CMI Mathematics Colloquium

August 23, 2023

Amenable and Hyperbolic Groups, Stable Random Fields and von Neumann Algebras

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Random fields indexed by amenable groups arise naturally in machine learning algorithms for structured and dependent data. On the other hand, mixing properties of such fields are extremely important tools for investigating asymptotic properties of any method/algorithm in the context of space-time statistical inference. In this work, we find a necessary and sufficient condition for weak mixing of a left-stationary symmetric stable random field indexed by an amenable group in terms of its Rosinski representation. The main challenge is ergodic theoretic - more precisely, the unavailability of an ergodic theorem for nonsingular (but not necessarily measure preserving) actions of amenable groups even along a tempered Følner sequence. We remove this obstacle with the help of a truncation argument along with the seminal work of Lindenstrauss (2001) and Tempelman (2015), and finally applying the Maharam skew-product. This work extends the domain of application of the speaker's previous paper connecting stable random fields with von Neumann algebras via the group measure space construction of Murray and von Neumann (1936). In particular, weak mixing has now become $W*$-rigid properties for stable random fields indexed by any amenable group, not just $\mathbb{Z}^d$. We have also shown that many stable random fields generated by natural geometric actions of hyperbolic groups on various negatively curved spaces are ergodic.

This talk is based on a joint work with Mahan Mj (TIFR Mumbai) and Sourav Sarkar (University of Cambridge).