CMI Mathematics Colloquium

September 13, 2023

On algebraicity of power series in positive characteristic

Xavier Caruso

Let $f(x) = \sum_{n\geq 0} a_n x^n$ be a power series with coefficients in a field k. In general, recognizing whether $f(x)$ is algebraic over $k(x)$, i.e. whether there exists a bivariate polynomial $P(x, y)$ such that $P(x, f(x))$ is a difficult question.

However, when k has positive characteristic, there is a beautiful criterion, known as Christol's theorem, allowing or reading algebraicity on the sequence $(a_n)_{n\geq 0}$ of the coefficients.

In this talk, I will discuss Christol's theorem, illustrate it with many examples and give several applications. I will also investigate how this could help to attack the case of characteristic zero.