DOT PRODUCT

Multiply 2 vectors to get a scalar.

Algebraic Explanation:

Assume x, y are vectors in some 3 dimensional space.

same idea extends in an n-dimensional space.

much more common in ML

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$x \cdot y = x_1 y_1 + x_2 y_2 + x_3 y_3$$

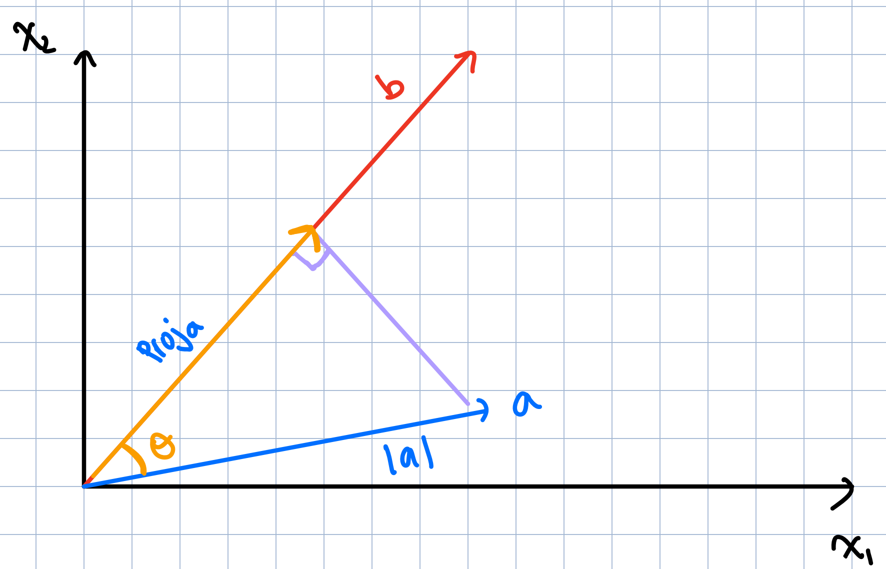
Alternatively, we could write

$$x \cdot y = \underbrace{\|x\| \, \|y\|}_{\text{magnitude/length of the vectors}} \cos\theta$$

angle between the two vectors

$$\|v\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

Geometric Explanation: (Assume we're working in 2 dimensions)



$$\cos\theta = \frac{\text{Proj } a}{|a|} \; ; \quad \text{Proj } a = |a| \cos\theta$$

$$a \cdot b = |a| |b| \cos\theta$$

PROPERTIES OF DOT PRODUCT:

1. COMMUTATIVE $\longrightarrow a \cdot b = b \cdot a$
2. DISTRIBUTIVE $\longrightarrow a \cdot (b + c) = (a \cdot b) + (a \cdot c)$
3. SCALAR MULTIPLICATION $\longrightarrow (c_1 a) \cdot (c_2 b) = c_1 c_2 (a \cdot b)$
4. NON ASSOCIATIVE $\longrightarrow \underbrace{(a \cdot b)}_{\text{scalar}} \cdot c \neq a \cdot \underbrace{(b \cdot c)}_{\text{scalar}}$

   → This doesn't work because both the scalars are different and so are the vectors that they are scaling.

5. PRODUCT RULE $\longrightarrow (a \cdot b)' = a' \cdot b + a \cdot b'$

   from calculus