STAT-S 352 Problem Set 6

Upload your answers through the Assignments tab on Canvas by 11:59 pm,

Tuesday, October 14, 2025.

Important Note: Answer all questions and include R code when necessary. In general in this course, give explanations and/or working for all answers unless otherwise stated. Show your work for full credit.

Reminder: As a student at IU, you are expected to uphold and maintain professional and academic honesty and integrity. Academic integrity violations include: cheating, fabrication, plagiarism, interference, violation of course rules, and facilitating academic dishonesty. When you submit an assignment with your name on it, you are signifying that the work contained therein is yours, unless otherwise cited or referenced. Any ideas or materials taken from another source must be fully acknowledged.

1. Recall the data from the file `expo.txt` you worked with in Problem Set 5. The file contains data I generated from an Exponential distribution. However, I've forgotten what rate parameter λ I used. Was it 0.5? Analyze this dataset one more time to help me answer this question. **Use Wald inference!**

   Review the MLE Normality Theorem and the examples we covered in class (Chapter 3 slides, pp. 7-27) and Week 7 R files. It may be useful to recall how the MLE for λ in the Exponential distribution was derived (Chapter 2 slides, pp. 66-67).

   (a) Use observed Fisher information (defined on p. 12 of Chapter 3 slides) to estimate the variance of the MLE for the rate parameter λ in the Exponential distribution, $\hat{V}(\hat{\lambda})$.

   (b) Compare the above result with the variance of the MLE for the rate parameter λ in a Poisson distribution. Are those two formulas the same?

   (c) Use computational method to estimate the variance of the MLE for λ in the Exponential distribution, $\hat{V}(\hat{\lambda})$.

   (d) Construct a 96% Wald confidence interval for λ. Interpret your confidence interval and make a conclusion about the true value of λ being 0.5.

   (e) Conduct a Wald test to the hypothesis that the value of λ I used was 0.5, i.e.,

   $$H_0 : \lambda = 0.5.$$

   Give a P-value and an appropriate conclusion.

2. The file `womensheights.txt` contains the heights (in cm) of what is supposed to be a representative sample of 3766 American adult women from the 2009 NHANES survey done by the CDC. The Normal distribution is usually a good model for heights; we'll parameterize this model using the mean μ and variance σ^2. Review the MLE Normality Theorem and the Normal example from the lecture slides (Chapter 2: p. 32; Chapter 3: pp. 30-31) and answer the following questions using the theory approach. (i.e., Use the formulas for MLEs, and the theoretical results of the distributions of the MLEs. Do not use `optim` or `optimize` for numerical optimization.)

   (a) What are the MLEs for μ and σ^2 respectively?

   (b) Find a 99% Wald confidence interval for μ, the average height of American adult women.

   (c) Find a 99% Wald confidence interval for σ^2, the variance of heights of American adult women.

   (d) According to Wikipedia, the average height of American adult women is 161.3 cm (2015-2018). Is the data here consistent with that statement? First, write the hypotheses. Then, perform a Wald test, giving a P-value and an appropriate conclusion.

   (e) Use `t.test` to perform a t test for the above hypothesis as well as produce a 99% confidence interval. Do you reach the same conclusion as in part (d)?

   (f) Wikipedia doesn't say (as far as I can see) what the standard deviation of women's heights is, but in Belgium it's 5.3 cm. Use our data to test the hypothesis that the SD of American adult women's heights is 5.3 cm, giving a P-value and an appropriate conclusion. (Hint: SD = 5.3 is equivalent to variance = ?)