APPROXIMATION

→ Builds on the results of balls in bins, check that out first.

$$P(\text{at least one collision}) = 1 - \underbrace{\prod_{i=0}^{n-1} \frac{N-i}{N}}_{P(\text{no-collision})}$$

$$P(\text{no collision}) = \prod_{i=0}^{n-1} \frac{N-i}{N} \quad ; \text{ no-collision} = A$$

$$\log(P(A)) = \log\left(\prod_{i=0}^{n-1} \frac{N-i}{N}\right)$$

$$= \sum_{i=0}^{n-1} \log\left(\frac{N-i}{N}\right)$$

$$= \sum_{i=0}^{n-1} \log\left(1 - \frac{i}{N}\right) \quad ; \log(1+x) \sim x \text{ for small } x$$

$$x = -\frac{i}{N} \quad ; \quad i < n << N \Rightarrow x \text{ is small}$$

$$= \sum_{i=0}^{n-1} -\frac{i}{N}$$

$$= -\frac{1}{N} \sum_{i=0}^{n-1} i \quad \text{sum of first n numbers}$$

$$\log(P(A)) = -\frac{1}{N} \frac{(n-1)(n)}{2}$$

$$P(A) = e^{-\frac{(n-1)(n)}{2N}} \Rightarrow \frac{n^2 - n}{-2N} \quad \text{[we can ignore n)}$$

$$P(A) = e^{-n^2/2N}$$

$$P(\text{atleast 1 collision}) = 1 - P(A)$$

$$= 1 - e^{-n^2/2N}$$