Kalman Filters in Robot Localization

Introduction to Artificial Intelligence : Seminar Report

Vipul Harsh
Nilesh Kulkarni
Sushant Hiray
Anand Soni

Topics

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Robot localization : Motivation

- The problem of **robot localization** consists of answering the question **where am I** from the robot's point of view.
- It is a key problem in making truly autonomous robots.

Problem Statements

- **Position Tracking Problem**

The problem is to keep track of the position while the robot is navigating through the environment. The robot knows its initial position.

- **Wake up robot or Global positioning problem**

The robot does not know its initial position. In order for this to work, the robot needs to work out an initial belief using the sensory measurements obtained from the environment.

Problem Statements

- **Kidnapped Robot problem**

The robot is aware of its localization (i.e, it has a certain belief about its location) and then it is moved to another location all of a sudden without the robot knowing about it. The robot first has to detect that it has been kidnapped and then come up with a belief for its location.

Available Information

- A-priori information
  - Maps
  - Cause-effect relationships (System Model)
- Navigational information
  - Relative measurement
  - Sensory measurement

Naïve idea of localization

Initial state
detects nothing:

Moves and
detects landmark:

Moves and
detects nothing:

Moves and
detects landmark:

Kalman Filters, What & Why?

- The Kalman filter is a **recursive data processing algorithm** that estimates the state of a noisy linear dynamic system.
- The **state of a system is modelled as a vector** consisting of variables that describe some properties of interest of the system.
- A KF has access to **measurements** of a system that are **linearly related to the state** and are corrupted by noise.

Kalman Filters, What & Why?

- The motivation behind the use of Kalman filters in robot localization is that it **uses** only **the current estimate** and **measurement** to obtain the next estimate and no history of observations and estimates is required (Though it accounts for history in some way).
- Due to its small computational requirement, elegant recursive properties
- Its status as the optimal estimator for one-dimensional linear systems with Gaussian error statistics

Kalman Filter, Assumptions

- State Variables are Multivariate Gaussian
- Linear System
- Characteristics of noise
  - **Independence**

    The noise in system is independent of the noise in measurements.
  - **White noise**

    The noise at the current time step is independent of the noise at any other time step.
  - **Zero-mean**

    The mean of the noise is zero.
  - **Gaussian noise**

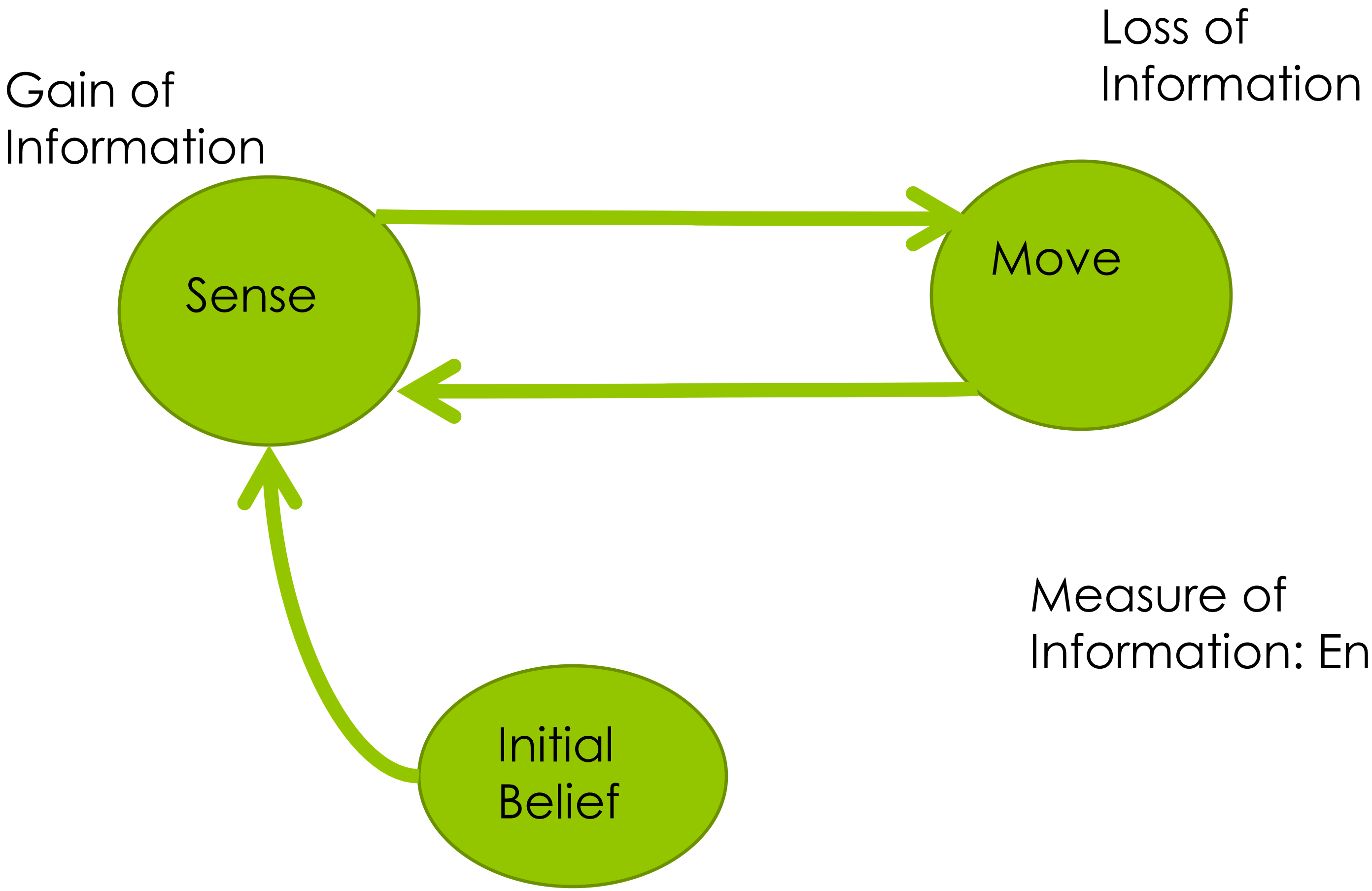
How localization works?

- The functioning of a Kalman filter can be divided into two phases:
  - **Prediction phase**

    The filter computes a prior state estimate and prior covariance of the estimate using the previous state estimate and the “action” taken by the robot.
  - **Correction phase**

    The filter uses the sensory measurements from the environment and the estimates computed in the prediction phase to obtain a posterior state estimate and the posterior estimate covariance.

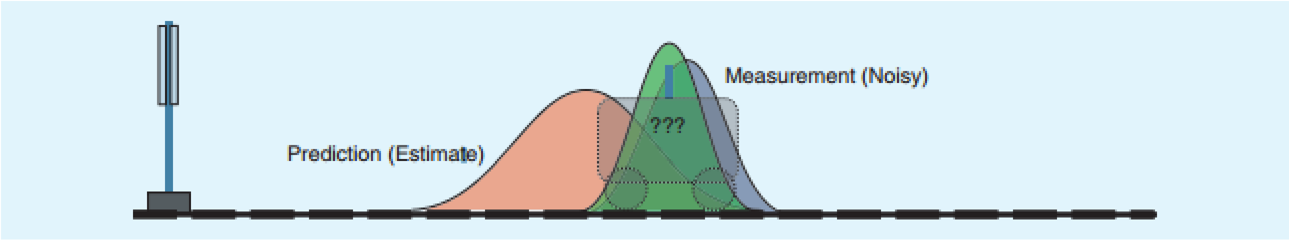
Kalman Filter Working



Measure of
Information: Entropy

Robot Localization in One Dimension

- Intuitively if the predicted position of the robot and the state information from the sensor are Gaussian random variables, then one can combine both to get the optimal estimate of the robot as a Gaussian which has mean lying between the two of them , and variance less then both.
- Because independently these Gaussians did not contribute as much information to the system, so more information implies less uncertainty in state



[FIG5] Shows the new pdf (green) generated by multiplying the pdfs associated with the prediction and measurement of the train's location at time $t = 1$. This new pdf provides the best estimate of the location of the train, by fusing the data from the prediction and the measurement.

Terminology

- **Measurement Residual or Innovation**
  - It is the difference between the observed measurement and the expected measurement from the current estimated location.
- **Kalman Gain**
  - Kalman gain factor determines the extent to which the innovation should be taken into account in the posterior state estimate.

Kalman Filters, Prior Equations

$$\hat{x}^{-}{}_{k} = F_k \hat{x}^{+}{}_{k-1} + B_k U_k$$
$$P^{-}{}_{k} = F_k P^{+}{}_{k-1} F_k{}^{T} + Q_k$$

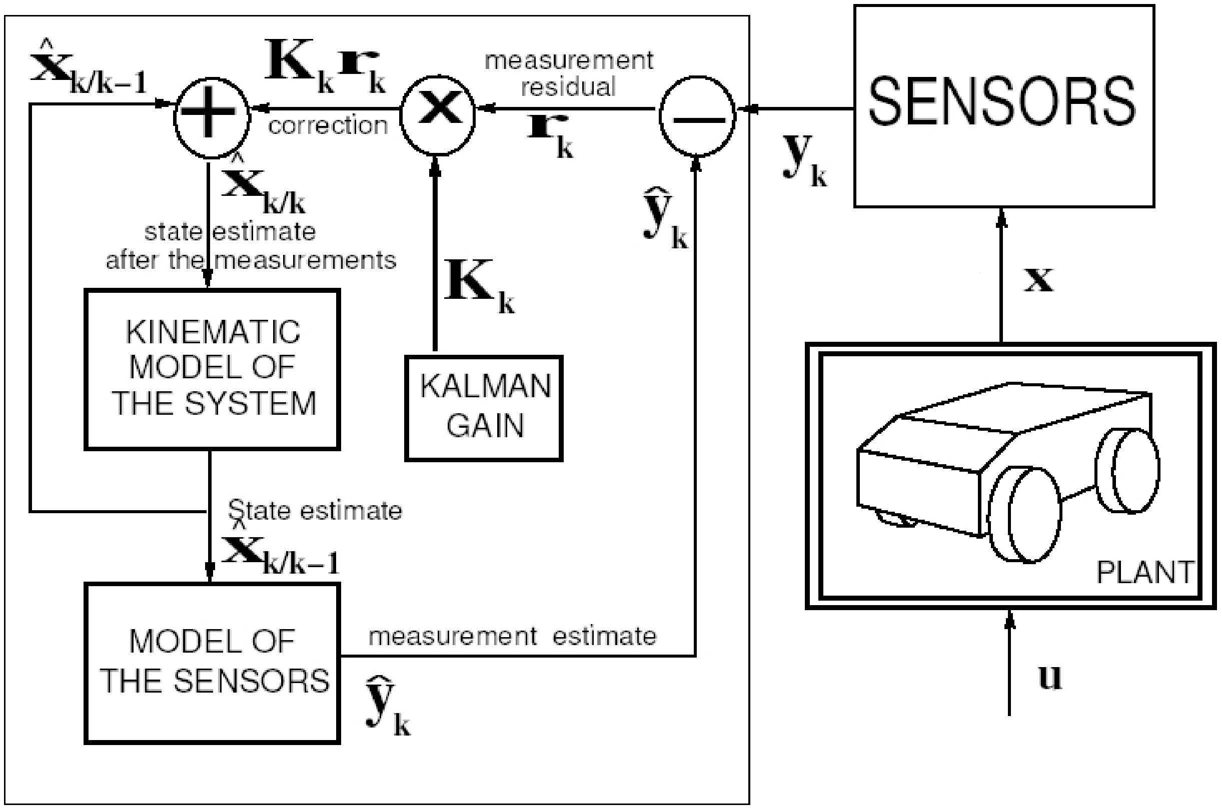
- $\hat{x}^{-}{}_{k}$ is the prior state estimate at time step k
- $P^{-}{}_{k}$ is the prior covariance matrix
- F_k relates state at time step k-1 to the state at time step
- B_k is the control input model
- U_k is the control input vector
- Q_k is covariance matrix of the state space noise

Kalman Filters, Posterior Equations

$$\hat{y}_k = z_k - H_k \hat{x}^-{}_k$$
$$S_k = H_k P^-{}_k H_k{}^T + R_k$$
$$K_k = P^-{}_k H_k{}^T S_k{}^{-1}$$
$$\hat{x}^+{}_k = \hat{x}^-{}_k + K_k \hat{y}_k$$
$$P^+{}_k = (I - K_k H_k) P^-{}_k$$

- $\hat{y}_k$ is the measurement residual
- z_k is the sensory measurement
- S_k is the residual covariance
- K_k is the optimal Kalman gain
- $\hat{x}^+{}_k$ is the updated posterior state estimate
- $P^+{}_k$ is the updated posterior estimate covariance

Kalman localization algorithm



Demo!

Kalman Filter : Observations

- If prediction uncertainty is larger, sensor updates will dominate state estimate
- If sensor uncertainty is larger, prediction propagation will dominate state estimate
- Improper estimates of the state and/or sensor covariance may result in a rapidly diverging estimator

Consistency and Uncertainty Convergence

- We calculate the 95% confidence intervals of the measurement residual using the prior covariance.
- If the measured residual lies out of this confidence interval, then the system might be inconsistent.
- After sometime, the covariance of the estimates does not change much

Kidnapped Robot Problem

- If a robot is kidnapped all of a sudden, the state predictions become inconsistent.
- **Validation Gates**
  - If the measured residual becomes inconsistent over a large number of successive time steps, then it is an indication to the robot that it has been kidnapped.
  - Inconsistency indicates that the state space estimates are inaccurate.

Kidnapped Robot Problem

- **Recovery**
  - When a robot is kidnapped the residual is very high and KF automatically gives more weight to the sensory measurements. Thus, the estimate slowly approaches the new location. During this period, the predicted location is inconsistent.
  - To speed this up, after detecting a kidnap, the robot can deliberately give more weight to the sensory measurements.
  - However, care must be taken to not lose all the information about the predicted state in the event of a wrong detection.

Conclusive Remarks

- If the noise is Gaussian distributed, then the KF estimator is optimal with respect to any kind of practical optimality criteria.
- We have discussed the problem of Robot Localization, the problem of state estimation of noisy dynamic systems using KF and finally how to apply KF techniques to solve Robot Localization Problems.

References

[1] Rudy Negenborn, Robot Localization and Kalman Filters.

Utrecht University, Thesis number INF/SCR-03-09, Septmeber 1, 2003

[2] Understanding the Basis of the Kalman Filter Via a Simple and Intuitive Derivation by Ramsey Faragher, *IEEE SIGNAL PROCESSING MAGAZINE September 2012*

[3] http://en.wikipedia.org/wiki/Kalman_filter

Questions??