Experiment 5: Electromagnetic Damping
Lab Report

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1 Introduction

This experiment lies perfectly at the intersection of two major branches of physics; classical mechanics and electromagnetism. The flywheel rotates due to the torque experienced by it, which is in turn generated by the pulling tension of the mass falling downwards. This is a purely mechanical problem (rotational dynamics). However, interesting observations are obtained when the flywheel is a conducting router and magnets are introduced near the rim of the flywheel while the mass is falling. The magnetic field induces eddy currents inside the flywheel, which eventually leads to a damping force. This damping force balances out the gravitational acceleration, making the object fall with a certain terminal velocity. This experiment attempts to delve a bit deeper into deriving these relations mathematically and then verifying them experimentally.

2 Aim

- **Part A:** To determine the moment of inertia and frictional torque of the flywheel with no electromagnetic damping, and verify the relation between acceleration of falling object (a) and moment of inertia (I) of flywheel.
- **Part B:** To introduce electromagnetic damping on the flywheel, find the terminal velocities (v_t) for different masses (keeping magnetic field (B) constant) and verify the relation between v_t and mass.
- **Part C:** To find terminal velocities (v_t) for a fixed value of mass and changing separation distances between the magnets (thus, changing value of B).

3 Apparatus

1. A flywheel disc mounted on a horizontal axle (to drop the masses from)
2. Two strong magnets (to introduce EM damping into the system)
3. Two stands for magnets with attached screw gauges (to measure the change in position of magnets for Part C)

4. Light, inextensible cord (to tie the masses to the flywheel)
5. Slotted masses (7 pieces, 50 gm each)
6. Small 50 gm mass with a hook (to attach slotted masses)
7. DSLR camera (to record videos of falling objects)
8. Tripod stand for DSLR camera (to ensure steady, good quality videos)
9. Gauss Meter and Hall Probe (to measure magnetic field strength (B) with varying separation distances between magnets)
10. Brightly colored stickers (to attach to falling masses and track them easily)
11. Metre tape (to attach stickers to, as a calibration stick reference for Tracker)
12. Vernier Caliper (to measure diameter of chosen axle)
13. Tracking software - Tracker (to extract data from recorded videos)

4 Theory

4.1 The Flywheel: Undamped Mechanics

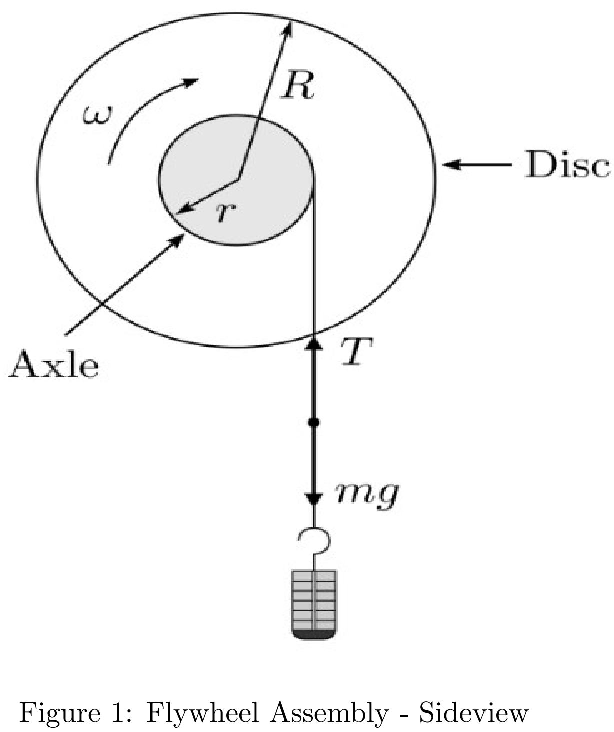


Figure 1: Flywheel Assembly - Sideview

The flywheel is an aluminium disk mounted on a horizontal axle, as shown in the figure above. The cord winds around the axle and is attached to a slotted mass (with a hook). When this slotted mass is allowed to fall, the aluminium disc rotates with it. The flywheel experiences two major forces while rotating with the falling mass; Friction and Tension. Both these forces act at a certain distance away from the center of the flywheel, thus associating them with their respective torques on the flywheel. The Tension force (T) on the flywheel acts at distance 'r' (=radius of chosen axle), while the torque due to friction acts all throughout the radius. It has a negative sign since it always acts in the direction opposite to the rotation. Thus, we have the following expression for net torque τ_{net} on the flywheel -

$$\begin{aligned}\tau_{\text{net}} &= \tau_T + (-\tau_f) \\ &= Tr + \tau_f\end{aligned}$$

But the generalization of Newton's second law of motion for rotational motion gives us another expression for the net torque on the flywheel -

$$\tau_{\text{net}} = I\alpha$$

where α is the **angular acceleration** and I is the **moment of inertia** of the flywheel. On equating the two expressions for net torque on the flywheel, we get -

$$I\alpha = Tr - \tau_f \tag{1}$$

To get rid of the angular acceleration 'α', we write it in terms of the linear acceleration of the falling mass 'a' and the radius of the chosen axle, as -

$$\alpha = \frac{a}{r} \tag{2}$$

To find an expression for Tension 'T' in the cord, we look at the forces experienced by the falling mass. The falling mass 'm' experiences two forces; downward gravitational attraction (mg) and upward tension (T) due to cord. Thus, its equation of motion (from Newton's 2nd law of motion) gives us the expression for tension (T) in the cord -

$$ma = mg - T \tag{3}$$

$$\implies T = m(g - a) \tag{4}$$

Now, we substitute equations (2) and (4) in equation (1) and simplify to get -

$$I\left(\frac{a}{r}\right) = mr(g - a) - \tau_f$$

Now we solve for the acceleration 'a' of the falling mass -

$$\begin{aligned}I\left(\frac{a}{r}\right) &= mr(g - a) - \tau_f \\ \implies Ia &= mr^2(g - a) - r\tau_f \\ \implies Ia &= mgr^2 - mar^2 - r\tau_f \\ \implies a(I + mr^2) &= mgr^2 - r\tau_f \\ \implies a &= g\left(\frac{mr^2}{I + mr^2}\right) - \left(\frac{r\tau_f}{I + mr^2}\right)\end{aligned}$$

Thus, we have our final expression relating 'a' and 'g' as follows -

$$\boxed{a = g\left(\frac{mr^2}{I + mr^2}\right) - \left(\frac{\tau_f r}{I + mr^2}\right)} \tag{5}$$

Realistically, the value of the falling masses 'm' and the radius of the chosen axle 'r' is very small, in comparison with the moment of inertia 'I' of the flywheel. Thus, we can approximate the denominator term in both the fractions on the right hand side of Equation (5), to 'I'. Then, we have -

$$a = g\left(\frac{mr^2}{I}\right) - \left(\frac{\tau_f r}{I}\right) \tag{6}$$

This equation will be used in plotting graphs and calculating the value of frictional torque and moment of inertia of the flywheel.

4.2 Magnets: Damped Mechanics

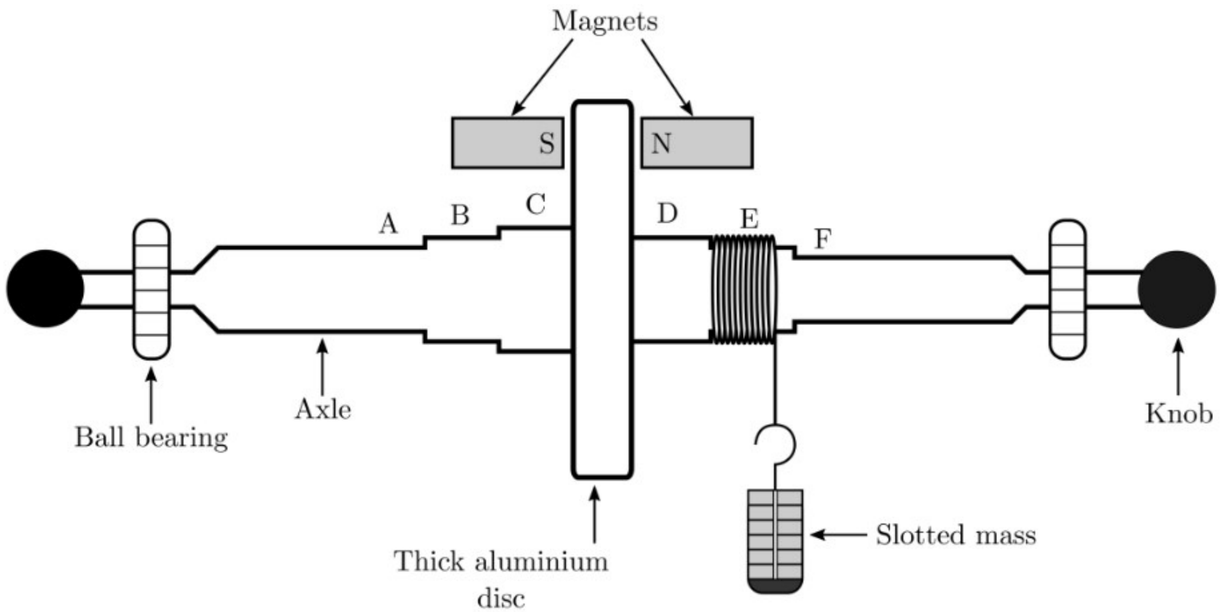


Figure 2: Flywheel assembly with magnets for EM damping

As shown in the figure above, we now introduce two magnets near the flywheel to see its effects on the falling mass. Before we look at a mathematical approach to calculate the damping, in this section we will try to justify qualitatively why there should be any resultant damping force on the flywheel, in this arrangement.

Since the aluminium flywheel is a conductor, it has many free electrons. When an electron passes through a magnetic field (no electric field, thus we are not including the electric field term in this expressions currently), it experiences a **Lorentz force**, given by the relation -

$$\mathbf{F} = e\mathbf{v} \times \mathbf{B}$$

Now we consider the effects of this Lorentz force on the free electrons of our aluminium flywheel, in the above set up. The magnetic field between the two magnets, is perpendicular to the plane of the flywheel. The initial velocity of the electrons is in the **azimuthal direction** (tangential to the flywheel). By using the right hand rule for cross products (as demonstrated by the image below), we can see that the lorentz force on the electrons will be along the **radial** direction (either towards the center or the rim of the flywheel).

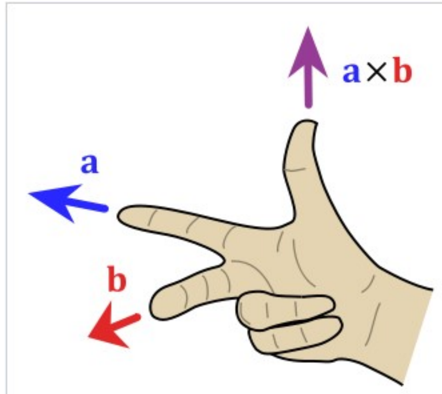


Figure 3: Right hand rule for cross products - Credit: Wikipedia

Thus, the lorentz force will cause the electrons to move radially outward, or inward. This separation of charges inside the flywheel will create a polarizing electric field, which will grow until it balances out the magnetic field's effect on the electrons. However, since our conductor (flywheel) extends outwards from the immediate vicinity of the of magnets, the electrons return to their original positions through a longer route (circuitous). This will lead to the creation of small, spread out **eddy currents** inside the flywheel.

These eddy currents will vary greatly over their magnitude and direction in various sections of the flywheel. However, they can be averaged out and approximated to be along the radial direction (since the deviations are very tiny). Since the eddy currents flow in the radial direction, by applying the right hand rule (Figure 3) on the electrons again, we get that there will be a Lorentz force on them, now in the azimuthal direction (since it needs to be perpendicular to both, the magnetic field and the eddy currents). From the directions of eddy current and magnetic field, we thus find that the lorentz force acts in a direction opposite to the flywheel's rotation.

We can determine the direction of this lorentz force without using the right hand rule too. There are two azimuthal directions, one along the direction of rotation of the flywheel, while one opposite to that. If the lorentz force acted along the direction of rotation, it will add energy to the system and make it go faster, thus violating energy conservation. Thus, the lorentz force must act such that it removes energy from the system i.e. like a damping force in the direction opposite of the flywheel's rotation.

Hence, the final expression for torque on the flywheel can be written as follows -

$$I\alpha = Tr - \tau_f - \tau_B$$

where τ_B is the damping lorentz force.

4.3 Calculating Magnetic Torque

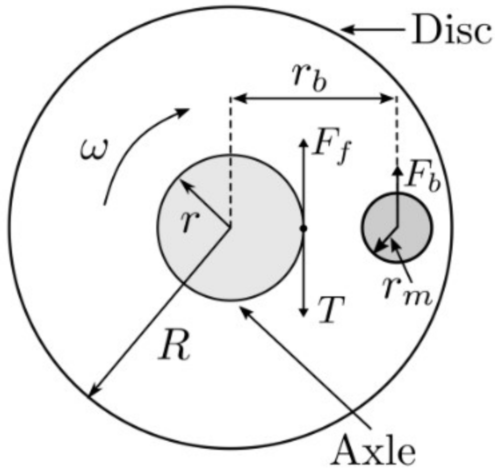


Figure 4: Side vide of axle to calculate magnetic torque

Having outline a qualitative approach to justifying the damping nature of the lorentz force in the previous section, we attempt to derive the result mathematically in this section. We use the approximate directions of eddy currents to do this calculation.
As shown in Figure 4, consider a small circle of radius r_m on the flywheel. This is where the magnetic field passes through the flywheel. All other distances are as specified in the diagram, while ω is the angular velocity of the flywheel. The velocity of the small circular portion is $v = r_b\omega$. The electromotive force ϵ_b induced by the magnetic field in the conducting flywheel is given by the expression -

$$\begin{aligned}\epsilon_b &= v \times B \times l \\ &= (r_b\omega) \times (B) \times (2r_m)\end{aligned}$$

The **eddy current** i can be obtained by dividing the above expression for ϵ_b, by the resistance (say, R^*) of the small circular part of the flywheel -

$$\begin{aligned}i &= \frac{\epsilon_b}{R^*} \\ &= \frac{(r_b\omega) \times (B) \times (2r_m)}{R^*}\end{aligned}$$

The **lorentz force** experienced by the currents is given by -

$$\begin{aligned}F_b &= i \times B \times l \\ &= \frac{4r_m^2\omega B^2 r_b}{R^*} \\ &= \left(\frac{4r_m^2 r_b}{R^*}\right)\omega B^2\end{aligned}$$

This forces creates our **damping torque** τ_B on the chosen disk, at a distance r_b. It is given by the expression -

$$\begin{aligned}\tau_B &= F_b r_b \\ &= \left(\frac{4r_m^2 r_b^2}{R^*}\right)\omega B^2 \\ &= C_1\omega B^2\end{aligned}$$

However, like mentioned earlier, we would need to substitute an averaging factor C instead of C_1 in the above expression. Thus, our final expression for the net torque on the flywheel as follows -

$$I\alpha = m(g-a)r - \tau_f - C\omega B^2$$

When $\alpha = 0$ (no angular acceleration), the downward linear acceleration 'a' of the mass is also 0, while the angular velocity of the flywheel 'ω' will reach a constant value 'ω_t' -

$$\begin{aligned}0 &= mgr - \tau_f - C\omega_t B^2 \\ \implies \omega_t &= \frac{mgr - \tau_f}{CB^2} \\ \implies \omega_t r &= \frac{mgr^2 - r\tau_f}{CB^2} \\ \implies v_t &= \frac{gr^2}{CB^2}\left(m - \frac{\tau_f}{gr}\right) \qquad (\because \omega_t r = v_t)\end{aligned}$$

Thus, we get our final expression for the terminal velocity of the flywheel as follows -

$$\boxed{v_t = \frac{gr^2}{CB^2}\left(m - \frac{\tau_f}{gr}\right)} \tag{7}$$

5 Object Tracking Procedure

In this experiment, we use a video tracking software named **"Tracker"** to track the positions and velocities of falling objects. It is a very important software, central to obtaining and analysing the data for this experiment. Thus, in this section of the report, I outline my procedure for loading recorded videos of falling objects onto Tracker and extracting .csv data files of y-position, time, y-velocity, etc. from them. Note that while this procedure is mostly applicable for all tracking using this software, there may be some degree of customization in the procedure for this particular experiment -

1. Open Tracker and go to "Video" section near the top left corner. Click on "Import", and import the video to be tracked, onto Tracker.

2. If necessary, use the "Rotate" filter on the video and increase brightness to achieve a suitable configuration.

3. Click on the calibration icon on the toolbar and select "calibration stick". Align the ends of this stick with the reference stickers in the video frame, and enter in the value of the actual distance between these stickers (0.5 m in this experiment).

4. Move through the start and end frames of the video to decide upon a suitable starting and ending reference frame for tracking. In this experiment, ensure that the tracking stops before the mass starts going up after hitting the bottom point. Fix this start and end point.

5. Click on "Track" in the toolbar, and select a new "Point Mass". Go back to the starting frame, and zoom in to the mass to be tracked.

6. Press "Ctrl+Shift" and click at a suitable place on the mass to assign a template image for tracking the mass (preferably on the sticker). The software will keep on tracking the closest possible matches to this template image throughout the video, and extract position and time data values based on the calibration distance entered earlier.

7. Click on "Search" to begin the tracking. Ensure that all columns of data (like y-velocity, x-velocity, etc.) are selected for tracking.

8. In case of errors in tracking, click on "Accept" or "Skip" for a few times. If the error persists, delete the point mass. Assign a new template image to the tracker and try again.

9. Once the tracker goes through all the frames to analyze, click on "Export" data file. Ensure that the data file name ends with a .csv extension, all cells are exported and the delimiter is a "comma".

10. After exporting the data file, open it and put a "#" symbol before all column names. This is necessary for NumPy's `np.loadtxt()` function to correctly load the data onto python arrays during analysis. Remember to save the .csv file before closing it.

11. We now have the requisite data to plot and complete our experiment. Load it onto Python and plot the necessary quantities against each other.

6 Part A

In this part, we record videos and track falling masses without any electromagnetic damping.

6.1 Procedure

1. Choose an appropriate axle and record its diameter using the vernier caliper.

2. Make a loop at the end of the cord (to hang masses) and attach the other end to the chosen axle.

3. Stick several bright stickers to every single slotted mass and the hooked mass after recording their respective weights. This will help us in tracking the weights easily.

4. Hang the assembly of slotted masses onto the cord using the loop made, and wind up the cord around the chosen axle. Ensure that is stays on the same axle, and that it is wound tightly with no overlapping.

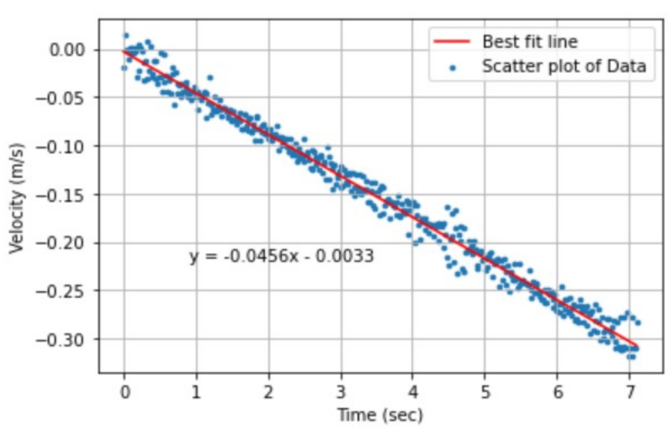
5. Tape the meter ruler to a vertical stand or a wall near the assembly. Put bright stickers 50 cm apart on this ruler. This will help us in the calibration of distances in our tracking software.

6. Setup the camera on a tripod stand, with the vertical setting. Ensure that the flywheel is at the top of the frame, while the length of the maximum stretched cord does not go below the frame.

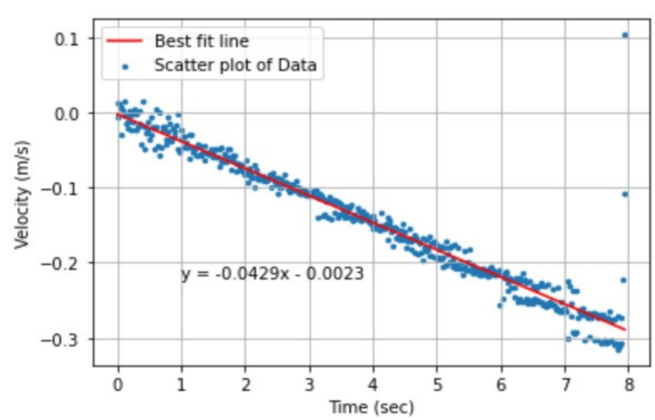
7. Ensure the 50 cm distance calibration stickers are also clearly visible in the frame. Use a spirit level to fix the levelling of the camera if necessary.
8. Start with the maximum mass. Ensure that it does not wobble, and hold it just below the flywheel. Start the video recording on the camera, and gently release the mass.
9. Once the mass reaches the bottom, stop the recording. In cases when the recorded video includes the masses rolling back up after hitting the lowest point, remember to shed that part of data.
10. Wind the cord tightly around the same axle again, and remove one of the slotted masses. Let the remaining masses fall again, and record a video.
11. Repeat this procedure for all masses, and record the videos on the DSLR camera. Export them onto the computer and name them correctly.
12. Using the procedure outlined in the previous section, track each video using Tracker. Export the data in a csv file.
13. Plot the y-velocity with time for each data set, find the line of best fit (using `np.polyfit()`). The slope of this line gives us the downward acceleration of that mass.
14. Collect acceleration values for all masses, and plot them against the respective masses to verify the relation between acceleration (a) of mass and moment of inertia (I) of flywheel.

6.2 Observations

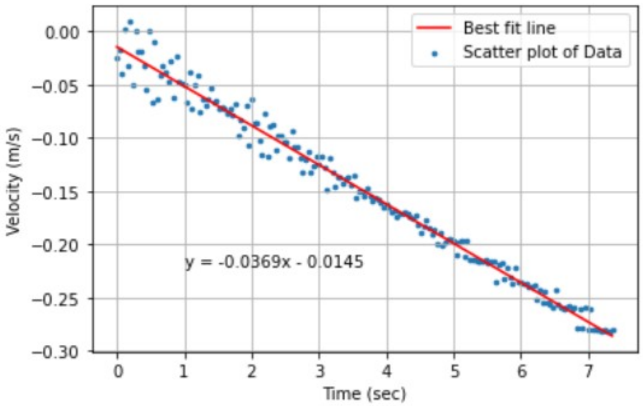
- Diameter of chosen axle (using vernier caliper) = M.S.R + V.S.R $\times$ L.C = 17 + 0 $\times$ 0.02 = 17 mm
- Thus, radius of axle: $r = d/2 = 17/2 = 8.5$mm
- The following velocity-time graphs for 6 different masses give us the values of acceleration, which we then plot against the respective values of masses to calculate the moment of inertia (I) of the flywheel and the frictional torque (τ_f) experienced by it -



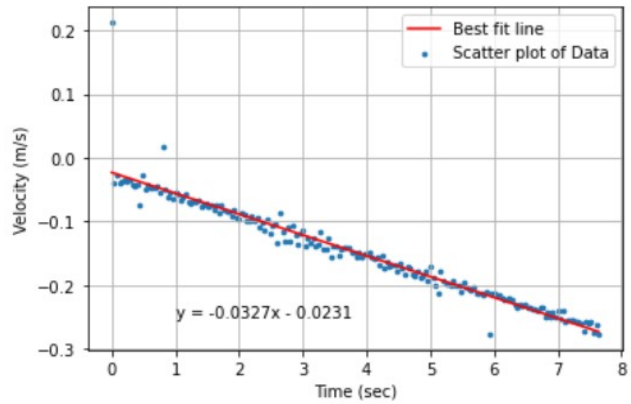
(a) v vs t for 401.8 gm mass; acceleration = $0.0456 m/s^2$



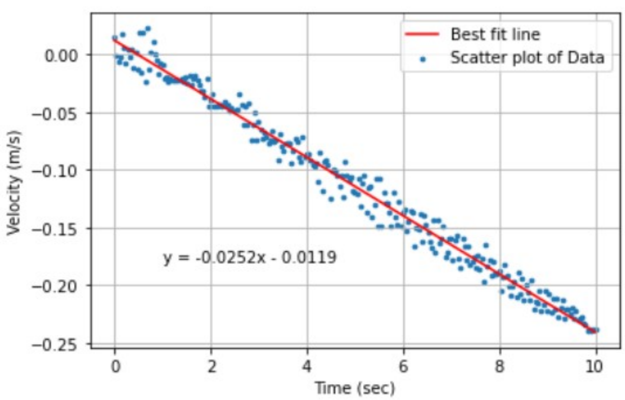
(b) v vs t for 351.4 gm mass; acceleration = $0.0429 m/s^2$



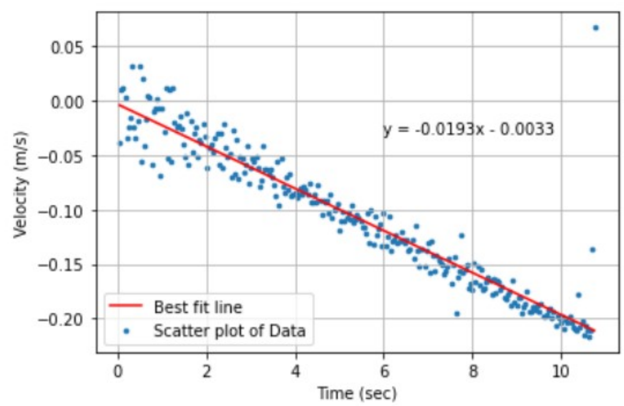
(c) v vs t for 301.1 gm mass; acceleration $= 0.0369 m/s^2$



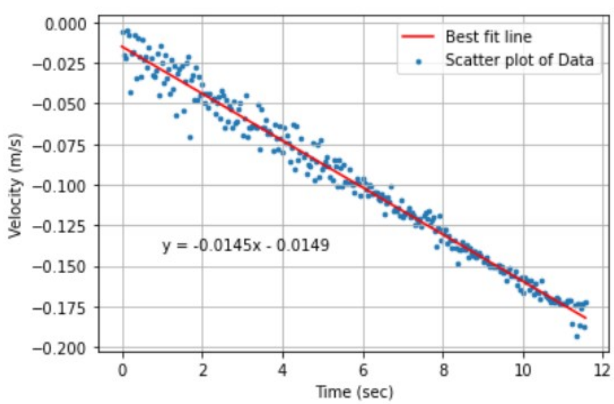
(d) v vs t for 250.6 gm mass; acceleration $= 0.0327 m/s^2$



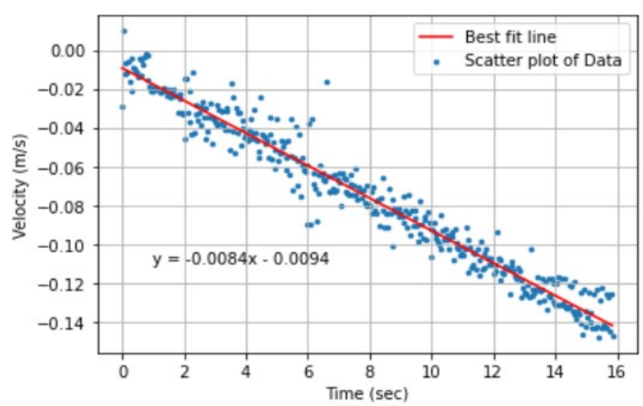
(e) v vs t for 200.6 gm mass; acceleration $= 0.0252 m/s^2$



(f) v vs t for 151.2 gm mass; acceleration $= 0.0193 m/s^2$



(g) v vs t for 101.2 gm mass; acceleration $= 0.0145 m/s^2$



(h) v vs t for 51.5 gm mass; acceleration $= 0.0084 m/s^2$

Figure 5: Velocity-time graphs for Part A

We can see the data points in that part of the graph, and they show a negative slope since the gravitational acceleration is downwards. The table below represents the slopes of the above graphs along with the respective mass values -

No. of masses	Mass (kg)	Acceleration (ms^{-2})
8	0.4018	0.0456
7	0.3514	0.0429
6	0.3011	0.0369
5	0.2506	0.0321
4	0.2006	0.0252
3	0.1512	0.0193
2	0.1012	0.0145
1	0.0515	0.0084

Table 1: Masses and accelerations for part A

The graph below shows the data from the above table -

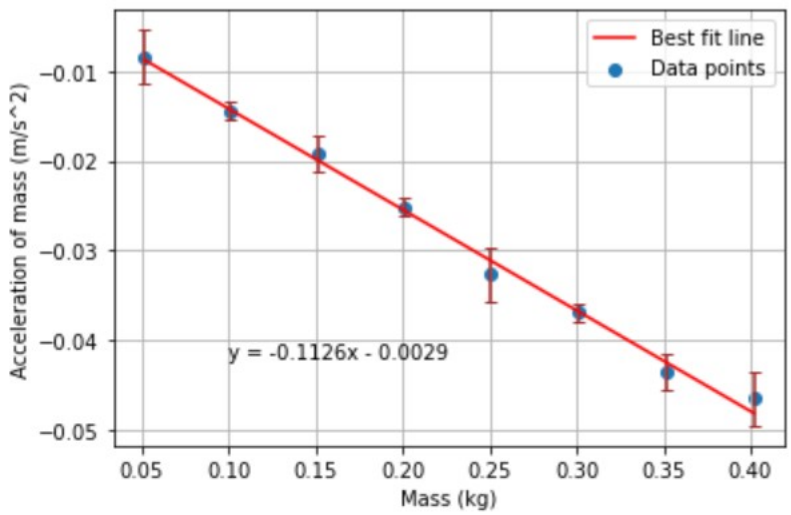


Figure 6: Acceleration vs mass to find I and τ_f

As demonstrated in the theory section, the above graph follows Equation(5) -

$$a = m\left(\frac{gr^2}{I}\right) - \left(\frac{\tau_f r}{I}\right)$$

The slope of the best fit line of the above graph will give us the moment of inertia (I) of the flywheel, while the intercept will give us the value of the friction torque on the flywheel.

Using r = 8.5 mm, g = -9.81 m/s^2, slope = -0.1126, intercept = 0.0029 -

$$\begin{aligned} I &= \frac{gr^2}{\text{slope}} \\ &= \frac{(-9.81) \times (8.5 \times 10^{-3})^2}{-0.1126} \\ &= 0.0063 \ \ kgm/s^2 \end{aligned}$$

$$\begin{aligned} \tau_f &= \frac{I \times \text{intercept}}{r} \\ &= \frac{(0.0063) \times (0.0029)}{8.5 \times 10^{-3}} \\ &= 0.0021 \ \ Nm \end{aligned}$$

6.3 Results

Thus, we have found the following values for moment of inertia (I) of the flywheel and the frictional torque (τ_f) acting on it -

$$\begin{aligned} I &= 0.0063 \ \ kgm/s^2 \\ \tau_f &= 0.0021 \ \ Nm \end{aligned}$$

7 Part B

In this part we fix magnets near the flywheel to introduce electromagnetic damping in the system and observe its effects on the terminal velocity of the falling masses.

7.1 Procedure

1. Insert both the magnets onto their holders on either sides of the flywheel. Ensure that they are aligned properly in a plane perpendicular to the flywheel.

2. Space the magnets equally on both sides of the flywheel. To do this, first touch both magnets to the flywheel and stop after hearing 3 clicks of the screw gauge, on both sides. Then, turn the screw gauge in the opposite direction in equal amounts for both magnets. This will ensure equal spacing on both sides.

3. Then, repeat the procedure of recording and tracking videos of different masses, as done in Part A. Obtain the .csv data files for all videos.

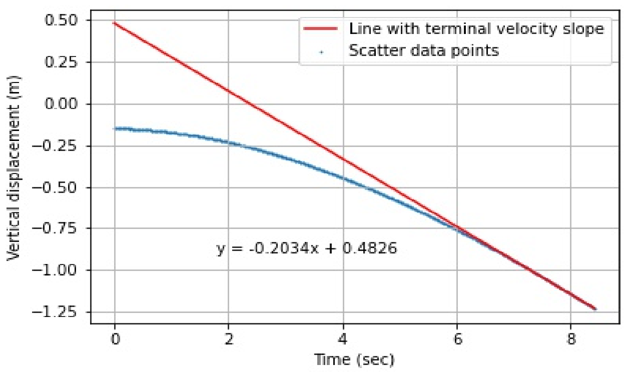
4. Plot the y-position vs time graphs for all masses. Create a separate array storing values of approximately the last 100 data points for both y and t. Find the line of best fit for this data and plot it on the same graph.

5. The slope of this line gives us the terminal velocity of that falling mass. Do this for all masses.

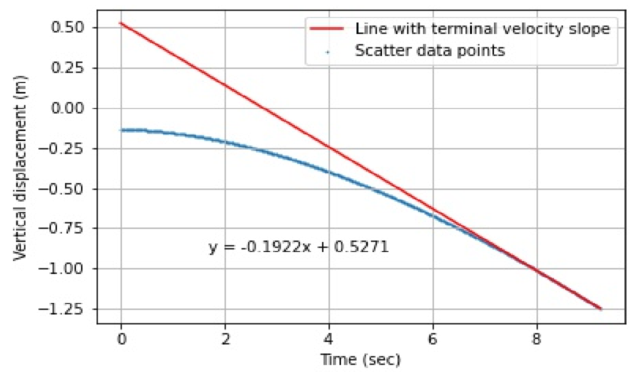
6. Finally, plot the obtained terminal velocities (v_t) with the respective masses to verify the relation between them.

7.2 Observations

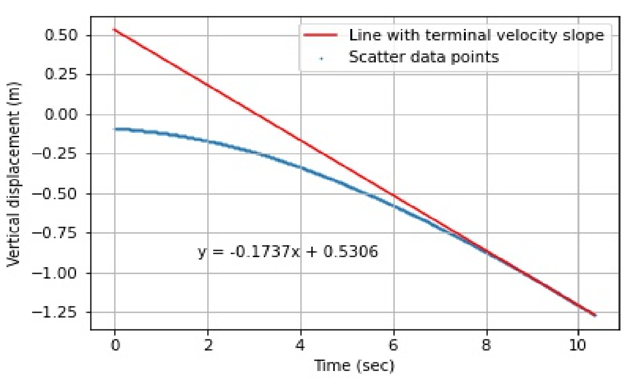
The same 8 masses were used to take data for this part of the experiment, as used in part A. Please refer to the second column of Table 1 for the actual values of the masses. The following displacement-time graphs of the 8 masses give us the respective terminal velocities achieved by them. Plotting these values against those masses will help us to experimentally verify the theoretically proposed relation between mass of object and its terminal velocity.
For plotting the line whose slope gives us the terminal velocity, I have used only the last 100 data points of y, t. Out of approximately 600-800 data points for every graph, it is reasonable to assume (and later this is verified) that all the masses reach terminal velocity before the last 100 data points. Thus, these red colored best fit lines in the graphs below are those which fit best to the last 100 data points of these graphs -



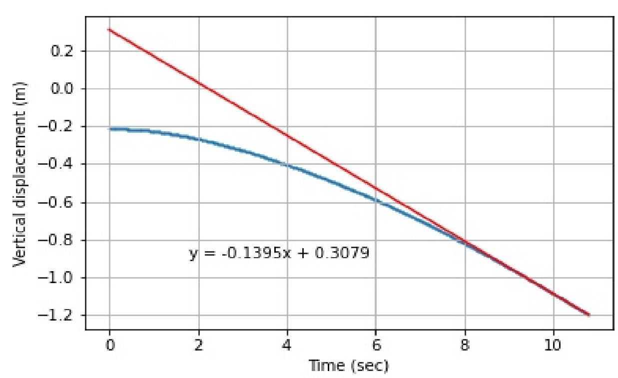
(a) y-position vs t for 401.8 gm mass; $v_t = 0.2034 m/s$



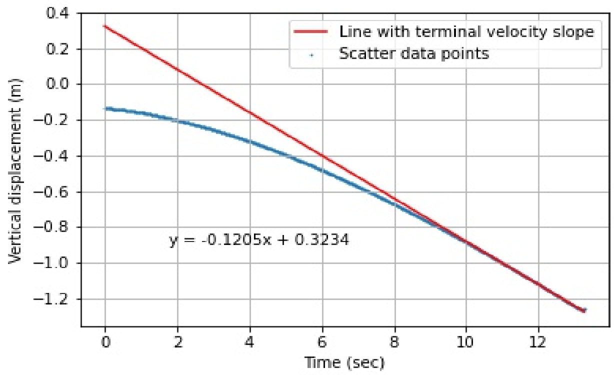
(b) y-position vs t for 351.4 gm mass; $v_t = 0.1922 m/s$



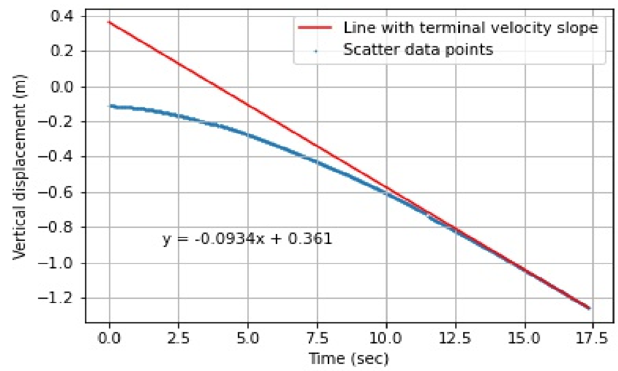
(c) y-position vs t for 301.1 gm mass; $v_t = 0.1737 m/s$



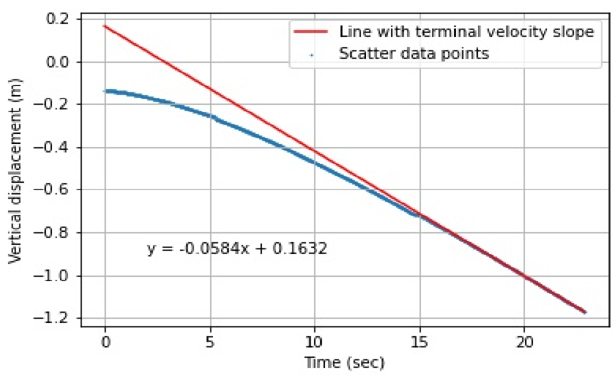
(d) y-position vs t for 250.6 gm mass; $v_t = 0.1395 m/s$



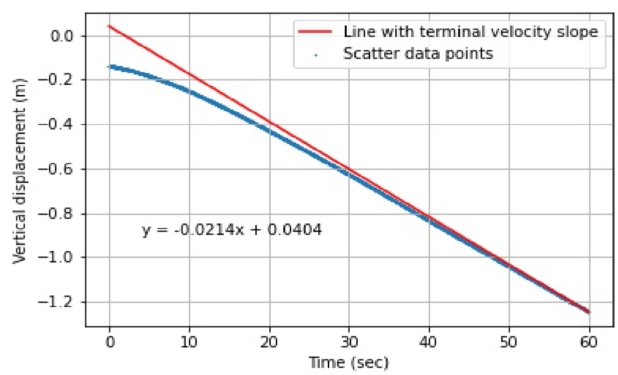
(e) y-position vs t for 200.6 gm mass; $v_t = 0.1205 m/s$



(f) y-position vs t for 151.2 gm mass; $v_t = 0.0934 m/s$



(g) y-position vs t for 101.2 gm mass; $v_t = 0.0584 m/s$



(h) y-position vs t for 51.5 gm mass; $v_t = 0.0214 m/s$

Figure 7: Displacement-time graphs for Part B

The table below shows us the final obtained terminal velocities (v_t) obtained for the corresponding masses -

No. of masses	Mass (kg)	Terminal Velocity (ms^{-1})
8	0.4018	0.2034
7	0.3514	0.1922
6	0.3011	0.1737
5	0.2506	0.1395
4	0.2006	0.1205
3	0.1512	0.0934
2	0.1012	0.0584
1	0.0515	0.0214

Table 2: Masses and terminal velocities for part B

On plotting the data in the above table to inspect the trend, we get the following graph -

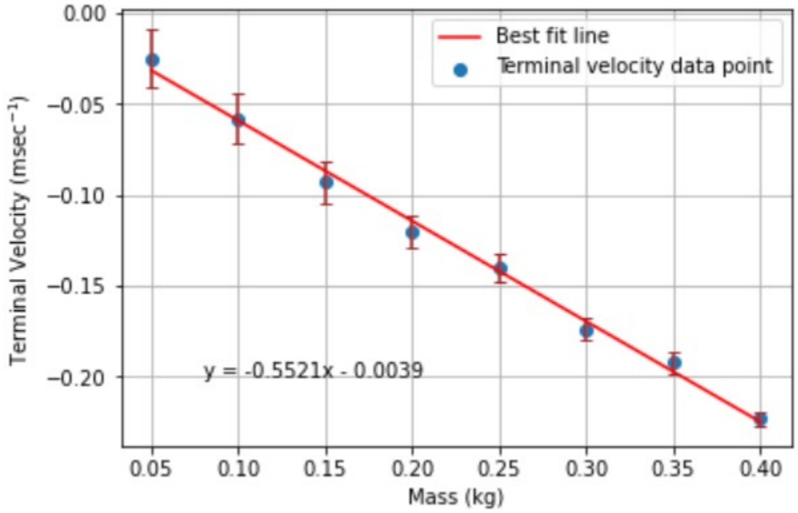


Figure 8: Terminal velocity (v_t) vs mass to verify theoretical relation

The above graph clearly shows a linear trend between v_t and mass. As demonstrated in the theory section, the above graph follows Equation(7) -

$$\begin{aligned} v_t &= \frac{gr^2}{CB^2}\left(m - \frac{\tau_f}{gr}\right) \\ &= m\left(\frac{gr^2}{CB^2}\right) - \frac{\tau_f r}{CB^2} \end{aligned}$$

We will get the value of CB^2 from both the slope and intercept of this graph. Using r = 8.5 mm, g = -9.81 m/s^2, slope = -0.5521, intercept = 0.0039, we get -

$$\begin{aligned} (CB^2)_{\text{slope}} &= \frac{gr^2}{\text{slope}} \\ &= \frac{(-9.81) \times (8.5 \times 10^{-3})^2}{-0.5521} \\ &= 0.00128 \;\; kgm^2s^{-1} \end{aligned}$$

$$\begin{aligned} (CB^2)_{\text{intercept}} &= \frac{\tau_f r}{\text{intercept}} \\ &= \frac{(0.0021) \times (8.5 \times 10^{-3})}{0.0039} \\ &= 0.00457 \;\; kgm^2s^{-1} \end{aligned}$$

The value from the intercept may be higher due to error in both, the value of τ_f from part A and the intercept error (see error bars). Thus, we will keep the value generated from the slope.

7.3 Results

Thus, we successfully verified a theoretically predicted linear trend between mass and terminal velocity. The value of the quantity CB^2 was found out from the slope of the v_t vs mass graph, and was found out to be 1.28×10^{-4} kgm^2s^{-1}.

8 Part C

In this part we choose a fixed value of mass and let it fall for different values of distances between flywheel and magnets, to observe the effects of changing magnetic magnetic field (B) on terminal velocity (v_t) of falling mass.

8.1 Procedure

Measuring variation of B with distance

1. Lower the magnets on the assembly such that the flywheel is not in the space between the magnets.
2. Place an object in between the magnets and tighten the screw gauges on both sides (upto 3 clicks). Now measure the length of this object (using vernier caliper) to find out the initial separation distance between the magnets.
3. Mark the midpoint of this same object, and keep it back between the magnets. Attach the Hall probe of the Gauss meter to a stand, and align it perfectly with this midpoint mark on the object. This will ensure that the probe is at the midpoint of the separation between magnets.
4. Remember to remove the cover of the Hall probe, and ensure that is exactly perpendicular to the plane of the magnetic field lines passing through. This is essential since the Hall probe actually measures the flux of magnetic field through its surface area, which is dependent on the inclination angle between field lines and area.
5. Record the value shown by Gauss meter at this distance. Record this separation distance too.
6. Move both magnets outwards by turning the screw gauges in equal amounts. Record the values of B after appropriate step sizes of distances (say 1mm).
7. After collecting this data, plot the magnetic field strength (B) against the separation distance, along with the log-log plots. These will tell us about the variation of B with separation distance.

Measuring terminal velocity

1. Fix an appropriate range of separation distances between magnets along with step sizes, such that the maximum and minimum distances allow the object to reach terminal velocity within the length of the cord. Also fix the value of the falling mass.
2. Start with the maximum separation distance, and record a video of the mass falling. Decrease the distance with the decided step size, and record a video of the falling mass again.
3. Repeat this procedure till the minimum separation distance is reached. Export the videos onto tracker and analyze them to get .csv data for y-position and time.
4. Plot y-position with time for all separation distances, and obtain values of terminal velocity for all graphs as done in Part B.
5. Plot obtained values of terminal velocities with respective magnetic field strengths to verify the relation between B and v_t.

8.2 Observations

For this first section of this part, we used the Hall probe and the Gauss meter to record the variation of magnetic field strength 'B' with the separation distance between magnets. The Gauss meter displays readings in the unit 'Gauss', which needs to be multiplied by a factor of 10^{-4} to get the values in 'Tesla' (SI Units). The table below shows us the recorded data for variation in 'B' for different separation distances between the magnets -

Distance between magnets (m)	Magnetic field strength B (gauss)	B (in Tesla)
0.012	200	0.0200
0.013	180	0.0180
0.014	163	0.0163
0.015	147	0.0147
0.016	133	0.0133
0.017	121	0.0121
0.018	111	0.0111
0.019	101	0.0101
0.020	91	0.0091
0.021	83	0.0083
0.022	78	0.0078
0.023	71	0.0071
0.024	65	0.0065
0.025	60	0.0060

Table 3: Variation of B with distance

This data, when plotted on a graph is as follows -

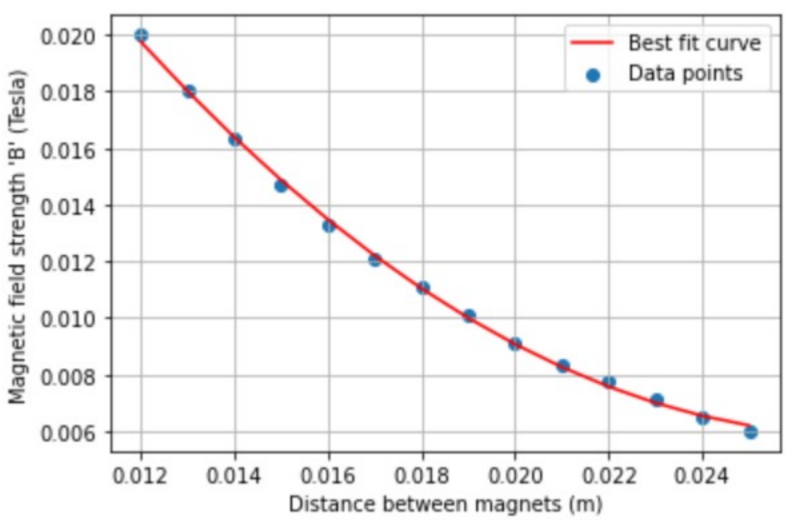


Figure 9: Variation of 'B' with separation distance

This is a quadratic best fit curve for the data. To get a better approximation of the relation between B and separation distance, we plot a log-log graph of the same data -

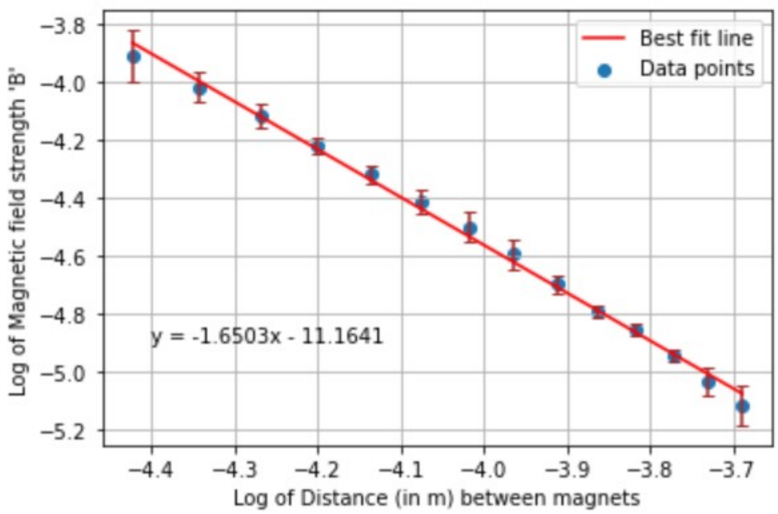


Figure 10: Log-log plot to determine power law

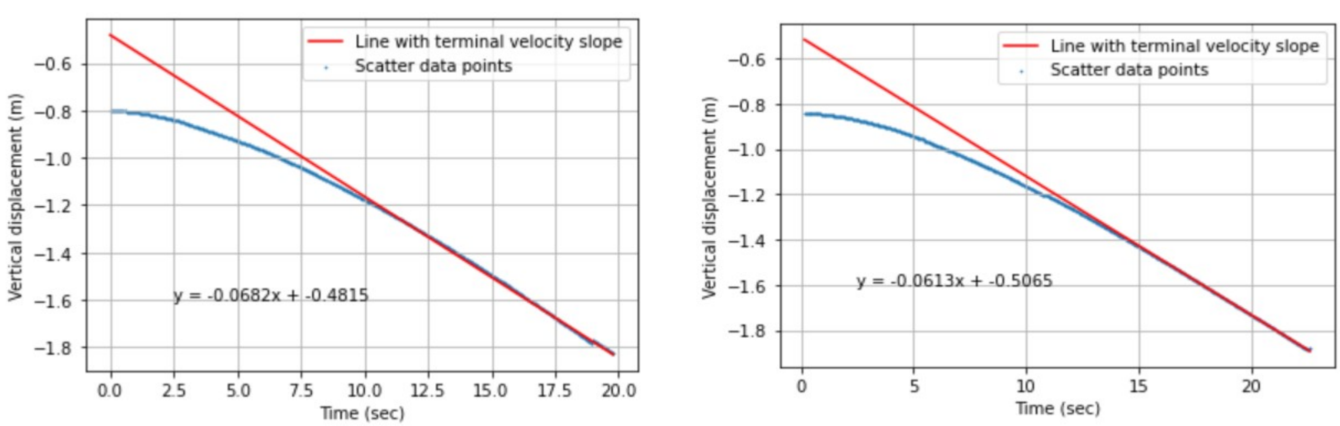
We can see that the slope of this log-log plot is **-1.6503**. If we hypothesize a power law relation "a" between magnetic field strength B and separation distance d, we can say the following -

$$
\begin{aligned}
& B \propto d^a \\
\implies & B = Cd^a \\
\implies & \log B = \log(Cd^a) \\
\implies & \log B = a\log(d) + \log C
\end{aligned}
$$

Here, we can see that the slope of a log-log plot should be "a", and the intercept should be $\log(C)$. Thus, $\mathbf{a = -1.6503}$ and $\log(C) = -11.641$ in this case. Thus, our expected power law relation is -

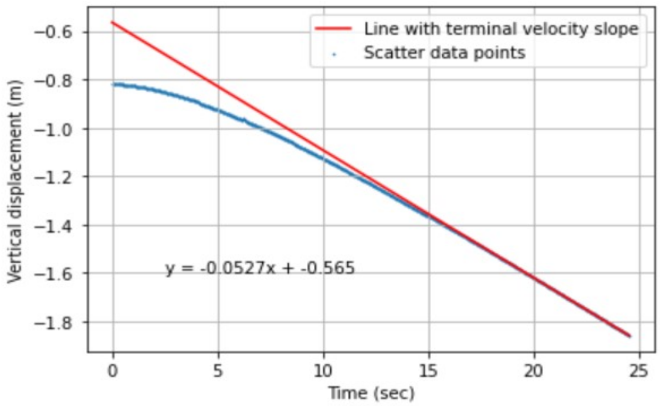
$$B = e^{-11.46} d^{-1.65}$$

Now, we plot the displacement time graphs for a constant mass of 100 grams, with varying separation distances (and thus, varying B) to get terminal velocities from their slopes. The same technique of finding best fit lines and their slopes for last 100 data points was used in this part also (similar to Part B). The graphs are as follows -

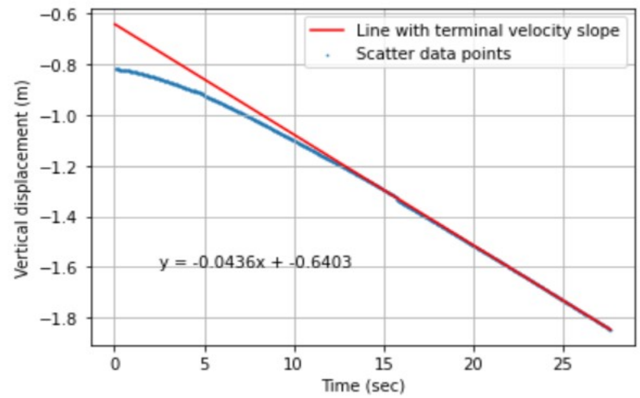


(a) y-position vs t for 25.64mm separation; $v_t = 0.0682 m/s$

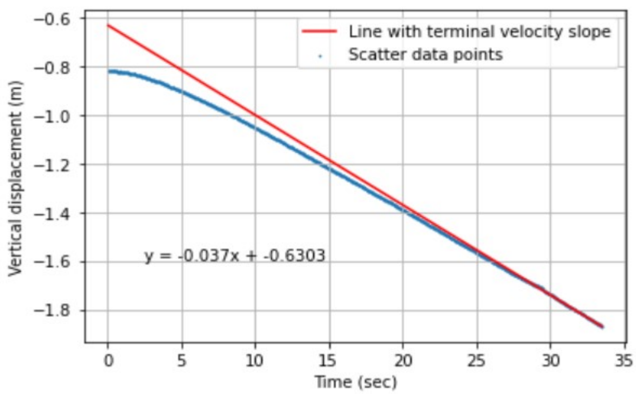
(b) y-position vs t for 24.64mm separation; $v_t = 0.0613 m/s$



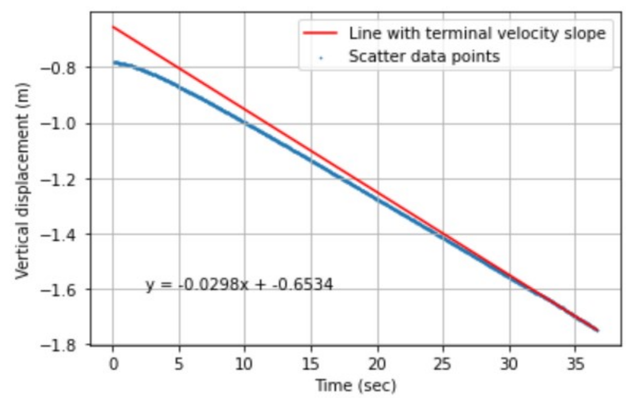
(c) y-position vs t for 23.64mm separation; $v_t = 0.0527 m/s$



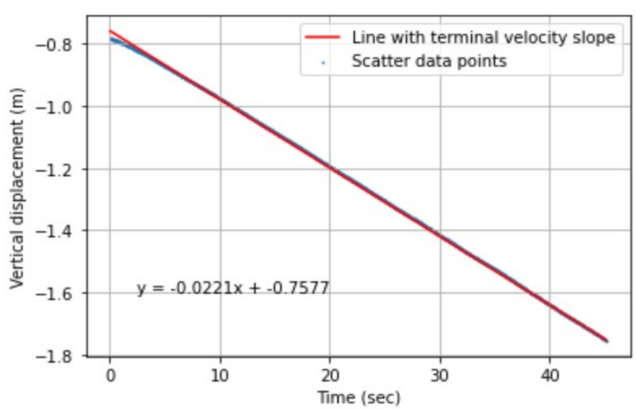
(d) y-position vs t for 22.64mm separation; $v_t = 0.0436 m/s$



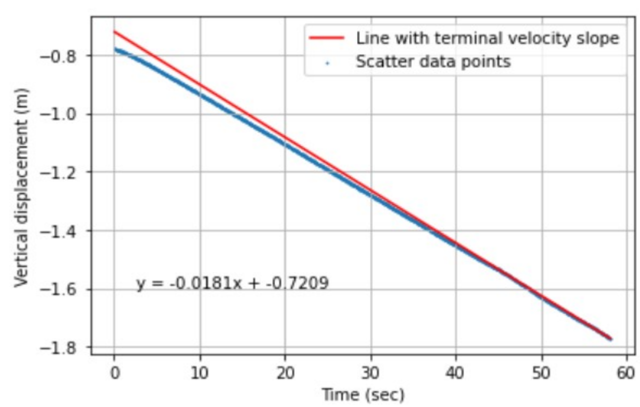
(e) y-position vs t for 21.64mm separation; $v_t = 0.0370 m/s$



(f) y-position vs t for 20.64mm separation; $v_t = 0.0298 m/s$



(g) y-position vs t for 19.64mm separation; $v_t = 0.0221 m/s$



(h) y-position vs t for 18.64mm separation; $v_t = 0.0181 m/s$

Figure 11: Displacement-time graphs for Part C

We now use our previously derived power law to calculate the corresponding magnetic fields from the separation distances. This is given along with the respective terminal velocity values in the table below -

Distance between magnets (mm)	Magnetic field strength B (μTesla)	Terminal Velocity (m/sec)
25.64	0.0671	0.0682
24.64	0.0716	0.0613
23.64	0.0767	0.0527
22.64	0.0824	0.0436
21.64	0.0888	0.0370
20.64	0.0960	0.0298
19.64	0.1042	0.0221
18.64	0.1136	0.0181

Table 4: v_t and calculated B for every separation distance

Now, we plot a graph between terminal velocity and magnetic field strength for this above data. We also plot the best fit quadratic curve to this data, to get -

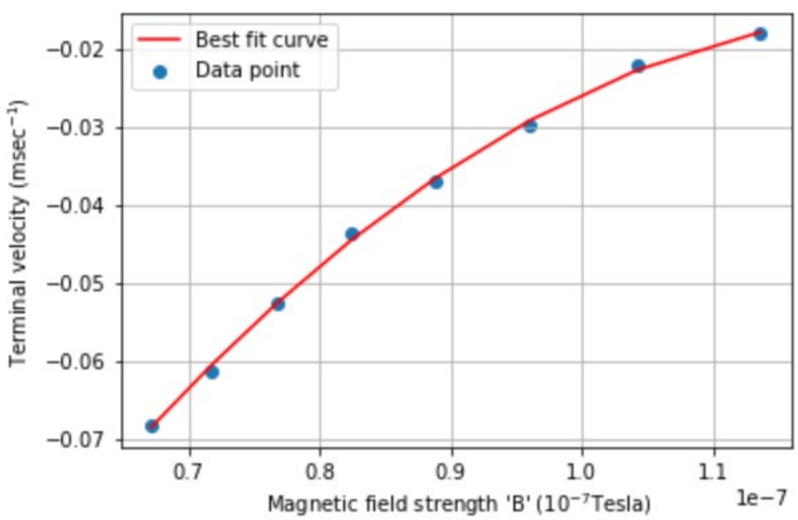


Figure 12: Terminal velocity v_t vs magnetic field B

Since we know from the theoretical relation (Equation(7)) that v_t should vary linearly with the inverse square of B, we plot that graph to verify the relation -

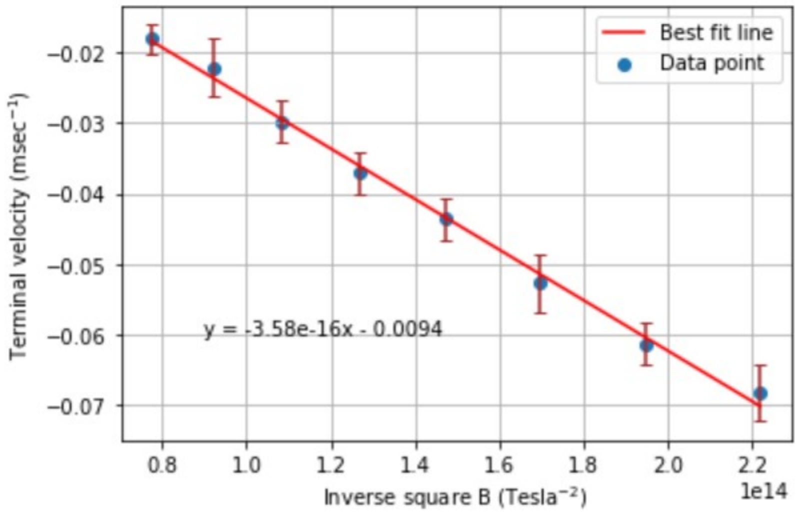


Figure 13: Terminal velocity v_t vs Inverse square B plot

Thus, we successfully verify that the trend is indeed linear.

8.3 Results

In this part of the experiment, we found the terminal velocity variation with the changing separation distance between magnets, and thus verified the theoretically expected variation of terminal velocity v_t with changing magnetic field B.

9 Error Analysis

The least count of the vernier caliper = $\Delta r = 0.02mm$. The error in slope and intercept values given by polyfit can be calculated as follows -

$$\begin{aligned}\frac{\Delta\text{slope}}{\text{slope}} &= \sqrt{(\Delta x)^2 + (\Delta m)^2} \\ &= \sqrt{(0.001)^2 + (0.0001)^2} \\ &= \sqrt{10^{-6}} \\ &= 1.00 \times 10^{-3}\end{aligned}$$

$$\begin{aligned}\frac{\Delta\text{intercept}}{\text{intercept}} &= \frac{0.0001}{0.0029} \\ &= 0.0345\end{aligned}$$

Using this we now calculate the error in I and τ_f -

$$I = \frac{gr^2}{\text{slope}}$$

$$\implies \frac{\Delta I}{I} = \sqrt{\left(\frac{2\Delta r}{r}\right)^2 + \left(\frac{\Delta \text{slope}}{\text{slope}}\right)^2}$$

$$\implies \frac{\Delta I}{I} = \sqrt{\left(\frac{2 \times 0.02}{8.5}\right)^2 + (10^{-3})^2}$$

$$\implies \frac{\Delta I}{I} = 2.314 \times 10^{-5}$$

$$\implies \Delta I = (2.314 \times 10^{-5}) \times 0.0063$$

$$\implies \Delta I = 1.458 \times 10^{-7} kgm/s^2$$

$$\tau_f = \frac{I \times \text{intercept}}{r}$$

$$\implies \frac{\Delta \tau_f}{\tau_f} = \sqrt{\left(\frac{\Delta \text{intercept}}{\text{intercept}}\right)^2 + \left(\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta r}{r}\right)^2}$$

$$= \sqrt{(0.0345)^2 + (2.314 \times 10^{-5})^2 + \left(\frac{0.02}{8.5}\right)^2}$$

$$\approx 0.0345$$

$$\implies \Delta \tau_f = 0.0345 \times 0.0029$$

$$\implies \Delta \tau_f = 1.005 \times 10^{-4}$$

10 Sources of Error

- The tracker frame rate was sometimes an important source of error, especially in part A where the masses were falling at a considerable velocity. The resolution of the camera was also another important constraint.
- Errors in tracking involve correction selection of template, periodic re-assignment of point mass when the software is unable to track the template image, uncertainty in calibration sticks, etc. Sometimes, setting calibration at an angle to the frame also introduces errors in the data.
- Parallax error (very less magnitude) for calibrating the distances while tracking, since the calibration stick and the falling mass may be at different distances from the camera.
- Unstable positioning of the Hall probe is another source of error for the data in this experiment, along with varied sensitivity of the gauss meter.
- Positive or negative zero error in distance measuring instruments like vernier caliper and screw gauge may also introduce error into our data.

11 Discussion

11.1 Tracking software

1. Tracker identifies and tracks the position of the specified point by finding out matches for a certain template image given by us while setting up the tracking. The color of this object needs to be in high contrast to the background of the video, for tracker to identify this object clearly.

2. Another tricky aspect of the tracking done in this experiment is setting the frame-rate of Tracker and resolution of the DSLR camera used to record videos. We tried various combinations of frame-rates for the tracker, and found that for lower masses, when the change between two frames isn't very rapid, it is computationally more efficient feasible to use an increased frame rate. This may also decrease any floating point errors in all the extra data points it has to track for a lower frame rate, which is not necessary for a slightly higher frame rate (around 5-8 frames/sec).

3. Increasing the length of the rectangular area inside which the search function of Tracker searches for a match to the template helped significantly in better tracking. Customizing the template image's search object according to every video (like oval shape, elliptical and sometimes circular) also supported better tracking.

4. Lastly, choosing start and end frames wisely was crucial for the successful tracking of videos in this experiment. For ideal start frames, there should be zero initial velocity but it should start gaining velocity immediately after that frame. For ideal end frames, the mass should reach exactly the bottom-most point and not come back up. If this happens, there is an unwanted spike in the y, V_y data which messes with the best fit curves to be generated later in Python.

11.2 Experimental

1. The first experimental constraint for this experiment is to get a plain background (with minimal shadows/light flickering movements) to ensure that the videos are good enough to be tracked. We used brightly colored orange stickers and pasted them on the masses falling to get a good color contrast with the plain white background to deal with this.

2. Alignment of the camera with the falling masses, the flywheel and the calibration stickers is another critical aspect in performing this experiment. We had to retake several of our videos just because we didn't ensure proper alignment of these three entities in them.

3. Choosing the right combination of masses and separation distances between magnets is very important to get good results from analysing the data recorded in this experiment. If the masses are too high in parts B and C, then the Lorentz force on the flywheel would be insufficient to balance out the gravitational attraction experiences by the falling masses. Thus, terminal velocity can only be reached upto a certain critical mass limit. Finding out this critical mass limit (dependent on strength of magnets, etc.) could be an interesting extension worth probing more deeply into, for this experiment. On the other hand, if the mass is too less then the magnets damp out the gravitational force very quickly, and thus we cannot observe the gradual flattening of the velocity curve, since the mass falls too slowly and with terminal velocity immediately after release.

4. Recording the correct values of magnetic field using the Gauss meter was also crucial to Part C of this experiment. The Hall probe actually measures magnetic flux of the field lines passing through its surface area. Thus, it is imperative that it is kept exactly perpendicular to the field lines and exactly at the middle of both magnets to record correct data.

5. **Working of the Hall Probe** - The hall probe has an internal current, and it measures the external magnetic field by observing its interaction with its internal current. Similar to the qualitative explanation of damping in the theory section of this report, the moving electrons inside the probe experience a Lorentz force causing charge separation and thus, polarization along the length of the probe. This sets up an electric field which eventually counters the external magnetic field. At equilibrium, the steady state voltage of this electric can be measured, and thus the Hall probe helps us to find out the external magnetic field's magnitude.

12 References

- Ashoka University, Physics Lab 2, Handout - Electromagnetic damping
- Right Hand rule - Wikipedia (image credits)

13 Appendix

All relevant data collected for this experiment is attached as .csv files along with this report.