CMI Mathematics Colloquium

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0-cycles and K-theory

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In 1960, Serre proved that for a commutative ring R of dimension d, every projective R-module of rank $> d$ has a free direct summand, and in 1994, Murthy studied this question for projective modules of rank equal to d, when R is a smooth commutative ring over an algebraically closed field k.

In the talk, we recall the definition of a projective module and the main statement of Murthy. We discuss the main idea of Murthy's proof and a generalization of his result for a singular commutative ring R over k. In particular, we define the group of 0-cycles $\mathrm{CH}_0(R)$ and the Grothendieck group $K_0(R)$ associated to R, and construct a cycle class map $\rho_R : CH_0(R) \to K_0(R)$. One of the steps in Murthy's proof is that the kernel of ρ_R is divisible by $n!$ for smooth rings over k. We prove a similar result for singular rings over k. If time permits, as an application, we derive Bloch's formula for such rings.

The talk is based on a joint work with Prof. A. Krishna.