Problem-1

Let $x(t)$ be a pulse defined as $u(t) - u(t-T)$ where T = 0.001 seconds.

1. Calculate the Fourier transform$X(f)$of $x(t)$. Sketch the corresponding magnitude and phase spectra.
2. Suppose we sample $x(t)$, with a sampling pulse train $c(t) = \Sigma_{n=-\infty}^{\infty}\delta(t - nT_s)$, with sam- pling rate F s = 2000Hz (2000 samples/second) and T s = 1/F s . What is DTFT of$xTs(t) = x(t)c(t)$? Sketch the magnitude spectrum corresponding to the computed DTFT. Compare it with the Fourier transform of x (t) .
3. Do you see any form of aliasing in DTFT? Can you justify your answer? Will change in F s help you in avoiding aliasing due to discretization?
4. Suppose before sampling, the function x (t) is passed through an ideal low-pass filter having cut-off frequency of 1000Hz. What will be the effect of filtering on sampled signal?
5. In general it is difficult to find DTFT of a given discrete time function in a closed form. We usually refer to DFT for the meaningful calculation of DTFT. Let us take samples of x (t) for a finite duration, say .01 sec. What would be the DFT of this finite length sequence $x(n)(n = 0, 1, ..., N-1)$?
6. If you pad ten zeros to x (n) what will be the DFT of this zero padded sequence? Can we keep padding zeros at the end and argue that the resolution of DFT will keep increasing? What best can you get out of DFT?

solution

For calculating Fourier transform of given function

$$x(t) = u(t) - u(t-T) \text{ where T} = 0.001 \text{ sec.}$$

Fourier Transform of function is follow

$\Rightarrow X(F) = \int_0^T e^{-j\omega t}dt$

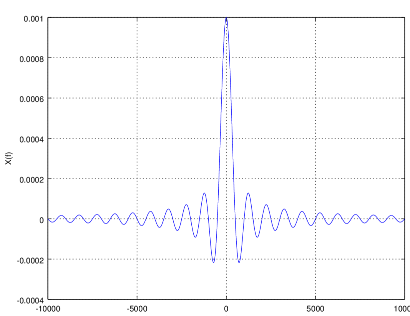
$\Rightarrow X(F) = -\frac{1}{j\omega}(\frac{1}{e^{j\omega T}} - 1)$

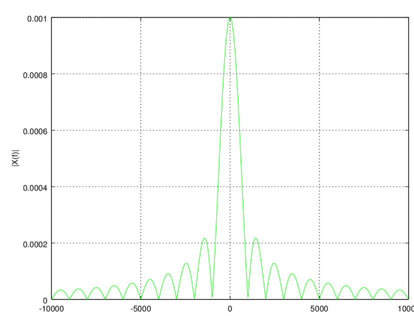
$\Rightarrow X(F) = \frac{j}{\omega}(1 - \frac{1}{e^{j\omega T}})$

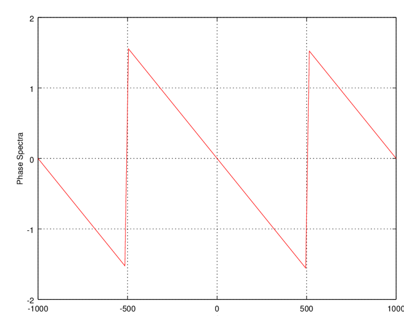
$\Rightarrow X(F) = \frac{j}{\omega}(cos(j\omega T) - jsin(j\omega T)) - \frac{j}{\omega}$

$\Rightarrow X(F) = \frac{j}{\omega}(1 + cos(j\omega T)) + sin(j\omega T)$

$$\boxed{\text{magnitude} = \sqrt{RealPart^2 + ImgPart^2}} \quad \boxed{\text{phase} = tan^{-1}(\frac{ImgPart}{RealPart})}$$







Problem-2

`Consider a square jogging track surrounding a lake as shown in the Figure1. You usually start your daily walk from the location ( x(0), y(0) ) at time t = 0.`

1. Assuming that you walk with a constant speed of 8km/hr and given that the sides of the square are 2kms long, find out the functions x(t) and y(t) describing co-ordinates (w.r.t. center of the pond (0, 0)). Let $g(t) = x(t) + jy(t)$ be the path traveled by you in an hour.
2. We may think of the function p(t) which is a periodic extension of g(t), in other words p(t) is the path traced by you if you keep walking from eternity to eternity. Calculate the first three Fourier series coefficients of p(t).
3. Suppose you decide to reconstruct the function p(t) with only the coefficient corresponding to the fundamental frequency. What will be this new path, sketch the reconstructed path.
4. On one fine day, you decided to start your walk from the location (-1, 1). What would be the path traveled by you in the form of $g_1(t) = x_1(t) + jy_1(t)$, where $x_1(t)$ and $y_1(t)$ are the components of your position vector.
5. What is the Fourier series of $p_1(t)$, a periodic extension of $g_1(t)$, in terms of the Fourier series of p(t)?

Solution

$x(t)$	$y(t)$	t
-1	-8t	$0 \le t \le 1/8$
8t - 2	-1	$1/8 \le t \le 3/8$
1	8t - 4	$3/8 \le t \le 5/8$
-8t + 6	1	$5/8 \le t \le 7/8$
-1	-8 + 8t	$7/8 \le t \le 1$

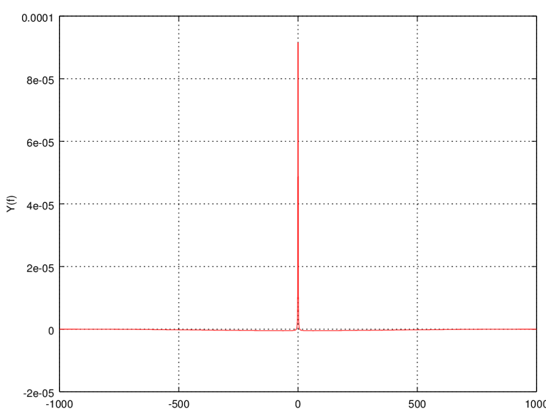
$$\boxed{\frac{1}{T}\int_0^T g(t)e^{-jk\omega_0 t}dt}$$

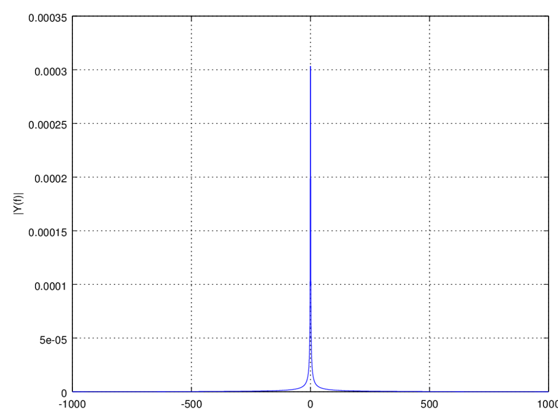
$$X(0) = -0.375$$
$$X(1) = -0.808 + j0.808$$
$$X(2) = 0.001$$

$x(t)$	$y(t)$	t
-1	1-8t	$0 \le t \le 2/8$
8t - 3	-1	$2/8 \le t \le 4/8$
1	8t - 5	$4/8 \le t \le 6/8$
-8t + 7	1	$6/8 \le t \le 1$

$$X(0) = -0.001$$
$$X(1) = -0.808 + j0.808$$
$$X(2) = 0.001$$

Fourier transform $p_1(t) = p(t) +$ remaining fourier





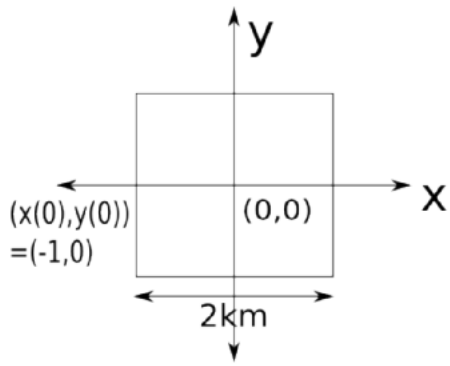


Figure 1: Jogging track surrounding a square lake

Problem-3

A spring-damper based weighing scale for measuring mass from 1 to 10 kgs is shown in Figure. Initially at rest, when an object with mass M kgs is put on the hook (at t = 0), it exerts a downward force of $Mgu(t)$ newtons on the system, where $g \simeq 10 meter/sec^2$ is the acceleration due to gravity. This force causes a displacement of x(t) meters of the pointer (time t in seconds), which after stabilizing enables us to read the mass of the object on the scale. The input downward force Mgu(t) newtons is balanced by the inertial force $M\frac{d^2x(t)}{dt^2}$ newtons, force due to the spring $kx(t)$ newtons and force due to the damper $b\frac{dx}{dt}(t)$ newtons. The parameters of the system shown in Figure 2 are k = 1000 newton/meter and b = 100 newton-sec/meter. Ignoring all other factors, for eg. mass of the scale itself, answer the following questions. (Suggestion: Substitute numerical values only in the end. While analyzing, ignore mass of the hook.)

Solution

$$M\frac{d^2x(t)}{dt^2} + kx(t) + bdx(t) = Mgu(t)$$

now we take laplace tranform of this above equation
$x(t) \Leftrightarrow X(S)$
$Ms^2X(S) = \frac{Mg}{s(Ms^2+bs+k)}$
put the values of M, b, k, and g,
$X(s) = \frac{100}{s(10s^2+100S+1000)}$
now we find redidues and poles for this above equations

[r,p,k,e] = residue([nominator cofficient],[denominator cofficient])
r-residue, p-poles
for here [r,e]=residue([100],[10,100,1000,0])

$$X(s) = \frac{-0.05+0.02887i}{s-(-5.0+8.66025i)} + \frac{-0.05-0.02887i}{s-(-5-8.66025i)} + \frac{0.1}{s}$$

now we take inverse La-place

$$\boxed{\frac{1}{s-a} \Leftrightarrow e^{-at}u(t)}$$

now after taking inverse of our equation

$x(t) = (-0.05-0.02887i)e^{-(5+8.66025i)t}u(t) + (-0.05+0.02887i)e^{-(5-8.66025i)}u(t) + (0.1)u(t)$
now
$t \to \infty$
$x(\infty) = 0.1$

answer = 0.1 meter

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octave:1> [r,p]=residue([100],[10,100,1000,0])
r =

  -0.05000 + 0.02887i
  -0.05000 - 0.02887i
   0.10000 + 0.00000i

p =

  -5.00000 + 8.66025i
  -5.00000 - 8.66025i
   0.00000 + 0.00000i
```

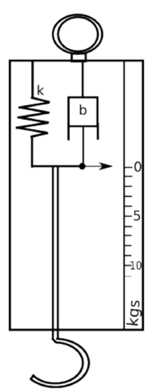

Figure 2: Weighing Balance as a Second Order LTI System