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Marvels and mysteries of rational base numeration systems

Jacques Sakarovitch

The definition of numeration systems with rational base, in a joint work with S. Akiyama and Ch. Frougny (Israel J. Math. 2008), has allowed to make some progress in a number theoretic problem, by means of automata theory and combinatorics of words.

At the same time, it raised the problem of understanding the structure of the sets of the representations of the integers in these systems from the point of view of formal language theory. At first sight, these sets look rather chaotic and do not fit well in the classical Chomsky hierarchy of languages.

On the other hand, these sets also exhibit remarkable regularity properties. In the subsequent years, these regularities have been studied in a series of joint papers with my student V. Marsault. In particular, we have shown that periodic signatures are characteristic of the representation languages in rational base numeration systems (Indagationes Mathematicae, 2017) and studied, jointly with S. Akiyama, a kind of autosimilarity property that also leads to the construction of Cantor-like sets (Discrete Mathematics and Theoretical Computer Science 2018).

The representation languages still keep most of their mystery. The partial results which will be presented call for further investigations on the subject even stronger.