

# Motivation — Header overhead in VoIP



Field	Size in bytes	Size in bits
VoIP Traffic Data	31	248
L3 Overhead		
RTP + UDP + IP	(12-72) + (6-8) + 20	
	38–100	304-800
ROHC (on avg.)	3	24
MAC Headers	6	48
HARQ overhead	2	16
MAP overhead (all users)		672
DL MAP (per user)		44
UL MAP (per user)		36

### Motivation — Header overhead in VoIP



MCS	0	1	2	3	4	5	6	7	8	9
Bits per slot	8	12	24	48	72	96	114	192	216	240

- MAC PDU is sent using a MCS corresponding to the user's channel state.
- MAP is sent using a MCS corresponding to a bad channel state, so that all users can decode the MAP.
- Sending bits in MAP is more costly.



# Motivation — Header overhead in VoIP



Field	Cian in hydro	Size in bits	Size in symbols		
rieid	Size in bytes	Size in dits	MCS 0	MCS 2	MCS 9
VoIP Traffic Data	31	248	31	10.33	1.03
L3 Overhead					
RTP + UDP + IP	(12-72) + (6-8) + 20				
	38–100	304-800			
ROHC (on avg.)	3	24	3	1	0.1
MAC Headers	6	48	6	2	0.2
HARQ overhead	2	16	2	0.67	0.067
MAP is sent us					
MAP overhead (all users)		672	112	112	112
DL MAP (per user)		44	7.33	7.33	7.33
UL MAP (per user)		36	6	6	6
% DL MAP overhead			23.65	70.96	709.6
% UL MAP overhead			19.35	58.08	580.3

## Reducing Signalling Overhead



- VoIP traffic is periodic. During voice activity, there is on 31 byte packet every 20ms.
- If the user is moving slowly, the channel state does not change significantly in 20ms. Therefore, the number of slots allocated to the user do not change.
- Can we keep the location of the slots allocated to the user fixed, and not incur any MAP overhead?

### Persistent Scheduling (Basic Idea)

- Send allocation (number of slots and their location) information to the user once.
- Use this allocation until told otherwise.



### Persistent Scheduling (Details)



- Assume that packet length is fixed.
- At the beginning of each frame, the base-station and the user know
  - the current persistent allocation
  - the current SNR of the user
- Both the base-station and the user can determine the best MCS to use for the current SNR.

# Persistent Scheduling (Details)



• The base-station has three options:

	Option	overhead
1.	If the persistent allocation is large enough to accommodate the current packet using the current MCS, send the current packet at the persistently allocated slot.	No overhead
2.	Change the current allocation to send the current packet, but do not change the persistent allocation	overhead_Current
3.	Change the persistent allocation and send the current packet using this new persistent allocation	overhead_Persistent

# Comparison with Dynamic Scheduling



- The current allocation is signalled each time.
- There is a signalling overhead (overhead\_Dynamic)
- The change in persistent allocation must be sent at a higher reliability than the change in current allocation.

overhead\_Dynamic = overhead\_Current < overhead\_Persistent

Questions: Is persistent scheduling better than dynamic scheduling? How to choose optimally when and how to change the persistent allocation?

# Modelling Assumptions – First Stage



• Consider the optimal allocation problem for a single-user.

### VoIP traffic

- Voice activity is always present.
- VoIP packets are of fixed length (No packet size variation due to ROHC).

### **Channel Variation**

- Quantize the channel state according to the corresponding MCS.
- The quantized channel state is a Markov chain.
- The base-station knows the channel state transition matrix of each user.



### Notation



Notation	Description	Values		
S	Current MCS of the user	$\{1, 2, \dots, 10\}$		
c(s)	Number of slots needed to transmit the MAC PDU in the current channel state.	$31 \times 8/(8, 12, \dots, 240)$		
Q	MCS state transition matrix	Estimate for each user.		
p	current persistent allocation	$\{1, 2, \dots, 10\}$		
$\mathfrak{p}'$	new persistent allocation	$\{1, 2, \dots, 10\}$		
a	current allocation	$\{1, 2, \dots, 10\}$		
$O_{C}$	overhead to signal a different current allocation	$44 \times 4/24 = 7.33$		
Ор	overhead to signal a different persistent allocation	$2 \times O_C$		

### Model



#### Given:

- slots needed per packet, c
- MCS transition matrix, Q
- overhead to signal current and persistent MCS, O<sub>C</sub> and O<sub>P</sub>
- Determine optimal allocation rule  $\gamma$ 
  - Input: (current state, current persistent allocation) = (s, p)
  - $\circ$  Output: (current allocation, new persistent allocation) =  $(\alpha, p')$  to minimize the infinite horizon expected discounted cost
- Instantaneous cost  $\rho(\alpha, p, p') =$ 
  - c(a) (slots used by current allocation)
  - +  $O_C$  if  $a \neq p'$
  - +  $O_P$  if  $p \neq p'$
- Feasible allocation rule  $\gamma(s, p) = (a, p')$  only if  $a \leq s$

## Dynamic Program



$$V(s,p) = \min_{\substack{\alpha \leqslant s \\ p'}} \rho(\alpha, p, p') + \beta \sum_{s'} Q(s', s) V(s', p')$$

Find the fixed point of this functional equation.

### Simplification

If you change the current allocation, it should be equal to the current state.

$$\begin{split} V(s,p) &= \min_{\substack{p' \\ \alpha \in \{s,p'\}}} \rho(\alpha,p,p') + \beta \sum_{s'} Q(s',s) V(s',p') \\ &= \min_{\substack{p' \\ p'}} \Big\{ \min\{c(s),c(p') + O_C\} \\ &+ O_P \mathbf{1} \left[ p \neq p' \right] + \beta \sum_{s'} Q(s',s) V(s',p') \Big\} \end{split}$$

## **Computational Study**



- Data
  - time traces of SNR of 40 users at 5ms intervals for 20s (4000 samples)
  - thresholds to determine MCS based on SNR
- Compute time trace of MCS
- Assuming that the MCS evolution is first order Markov and determine the transition matrix
- Assume that this transition matrix is known *a priori* and determine the optimal allocation rule for each user.

## Advantage of Persistent Allocation

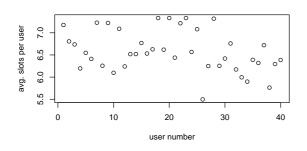


- Use the "optimal" allocation rule on the actual time trace of MCS and determine the average resources used.
- Use dynamic allocation on the actual trace and determine the average resources used.

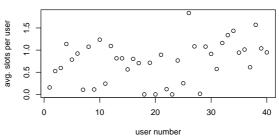
# Advantage of Persistent Allocation



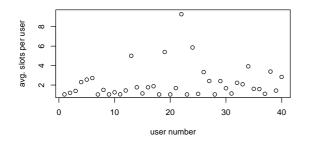
#### improvement over dynamic allocation



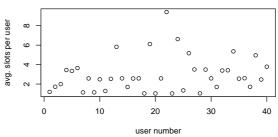
#### avg. overhead for persistent allocation



#### minimum slots needed

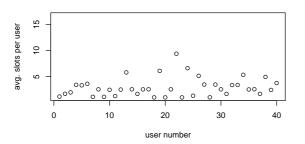


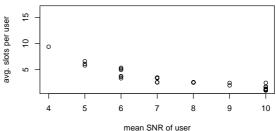
#### avg. slots used by persistent allocation

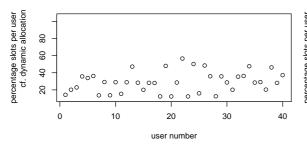


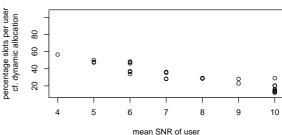
### Performance of Persistent Allocation





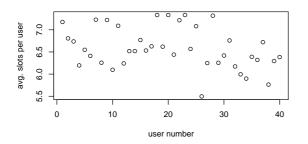


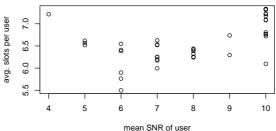


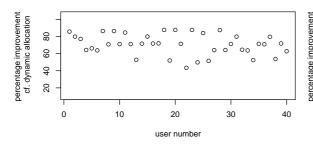


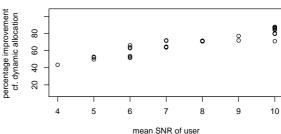
### Performance Improvement











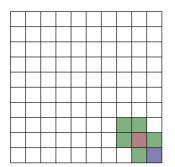
## Online Implementation

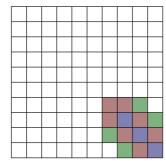


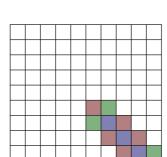
- Need to estimate the transition matrix Q
- classify users on the basis on their transition matrix.
- Compare the optimal allocation rules of users belonging to the same class.

### **User Classes**





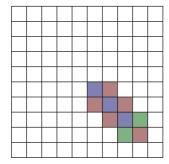


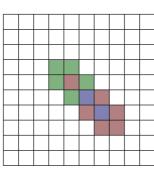


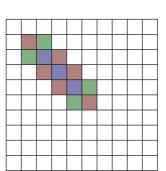
Users of Class I Users 1, 7, 9, 11, 18, 20, 23, 28



Users of Class III Users 3, 8, 12, 35, 36, 39







Users of Class IV Users 14, 16, 17, 21

Users of Class V Users 4, 5, 6, 27, 29, 30, 32, 33, 40

Users of Class VI Users 13, 19, 22, 24, 34



$$\sim 10^{-2}$$

### **User Classes**



- User can be classified based on their transition matrix (or based on the PDF of their MCS)
- Users in the same class have (almost always) the same optimal allocation rule.
- So, for an online implementation, we just need to identify the user class and use the optimal allocation rule for that user class.

This observation is based on data from only 40 slow moving users. Need to verify against more data.

### Single Allocation Rule



- Combine the transition matrices of all users to obtain an user average transition matrix.
- Use this transition matrix to compute an optimal allocation policy.
- Use this allocation policy for all users.

Avg. overhead of optimal policy : 0.75 symbols Avg. overhead of average policy : 1.34 symbols Agv. size of VoIP traffic : 2.22 symbols

### **Average Policy**

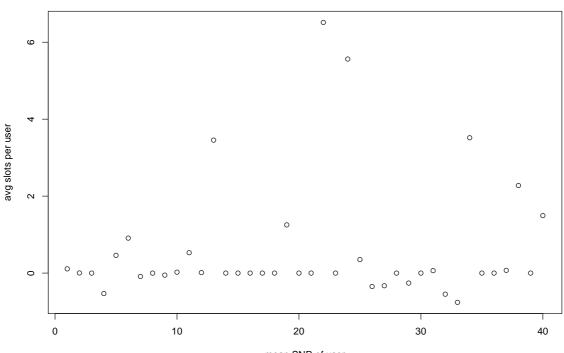


$$\gamma = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) & (1,7) & (1,8) & (1,9) & (1,10) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) & (2,7) & (2,8) & (2,9) & (2,10) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) & (3,7) & (3,8) & (3,9) & (3,10) \\ (4,1) & (4,2) & (3,3) & (4,4) & (4,5) & (4,6) & (4,7) & (4,8) & (4,9) & (4,10) \\ (5,1) & (5,2) & (3,3) & (4,4) & (5,5) & (5,6) & (5,7) & (5,8) & (5,9) & (5,10) \\ (6,6) & (6,6) & (6,6) & (4,4) & (5,5) & (6,6) & (6,6) & (6,6) & (6,6) \\ (7,7) & (7,7) & (7,7) & (7,7) & (4,4) & (5,5) & (6,6) & (7,7) & (7,7) & (7,7) \\ (8,1) & (8,2) & (8,3) & (4,4) & (5,5) & (6,6) & (7,7) & (8,8) & (8,9) & (8,10) \\ (9,1) & (9,2) & (9,3) & (4,4) & (5,5) & (6,6) & (7,7) & (8,8) & (9,9) & (9,10) \\ (10,1) & (10,2) & (10,3) & (4,4) & (5,5) & (6,6) & (7,7) & (8,8) & (9,9) & (10,10) \end{bmatrix}$$

## Comparison



#### Distance from optimality of average policy



mean SNR of user Average difference 0.592223338841215



### Extensions/Improvements



- **Effect of ROHC:** Variation in packet lengths can be easily taken into account.
- Order of channel: Assuming that the channel is first order Markov. Actual data appears to be third order Markov. Higher order Markov model can be incorporated into the model, but will make the allocation rule more complicated ( $1000 \times 1000$  lookup table, rather than  $10 \times 10$  lookup table).
- **Frequency of CQI reports:** Currently assuming that channel state information is available every 5ms. If not, the channel model will be coarser and we will loose something in performance.
- Need to verify against more data.





# **Backup Slides**



### Simulation Results

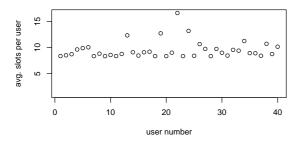


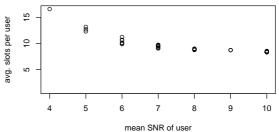
- Performance Comparison
- Optimal Allocation Policies
- Simulation Data

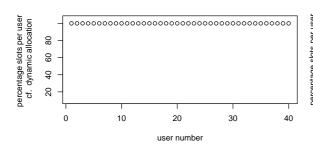


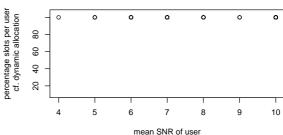
# Performance of Dynamic Allocation





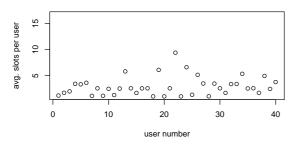


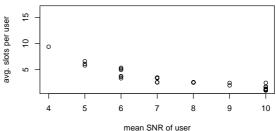


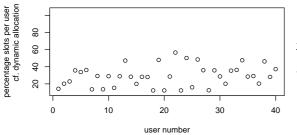


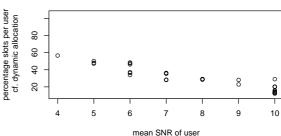
### Performance of Persistent Allocation





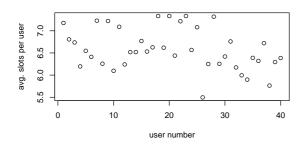


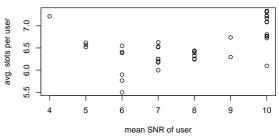


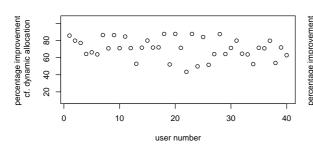


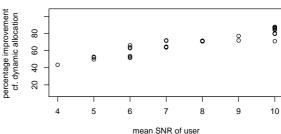
### **Performance Comparison**













### **Optimal Allocation Policies**



### **Average Policy**

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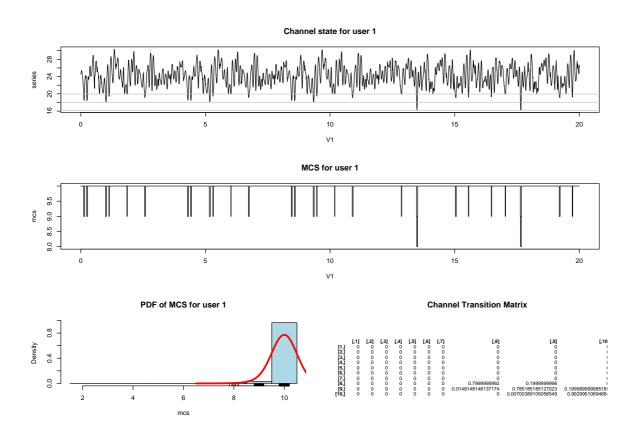
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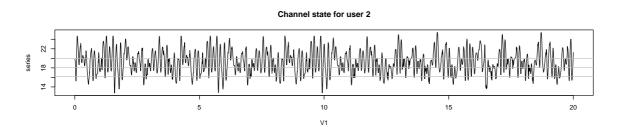
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                                                              (6, 6)
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                                                                                           (5,5)
                                                                                                  (6,6)
                                                                                                        (6,6)
                         0.0727 0.924
                                         0.00349
                                                              (6, 6)
                                                                      (6, 6)
                                                                              (6,6)
                                                                                     (6,6)
                                                                                           (5,5)
                                                                                                  (6,6) (7,7)
                                                                                                               (6, 6)
                                                                                                                      (6, 6)
                                                                                                                              (6, 6)
                                 0.2
                                         0.8
                                                              (7,7)
                                                                      (7,7)
                                                                              (7,7)
                                                                                     (7,7) (5,5) (6,6) (7,7) (8,8)
                                                                                     (4,4)
                                                                                           (5,5) (6,6) (7,7)
                                                              (10,1) (10,2)
                                                                             (10,3) (4,4) (5,5) (6,6) (7,7) (8,8) (9,9)
```

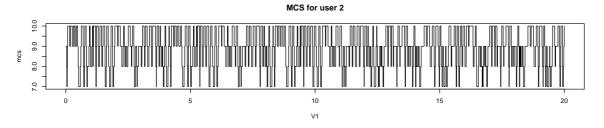
```
(1,4) (1,5) (1,6) (1,7)
                                                                                            (2,5)
                                                                       (3, 2)
                                                               (4,1)
                                                                       (4, 2)
                                                                              (3,3)
                                                                                     (4,4) (4,5)
                                                                                                  (4,6)
                                                                                                                (4,8)
                  0.75
                            0.25
                                                               (5,5)
                                                                      (5,5)
                                                                              (5,5)
                                                                                     (4,4) (5,5)
                                                                                                  (5,5) (5,5)
                                                                                                               (5,5)
    O =
                                                                                                                                              (33)
                            0.904
                                                               (6, 6)
                                                                       (6,6)
                                                                                            (5,5)
                            0.072 0.911
                                           0.0166
                                                               (6, 6)
                                                                      (6, 6)
                                                                              (6, 6)
                                                                                     (6,6) (5,5)
                                                                                                  (6,6)
                                   0.198
                                           0.802
                                                               (7,7)
                                                                      (7,7)
                                                                              (7,7)
                                                                                     (7,7) (5,5)
                                                                                                  (6,6)
                                                                      (9, 2)
                                                                                     (4,4) (5,5) (6,6) (7,7)
                                                                                                               (8, 8)
                                                              (10,1) (10,2)
                                                                             (10,3) (4,4) (5,5) (6,6) (7,7) (8,8) (9,9) (10,10)
                                                                                          (1,4) (1,5) (1,6)
                                                                   (2,1)
                                                                                          (2,4) (2,5)
                                                                                                      (2,6)
     · · 0.793
                  0.207
                                                                   (3,3)
                                                                                          (3,3) (3,3)
                                                                                                      (3,3)
                                                                                                             (3,3)
     · · 0.0367
                 0.841
                           0.119
                                   0.00386
                                                                   (4,4)
                                                                           (4,4)
                                                                                  (4,4)
                                                                                          (4,4) (4,4) (4,4) (4,4)
                  0.0845
                           0.853
                                   0.0628
                                                                   (5,5)
                                                                           (5,5)
                                                                                          (4,4) (5,5)
                                                                                                      (5,5)
Q =
                                                                                                                                              (34)
                  0.00156 0.0739 0.922
                                            0.00233
                                                                   (5,5)
                                                                           (5,5)
                                                                                  (5,5)
                                                                                          (4,4) (5,5)
                                                                                                       (6,6)
                                                                                                             (5,5)
                                   0.2
                                            0.8
                                                                   (6, 6)
                                                                           (6, 6)
                                                                                   (6,6)
                                                                                          (6,6) (5,5) (6,6)
                                                                                                             (7,7)
                                                                                                                    (6, 6)
                                                                           (8, 2)
                                                                                          (4,4) (5,5)
                                                                                                      (6,6)
                                                                                          (4,4) (5,5) (6,6)
                                                                   (10,1) (10,2) (10,3) (4,4) (5,5) (6,6) (7,7) (8,8) (9,9) (10,10)
                                                                   (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (1,7) (1,8) (1,9)
                                                                   (2,1) (2,2)
                                                                               (2,3) (2,4) (2,5) (2,6)
                                                                                                          (2.7)
                                                                                                                 (2.8)
                                                                               (3,3) (3,4) (3,5) (3,6) (3,7)
                                                                                                                 (3.8)
                                                                                             (4,5)
                                                                                                   (4,6)
                                                                        (5, 2)
                                                                               (3,3)
                                                                                             (5,5)
                                                                                                   (5,6)
   O =
                                                                                                                                              (35)
                    0.724 0.276
                                                                                             (5,5)
                                                                                                   (6,6)
                                                                                                          (6,6)
                                                                                                                               (6,6)
                    0.025 0.927 0.0471
                                          0.000417
                           0.102 0.859
                                          0.0393
                                                                                                   (6,6)
                                                                                                                 (8,8)
                                  0.175
                                          0.813
                                                    0.0117
                                                                                                   (6,6)
                                          0.273
                                                    0.727
                                                                   (8,8) (8,8) (8,8) (8,8) (8,8) (6,6) (7,7)
                                                                    [(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
                                                                                  (2,3) (2,4)
                                                                                               (2,5) (2,6)
                                                                                                                   (2, 8)
                                                                                  (3,3) (3,4) (3,5) (3,6)
                                                                                                            (3,7)
                                                                                                                   (3, 8)
                                                                                  (3,3) (4,4) (4,5) (4,6)
                                                                                                            (4,7)
            . 0.667
                       0.333
                                                                                 (5,5) (4,4) (5,5) (5,6)
                                                                                                            (5,5)
                                                                                                                                              (36)
             . 0.0116 0.78
                              0.208
                                                                                               (5,5)
                                                                                                     (6,6)
                       0.026 0.91
                                       0.0642
                                                                                               (5,5)
                                                                                                      (6,6)
                                                                                                                                 (7,7)
                              0.112
                                       0.825
                                                                                               (7,7)
                                                                                                     (6,6) (7,7)
                                                                                                                   (8.8)
                                                                                                                                 (7,7)
                                               0.0625
                                                                                        (7,7)
                              0.00445 0.158
                                                       0.029
                                               0.808
                                                                                                     (6,6)
                                                       0.69
                                               0.31
                                                                     (8,8) (8,8) (8,8) (8,8) (8,8) (6,6) (7,7)
                                                                                                                   (8,8) (9,9)
```

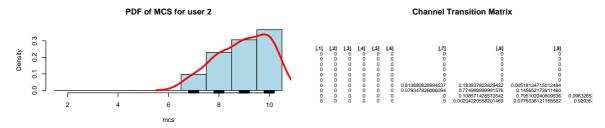




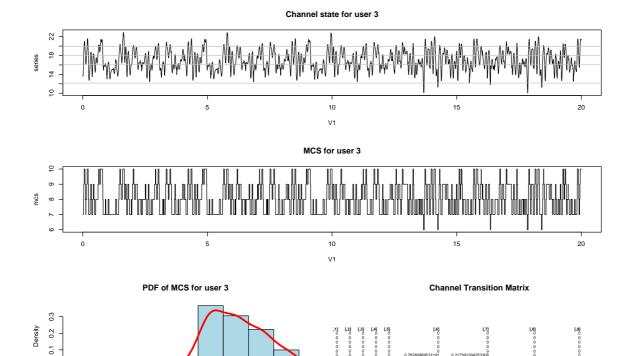






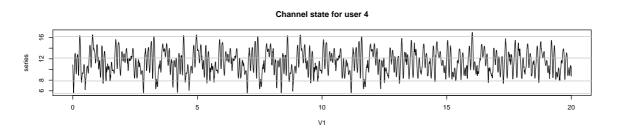


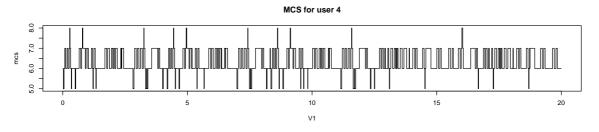


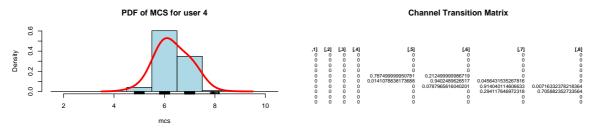


mcs



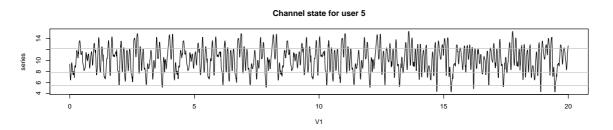


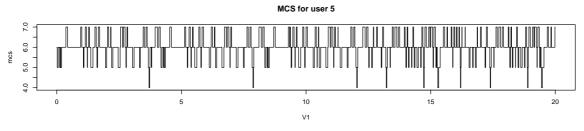


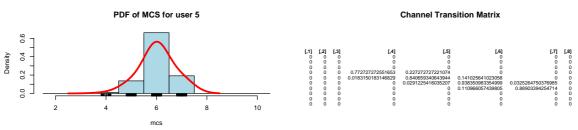






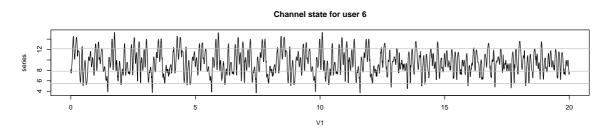


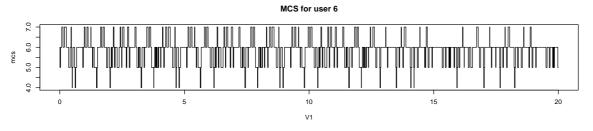


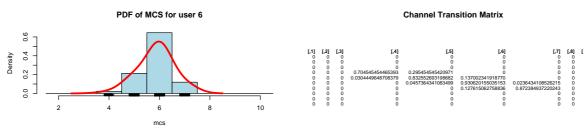






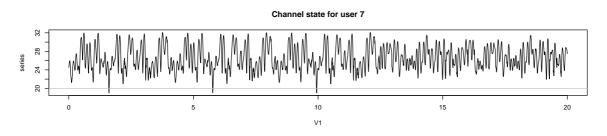


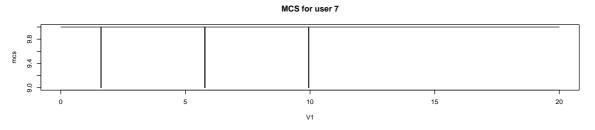


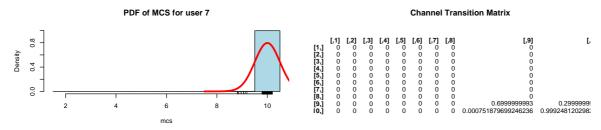




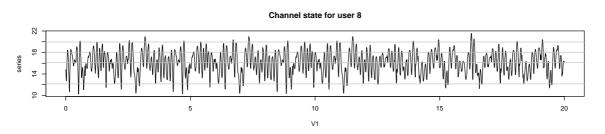


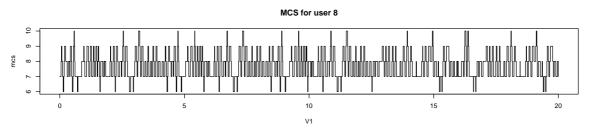


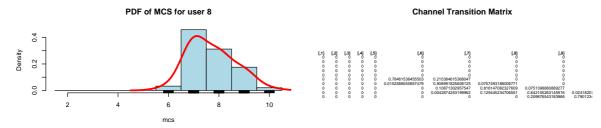






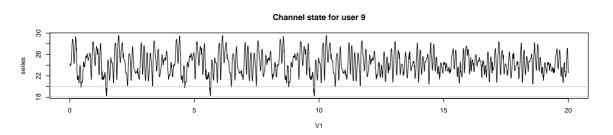


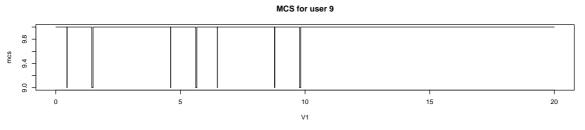


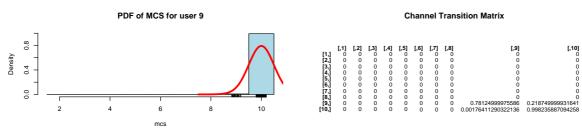






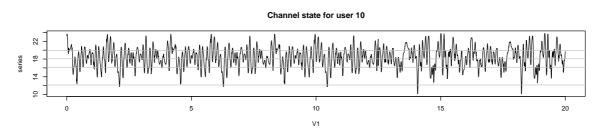


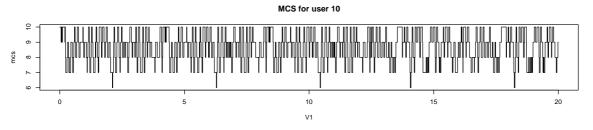


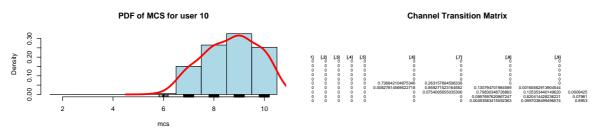






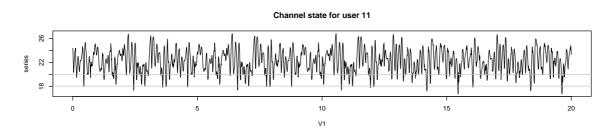


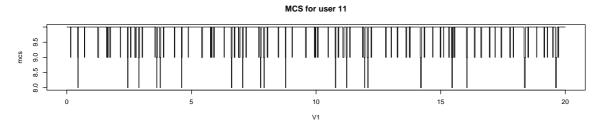


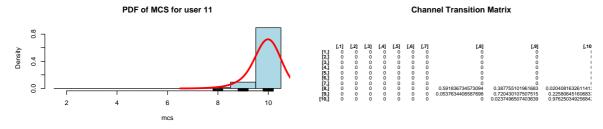






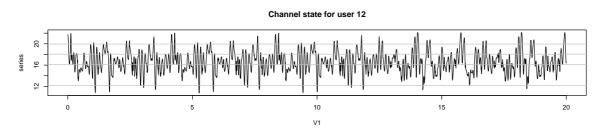


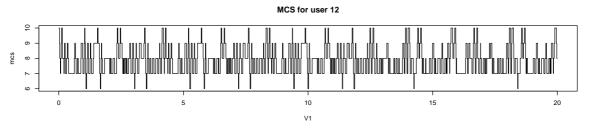


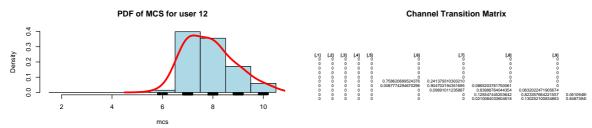




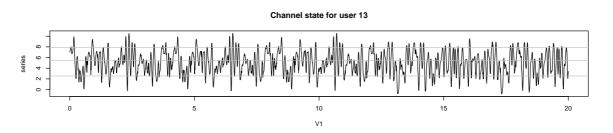


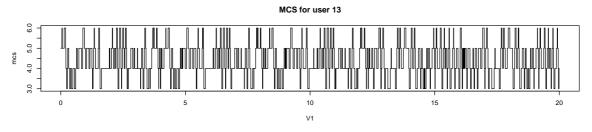


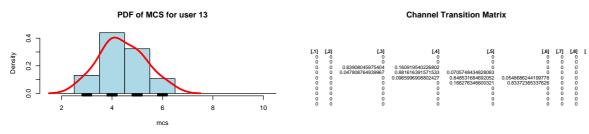






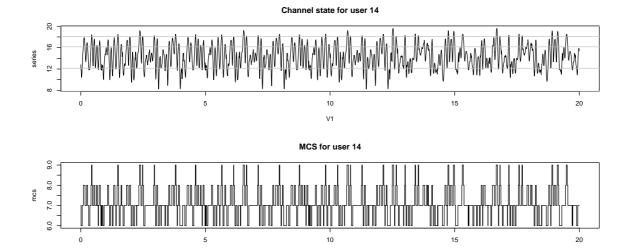


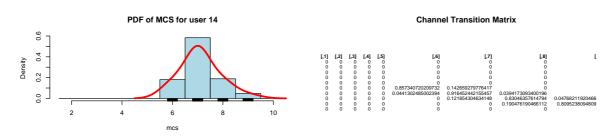








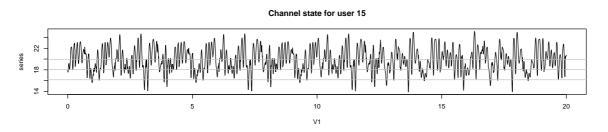


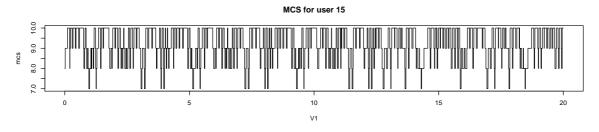


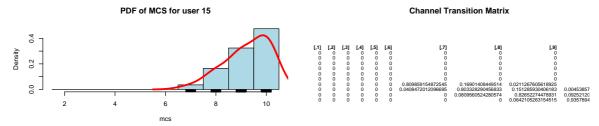
V1





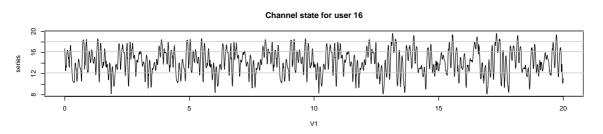


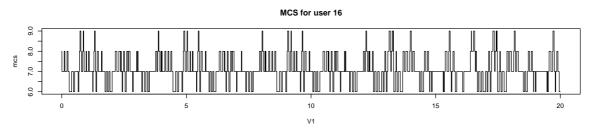


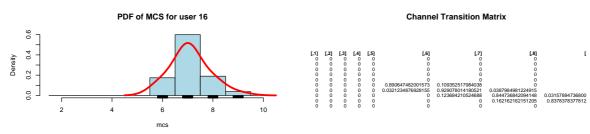






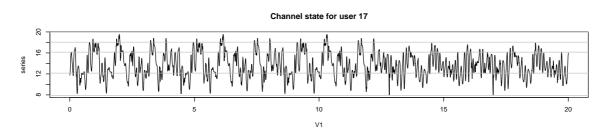


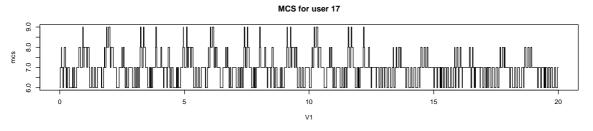


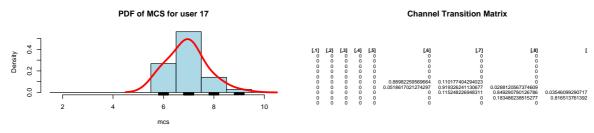






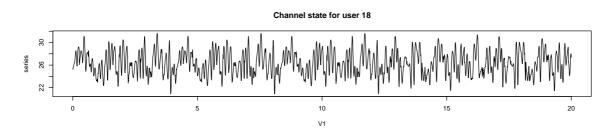


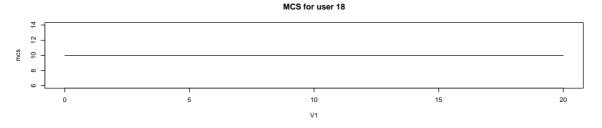


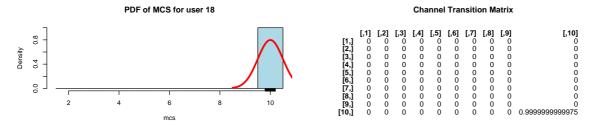




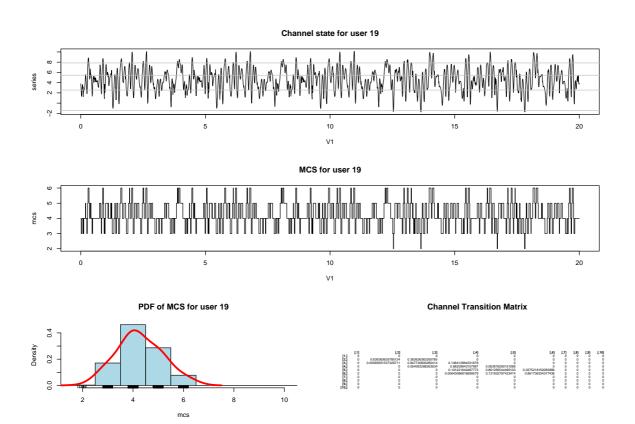




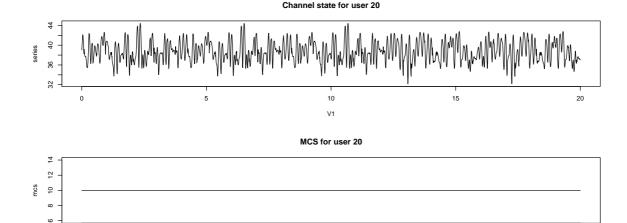


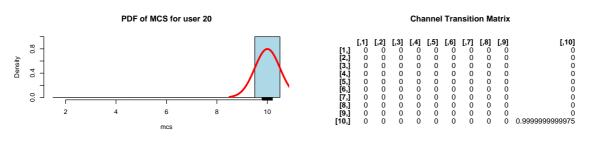












10

V1

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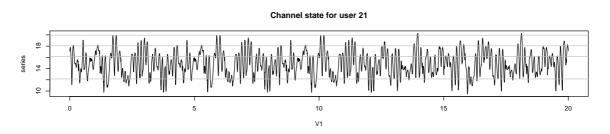


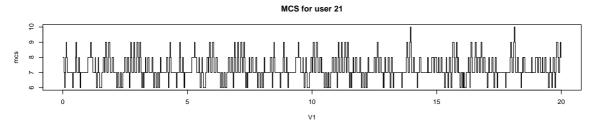
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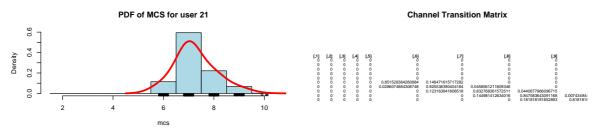
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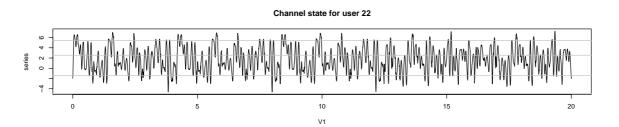


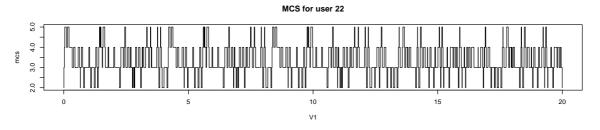


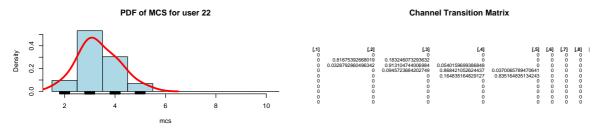




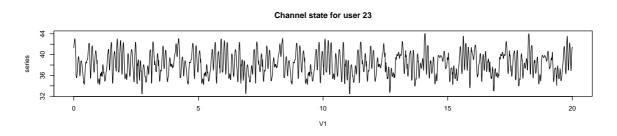


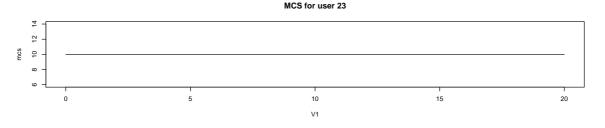


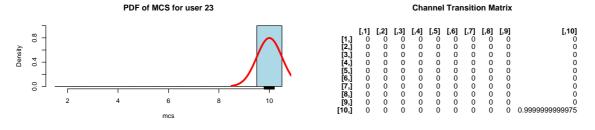




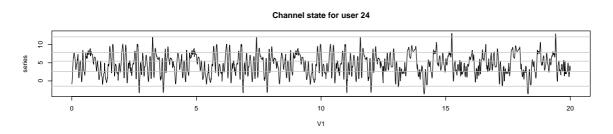


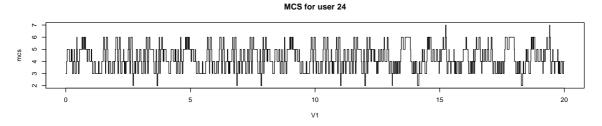


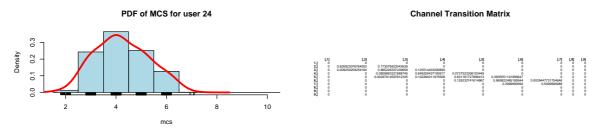






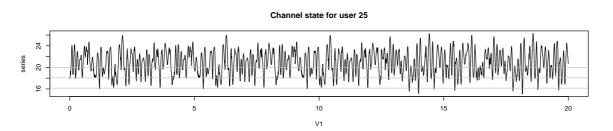


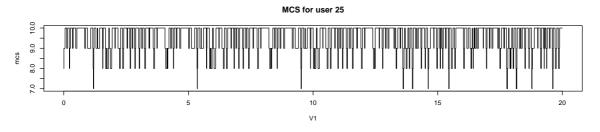


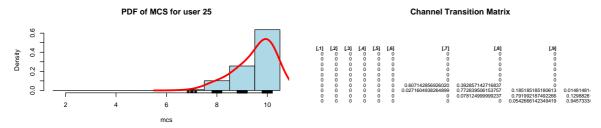






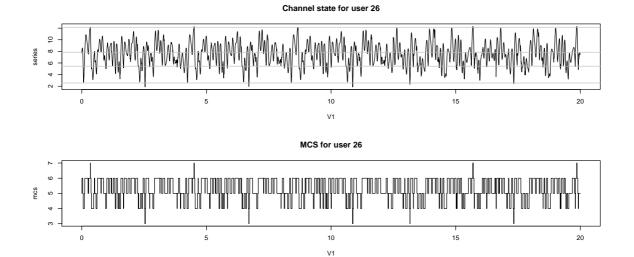


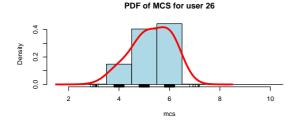










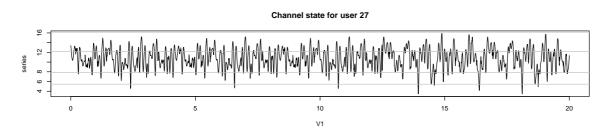


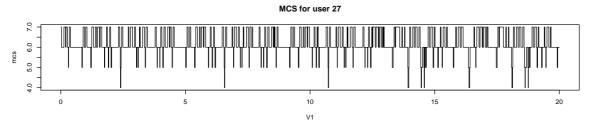


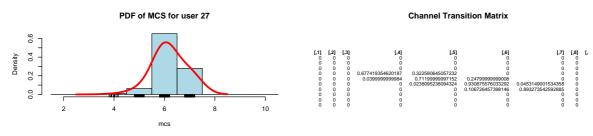
**Channel Transition Matrix** 





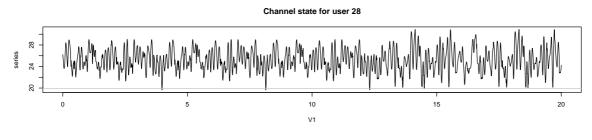


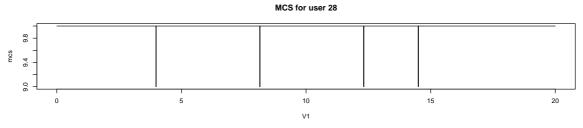


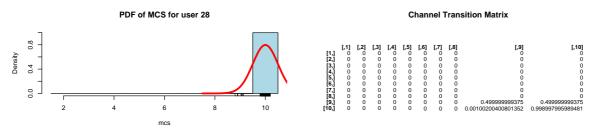






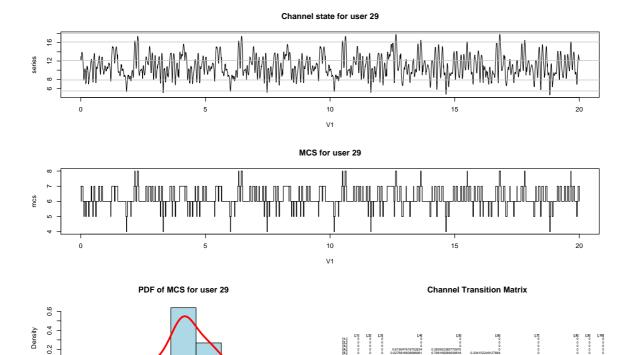












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mcs



