Experiment 6: Free Fall
Lab Report

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1 Introduction

We regularly observe objects freely falling from heights under the influence of earth's gravity. The development of our motor skills has been historically shaped and fine-tuned by natural selection, to help us gauge and catch falling objects. We know from experience that a leather ball falls very slowly compared to a lump of feathers of the same mass. However, experiments conducted in vaccuum show that two very different objects of the same mass indeed fall in the same amount of time (when dropped from the same height). The differentiating factor then, is the drag force (resistance) offered by air in the atmosphere. It acts differently on different objects, thus affecting their falling velocities and times. This experiment delves a bit deeper into understanding the effect of air resistance on freely falling bodies, and the dependence of terminal velocity on the mass of the falling object.

2 Aim

- **Part A:** To drop objects of known masses from a height and calculate the value of gravitational acceleration 'g' by analysing their free-fall motion.
- **Part B**: To drop a cupcake liner with increasing masses of magnets and study the effect of air resistance on making falling masses achieve terminal velocity.

3 Apparatus

1. Table Tennis ball (lighter falling mass)
2. Squash ball (heavier falling mass)
3. Step-ladder (to drop objects from a considerable height)
4. Brightly colored stickers (for better length calibration and tracking)
5. A plain white wall with minimal air currents (to model normal air resistance and ensure good quality videos)

6. Electronic weighing machine (to weigh magnets and balls)
7. 2 Cupcake liners (falling objects with high air resistance)
8. A set of 47 small, lightweight magnets (to achieve incremental masses of falling objects)
9. A DSLR Camera (Nikon D5700)
10. A Tripod stand (to keeo the DSLR stable)
11. Video Tracking software - *Tracker*

4 Theory

Newton's laws of motion are one of the most foundational principles of contemporary physics. They describe the motion of all classical objects with very high accuracy. Until Einstein's formulation of the two theories of relativity (special and general), Newton's laws of motion were our best tool for predicting the motion of everything, from falling apples to moving celestial bodies. Although the corrections made to these laws by relativity are important, their effect is significant only for bodies moving at speeds comparable to the light speed. Thus, for most everyday motion we observe, Newton's laws still possess enough explanatory power to predict it.

4.1 Expressions for y and v_y of free-falling object (air resistance neglected)

Newton's Second Law of Motion gives the following expression to find the acceleration of a body of mass m when a force F acts on it -

$$\mathbf{F} = m\mathbf{a}$$
$$\implies \mathbf{a} = \left(\frac{1}{m}\right)\mathbf{F}$$

For a freely falling object (ignoring air resistance here), the force acting on the mass is simply the gravitational attraction from the earth, mg. Hence, the acceleration will be -

$$\mathbf{a_y} = \left(\frac{1}{m}\right)(-mg)$$
$$\implies \mathbf{a_y} = -g$$

The negative sign of the force, and thus the acceleration is by virtue of the fact that both of them point downwards, towards the surface of the Earth. However, it is a matter of just convention. The subscript y indicates that the mass is falling vertically downwards, i.e. in the y direction. Since acceleration is defined as the rate of change of an object's velocity, we solve the following first order differential equation to get an expression for the velocity of the free-falling mass. We assume the mass's initial velocity to be u and final velocity to be v. Thus, we have -

$$\begin{aligned}
\mathbf{a_y} &= -g \\
\implies \frac{d\mathbf{v_y}}{dt} &= -g \\
\implies d\mathbf{v_y} &= -gdt \\
\implies \int_u^v d\mathbf{v_y} &= \int_0^t -gdt \\
\implies v - u &= -gt \\
\implies v &= u - gt
\end{aligned}$$

Thus, we have our expression for the velocity of the falling mass as follows -

$$\boxed{v = u - gt} \tag{1}$$

On integrating this expression with respect to time again, we get the vertical displacement y of the falling object. Let the object's initial height be y_0 and final height be some y. Thus, we have -

$$\begin{aligned}
v &= u - gt \\
\implies \frac{d\mathbf{y}}{dt} &= u - gt \\
\implies d\mathbf{y} &= (u - gt)dt \\
\implies \int_{y_0}^y d\mathbf{y} &= \int_0^t (u - gt)dt \\
\implies y - y_0 &= ut - \frac{1}{2}gt^2 \\
\implies y &= y_0 + ut - \frac{1}{2}gt^2
\end{aligned}$$

Thus, we have our final expression for the vertical displacement y of the falling object -

$$\boxed{y = y_0 + ut - \frac{1}{2}gt^2} \tag{2}$$

4.2 Introducing air resistance

However, the falling objects do not follow the motion described by Equations (1) and (2) in real life. Due to presence of air in the atmosphere, the air molecules (like any other fluid) exert a resistive, drag force (F_d) on any falling object. This decreases the net downwards force on the falling object, since a part of its weight is balanced by air resistance. Hence, the net force F on the falling object is actually -

$$F_{\text{net}} = mg - F_d$$

This damping force (F_d) is directly proportional to the velocity of the falling object. Thus, as the object achieves a certain velocity (v_t), the air resistance completely balances out the weight of the object. Thus, the object then falls with a constant velocity. This velocity (v_t) is called **terminal velocity**.

The dependence of F_d on velocity is usually considered to be v, since most of our modelling involves the assumption that the air has a *laminar flow*. However, when the air flow is *turbulent*, this dependence of F_d becomes directly proportional to v^2. Here, we only consider the *laminar flow* case.

4.3 Mathematically modelling F_d

Consider a freely falling object with a cross sectional area A, **in the direction of falling**. In a small time interval Δt, it displaces air of mass $m_{\text{air}} = \rho_{\text{air}} A v \Delta t$ (where v is the velocity of the object at that instant). Assuming this 'block' of the displaced air was stationary initially, it will now start moving with the same velocity v (elastic collisions). Thus, the change in momentum $\Delta \mathbf{p}$ of the air is given by $\Delta \mathbf{p}_{\text{air}} = m_{\text{air}} v$. Thus, we have the total change in momentum of air as -

$$\begin{aligned}
\Delta p_{\text{air}} &= \rho_{\text{air}} A v^2 \Delta t \\
\implies \frac{\Delta p_{\text{air}}}{\Delta t} &= \rho_{\text{air}} A v^2 \\
\implies F_d &= \rho_{\text{air}} A v^2
\end{aligned}$$

Since rate of change of momentum is equal to the force (Newton's second law of motion), we have the above expression for F_d. This expression works because the falling object experiences the same *decrease* in momentum, as the air block experiences *increase* in. One last critical factor we need to consider in this, is the geometry/shape of the falling object. This is specific to the kind of object, and thus is like a constant C from the material. This C is known as the *drag coefficient* of the object. Incorporating this and a negative sign (since it's a damping force), we have the final expression for F_d as -

$$\begin{aligned}
F_d &= -C \rho_{\text{air}} A v^2 \\
\implies F_d &= -\alpha v^2
\end{aligned}$$

Here, since $C \rho_{\text{air}} A$ is a constant, we encapsulate it into a single constant α. We can easily argue that $\alpha > 0$. Density of air (ρ_{air}) is a positive value, since mass and volume of air are positive. A is facing cross-sectional area of the object, and thus has to be positive. The *drag coefficient* (C) also has to be positive, since the air resistance sucks energy out of the system. If it wasn't positive, then the object would fall with an acceleration greater than g, violating conservation of energy. Due to all these factors, we can safely put a constraint on α such that always, $\alpha > 0$.

As the velocity of the falling object increases, the drag force on it also increases upto a point where it balances out the weight of the object. Thus, the condition for reaching **terminal velocity** v_t is given by -

$$\begin{aligned}
F_{\text{net}} &= 0 \\
\implies mg - F_d &= 0 \\
\implies mg &= F_d \\
\implies mg &= C \rho_{\text{air}} A v_t^2 \\
\implies v_t &= \sqrt{\frac{mg}{C \rho_{\text{air}} A}} \\
\implies v_t &= \sqrt{\frac{mg}{\alpha}}
\end{aligned}$$

The net force on a freely falling object of mass m will now be given by (considering air resistance) -

$$ma = mg - \alpha v^2$$

Now, we solve this equation to get an expression for $v(t)$ as follows -

$$\begin{aligned} m\frac{dv}{dt} &= mg - \alpha v^2 \\ \implies \frac{dv}{dt} &= g - \frac{\alpha}{m}v^2 \end{aligned}$$

Earlier, we derived the expression: $v_t = \sqrt{\frac{mg}{\alpha}}$ for terminal velocity. On rearranging, we get: $\frac{\alpha}{m} = \frac{g}{v_t^2}$. Substituting in this, we get -

$$\begin{aligned} \frac{dv}{dt} &= g - \frac{g}{v_t^2}v^2 \\ \implies \frac{dv}{dt} &= g\left(1 - \frac{v^2}{v_t^2}\right) \\ \implies \frac{dv}{1 - \frac{v^2}{v_t^2}} &= gdt \\ \implies \int_0^v \frac{dv}{1 - \frac{v^2}{v_t^2}} &= \int_0^t gdt \end{aligned}$$

Here, we need to substitute both $\frac{v}{v_t} = k \implies dv = v_t dk$ here. Thus, we get -

$$\begin{aligned} \int_0^k \frac{v_t}{1-k^2}dk &= \int_0^t gdt \\ \implies v_t\int_0^k \frac{1}{1-k^2}dk &= \int_0^t gdt \\ \implies \frac{v_t}{2}\int_0^k \left(\frac{1}{1+k} + \frac{1}{1-k}\right)dk &= gt \\ \implies v_t\left[\frac{1}{2}\left(\log(1+k) - \log(1-k)\right)\right]_0^k &= gt \\ \implies v_t\left[\frac{1}{2}\log\left(\frac{1+k}{1-k}\right)\right]_0^k &= gt \end{aligned}$$

Now, we know that the inverse of the hyperbolic tangent, $\tanh^{-1}$ is equal to the term inside the limits on the left hand side of the equation. We have -

$$\tanh^{-1} k = \frac{1}{2}\log\left(\frac{1+k}{1-k}\right)$$

Thus, we substitute that into the left hand side and solve to get -

$$\begin{aligned} v_t \left[\tanh^{-1} k\right]_0^k &= gt \\ \implies v_t \left(\tanh^{-1} k\right) &= gt \\ \implies v_t \left(\tanh^{-1} \frac{v}{v_t}\right) &= gt \\ \implies \tanh^{-1} \frac{v}{v_t} &= \frac{gt}{v_t} \\ \implies \frac{v}{v_t} &= \tanh\left(\frac{g}{t} v_t\right) \\ \implies v &= v_t \tanh\left(\frac{g}{t} v_t\right) \end{aligned}$$

Thus, we have our final equation for velocity of the freely falling object (with air resistance) as -

$$\boxed{v = v_t \tanh\left(\frac{g}{t} v_t\right)} \tag{3}$$

A schematic plot of this equation (with realistic values of constants) is given below -

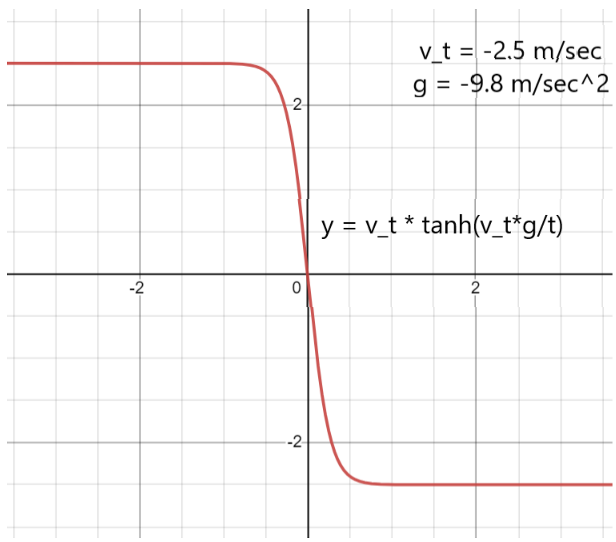


Figure 1: Theoretical plot for v_y vs t

Thus, we should expect similar graphs for the second part of this experiment, where we experimentally verify this analytically derived relation.

5 Object Tracking Procedure

In this experiment, we use a video tracking software named **"Tracker"** to track the positions and velocities of falling objects. It is a very important software, central to obtaining and analysing the data for this experiment. Thus, in this section of the report, I outline my procedure for loading recorded videos of falling objects onto Tracker and extracting .csv data files of y-position, time, y-velocity, etc. from them. Note that while this procedure is mostly applicable for all tracking using this software, there may be some degree of customization in the procedure for this particular experiment -

1. Open Tracker and go to "Video" section near the top left corner. Click on "Import", and import the video to be tracked, onto Tracker.
2. If necessary, use the "Rotate" filter on the video and increase brightness to achieve a suitable configuration.
3. Click on the calibration icon on the toolbar and select "calibration stick". Align the ends of this stick with the reference stickers in the video frame, and enter in the value of the actual distance between these stickers (0.5 m in this experiment).
4. Move through the start and end frames of the video to decide upon a suitable starting and ending reference frame for tracking. In this experiment, ensure that the tracking stops before the mass starts going up after hitting the bottom point. Fix this start and end point.
5. Click on "Track" in the toolbar, and select a new "Point Mass". Go back to the starting frame, and zoom in to the mass to be tracked.
6. Press "Ctrl+Shift" and click at a suitable place on the mass to assign a template image for tracking the mass (preferably on the sticker). The software will keep on tracking the closest possible matches to this template image throughout the video, and extract position and time data values based on the calibration distance entered earlier.
7. Click on "Search" to begin the tracking. Ensure that all columns of data (like y-velocity, x-velocity, etc.) are selected for tracking.
8. In case of errors in tracking, click on "Accept" or "Skip" for a few times. If the error persists, delete the point mass. Assign a new template image to the tracker and try again.
9. Once the tracker goes through all the frames to analyze, click on "Export" data file. Ensure that the data file name ends with a .csv extension, all cells are exported and the delimiter is a "comma".
10. After exporting the data file, open it and put a "#" symbol before all column names. This is necessary for NumPy's `np.loadtxt()` function to correctly load the data onto python arrays during analysis. Remember to save the .csv file before closing it.
11. We now have the requisite data to plot and complete our experiment. Load it onto Python and plot the necessary quantities against each other.

6 Part A: Calculating value of 'g'

In this part, we drop 2 different masses (a Squash ball and a Table Tennis ball) from a certain height and track their motion to calculate the value of gravitational acceleration g.

6.1 Procedure

1. Find a location with a plain (preferably white) background wall with no/minimum wind currents flowing. Carry the step-ladder to that location carefully, and set it up.
2. Fix the DSLR onto the tripod, at a reasonable distance away from the wall. All elements of the experiment must happen inside the frame specified by the camera.

3. Climb onto the step-ladder at a comfortable height, and put a sticker on the background wall to mark the location from where the balls will be released. Ensure this is inside the camera frame.

4. At suitable locations on the wall, fix two bright stickers at a known distance apart from each other. These will be used as a calibration in the tracking software. Ensure both of these are well inside the frame too.

5. Climb onto the step ladder and hold the ball at the marked position. Gently release it. Ensure that it is released exactly vertically i.e. no horizontal component of velocity is given to it while releasing.

6. Record 10-15 videos of one such free-falling ball from a height.

7. Export all videos onto a computer, and track the motion of the ball using *Tracker* to generate .csv data files for all videos. The tracking procedure is explained clearly in Section 5 of this report.

8. Plot the v_y vs time graphs for all data sets and calculate the slope of their best fit lines to get the respective values of g.

9. Plot a histogram of all these values to find out the class with the highest frequency. The arithmetic mean of all g values falling in this class is the g value obtained from this experiment.

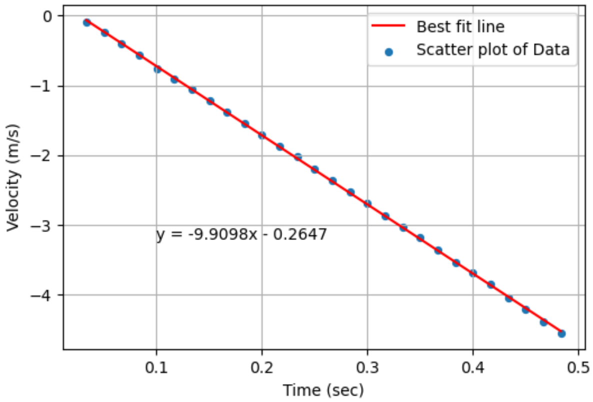
10. Repeat the above procedure for the other ball too.

6.2 Observations

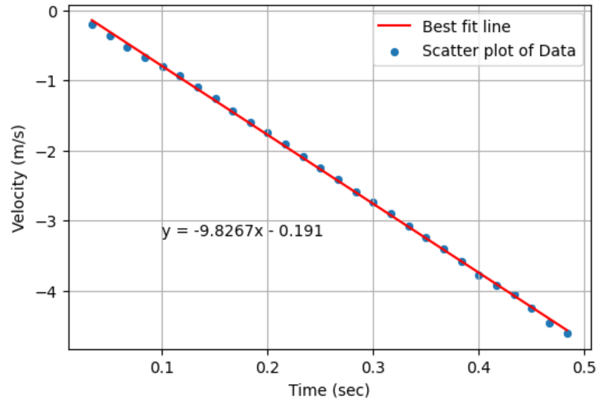
In this part we calculate the acceleration due to gravity, g, by tracking the free-fall motion of two balls - a Table Tennis ball and a Squash ball. Their individual observations are as shown in the subsequent sections below.

6.2.1 Squash Ball

The mass of the squash ball was 23.62 grams, recorded by an electronic weighing machine with least count = 0.1 gram. Its diameter was 3.74 cm (thus, radius $r = 3.74/2 = 1.87$ cm), measured by a vernier caliper of least count 0.02 mm. The Squash ball was dropped and recorded 16 times. Its motion was tracked for all trials, and the vertical velocity v_y was plotted with time. The slopes of the best-fit lines (made using matplotlib's inbuilt `np.polyfit()` function) give us the acceleration for that data-set. The v_y vs time graphs for the 16 data-sets are as follows -



(a) Data-set 1



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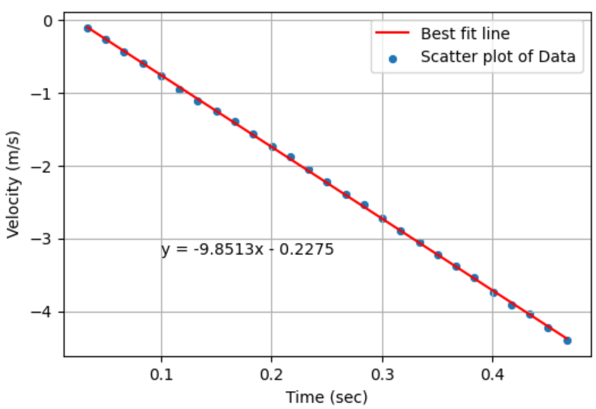

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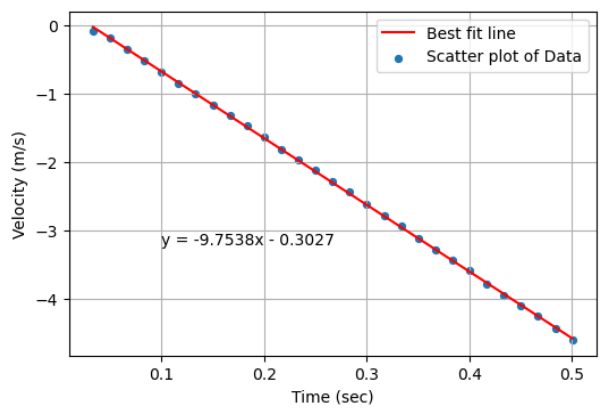

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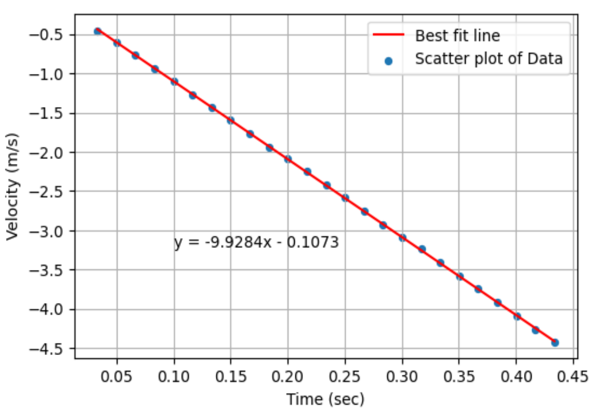

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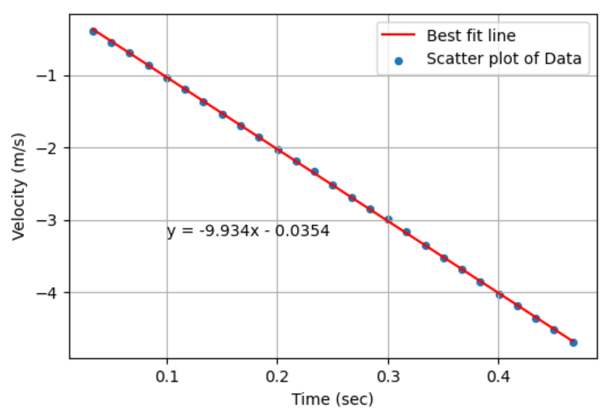

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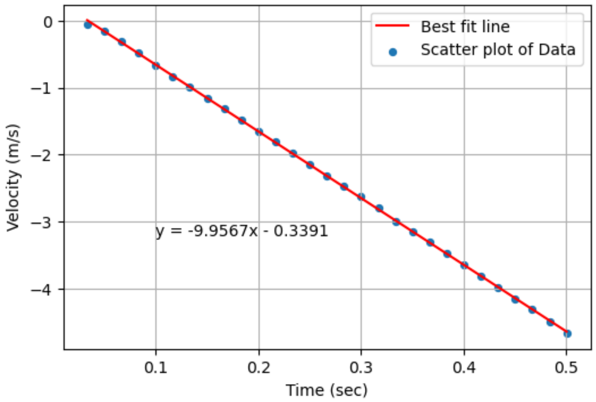

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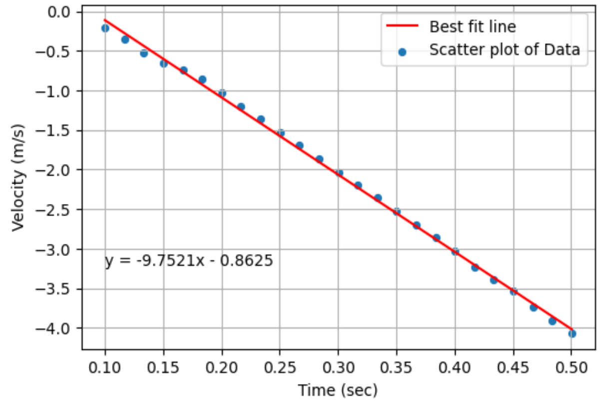

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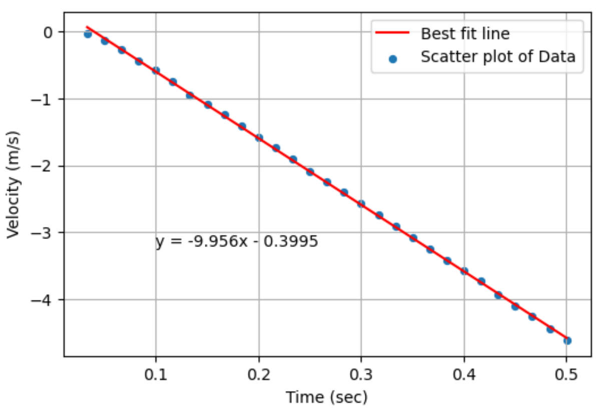

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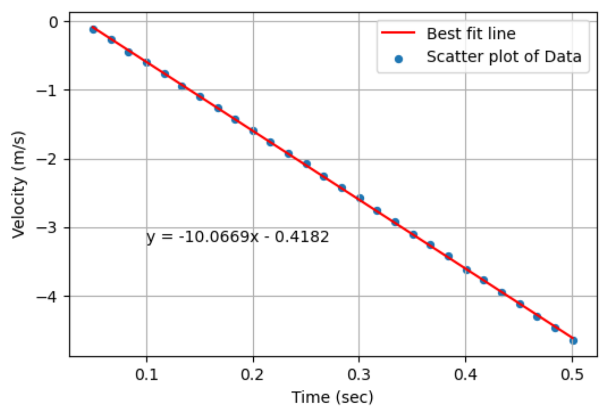

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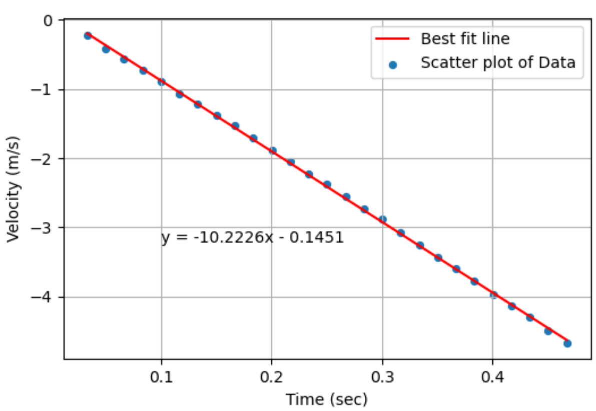

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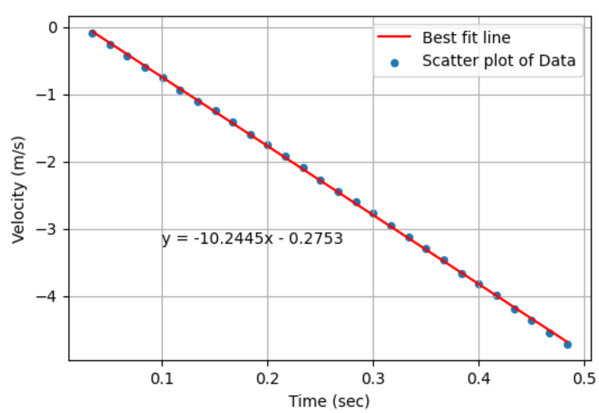
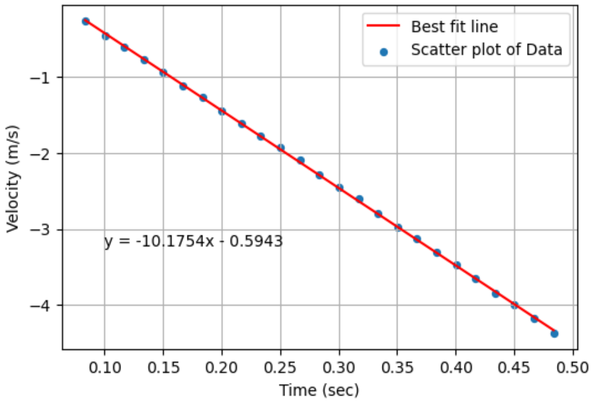
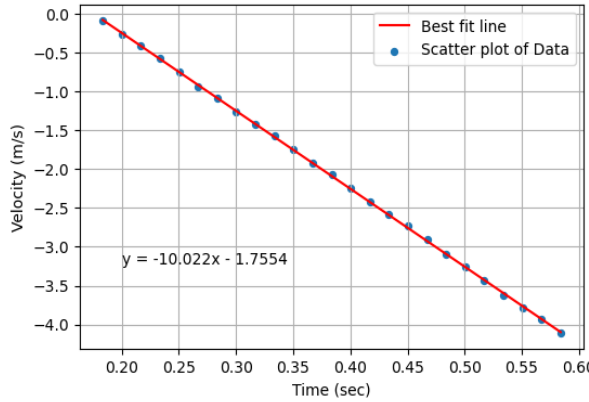


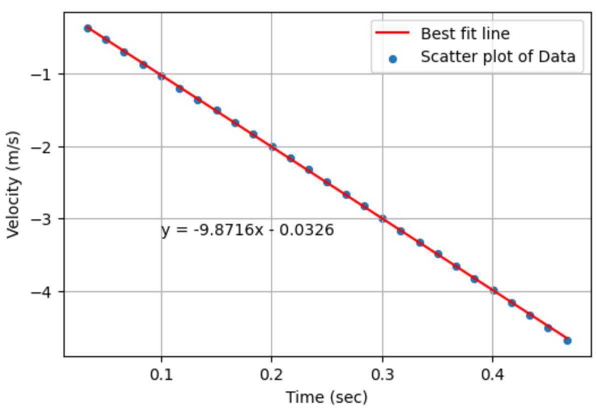
(l) Data-set 12



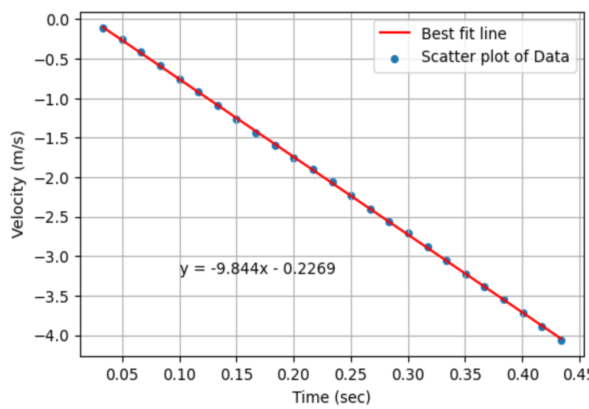
(m) Data-set 13



(n) Data-set 14



(o) Data-set 15



(p) Data-set 16

Figure 2: Velocity-time graphs for Squash Ball

The slopes of the above graphs give us corresponding values of acceleration. These are enlisted in the table below -

Trial Number	Value of g from slope (m/s^2)	Trial Number	Value of g from slope (m/s^2)
1	-9.9098	9	-9.9560
2	-9.8267	10	-10.0669
3	-9.8513	11	-10.2226
4	-9.7538	12	-10.2445
5	-9.9284	13	-10.1754
6	-9.9340	14	-10.0220
7	-9.9567	15	-9.8716
8	-9.7521	16	-9.8440

Table 1: Values of g for Squash Ball trials

Now, in order to get a single value of g which is a good representative of all these 16 trials, we plot a histogram of the values in Table 1. Then, we take the arithmetic mean of the values inside the class with highest frequencies. This will be our reported value of g for the squash ball. The histogram is as follows -

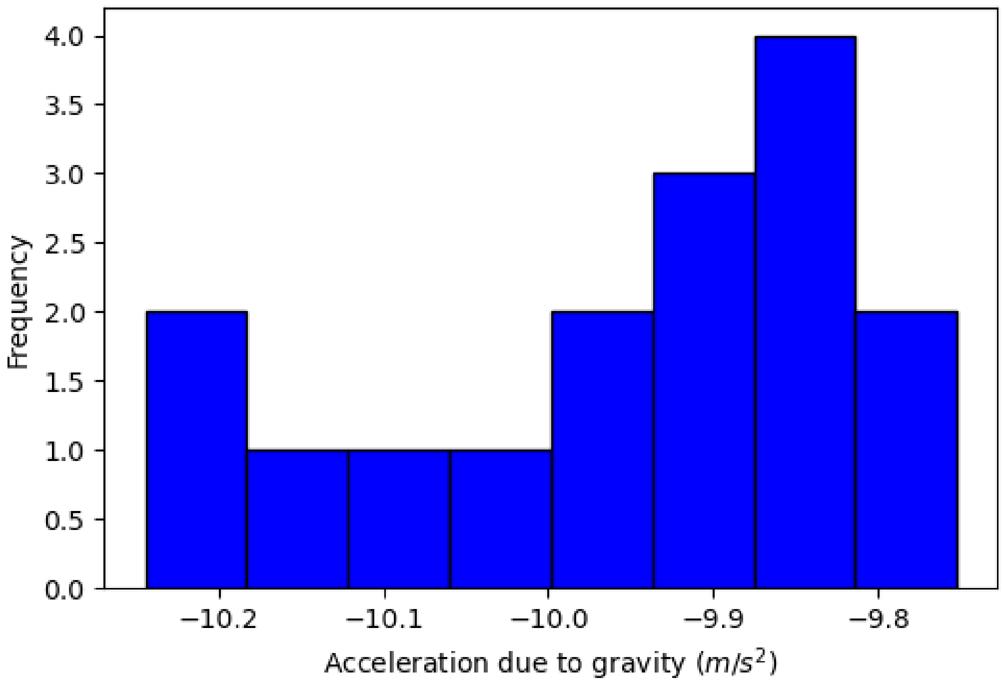


Figure 3: Histogram of g values for Squash Ball

The highest frequency for a single class according to the above histogram is **4**. The values of this class lie between **-9.82** to **-9.88** m/s^2. The arithmetic mean of the values is as follows -

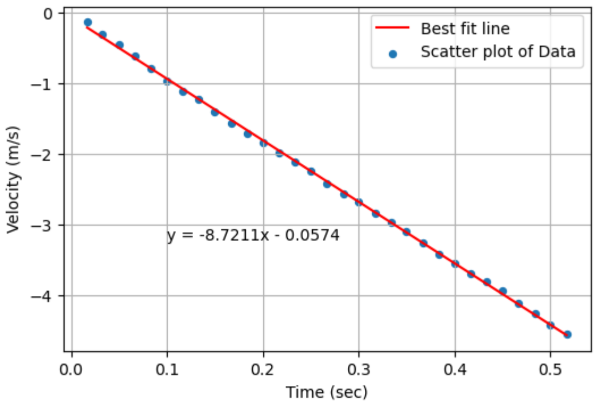
$$\begin{aligned} g_{\text{avg}} &= -\left(\frac{9.8267 + 9.8513 + 9.8716 + 9.8440}{4}\right) \\ &= -9.8484 \;\; m/s^2 \end{aligned}$$

Thus, the average value of gravitational acceleration obtained by tracking the free-falling motion of a squash ball is -

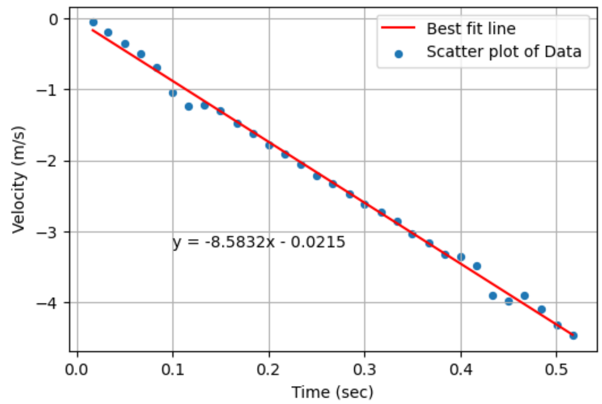
$$\boxed{g = -9.8484 \;\; m/s^2} \tag{4}$$

6.2.2 Table Tennis Ball

The mass of the table tennis ball was 2.56 grams, recorded by an electronic weighing machine with least count = 0.1 gram. Its diameter was 3.76 cm (thus, radius $r = 3.76/2 = 1.88$ cm), measured by a vernier caliper of least count 0.02 mm. The table tennis ball was dropped and recorded 14 times. Its motion was tracked for all trials, and the vertical velocity v_y was plotted with time. The slopes of the best-fit lines (made using matplotlib's inbuilt `np.polyfit()` function) give us the acceleration for that data-set. The v_y vs time graphs for the 14 data-sets are as follows -



(a) Data-set 1



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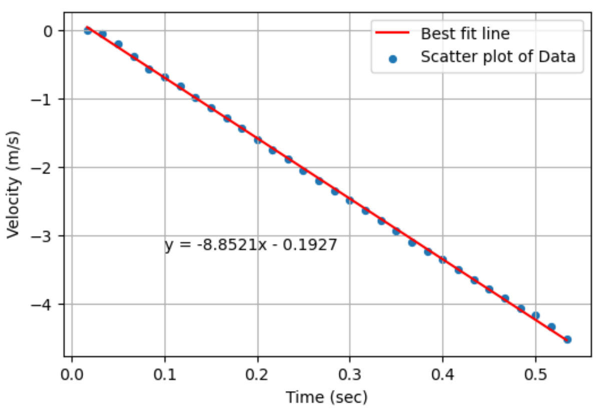

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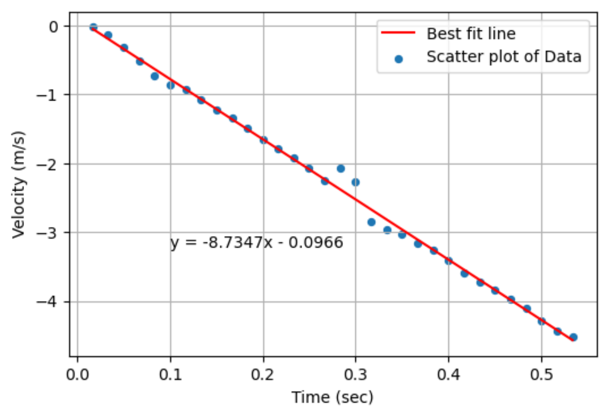

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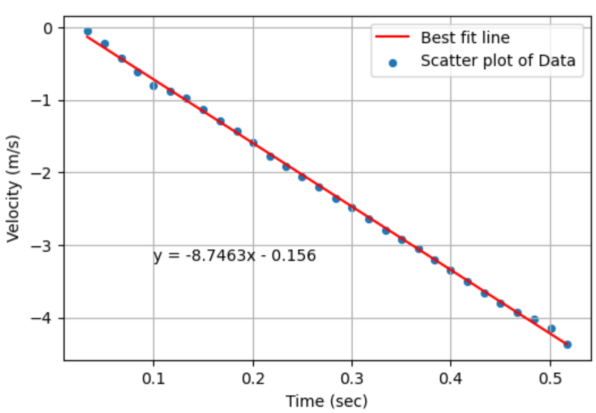

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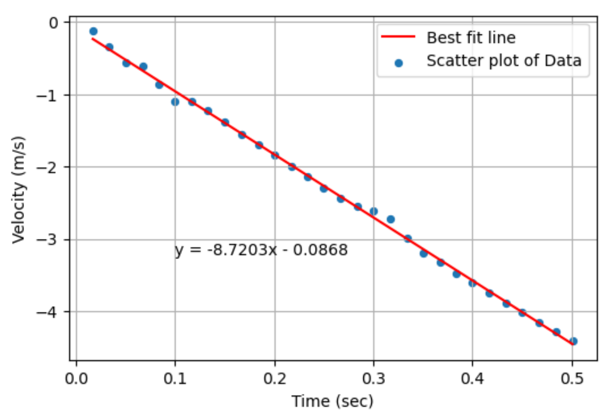

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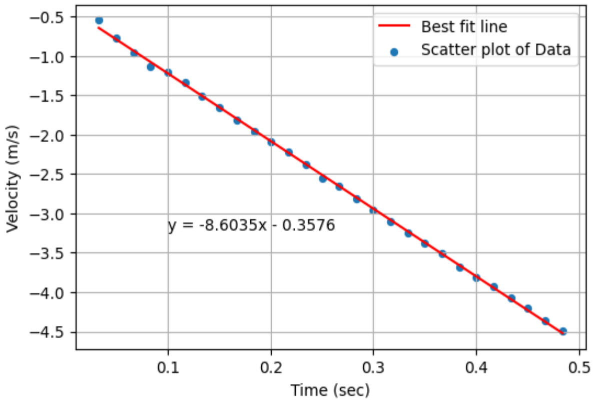

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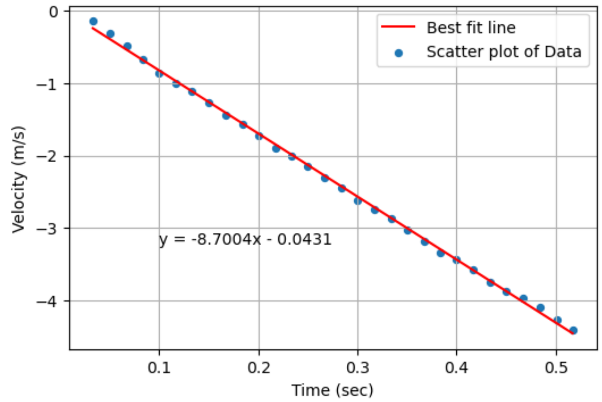
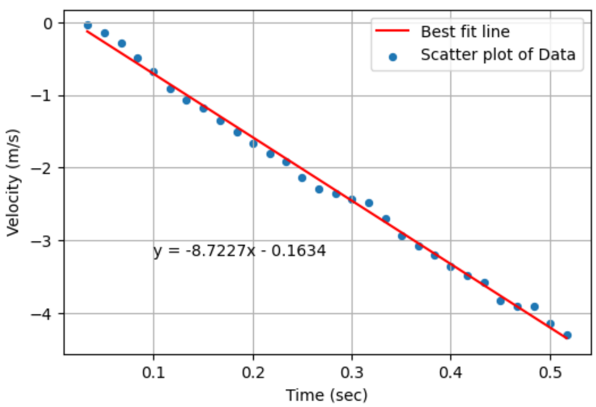
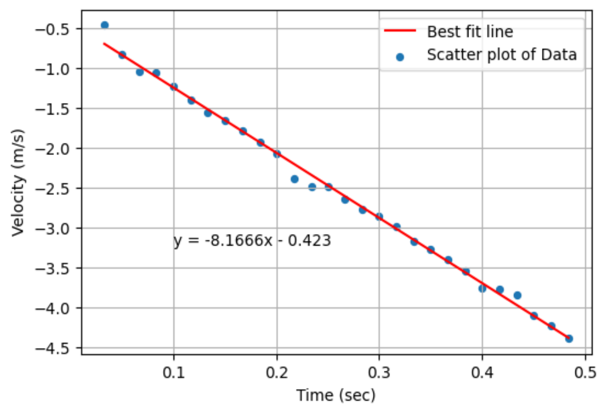


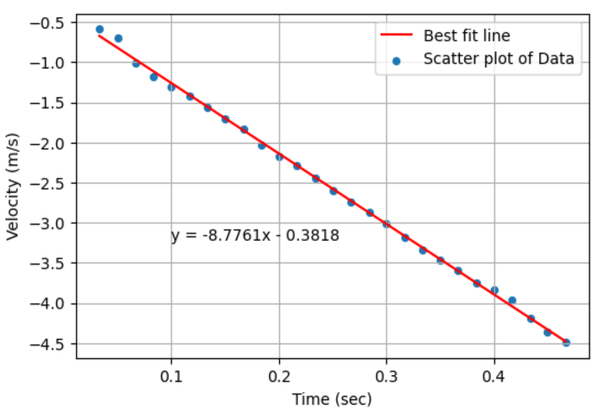
(h) Data-set 8



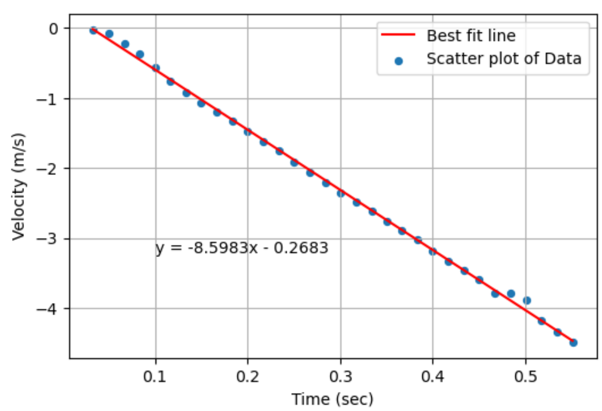
(i) Data-set 9



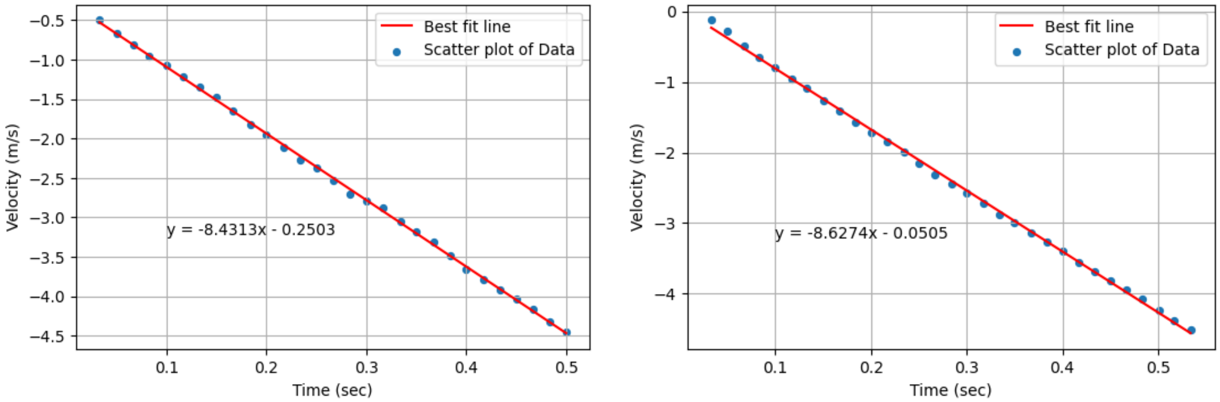
(j) Data-set 10



(k) Data-set 11



(l) Data-set 12



(m) Data-set 13

(n) Data-set 14

Figure 4: Velocity-time graphs for Table Tennis Ball

The slopes of the above graphs give us corresponding values of acceleration. These are enlisted in the table below -

Trial Number	Value of g from slope (m/s^2)
1	-8.7211
2	-8.5832
3	-8.8521
4	-8.7347
5	-8.7463
6	-8.7203
7	-8.6035

Trial Number	Value of g from slope (m/s^2)
8	-8.7004
9	-8.7227
10	-8.1666
11	-8.7761
12	-8.5983
13	-8.4313
14	-8.6274

Table 2: Values of g for Table Tennis Ball trials

Now, in order to get a single value of g which is a good representative of all these 14 trials, we plot a histogram of the values in Table 2. Then, we take the arithmetic mean of the values inside the class with highest frequencies. This will be our reported value of g for the table tennis ball. The histogram is as follows -

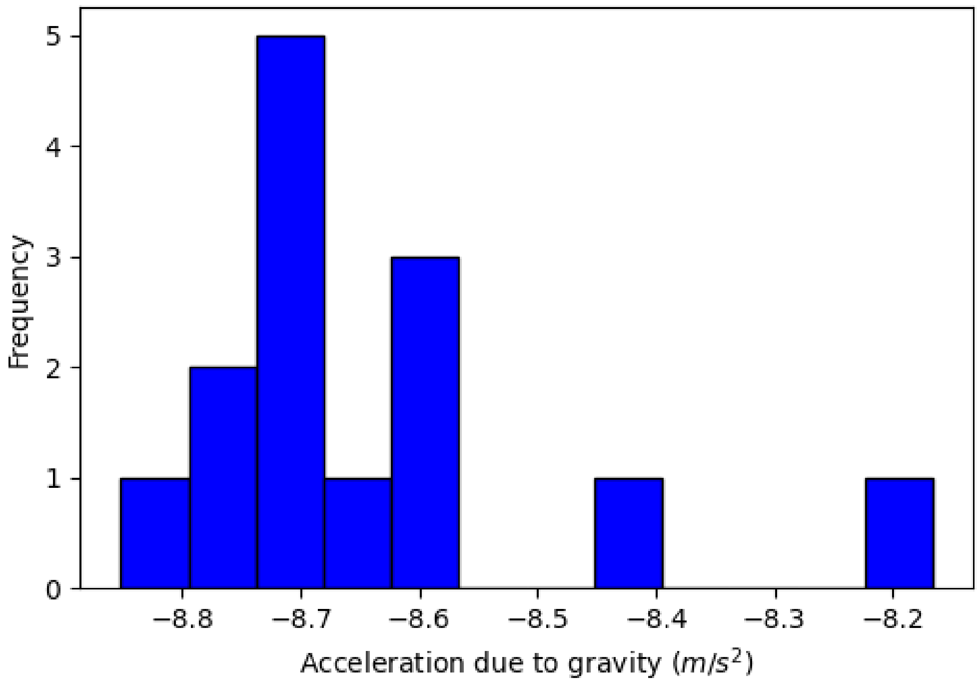


Figure 5: Histogram of g values for Table Tennis Ball

The highest frequency for a single class according to the above histogram is **5**. The values of this class lie between **-8.69** to **-8.74** m/s^2. The arithmetic mean of the values is as follows -

$$\begin{aligned} g_{\text{avg}} &= -\left(\frac{8.7004 + 8.7203 + 8.7211 + 8.7227 + 8.7347}{4}\right) \\ &= -8.7198 \ \ m/s^2 \end{aligned}$$

Thus, the average value of gravitational acceleration obtained by tracking the free-falling motion of a squash ball is -

$$\boxed{g = -8.7198 \ \ m/s^2} \tag{5}$$

6.3 Results

Thus, we can see that the values of acceleration due to gravity (g) we get from the squash ball trials and the table tennis ball trials are quite different. The squash ball value (-9.84 m/s^2) is only slightly higher than the true local value (-9.79 m/s^2). But the value obtained from the table tennis ball trials (-8.7198 m/s^2) is significantly lower than the actual value. This may be due to its significantly lower weight, due to which experiences more air drag. This is explained more elaborately in the Discussion Section.

7 Part B: Relating Terminal Velocity with Falling Mass

In this part, we drop a cupcake liner with increasing masses of magnets from a fixed height and track its motion to find a relation between the falling mass and its achieved terminal velocity.

7.1 Procedure

1. Use the same setup as used in Part A of this experiment. Be extra vigilant about having no wind in the area, since the cupcake liner is especially sensitive to even small air currents, which will affect its motion thus introducing errors in the data.

2. Release the cupcake liner from the fixed height and record its free fall.

3. Then, take three small magnets and stick them onto the cupcake liner, two above and one below. Now, release this assembly from the same height and record a video.

4. Increase the magnets by 3 (and thus, the falling mass) in every subsequent iteration. Record videos for all masses.

5. Ensure a stable distribution of magnets above and below the surface of the cupcake liner. This is more important as we go towards using maximum number of magnets.

6. Transfer all videos to a computer, track the motion of the cupcake liner and magnet assembly using *Tracker* and export .csv data files. The tracking procedure is explained clearly in Section 5 of this report.

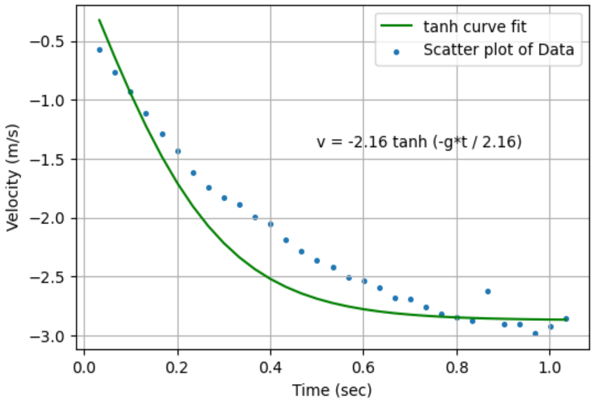
7. Plot the v_y vs time graphs for each data file. Fit the graphs to a *tanh* function with suitable parameters, as shown in the theory. This will enable us to derive a relation between the falling mass and its corresponding terminal velocity.

7.2 Observations

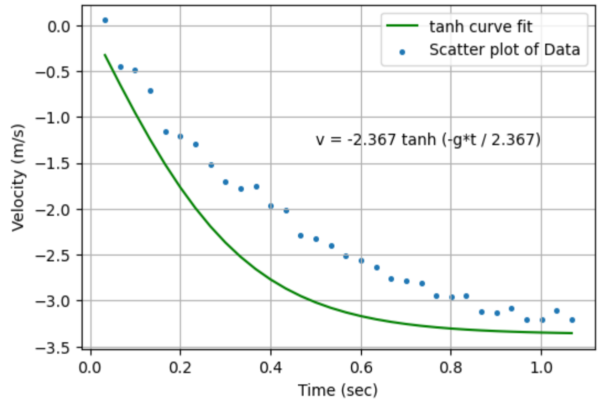
The mass of the cupcake liner released from a specific height was 0.5 grams. We released the cupcake liner with no added mass of magnets, recorded and tracked the motion. However, due to the extreme lightweight-ness of the cupcake liner, even small currents in the surrounding air hampered its free-fall motion. These random perturbations gave horizontal components of velocity to the cupcake liner, which was not desirable. Thus, we started with 12 magnets attached to the cupcake liner. We then added 3 magnets per trial for the subsequent trials and went till 42 magnets. The mass of a single magnet was 0.1 grams, taken on a weighing machine with least count 0.1 mg. The v_y vs time graphs are expected to follow the relation given by Equation 3 -

$$\boxed{v = v_t \tanh\left(\frac{g}{t} v_t\right)} \tag{6}$$

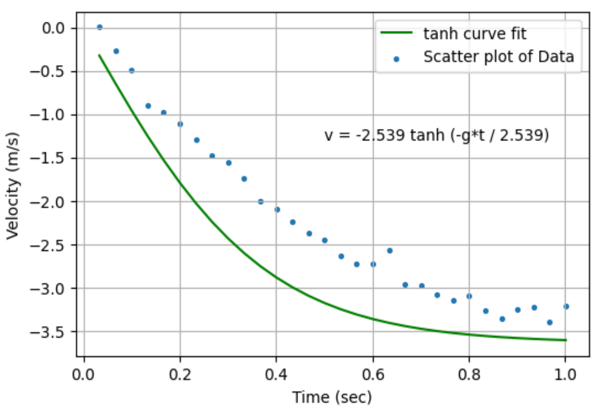
Thus, we plot the v_y vs time graphs along with the best-fit tanh functions for all 10 data sets as follows -



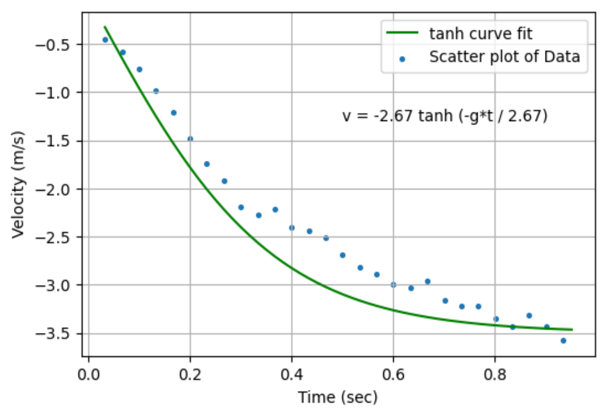
(a) Data set for cupcake liner + 12 magnets



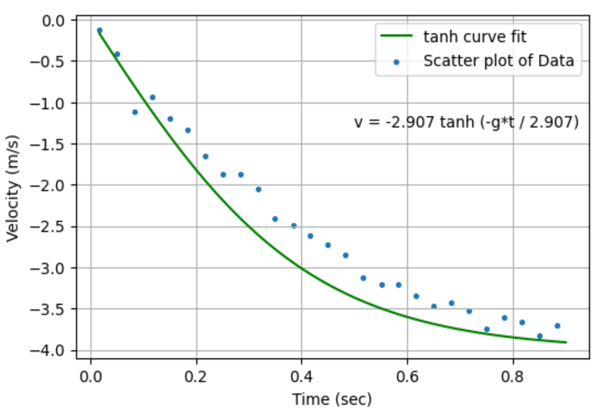
(b) Data set for cupcake liner + 15 magnets



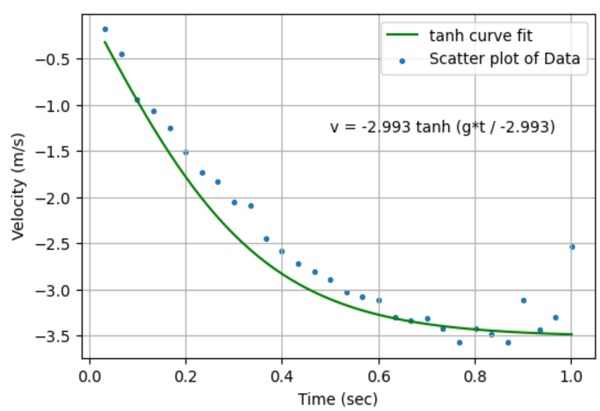
(c) Data set for cupcake liner + 18 magnets



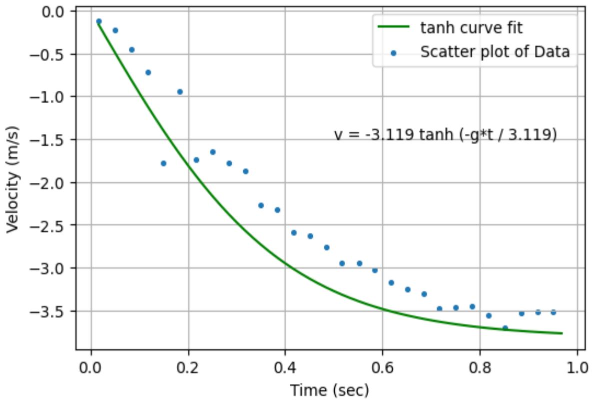
(d) Data set for cupcake liner + 21 magnets



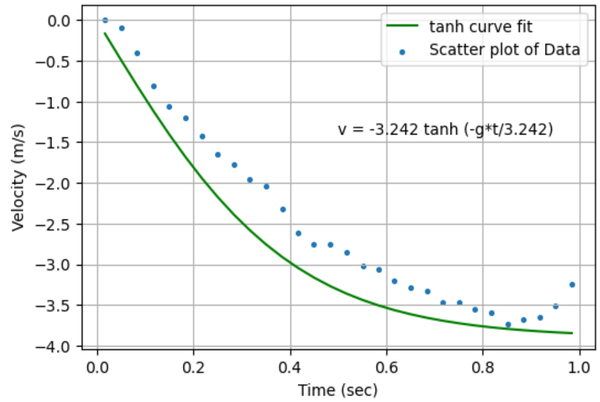
(e) Data set for cupcake liner + 27 magnets



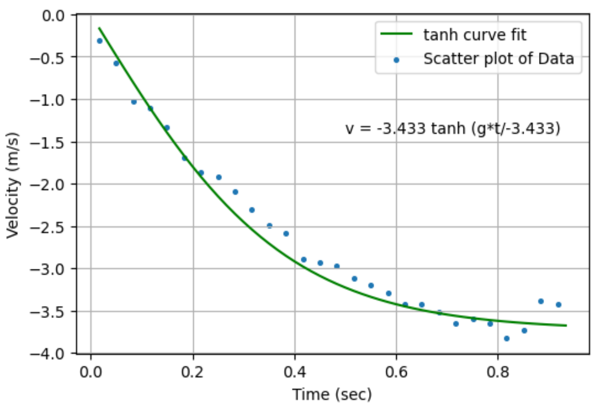
(f) Data set for cupcake liner + 30 magnets



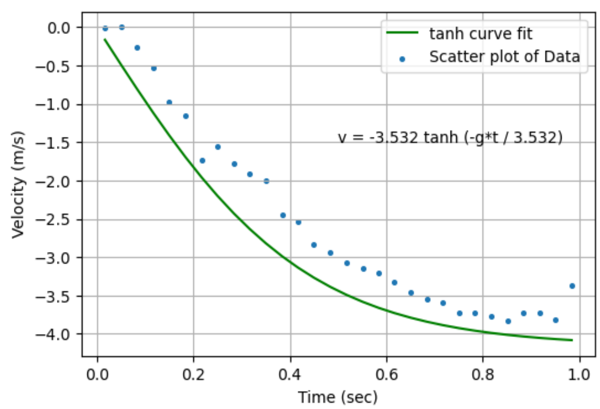
(g) Data set for cupcake liner + 33 magnets



(h) Data set for cupcake liner + 36 magnets



(i) Data set for cupcake liner + 39 magnets



(j) Data set for cupcake liner + 42 magnets

Figure 6: Velocity-time graphs for cupcake liner with magnets

The coefficient of the tanh function gives us the corresponding terminal velocity for that amount of mass added to the cupcake liner. This will be useful in experimentally determining a relation between falling mass m and terminal velocity v_t. The table below shows the falling masses and corresponding terminal velocities obtained from the coefficient of tanh from the best fit plots of all data sets, as shown in the graphs above -

No. of magnets added to cupcake liner = n	Total falling mass (gm) = (0.1)*n + 0.5	Terminal velocity achieved (m/sec) (downward)
12	1.7	2.160
15	2.0	2.367
18	2.3	2.539
21	2.6	2.670
27	3.2	2.907
30	3.5	2.993
33	3.8	3.119
36	4.1	3.242
39	4.4	3.433
42	4.7	3.532

Table 3: Falling masses and terminal velocities for Part B

To identify a clear relation between the above two quantities, we now plot a log-log plot of them and gain a power law relation. This can be mathematically understood as follows -

$$
\begin{aligned}
v_t &\propto m^i \\
\implies v_t &= k m^i \\
\implies \log(v_t) &= \log(k m^i) \\
\implies \log(v_t) &= i \log(m) + \log(k)
\end{aligned}
$$

Thus, the slope of the best-fit line of the log-log plot will be equal to that index i.e. power of m to which terminal velocity is proportional. The log-log plot is as follows -

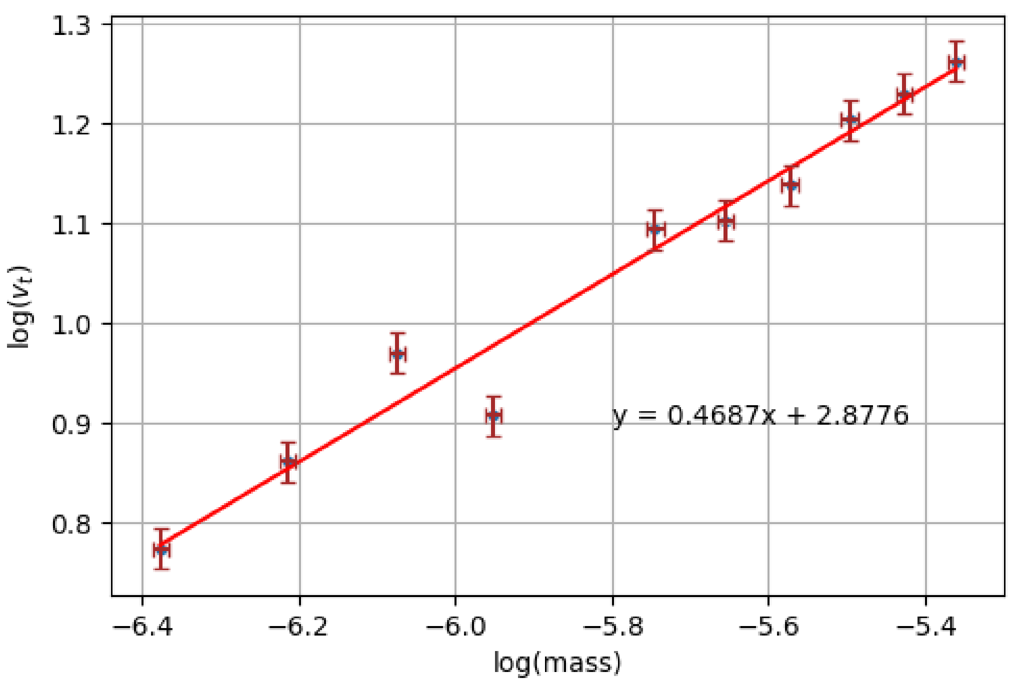


Figure 7: log-log plot to determine power law

From the best fit trend-line's equation, we get that the slope is **0.4687**. We have derived the relation $v_t = \sqrt{\frac{mg}{\alpha}}$ in the theory section. Hence, the expected power law (theoretically) is **0.5**. We can see that the obtained value of the power law is quite close to the theoretically expected value.

7.3 Results

In this part, we plotted the v_t vs time graphs for a variety of falling masses, tracked their motion and plotted a log-log graph between v_t and falling mass m to experimentally derive a relation between the two quantities. We experimentally obtained a power law relation as follows -

$$\boxed{v_t \propto m^{0.4687}}$$

8 Precautions and Error Sources

1. Ensure that the camera is aligned correctly i.e. it is perfectly perpendicular to the line along which the objects fall. If it is not aligned well, there will be an error introduced in calibration of distances in the *Tracker*.

2. Ensure that the entire trajectory of the falling mass lies inside the frame of the camera. This is important to get enough correct data points from the tracking.

3. Ensure that no horizontal or vertical velocity is imparted to the falling mass while releasing, since this will hamper its free fall motion.

4. Ensure that the stickers used for the calibration stick are more or less in the same plane as the falling mass. This will minimize the parallax error in the distance calibration.

9 Error Analysis

9.1 Part A

The error propagated in the calculation of the g value is given by the expression -

$$\sigma' = \frac{\sigma}{\sqrt{n}}$$

where σ is the standard deviation of the histogram, and n is the number of entries in the histogram.

The relative percentage error in the value of g is given by the expression -

$$\% \text{ error in g} = \frac{g_{\text{obtained}} - g_{\text{actual}}}{g_{\text{actual}}} \times 100$$

9.1.1 Squash Ball Trials

For these trials, $\sigma = 0.1483$, $n = 16$, $g_{\text{obtained}} = -9.8484\ m/s^2$,and $g_{\text{actual}} = -9.7923\ m/s^2$. Thus we have -

$$\begin{aligned}
\sigma' &= \frac{0.1483}{\sqrt{16}} \\
&= 0.037 \\
\%\ \text{error in g} &= \frac{-9.8484 - (-9.7923)}{-9.7923} \times 100 \\
&= 0.0057 \times 100 \\
&= 0.57\%
\end{aligned}$$

9.1.2 Table Tennis Ball Trials

For these trials, $\sigma = 0.1650$, $n = 14$, $g_{\text{obtained}} = -8.7198\ m/s^2$,and $g_{\text{actual}} = -9.7923\ m/s^2$. Thus we have -

$$\begin{aligned}
\sigma' &= \frac{0.1650}{\sqrt{14}} \\
&= 0.044 \\
\%\ \text{error in g} &= \frac{-8.7198 - (-9.7923)}{-9.7923} \times 100 \\
&= -0.1095 \times 100 \\
&= -10.95\%
\end{aligned}$$

This again confirms our presupposed notion that the table tennis ball trials yield much more error than squash ball. This is explained in the discussion section.

9.1.3 Cupcake Liner

The major source of error is the power law value obtained, since it involves the error in curve fitting the best tanh functions. From the expression for index i, we can get the following -

$$\begin{aligned}
\log(v_t) &= i \log(m) + \log(k) \\
\implies i &= \frac{\log(v_t) - \log(k)}{\log(m)} \\
\implies \left(\frac{\Delta i}{i}\right)^2 &= \left(\frac{\Delta v_t}{\log(v_t) v_t}\right)^2 + \left(\frac{\Delta m}{\log(m) m}\right)^2
\end{aligned}$$

For every data-set, we can calculate the propagated error in index i by using values of variables specific to that data-set, using the above derived expression.

The relative error associated with i is given by -

$$\begin{aligned}
\frac{\Delta i}{i} &= \frac{i_{\text{theory}} - i_{\text{obtained}}}{i_{\text{theory}}} \\
\implies &= \frac{0.5 - 0.4687}{0.5} \\
&= 0.0626 \\
\%\text{error} &= 0.0626 \times 100 = 6.26\%
\end{aligned}$$

10 Discussion

1. The most obvious point of discussion in this experiment is the value of g obtained from the table tennis ball trials. The values of the radii of both balls tell us that the TT ball is comparable in size to the squash ball. Since air resistance depends on cross sectional area perpendicular to line of falling, both balls experience similar magnitudes of air drag. But since the mass of the table tennis ball is much lighter than that of the squash ball, the net g value experienced by the table tennis ball is much lower than that experienced by the squash ball. Thus, we can see why the value of g obtained from tracking the motion of the table tennis ball is significantly lower than the true value.

2. The tracking software uses template images and a search function to track the moving object's motion. For fast-moving objects, the image of the object becomes blurry and no longer matches with the original template image, thus making it hard to track. We countered this by putting bright stickers on moving objects, choosing a plain background, increasing frame rate of tracker, using the DSLR in sports mode, customizing the search area shape for every tracked video (elliptical, circular, rectangular, etc.), choosing locations with no/minimal wind, etc.

3. An interesting idea that came up in our discussions was to glean maximum information from the g values obtained in the case of the table tennis ball. Since we know it's cross sectional area, and the effective g value it experiences, we can use the difference in g_{net} and g_{actual} to estimate the drag force experienced by the falling mass in every trial. This can later be extended to objects of variable areas too, to estimate the drag force experienced non-uniform falling masses such as the cupcake liner and magnets assembly. This is a worthy addition that can be done in future replications of this experiment to make it more complete.

4. While incrementally increasing the number of falling magnets, we later noticed that the cupcake liner tore while falling in the trial with 24 magnets. Thus, that data was rendered useless, and not used by me in this report. We also recorded data and plotted graphs for number of magnets greater than 42 (till 47) and less than 12. For more magnets, we noticed that the falling assembly was barely reaching any terminal velocity, while for less magnets it was too lightweight and thus was fluttering around quite a lot due to small air currents. Thus, I have not included these data sets in my report.

5. Many data-sets gave very condensed, overlapping scatter points when plotted. This introduced unnecessary errors for the `np.polyfit()` function to find the best fit curve. I tried to fix this by plotting a smoother version of the data-set, by sampling out only alternate data points. This did not change the nature of the data set points at all (since the variation over a single point is negligible), but made it easier for the polyfit function to get a better fitting curve for the smoother data set.

11 Appendix

All data files (in *.csv* format) are attached with this report in a zipped folder.

12 References

- Ashoka University, PHY-2010 Lab Handout, Free Fall experiment