INTRODUCTION TO OPTIMIZATION

Introduction:

An optimization problem consists of the following parts:

→ An objective function
→ constraints → inequality constraints, equality constraints
→ Feasible set
→ An optimum solution (if one exists)

Example: Lets say we we're renovating our house. We want to renovate two rooms (say bedroom & kitchen). We want to paint and re-do the electrical wiring. The cost of painting one room is \$500. The cost of redoing the wiring is \$1000. Our total budget is \$2000! Both rooms must be renovated. It takes 3 days to paint 1 rooms & 4 days to rewire one room. We want our total time to be 8 days.

↳ either painted or re-wired.

$x_1 \longrightarrow$ # rooms being painted
$x_2 \longrightarrow$ # rooms being rewired

what we're trying to minimize

minimize cost $= 500x_1 + 1000x_2$ ⟶ objective function

such that:

constraints

$$500x_1 + 1000x_2 \leq 1750$$
$$x_1 + x_2 \geq 2$$

Inequality constraints

$$3x_1 + 4x_2 = 8$$

equality constraints

Feasible set: All possible set of x_1 and x_2 that satisfy all the above mentioned constraints.

Optimum solution ⇒ min (feasible set)

take all the values from our feasible set and plug that into our objective function. The minimum value becomes our optimized solution.

I read this as: give me the argument that minimizes the feasible set.

minimizer

if you want the arguments that give you the optimized solution, the equation would be

→ x_1, x_2

argmin (feasible set)

MATHEMATICAL NOTATION: [Assuming you're familiar with vectors]

Problem :

optimization variable

objective function

$$\min_{x \in \mathbb{R}^n} f_0(\vec{x})$$

objective could also be max

inequality constraints

$$\text{s.t } f_i(\vec{x}) \leq c_i \quad \forall i \in \{1, \ldots\ldots, m\}$$
$$h_j(\vec{x}) = d_i \quad \forall j \in \{1, \ldots\ldots, p\}$$

equality constraints

feasible ← $\Omega = \left\{ \vec{x} \in \mathbb{R}^n \;\middle|\; \begin{array}{ll} f_i(\vec{x}) \leq c_i & \forall i \in \{1, \ldots\ldots, m\} \\ h_j(\vec{x}) = d_i & \forall j \in \{1, \ldots\ldots, p\} \end{array} \right\}$

$\therefore$ Problem $\Rightarrow \min_{x \in \Omega} f_0(\vec{x})$ → rewriting the problem