Real-Time Gunshot Localization Using Acoustics

Presented by: Aditya Rajeev Harakare

Authors: Aniruddha Sinha and Hemendra Arya

Under the Guidance of: Prof. Rajbabu Velmurugan



Department of Electrical Engineering
Indian institute of technology Bombay

Introduction

Signatures left by a Gunfire



Sound of charge explosion in gun muzzle,

Light of the charge explosion,



Sound of the bullet travelling, which is only perceptible if the bullet's speed is supersonic,

- 'Sight' of the bullet travelling,
- Sound of the bullet hitting the target
- Smell of the gunfire

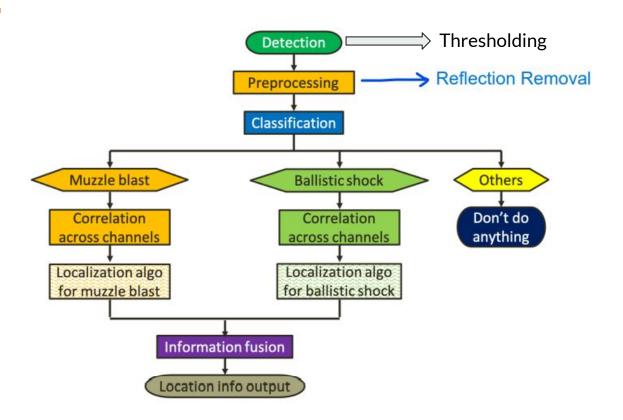
Classification of acoustic targeting devices based on mobility:



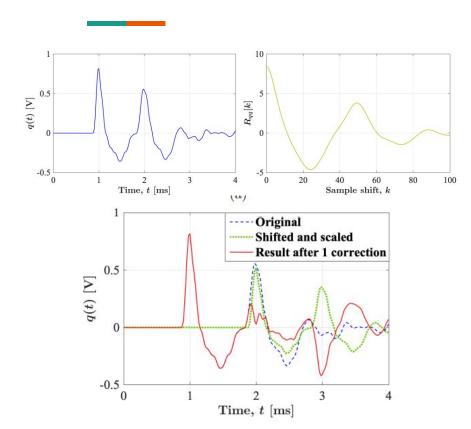
Static Devices (city, forward camps in forests)

- Vehicle-mounted
- Helmet mounted or shoulder worn

Gunshot Localization Algorithm Flow



Reflection Minimization



Find autocorrelation

$$R_{qr}[k] := \sum_{i=\max(0,-k)}^{N-1-\min(0,-k)} q[i+k]r[i].$$

Find

γ = Ratio of the second peak of Rqq to the peak at zero shift

 α = attenuation ratio

$$\alpha = 0.5(1 - \sqrt{1 - 4\gamma^2})/\gamma.$$

Delay, scale and subtract from the original signal = 1 correction

$$\hat{q}[i] = \sum_{i=0}^{M} (-\alpha)^{j} q[i - jk^{*}].$$

Gunshot Event Categorization

Muzzle Blast signature:

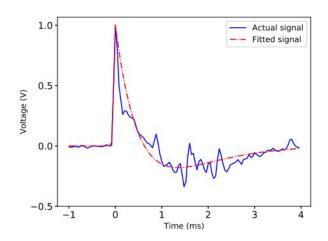
$$f_{\text{Friedlander}}(t) = \begin{cases} 0, & t \le t_0, \\ A(t - t_0)/t_r, & t_0 < t \le t_0 + t_r, \\ A[1 - (t - t_0 - t_r)/t_d] e^{-(t - t_0 - t_r)/t_d}, & t > t_0 + t_r. \end{cases}$$

A = peak amplitude

 $t_{0} = TOA$

t = Rise Time (Can be neglected)

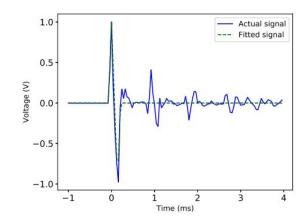
t = Decay time



'N' wave signature for ballistic shocks:

$$\text{Friedlander}(t) = \left\{ \begin{array}{l} 0, & t \leq t_0, \\ A(t-t_0)/t_r, & t_0 < t \leq t_0 + t_r, \\ A\left[1-(t-t_0-t_r)/t_d\right] \operatorname{e}^{-(t-t_0-t_r)/t_d}, & t > t_0 + t_r. \end{array} \right.$$

$$\text{Friedlander}(t) = \left\{ \begin{array}{l} 0, & t \leq T_0, \\ B(t-T_0)/T_r, & T_0 < t \leq T_0 + T_r, \\ B\left[1-2(t-T_0-T_r)/T_d\right], & T_0 + T_r < t < T_0 + T_r + T_d, \\ B\left[(t-T_0-T_r-T_d)/T_r-1\right], & T_0 + T_r + T_d < t < T_0 + T_r + T_d, \\ B\left[(t-T_0-T_r-T_d)/T_r-1\right], & T_0 + T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d < t < T_0 + T_r + T_d, \\ D(t-T_0-T_r-T_d)/T_r + T_d < t < T_0 + T_r + T_d < T_0 + T_0 + T_0 + T_0 < T_0 + T_0 + T_0 + T_0 < T_0 + T_0 + T_0 + T_0 + T_0 < T_0 + T_$$



Fit both signatures.

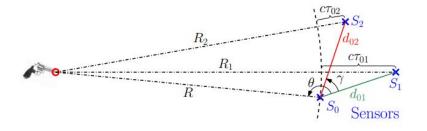
Determine goodness of fitting (chi square measure)

Characterise the event into muzzle blast, ballistic shock or none

Algorithms for localizing gun-shooter from muzzle blast

Wavefront Curvature Method

- Planar localization technique assuming circular wavefront expansion
- Min sensors required = 3



$$R = \frac{d_{01}}{2} \frac{1 - \eta_{01}^2}{\cos \theta + \eta_{01}}, \quad R = \frac{d_{02}}{2} \frac{1 - \eta_{02}^2}{\cos(\theta - \gamma) + \eta_{02}}.$$

- 2 mathematically possible solutions Near one to be ignored
- Error:

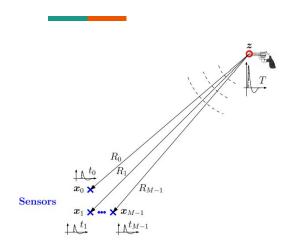
$$\epsilon_{wc} = |R_2 - R_1 - c\tau_{12}|.$$

$$\theta = \tan^{-1} \left(\frac{\sin \gamma}{\cos \gamma - \delta} \right) \pm \cos^{-1} \left(\frac{\delta \eta_{01} - \eta_{02}}{\sqrt{1 + \delta^2 - 2\delta \cos \gamma}} \right), \qquad \delta := \frac{d_{02} (1 - \eta_{02}^2)}{d_{01} (1 - \eta_{01}^2)}.$$

$$\delta := \frac{d_{02}(1 - \eta_{02}^2)}{d_{01}(1 - \eta_{01}^2)}$$

Algorithms for localizing gun-shooter from muzzle blast

Non-linear least squares method



$$ct_i = \mathcal{D}(\boldsymbol{x}_i, \boldsymbol{z}) + cT, \qquad i \in \{0, 1, \dots, M-1\}.$$

C: ambient speed of sound

T: Instant of gunfire at the position z

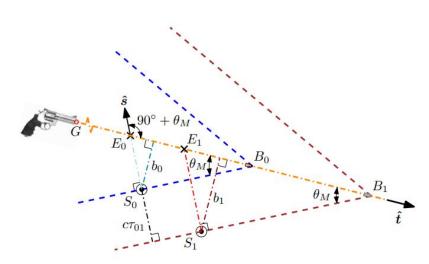
$$oldsymbol{z}^{(k+1)} = oldsymbol{z}^{(k)} - \left(J\left(oldsymbol{z}^{(k)}
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ight)^{\mathrm{T}}$$

Initial guess solution z(0) from WCM.

$$J_{ij}(\boldsymbol{z}) := \frac{\partial r_i}{\partial z_j} = \frac{x_{ij} - z_j}{\mathscr{D}(\boldsymbol{x}_i, \boldsymbol{z})} - \frac{x_{0j} - z_j}{\mathscr{D}(\boldsymbol{x}_0, \boldsymbol{z})}.$$

*Other algorithms like Bancroft's method and Localization using multiple direction-of-arrival vectors briefly discussed in the paper

Algorithms for localizing gun-shooter from shock wave



Mach Number:

$$M = V_s/c > 1$$

Mach Angle:

$$\theta_{_M} = \sin^{-1}(1/M)$$

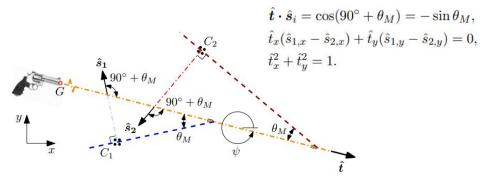
Whitham's formula:

$$T_d = \frac{1.82Mb^{1/4}d}{c(M^2 - 1)^{3/8}\ell^{1/4}}.$$

d = caliber (diameter) of bullet l = length (diameter) of bullet b = miss distance

Estimation of bullet trajectory direction

- 1. Direction of bullet trajectory (needs to be solved for approx bearing angle)
- 2. Finding a point on the trajectory
- 3. Locating the shooter
- Need at least two spatially-separated clusters of sensors for solving any and all of the above localization problems
- Linearity of the wavefront is a fact instead of an assumption for the ballistic shock
 - => No requirement of the sensor array to be far from the bullet trajectory
- Sensors on either side of the trajectory



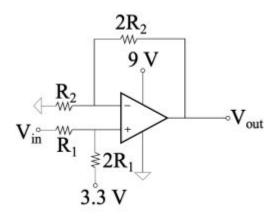
Hardware and Software for Acoustic Gunshot Localization





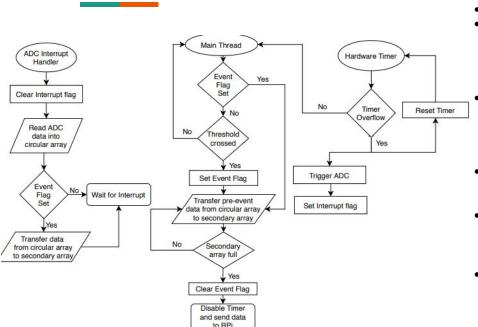
Sennheiser MD-42 microphones sensitivity of 2 mV/Pa

Tetrahedron (3D)/Square (2D) 1m distance b/w mics



Biasing Circuit Shifting Voltage range from 0 to 3.3 V to -1.65 to +1.65 V

Data Acquisition and Processing on Tiva Board Online

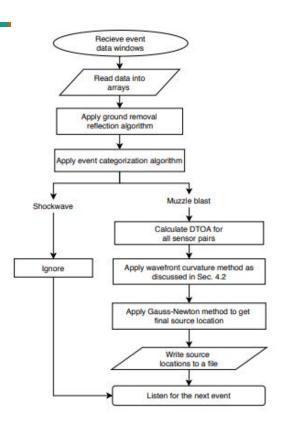


- Tiva C Series TM4C1294 Connected LaunchPad Evaluation Board
- 2 12-bit ADC modules
- Functions:
 - Continuously 'listen' on all microphone channels (4)
 - Detect the acoustic event on any microphone channel
 - Acquire data in a brief time window
 - o Pass on the collected data over Ethernet to a downstream microprocessor (Rpi)

DC offset correction for each microphone:

- event checking algorithm starts only after 6.25 ms post boot (500 samples @80Khz)
- Average of the acquired voltage values for this initial 6.25 ms is calculated for each channel to get their respective DC voltages
- Pre-event data recording:
 - 0.3ms experimental = 24 samples @80kHz
 - o 100 samples buffer implemented
- Event Detection on any microphone:
 - o Transfer sampled data to 25th entry of secondary buffer
 - 1-24 entries of secondary buffer <= recent 24 entries of circular buffer
 - done for all four channels, even though the event is detected on only one channel
- Secondary buffer size:
 - 4ms gunshot event
 - 0.3ms pre-event
 - \circ $\sqrt{2/c}$ = 4.2ms travel time
 - Total 8.5ms; 8.75ms = 700 samples
 - Buffer size = 4x700
- Ethernet Transfer:
 - Secondary array filled; 2 packets transmitted for each channel:
 - Channel Number
 - i Channel Data

Localization on RPi Board Online



- Raspberry Pi Compute Module 3+
- Solution:
- Getting the 2-D coordinates of the source from the wavefront curvature method
- providing the solution as initial guess for the classical non linear least square estimation method
- 100ms pause for Rpi to process
- minimum time between two gunshot events= 100ms
- Time required to report the location estimate
 = 100ms

Results and Discussions

Offline | Blast Wave | Tetrahedral Array

- 25kHz Sampling
- Calibrating Microphone distances based on the reverse nonlinear least-squares method
- Choosing Reference microphone:
 - For any reference sensor, the remaining sensors yield corresponding normalized DTOA values (ŋ's)
 - Identify η max for each sensor
 - Choose sensor with lowest η_max as reference
- Max error in:
 - Bearing Angle = 3°
 - Range = 200%

Online | Blast Wave | Square Array

- WCM for initial guess
- nonlinear least squares for refinement
- Procedure:

(4 triplets)

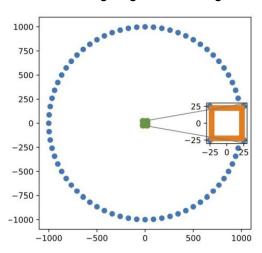
- Choose best reference for each triplet
- WCM for preliminary estimate
- Calculate error for each triplet
- Choose the triplet which gives minimum error
- Identify best reference among all hydrophones
- Non-linear least square method for refined estimate
- Max error in:
 - Bearing Angle = 2°
 - Range = 40%

Parameter	Normalized Range Error	
	Improved Online Implementation	First Offline Implementation
Maximum	0.40	2.22
Median	0.21	0.36
Mean	0.21	0.60



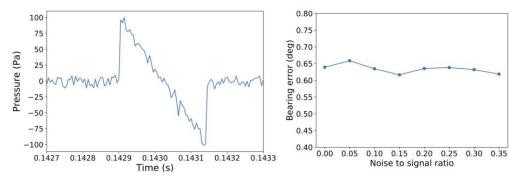
Shock Wave | Security System | Simulation

- 500 gunshot events simulated
- Max error in:
 - Bearing Angle = 1.75 degrees



Effect of Noise

 Addition of zero-mean Gaussian noise to the ideal 'N'-wave signal



No effect (cross-correlation is independent of Noise)

Thank You