt{2}$

(c) $3\sqrt{2}$ (d) None of these

**2.** (a) Resolve into partial fraction: $\frac{2x+3}{(x-3)(x+1)}$

(b) If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then prove that $a^{b+c}.b^{c+a}.c^{a+b} = 1$.

**3.** (a) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

(b) Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.7.\cdots(2n-1)}{\lfloor n}.2^n x^2$, where $n$ is a positive integer.

**4.** (a) Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

(b) Prove that $\frac{\sin^3 A+\cos^3 A}{\sin A+\cos A} + \frac{\sin^3 A-\cos^3 A}{\sin A-\cos A} = 2$.

**5.** (a) For the two vectors $\vec{a} = \hat{\imath} + 3\hat{\jmath} - 7\hat{k}$ and $\vec{b} = 5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, find the dot product $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$.

(b) A particle acted on by constant forces $4\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + \hat{\jmath} - \hat{k}$ is displaced from the point $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ to the point $5\hat{\imath} + 4\hat{\jmath} + \hat{k}$. Find the total work done by the forces.

**6.** (a) Find the equation of a straight line which makes intercepts of $a$ and $b$ with $x$-axis and $y$-axis respectively.

(b) Find the distance between the parallel lines $8x + 15y - 36 = 0$ and $8x + 15y + 32 = 0$.

**7.** (a) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

(b) If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then find $AB$ and $BA$ if possible and show that $AB \neq BA$.

8. (a) If the sides of $\Delta ABC$ are 3, 4 and 5 unit respectively, then find the smallest angle of the triangle.

   (b) Find the equation of the circle whose centre is (1, 2) and which passes through the point of intersection of $3x + y = 14$ and $2x + 5y = 18$.

9. (a) Find the 6th term of $(a + 2b)^8$.

   (b) Prove $\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$.

10. (a) If the co-ordinates of end point of a diameter of a circle be $(x_1, y_1)$ and $(x_2, y_2)$, find the equation of the circle.

    (b) $\vec{a}$ has magnitude $3\sqrt{2}$ and $\vec{b}$ has magnitude 5 and the angle between $\vec{a}$ and $\vec{b}$ is 135°. Find the value of $\vec{a} \cdot \vec{b}$.

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