AHP with Consensus Learning

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Overview

1. User-User Comparison Matrix
   - Single Label User-User Comparison Matrix
   - User correlations for set of labels
2. Label Correlations
   - Single user

User comparison for single label

Setup : users tags videos with different labels.
We try to compare between the users for each label. The quantity learned would be user reliability. We generate system's relative reliability / confidence of one user over the other, learning a matrix with elements standing for reliability values.

User Reliability Matrix

A simple way to build reliability matrix -
Users u_i , u_j and label l. The space of videos with having label l, watched by both users. n_i videos labelled l by u_i, similarly n_j and $n_{ij} = n_{ji}$ videos which are labelled by both as l.
Relative reliability is given by following ratio -

$$\begin{aligned} r_{ij} &= \frac{{}^{n_{ij}+1}/_{n_i+1}}{{}^{n_{ji}+1}/_{n_j} + 1} \\ &= {}^{n_j+1}/_{n_i+1} \end{aligned} \tag{1}$$

where ${}^{n_{ij}+1}/_{n_i+1}$ denotes fraction of videos labelled by u_i over which he receives consensus with u_j. Thus it describes relative reliability of u_i over u_j. 1 is added to avoid division by 0.

Interpretation

$R = [r_{ij}]$ is the relative reliability matrix (reliability preference).

$$r_{ij} = {}^{n_j+1}/_{n_i+1} \tag{2}$$

This value also stands for inverse of fraction of videos labelled as l amongst the common videos watched by the two users. **The lesser the number of videos labelled, the more is the reliability; since less tagging means restricted use of tag l - means tag is used only when it is really required.** This would form the basis of trust on the users.

Properties of R_{ij}

The previous definition of elements of relative reliability (reliability preference) matrix has following properties :

1. $r_{ij} = {}^{n_j+1}/_{n_i+1} = \frac{1}{r_{ji}}$, reciprocal matrix without any assumption
2. Consider the ideal case, when all videos containing a given label l is watched by all users. Then n_i represents absolute value of videos labelled l by user u_i. Thus we get $r_{ij} * r_{jk} = \frac{n_j}{n_i} * \frac{n_k}{n_j} = \frac{n_k}{n_i} = r_{ik}, \forall i, j, k$, which tells matrix would be consistent in ideal case.
3. The previous point gives that the principal eigen vector of this matrix would be $r = (r_1, ..., r_t), r_i = {}^1/_{n_i}$, (with $\lambda_{max} = t$, $t =$ total users), which means the reliability for user u_i is ${}^1/_{n_i}$, inverse of number of videos labelled as l.

Remarks on R_{ij}

In case of non ideal environment, when not all videos, labelled as I, are watched by all users, we can expect that the matrix is near consistent and we could identify pairs of users for which we need to revise our relative reliability measure. This could be achieved by following -

- Use method outlined in AHP paper to identify which value needs to be updated to increase the consistency to maximum, so as to use eigen vector as reliability measure
- Based on interpretation of matrix R, we could find users which have few number of videos in common and suggest such videos to users based on this
- ? Not sure if first point would actually mean the second point

Remarks on R_{ij}

Is the definition of r_{ij} really capturing relative reliability ?

- We have not really accounted for consensus, except in beginning of formulating r_{ij}
- Cases in which number of videos tagged is same but the overlaps are different are not captured
- Cases in which no common videos are present is treated same as in which all videos are labelled as I by both users

Set of Labels

In this case, we would like to capture similar notion of user reliability over a set of labels, $L = \{l_k\}$. We extend our previous formulation to this case with n_i being the number of common videos which are labelled by either of labels from L -

$$r_{ij} = \frac{n_j}{n_i} \tag{3}$$

which stands for inverse of ratio of videos labelled by label from L amongst the common videos watched.

Remarks

For the set of labels case :

- All desired properties as in case of single labels hold
- Lesser information is used to infer reliability
- Information of overlap between labels in same videos and across videos is lost

Label correlation for single user

We wanted some confusion matrix between labels for some user.
Based on the data that we have i.e. set of labels for each video is the set, it becomes difficult to come up with a measure of confusion based on only these parameters, since

- Are the labels actually correlated or the labeller is confused cannot be inferred from just the data
- Videos could have unrelated materials
- Correlation / confusion is bidirectional - we need something unidirectional for analysis as previously done

We could but define bias over one label than other.

Confusion

In order to detect confusion between labels we would need data across all users to check for whether some users are confused or if the labels are correlated in some sense.
Need more analysis to reach correct inference.

References

Thomas L. Saaty
Decision-making with the AHP: Why is the principal eigenvector necessary

The End