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A probabilistic proof of two combinatorial identities

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This talk aims at telling several stories, mathematical or otherwise. In particular, let X be a denumerable probability space, n an integer ≥ 2, endow X^n with the product probability. The computation of the probability of the subset $X(n)$ of n-tuples of pairwise distinct elements is part of the folklore in statistical mechanics and in combinatorics, but does not seem to be as well-known as it could be. We give an elementary proof for a slightly more general computation, and obtain along the way that for each positive integer n, the sum of the terms $(-1)^k c_k(n)$, where $c_k(n)$ denotes the number of connected graphs with n vertices and k edges equals $(-1)^{n-1}(n-1)!$.(There is another proof using generatingfunctionology).