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About : Assignment 3, APS1080 (Introduction to RL)

Topic : TD(n) and Mountain Car

In [1]:

```
import matplotlib.pyplot as plt
import gym
from IPython import display as ipythondisplay
import numpy as np
import json
import pickle
```

Exercise 1: Prelims

Here we explore the mountain car environment

In [8]:

```
env = gym.make('MountainCar-v0')

print('Observation Space: ', env.observation_space)
print('Action Space: ', env.action_space)
```

```
Observation Space:  Box([-1.2  -0.07], [0.6  0.07], (2,), float32)
Action Space:  Discrete(3)
```

The environment's state space is represented by two variables: car position and car velocity. The car position can vary from -1.2 to 0.6 and velocity varies from -0.07 to 0.07. The agent can choose to take 3 actions: accelerate left (0), accelerate right (2) or don't accelerate (1). The environment's model is not available, hence, we cannot use DP.

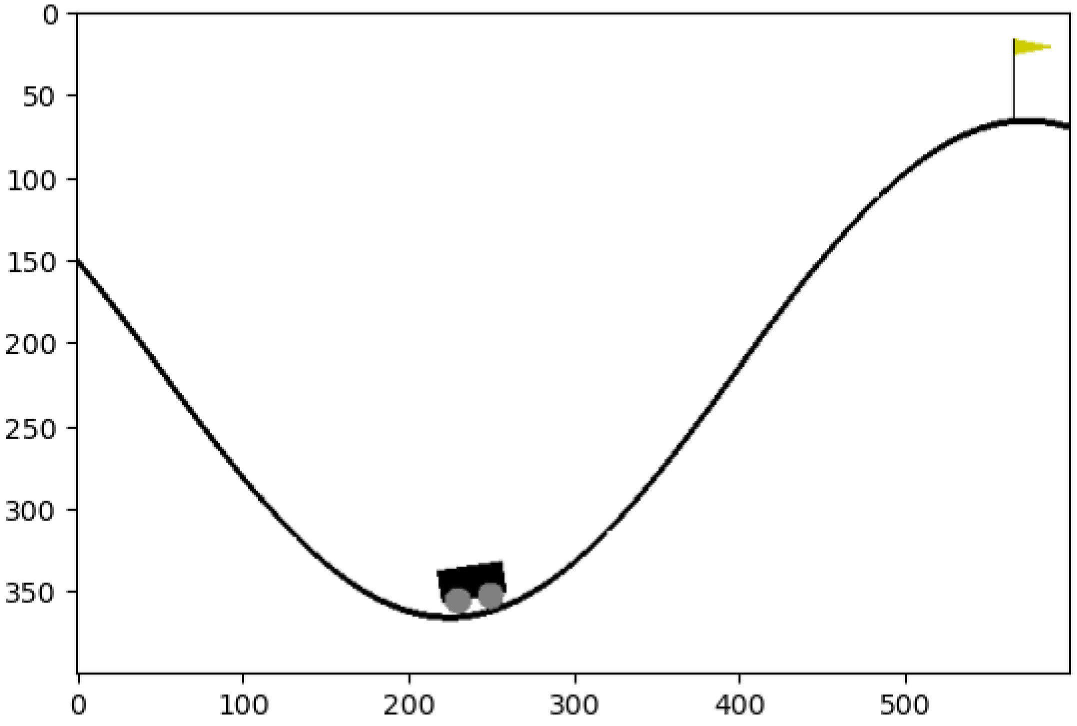
In [9]:

```
env.reset()
prev_screen = env.render(mode='rgb_array')
plt.imshow(prev_screen)

for _ in range(10):
    env.step(env.action_space.sample()) # take a random action
    screen = env.render(mode='rgb_array')
    plt.imshow(screen)

    ipythondisplay.clear_output(wait=True)
    ipythondisplay.display(plt.gcf())

env.close()
```



Check Controller Performance

The below function is used to check the RL controller over n (=1000) trials

In [2]:
```
def control_performance(env_name, pos_fac, vel_fac, policy, no_actions, trial

    env           = gym.make(env_name)
    reward_list   = []
    steps_list    = []
    reached_goal  = 0

    for i in range(trials):
        done            = False
        episode_reward  = 0
        obs             = env.reset()
        obs             = process_state(obs, pos_fac, vel_fac)
        steps           = 0

        while not done:
            action                      = np.random.choice(no_actions, p=policy[o
            obs, reward, done, info     = env.step(action)
            steps                      += 1
            episode_reward             += reward
            obs                         = process_state(obs, pos_fac, vel_fac)

        if steps<200:
            steps_list.append(steps)
            reached_goal += 1

        reward_list.append(episode_reward)

    return reached_goal, steps_list
```

Defining some helper functions

In [3]:
```
def epsilon_greedy_action_policy(obs, epsilon, no_actions, policy):
    '''chooses epsilon greedy action given a policy'''
```

```
    if np.random.rand()<epsilon:
        action = np.random.choice(no_actions, p=[1/no_actions, 1/no_actions,
    else:
        action = np.random.choice(no_actions, p=policy[obs])

    return action


def process_state(state, p_fac, v_fac):
    return (round(state[0]*p_fac), round(state[1]*v_fac))


def epsilon_greedy_action_Q(obs, epsilon, no_actions, Qpi_sa):
    '''chooses epsilon greedy action given Q function'''

    if np.random.rand()<epsilon:  # random action
        action = np.random.choice(no_actions, p=[1/no_actions, 1/no_actions,
    else:                         # greedy actions
        qpi_list = []
        for action in range(no_actions):
            qpi_list.append(Qpi_sa[obs, action])

        maxa_list = np.argwhere(qpi_list == np.amax(qpi_list))
        maxa_list_indx = []

        # indices that have max. q values
        for item in maxa_list:
            maxa_list_indx.append(item[0])

        act_probab = [0, 0, 0]
        for i in range(no_actions):
            if i in maxa_list_indx:
                act_probab[i] = 1/len(maxa_list_indx)
            else:
                act_probab[i] = 0

        action = np.random.choice(no_actions, p=act_probab)

    return action

def get_policy(state_list, Qpi_sa):
    ''''
    based on the Q function, returns the policy
    '''

    policy = {}

    for state in state_list:
        qpi_list = []

        for action in range(no_actions):
            qpi_list.append(Qpi_sa[state, action])

        maxa_list      = np.argwhere(qpi_list == np.amax(qpi_list))
        maxa_list_indx = []

        # indices that have max. q values
        for item in maxa_list:
            maxa_list_indx.append(item[0])

        # updating the new policy policy
        policy[state] = [0, 0, 0]
        for i in range(no_actions):
            if i in maxa_list_indx:
                policy[state][i] = 1/len(maxa_list_indx)
```

```
            else:
                policy[state][i] = 0

    return policy
```

Random Policy

In [83]:

```
random_policy_pi = {}

for item in state_list:
    random_policy_pi[item] = [1/no_actions, 1/no_actions, 1/no_actions] # act

success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, rand
print('Success rate of random policy over 1000 runs: ', (success/1000)*100, '
if len(steps)!=0:
    print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

```
Success rate of random policy over 1000 runs:  0.0 %
```

Task 1: TD(0) On-Policy SARSA

Intialization of an arbitrary state-action value function

In [22]:

```
Qpi_sa      = {}
state_list = [] # contains the list of states
no_actions = 3    # 0, 1 and 2

# multiplicative factor of 100
pos_min     = -120
pos_max     = 60
pos_inc     = 1
pos_fac     = 100

# multiplicative factor of 100
vel_min     = -7
vel_max     = 7
vel_inc     = 1
vel_fac     = 100

for pos in range(pos_min, pos_max+1, pos_inc):
    for vel in range(vel_min, vel_max+1, vel_inc):
        state_list.append((pos, vel))
        for action in range(no_actions):
            Qpi_sa[(pos, vel), action] = 0
```

Intialization of an arbitrary policy based on Qpi

In [23]:

```
policy_pi = {}

for key in Qpi_sa.keys():
    policy_pi[key[0]] = [1/no_actions, 1/no_actions, 1/no_actions] # action i
```

In [17]:

```
env          = gym.make('MountainCar-v0')

episodes     = 0
epsilon      = 0.1
alpha        = 0.5
gamma        = 0.9
converged    = False
```

```
while not converged:
    episodes += 1

    cur_obs     = env.reset()
    cur_obs     = process_state(cur_obs, pos_fac, vel_fac)
    cur_action  = epsilon_greedy_action_policy(cur_obs, epsilon, no_actions, 
    done        = False

    while not done:
        next_obs, reward, done, info = env.step(cur_action)
        next_obs                     = process_state(next_obs, pos_fac, vel_f
        next_action                  = epsilon_greedy_action_policy(next_obs,
        Qpi_sa[cur_obs,cur_action]   = Qpi_sa[cur_obs,cur_action] + alpha*(re
        cur_obs                      = next_obs
        cur_action                   = next_action

    new_policy  = get_policy(state_list, Qpi_sa)
    if new_policy==policy_pi:
        converged = True

    policy_pi = new_policy
```

In [85]:

```
pickle.dump(policy_pi, open("task1_onpolicy-sarsa.p", "wb" ))
print('Episodes took to converge for on-policy SARSA: ', episodes)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
print('Success rate of on-policy SARSA control over 1000 runs: ', (success/10
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

```
Episodes took to converge for on-policy SARSA:  11413
Success rate of on-policy SARSA control over 1000 runs:  6.5 %
Average steps taken to reach the goal:  178.56923076923076
```

Task 1: TD(0) On-Policy Expected SARSA

Intialization of an arbitrary state-action value function

In [86]:

```
Qpi_sa     = {}
state_list = [] # contains the list of states
no_actions = 3   # 0, 1 and 2

# multiplicative factor of 100
pos_min    = -120
pos_max    = 60
pos_inc    = 1
pos_fac    = 100

# multiplicative factor of 100
vel_min    = -7
vel_max    = 7
vel_inc    = 1
vel_fac    = 100

for pos in range(pos_min, pos_max+1, pos_inc):
    for vel in range(vel_min, vel_max+1, vel_inc):
        state_list.append((pos, vel))
        for action in range(no_actions):
            Qpi_sa[(pos, vel), action] = 0
```

In [87]:

```
policy_pi = {}
```

```
for item in state_list:
    policy_pi[item] = [1/no_actions, 1/no_actions, 1/no_actions] # action is
```

In [88]:

```
env          = gym.make('MountainCar-v0')

episodes     = 0
epsilon      = 0.1
alpha        = 0.5
gamma        = 0.9
converged    = False

while not converged:
    episodes += 1

    # Policy Evaluation
    cur_obs      = env.reset()
    cur_obs      = process_state(cur_obs, pos_fac, vel_fac)
    cur_action   = epsilon_greedy_action_policy(cur_obs, epsilon, no_actions, 
    done         = False

    while not done:
        next_obs, reward, done, info = env.step(cur_action)
        next_obs                     = process_state(next_obs, pos_fac, vel_f
        next_action                  = epsilon_greedy_action_policy(next_obs,
        expect_q_sum                 = 0
        for action in range(no_actions):
            expect_q_sum             += policy_pi[next_obs][action]* Qpi_sa[n

        Qpi_sa[cur_obs,cur_action]   = Qpi_sa[cur_obs,cur_action] + alpha*(re
        cur_obs                      = next_obs
        cur_action                   = next_action


    # Policy Improvement
    new_policy_pi   = get_policy(state_list, Qpi_sa)
    if new_policy_pi==policy_pi:
        converged = True

    policy_pi = new_policy_pi

pickle.dump(policy_pi, open("task1_onpolicy-expectedsarsa.p", "wb" ))
```

In [90]:

```
print('Episodes took to converge for on-policy Expected SARSA: ', episodes)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
print('Success rate of on-policy Expected SARSA control over 1000 runs: ', (s
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

```
Episodes took to converge for on-policy Expected SARSA:  9518
Success rate of on-policy Expected SARSA control over 1000 runs:  7.3 %
Average steps taken to reach the goal:  171.32876712328766
```

Task 1: TD(0) Off-Policy Expected SARSA with a greedy control policy

Intialization of an arbitrary state-action value function

In [118…

```
Qpi_sa      = {}
state_list = [] # contains the list of states
no_actions = 3    # 0, 1 and 2

# multiplicative factor of 100
pos_min     = -120
```

```python
pos_max     = 60
pos_inc     = 1
pos_fac     = 100

# multiplicative factor of 100
vel_min     = -7
vel_max     = 7
vel_inc     = 1
vel_fac     = 100

for pos in range(pos_min, pos_max+1, pos_inc):
    for vel in range(vel_min, vel_max+1, vel_inc):
        state_list.append((pos, vel))
        for action in range(no_actions):
            Qpi_sa[(pos, vel), action] = 0
```

Initialization of behaviour (B) and target (Pi) policy

In [119…

```python
policy_pi = {}              # target policy

for item in state_list:
    policy_pi[item] = [1/no_actions, 1/no_actions, 1/no_actions] # action is

policy_b  = policy_pi       # behaviour policy
```

In [120…

```python
def update_behaviour_policy(pol_pi, state_list, no_actions, epsilon):
    '''
    makes the behaviour policy exploratory with respect to policy pi
    '''
    pol_b = {}

    for state in state_list:
        nonzeroind    = np.nonzero(policy_pi[state])[0]
        pol_b[state] = [0,0,0]

        for action in range(no_actions):
            if action not in nonzeroind:
                pol_b[state][action] = epsilon/no_actions
            else:
                pol_b[state][action] = policy_pi[state][action] - ((no_action

    return pol_b
```

In [121…

```python
env          = gym.make('MountainCar-v0')

episodes     = 0
epsilon      = 0.1
alpha        = 0.5
gamma        = 0.9
converged    = False

while not converged:
    episodes += 1

    # Policy Evaluation
    cur_obs       = env.reset()
    cur_obs       = process_state(cur_obs, pos_fac, vel_fac)
    cur_action    = np.random.choice(no_actions, p=policy_b[cur_obs])      # act
    done          = False

    while not done:
```

```
        next_obs, reward, done, info = env.step(cur_action)
        next_obs                     = process_state(next_obs, pos_fac, vel_f
        next_action                  = np.random.choice(no_actions, p=policy_
        expect_q_sum                 = 0
        for action in range(no_actions):
            expect_q_sum             += policy_pi[next_obs][action]* Qpi_sa[n

        Qpi_sa[cur_obs,cur_action]   = Qpi_sa[cur_obs,cur_action] + alpha*(re
        cur_obs                      = next_obs
        cur_action                   = next_action


    # Policy Improvement
    new_policy_pi  = get_policy(state_list, Qpi_sa)
    if new_policy_pi==policy_pi:
        converged = True

    policy_pi = new_policy_pi
    policy_b  = update_behaviour_policy(policy_pi, state_list, no_actions, ep

pickle.dump(policy_pi, open("task1_offpolicy-expectedsarsa.p", "wb" ))
```

In [122…

```
print('Episodes took to converge for off-policy Expected SARSA: ', episodes)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
print('Success rate of off-policy Expected SARSA control over 1000 runs: ', (
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

```
Episodes took to converge for off-policy Expected SARSA:  9189
Success rate of off-policy Expected SARSA control over 1000 runs:  12.5 %
Average steps taken to reach the goal:  174.88
```

Task 1 Comparison

Controller Type	Episodes Taken to Converge	Success Rate	Avg. Steps to Goal
On-policy SARSA	11413	6.5 %	178.57
On-policy Expected SARSA	9518	7.3 %	171.32
Off-policy Expected SARSA	9189	12.5 %	174.88

It is evident that expected SARSA converges faster to the optimal policy as compared to SARSA. The expected versions also have better success rates. There is a significant performance difference between on-policy and off-policy expected SARSA. The off-policy version has ~71% additional success rate over on-policy. This can be attributed to sufficient exploration in off-policy expected SARSA.

Exercise 2: TD(n) - nstep SARSA control

Contains an implementation of control algorithms that use TD(n) SARSA. The implementation is tested in Task 2

In [4]:

```
def n_sarsa(epsilon, alpha, gamma, n_step):

    Qpi_sa     = {}
    state_list = [] # contains the list of states
    no_actions = 3   # 0, 1 and 2

    # multiplicative factor of 100
    pos_min    = -120
    pos_max    = 60
```

```
    pos_inc     = 1
    pos_fac     = 100

    # multiplicative factor of 100
    vel_min     = -7
    vel_max     = 7
    vel_inc     = 1
    vel_fac     = 100

    for pos in range(pos_min, pos_max+1, pos_inc):
        for vel in range(vel_min, vel_max+1, vel_inc):
            state_list.append((pos, vel))
            for action in range(no_actions):
                Qpi_sa[(pos, vel), action] = 0

    policy_pi = {}
    for item in state_list:
        policy_pi[item] = [1/no_actions, 1/no_actions, 1/no_actions] # action


    env          = gym.make('MountainCar-v0')

    episodes     = 0
    converged    = False

    while not converged:
        episodes += 1

        ## Policy Evaluation ##
        cur_obs      = env.reset()
        cur_obs      = process_state(cur_obs, pos_fac, vel_fac)
        cur_action   = epsilon_greedy_action_policy(cur_obs, epsilon, no_actio
        done         = False
        t            = 0
        t_term       = pow(10,8)              # the termination step
        trans_list   = []
        trans_list.append([cur_obs, cur_action])

        while not done:
            next_obs, reward, done, info = env.step(cur_action)
            if done:
                t_term = t+1
            next_obs                    = process_state(next_obs, pos_fac, v
            next_action                 = epsilon_greedy_action_policy(next_
            cur_obs                     = next_obs
            cur_action                  = next_action
            trans_list.append([next_obs, next_action, reward])

            tau = t-n_step+1
            if tau>=0:
                G   = 0
                for i in range(tau+1, min((tau+n_step+1), t_term+1)):
                    G += pow(gamma, i-tau-1)*trans_list[i][2]

                if tau+n_step<t_term:
                    G += pow(gamma, n_step)*Qpi_sa[trans_list[tau+n_step][0],

                Qpi_sa[trans_list[tau][0], trans_list[tau][1]]  = Qpi_sa[tran

            t += 1

        new_policy  = get_policy(state_list, Qpi_sa)
        if new_policy==policy_pi:
            converged = True
```

```
        policy_pi = new_policy

    return policy_pi, episodes
```

Exercise 2: TD(n) Off-Policy Tree Backup Control

The below implementation is tested in bonus task 3

In [10]:

```
def tree_backup(epsilon, alpha, gamma, n_step):
    Qpi_sa      = {}
    state_list = []  # contains the list of states
    no_actions = 3   # 0, 1 and 2

    # multiplicative factor of 100
    pos_min     = -120
    pos_max     = 60
    pos_inc     = 1
    pos_fac     = 100

    # multiplicative factor of 100
    vel_min     = -7
    vel_max     = 7
    vel_inc     = 1
    vel_fac     = 100

    for pos in range(pos_min, pos_max+1, pos_inc):
        for vel in range(vel_min, vel_max+1, vel_inc):
            state_list.append((pos, vel))
            for action in range(no_actions):
                Qpi_sa[(pos, vel), action] = 0

    policy_pi = {}
    for item in state_list:
        policy_pi[item] = [1/no_actions, 1/no_actions, 1/no_actions] # action

    env          = gym.make('MountainCar-v0')
    episodes     = 0
    converged    = False

    while not converged:
        episodes += 1

        cur_obs      = env.reset()
        cur_obs      = process_state(cur_obs, pos_fac, vel_fac)
        cur_action   = epsilon_greedy_action_policy(cur_obs, epsilon, no_actio
        done         = False
        t            = 0
        t_term       = pow(10,8)              # the termination step
        trans_list   = []
        trans_list.append([cur_obs, cur_action])
        cur_policy   = get_policy(state_list, Qpi_sa)

        while not done:
            next_obs, reward, done, info = env.step(cur_action)
            if done:
                t_term = t+1
            next_obs                     = process_state(next_obs, pos_fac, ve
            next_action                  = epsilon_greedy_action_policy(next_
            cur_obs                      = next_obs
            cur_action                   = next_action
            trans_list.append([next_obs, next_action, reward])

            tau = t-n_step+1
```

```
            if tau>=0:
                policy_pi    = get_policy(state_list, Qpi_sa)
                G            = 0

                if t+1>=t_term:
                    G += trans_list[t_term][2]
                else:
                    expect_q_sum  = 0
                    for action in range(no_actions):
                        expect_q_sum   += policy_pi[trans_list[t+1][0]][actio
                    G += trans_list[t+1][2] + gamma*expect_q_sum

                for k in range(min(t, t_term-1), tau, -1):
                    expect_sum  = 0
                    for action in range(no_actions):
                        if action!=trans_list[k][1]:
                            expect_sum += policy_pi[trans_list[k][0]][action]
                    G += trans_list[k][2] + gamma*expect_sum + gamma*policy_p

                Qpi_sa[trans_list[tau][0], trans_list[tau][1]]  = Qpi_sa[tran

            t += 1

        new_policy  = get_policy(state_list, Qpi_sa)
        if new_policy==policy_pi:
            converged = True
        policy_pi = new_policy

    return policy_pi, episodes
```

Task 2: TD(2), TD(3), TD(4)

In [7]:

```
epsilon      = 0.1
alpha        = 0.5
gamma        = 0.9
no_actions   = 3
```

In [ ]:

```
n_step       = 2
policy, eps = n_sarsa(epsilon, alpha, gamma, n_step)
pickle.dump(policy, open("task2_td2-nsarsa.p", "wb" ))
print('Episodes took to converge for TD(2) SARSA: ', eps)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
print('Success rate of TD(2) SARSA control over 1000 runs: ', (success/1000)*
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

In [ ]:

```
n_step       = 3
policy, eps = n_sarsa(epsilon, alpha, gamma, n_step)
pickle.dump(policy, open("task2_td3-nsarsa.p", "wb" ))
print('Episodes took to converge for TD(3) SARSA: ', eps)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
print('Success rate of TD(3) SARSA control over 1000 runs: ', (success/1000)*
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

In [ ]:

```
n_step       = 4
policy, eps = n_sarsa(epsilon, alpha, gamma, n_step)
pickle.dump(policy, open("task2_td4-nsarsa.p", "wb" ))
print('Episodes took to converge for TD(4) SARSA: ', eps)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
```

```
print('Success rate of TD(4) SARSA control over 1000 runs: ', (success/1000)*
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

Bonus Task 3: Tree Backup

In [11]:

```
epsilon      = 0.1
alpha        = 0.5
gamma        = 0.9
no_actions   = 3
```

In [12]:

```
n_step       = 2
policy, eps = tree_backup(epsilon, alpha, gamma, n_step)
pickle.dump(policy, open("task3_td2-treebackup.p", "wb" ))
print('Episodes took to converge for TD(2) tree backup: ', eps)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
print('Success rate of TD(2) tree backup control over 1000 runs: ', (success/
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

```
Episodes took to converge for TD(2) tree backup:  1
Success rate of TD(2) tree backup control over 1000 runs:  0.0 %

---------------------------------------------------------------------------
ZeroDivisionError                         Traceback (most recent call last)
File /opt/anaconda3/envs/milamoth/lib/python3.7/site-packages/IPython/core/int
eractiveshell.py, in run_code:
Line 3441:  exec(code_obj, self.user_global_ns, self.user_ns)

In  [12]:
Line 7:     print('Average steps taken to reach the goal: ', sum(steps)/len(st
eps))

ZeroDivisionError: division by zero
---------------------------------------------------------------------------
```

In [ ]:

```
n_step       = 3
policy, eps = tree_backup(epsilon, alpha, gamma, n_step)
pickle.dump(policy, open("task3_td3-treebackup.p", "wb" ))
print('Episodes took to converge for TD(3) tree backup: ', eps)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
print('Success rate of TD(3) tree backup control over 1000 runs: ', (success/
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

In [ ]:

```
n_step       = 4
policy, eps = tree_backup(epsilon, alpha, gamma, n_step)
pickle.dump(policy, open("task3_td4-treebackup.p", "wb" ))
print('Episodes took to converge for TD(4) tree backup: ', eps)
success, steps = control_performance('MountainCar-v0', pos_fac, vel_fac, poli
print('Success rate of TD(4) tree backup control over 1000 runs: ', (success/
print('Average steps taken to reach the goal: ', sum(steps)/len(steps))
```

COMMENT: I have implemented n-step SARSA and n-step Tree Backup, however, there are some convergence issues