Question 1

(d). A CT scanner designer has to select of Δt, and $\Delta\theta$ for taking the scans. The trade off is between quality of image taken and exposure time of the patient.

Let us analyse the effects on one slice of body. Then the factors which influence image reconstruction quality and exposure time is Δt and $\Delta\theta$. Consider that we have W as the maximum body width to be convered, so t spreads from $-W/2$ to $W/2$. θ varies from 0 to 180. Thus number of effective x-ray beams shot would be $\frac{W * 180}{\Delta t * \Delta\theta}$. The value of denominator could be chosen based on exposure time desired. Thus we see that Δt almost equal to $\Delta\theta$ for a fixed value of denominator ensures lower value of effective x-ray beams and a good spread of measurements over t and θ. Values of $\Delta\theta$ and Δt which are not well spread would not be desirable to generate effective reconstruction. Following tells the effect of $\Delta\theta$ variation with Δt fixed -

Figure 1: Original Image

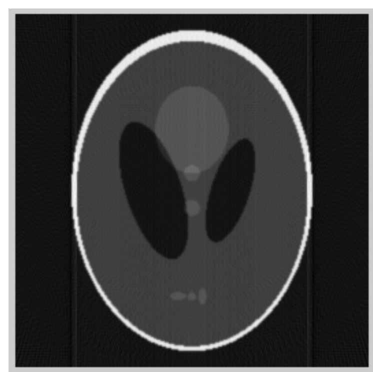

Figure 2: $\Delta\theta = 1$ degrees

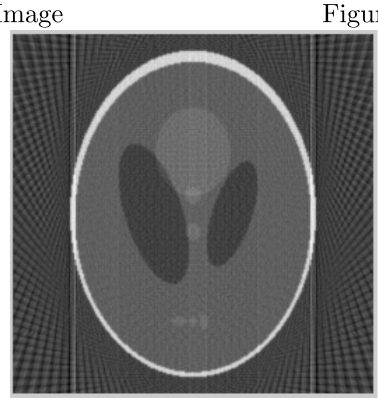

Figure 3: $\Delta\theta = 3$ degrees

Clearly the spread of t, θ should be almost the same.

(e). In the ART technique of image reconstruction, $Ax = b$, x is the vectorized values of attenuation coefficients. We estimate this using the ART method. Each b_i correspond to the measurement (integral) received for a given t, θ. The pixel grid selection would be based on number of t, θ measurements that could be taken, since that is equal to the number of unknowns. The number of measurements is related to the exposure time of patient to the radiation. So appropriate pixel grid, depending directly on number of t, θ measurements, which reduces the exposure time of the patient as well as gives good image reconstruction would be desirable. The Δs values tells about the step size of integral and hence should be chosen such that good sharp image is obtained with less noise effects.

Each row of A, a_i captures the integral.
For a given image grid, if $\Delta s << 1$ pixel width, then the situation if similar to calculating integrating a function, for which we know values at fixed points, using very small step size. It would correspond to calculating area of reactangles associated with each value of function. Thus if noise is present, then it would not get smoothed. We would get sharp edges in recontructed images.
If $\Delta s >> 1$ pixel width, then the situation is like calculating integration using large step size. Thus we could be smooting the function too much, producing blurred edges in the image.
Thus Δs should be such that we get good crisp images, with less noise effects.

(c). The plots for $\Delta s = 0.5$ pixel width appear to be smoothest, while $\Delta s = 3$ pixel width appear to be roughest. Reason would be the integral calculated for small step size is better approximation of integral and hence no abrupt discontinuity in values. Thus the smoother function. Similar observation for images. The smoother image is one corresponding to smaller Δs.