MA201 Mid-Term Examination
Simulation Problems*

Autumn 2014
Indian Institute of Information Technology, Vadodara

September 17, 2014

1. We break a stick at random in two places, what is the probability that the three pieces will form a triangle?

2. Suppose you are on the bank of a straight river and walk a kilometer in a randomly chosen direction.

   (a) How far from the river will you be?

   (b) After you walk the kilometer, at that point you choose a new random direction and then you begin to walk another kilometer. What is the probability that you will reach the river bank before the second kilometer is complete?

3. Generate random variates for a random variable X corresponding to the density function $f_X(x)$ given by,

$$f_X(x) = \begin{cases} kx & 0 \leq x < 1 \\ 2k - kx & 1 \leq x \leq 2. \end{cases} \tag{1}$$

   Verify the shape of the density function with histogram of generated variates. Also estimate the mean and variance of the generated variable. How far are these estimates from the analytical values?

4. A forest consists of 1000 trees forming a perfect 50×20 rectangle, Figure 1. The northwestern (top-left) corner tree catches fire. Wind blows from the west, therefore, the probability that any tree catches fire from its burning left neighbor is 0.8. The probability to catch fire from a tree immediately to the right, above, or below is only 0.3.

   (a) Conduct a Monte Carlo study to estimate the probability that more than 30% of the forest will eventually be burning.

   (b) Based on the same study, predict the total number of affected trees X.

   (c) Estimate Std(X) and comment on the accuracy of your estimator of X.

   (d) A wooden house is located in the northeastern corner of the forest. Would you advise the owner that her house is in real danger?

*You are encouraged to work in groups. But a group can not be of more than 4 students. During examination each one of you, independent of the groups, is required to submit a hard copy of the results that **you have** acquired along with **your own** comments and/or conclusions. There will be a viva-voce following the exam for simulations problems. You will be informed about the viva schedule in due time.

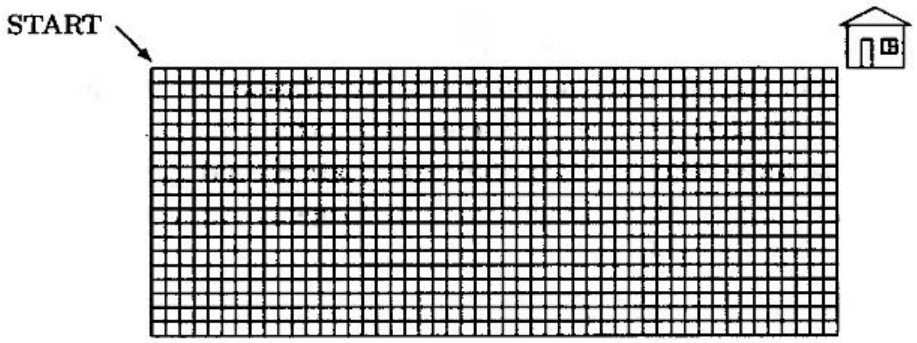

Figure 1: This figure is taken from the book by M Baron [1].

References

[1] M. Baron. *Probability and Statistics for Computer Scientists.* Taylor & Francis, 2006.