# Information Theoretic Privacy in Electric Grids with Smart Meters


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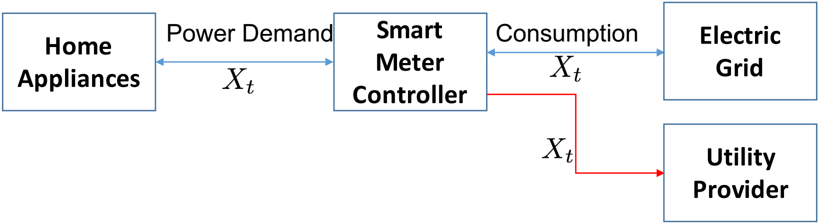
# Electric Grid with Smart Meters




Real-Time Information of Energy Consumption

Advantages

- Demand-Response Systems
- Reduce energy consumption

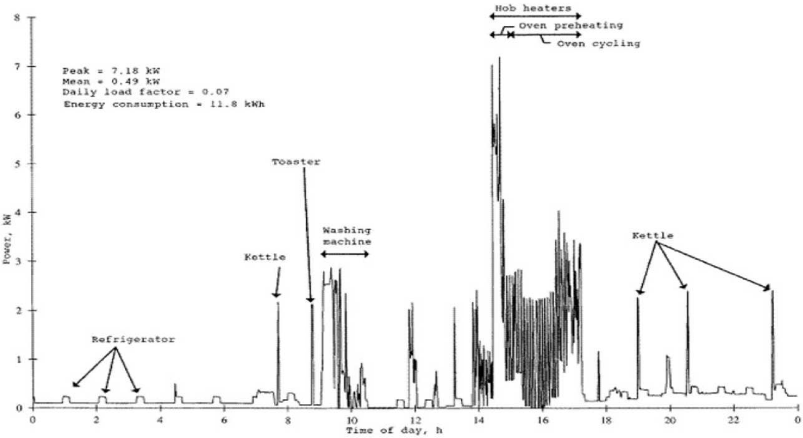
Disadvantages

- User Privacy
- Safety Hazards

# Non-Intrusive Load Monitoring

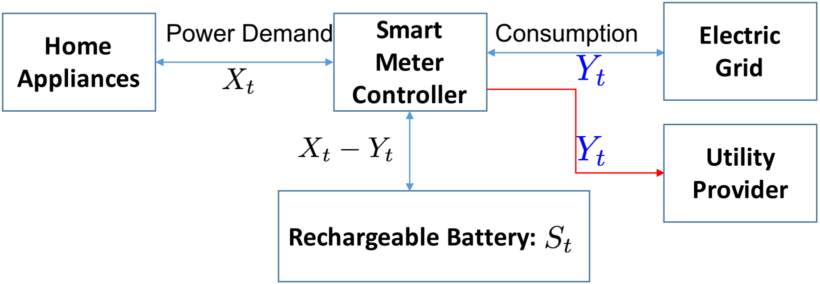
G. Hart, "Nonintrusive appliance load monitoring". Proceedings of the IEEE, 1992

User Energy Consumption Profile Leaks Sensitive Information



# Rechargeable Battery for Information Masking

G. Kalogridis et al. Privacy for Smart Meters: Towards Undetectable Appliance Load Signatures, 2010



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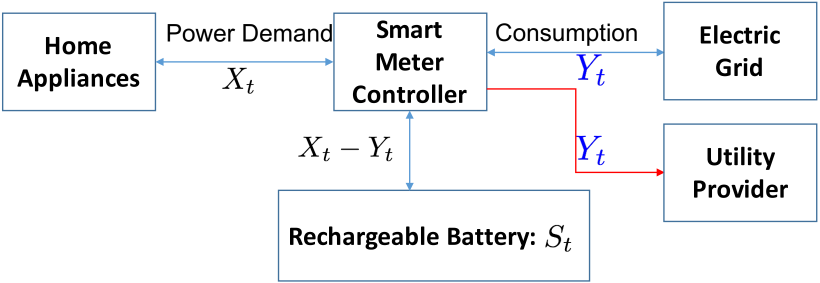


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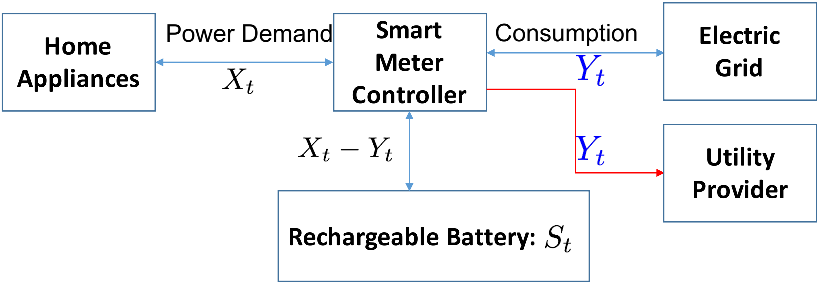


Variables

- User Load: $X_t$
- Output Load: $Y_t$
- Battery State: $S_t$

Constraints

- Battery Update: $S_{t+1} = S_t - X_t + Y_t$
- Storage: $S_t \in [0, B_{\max}]$

# Literature Review

## Rechargeable Battery based Solutions

- G. Kalogridis, C. Efthymiou, et. al , *Privacy for smart meters: Towards undetectable appliance load signatures*, IEEE SmartGridComm, 2010
- S. McLaughlin, P. McDaniel, and W. Aiello, "Protecting Consumer Privacy from Electric Load Monitoring," IEEE CCS 2011
- L. Yang, et. al, *Cost-effective and privacy preserving energy management for smart meters*, IEEE Trans. on Smart Grid, Jan. 2015.
- D. Egarter, C. Prokop, and W. Elmenreich, *Load hiding of household's power demand*, IEEE SmartGridComm 2014
- G. Giaconi and D. Gunduz, *Smart Meter Privacy with Renewable Energy and a Finite Capacity Battery* 2016
- D. Varodayan and A. Khisti, *Smart meter privacy using a rechargeable battery: Minimizing the rate of information leakage*, ICASSP 2011
- S. Li, A. Khisti and A. Mahajan, *Information-Theoretic Privacy for Smart Metering Systems with a Rechargeable Battery*, Trans-IT 2016 (Submitted)

# Literature Review

Inaccurate Readings

- L. Sankar, S. R. Rajagopalan, S. Mohajer, and H. V. Poor, *Smart meter privacy: A theoretical framework*, IEEE Trans. Smart Grid, 2013.
- Z. Zhang et. al, *Cost-friendly Differential Privacy for Smart Meters: Exploiting the Dual Roles of the Noise*, IEEE Trans. Smart Grid, 2016

Experimental Studies:

- M. Lisovich, D. Mulligan, and S. Wicker, *Inferring personal information from demand-response systems* IEEE Security & Privacy, 2010
- Kim et. al. *Unsupervised Disaggregation of Low Frequency Power Measurements*, SIAM Int'l Conf. on Data Mining, Mesa, AZ, Apr. 2011.

Policy Issues:

- Ann Cavoukian, *Operationalizing Privacy by Design: The Ontario Smart Grid Case Study*, Ontario Information & Privacy Commissioner 2011
- E. L. Quinn, *Smart metering and privacy: Existing laws and competing policies*, Colorado Public Utilities Commission, Tech. Rep., 2009.

# Information Theoretic Model

- Input Load: $X_t \overset{iid}{\sim} P_X(\cdot)$, $X_t \in \mathcal{X}$.
- Output Load: $Y_t \in \mathcal{Y}$, $\quad \mathcal{X} \subseteq \mathcal{Y}$.
- Battery State: $S_t \in \mathcal{S}$, $S_1 \sim P_S(\cdot)$
- Battery Update: $S_{t+1} = S_t - X_t + Y_t$
- Battery Charging Policy:
  $\mathbf{q} = \left( q_1(y_1|x_1, s_1), q_2(y_2|x_1^2, s_1^2, y_1), \ldots, q_t(y_t|x_1^t, s_1^t, y_1^{t-1}), \ldots \right)$
- Feasible Policy
  - Causal
  - Storage Constraint:

$$\sum_{y \in \mathcal{Y} : s_t + y - x_t \in \mathcal{S}} q_t(y|x_1^t, s_1^t, y_1^{t-1}) = 1$$

- Set of Feasible Policies: $\mathcal{Q}_A = \{\mathbf{q} : \mathbf{q}$ is feasible.$\}$

# Information Theoretic Model

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- Leakage Rate:

$$L(\mathbf{q}) = \limsup_{n\to\infty} \frac{1}{n} I^{\mathbf{q}}(X_1^n, S_1; Y^n)$$

- Optimal Leakage:

$$L^\star = \inf_{\mathbf{q}\in\mathcal{Q}_A, P_{S_1}} L(\mathbf{q})$$

# Information Theoretic Model

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Communication Channel Viewpoint

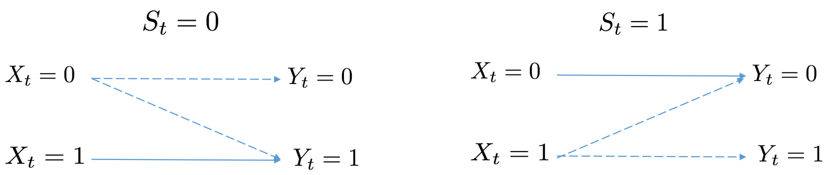
$$X_t \overset{iid}{\sim} P_x(\cdot) \longrightarrow \boxed{\begin{array}{c}\text{Battery System}\\ q_t(Y_t|X^t, S^t, Y^{t-1})\\ S_{t+1} = S_t + Y_t - X_t\end{array}} \xrightarrow{\quad Y_t \quad}$$

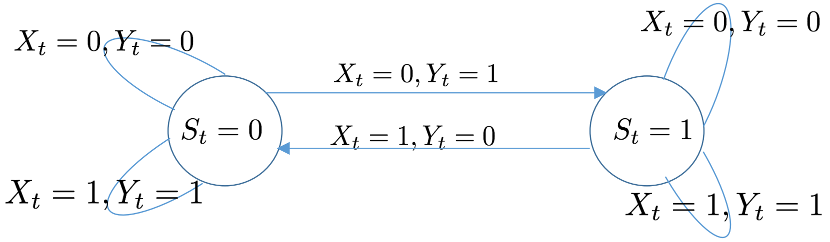
$$L(\mathbf{q}) = \limsup_{n\to\infty} \frac{1}{n} I^{\mathbf{q}}(X_1^n, S_1; Y^n)$$

i.i.d. $X_t$ + "Memoryless Channel" $\not\Longrightarrow$ i.i.d. $Y_t$

# Binary Case

$\mathcal{X} = \mathcal{Y} = \mathcal{S} = \{0, 1\}$, $P_X(\cdot) = P_{S_1}(\cdot) =$ Ber(0.5)



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Policy 1: $Y_t = X_t$

$S_t = 0$

$$X_t = 0 \longrightarrow Y_t = 0$$

$$X_t = 1 \longrightarrow Y_t = 1$$

$S_t = 1$

$$X_t = 0 \longrightarrow Y_t = 0$$

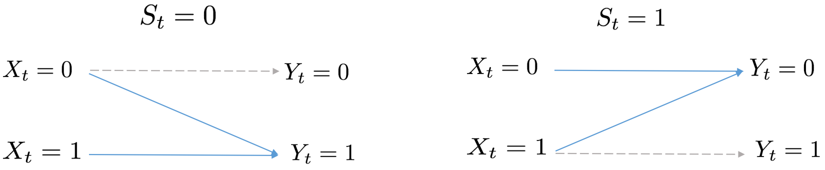
$$X_t = 1 \longrightarrow Y_t = 1$$

$$L(\mathbf{q}) = \frac{1}{n} I(X^n; Y^n) = 1$$

# Binary Case

$\mathcal{X} = \mathcal{Y} = \mathcal{S} = \{0, 1\}$, $P_X(\cdot) = P_{S_1}(\cdot) =$ Ber(0.5)

Policy 2: $Y_t = \bar{S}_t$




$$L(\mathbf{q}) = ??$$

Recall: $S_{t+1} = S_t - X_t + Y_t$

Policy 3: Best Effort Algorithm (G. Kalogridis, C. Efthymiou)

$$q(y_t|y_{t-1}, x_t, s_t) = \begin{cases} \mathbb{I}_{y_t}(y_{t-1}), & y_{t-1} + s_t - x_t \in \{0, 1\} \\ \mathbb{I}_{y_t}(\bar{y}_{t-1}), & \text{else.} \end{cases}$$

Varodayan-Khisti (2011):

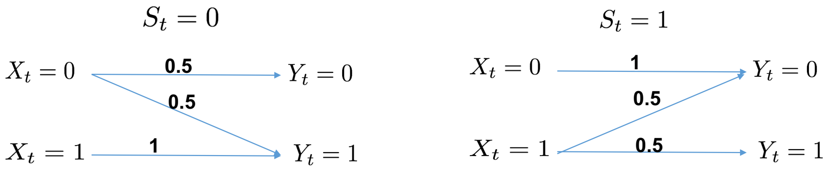
$$L(\mathbf{q}) \approx 0.68$$

(Numerical Evaluation using BCJR Algorithm)

# Binary Case

$\mathcal{X} = \mathcal{Y} = \mathcal{S} = \{0, 1\}$, $P_X(\cdot) = P_{S_1}(\cdot) =$ Ber(0.5)

Policy 4: Randomized Equiprobable Policy (Varodayan-Khisti 2011)




$$L(\mathbf{q}) = ??$$

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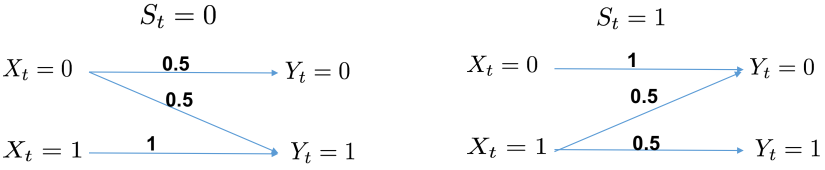
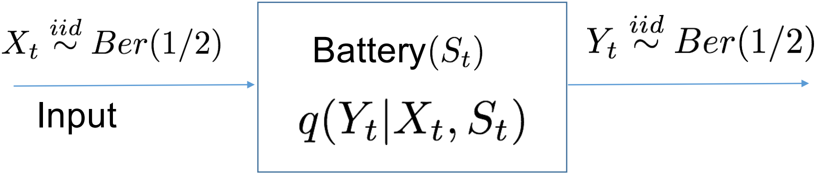



$$L(\mathbf{q}) = 0.5$$

# Binary Case

$\mathcal{X} = \mathcal{Y} = \mathcal{S} = \{0, 1\}$, $P_X(\cdot) = P_{S_1}(\cdot) =$ Ber(0.5)

## Key Properties: Randomized Equiprobable Policy




- **State Invariance**: $S_t \perp Y_1^{t-1}$, $S_t \sim \mathrm{Ber}(0.5)$, $\forall t \geq 1$
- $(X_t, S_t, Y_t) \perp Y_1^{t-1}$, $(X_t, S_t, Y_t) \sim (X_1, S_1, Y_1)$
- $Y^n \overset{\text{iid}}{\sim} \mathrm{Ber}(0.5)$
- $\frac{1}{n} I(S_1, X^n; Y^n) = I(S_1, X_1; Y_1) = \frac{1}{2}$.

# Main Result

Li-Khisti-Mahajan, IT-Trans (Submitted)

## Theorem

*For the case of iid inputs $X_t \sim P_X(\cdot)$, the optimal leakage rate is given by the following:*

$$L^\star = \min_{P_S(\cdot)} I(S - X; X), \quad S \in \mathcal{S},\ S \perp X$$

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*The leakage rate is achieved via the following:*

- *Initial State: $S_1 \sim P_S^\star(\cdot)$ (the optimizing distribution for $L^\star$)*
- *Battery Policy:*

$$q^\star(y|x,s) = \frac{P_X(y)P_S^\star(y+s-x)}{\sum_{y\in\mathcal{Y}} P_X(y)P_S^\star(y+s-x)}, \quad y+s-x \in \mathcal{S}$$

*Furthermore $I^q(S-X;X) = I^q(S,X;Y)$.*

# Achievability

Battery Policy:

$$q(y|x,s) = \frac{P_X(y)P_S(y+s-x)}{\sum_{y\in\mathcal{Y}} P_X(y)P_S(y+s-x)}, \quad y+s-x \in \mathcal{S}$$

1. State Invariance: $S_t \perp Y_1^{t-1}, \qquad S_t \sim P_S(\cdot)$

$$P_S^\star(s_2)P_X(y_1) = \sum_{s_1\in\mathcal{S},x_1\in\mathcal{X}} \mathbb{I}_{s_2}(s_1-x+y_1)\cdot q(y_1|x_1,s_1)\cdot P_X(x_1)\cdot P_S^\star(s_1)$$

# Achievability

Battery Policy:

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   - $(X_t, S_t, Y_t) \perp Y^{t-1}, \qquad (X_t, S_t, Y_t) \sim (X_1, S_1, Y_1)$
   - $Y^n$ is i.i.d. $\sim P_X(\cdot)$.
   - Single-Letter Leakage $\frac{1}{n}I(S_1, X_1^n; Y_1^n) = I(S_1, X_1; Y_1)$

2. Sufficiency of $X - S$: $q(y|x,s) = q(y|w = x-s)$
3. Optimality: $q^\star(\cdot)$ minimizes $I(S,X;Y)$ subject to State-Invariance.

# Converse

**Claim**

*For any* $\mathbf{q} \in \mathcal{Q}_A$ *we have that:*

$$\lim_{n\to\infty} \frac{1}{n} I^{\mathbf{q}}(S_1, X^n; Y^n) \geq L^\star \triangleq \min_{P_S(\cdot)} I(S - X; X)$$

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## Additive Cost

$I(S_1, X^n; Y^n) \geq \sum_{i=1}^n I(X_i, S_i; Y_i | Y^{i-1})$

## Telescoping Sum Lemma

$$\sum_{i=1}^n I(X_i, S_i; Y_i | Y^{i-1}) \geq \sum_{i=2}^n I(S_i - X_i; X_i | Y^{i-1}) + H(S_1 - X_1) - H(S_n - X_n | Y^{n-1})$$

# Converse

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## Term by Term Lower Bound

$$\begin{aligned}\sum_{i=2}^{n} I(S_i - X_i; X_i | Y^{i-1}) &\geq \sum_{i=2}^{n} \min_{p(s_i|y^{i-1})} I(S_i - X_i; X_i | Y^{i-1}) \\ &= (n-1) \min_{P_S(\cdot)} I(S - X; X) \\ &= (n-1) L^\star\end{aligned}$$

# Further Remarks: IID Case

Optimal Policy:

- **State Invariance**: $S_t \perp Y_1^{t-1}$
- $Y^n$ is i.i..d. with distribution $P_X(\cdot)$

$$L^\star = \min_{P_S(\cdot)} I(S - X; X)$$

- $I(S - X; X)$ is a convex function in $P_S(\cdot)$ for a fixed $P_X(\cdot)$ $\Rightarrow$ Blahut-Arimoto Algorithm.
- For the optimal policy it suffices to have $\mathcal{X} = \mathcal{Y}$.
- For binary case with equiprobable inputs, Randomized Equiprobale policy is optimal.
- Extends to Continuous Valued Random Variables.

# Numerical Example

- $X_t \sim \mathsf{Bin}(N, 0.5)$
- Corresponds to $N$ devices, each device is on/off with prob. $0.5$
- $N = 6$, $\mathcal{X} = \mathcal{Y}$
- $L^\star = \min_{P_S(\cdot)} I(S - X; X)$
  - $\mathcal{S} = \{0, 1, ...6\}$
  - $L^\star = 0.1638$
  - $P_S^\star(\cdot)$=$\{0.0586, 0.1332, 0.1972, 0.2220, 0.1972, 0.1332, 0.0586\}$

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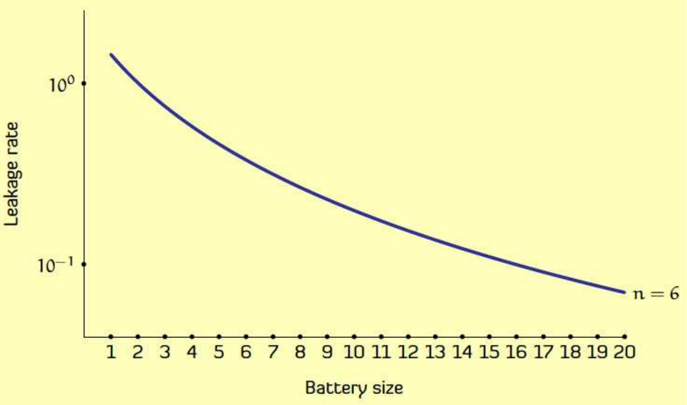

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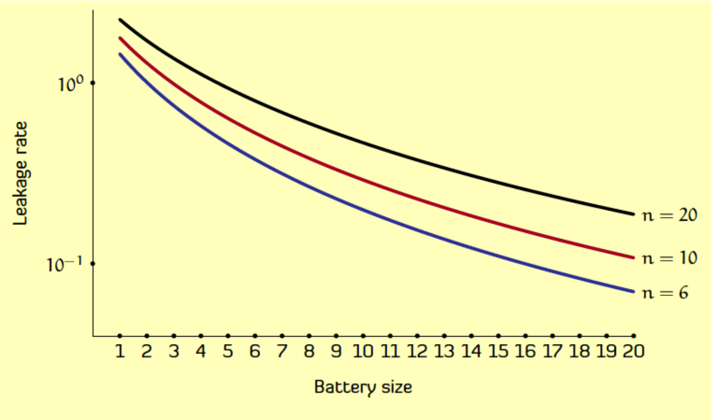

# Markovian Demands

First Order Markov Source:

$$X^n \sim P_X(x_1) \prod_{t=2}^{n} Q(x_t | x_{t-1})$$

- Belief State: $\pi_t(x, s) = \Pr(X_t = x, S_t = s | Y^{t-1} = y^{t-1})$
- Optimal Policy is of the form: $q(y | x, s, \pi_t)$

Dynamic Programming Decomposition:

- State: Posterior Distribution $\pi_t(x, s)$
- Action: Battery Policy $q(y | x, s, \pi_t)$

# Dynamic Programming Decomposition

Similar to Permuter-Cuff-Van Roy and Weissman (2007), Tatikonda-Mitter (Trans-IT 2009)

(First Order) Markovian Source:

$$L^\star = \min_{\mathbf{q} \in \mathcal{Q}_a} \left[ \limsup_{n \to \infty} \frac{1}{n} I^{\mathbf{q}}(S_1, X^n; Y^n) \right]$$

- State Space: $P_{X,S}(x, s)$
- Action Space: $\left\{ a(y|x, s) \text{:Battery Storage Constraint Satisfied} \right\}$
- State: $\pi_t(x, s) = \Pr(X_t = x, S_t = s | Y^{t-1} = y^{t-1})$
- Dynamics: $\pi_{t+1} = \Phi(\pi_t, a_t, y_t)$
- Per step cost: $I(\pi_t, a_t)$

$$I(X_t, S_t; Y_t | y^{t-1}) = \sum_{x,s,y} \pi_t(x, s) a_t(y|x, s) \log \frac{a_t(y|x, s)}{\sum_{x,s} \pi_t(x, s) a_t(y|x, s)}$$

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(First Order) Markovian Source:

- State: $\pi_t(x,s) = \Pr(X_t = x, S_t = s | Y^{t-1} = y^{t-1})$
- Dynamics: $\pi_{t+1} = \Phi(\pi_t, a_t, y_t)$
- Per step cost: $I(\pi_t, a_t)$

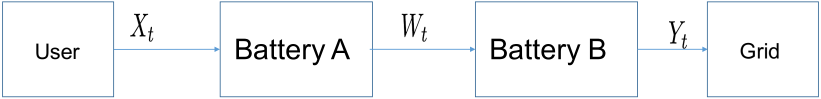
## Infinite Horizon Average Cost DP

Let $J^\star \in \mathbb{R}$ and $v : \mathcal{P}_{X,S}(\cdot) \to \mathbb{R}$ be such that:

$$J^\star + v(\pi) = \inf_{a \in \mathcal{A}} \left[ I(a;\pi) + \sum_{x,s,y} \pi(x,s) a(y|x,s) v(\Phi(\pi,a,y)) \right], \forall \pi$$

- $J^\star$ is the optimal leakage rate
- $q(y|x,s,\pi) = a^\pi(y|x,s)$ is the optimal policy

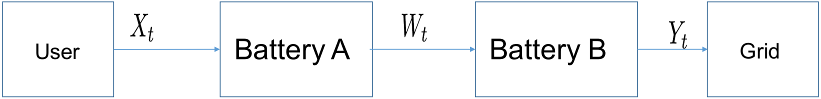
Joint Work with Si-Hyeon Lee (Toronto) and Helena Liu (Toronto)




- Input $X_t \overset{iid}{\sim} P_X(\cdot)$, $X_t \in \mathcal{X}$
- Battery A State: $S_{A,t} \in \mathcal{S}$, Output $W_t \in \mathcal{W}$
  - State Update: $S_{A,t+1} = S_{A,t} - X_t + W_t$
  - Battery Policy: $q_t^A(w_t | x^t, s_{A,1}^t w_1^{t-1})$

# Extension: Two Batteries in Cascade

## Joint Work with Si-Hyeon Lee (Toronto) and Helena Liu (Toronto)

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  - Battery Policy: $q_t^A(w_t|x^t, s^t_{A,1} w_1^{t-1})$
- Battery B State: $S_{B,t} \in \mathcal{S}$, Output $Y_t \in \mathcal{Y}$
  - State Update: $S_{B,t+1} = S_{B,t} - W_t + Y_t$
  - Battery Policy: $q_t^B(y_t|w^t, s^t_{B,1}, y_1^{t-1})$

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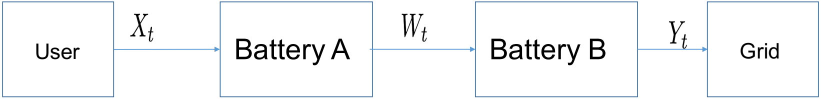


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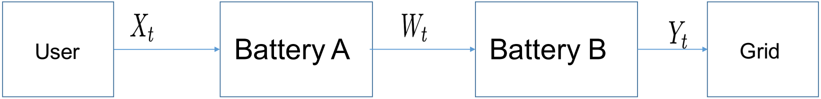
  - Battery Policy: $q_t^B(y_t | w^t, s_{B,1}^t, y_1^{t-1})$
- Leakage Rate

$$L(\mathbf{q}^A, \mathbf{q}^B) = \lim_{n \to \infty} \frac{1}{n} I(S_{A,1}, S_{B,1}, X^n; Y^n)$$

# Extension: Two Batteries in Cascade

Joint Work with Si-Hyeon Lee (Toronto) and Helena Liu (Toronto)




- Input $X_t \overset{iid}{\sim} P_X(\cdot)$, $X_t \in \mathcal{X}$
- Battery A State: $S_{A,t} \in \mathcal{S}$, Output $W_t \in \mathcal{W}$
  - State Update: $S_{A,t+1} = S_{A,t} - X_t + W_t$
  - Battery Policy: $q_t^A(w_t|x^t, s^t_{A,1} w_1^{t-1})$
- Battery B State: $S_{B,t} \in \mathcal{S}$, Output $Y_t \in \mathcal{Y}$
  - State Update: $S_{B,t+1} = S_{B,t} - W_t + Y_t$
  - Battery Policy: $q_t^B(y_t|w^t, s^t_{B,1}, y_1^{t-1})$
- Leakage Rate

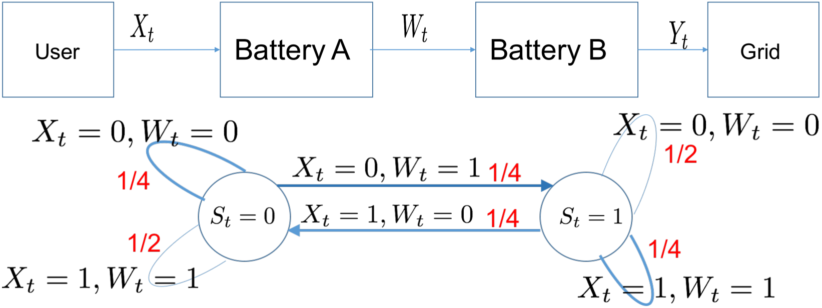
$$L(\mathbf{q}^A, \mathbf{q}^B) = \lim_{n\to\infty} \frac{1}{n} I(S_{A,1}, S_{B,1}, X^n; Y^n)$$

$$L^\star = \inf_{\mathbf{q}^A, \mathbf{q}^B} L(\mathbf{q}^A, \mathbf{q}^B)$$

# Binary Case

Liu-Lee-Khisti (2016, Submitted)

$\mathcal{X} = \mathcal{Y} = \mathcal{W} = \mathcal{S}_A = \mathcal{S}_B = \{0,1\}$, $P_X(\cdot) = P_{S_A}(\cdot) = P_{S_B}(\cdot) = \text{Ber}(0.5)$



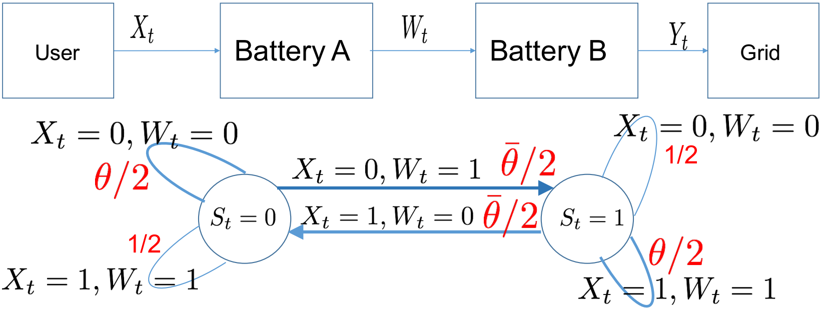

$$\frac{1}{n} I(S_{A,1}, X^n; W^n) = \frac{1}{n} I(S_{B,1}, W^n; Y^n) = 0.5$$

$$\frac{1}{n} I(S_{A,1}, S_{B,1} X^n; Y_1^n) \approx 0.3443$$

# Binary Case

Liu-Lee-Khisti (2016, Submitted)

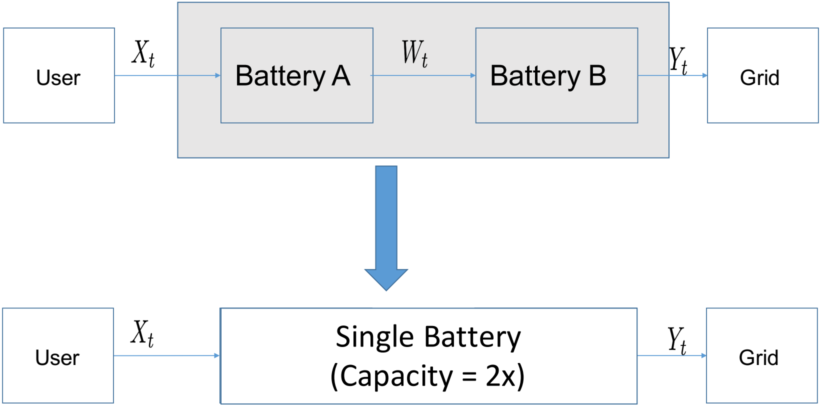
$\mathcal{X} = \mathcal{Y} = \mathcal{W} = \mathcal{S}_A = \mathcal{S}_B = \{0,1\}$, $P_X(\cdot) = P_{S_A}(\cdot) = P_{S_B}(\cdot) = \mathrm{Ber}(0.5)$




$$\theta = 0.4, \frac{1}{n} I(S_{A,1}, X^n; W^n) = \frac{1}{n} I(S_{B,1}, W^n; Y^n) \approx 0.514$$

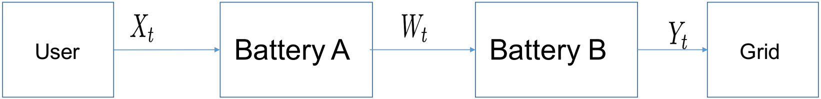
$$\frac{1}{n} I(S_{A,1}, S_{B,1} X^n; Y^n) \approx 0.323$$

# Cooperation Based Lower Bound




- $\mathcal{S} = \{0, 1, 2\}$
- $P_S^\star(\cdot) \approx \{0.275, 0.45, 0.275\}$
- $L^\star = \min I(S - X; X) \approx 0.306$

# Upper Bound




- $\mathcal{Q}_A^\star$: Invariant Policies for battery $A$
- $\mathcal{Q}_B^\star$: Invariant Policies for battery $B$

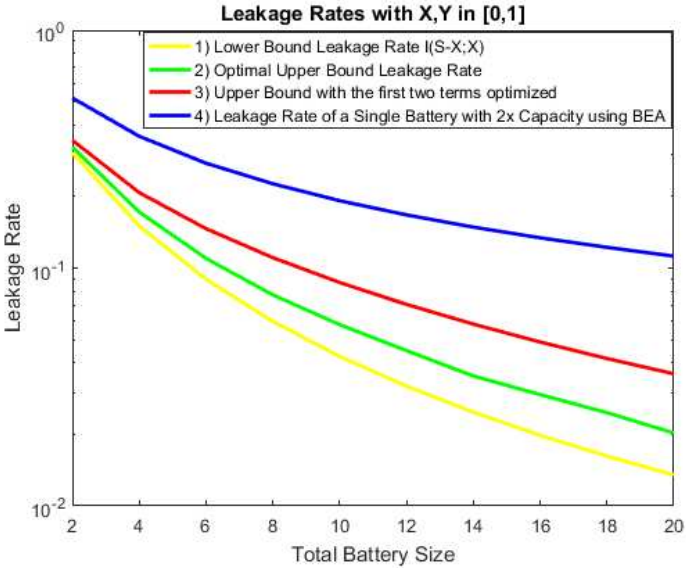
**Theorem (Upper Bound)**

*For any* $\mathbf{q}^A \in \mathcal{Q}_A^\star$ *and* $\mathbf{q}^B \in \mathcal{Q}_B^\star$ *the following leakage rate is achievable:*

$$L^+ \leq I(S_A, X; W) + I(S_B, W; Y) - I(S_A, S_B, X, Y; W)$$

# Numerical Results

$\mathcal{X} = \mathcal{Y} = \{0, 1\}$, $P_X(\cdot) = [0.5, 0.5]$



# Conclusions

Information Theoretic Framework for Privacy in Smart-Metering Systems

- Privacy Metric: Mutual Information
- Battery Policy $\equiv$ State Constrained Comm. Channel
- IID Inputs
  - Explicit Battery Policy and Single-Letter Leakage Rate
  - State Invariance
- Markov Inputs: Dynamic Programming
- Cascaded Batteries: Single-Letter Achievability
- Our results extend to:
  - Continuous Valued Variables (Input/Output/State)
  - Higher Order Markov Process
- Future Work: Time Varying Input Distribution