Tutorial-1

Name→ Aditya Kotiyal

Section→ CST SPL-1

Semester→ 4

Class Roll NO→ 59

University Roll NO→ 2017450

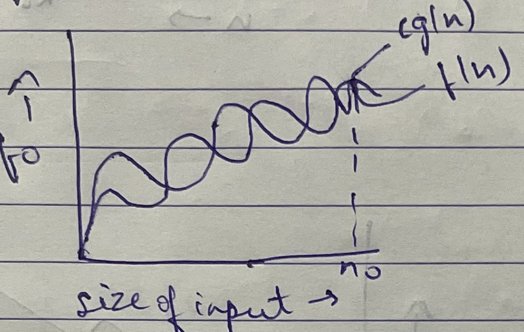
Date→ 10 March 2022

Date.
Page No.

1. Asymptotic Notations

They help you find the complexity an algorithm when input is very large.

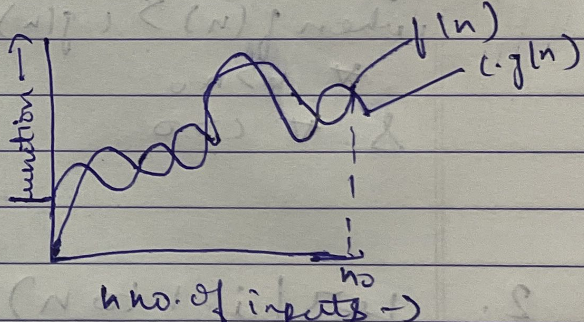
1) Big O (o)



$f(n) = O(g(n))$
iff $f(n) \le c\,g(n)$
$\forall\, n \geq n_0$
for some w.r.t $c > 0$

⇒ $g(n)$ is tight upper bound of $f(n)$

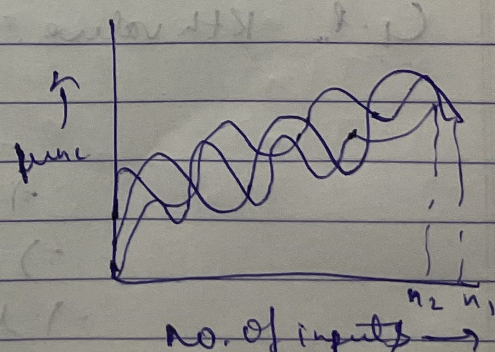
2) Big Omega (Ω)



$f(n) = \Omega(g(n))$
$g(n)$ is 'tight' lower bound of $f(n)$
$f(n) = \Omega(g(n))$
iff $f(n) \geq c\,g(n)$
$\forall\, n \geq n_0$ for some constant $c > 0$

3) Theta (θ)

$f(n) = \Theta(g(n))$
$g(n)$ is both 'tight' upper & lower bound of fn $f(n)$
$f(n) = \Theta(g(n))$
iff $c_1 g(n) \le f(n) \le c_2 \cdot g(n)$
$\forall\, n \geq \max(n_1, n_2)$
for some constant $c_1 > 0$ & $c_2 > 0$



4) Small o(o)

$f(n) = o(g(n))$

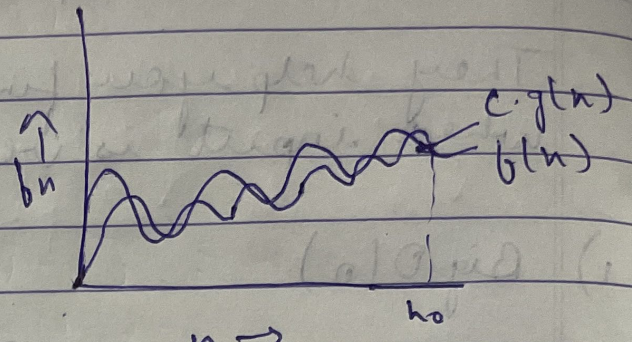
$g(n)$ is upper bound of fn $f(n)$.

$f(n) = o(g(n))$

when $f(n) < c \cdot g(n)$

$\forall\ n > n_0$

& $\forall\ c > 0$



5) Small omega (ω)

$f(n) = \omega(g(n))$

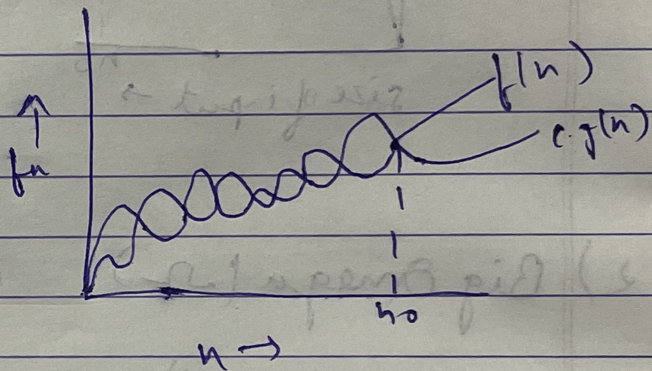
$g(n)$ is lower bound of fn $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c \cdot g(n)$

$\forall\ n > n_0$

& $\forall\ c > 0$



2. for (i = 1 to n) ~~{i = i * 2}~~ // i = 1, 2, 4, 8 -- n

{i = i * 2} // O(1)

$\Rightarrow \sum_{i=1}^{n} 1 + 2 + 4 + 8 + ---- n$

G.P Kth value $\Rightarrow T_k = ar^{k-1}$

$\Rightarrow 1 \times 2^{k-1}$

$\Rightarrow 2n = 2^k$

$\Rightarrow \log 2n = k \log 2$

$\Rightarrow \log 2 + \log n = k \log 2$

$\Rightarrow \log n + 1 = k$

Date.
Page No.

$\Rightarrow O(k) = O(1 + \log n)$
$= O(\log n)$

3. $T(n) = 3T(n-1)$ — 1
put $n = n-1$
$T(n-1) = 3T(n-2)$ — 2
from 1 & 2

$\Rightarrow T(n) = 3(3T(n-2))$
$= 9T(n-2)$ — 3

putting $n = n-2$ in 1

$T(n) = 3(T(n-3))$ — 4
$\Rightarrow T(n) = 27(T(n-3))$
$\Rightarrow T(n) = 3^k(T(n-k))$
putting $n-k=0$
$\Rightarrow n = k$
$\Rightarrow T(n) = 3^n[T(n-n)]$
$\Rightarrow T(n) = 3^n T(0)$
$\Rightarrow T(n) = 3^n \times 1$ $[T(0)=1]$
$\Rightarrow T(n) = O(3^n)$

4. $T(n) = 2T(n-1) - 1$ — 1
Let $n = n-1$
$\Rightarrow T(n-1) = 2T(n-2) - 1$ — 2

from 1 & 2

$T(n) = 2[2T(n-2) - 1] - 1$
$\Rightarrow T(n) = 4T(n-2) - 2 - 1$ — 3

let $n = n-2$

$\Rightarrow T(n-2) = 2T(n-3) - 1$ — (4)

~~let n~~ from (3) & (4)

$\Rightarrow T(n) = 4[2T(n-3) - 1] - 2 - 1$

$\Rightarrow T(n) = 8T(n-3) - 4 - 2 - 1$

$\Rightarrow T(n) = 2^K T(n-K) - 2^{K-1} - 2^{K-2}$

$\Rightarrow GP = 2^{K-1} + 2^{K-2} + 2^{K-3} + \cdots$

$a = 2^{K-1}$

$r = 1/2$

$\Rightarrow S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{2^{K-1}(1-(1/2)^n)}{1/2}$

$= 2^K(1-(1/2)^K)$

$= 2^K - 1$

let $n-K = 0$

$\Rightarrow n = K$

$\Rightarrow T(n) = 2^n T(n-n) - (2^n - 1)$

$\Rightarrow T(n) = 2^n - 1 - (2^n - 1)$

$\Rightarrow T(n) = 2^n - (2^n - 1)$

$\Rightarrow T(n) = O(1)$

Date.
Page No.

5. $i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$s = 1 * 3 * 6 * 10 * 15 * 21 \text{ --- } n$

Sum of $s = 1 + 3 + 8 + 10 + \text{---} + n$ — ①

also $s = 1 + 3 + 8 + 10 + \text{---} T_{n-1} + T_n$ — ②

from ① - ②

$0 = 1 + 2 + 3 + 4 + \text{---} - n - T_n$

$\Rightarrow T_k = 1 + 2 + 3 + 4 + \text{---} k$

$\Rightarrow T_k = \frac{1}{2} k(k+1)$

$\Rightarrow$ for k iterations

$1 + 2 + 3 + \text{---} + k <= n$

$\Rightarrow \frac{k(k+1)}{2} <= n$

$\Rightarrow \frac{k^2 + k}{2} <= n$

$\Rightarrow O(k^2) <= n$

$\Rightarrow k = O(\sqrt{n})$

$\Rightarrow T(n) = O(\sqrt{n})$

Date.
Page No.

6. as $i^2 <= n$

$\Rightarrow i <= \sqrt{n}$

$i = 1, 2, 3, 4 \dots, \sqrt{n}$

$\sum_{i=1}^{n} 1+2+3+4+ \dots + \sqrt{n}$

$\Rightarrow T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$\Rightarrow T(n) = \frac{n \times \sqrt{n}}{2}$

$\Rightarrow T(n) = O(n)$

7. for $K = K * 2$

$K = 1, 2, 4, 8, \dots n$

$\Rightarrow$ GP $\rightarrow$ $a = 1, r = 2$

$= \frac{a(r^n - 1)}{r-1}$

$= \frac{1(2^k - 1)}{1}$

$n \Rightarrow 2^k$

$\Rightarrow \log n = K$

$\Rightarrow$

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	$\log n * \log n$
$\vdots$	$\vdots$	$\vdots$
n	$\log n$	$\log n * \log n$

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

8. $T(n) = T(n/3) + n^2$

$\Rightarrow a = 1, \; b = 3 \qquad f(n) = n^2$

$c = \log_3 1 = 0$

$\Rightarrow n^0 = 1 > (f(n) = n^2)$

$\Rightarrow T(n) = \Theta(n^2)$

9.

for $i=1 \Rightarrow j = 1, 2, 3, 4, \dots n \qquad = n$
for $i=2 \Rightarrow j = 1, 3, 5, \dots n \qquad = n/2$
for $i=3 \Rightarrow j = 1, 4, 7, \dots n \qquad = n/3$
$\vdots \qquad \vdots$
for $i=n \Rightarrow j = 1 \dots \qquad 1$

$\Rightarrow \sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \text{-----} + 1$

$\Rightarrow \sum_{j=n}^{1} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \text{----} + \frac{1}{n}\right]$

$\Rightarrow \sum_{j=n} n[\log n]$

$\Rightarrow T(n) = [n \log n]$

$T(n) = O(n \log n)$

10. as given n^k & c^n

relation b/w n^k & c^n is

$n^k = O(c^n)$

as $n^k \leq a c^n$

$\forall n \geq n_0$ & some constant $a > 0$

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a 2^1$

$\Rightarrow n_0 = 1$ & $c = 2$