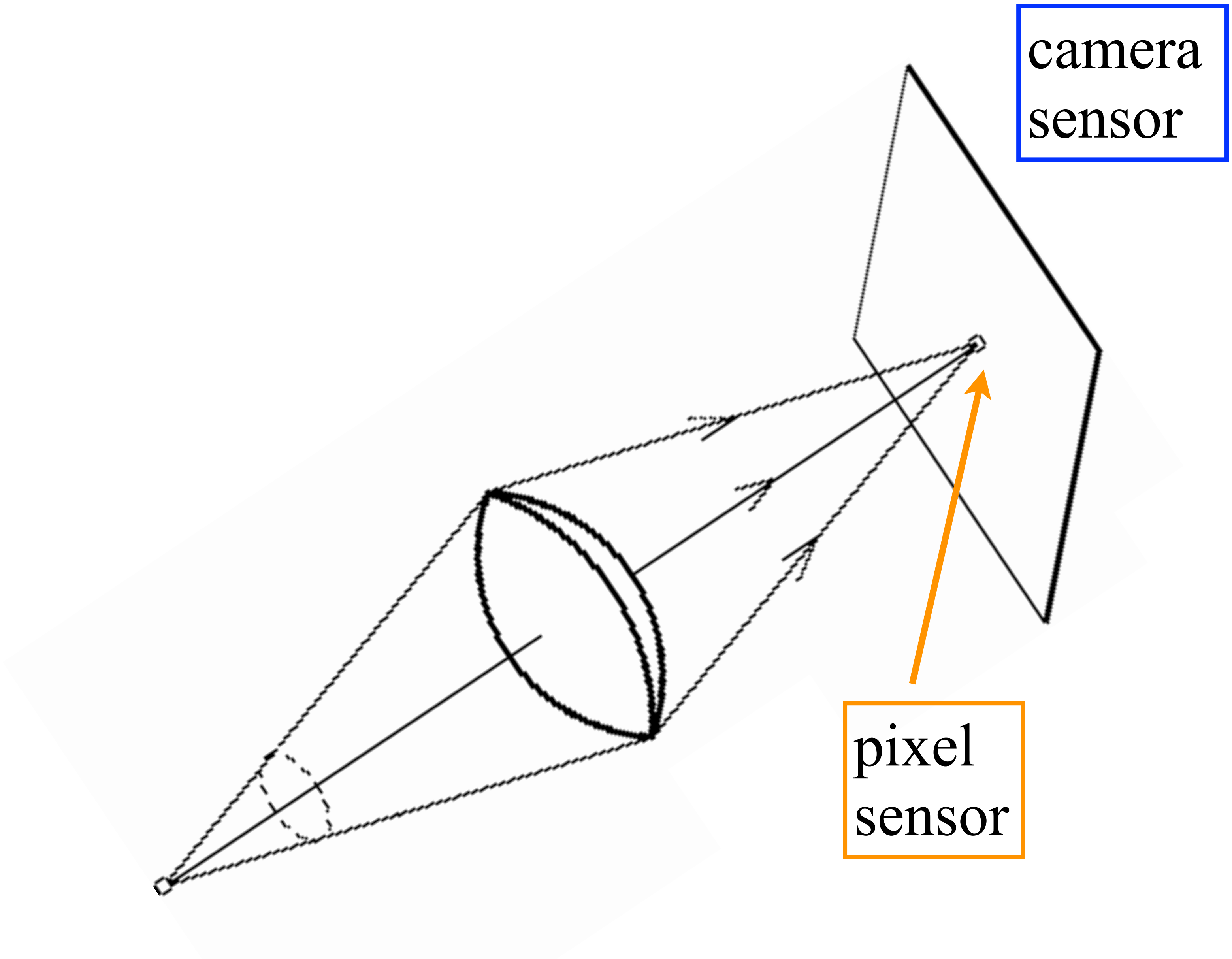
CS 477/577

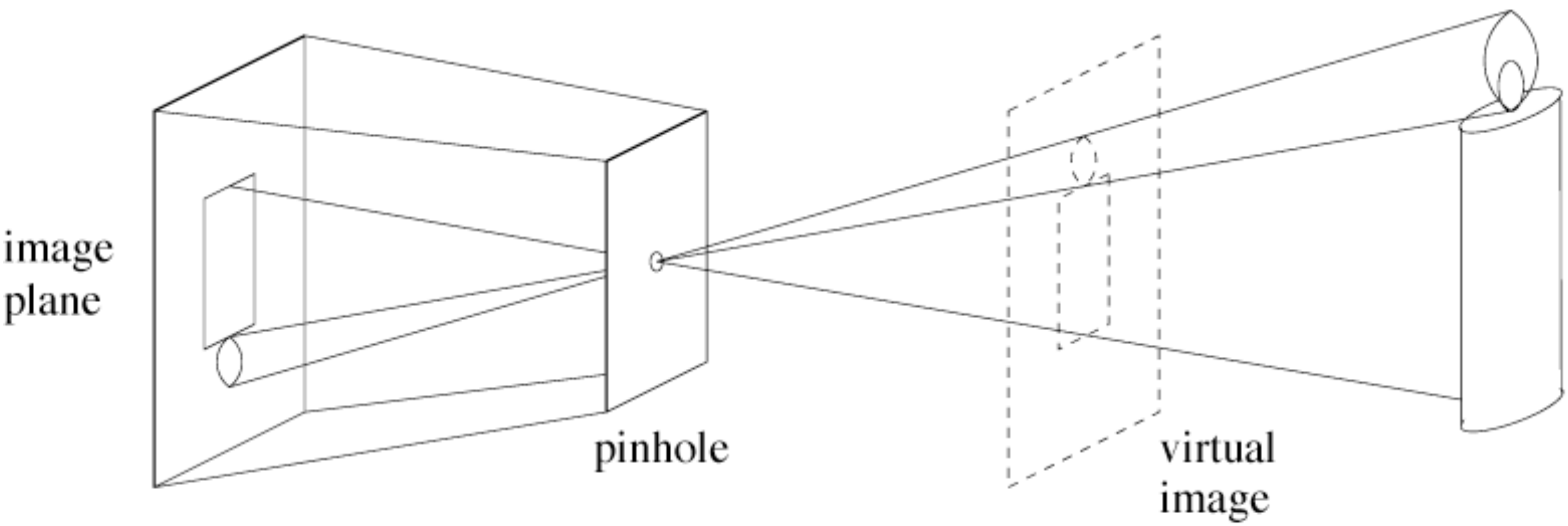
Asynchronous (pre-recorded) lecture five

Image formation (geometric) I: Perspective Projection

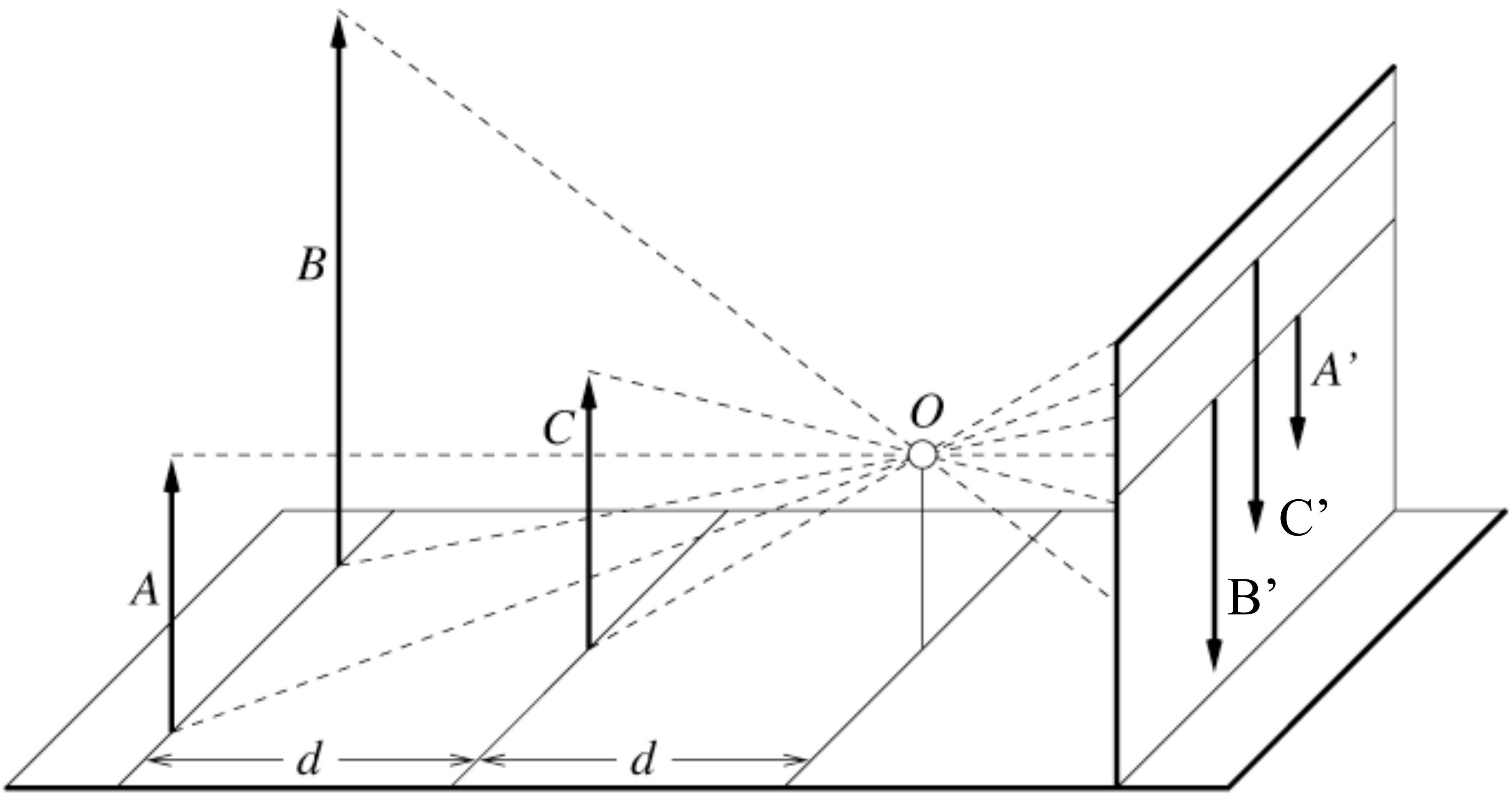

camera
sensor
pixel
sensor

Pinhole cameras

- Abstract camera model--box with a small hole in it
- Pinhole cameras work great for deriving geometry—a real camera needs a lens



Distant objects are smaller



Geometric properties of projection

- Points go to points
- Lines go to lines
- Polygons go to polygons
- Degenerate cases
  - line through focal point projects to a point
  - plane through focal point projects to a line

Vanishing points

More generally, all lines in a particular direction converge to the same vanishing point.

Photo by Neil Kremer

Vanishing points

- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction

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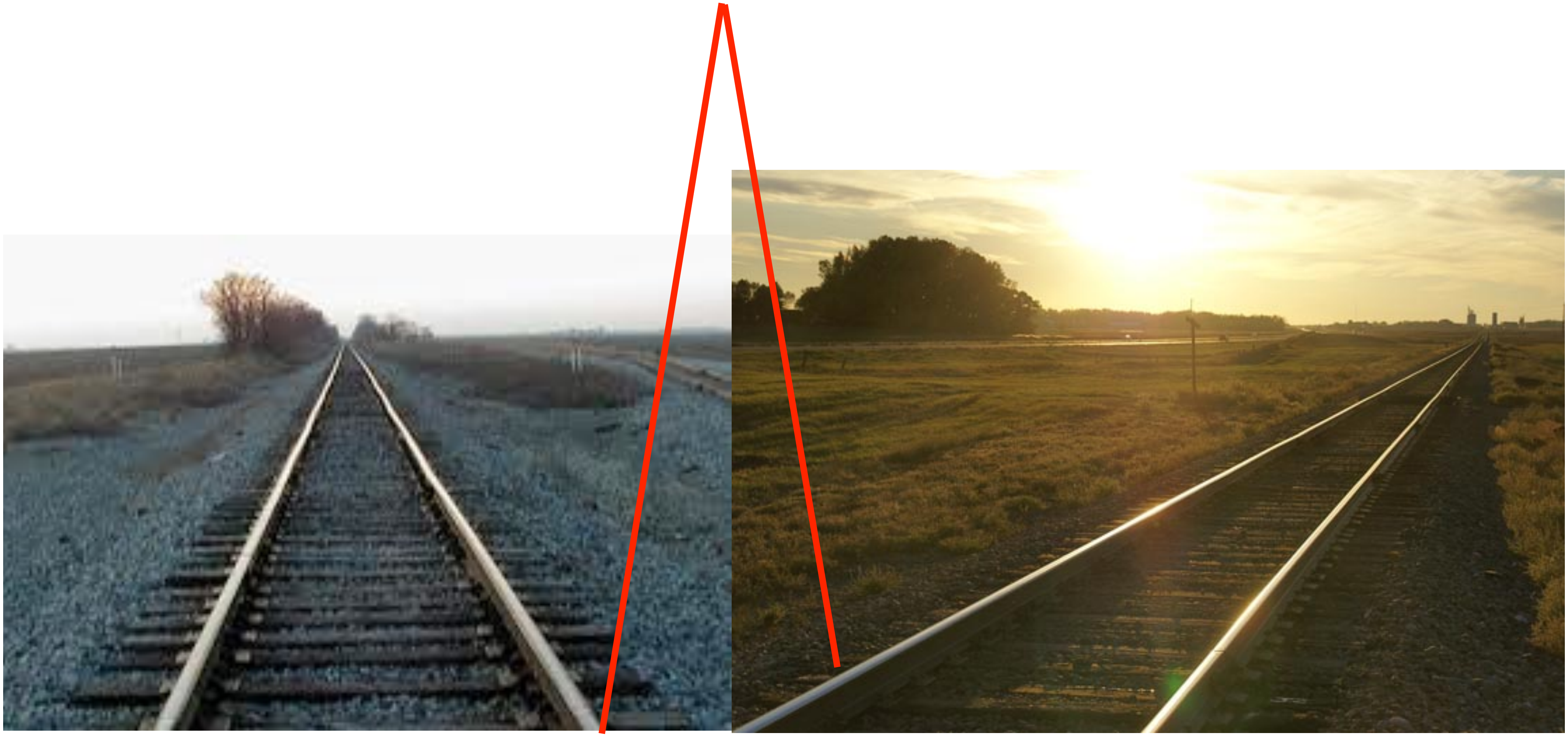

If we have a third set of tracks, would this look right?

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Important

Vanishing points

- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
  - Exception is lines that are parallel to the camera plane

- The vanishing points for directions **on a plane** are co-linear
  - The horizon for that plane

Is the picture a fake?

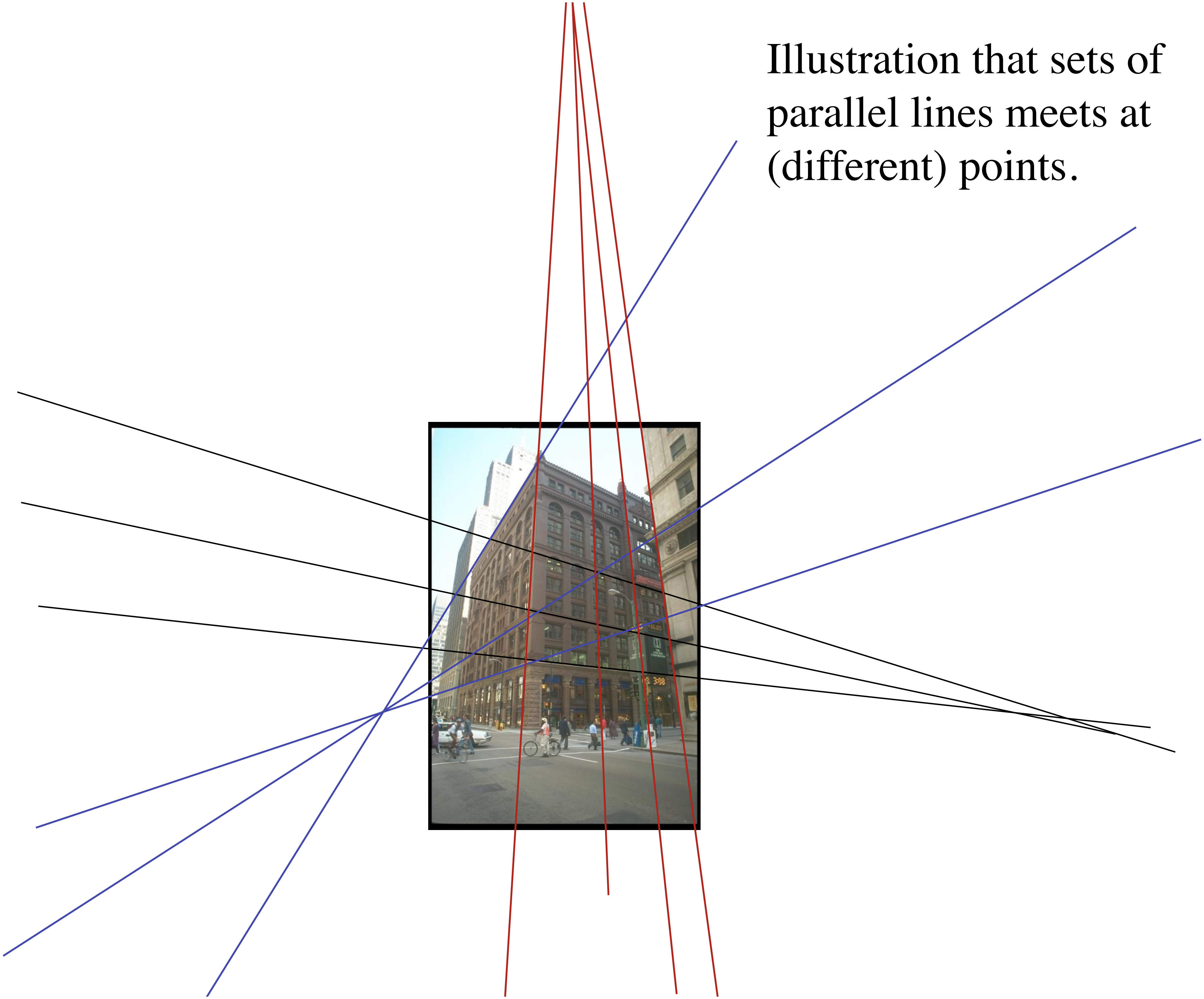
- If scale and perspective don’t work correctly, perhaps the image is a fake!

- We can check if:
  - Each set of parallel lines (=direction) meets at a different point
  - Sets of parallel lines on the same plane lead to *collinear* vanishing points.

Do sets of parallel lines meets at (different) points in this image?

Can you identify three different sets of parallel lines in this image?

Illustration that sets of parallel lines meets at (different) points.



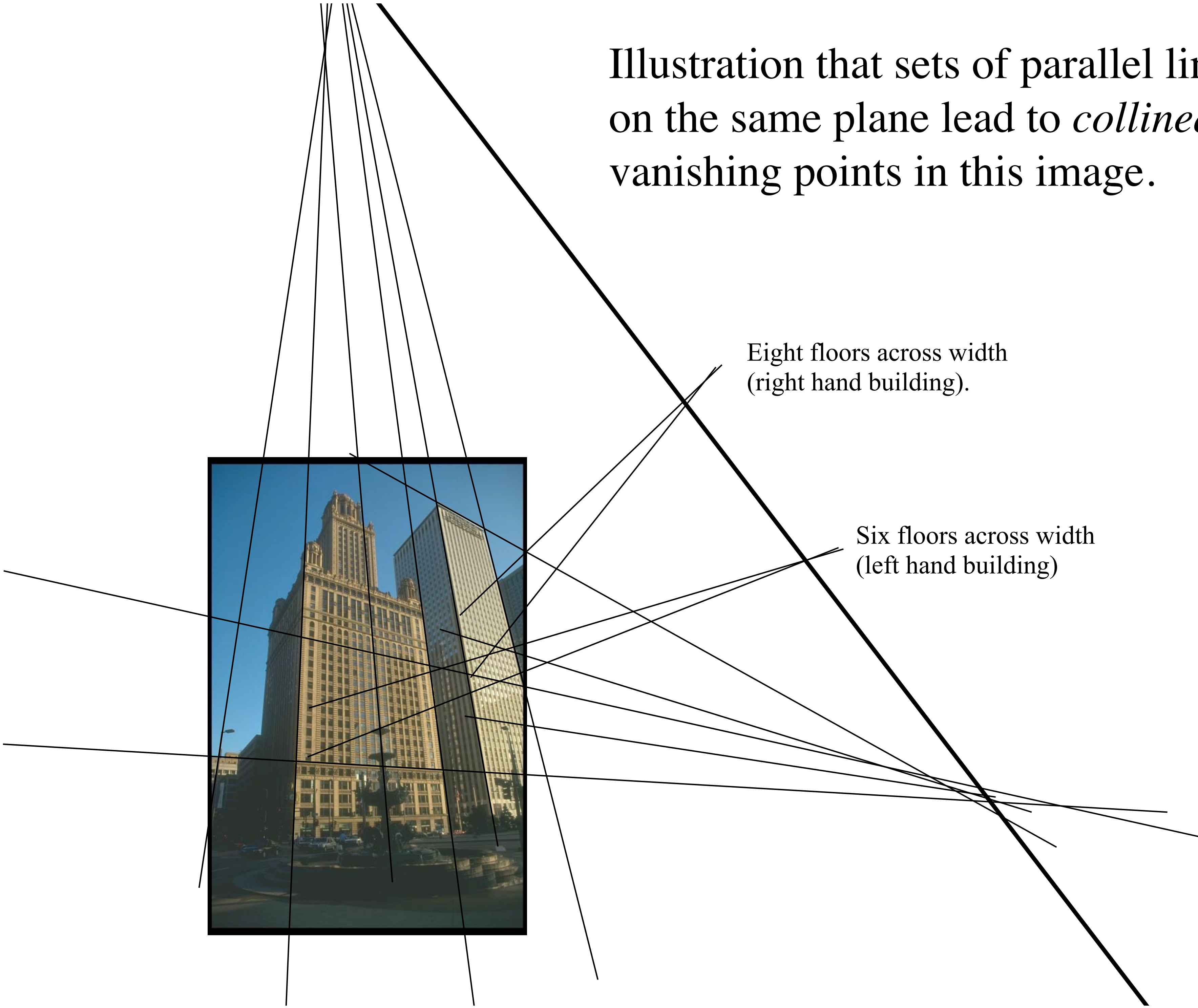
Vanishing points (cont)

- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane
  - Standard horizon is the horizon of the ground plane.

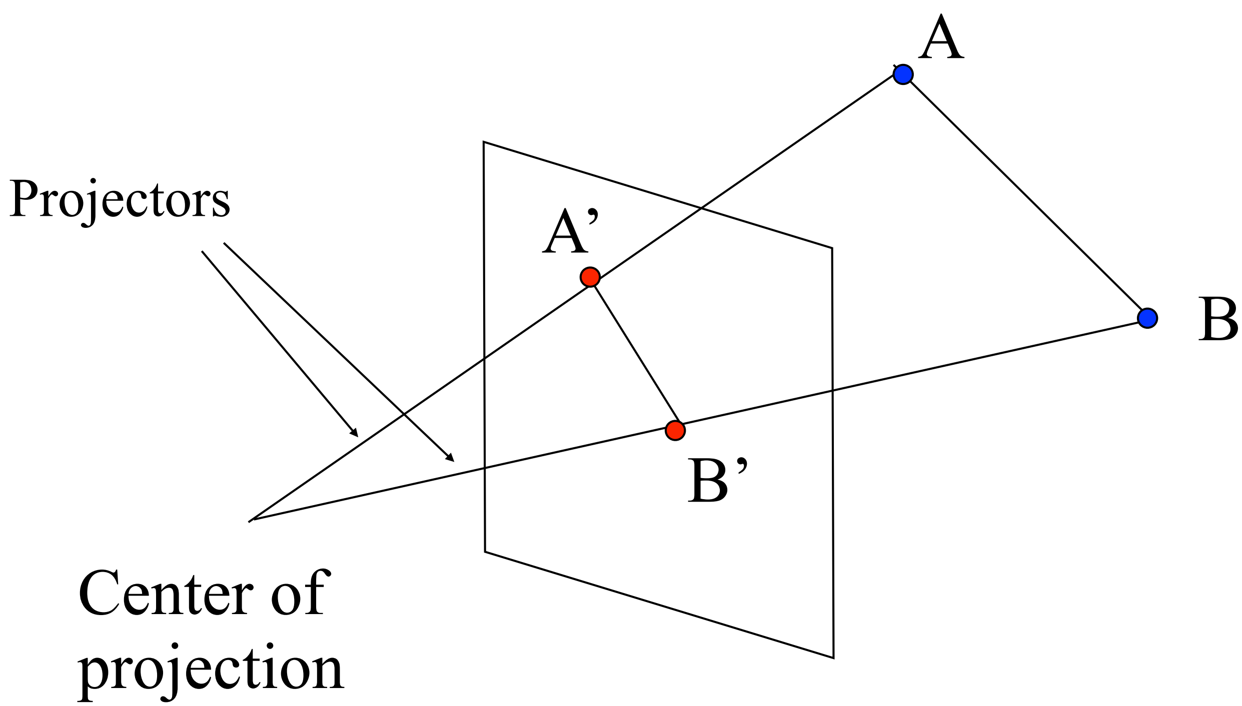
Do sets of parallel lines on the same plane lead to *collinear* vanishing points?

Can you identify three sets of parallel lines on the same plane in this image?

Illustration that sets of parallel lines on the same plane lead to *collinear* vanishing points in this image.



More on projections



The projection of A is A'

What is the projection of A'?

More on projections

- Want to think about geometric image formation as a mathematical transformation taking points in the 3D world and mapping them into an image plane.

- Mathematical definition of a projection: PP=P
  - (Doing it a second time has no effect).

- Transformation looses information (e.g., depth)
  - Given a 2D image, there are many possible 3D worlds
  - Projections are not invertible!
    - Exception is P=I

More on projections

- Transformation looses information (e.g., depth)
  - Given a 2D image, there are many possible 3D worlds
  - Projections are not invertible!
    - Exception is P=I
  - "Proof"

    Projections for cameras are not the identity because points not on the image plane must get there. Given

    $PP = P$

    and assuming that P is invertable, we can multiply both sides by P^{-1}

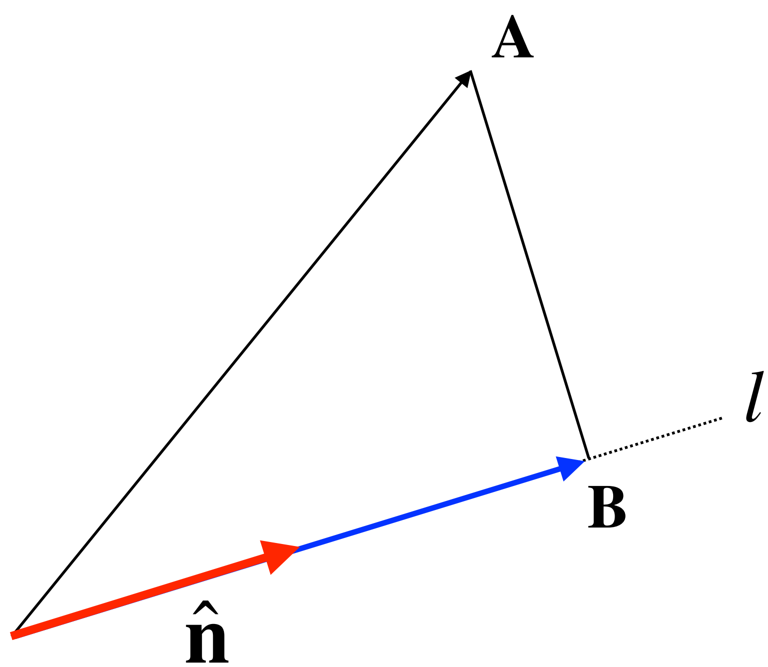
    $P^{-1}PP = P^{-1}P$

    which gives

    $P = I$

    In other words, the only invertable projection is I.

2D example of projection

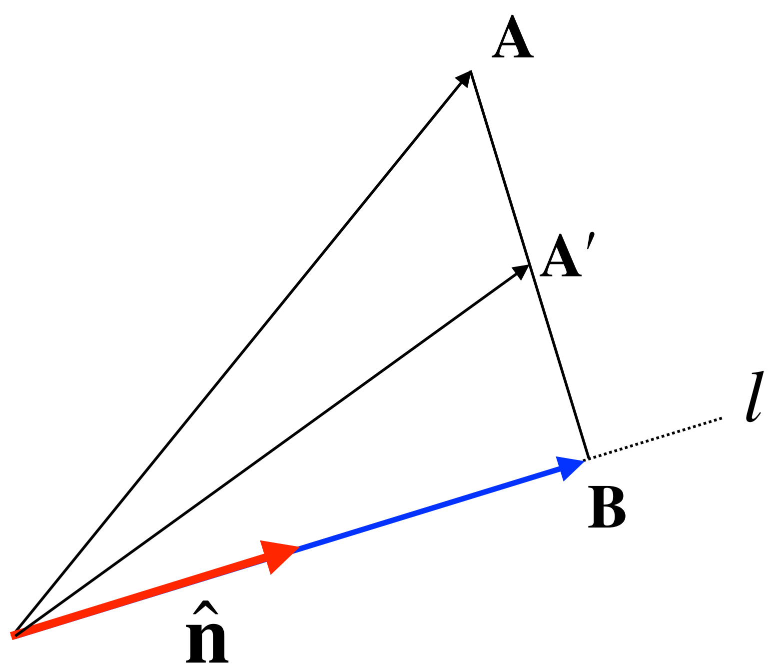


B is the perpendicular projection of **A** onto line (l)

Projecting B onto l gives **B.**

If you know l and **B**, is **A** unique?

2D example of projection



B is the perpendicular projection of **A** onto line (l)

Projecting **B** onto l gives **B.**

A′ also projects to B.

Summary on projections

- PP=P
  - (Doing it a second time has no effect).

- Projections looses information (e.g., depth)
  - Given a 2D image, there are many possible 3D worlds
  - Projections are not invertible!
    - Exception is P=I

Summary on projection via pin-hole camera model

- As objects move further away, they get smaller
  - Assuming motion is not parallel to camera plane

- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
  - Exception is lines that are parallel to the camera plane

- The vanishing points for directions **on a plane** are co-linear
  - The horizon for that plane

Preview of next topic