International Astronomy and Astrophysics Competition
Pre-Final Round 2022

Important: Read all the information on this page carefully!

General Information

- We recommend to print out this problem sheet. Use another paper to draft the solutions to the problems and write your final solution (with steps) on the provided space below the problems.
- You may use extra paper if necessary, however, the space under the problems is usually enough.
- Typing the solution on a computer is allowed but not recommended (no extra points).
- The six problems are separated into three categories: 2x basic problems (A; four points), 2x advanced problems (B; six points), 2x research problems (C; eight points). The research problems require you to read a short scientific article to answer the questions. There is a link to the PDF article.
- You receive points for the correct solution **and** for the performed steps. Example: You will not get all points for a correct value if the calculations are missing.
- Make sure to **clearly** mark your final solution values (e.g. underlining, red color, box).
- You can reach up to 36 points in total. You qualify for the final round if you reach at least 18 points (junior, under 18 years) or 24 points (youth, over 18 years).
- It is not allowed to work in groups on the problems. Help from teachers, friends, family, or the internet is prohibited. Cheating will result in disqualification! (Textbooks and calculators are allowed.)

Uploading Your Solution

- Please upload a file/pictures of (this sheet with) your written solutions: https://iaac.space/login
- Only upload **one single PDF file!** If you have multiple pictures, please compress them into one single file. Do not upload your pictures in a different format (e.g. no Word and Zip files).
- The deadline for uploading your solution is **Sunday 26. June 2022, 23:59 UTC+0**.
- The results of the pre-final round will be announced on Monday 4. July 2022.

Good luck!

Problem A.1: Looking back with the JWST (4 Points)

The James Webb Space Telescope (JWST) will allow us to look back in time and observe the early universe. You are a scientist trying to observe an object that emitted its light a long time ago.

(a) Explain why the light you receive from the object is *red-shifted*.

The object has a redshift of 7.6 and the JWST observes the object at a wavelength of 2 micrometres (mid-infrared light).

(b) What is the wavelength of the light emitted by the object?
(c) What type of radiation was originally emitted by the object?

Answers:-

(a)
- Our universe is constantly expanding at an ever-increasing rate, ever since the Big-Bang. (space-time itself stretches out)
- Since our object emitted its light a long time ago, the space between us and the object must have stretched too, in the time it took for the light to reach us.
- Thus, the light emitted also got stretched with the space, increasing its wavelength. It also became more 'redder', due to this.
- Since red light has the longest wavelength in the visible spectrum, we say that the light received is thus, red-shifted.

(b) Expression for redshift (z) : $\boxed{1+z = \frac{\lambda_{observed}}{\lambda_{emitted}}}$

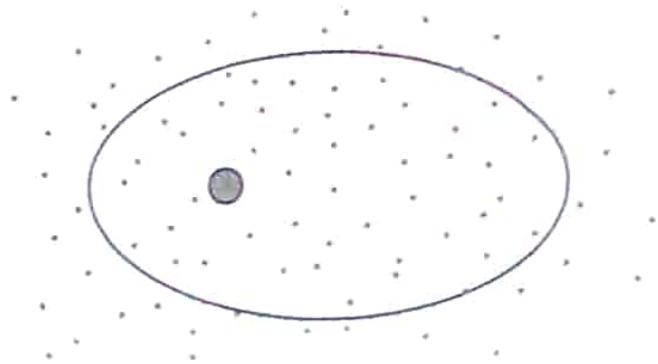
$$\therefore \lambda_{emitted} = \frac{2\times10^{-6}\,m}{1+7.6} = \frac{2}{8.6}\times10^{-6}\,m = 2.326\times10^{-7}\,m$$

$$\boxed{\lambda_{emitted} = 232.6\ nm}$$

(c) From (b), we get that the originally emitted light was $\boxed{\text{Ultra-Violet (UV)}}$ radiation.

Problem A.2: Counting Asteroids (4 Points)

An extraterrestrial civilisation lives on a planet with a very elliptical orbit. Additionally, thousands of large asteroids orbit their solar system. The civilisation uses the light from their home star to count the number of asteroids in the direct line between the star and their planet.



For a first measurement, they count the asteroids for 60 days and detect 1000 objects. Several months later, they start a second measurement: This time, they count for 80 days.

How many asteroids will they detect during the second measurement? Explain why.
(Note: *Assume that the asteroids are homogeneously distributed in their solar system.*)

Solution :- Main principle used: "Kepler's 3rd law of Planetary Motion"

Expression : $T^2 \propto a^3 \Rightarrow \boxed{\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3}$, where 'T' is the time period of orbit, 'a' = length of radius vector (semi-major axis)

(This is valid for all elliptical orbits)

Since distribution of asteroids is homogeneous along the lines joining the planet & home star, we can say, $N \propto a \Rightarrow N = Ka$

$\therefore \boxed{\frac{N_1}{N_2} = \frac{a_1}{a_2}}$ $N_1 = 1000$ (given)
$N_2 = ?$ (to find out)

On substituting this in the first equation, we have $\Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{N_1}{N_2}\right)^3$

$\therefore (N_2)^3 = (N_1^3)\left(\frac{T_2}{T_1}\right)^2 = (1000)^3\left(\frac{80}{60}\right)^2 = 1.778 \times 10^9$

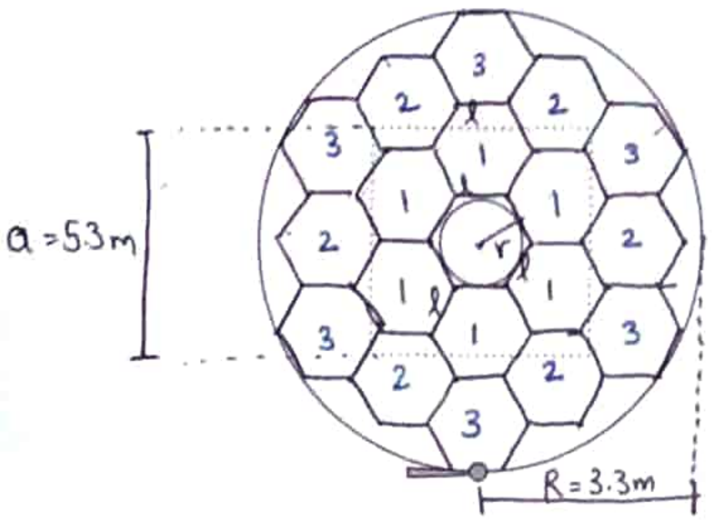
$\therefore N_2 = (1.778 \times 10^9)^{1/3} = 1211.413 \simeq \boxed{1212 \text{ asteroids}}$

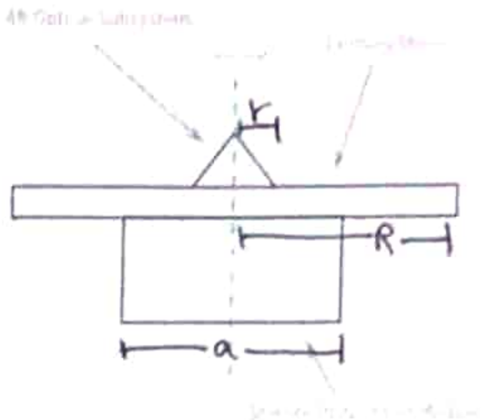
$\therefore$ They will detect approximately 1212 asteroids in 80 days.

Problem B.1: Rotating the JWST (6 Points)

The JWST has a propulsion system to adjust the orbit and orientation of the telescope.

For this problem, we assume that the JWST only consists of the 18 primary mirror segments (with a weight of 40 kg each, m_1) forming a cylinder with a radius of 3.3 m (R), the Aft optical subsystem with a weight of 120 kg (m_2) forming a cone with a radius of 65 cm (r), and the science instrument module with a weight of 1400 kg (m_3) forming a cuboid with a side length of 5.3 m (a):





(a) Derive a general expression for the moment of inertia I of the telescope's shape with respect to the dimensions R, r, a and the masses m_1, m_2, m_3. (Hint: Derive the moment of inertia for the individual components first. The rotational axis is the axis of symmetry.)

(b) Calculate the numerical value of I for the JWST. (Use only the values from the text above.)

To perform calibration measurements, the researchers need to rotate the telescope by 90 degrees. For that, they fire the MRE-1 thrusters at the bottom edge of the primary mirror (see figure) for 0.5 seconds with a thrust of 2.5 newtons.

(c) How long does it take for the telescope to rotate by 90 degrees?

Solution :

(a) To derive a general expression for I_{JWST} →

$$I_{JWST} = \underbrace{I_{mirrors}}_{6(I_1+I_2+I_3)} + \underbrace{I_{\substack{optical \\ subsystem}}}_{cone} + \underbrace{I_{SIM}}_{cube}$$ (all w.r.t rotational axis)

(hexagonal sheets)

(i) To find expression for $I_{mirror} \longrightarrow$

$\boxed{I_{\substack{hexagonal \\ sheet}} = 2.5\, m_1 l^2}$ (m_1 = mass of mirror, l = side length)

[wrt 'z' axis passing through center of hexagon]

→ We need to find and add up the momenta of intertia of all 18 mirror (hexagonal) sheets w.r.t the central, rotational axis.

→ Refer to given figure, the mirrors have been divided into 3 groups of 6 each, and mirrors from a group are equidistant from the central axis. We calculate 'I' for each group separately (using the shift-axis of rotation theorem) and add them up to get "$I_{mirrors}$".

$\therefore I_1$ (Group numbered '1') $= 6 \times \left(2.5\, m_1 l^2 + \underbrace{m_1(\sqrt{3} l)^2}_{\text{due to shift in axis of rotation}}\right)$

$$I_1 = 33\, m_1 l^2$$

$\therefore I_2$ (Group numbered '2') $= 6 \times \left(2.5\, m_1 l^2 + \underbrace{m_1 (3l)^2}_{\text{shift}}\right)$

$$I_2 = 69\, m_1 l^2$$

I_3 (Group numbered '3') $= 6 \times \left(2.5\, m_1 l^2 + \underbrace{m_1 (2\sqrt{3} l)^2}_{\text{shift}}\right)$

$$I_3 = 87\, m_1 l^2$$

* $\sqrt{3} l$, $3l$, $2\sqrt{3} l$ these distances were computed using basic trigonometry in hexagons

$$I_{mirrors} = I_1 + I_2 + I_3 = (33 + 69 + 87) m_1 l^2$$
$$= 189\, m_1 l^2$$

From the figure, we can calculate that $5\sqrt{3} l = 2R \Longrightarrow \boxed{l = \frac{2R}{5\sqrt{3}}}$

Substituting.... $\therefore \boxed{I_{mirrors} = 10.08\, m_1 R^2}$ $\left(\frac{252}{25} = 10.08\right)$

(ii) To find expression for $I_{\substack{optical \\ subsystem}}$ $\longrightarrow$

Since it is a regular cone with rotational axis = symmetry axis, we can use the standard expression :

$$I_{cone} = \frac{3mr^2}{10}$$

$$\therefore \boxed{I_{\substack{optical \\ subsystem}} = \frac{3m_2r^2}{10}}$$

(iii) To find expression for I_{SIM} $\longrightarrow$

It is a cube, with rotational axis = symmetry axis.

$\therefore$ standard expression : $I_{cube} = \dfrac{ma^2}{6}$

$$\therefore \boxed{I_{SIM} = \frac{m_3a^2}{6}}$$

(iv) To add these up and find expression for I_{JWST} $\longrightarrow$

$$\therefore I_{JWST} = I_{mirrors} + I_{\substack{optical \\ subsystem}} + I_{SIM}$$

$$\boxed{I_{JWST} = \frac{252}{25}m_1R^2 + \frac{3}{10}m_2r^2 + \frac{1}{6}m_3a^2}$$

(b) Calculating numerical value of I_{JWST} $\longrightarrow$

$\therefore$ $m_1 = 40\,kg$ $R = 3.3\,m$

$m_2 = 120\,kg$ $r = 0.65\,m$

$m_3 = 1400\,kg$ $a = 5.3\,m$

$$\Rightarrow I_{JWST} = \frac{252}{25}(40)(3.3)^2 + \frac{3}{10}(120)(0.65)^2 + \frac{1}{6}(1400)(5.3)^2$$

$$= 4390.848 + 15.210 + 6554.333$$

$$= 10960.391$$

$$I_{JWST} \approx 10960.4\ kgm^2 \Rightarrow \boxed{I_{JWST} = 10960.4\ kgm^2}$$

(c) To calculate rotation time $\longrightarrow$

Thrust force = 2.5 N   Thrust-time = 0.5 seconds

$$\theta_{total} = 90^\circ = \frac{\pi}{2} \text{ radians} = \underbrace{\theta_1}_{\substack{\text{during} \\ \text{thrust} \\ (\alpha > 0)}} + \underbrace{\theta_2}_{\substack{\text{after} \\ \text{thrust} \\ (\alpha = 0, \text{ constant } \omega)}}$$

$$\therefore \theta_1 = \underbrace{\omega_0}_{0}(t) + \frac{1}{2}\alpha(t)^2$$

$$\theta_1 = \frac{1}{2}(\alpha)(0.5)^2 \Longrightarrow \boxed{I \cdot \alpha = \tau \quad \therefore \alpha = \frac{\tau}{I} = \frac{F \times R}{I} = \frac{2.5 \times 3.3}{10960.4}}$$

$$\Rightarrow \theta_1 = \frac{1}{2}\left(\frac{2.5 \times 3.3}{10960.4}\right)(0.5)^2$$

$$\theta_1 \approx 9.4 \times 10^{-5} \text{ radians} \Rightarrow \boxed{\theta_1 \text{ is negligible w.r.t } \theta_{total} = \pi/2}$$

$$\therefore \theta_{total} \approx \theta_2$$

$$\therefore \boxed{\theta_2 = \frac{\pi}{2}}$$

$$\therefore \theta_2 = \omega_{attained} \cdot t, \quad \text{but } \omega_{attained} = ?$$

$$\therefore \omega_{attained} = \omega_0 + \alpha \cdot t_{thrust}$$

$$= 0 + \frac{2.5 \times 3.3}{10960.4} \times 0.5$$

$$\boxed{\omega_{attained} = \frac{2.5 \times 3.3 \times 0.5}{10960.4}}$$

$$\therefore \theta_2 = \frac{\pi}{2} = \omega_{attained} \cdot t$$

$$\Longrightarrow t = \frac{\theta_2}{\omega_{attained}} = \frac{\pi}{2} \times \frac{10960.4}{2.5 \times 3.3 \times 0.5}$$

$$t = 4173.710 \text{ sec}$$

$$t = 1.159 \text{ hours}$$

$$t \approx 1 \text{ hour } 9 \text{ min } 30 \text{ sec}$$

$$t \approx 69 \text{ min } 30 \text{ sec.}$$

$\therefore$ $\boxed{\text{Time for rotation of JWST by } 90^\circ = 69.5 \text{ min}}$

Problem B.2: Changing Temperature (6 Points)

The energy of our Sun is responsible for life on Earth. We are very lucky that the Sun has the right conditions and that the Earth is at the exact right position to create habitable temperatures.

(a) Find an equation for the surface temperature of the Earth $T_E(R, T)$ with respect to the radius R and the surface temperature T of the Sun.
(Note: *Approach the Earth and the Sun as black bodies; then, account for the Earth's albedo of 30% and add an atmosphere correction factor of 1.13 to the surface temperature of the Earth.)*

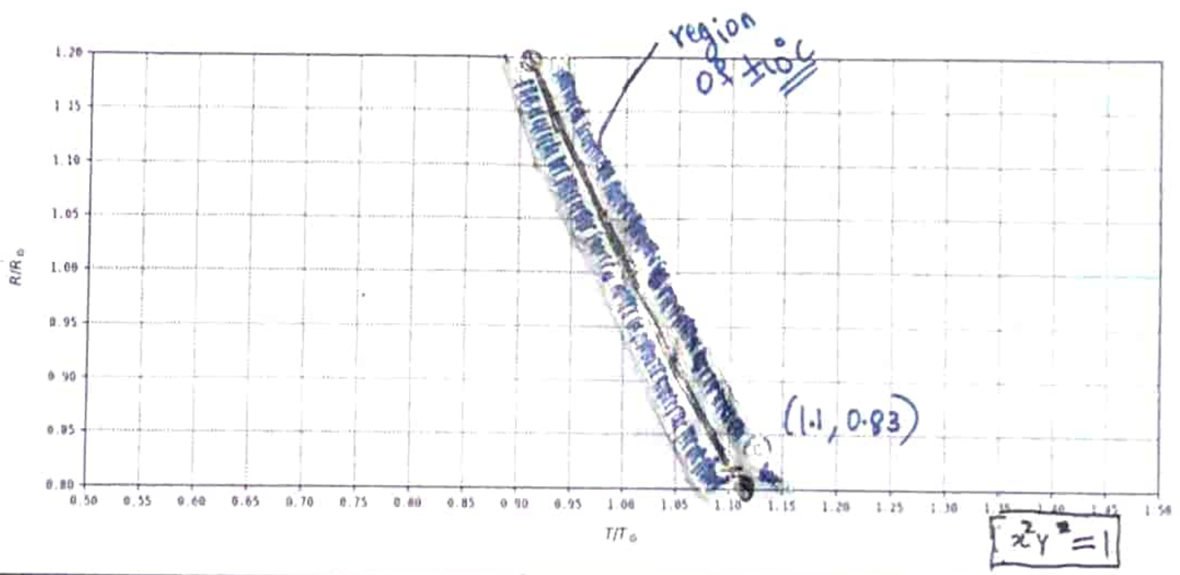
The radius of the Sun is 696 x 10^3 km, and the surface temperature is 5772 K:

(b) Confirm with your equation that Earth's current surface temperature is 15 °C.

The two axes of the diagram below display a relative change in the surface temperature (x-axis) and radius (y-axis) of the Sun.

(c) Draw a black line in the diagram for all pairs (R, T) that still result in a temperature of 15 °C on the Earth. If the Sun's temperature increases by 10%, how much needs the radius to decrease to maintain 15 °C on Earth? $r_e' = 0.83\, r_e$

(d) Draw a grey area in the diagram for all (R, T) that result in a temperature $\pm$ 10° from 15 °C.



Solution :-

(a) Stefan-Boltzmann equation for energy emitted by black body →

$$\boxed{U = \sigma A T^4}$$

σ = Stefan-Boltzmann constant
A = Area of body   T = Temperature of body

$\therefore$ U_{sun} (emitted) $= \sigma A_{sun} T_{sun}^4$

U_{sun} at a point on a sphere with radius = 1AU $\Rightarrow$ $= \dfrac{\sigma \cdot A_{sun} T_{sun}^4}{4\pi (1AU)^2}$

$\Rightarrow$ $U_{\substack{\text{incident}\\ \text{on Earth}}} = \dfrac{\sigma A_{sun} T_{sun}^4}{4\pi (1AU)^2} \times \underbrace{\pi r_{earth}^2}_{\text{cross sectional area exposed}}$

$\Rightarrow$ $U_{\substack{\text{absorbed}\\ \text{by earth}}} = \underbrace{\left(1 - \frac{3}{10}\right)}_{\text{Albedo}} \left(\dfrac{\sigma A_s T_s^4}{4\pi (AU)^2} \times \pi r_e^2\right)$

$= \dfrac{7 \sigma A_s T_s^4 \cdot \cancel{\pi} r_e^2}{10 \cdot 4\cancel{\pi} (1AU)^2} = \dfrac{7\sigma A_s T_s^4 r_e^2}{40 (1AU)^2}$

Now, $U_{earth} = \sigma A_e T_e^4$

Atmospheric correction factor $\longrightarrow \dfrac{T_e}{1.13}$ $\therefore U_{earth} = \sigma A_e \left(\dfrac{T_e}{1.13}\right)^4$

Condition for black body $\longrightarrow$ $\boxed{U_{absorbed} = U_{emitted}}$

$\therefore$ $\dfrac{7\cancel{\sigma} (4\cancel{\pi} R_s^2) T_s^4 \cancel{r_e^2}}{40 (1AU)^2} = \dfrac{\cancel{\sigma} (\cancel{4\pi r_e^2}) (T_e)^4}{(1.13)^4}$

$\Rightarrow$ $\dfrac{7 T_s^4 R_s^2}{40 (AU)^2} = \left(\dfrac{T_e}{1.13}\right)^4 \Longrightarrow T_e^4 = \dfrac{7 (1.13 T_s)^4 R_s^2}{40 (AU)^2}$

$\therefore$ T_{earth} (in K) $= 1.13\, T_{sun} \left(\dfrac{7 R_{sun}^2}{40 (1AU)^2}\right)^{\frac{1}{4}}$ $\therefore$ $\boxed{T_e = 1.13 \times \left(\dfrac{7 R^2}{40 (1AU)^2}\right)^{\frac{1}{4}} \times T}$

(b) Verifying the surface temperature of Earth →

$$\therefore \boxed{T_e(R,T) = 1.13T\left(\frac{7R^2}{40(1AU)^2}\right)^{\frac{1}{4}}} \quad \text{(part (a))}$$

$$\therefore T_e(696\times10^3 \text{ km}, 5772\text{K}) = 1.13\times5772\times\left(\frac{7\times(696\times10^6)^2}{40(1.496\times10^{11}\text{m})^2}\right)^{\frac{1}{4}}$$

$$T_E = 288.07 \text{ K}$$

$$T_E \approx 14.92^\circ C \approx 15^\circ C$$

$\therefore$ We can verify that $\boxed{T_E \approx 15^\circ C}$

(c) Graph equation →

$\frac{R}{R_s}$ vs $\frac{T}{T_s}$ graph : Since $T_e = 15^\circ c = 288.15$ K.....

$$\therefore 288.15 = 1.13\,T\left(\frac{7R^2}{40(1AU)^2}\right)^{\frac{1}{4}}$$

$$\therefore (288.15)^4 = (1.13)^4 T^4\left(\frac{7R^2}{40(1AU)^2}\right)$$

$$\Rightarrow R^2\left(\frac{7(1.13)^4}{(288.15)^4\,40(1AU)^2}\right) = \frac{1}{T^4}$$

$$\Rightarrow R^2(1.849\times10^{-33}) = \frac{1}{T^4}$$

To get in terms of $\frac{R}{R_s}$ & $\frac{T}{T_s}$....

$$\Rightarrow \left(\frac{R}{R_s}\right)^2\left(1.849\times10^{-33}\times R_s^2\right)\times T_s^4 = \frac{T_s^4}{T^4}$$

$$\Rightarrow \left(\frac{R}{R_s}\right)^2(0.99434) = \frac{1}{(T/T_s)^4} \Rightarrow y^2(0.99434) = \frac{1}{x^4}$$

$$\Rightarrow \boxed{x^4y^2 = 1.0057}$$

$$\Rightarrow x^2y = 1.0028$$

$$\boxed{x^2y \approx 1}$$

* T_s increases by 10%. →
at 1.1 T, we can see that 0.83 R falls on the line.

$\therefore \boxed{re' = 0.83 re}$

* $\therefore$ If the sun's temp increases by 10%, its radius needs to decrease to 0.83 times its current value, to maintain 15°C on Earth.

Problem C.1 : The Surface of Planets (8 Points)

This problem requires you to read the following recently published scientific article:

Inferring Shallow Surfaces on Sub-Neptune Exoplanets with JWST.

Shang-Min Tsai et al 2021 ApJL 922 L27. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ac399a/pdf

Answer the following questions related to this article:

(a) What is a *proxy*? What proxy is this study trying to find, and what are they doing differently compared to previous studies?

Refer page 8a/9

(b) Explain the meaning and use of the following acronyms: HELIOS, Exo-FMS, HAZMAT, NIRSpec.

Refer page 8a/9

(c) Make a sketch of the components used to model the planet (including the pressure-longitude grid and the equatorial regions):

Refer page 8b/9

(d) Explain the components of Figure 1. Why was it included in the paper?

Refer page 8b/9

(e) Why is CH_4 not a suitable proxy for the surface pressure?

Refer page 8b/9

(f) You detect CH_3OH but non NH_3 in the atmosphere of a sub-Neptune planet. What type of surface does this planet have?

Refer page 8b/9

Problem C1: The surface of Planets

(a) • A proxy is an indicator, a measurable pseudo-property of some other non-measurable property of an object.
• The presence/absence of a proxy detection tells us about the presence/absence of that non-observable characteristic, indirectly, as a a means of inference.
• This study is trying to find a proxy for detecting shallow surfaces on sub-Neptune exoplanets.
• while previous studies used only 1D models & neglected day-night interactions, this study uses 2D models, considers day-night changes & reevalutates the use of atmospheric chemistry as a viable proxy for detecting shallow surfaces.

(b) i) HELIOS:
It is an open-source radiative transfer code, written and used for studying exoplanetary atmospheres.
ii) Exo-FMS:
• FMS (Flexible Modelling System)
• Exo-FMS is a code-system to model exoplanets and their atmospheres (especially RTs & GCMs)
iii) HAZMAT:
• HAbitable Zones & M dwarf Activity across Time (HAZMAT), was a prior study which showed the far- & near-UV emission from M stars at various stages of a stellar lifetime.
iv) NIRSpec:
• The Near Infrared Spectograph (NIRSpec) enables scientists to obtain simultaneous spectra of more than 100 objects in a 9-square-arcmin field of view.

(c) VULCAN = 2D photochemical model for atmospheric chemistry

(d)
- Figure 1 is a pressure-temperature (P-T) graph of the specimen K2-18b, with different surface pressure levels.
- It was included in the paper to prove the fact that once the atmosphere is sufficiently opaque (albedo 0.1-0.3), the presence of a shallow surface has no real impact on the P-T profile & we can safely truncate the surface pressures to be 1 bar.

(e)
- CH_4 is not a suitable proxy for the surface pressure because →

i) It's unstable, converts to CO & CO_2 over time
ii) It also photodissociates rapidly
iii) It is still evolving constantly after millions of years with a quiet M star, generating ambiguity
iv) Also produces methanol (CH_3OH) as a by-product of dissociation.

(f)
- A positive detection of CH_3OH with a negative detection of NH_3 indicates that this sub-Neptune planet has a $\boxed{\text{shallow \& dry surface}}$

Problem C.2 : Black Holes and the JWST (8 Points)

This problem requires you to read the following recently published scientific article:

The Age of Discovery with the James Webb:
Excavating the Spectral Signatures of the First Massive Black Holes.
Inayoshi, K. et al. arXiv:2204.09692 [astro-ph.GA] (2022). Link: https://arxiv.org/pdf/2204.09692.pdf

Answer the following questions related to this article:

(a) What are massive black holes (BH)? Why is the observation of young massive BHs important?

Refer page 9a/9

(b) What is the *spectral energy distribution (SED)*?

Refer page 9a/9

(c) Figure 2 shows the total SED with three OI peaks: Where do they come from?

Refer page 9a/9

(d) What are broad-band filters, and what is their use in astronomy?

Refer page 9a/9

(e) Explain the increase of all lines for high z in Figure 3, top-left panel.

Refer page 9b/9

(f) Explain the meaning and use of the magenta rectangle in Figure 4.

Refer page 9b/9

Problem C2: Black Holes and the JWST

Answers: –

(a)
- Massive black holes (BHs) are those who masses are greater than $10^8 M_\odot$ (solar masses).
- The massive BHs formed when the universe was very young are of particular interest & importance, since that fact constrains their origin & formation pathway.
- Their study also helps in building BH seeding & growth models, to help us understand BH-life cycle better.

(b)
- The spectral energy distribution (SED) is a graph of the energy emitted by the object, as a function of varying wavelengths.
- The SED plotted in this study includes 3 components →
  - i) Radiation flux from unresolved disk of BH
  - ii) Nebular emission lines from irradiated gas parcels
  - iii) Radiation from dense accreting disk in RHD simulation

(c)
- The three O_I peaks in the SED come from the Lyβ fluorescence which happens when a population in n=3 of hydrogen is built up by collisional excitation & thus tightly correlates to enhancement of Balmer lines.

(d)
- Broad-band filters are those which only allow Hα, Hβ and O_{III} spectral lines to pass through & block all other wavelengths.
- They are used to observe the night sky, as they get rid of light pollution, natural skyglow, sodium and mercury vapour light, etc.
- Thus, as they provide a good S/N ratio, they are important in astronomy.

9a/9

(e) • Figure 3, top-left panel's image shows the colors for $z \sim 8$, in the broad-band filter F200W − F356W.

• This is chosen such that the continuum flux is red in the image.

• In this filter, all 3 lines increase for a high 'z' value due to the entrance of the prominent $H\alpha$ emission into the high-λ filters.

(f) • The magenta rectangles in Figure 4 indicate the colour-cut conditions for the color-selection of the seed BHs in 2 redshift ranges.

• These colour-cuts are →

$$z \sim 8 : \begin{cases} F200W - F356W > 0, \\ F356W - F560W > 0.8, \end{cases}$$

and

$$z \sim 10 : \begin{cases} F277W - F444W > 0, \\ F444W - F770W > 0.6 \end{cases}$$