QF605 Fixed Income Project Report

A PROJECT REPORT SUBMITTED
FOR THE REQUIREMENTS OF THE DEGREE OF

Master of Science in Quantitative Finance

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1 Part I - Bootstrapping Swap Curve

The analysis for Part I has been covered in the 'Part1.ipynb' file.

2 Part II - Swaption Calibration

The displaced diffusion is a model whose weights can be tweaked to make it more "normal" or "lognormal". To best make use of this property of the model, we use market prices and run a least squares-based algorithm to find the beta and sigma parameters that allow the model to represent a given tenor and expiry combination. The following are the parameters found:

	Sigma				
Tenor	1	2	3	5	10
Expiry					
1	0.403533	0.36408	0.333147	0.290906	0.252791
5	0.307549	0.313167	0.308511	0.270026	0.247525
10	0.299765	0.29814	0.29437	0.265877	0.24269

Figure 1: Sigma

	Beta				
Tenor	1	2	3	5	10
Expiry					
1	0.199527	0.199186	0.199027	0.199487	0.200042
5	0.19851	0.198085	0.365238	0.345461	0.336272
10	0.693301	0.254392	0.237191	0.315506	0.261165

Figure 2: Beta

The SABR model has more parameters and does a better a job than displaced diffusion than to fit a given market vol smile. The parameters below have been found using a similar least squares-based approach as above. Both market vol and market prices can be used for this calibration. Here, we have used market prices.

	Alpha				
Tenor	1	2	3	5	10
Expiry					
1	0.139072	0.18467	0.196872	0.178059	0.170897
5	0.166579	0.199577	0.210445	0.191014	0.176914
10	0.1778	0.195426	0.20686	0.202027	0.180694

Figure 3: Alpha

	Rho				
Tenor	**1**	**2**	**3**	**5**	**10**
Expiry					
1	-0.632616	-0.524618	-0.482396	-0.413988	-0.264048
5	-0.58467	-0.546605	-0.549637	-0.510837	-0.435193
10	-0.545996	-0.544315	-0.549436	-0.563482	-0.509539

Figure 4: Rho

	Nu				
Tenor	**1**	**2**	**3**	**5**	**10**
Expiry					
1	2.044426	1.673683	1.43482	1.062192	0.778492
5	1.336251	1.059152	0.933903	0.669562	0.49528
10	1.005423	0.923314	0.865705	0.717435	0.577074

Figure 5: Nu

Once the parameters have been found, we can look to price new options that are not quoted by the market. Due to the better fit that the SABR model provides, we would consider the volatility provided by it to be more accurate. Still, care must be taken when looking at far OTM options as the SABR model may not fit such volatility very well.

Payer Swaption 2y $\times$ 10y Here, the forward rate is 0.0389.

Receiver Swaption 8y $\times$ 10y Here, the forward rate is 0.0475.

3 Part III - Convexity Correction

3.1 Calculating PV of CMS Products

In Part II we calculated the SABR parameters at 1y, 5y, and 10y expiries. In order to calculate the Present Value of the Constant Maturity Swap (CMS) products in this part, we need to identify the SABR parameters for the various other expiries required for such products. We have calibrated the SABR Parameters α, ν, ρ for the remaining expiries with the help of Cubic Spline interpolation. The results of the interpolation can be seen below:

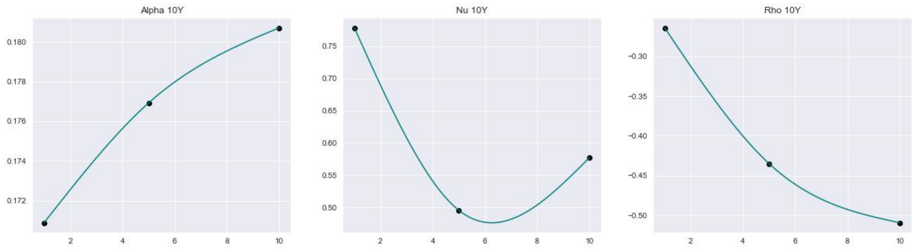

Figure 6: Interpolation of SABR Parameters using Cubic Spline - CMS10Y

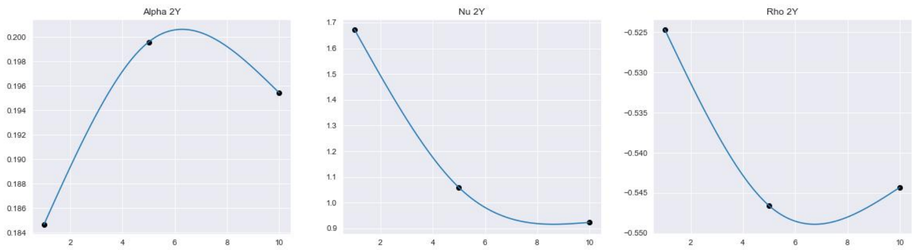

Figure 7: Interpolation of SABR Parameters using Cubic Spline - CMS2Y

Static replication was used to calculate the CMS rates. PV of the CMS legs were calculated using the following:

$$CMS\ PV = \sum_{i=0.5}^{N} D_0(0,T_i)\Delta CMS\left(S_{n,N}(T_i)\right)$$

The final results are as follows:

- PV of a leg receiving CMS10y semi-annually over the next 5 years is: 0.209139
- PV of a leg receiving CMS2y quarterly over the next 10 years is: 0.385202

3.2 Comparison of CMS Rates with Forward Swap Rates

Below are the CMS rates calculated for the various maturities and tenor:

ExpiryXTenor	CMS	FSR	difference
1Y x 1Y	0.032052	0.031950	0.000102
1Y x 2Y	0.033423	0.033200	0.000223
1Y x 3Y	0.034242	0.033947	0.000295
1Y x 5Y	0.035494	0.035184	0.000309
1Y x 10Y	0.038878	0.038335	0.000544
5Y x 1Y	0.039964	0.039177	0.000787
5Y x 2Y	0.041222	0.039974	0.001247
5Y x 3Y	0.041564	0.039969	0.001595
5Y x 5Y	0.042894	0.040982	0.001912
5Y x 10Y	0.046686	0.043508	0.003178
10Y x 1Y	0.043397	0.042074	0.001324
10Y x 2Y	0.045182	0.042995	0.002187
10Y x 3Y	0.047018	0.043971	0.003047
10Y x 5Y	0.050319	0.046109	0.004209
10Y x 10Y	0.061402	0.053296	0.008105

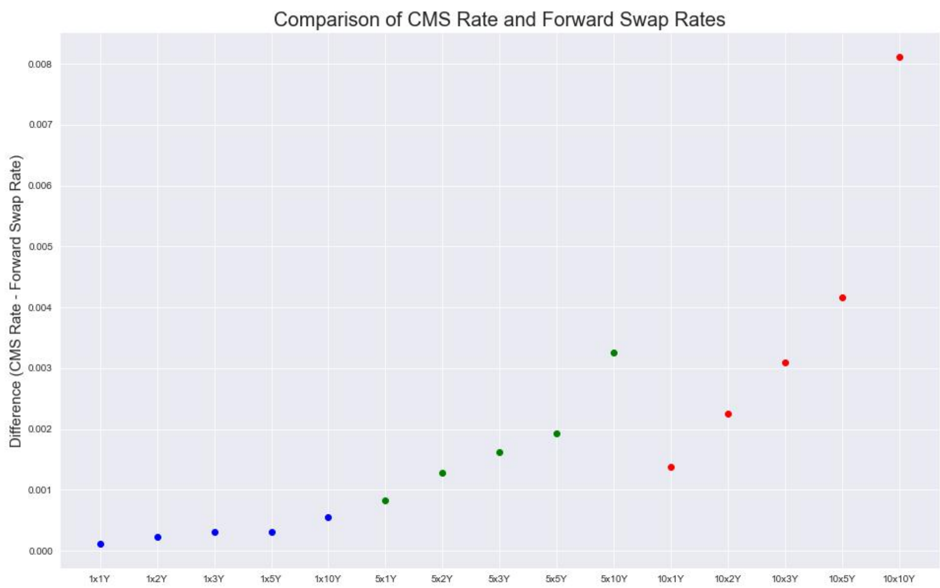

Figure 8: Difference between CMS Rates and Forward Swap Rates

The forward swap rates used for the comparison have been obtained from Part I. On comparison of the CMS rates with the Forward Swap Rates we observe that the difference between the two increases as the expiry increases. However, the same cannot be said about the effect of convexity correction on the tenor.

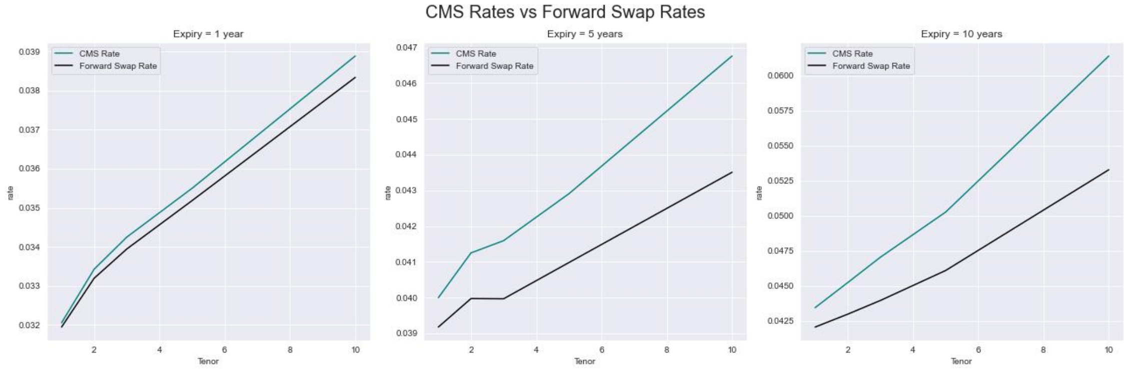

Figure 9: Comparison of CMS Rates and Forward Swap Rates across expiries

4 Part IV - Decompounded Options

A decompounded option at time T = 5y pays:

$$CMS\,10y^{1/p} - 0.04^{1/q}$$

To calculate the present value (PV) of the payoff function, we used Leibniz's Rule to calculate the PV of the IRR on the payer and receiver swaption to derive the below expression:

$$V_0 = D_0(0,T)g(F) + \int_0^F h''(K)\,V^{rec}\,(K)\,dK + \int_F^\infty h''(K)\,V^{pay}\,(K)\,dK$$

For the payoff of $CMS\,10y^{1/p} - 0.04^{1/q}$ at time 5y, we used 5y*10y forward swap rate (F) and OIS as the discount factor of $D_0(0,5)$ calculated from Part I, and substituted the first and second derivation of $h(K)$:

$$g(F) = F^{1/4} - 0.2 \qquad g'(F) = \frac{1}{4}F^{3/4} \qquad g''(F) = -\frac{3}{16}F^{7/14}$$

$$h(K) = \frac{g(K)}{IRR(K)}$$

$$h'(K) = \frac{IRR(K)g'(K) - g(K)IRR'(K)}{IRR\,(K)^2}$$

$$h''(K) = \frac{IRR(K)g''(K) - IRR''(K)g(K) - 2 * IRR'(K)g'(K)}{IRR(K)^2} + \frac{2 * IRR'(K)^2 g(K)}{IRR(K)^3}$$

We can obtain the payoff at 5y = 0.24954

Suppose the payoff is now $(CMS\,10y^{1/p} - 0.04^{1/q})^+$

For the payoff to be positive:

$$F^{1/4} > 0.2$$
$$F > 0.2^4$$
$$F > 0.0016 = L,$$

Hence,

$$CMS\,Caplet = D(0,T)\int_L^\infty g'(K)f(K)dK$$
$$= \int_L^\infty h(K)\frac{\partial^2 V^{pay}(K)}{\partial K^2}dK$$
$$= h'(L)V^{pay}(L) + \int_L^\infty h''(K)V^{pay}(K)dK$$

We have derived the pv of this payoff to be 0.03098