Part I - Bootstrapping Swap Curves

(1) In the IR Data.xlsm spreadsheeet, OIS data is provided. `Bootstrap the OIS discount factor` D_0 (0, T) and plot the discount curve for T $\in$ [0, 30].

In [1]:
```
import pandas as pd

from scipy.optimize import brentq
import matplotlib.pyplot as plt
import numpy as np

%matplotlib inline

from scipy import interpolate #wrote my own function to interpolate
```

For OIS

Day Count Convention = 30/360
O/N Leg Freq = Daily
Fixed Leg Freq = Annual

In [2]:
```
ois_data = pd.read_csv('Data/OIS.csv')
ois_data
```

Out[2]:

	Tenor	Product	Rate
0	6m	OIS	0.250%
1	1y	OIS	0.300%
2	2y	OIS	0.325%
3	3y	OIS	0.335%
4	4y	OIS	0.350%
5	5y	OIS	0.360%
6	7y	OIS	0.400%
7	10y	OIS	0.450%
8	15y	OIS	0.500%
9	20y	OIS	0.525%
10	30y	OIS	0.550%

In [3]:
```
# Data Cleaning
ois_data["OIS_rate"] = [float(i.strip('%')) for i in ois_data.Rate]
ois_data.OIS_rate = ois_data.OIS_rate/100

ois_data["Tenor"] = [0.5, 1, 2, 3, 4, 5, 7, 10, 15, 20, 30] # only a few so I hardcoded it
```

In [4]:
```
ois_data.head(3)
```

Out[4]:

	Tenor	Product	Rate	OIS_rate
0	0.5	OIS	0.250%	0.00250
1	1.0	OIS	0.300%	0.00300
2	2.0	OIS	0.325%	0.00325

General Idea: In order to plot discount curve, we need to do bootstrapping, and in order to do that, we need to first calculate all the daily compounded overnight rates f_i throughout the period T.

$$D_o(0,6m) = \frac{1}{(1+\frac{f_0}{360})^{180}}$$
$$D_o(0,1y) = D_0(0,6m) \times D_0(6m,1y) = D_o(0,6m) \times \frac{1}{(1+\frac{f_1}{360})^{180}}$$

Let f_0 denote the daily compounded overnight rate for [0, 6m]. Using the 6m OIS, we have:

$$PV_{fix}^{6mOIS} = PV_{flt}^{6mOIS}$$

$$D_o(0,6m) \times 0.5 \times 0.25\% = D_o(0,6m) \times 0.5 \times [(1+\frac{f_0}{360})^{180} - 1]$$

We can now solve for f_0.

In [5]:
```
f_0 = (((ois_data.OIS_rate[0]+1)**(1/180)) - 1) * 360
f_0
```

Out[5]: 0.004993795032932624

In [6]:
```
# crossed checked with root search method to find f_0 and the results are very similar
test_rs = lambda y: ((1/(1+y/360))**180 * 0.5 * ois_data.OIS_rate[0]) - ((1/(1+y/360))**180 * ((1 + y/360)**180 - 1))
rs = brentq(test_rs, 0, 1)
rs
```

Out[6]: 0.0024984474705118964

In [7]:
```
x0 = (1 + (f_0/360))**180 # for ease of troubleshooting
D_6m = 1 / x0
D_6m
```

Out[7]: 0.9975062344139736

Having obtained this, we can then let f_1 denote the daily compounded overnight rate for [6m, 1y]. Using the 1y OIS, we have:

$$PV_{fix}^{1yOIS} = PV_{flt}^{1yOIS}$$

$$D_o(0, 1y) \times 0.3\% = D_o(0, 1y) \times [(1 + \frac{f_0}{360})^{180}(1 + \frac{f_1}{360})^{180} - 1]$$

We can now solve for f_1 .

In [8]:
```
f_1 = (((1+ois_data.OIS_rate[1])/x0)**(1/180) - 1) * 360
f_1
```

Out[8]: 0.0009972589437445833

In [9]:
```
D_1y = D_6m * 1/(1+f_1/360)**180
D_1y
```

Out[9]: 0.9970089730807465

Next, we move on to the 2y OIS. Let f_2 denote the daily compounded overnight rate for [1y, 2y], we have

$$PV_{fix}^{2yOIS} = PV_{flt}^{2yOIS}$$

$$[D_o(0, 1y) + D_o(0, 2y)] \times 0.325\% = D_o(0, 1y) \times [(1 + \frac{f_0}{360})^{180}(1 + \frac{f_1}{360})^{180} - 1] + D_o(0, 2y) \times [(1$$

- Use root search method to solve for f first then substitute it back to find D_o

$$D_o(0,2y) = D_o(0,6m) \times D_o(6m,1y) \times D_o(1y,2y) = \frac{1}{(1+\frac{f_0}{360})^{180}} \times \frac{1}{(1+\frac{f_1}{360})^{180}} \times \frac{1}{(1+\frac{f_2}{360})^{360}}$$

=> $D_o(0,2y) = D_o(0,1y) \times D_o(1y,2y) = D_o(0,1y) \times \frac{1}{(1+\frac{f_2}{360})^{360}}$

=>

$$D_o(0,3y) = D_o(0,1y) \times D_o(1y,2y) \times D_o(2y,3y) = D_o(0,2y) \times D_o(2y,3y) = D_o(0,2y) \times \frac{1}{(1+\frac{f_3}{360})^{3}}$$

Hence, generally the below equation is used to root search the overnight rate f_t.

$$[\sum_{t=1}^{T-n} D_o(0,t) + D_o(0,T)] \times S_T = D_o(0,1y)[(1+\frac{f_0}{360})^{180}(1+\frac{f_1}{360})^{180} - 1] + \sum_{t=1}^{T-n} D_o(0,t)[(1+$$

where $D_o(0,T) = D_o(0,T-n) \times D_o(T-n,T) = D_o(0,T-n) \times \frac{1}{(1+\frac{f_T}{360})^{360*n}}$

In [10]:

```
# Include full time range as OIS is annual frequency

tmp_df = pd.DataFrame(data=np.arange(1,30.5,1), columns=['Tenor'])
data = []
data.insert(0,{'Tenor':0.5})
tmp_df = pd.concat([pd.DataFrame(data), tmp_df], ignore_index=True)
ois_df = pd.merge(tmp_df, ois_data, on="Tenor", how="left").drop('P
roduct', axis =1 )

# Prepare columns
ois_df['OIS_DF'] = np.nan
ois_df['f'] = np.nan

ois_df['f'][0] = f_0
ois_df['f'][1] = f_1
ois_df['OIS_DF'][0] = D_6m
ois_df['OIS_DF'][1] = D_1y
```

```
<ipython-input-10-087343347f26>:13: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pa
ndas-docs/stable/user_guide/indexing.html#returning-a-view-versus-
a-copy
  ois_df['f'][0] = f_0
<ipython-input-10-087343347f26>:14: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pa
ndas-docs/stable/user_guide/indexing.html#returning-a-view-versus-
a-copy
  ois_df['f'][1] = f_1
<ipython-input-10-087343347f26>:15: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pa
ndas-docs/stable/user_guide/indexing.html#returning-a-view-versus-
a-copy
  ois_df['OIS_DF'][0] = D_6m
<ipython-input-10-087343347f26>:16: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pa
ndas-docs/stable/user_guide/indexing.html#returning-a-view-versus-
a-copy
  ois_df['OIS_DF'][1] = D_1y
```

In [11]:

```
ois_df.head(3)
```

Out[11]:

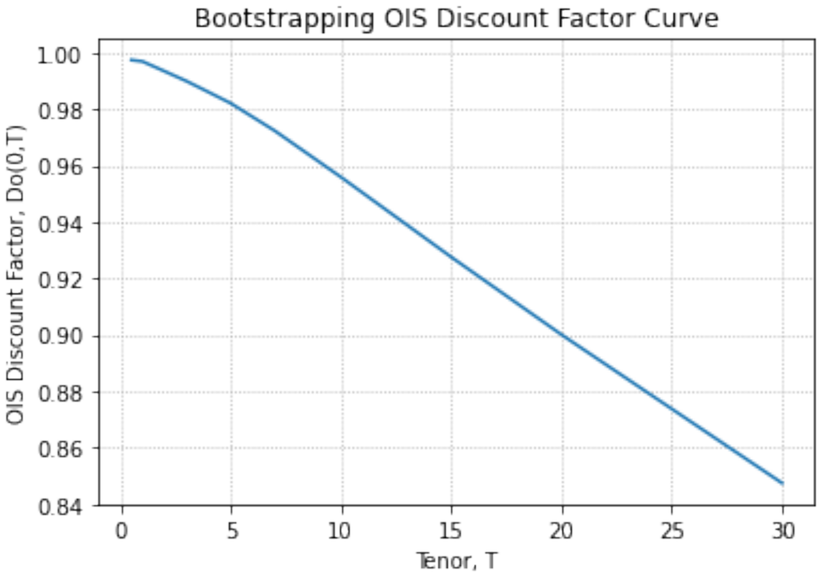
	Tenor	Rate	OIS_rate	OIS_DF	f
0	0.5	0.250%	0.00250	0.997506	0.004994
1	1.0	0.300%	0.00300	0.997009	0.000997
2	2.0	0.325%	0.00325	NaN	NaN

In [12]:
```
# Define root search func:  pv_fix - pv_float = 0
#we set last tenor OIS_Discount(0,Ti) as x ,and do root search to s
olve the equation
def get_OIS_func(x):
    float_unknown_sum = ois_df['OIS_DF'][i-1] - x
    fix_discount_unknown_sum = 0
    diff = ois_df['OIS_DF'][i-1] - x

    for n in range(0,h-i+1):
        D = x + ((h-i+1-n-1)/(h-i+1))*diff # interpolate LIBOR df
        fix_discount_unknown_sum += D
    fix_unknown_sum = fix_discount_unknown_sum * ois_df.OIS_rate[h]


    return fix_unknown_sum + fix_known_sum - float_unknown_sum - fl
oat_known_sum
```

In [13]:
```
i = 2
while i <=30:
    h = i

    while np.isnan(ois_df.OIS_rate[h]):
        h += 1
    k =1
    fix_known_sum = 0

    while k < i:
        fix_known = ois_df['OIS_DF'][k]*ois_df.OIS_rate[h]
        fix_known_sum += fix_known
        k+=1
    float_known_sum = ois_df.OIS_rate[i-1]*ois_df['OIS_DF'][1:i].su
m()

    func = lambda x:(get_OIS_func(x))
    D_last = brentq(func, 1e-6, 1)

    for n in range(0,h-i+1):
        ois_df['OIS_DF'][i+n] = D_last + ((h-i+1-n-1)/(h-i+1))*(ois
_df['OIS_DF'][i-1] - D_last)
        ois_df['f'][i+n] = 360*((ois_df['OIS_DF'][i+n-1]/ois_df['OI
S_DF'][i+n])**(1/360)-1)
    i = h+1
```

```
<ipython-input-13-4baecf52a959>:20: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pa
ndas-docs/stable/user_guide/indexing.html#returning-a-view-versus-
a-copy
  ois_df['OIS_DF'][i+n] = D_last + ((h-i+1-n-1)/(h-i+1))*(ois_df['
OIS_DF'][i-1] - D_last)
<ipython-input-13-4baecf52a959>:21: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pa
ndas-docs/stable/user_guide/indexing.html#returning-a-view-versus-
a-copy
  ois_df['f'][i+n] = 360*((ois_df['OIS_DF'][i+n-1]/ois_df['OIS_DF'
][i+n])**(1/360)-1)
```

In [14]:
```
plt.plot(figsize=(24,16))
plt.plot(ois_df.Tenor, ois_df["OIS_DF"])
plt.xlabel("Tenor, T")
plt.ylabel("OIS Discount Factor, Do(0,T)")
plt.title("Bootstrapping OIS Discount Factor Curve")
plt.grid(linestyle='dotted')
```



In [15]:
```
def interpolate(L, R, Target):

    x_L = (Target - R)/(L-R)
    x_R = 1 - x_L

    return abs(x_L), abs(x_R)
```

(2) Using the IRS data provided, `bootstrap the LIBOR discount factor` D(0,T), and plot it for T ∈ [0,30].

- Assume that the `swap market is collateralized` in cash and overnight interest is paid on collateral posted.
- Therefore, we need to discount with OIS Discount Factors found in Part 1

For IRS

Day Count Convention = 30/360
Fixed Leg Freq = Semi-annual
Floating Leg Freq = Semi Annual

In [16]:
```
irs_data = pd.read_csv('Data/IRS.csv')
irs_data.head(3)
```

Out[16]:

	Tenor	Product	Rate
0	6m	LIBOR	2.50%
1	1y	IRS	2.80%
2	2y	IRS	3.00%

In [17]:
```
# Data Cleaning and Preparation
irs_data["IRS_rate"] = [float(i.strip('%')) for i in irs_data.Rate]
irs_data.IRS_rate = irs_data.IRS_rate/100

irs_data["Tenor"] = [0.5, 1, 2, 3, 4, 5, 7, 10, 15, 20, 30] # only
a few so I hardcoded it

accrusal_frac = 0.5
```

In [18]:
```
irs_data.head(3)
```

Out[18]:

	Tenor	Product	Rate	IRS_rate
0	0.5	LIBOR	2.50%	0.025
1	1.0	IRS	2.80%	0.028
2	2.0	IRS	3.00%	0.030

Interpolating all the semi-annual OIS Discount Factors

In [19]:
```
ois_df.head(3)
```

Out[19]:

	Tenor	Rate	OIS_rate	OIS_DF	f
0	0.5	0.250%	0.00250	0.997506	0.004994
1	1.0	0.300%	0.00300	0.997009	0.000997
2	2.0	0.325%	0.00325	0.993531	0.003495

In [20]:
```
semi_ois_df = [ois_df.OIS_DF[0], ois_df.OIS_DF[1]]

# Interpolating all the semi-annual OIS Discount Factors
for t in range(1,len(ois_df)):
    if (ois_df.Tenor[t] - ois_df.Tenor[t-1]) > 0.5:
        for x in np.arange(ois_df.Tenor[t-1] + 0.5, ois_df.Tenor[t]
+0.5, 0.5):
            # print(x)
            left_weight, right_weight = interpolate(ois_df.Tenor[t-
1], ois_df.Tenor[t], x)
            # print(left_weight, right_weight)

            tmp = left_weight*ois_df.OIS_DF[t-1] + right_weight*ois
_df.OIS_DF[t]
            semi_ois_df.append(tmp)

len(semi_ois_df)
# semi_ois_df
```

Out[20]:
```
60
```

Background Equations (Concept)

$PV_{fix} = K \sum_{i=1}^{n} \triangle_{i-1} D(0, T_i)$

$PV_{flt} = D(0, T_0) - D(0, T_n)$

$PV_{fix} = PV_{flt}$
$=> K \sum_{i=1}^{n} \triangle_{i-1} D(0, T_i) = D(0, T_0) - D(0, T_n)$

- Swap market is collateralized

Given the 6 months LIBOR rate L_{6m} = 2.5%,
D(0, 6m) = $\frac{1}{1+\triangle_{6m} L_{6m}} = \frac{1}{1+(0.5)(2.5\%)}$

In [21]:
```
d1 = 1/(1+accrusal_frac*irs_data.IRS_rate[0])
print("D(0,6m) = ", d1)
```

```
D(0,6m) =  0.9876543209876544
```

Bootstrapping D(0,1y)

$$PV_{fix} = PV_{flt}$$
$$D_o(0,6m) \times \triangle \times 2.8\% + D_o(0,1y) \times \triangle \times 2.8\% = D_o(0,6m) \times \triangle L_{6m} + D_o(0,1y) \times \triangle L(6m,1$$

$$\implies \triangle S_{1y}[D_o(0,6m) + D_o(0,1y)] = \triangle[D_o(0,6m)L_{6m} + D_o(0,1y)L(6m,1y)]$$

$$\implies L(6m,1y) = \frac{S_{1y}[D_o(0,6m)+D_o(0,1y)]-D_o(0,6m)L_{6m}}{D_o(0,1y)}$$

In [22]:
```
l1 = (irs_data.IRS_rate[1]*(ois_df.OIS_DF[0] + ois_df.OIS_DF[1]) -
ois_df.OIS_DF[0]*irs_data.IRS_rate[0]) / ois_df.OIS_DF[1]
l1
```

Out[22]:
```
0.031001496259351672
```

Knowing that forward LIBOR rate is given as

$$L(6m,1y) = \frac{1}{\triangle}\frac{D(0,6m)-D(0,1y)}{D(0,1y)}$$

$$\implies D(0,1y) = \frac{D(0,6m)}{1+\triangle L(6m,1y)}$$

In [23]:
```
d2 = d1/(1 + accrusal_frac*l1)
print("D(0,1y) = ", d2)
```

```
D(0,1y) =  0.9725786246900278
```

Bootstrapping D(0,2y):

$$PV_{fix} = PV_{flt}$$

$$\triangle 3\%[D_o(0,6m) + D_o(0,1y) + D_o(0,1.5y) + D_o(0,2y)] =$$
$$\triangle[D_o(0,6m)L(0,6m) + D_o(0,1y)L(6m,1y) + D_o(0,1.5y)L(1y,1.5y) + D_o(0,2y)L(1.5y,2y)]$$

- $L(1y, 1.5y) = \frac{1}{\triangle} \frac{D(0,1y) - D(0,1.5y)}{D(0,1.5y)}$
- $D(0, 1.5y) = \frac{D(0,1y) + D(0,2y)}{2}$

$=> L(1y, 1.5y) = \frac{1}{\triangle} \frac{D(0,1y) - [0.5D(0,1y) + 0.5D(0,2y)]}{0.5D(0,1y) + 0.5D(0,2y)} = \frac{1}{\triangle} \frac{0.5D(0,1y) - 0.5D(0,2y)]}{0.5D(0,1y) + 0.5D(0,2y)} = \frac{1}{\triangle} \frac{D(0,1y) - D(0,2y)]}{D(0,1y) + D(0,2y)}$

- $L(1.5y, 2y) = \frac{1}{\triangle} \frac{D(0,1.5y) - D(0,2y)}{D(0,2y)}$
- $D(0, 1.5y) = \frac{D(0,1y) + D(0,2y)}{2}$

$=> L(1.5y, 2y) = \frac{1}{\triangle} \frac{[0.5D(0,1y) + 0.5D(0,2y)] - D(0,2y)}{D(0,2y)} = \frac{1}{\triangle} \frac{0.5[D(0,1y) - D(0,2y)]}{D(0,2y)}$

In [24]:
```
def calculateInterpolatedDF(lw, rw, lhs, rhs, target_DF):

    c = 1/accrusal_frac
    tmp_semi_ois_df = semi_ois_df[2*lhs-1:2*rhs]
    sum_RHS = 0

    for l in range(1, len(lw)):
        lhs_intp_df = (lw[l-1]*libor_df[2*lhs-1] + rw[l-1]*target_D
F)
        rhs_intp_df = (lw[l]*libor_df[2*lhs-1] + rw[l]*target_DF)

        sum_RHS += tmp_semi_ois_df[l-1]*c*((lhs_intp_df - rhs_intp_
df)/rhs_intp_df)

    return sum_RHS
```

In [25]:
```
libor_df = [d1, d2]
forward_libor = [irs_data.IRS_rate[0], l1]
# print(libor_df); print(forward_libor)

flt_sum = semi_ois_df[0]*forward_libor[0] + semi_ois_df[1]*forward_libor[1]

for t in range(2,11):
    # print(t)
    idx = int(irs_data["Tenor"][t])
    pv_fix = irs_data.IRS_rate[t]*sum(semi_ois_df[:2*idx])

    lw = []
    rw = []

    # For loop just to calculate weights in each interval
    for x in np.arange(irs_data["Tenor"][t-1], irs_data["Tenor"][t]+0.5, 0.5):
        # print(x)
        LHS, RHS = int(irs_data["Tenor"][t-1]), int(irs_data["Tenor"][t])
        left_weight, right_weight = interpolate(LHS, RHS, x)
        # print(left_weight, right_weight)
        lw.append(left_weight)
        rw.append(right_weight)

    tmp_df = brentq(lambda x: pv_fix - (flt_sum + calculateInterpolatedDF(lw, rw, LHS, RHS, x)), 1e-6, 1)
    # print(tmp_df)

    for l in range(1, len(lw)):
        libor_df.append((lw[l]*libor_df[2*LHS-1] + rw[l]*tmp_df))

    # print(libor_df)

    flt_sum +=  calculateInterpolatedDF(lw, rw, LHS, RHS, libor_df[-1])
```

In [26]:
```
len(libor_df), forward_libor
```

Out[26]:
```
(60, [0.025, 0.031001496259351672])
```

In [27]:
```python
irs_df = pd.DataFrame()
irs_df["Tenor"] = np.arange(0.5, 30.5, 0.5)
irs_df["LIBOR_DF"] = libor_df

irs_df = pd.merge(irs_df, irs_data, on="Tenor", how="left").drop(["
Product", "Rate"], axis=1)

irs_df.head(3)
```

Out[27]:

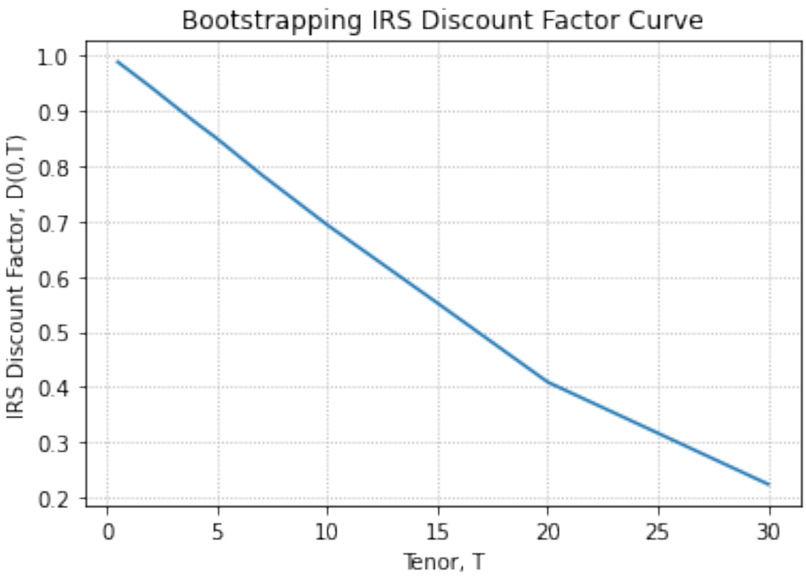
	Tenor	LIBOR_DF	IRS_rate
0	0.5	0.987654	0.025
1	1.0	0.972579	0.028
2	1.5	0.957406	NaN

In [28]:
```python
irs_df.head(3)
```

Out[28]:

	Tenor	LIBOR_DF	IRS_rate
0	0.5	0.987654	0.025
1	1.0	0.972579	0.028
2	1.5	0.957406	NaN

In [29]:
```python
plt.plot(figsize=(24,16))
plt.plot(irs_df["Tenor"], irs_df["LIBOR_DF"])
plt.xlabel("Tenor, T")
plt.ylabel("IRS Discount Factor, D(0,T)")
plt.title("Bootstrapping IRS Discount Factor Curve")
plt.grid(linestyle='dotted')
```



(3) Calculate the following forward swap rates:

- 1y×1y, 1y×2y, 1y×3y, 1y×5y, 1y×10y
- 5y×1y, 5y×2y, 5y×3y, 5y×5y, 5y×10y
- 10y×1y, 10y×2y, 10y×3y, 10y×5y, 10y×10y

Use linear interpolation on discount factors when necessary.

In [30]:
```
df_combined = pd.merge(irs_df, ois_df, on="Tenor", how="left")
df_combined.drop("Rate", axis = 1, inplace=True)
df_combined.head(3)
```

Out[30]:

	Tenor	LIBOR_DF	IRS_rate	OIS_rate	OIS_DF	f
0	0.5	0.987654	0.025	0.0025	0.997506	0.004994
1	1.0	0.972579	0.028	0.0030	0.997009	0.000997
2	1.5	0.957406	NaN	NaN	NaN	NaN

In [31]:
```
df_combined["OIS_DF"] = df_combined["OIS_DF"].interpolate()
# df_combined["Forward LIBOR"] = df_combined["Forward LIBOR"].inter
polate()

df_combined.head(3)
```

Out[31]:

	Tenor	LIBOR_DF	IRS_rate	OIS_rate	OIS_DF	f
0	0.5	0.987654	0.025	0.0025	0.997506	0.004994
1	1.0	0.972579	0.028	0.0030	0.997009	0.000997
2	1.5	0.957406	NaN	NaN	0.995270	NaN

In [32]:
```
df_combined['Foward_LIBOR'] = np.NAN
df_combined['Foward_LIBOR'][0] = irs_data.IRS_rate[0]
df_combined['Foward_LIBOR'][1] =l1

df_combined.head()
```

Out[32]:

	Tenor	LIBOR_DF	IRS_rate	OIS_rate	OIS_DF	f	Foward_LIBOR
0	0.5	0.987654	0.025	0.00250	0.997506	0.004994	0.025000
1	1.0	0.972579	0.028	0.00300	0.997009	0.000997	0.031001
2	1.5	0.957406	NaN	NaN	0.995270	NaN	NaN
3	2.0	0.942234	0.030	0.00325	0.993531	0.003495	NaN
4	2.5	0.926412	NaN	NaN	0.991773	NaN	NaN

In [33]:
```
# while loop to solve forward LIBOR for each tenor
i = 3
while i <= 60:
    h = i

    for n in range(0,h-i+2): # Populate Forward LIBOR
        df_combined['Foward_LIBOR'][i-1+n] = (df_combined['LIBOR_DF'][i-2+n]-df_combined['LIBOR_DF'][i-1+n]) /\
        (0.5*df_combined['LIBOR_DF'][i-1+n])

    i = h+2 # increase by 1 year (0.5*2)
```

In [34]:
```
df_combined.rename(columns= {"Tenor": "Tenor (year)",
                             "LIBOR_DF": "LIBOR Discount Factor",
                             "IRS_rate": "IRS Rate",
                             "OIS_rate": "OIS Rate",
                             "OIS_DF": "OIS Discount Factor",
                             "f": "OIS Overnight Rate",
                             "Foward_LIBOR": "Forward LIBOR"},
                   inplace=True)
```

In [35]:
```
def par_swap_rate_solver(start,duration):
    end = start + duration
    start_i = start*2 #find the start index
    end_i = end*2-1 #find the end index
    float_leg_sum = 0 # initialize
    fix_discount_sum = 0 # initialize
    while start_i <= end_i:
        float_leg = df_combined['Forward LIBOR'][start_i]*0.5*df_combined['OIS Discount Factor'][start_i] # float leg value at each tenor
        float_leg_sum += float_leg
        fix_discount = 0.5*df_combined['OIS Discount Factor'][start_i] #fix discount at each tenor * 0.5
        fix_discount_sum += fix_discount
        start_i += 1
    par_swap_rate = float_leg_sum/fix_discount_sum # formula in the markdown
    return par_swap_rate
```

In [36]:
```
df_combined.head(3)
```

Out[36]:

	Tenor (year)	LIBOR Discount Factor	IRS Rate	OIS Rate	OIS Discount Factor	OIS Overnight Rate	Forward LIBOR
0	0.5	0.987654	0.025	0.0025	0.997506	0.004994	0.025000
1	1.0	0.972579	0.028	0.0030	0.997009	0.000997	0.031001
2	1.5	0.957406	NaN	NaN	0.995270	NaN	0.031695

In [37]:
```
df_combined.to_csv('df_comb.csv',sep=',')
```

In [38]:
```
# print(df_combined)
```

In [39]:
```
df_FowardSwap = pd.DataFrame([['1Y','1Y',par_swap_rate_solver(1,1)],['1Y','2Y',par_swap_rate_solver(1,2)],['1Y','3Y',par_swap_rate_solver(1,3)],['1Y','5Y',par_swap_rate_solver(1,5)],['1Y','10Y',par_swap_rate_solver(1,10)],\
                             ['5Y','1Y',par_swap_rate_solver(5,1)],['5Y','2Y',par_swap_rate_solver(5,2)],['5Y','3Y',par_swap_rate_solver(5,3)],['5Y','5Y',par_swap_rate_solver(5,5)],['5Y','10Y',par_swap_rate_solver(5,10)],\
                            ['10Y','1Y',par_swap_rate_solver(10,1)],['10Y','2Y',par_swap_rate_solver(10,2)],['10Y','3Y',par_swap_rate_solver(10,3)],['10Y','5Y',par_swap_rate_solver(10,5)],['10Y','10Y',par_swap_rate_solver(10,10)]],\
                            columns=['Expiry','Tenor','Swap_Rate'])
```

In [40]:
```
df_FowardSwap.head()
```

Out[40]:

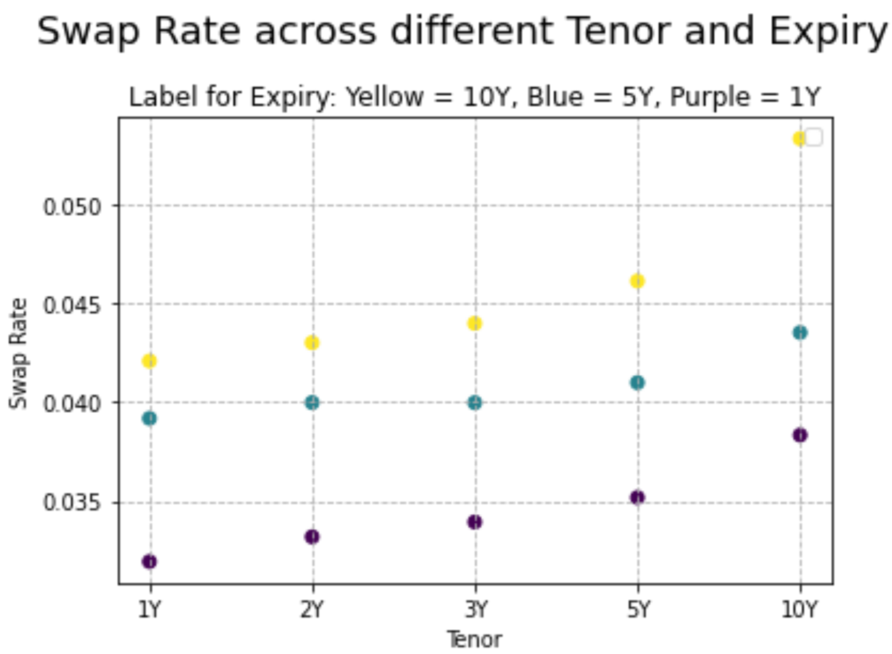
	Expiry	Tenor	Swap_Rate
0	1Y	1Y	0.031950
1	1Y	2Y	0.033200
2	1Y	3Y	0.033947
3	1Y	5Y	0.035184
4	1Y	10Y	0.038335

In [41]:
```
df_FowardSwap["color"] = [1, 1, 1, 1, 1, 5, 5, 5, 5, 5, 10, 10, 10, 10, 10]
```

In [42]:
```
df_FowardSwap['Expiry'] = df_FowardSwap['Expiry'].astype(str)
```

In [43]:
```
plt.plot(figsize=(24,16))
plt.scatter(df_FowardSwap.Tenor, df_FowardSwap.Swap_Rate, c=df_Fowa
rdSwap.color)
plt.xlabel("Tenor")
plt.ylabel("Swap Rate")
plt.suptitle("Swap Rate across different Tenor and Expiry", y=1.05,
fontsize=18)
plt.title("Label for Expiry: Yellow = 10Y, Blue = 5Y, Purple = 1Y")

plt.grid(linestyle='--')
plt.legend()
plt.show()
```

```
No artists with labels found to put in legend.  Note that artists
whose label start with an underscore are ignored when legend() is
called with no argument.
```



In [44]:
```
df_FowardSwap.to_csv('df_ForwardSwap.csv',sep=',')
```