Mid term Submission

Yash Choubey-201351006
Aditya Prakash-201351010
Manu Sharma-201351021

March 3, 2016

0.1 Given an arc length s, determine t at which this arc length is achieved.

First, we take six control points $->$ p1, p2, p3, p4, p5 and p6
These six points are represented by their x, y and z coordinates repectively
Then we define M as a 4X4 matrix to get the B-Spline curve

$$M = \begin{bmatrix} 1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

The equation used to find a B-Spline segment is:
P_i(t)=(1/6)*T*M*P

Where
T = $[t^3\ t^2$ t 1]
P = $[P_{i-1} P_i P_{i+1} P_{i+2}]^T$

Now, the x,y and z coordinates of three such segments are calculated.
Segment 1 from p1, p2, p3, p4
Segment 2 from p2, p3, p4, p5
Segment 3 from p3, p4, p5, p6

Now we have the whole curve (comprising of 3 segments in) px, py and pz as the x, y and z coordinates of the curve.

CALCULATION: Arc length of the curve:
Instead of integrating the curve over the interval tmin to tmax We have 303 points on the curve. Calculate the distance between adjacent points and add to the length of the curve. L gives the arc length of the curve.

Take input from the user:s.
s is the length for which we want to find out t. s belongs to [0,L].

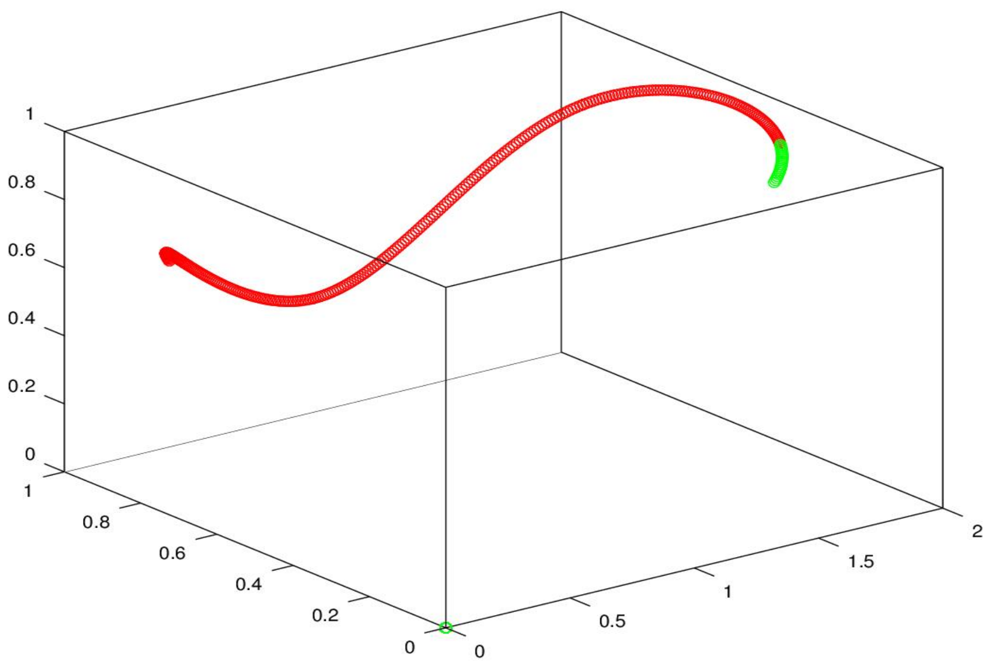
t = (s/L)$(t_{min} - t_{max})$ take $t_{min} = 0, t_{max} = 1$
So: t=s/L

Now we can find the point where Y(t) = X(s). Now increment the length of the curve till Y(t)=X(s). So the curve is red till s, and green afterwards.

Example: for s = 2.25

p1 = [0 0 0]
p2 = [0 1 1]
p3 = [1 1 0]
p4 = [2 1 1]
p5 = [2 0 1]
p6 = [1 1 0]

The curve is :



0.2 Begin with a parametrized curve C3(u) for u$\in$ [u_{min}, u_{max}] , and determine a parametrization by time t, say, C2(t) = C3(u) for t $\in$ [t min , t max] , so that the speed at time t is a specified function (t) .

We need to use the differential of the curve now. So we differentiate the matrix hence T = [t^3 t^2 t 1]
T' = [3*t^2 2*t 1 0]

We get the coordinates of the differentiated parameterized curve. So, pdx, pdy and pdz are the x,y and z coordinates of the new curve.

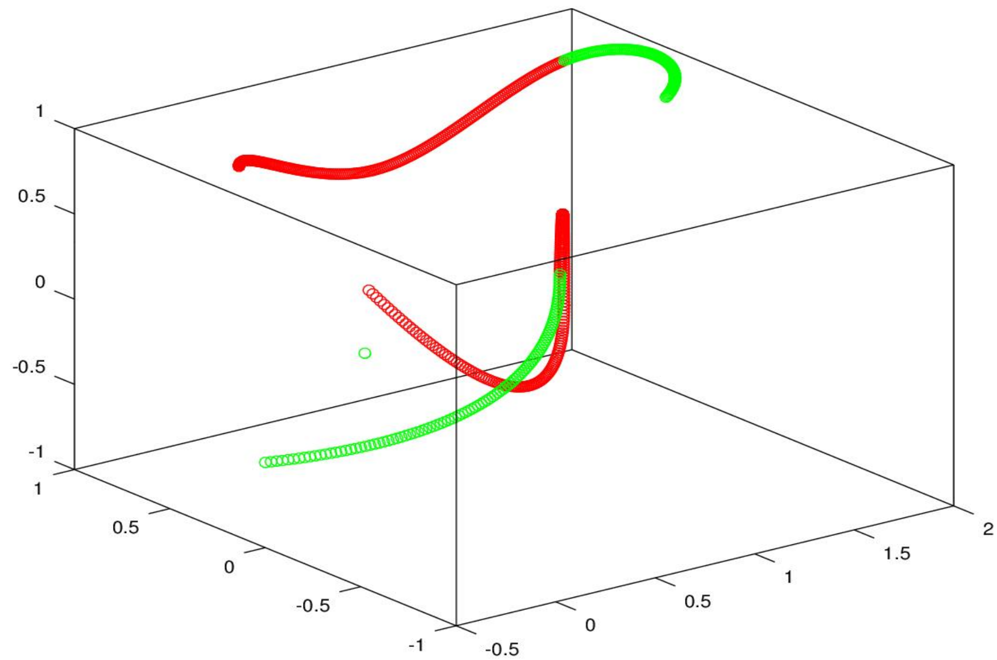
To get the arc length of the new curve, we keep adding the distance between two points to obtain L.

Function σ(t), t_{min} and t_{max} are provided by the user. We integrate the function over this interval to get 'len' which is the value of the length of the arc covered.

Here we equate:

$$\int_{t_{min}}^{t_{max}} \sigma(t)dt = \int_{u_{min}}^{u_0} |dC_3(u)/du|du$$

For the given length, we try to find out the u_0 for which the same length is achieved on our curve. The value of u_0 is stored. So the curve is plotted in red till u is reached and in green after u is reached.
In this case function $\sigma(t) = 9 * t^2 + 3 * t$
$t_{min} = 0$ and $t_{max} = 0.8$



The curve: