Experiment 3: Electronic Circuits
Lab Report

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1 Introduction

Electronic circuits form the basis of every electronic device we use in our day-to-day lives. Televisions, mobile phones, refrigerators, air-conditioners, cars, computers; every one of these products function due to the **intricate design** of many kinds of electronic circuits inside them. In this era of developing and using **technological prowess** to enhance the **lifestyle of humans**, it is important that we understand the basic kinds of electronic components, their functions, and usage in forming more **complex and relevant** electronic circuits.

We now take a brief look at two major ways in which a variety of basic electronic components (resistors, capacitors, and inductors) **respond** when a current is passed through them -

- **Resistance (R):** It is a measure of the **opposition** to the **flow of current** through the given electronic component. It is measured in **Ohms** (Ω). When an alternating current (AC) passes through a resistor, the voltage across that resistor drops **in-phase** with the current. The magnitude of this drop is given by **Ohm's Law**: $V = IR$.
  Resistance R remains **constant with varying frequency**.

- **Reactance (X):** It is a measure of the **opposition** to a **change in current** passing through the electronic component. It is also measured in **Ohms** (Ω). When an alternating current (AC) passes through a resistor, the voltage across that resistor drops **90° out of phase** with the current. It is most notably observed in **capacitors** and **inductors**. The expression for **Capacitive reactance** (X_C) and **Inductive reactance** (X_L) is as follows -

$$X_C = \frac{1}{2\pi f C} \tag{1}$$

$$X_L = 2\pi f L \tag{2}$$

  where C is the **capacitance** of the capacitor, L is the **inductance** of the inductor and f is the **frequency** of the input signal.
  As evident from the expressions, Reactance X **varies with frequency.**

Perfect resistors possess only resistance (R), no reactance. Perfect capacitors and inductors possess only their respective reactances (X_C, X_L), no resistance.

2 Aim

- **Part A:** To study the response of an RC circuit (to a sinusoidal waveform input) as a **low-pass** and **high-pass** filter.
- **Part B:** To study the output waveforms of an RC circuit (to a square waveform input) to verify the **integrator** and **differentiator** circuit.
- **Part C:** To study the variation of **current** and **voltage** with frequency in a **series** and **parallel LCR circuit** respectively, and to plot the quantities on a graph and compare the theoretical and experimental values of the **resonance frequency**.

3 Apparatus

1. A breadboard (on which the circuits are constructed)
2. A signal generator (to provide signals of specified waveforms and frequencies to the circuit)
3. A digital oscilloscope (to compare the input and output waveforms of the circuit, minimize noise, etc.)
4. Wires (to make the connections in a circuit)
5. 3 Digital Multimeters (to measure the current and voltage across specific components and at certain locations in the circuit.)
6. Resistors (1 $k\Omega$, 10 $k\Omega$, 15$k\Omega$, 27$k\Omega$, 33$k\Omega$, 75$k\Omega$, 100$k\Omega$, 330$k\Omega$, 470$k\Omega$, 980$k\Omega$)
7. Inductors (30 mH, 60 mH)
8. Capacitors (0.047 μF, 0.01 μF)

4 Theory

4.1 RC response to D.C. signal

Let a DC voltage V be applied to a series R-C circuit as shown

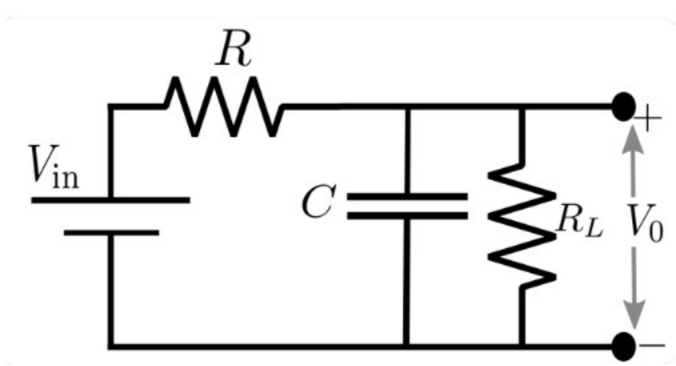


Figure 1: RC circuit with DC power supply

By **Kirchoff's Voltage Law** we can find the total voltage drop across the entire circuit by adding up the **voltage drop** across the individual electronic components (R and C). We use the expressions relating **voltage** and **current** using **material-dependent** properties like **resistance** and **capacitance** to do this, as follows -

$$RI + \frac{1}{C}\int I dt = V \tag{3}$$

Differentiating both side of the above equation w.r.t. time we get,

$$\begin{aligned} R\frac{dI}{dt} + \frac{I}{C} &= 0 \\ \implies \frac{dI}{dt} + \frac{I}{RC} &= 0 \end{aligned} \tag{4}$$

Let our **ansatz** be $I(t) = Ne^{kt} \implies \dot{I} = Nke^{kt}$.
Substituting this in the above equation, we get

$$\begin{aligned} Nke^{kt} + \frac{Ne^{Kt}}{RC} &= 0 \\ \Rightarrow Ik + \frac{I}{RC} &= 0 \\ \implies I\left(k + \frac{1}{RC}\right) &= 0 \end{aligned}$$

For the **non-trivial** solution, we cannot have $I = 0$. Thus, the only remaining option is - $k = -\frac{1}{RC}$, therefore -

$$I(t) = Ne^{\frac{-t}{RC}}$$

We can say that at ($t = 0$), the current started with its maximum value, I_0 (assuming capacitor is not charged initially). Thus, we have $I(t = 0)$ as N, which will be $\frac{V}{R}$. On substituting this in the expression for current $I(t)$ above, we get -

$$I(t) = \frac{V}{R}e^{\frac{-t}{RC}}$$

From observing the nature of the above expression, we can say that the **charging current** I is an **exponentially decaying** current. Using **Ohm's law** and expression relating V_C and $I(t)$ on this, we get -

$$\begin{aligned} V_R(t) &= I(t)R \\ \implies V_R(t) &= Ve^{-\frac{t}{RC}} \\ V_C(t) &= \frac{1}{C}\int_0^t I(t)dt \\ &= \frac{1}{C}\int_0^t \frac{V}{R}e^{\frac{-t}{RC}}dt \\ \implies V_C(t) &= V\left(1 - e^{-\frac{t}{RC}}\right) \end{aligned}$$

From the above expressions, we conclude that V_R is an **exponentially decaying** function while V_C is an **exponentially increasing** function. Thus, the **resistor** acts as a **charge and energy dissipating element** whereas **capacitor** is a **non- dissipating element** in the circuit. It is important to note that V is the **steady state voltage** across the capacitor.

4.2 RC Circuit as a Low-pass filter

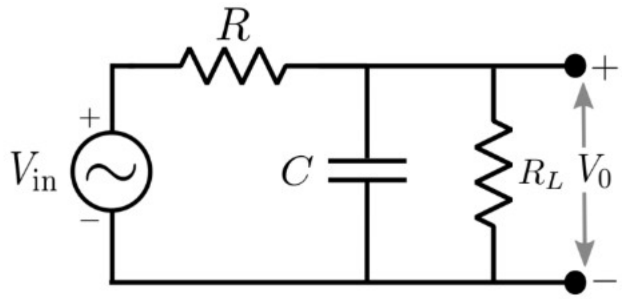


Figure 2: Low-pass filter RC circuit

As the name suggests, a **low-pass filter** is a specific kind of RC circuit that "filters out" the low frequencies from other unwanted higher frequencies in any input signal. The frequencies are classified into 'low' and 'high' by defining a certain **cutoff frequency** f_c.
A low-pass filter allows frequencies **lesser than** f_c to pass, while **attentuating** (blocking) all frequencies **greater than** f_c. Thus, we can see that value of this f_c becomes critical. It is dependent on the combination of resistance and capacitance used in the circuit (specifically on the circuit's **RC time constant**). Thus, we can filter out the **exact** range of frequencies needed by using the appropriate values of R, C in our circuit. The expression for the **cutoff frequency** of the circuit is as follows -

$$f_c = \frac{1}{2\pi RC} \tag{5}$$

We can understand the "low-pass filtering behavior" of the circuit from the nature of equations (1) and (5) as follows -

- At low frequencies, the value of X_C becomes very high. For $f \to 0$, we get $X_C \to \infty$ (see equation 1). Thus, at low frequencies, the capacitor **strongly opposes** a change in the current passing through it. Thus, it acts as an open circuit, and allows signals with low frequencies to pass through. We can measure these signals across the load resistor R_L at the output.

- At high frequencies, the value of X_C becomes very low. For $f \to \infty$, we get $X_C \to 0$ (see equation 1). Thus, at high frequencies, the capacitor **doesn't oppose** a change in the current passing through it. Thus, it acts as a **shorted** circuit, and doesn't allow signals with high frequencies to pass through. Thus, these are filtered out.

For calculating Gain in a Low Pass Filter we use -

$$\text{Gain (in dB)} = -20 \log \left(\frac{V_0}{V_{in}} \right)$$

4.3 RC circuit as an integrator

At frequencies higher than f_c, the same **RC circuit** acts like an **integrator.** At high frequencies, since the RC circuit is essentially **shorted** by the capacitor, it gets charged gradually due to the current passing through it (low opposition to change in current at high f). This charging reflects as an **integration** of the input signal's waveform, and the integrated waveform can be observed on the oscilloscope at the output. Thus, at high frequencies (higher than f_c), a **cosine wave input** produces a **sine wave output**, or a **square wave input** produces a **triangular wave output**. The integration becomes more and more accurate as the frequency increases. This **integrating behavior** can be mathematically shown as follows -

Consider the circuit as shown in Figure 1. The instantaneous current I passing through the capacitor is given by -

$$I = \frac{dQ}{dt}$$

The expression which relates the charge Q passing through a capacitor of capacitance C with the voltage V_C across it is -

$$Q = CV_C$$

Thus, by combining both the above equations, we get -

$$I = C\frac{dV_C}{dt}$$

To relate the resistance R with the current and voltage, we use **Ohm's Law** to get -

$$I = \frac{V_{\text{in}}}{R}$$

Now, we know that at high frequencies, the **capacitive reactance** X_C of the circuit is very low (ideally negligible). Thus, the overall resistance of the circuit is very close to the resistance R of the resistor. Thus, we can consider the **input voltage** to be **equivalent** to the **voltage across the resistor** V_R. Therefore, we have -

$$I = \frac{V_{\text{in}}}{R} = \frac{V_R}{R} = C\frac{dV_C}{dt}$$

Since we are measuring the **output voltage** across the capacitor, $V_C = V_0$. On solving the above equality for V_C, we get -

$$\begin{aligned} dV_0 &= \frac{1}{RC} V_{\text{in}} dt \\ \implies V_0 &= \frac{1}{RC} \int V_{\text{in}} dt \\ \implies V_0 &\propto \int V_{\text{in}} dt \end{aligned}$$

Thus, we can clearly see that the **output voltage** is **directly proportional** to the **integral of the input voltage**, thus justifying the "integrating behavior" of the circuit mathematically.

4.4 RC circuit as a High-pass filter

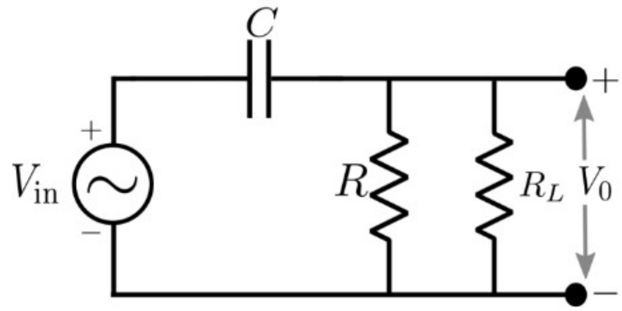


Figure 3: High-pass filter RC circuit

Analogically, a **high-pass filter** is a specific kind of RC circuit that "filters out" the high frequencies from other unwanted (lower) frequencies in any input signal. The frequencies are classified into 'low' and 'high' by the same **cutoff frequency** f_c.
A high-pass filter allows frequencies **higher than** f_c to pass, while **attentuating** (blocking) all frequencies **lesser than** f_c. Here too, we can see that value of this f_c becomes critical. Like for the low pass filter, the expression for the **cutoff frequency** of the circuit is the same -

$$f_c = \frac{1}{2\pi RC}$$

Similarly, we can understand the "high-pass filtering behavior" of the circuit from the nature of equations (1) and (5) as follows

- At low frequencies, the value of X_C becomes very high. For $f \to 0$, we get $X_C \to \infty$ (see equation 1). Thus, at low frequencies, the capacitor **strongly opposes** a change in the current passing through it. Thus, the current faces high resistance to travel to the load resistor. Since output voltage is measured across the **load resistor** R_L, there is negligible/no output signal obtained at frequencies lower than f_c. Thus, these are filtered out.

- At high frequencies, the value of X_C becomes very low. For $f \to \infty$, we get $X_C \to 0$ (see equation 1). Thus, at high frequencies, the capacitor **doesn't oppose** a change in the current passing through it. Thus, it acts as a **shorted** circuit, and provides a path of **almost** zero resistance to the current. Therefore, we can measure those high frequencies across the load resistor as output signals.

For calculating Gain in a High Pass Filter we use -

$$\text{Gain (in dB)} = 20log\left(\frac{V_0}{V_{in}}\right)$$

4.5 RC circuit as a differentiator

At frequencies lower than f_c, the same **RC circuit** acts like a **differentiator.** Analogically to the integrator circuit, since the **high-pass filter** circuit just has the positions of R and C exchanged, the opposite effect must be observed. Thus, at low frequencies (lower than f_c), a **sine wave input** produces a **cosine wave output**, or a **square wave input** produces a **short duration pulse** or **spiked output**. The differentiation becomes less and less accurate as the frequency increases. This **differentiating behavior** can be mathematically shown as follows -

Consider the circuit as shown in Figure 3. The total voltage drop is the sum of individual voltage drops across the capacitor and resistor -

$$V_{\text{in}} = V_C + V_R$$

Using the I, R and V relation for capacitor and resistors, we get -

$$V_{\text{in}} = \frac{Q}{C} + R\frac{dQ}{dt}$$

Since both C, R are very small, and the C term is in the denominator, it dominates over the second term. Thus, we get -

$$V_{in} = \frac{Q}{C}$$

Now, we rearrange this expression to solve for Q and differentiate w.r.t. time to get -

$$\frac{dQ}{dt} = C\frac{dV_{\text{in}}}{dt}$$

Using **Ohm's Law** for the output voltage V_0 across the resistor, we have -

$$V_0 = R\frac{dQ}{dt}$$

Substituting the expression for current into the above equation, we get -

$$V_0 = RC\frac{dV_{\text{in}}}{dt}$$
$$\implies V_0 \propto \frac{dV_{\text{in}}}{dt}$$

Thus, we can clearly see that the **output voltage** is **directly proportional** to the **derivative of the input voltage**, thus justifying the "differentiating behavior" of the circuit mathematically.

4.6 LCR series resonance circuit

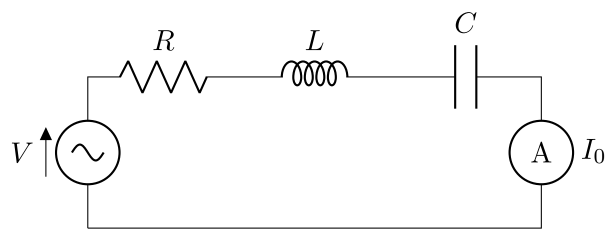


Figure 4: LCR Series Resonance circuit

In a series LCR resonant circuit, the quantity that is measured and plotted is always **current**, since it remains constant throughout the circuit. The total **voltage drop** across the circuit is given by the sum of the **voltage drops** across each of the 3 electronic components. These can be calculated by the respective relationships between V and I using constants **characteristic of that particular electronic component** such as R, C and L. Thus, we have -

$$\begin{aligned}(V_{\text{drop}})_{total} &= (V_{\text{drop}})_R + (V_{\text{drop}})_L + (V_{\text{drop}})_C \\ &= IR + L\frac{dI}{dt} + \frac{1}{C}\int_0^t I dt\end{aligned}$$

We have the following relation between charge Q and current I -

$$I = \frac{dQ}{dt}$$

On subsituting this relation in our expression for total voltage, and consider *free oscillations* (RHS=0), we get the following **second order L.D.E** -

$$\begin{aligned}L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} &= 0 \\ \implies \ddot{Q} + \frac{R}{L}\dot{Q} + \frac{Q}{CL} &= 0\end{aligned}$$

Using the **Ansatz** $Q(t) = Ce^{kt}$, we can get the characteristic equation of the above D.E as follows -

$$k^2 + \frac{R}{L}k + \frac{1}{CL} = 0$$

On solving this quadratic equation for k, we get the following two solutions -

$$\begin{aligned}k_1 &= \frac{-\frac{R}{L} + \sqrt{\frac{R^2}{L^2} - \frac{4}{CL}}}{2} \\ k_2 &= \frac{-\frac{R}{L} - \sqrt{\frac{R^2}{L^2} - \frac{4}{CL}}}{2}\end{aligned}$$

For this kind of a discriminant, there are 3 kinds of standard solutions; $D < 0, D = 0$ and $D > 0$. For this kind of a series resonant LCR circuit, we try to set a Q-factor greater than 0.5, which implies the **under-damped case**. Thus, we consider the case $D < 0$ and solve further. Since $D < 0$, the solutions will be **complex conjugates** of the form -

$$\begin{aligned}k_1 &= -\frac{R}{2L} + i\omega_1 \\ k_2 &= -\frac{R}{2L} - i\omega_1\end{aligned}$$

where -

$$\omega_1 = \frac{\sqrt{\frac{R^2}{L^2} - \frac{4}{CL}}}{2}$$

The general solution of our D.E is as follows -

$$Q(t) = e^{-Rt/2L}\left(a_1 \cos\omega_1 t + a_2 \sin\omega_1 t\right)$$

Now, we substitute the following normalizing expressions and ideal conditions (R=0) -

$$\sqrt{a_1^2 + a_2^2} = A$$
$$A\cos\phi = a_1$$
$$A\sin\phi = a_2$$

After this substitution, we can write -

$$Q(t) = A\cos\left(\frac{t}{\sqrt{LC}} - \phi\right)$$
$$= A\cos\left(\omega t - \phi\right)$$

Now, to solve for current I, we differentiate the above expression once with respect to time, to get

$$I(t) = -A\omega\sin\left(\omega t - \phi\right)$$

We get the value of our **resonance frequency** by carefully examining the value of ω in this expression -

$$\omega_R = \frac{1}{\sqrt{LC}}$$
$$f_R = \frac{1}{2\pi\sqrt{LC}}\ \mathrm{Hz}$$

This same value can be derived by solving the **resonance condition** $X_L = X_C$ for ω, as given in the lab handout. As is evident, maximum current will flow from the current at resonance condition, as the impedance Z is minimum. The **voltage drop** is also the maximum across the resistance R at resonance condition. In a **capacitor**, it takes time for the voltage across the two plates to build up and the current flows only till a voltage difference exists between them.

4.7 Phase shift around resonance

In a **capacitor**, it takes time for the voltage across the two plates to build up and the current flows only till a voltage difference exists between them.

$$Q = CV$$

and

$$I = \frac{dQ}{dt} \implies I = CdVdt$$

If $V(t) = sin(wt)$,

$$I = C\frac{dV}{dt} = C(cos(\omega t)) = C(sin(\omega t + 90))$$

Therefore, in a **capacitor**, the current leads the voltage.

In an **inductor**, a voltage difference appears as soon as it is applied, however, it takes time for current to start flowing through the inductor, as it converts electrical energy to magnetic energy.

$$V_L = L\frac{dI}{dt}$$

If source voltage is $V = sin(\omega t)$ then,

$$I = \frac{sin(\omega t)}{R}$$

hence,

$$V_L = \frac{L}{R}\frac{dsin(\omega t)}{dt}$$

$$\implies V_L = \frac{L}{R}cos(\omega t) = \frac{L}{R}sin(\omega t + 90)$$

Therefore, in a **inductor**, the voltage leads over the current in this case.

4.8 LCR parallel resonance circuit

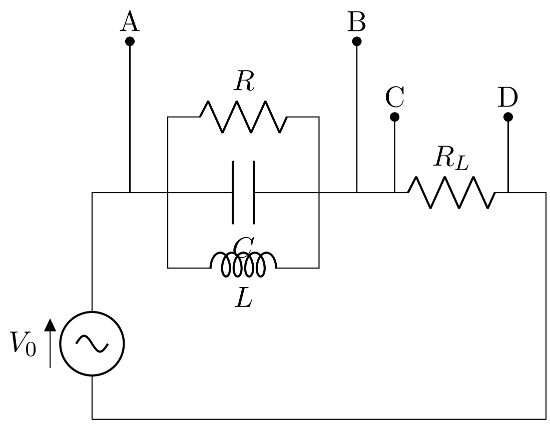


Figure 5: LCR Parallel Resonance circuit

In the above circuit, we measure the output voltage V_0 across points A and B. The voltage across the load resistor (C and D) is denoted by V_L. Similar to the **series resonant circuit**, a parallel LCR circuit as shown in the above figure shows resonance at the condition $X_C = X_L$. The **key difference** between parallel and series LCR circuit is the quantities that remain constant. While current I remains constant throughout the series circuit and voltage V drops, current I passing through every electronic component varies while the voltage V remains constant, in a **parallel LCR circuit.** The expression for net **impedance** Z of a parallel circuit can be obtained as follows -

$$I_{\text{in}}^2 = I_R^2 + (I_L^2 - I_C^2)$$

$$\implies I_{\text{in}} = \sqrt{I_R^2 + (I_L^2 - I_C^2)}$$

$$\implies \frac{V}{Z} = \sqrt{\frac{V^2}{R^2} + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2}$$

$$\implies V\left(\frac{1}{Z}\right) = V\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

Thus, the **net impedance** Z of a **parallel resonant LCR circuit** is given by -

$$Z_{\text{parallel}} = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

From this expression, we can clearly see that when $X_L = X_C$, $Z = R$ (min. value possible). The **capacitor** and **inductor** form an open circuit at resonance, and the overall circuit becomes **purely resistive**.

5 Part A

In this part we construct the specified RC circuits and study their behavior as **low-pass** and **high-pass** filters.

5.1 Procedure

1. Construct a circuit as shown in Figure 2. Ensure that the load resistor (R_L) is **in parallel** to the capacitor, across which the output will be measured.
2. Connect the signal generator and a Digital Storage Oscilloscope (DSO) at the input and across the output of the circuit respectively. This will aid us in comparing the input and output waveforms, and measuring their difference.
3. Calculate the cut-off frequency by substituting the values of R and C used into Equation (5). Let this value be f_c^{th}.
4. Connect digital multimeters across the input and the output, to measure V_{in}, V_0. Ensure they are in the correct setting and sensitivity.
5. Using the signal generator, apply a **sinusoidal** input waveform with a suitable amplitude and frequency.
6. Vary the frequency from 0 Hz to more than $2f_c^{\text{th}} Hz$, and measure the values of V_{in}, V_0 for every input frequency.
7. Record these three data sets (input frequency,V_{in}, and V_0) in three different NumPy arrays.
8. Generate an array for the values of **Gain** from the corresponding values of V_{in}, and V_0 obtained.
9. Plot a graph of **Gain vs input frequency** f and **Gain vs** $\log f$, and study their nature to verify the functioning of our low-pass filter.
10. Now, interchange the position of R and C in the circuit to construct a circuit as shown in Figure 3.
11. Repeat the same procedure, record the same data sets and plot the same graph for this circuit too.

5.2 Observations

The values of components used in the circuit are -

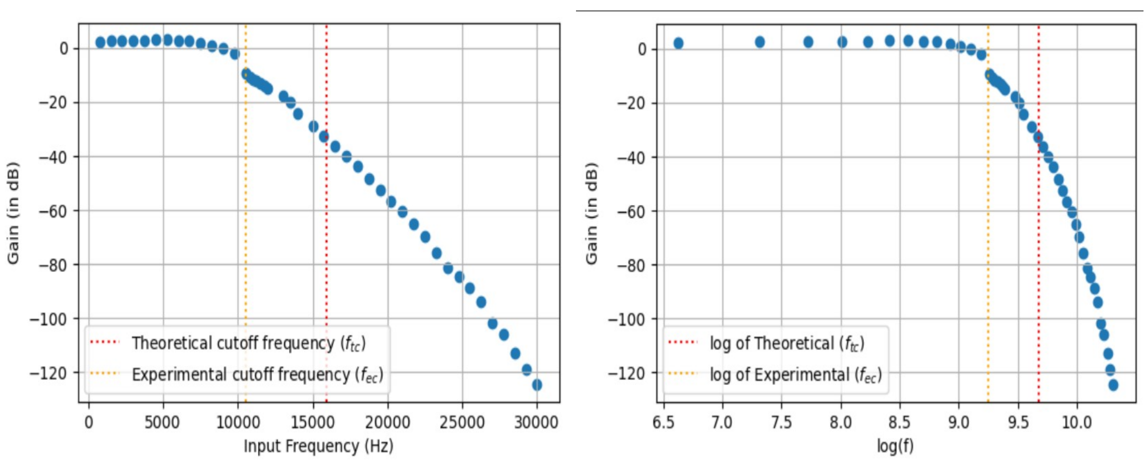
$$R = 1k\Omega,\ C = 0.01\mu F$$

The **theoretical value** of the **cutoff frequency** (f_{tc}) is -

$$\begin{aligned} f_{tc} &= \frac{1}{2\pi RC} \\ &= \frac{1}{2 \times (1000) \times (0.01 \times 10^{-6}} \\ &= 15.915\ k\text{Hz} \end{aligned}$$

5.2.1 Low-Pass filter observations

The following graphs help us to calculate the **experimental values** of the **cutoff frequency** f_{tc}-



(a) Gain vs Input frequency f for low pass

(b) Gain vs log(Input frequency f) for low pass

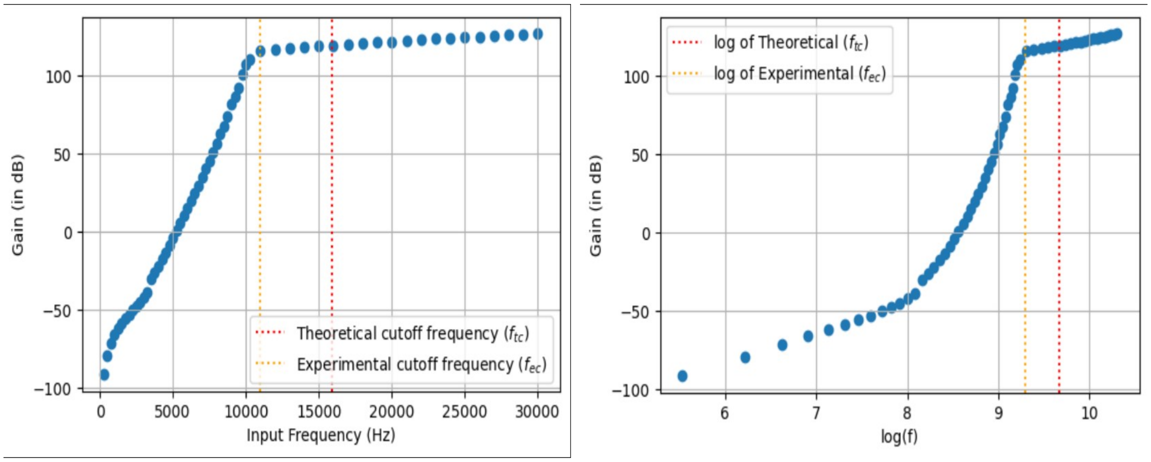
Figure 6: Gain vs input frequency for Low-Pass filter RC circuit

We can **clearly see** and verify the **low-pass behavior** of this RC circuit from the graphs above, since the **Gain** is decreasingly rapidly after the input frequency crosses and becomes more than the **cutoff frequency**.
The approximate **experimental value** of the **cutoff frequency** was 11 kHz (from where the drop in gain starts). Thus, we get -

$$f_{ec} = 11\ k\text{Hz}$$

5.2.2 High-pass filter observations

The following graphs help us to calculate the **experimental values** of the **cutoff frequency** f_{tc}-



(a) Gain vs Input frequency f for high pass

(b) Gain vs log(Input frequency f) for high pass

Figure 7: Gain vs input frequency for High-Pass filter RC circuit

We can **clearly see** and verify the **high-pass behavior** of this RC circuit from the graphs above, since the **Gain** stops increasing rapidly after the input frequency crosses and becomes more than the **cutoff frequency**.
The approximate **experimental value** of the **cutoff frequency** was 11.5 kHz (from where the increase in gain stops/slows down). Thus, we have -

$$f_{ec} = 11.5\ k\text{Hz}$$

5.3 Error Analysis

The error percentage in values of cutoff frequency can be found by the expression -

$$\%\text{error} = \frac{f_{tc} - f_{ec}}{f_{tc}} \times 100$$

For **low-pass** and **high-pass filter** data, we have -

$$\begin{aligned}
\%\text{e}_{\text{low-pass}} &= \frac{15.915k\text{Hz} - 11k\text{Hz}}{15.915k\text{Hz}} \times 100 \\
&\approx 30.8\% \\
\%\text{e}_{\text{high-pass}} &= \frac{15.915k\text{Hz} - 11.5k\text{Hz}}{15.915k\text{Hz}} \times 100 \\
&\approx 27.7\%
\end{aligned}$$

Such high values of error can be attested to multiple systematic sources of **error propagation** such as faulty **multimeter** readings, range of readings for a single input frequency, high **resistance** in wires, errors in **manufacturing** of electronic components of given values, etc. Some **random errors** like objects in the **vicinity** of the multimeter which cause varying amounts of **interference** in every reading may also contribute to a large part of propagating this error. Details on some specific error sources can be found in the **Discussion Section**.

5.4 Result

From this part of the experiment, we can clearly verify the **low-pass** and **high-pass** behavior of the given **RC circuit** (albeit with a relatively high error percentage).

6 Part B

In this part we use the same constructed **RC circuits** as for the previous part of the experiment and aim to verify the functioning of **low-pass filter** and **high-pass filter** circuits as **integrators** and **differentiators** respectively.

6.1 Procedure

1. Use the same circuit as for **Low-pass filter** in the first subpart of this part. To check if the DSO and signal generator are working perfectly, first connect the signal generator directly to the DSO, and observe if the input waveform is as expected. If it is, then proceed with the experiment. If not, fix the DSO and signal generator first.

2. Instead of feeding a sinusoidal input waveform, feed a square waveform to the circuit, from the signal generator. Use a suitable amplitude and frequency.

3. Observe the input and output waveforms on the DSO. Record images of the waveforms at some arbitrary low, medium and high frequency as data to be reported.

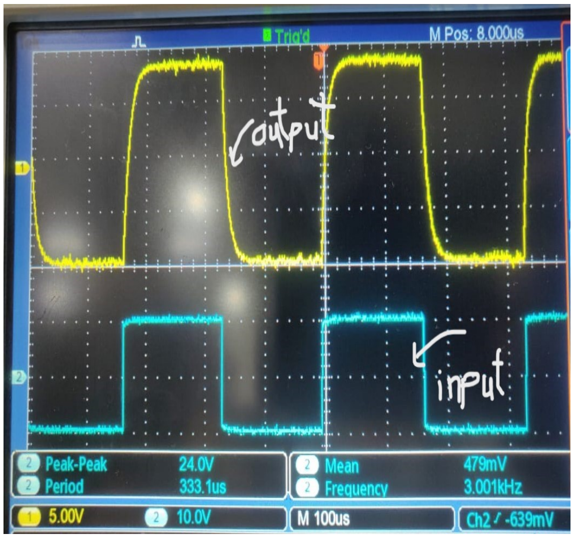
4. Exchange the positions of R and C to make a **high-pass filter** circuit. Repeat the same procedure and take images of the input and output waveforms at low, medium and high frequencies to report as data.

6.2 Observations

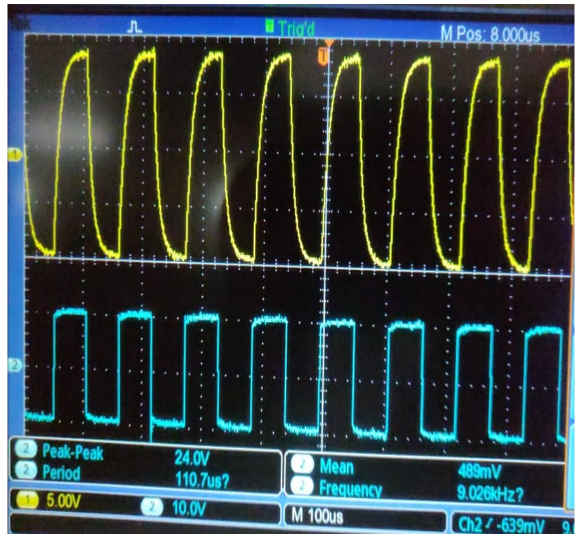
The **blue waveforms** represent the **input signal** while the **yellow waveforms** represent the **output signal** of the circuits.

6.2.1 Integrator RC circuit -

The following images of input and output waveforms (at low, medium and high frequencies) were taken to verify the **integrating behavior** of the **low-pass filter** RC circuit -



(a) Integrator at low frequency = 3kHz



(b) Integrator at med frequency = 9kHz

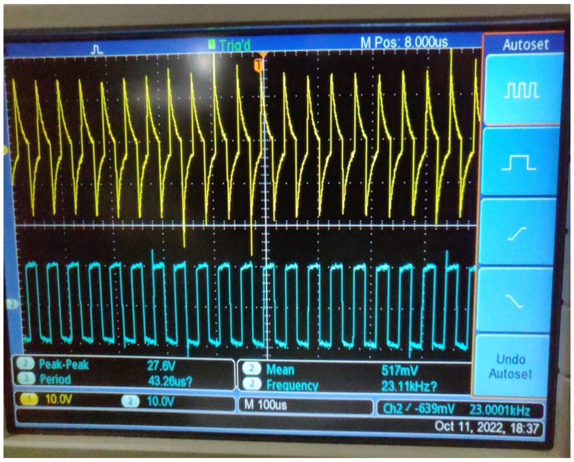
(c) Integrator at high frequency = 23kHz

Figure 8: Integrator RC circuit for increasing frequencies

The integration of a square wave is a triangular wave, since the area under a square wave (i.e. the integral) varies in a triangular fashion.
Thus, the above three images clearly show us the **integrating behavior** of the **low-pass filter** RC circuit. We can also verify that as the value of input frequency increases, the integrator gives increasingly accurate output waveforms, as is expected theoretically.

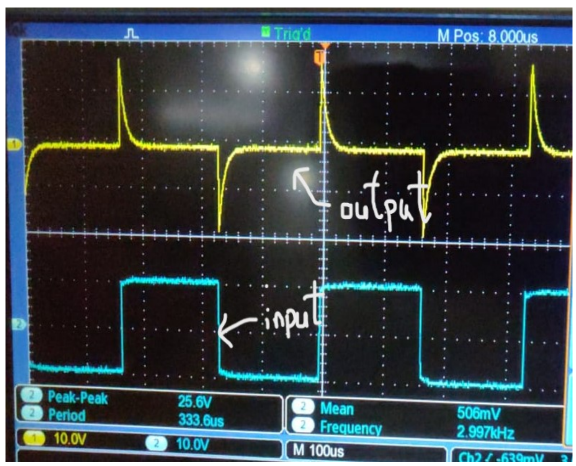
6.2.2 Differentiator RC circuit -

The following images of input and output waveforms (at low, medium and high frequencies) were taken to verify the **differentiating behavior** of the **high-pass filter** RC circuit -



(a) Differentiator at high frequency = 23kHz

(b) Differentiator at med frequency = 9kHz



(c) Differentiator at low frequency = 3kHz

Figure 9: Differentiator RC circuit for decreasing frequencies

The differentiation a square wave is an **alternating step-function (spike)**, since the rate of change of a square ave only spikes when there's a sudden change in ordinate, it is zero when the ordinate is constant.
Thus, the above three images clearly show us the **differentiating behavior** of the **high-pass filter** RC circuit. We can also verify that as the value of input frequency decreases, the differentiator gives increasingly accurate output waveforms, as is expected theoretically.

6.3 Result

The "integrating" and "differentiating" behavior of **low-pass** and **high-pass** filter RC circuits was verified experimentally by observing the corresponding input and output waveforms.

7 Part C

7.1 Procedure

1. Construct a **series-LCR circuit** as shown in Figure 4. Connect the signal generator at the input to feed in signals of varying frequencies into the circuit, and a DSO across the output to measure change in waveform.

2. For appropriate step lengths of increasing frequency, note down the current values from the digital multimeter at every frequency. Record them in two separate NumPy arrays.

3. Plot a graph of I vs f and I vs $\log f$. Record the frequency at which the maxima of the graph lies. This will be the **experimental resonance frequency** - f_{ER}.

4. Calculate the **theoretical resonance frequency** f_{TR} from the values of R, L, C used. Find the values of f_1, f_2, bandwidth and **Q-factor** from the graph.

5. Now, construct a **parallel-LCR circuit** as shown in Figure 5.

6. It might be the case that the parallel LCR circuit gives different values of error for different load resistances, R_L. To minimize this error, first find that value of R_L which gives **minimum deviation** from the theoretically expected value of the resonance.

7. To do this, first collect resistors of all **available resistance values** in the lab, above $1K\Omega$ and attach them in place of R_L. For every such resistance, record that **frequency** of the input signal which gives the peak value of Voltage. Do this by **modulating the frequency** values and recording at what value of f it **changes trend** i.e. starts decreasing after increasing till a point.

8. Record observations for such '**experimental resonance frequencies**' for all possible R_L values, and plot a log and **log-log graph** of f vs R_L with the theoretical resonance frequency. Based on this, identify the R_L value which gives **least deviation**. Use this value of R_L for recording the actual data for this part.

9. For appropriate **step lengths** of increasing frequency, note down the **voltage values** from the digital multimeter at every frequency. Record them in two separate NumPy arrays.

10. Plot a graph of V vs f and V vs $\log f$. Record the frequency at which the maxima of the graph lies. This will be the **experimental resonance frequency** - f_{ER}.

11. Calculate the **theoretical resonance frequency** f_{TR} from the values of R, L, C used. Find the values of f_1, f_2, bandwidth and **Q-factor** from the graph.

7.2 Observations

7.2.1 Series LCR circuit -

The values of components used in this circuit are -

$$R = 1k\Omega,\ C = 0.047\mu F,\ L = 30mH$$

Calculation from theoretical values -

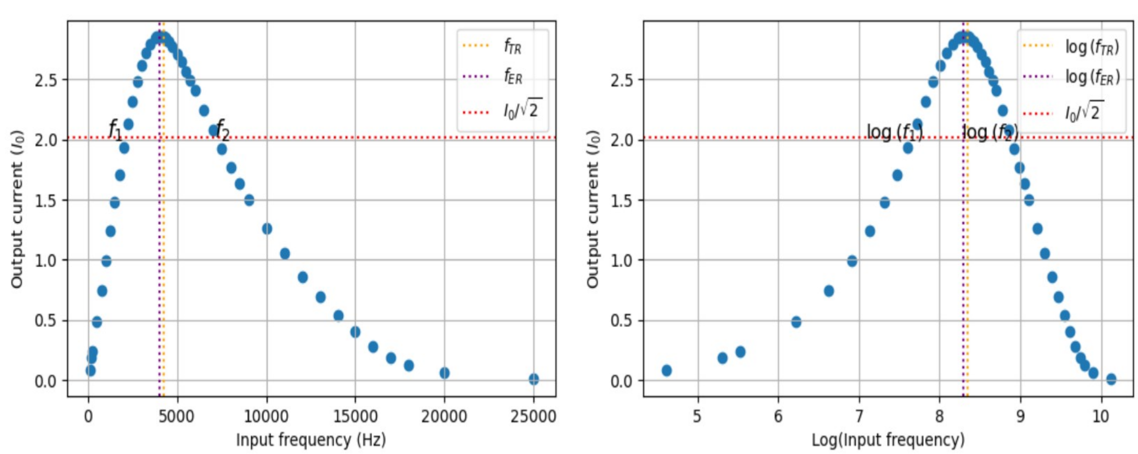
- **Theoretical Resonance Frequency** (f_{TR}):

$$
\begin{aligned}
f_{TR} &= \frac{1}{2\pi\sqrt{LC}} \\
&= \frac{1}{2\pi \times \sqrt{(30 \times 10^{-3}) \times (0.047 \times 10^{-6})}} \\
&= 4238.5 \text{ Hz}
\end{aligned}
$$

- **Theoretical Q-Factor** (Q_{sth}):

$$
\begin{aligned}
Q_{\text{sth}} &= \frac{1}{R}\sqrt{\frac{L}{C}} \\
&= \frac{1}{1000}\sqrt{\frac{30 \times 10^{-3}}{0.047 \times 10^{-6}}} \\
&= 0.798
\end{aligned}
$$

The following graphs were obtained on plotting **current** I with the **input frequency** f and **log of input frequency** $\log(f)$ -



(a) I vs f for series resonant LCR circuit

(b) I vs $\log(f)$ for series resonant LCR circuit

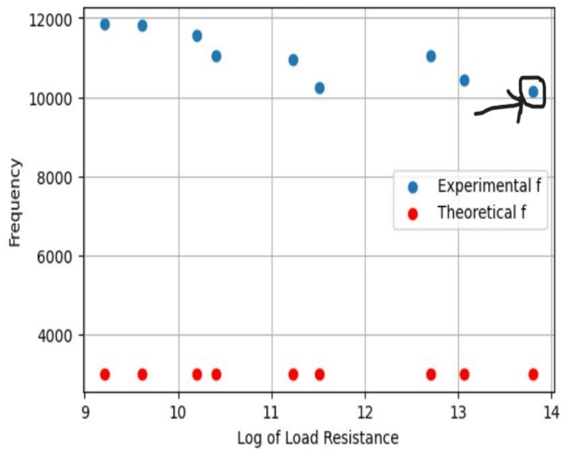
Figure 10: Graph for bandwidth, resonant frequency and Q-factor calculation

The values obtained: $f_1 = 2200$ Hz, $f_2 = 7500$ Hz

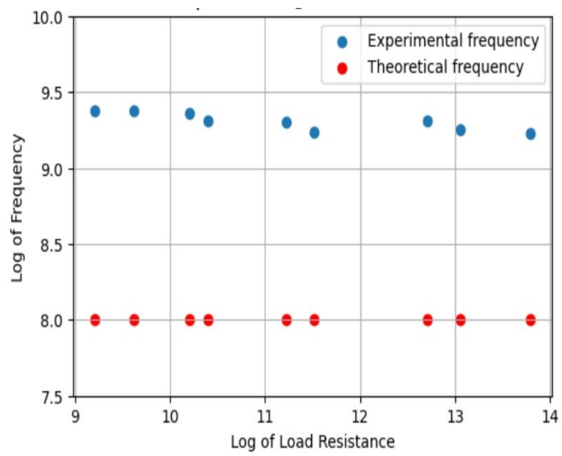
- **Bandwidth** $= f_2 - f_1 = 5300$ Hz
- **Experimental** Resonance frequency (f_{ER}) $= 4200$ Hz
- **Q-factor** (experimentally) $= \frac{f_{ER}}{f_2 - f_1} = \frac{4200}{7500-2200} = 0.792$

7.2.2 Finding most optimum R_L value -

As described in the procedure, the following plots show the f vs $\log(R_L)$ and $\log(f)$ vs $\log(R_L)$ graphs -



(a) f vs $\log{(R_L)}$



(b) $\log{(f)}$ vs $\log{(R_L)}$

Figure 11: Plots to find R_L which gives least error

From both these graphs above, especially Figure 8(a), we can see that the **least different** between experimental resonance frequency and **theoretical resonance frequency** was obtained when R_L was very high. This high value is $R_L = 0.98M\Omega$.

Thus, we conclude that the most optimum value of **load resistance** R_L is **0.98** $M\Omega$. We now use this value of R_L in our **parallel resonant LCR circuit**.

7.2.3 Parallel LCR circuit -

The values of components used in this circuit are -

$$R = 1k\Omega,\ C = 0.047\mu F,\ L = 60mH$$

Calculation from theoretical values -

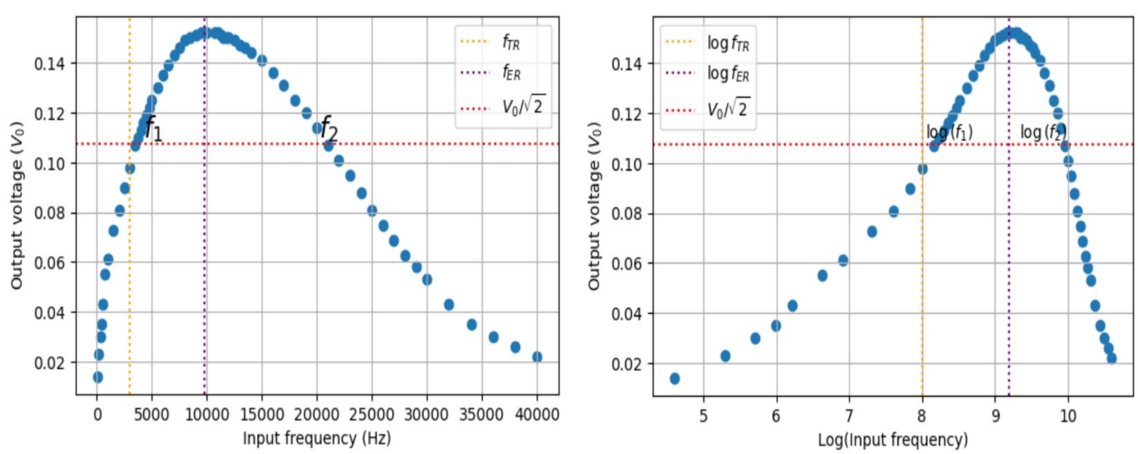
- **Theoretical Resonance Frequency** (f_{TR}):

$$\begin{aligned} f_{TR} &= \frac{1}{2\pi\sqrt{LC}} \\ &= \frac{1}{2\pi \times \sqrt{(60 \times 10^{-3}) \times (0.047 \times 10^{-6})}} \\ &= 2997.06 \text{ Hz} \end{aligned}$$

- **Theoretical Q-Factor** (Q_{pth}):

$$\begin{aligned} Q_{\text{pth}} &= R\sqrt{\frac{C}{L}} \\ &= 1000\sqrt{\frac{0.047 \times 10^{-6}}{60 \times 10^{-3}}} \\ &= 0.885 \end{aligned}$$

The following graphs were obtained on plotting **voltage** V_0 with the **input frequency** f and **log of input frequency** $\log(f)$ -



(a) V_0 vs f for parallel resonant LCR circuit (b) V_0 vs $\log(f)$ for parallel resonant LCR circuit

Figure 12: Graph for bandwidth, resonant frequency and Q-factor calculation

The values obtained: $f_1 = 3750$ Hz, $f_2 = 21000$ Hz

- **Bandwidth** $= f_2 - f_1 = 17250$ Hz
- **Experimental** Resonance frequency $(f_{ER}) = 11000$ Hz
- **Q-factor** (experimentally) $= \frac{f_{ER}}{f_2 - f_1} = \frac{1100}{21000-3750} = 0.638$

7.3 Error Analysis

7.3.1 Series LCR -

The % error in **Resonance frequency** can be given by -

$$\begin{aligned}\%\text{error} &= \frac{f_{TR} - f_{ER}}{f_{TR}} \times 100 \\ &= \frac{4238.5 - 4200}{4238.5} \times 100 \\ &= 0.9\%\end{aligned}$$

The % error in **Q-factor** can be given by -

$$\begin{aligned}\%\text{error} &= \frac{Q_t - Q_e}{Q_t} \times 100 \\ &= \frac{0.798 - 0.792}{0.798} \times 100 \\ &= 0.75\%\end{aligned}$$

These values of error can be justified very easily even by the manufacturing errors in the values of components, since it is a relatively low **error percentage**.

7.3.2 Parallel LCR -

The **error percentage** in the **parallel LCR** circuit was observed to be very high, especially the error in **resonance frequency**. This is a point of contention in the set-up of the experiment, and needs further detailed probing into every element used in the experimental set-up and procedure to understand and justify. The error for **Q-factor** is also relatively high, and can be calculated as follows -

$$\begin{aligned}\%\text{error} &= \frac{Q_t - Q_e}{Q_t} \times 100 \\ &= \frac{0.885 - 0.638}{0.885} \times 100 \\ &\approx 27.9\%\end{aligned}$$

7.4 Result

The behavior of **series and parallel resonant LCR circuit** was studied by constructing the correct circuits and recording observations to calculate **resonance frequency**, **bandwidth**, and **Q-factor**.

8 Discussion

1. The most important piece of discussion in this entire experiment is the **parallel resonant** circuit. It gives a very **error percentage** inspite of changing every single variable parameter possible and keeping the others constant. We found out a value of load resistance R_L that gave us the **minimum deviation** from the theoretically expected value, as explained in the procedure above, but even that R_L gave us readings with **significant errors**.

2. The below graph shows the data we obtained initially for **parallel** resonant circuit with $10k\Omega$ **load resistance.** -

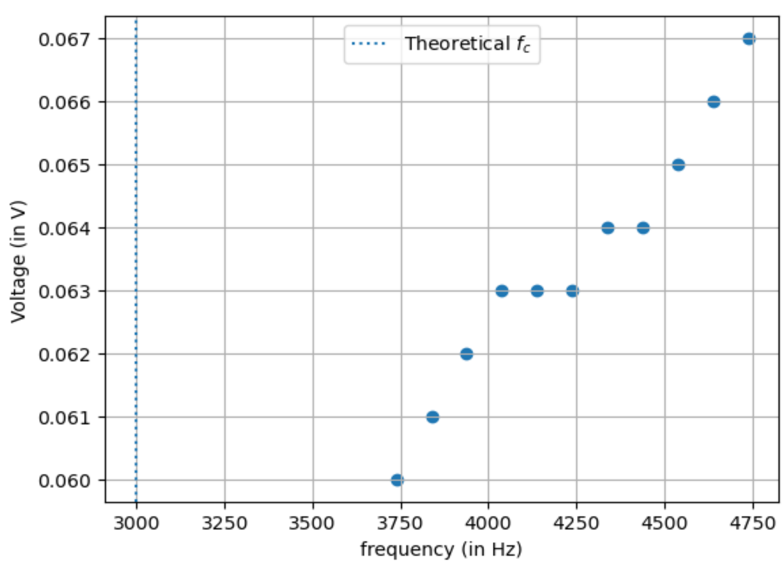


Figure 13: High error for arbitrary R_L

We can clearly see the high error percentage in the theoretical and experimental **resonance frequencies**, as reflected in the huge gap between the two plots.

3. Next, we took all the readings with different R with value $10k\Omega$, and then changed the values of L, C to 60 mH and 0.47 μF respectively to again take the entire data. But while using all of these different values of electronic components changed the **theoretical resonance frequency**, the **experimental value of resonance frequency** remained **more or less** constant, somewhere $\approx 11k$Hz.

4. Another significant **source of error** was the **arbitrary change** in values of I or V when different electrical and non-electrical components (like laptop, hand, watch, etc.) were brought near the multimeter. This is still left unexplained.

5. The **major learning** from this experiment was to get familiarity with creating **physical electronic circuits** from the circuit diagrams given in the handout (especially parallel and series circuit) The use of a DSO and a **signal generator** was also learnt in this experiment.

6. An additional element of error analysis that can be done in this experiment is by noting down the **uncertainty** quoted by the manufacturer of all the various **electronic components** used in this experiment. Incorporating that data into our error analysis will improve the quality of our analysis manifold.

9 Precautions -

1. Ensure that the connections are tight and clean. Do not make unnecessarily complicated connections which are difficult to follow and track around.

2. Ensure that the multimeter is set at the correct sensitivity while measuring data.

3. DO NOT keep any objects near the multimeter while taking readings. They may cause an unwanted interference with the readings. If an object is already kept near it for some of the readings, then let it remain there for all of the readings.

4. Do not increase the current too much, it will hamper the functioning of the instruments.

10 References

- Lab Handout,"Electronic Circuits", Ashoka University
- Damped Harmonic Oscillator, derivation, LibreTexts Math

11 Appendix

- The appendix of this report contains all the **observation tables** of the data we recorded for all parts of this experiment. This was used to **plot the graphs** which are included in the report.
- Since the units of the physical quantities cannot be included in the graphs due to **space constraints**, I mention that the below data has been taken in the following units -
  Voltage in Volts (V), **Current in Ampere** (A), **Frequency in Hertz** (Hz),
  Resistance in Ohms (Ω)

11.1 Part A

11.1.1 Low-Pass Filter

V_{in}	V_{out}	f	$log(f)$	Gain
5.71	5.08	750	6.62	2.34
5.71	5.05	1500	7.31	2.46
5.71	5.01	2250	7.72	2.62
5.71	4.98	3000	8.01	2.74
5.69	4.94	3750	8.23	2.83
5.66	4.9	4500	8.41	2.88
5.61	4.86	5250	8.57	2.87
5.52	4.81	6000	8.7	2.75
5.38	4.77	6750	8.82	2.41
5.17	4.72	7500	8.92	1.82
4.9	4.67	8250	9.02	0.96
4.55	4.62	9000	9.1	-0.31
4.14	4.58	9750	9.19	-2.02
2.79	4.52	10500	9.26	-9.65
2.64	4.5	10800	9.29	-10.67
2.5	4.49	11000	9.31	-11.71
2.42	4.47	11250	9.33	-12.27
2.33	4.46	11500	9.35	-12.99
2.21	4.44	11750	9.37	-13.95
2.07	4.42	12000	9.39	-15.17
1.8	4.37	13000	9.47	-17.74
1.56	4.31	13500	9.51	-20.33

V_{in}	V_{out}	f	$log(f)$	Gain
1.26	4.27	14000	9.55	-24.41
0.99	4.2	15000	9.62	-28.9
0.81	4.15	15750	9.66	-32.68
0.66	4.09	16500	9.71	-36.48
0.54	4.03	17250	9.76	-40.2
0.44	3.98	18000	9.8	-44.05
0.35	3.92	18750	9.84	-48.32
0.28	3.86	19500	9.88	-52.47
0.22	3.8	20250	9.92	-56.98
0.18	3.75	21000	9.95	-60.73
0.14	3.69	21750	9.99	-65.43
0.11	3.63	22500	10.02	-69.93
0.08	3.57	23250	10.05	-75.97
0.06	3.51	24000	10.09	-81.38
0.05	3.46	24750	10.12	-84.74
0.04	3.4	25500	10.15	-88.85
0.03	3.34	26250	10.18	-94.25
0.02	3.28	27000	10.2	-102.0
0.016	3.23	27750	10.23	-106.15
0.011	3.17	28500	10.26	-113.27
0.008	3.12	29250	10.28	-119.32
0.006	3.06	30000	10.31	-124.69

11.1.2 High-Pass Filter

V_{in}	V_{out}	f	$log(f)$	Gain
5.74	0.06	250	5.52	-91.22
5.74	0.11	500	6.21	-79.09
5.74	0.16	750	6.62	-71.6
5.73	0.21	1000	6.91	-66.13
5.72	0.26	1250	7.13	-61.82
5.69	0.3	1500	7.31	-58.85
5.64	0.35	1750	7.47	-55.59
5.54	0.39	2000	7.6	-53.07
5.4	0.44	2250	7.72	-50.15
5.19	0.48	2500	7.82	-47.61
4.92	0.51	2750	7.92	-45.33
4.57	0.56	3000	8.01	-41.99
4.16	0.59	3250	8.09	-39.06
2.79	0.63	3500	8.16	-29.76
2.42	0.67	3750	8.23	-25.68
2.08	0.7	4000	8.29	-21.78
1.75	0.73	4250	8.35	-17.49
1.46	0.76	4500	8.41	-13.06
1.21	0.8	4750	8.47	-8.28
0.99	0.83	5000	8.52	-3.53
0.82	0.86	5250	8.57	0.95
0.67	0.89	5500	8.61	5.68
0.54	0.91	5750	8.66	10.44
0.44	0.94	6000	8.7	15.18
0.35	0.97	6250	8.74	20.39
0.29	0.99	6500	8.78	24.56
0.23	1.02	6750	8.82	29.79
0.18	1.04	7000	8.85	35.08
0.14	1.07	7250	8.89	40.68
0.11	1.09	7500	8.92	45.87

V_{in}	V_{out}	f	$log(f)$	Gain
0.067	1.13	8000	8.99	56.51
0.05	1.15	8250	9.02	62.71
0.04	1.17	8500	9.05	67.52
0.029	1.19	8750	9.08	74.29
0.02	1.2	9000	9.1	81.89
0.016	1.22	9250	9.13	86.68
0.012	1.24	9500	9.16	92.76
0.008	1.26	9750	9.19	101.19
0.006	1.27	10000	9.21	107.1
0.005	1.29	10250	9.24	111.06
0.004	1.34	11000	9.31	116.28
0.004	1.39	12000	9.39	117.02
0.004	1.44	13000	9.47	117.72
0.004	1.49	14000	9.55	118.4
0.004	1.54	15000	9.62	119.06
0.004	1.59	16000	9.68	119.7
0.004	1.64	17000	9.74	120.32
0.004	1.69	18000	9.8	120.92
0.004	1.74	19000	9.85	121.51
0.004	1.8	20000	9.9	122.18
0.004	1.85	21000	9.95	122.73
0.004	1.9	22000	10.0	123.27
0.004	1.96	23000	10.04	123.89
0.004	2.01	24000	10.09	124.39
0.004	2.06	25000	10.13	124.88
0.004	2.11	26000	10.17	125.36
0.004	2.16	27000	10.2	125.83
0.004	2.21	28000	10.24	126.29
0.004	2.25	29000	10.28	126.65
0.004	2.29	30000	10.31	127.0

11.2 Part C

11.2.1 Series Resonant Circuit -

I	f	$log(f)$
0.08	100.0	5.52
0.19	200.0	6.21
0.24	250.0	6.62
0.49	500.0	6.91
0.74	750.0	7.13
0.99	1000.0	7.31
1.24	1250.0	7.47
1.48	1500.0	7.6
1.71	1750.0	7.72
1.93	2000.0	7.82
2.13	2250.0	7.92
2.32	2500.0	8.01
2.48	2750.0	8.09
2.62	3000.0	8.16
2.72	3250.0	8.23
2.79	3500.0	8.29

I	f	$log(f)$
2.84	3750.0	8.35
2.85	3850.0	8.41
2.85	3950.0	8.47
2.85	4050.0	8.52
2.85	4150.0	8.57
2.85	4238.48	8.61
2.85	4250.0	8.66
2.84	4350.0	8.7
2.82	4450.0	8.74
2.81	4550.0	8.78
2.77	4750.0	8.82
2.71	5000.0	8.85
2.65	5250.0	8.89
2.57	5500.0	8.92
2.49	5750.0	8.96
2.41	6000.0	8.99

I	f	$log(f)$
2.24	6500.0	9.02
2.08	7000.0	9.05
1.92	7500.0	9.08
1.77	8000.0	9.1
1.63	8500.0	9.13
1.5	9000.0	9.16
1.26	10000.0	9.19
1.05	11000.0	9.21
0.86	12000.0	9.24
0.69	13000.0	9.31
0.54	14000.0	9.39
0.4	15000.0	9.47
0.28	16000.0	9.55
0.19	17000.0	9.62
0.12	18000.0	9.68
0.06	20000.0	9.74
0.01	25000.0	9.8

11.2.2 Parallel Resonant Circuit -

I	f	$log(f)$
0.014	100	4.61
0.023	200	5.3
0.03	300	5.7
0.035	400	5.99
0.043	500	6.21
0.055	750	6.62
0.061	1000	6.91
0.073	1500	7.31
0.081	2000	7.6
0.09	2500	7.82
0.098	3000	8.01
0.107	3500	8.16
0.11	3750	8.23
0.113	4000	8.29
0.116	4238	8.35
0.116	4250	8.35
0.119	4500	8.41
0.122	4750	8.47
0.125	5000	8.52
0.13	5500	8.61

I	f	$log(f)$
0.135	6000	8.7
0.139	6500	8.78
0.143	7000	8.85
0.146	7500	8.92
0.149	8000	8.99
0.15	8500	9.05
0.151	9000	9.1
0.152	9500	9.16
0.152	10000	9.21
0.152	10750	9.28
0.152	11000	9.31
0.151	11250	9.33
0.15	11500	9.35
0.15	11750	9.37
0.15	12000	9.39
0.149	12500	9.43
0.147	13000	9.47
0.146	13500	9.51
0.144	14000	9.55
0.141	15000	9.62

I	f	$log(f)$
0.136	16000	9.68
0.131	17000	9.74
0.125	18000	9.8
0.12	19000	9.85
0.114	20000	9.9
0.107	21000	9.95
0.101	22000	10.0
0.095	23000	10.04
0.088	24000	10.09
0.081	25000	10.13
0.075	26000	10.17
0.069	27000	10.2
0.063	28000	10.24
0.058	29000	10.28
0.053	30000	10.31
0.043	32000	10.37
0.035	34000	10.43
0.03	36000	10.49
0.026	38000	10.55
0.022	40000	10.6

11.2.3 Finding the optimum R_L -

R_L	Experimental Frequency (f)	$log(R_L)$	$log(f)$
10000	11838	9.21	9.38
15000	11808	9.62	9.38
27000	11578	10.2	9.36
33000	11048	10.4	9.31
75000	10938	11.23	9.3
100000	10238	11.51	9.23
330000	11038	12.71	9.31
470000	10438	13.06	9.25
980000	10138	13.8	9.22