Team Decision Theory: Characterization of Information Structures, Basic Concepts and Solution Methods

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CDC tutorial on
Information Structures in Optimal Control
10 Dec, 2012

Outline of the talk

Dynamic programming for centralized control

Dynamic programming for decentralized control

Salient features of common information approach

- Combining with person-by-person approach
- Contructing common information

Using decentralized dynamic programming to specific applications

Conclusion

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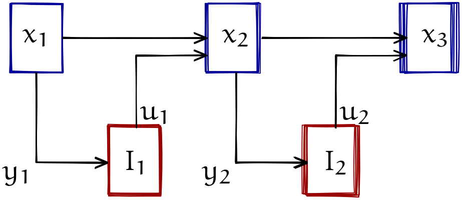

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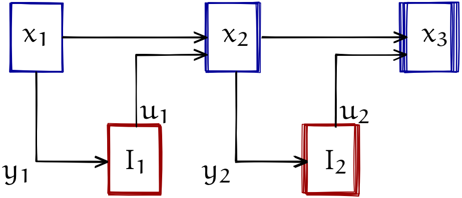

- $I_t = \{y_{[1:t]}, u_{[1:t-1]}\}$ (perfect recall)
- $I_t \subseteq I_{t+1}$ (classical info-structure)

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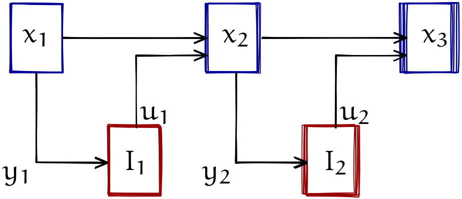

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Conceptual difficulties

- $\gamma_t : I_t \mapsto u_t$
- $\min J(\gamma_1, \ldots, \gamma_T)$

Identifying an information state is a key step in centralized dynamic programming

Properties of information state

- π_t is a function of the information I_t
- π_{t+1} is a function of π_t and new information (u_t, y_{t+1})
- π_t is a sufficient statistic for predicting future observations

$$\mathcal{P}(y_{t+1} \mid I_t) = \mathcal{P}(y_{t+1} \mid \pi_t)$$

- π_t is sufficient for performance evaluation

$$\mathcal{E}[c(x_t, u_t) \mid I_t, u_t] = \mathcal{E}[c(x_t, u_t) \mid \pi_t, u_t]$$

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Examples of information states

- In partially observable Markov decision problems (POMDPs), the belief state $\mathcal{P}(x_t \mid I_t)$ is an information state.
- In linear quadratic and Gaussain (LQG) problems, the state estimate $\hat{x}_{t|t}$ is an information state.

DP uses the information state to sequentially decompose the optimization problem

Structural result

Restricting attention to control laws of the form $u_t = \gamma_t(\pi_t)$ does not entail any loss of optimality.

- In some cases, also derive qualitative properties of the optimal policy (monotonicity, convexity, threshold, etc.)

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Dynamic programming decomposition

Recursively define

$$V_t(\pi_t) = \inf_{u_t \in \mathbb{U}_t} \mathcal{E}\left[c(x_t, u_t) + V_{t+1}(\pi_{t+1}) \mid \pi_t, u_t \right]$$

If the infimum is achieved, then the arg min at time t gives the optimal control action at information state π_t.

- Functional vs parameteric optimization

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There is no obvious way to generalize DP to decentralized control

DP relies on classical information structure:

$$V_t(I_t) = \inf_{u_t \in \mathbb{U}_t} \mathcal{E}\left[c(\cdot) + \inf_{u_{t+1} \in \mathbb{U}_{t+1}} \mathcal{E}[c(\cdot) + \cdots \mid I_{t+1}] \;\middle|\; I_t\right]$$

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Techniques from centralized control not directly applicable to decentralized control.

All DP approaches exploit the common or shared information between DMs

Examples of information structures with sharing

- Periodic sharing information structure
  (Chong and Athans, 1976; Ooi et at, 1997)
- Delayed sharing information structure
  (Yoshikawa 1975, Aicardi et al 1987, Nayyar Mahajan Teneketzis 2011)
- Belief sharing (Yüksel 2009)
- Control sharing information structure
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- Specific information structures
  (Walrand Varaiya 1982, Veeravalli Başar Poor 1993)
- Partial history sharing
  (Nayyar 2011, Nayyar Mahajan Teneketzis (accepted 2013))

A unified framework for dynamic programming*

- $\gamma^i_t : I^i_t \mapsto u^i_t$

- $\min J(\gamma_1, \dots, \gamma_T)$

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- $\gamma_t^i : I_t^i \mapsto u_t^i$

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Identify a coordinator based on common information

- has access to the common information $C_t = \bigcap_{i=1}^{n} I_t^i$
- chooses prescriptions $\varphi = (\varphi_t^1, \dots, \varphi_t^n)$ where $\varphi_t^i : I_t^i \setminus C_t \mapsto u_t^i$.

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- $\gamma_t^i : I_t^i \mapsto u_t^i$

Identify sufficient statistic/information state

- $\gamma_t^i : S_t^i \mapsto u_t^i$
- Use a person-by-person approach

- $\min J(\gamma_1, \dots, \gamma_T)$

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The common information approach to reformulate the system from the perspective of a coordinator

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4. Show the equivalence between the two models
5. Translate the results back to the original decentralized system

Example 1: Common information approach to a static team*

$\omega \mapsto (C, M^1, M^2),$

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- Same cost $\ell(\omega, u^1, u^2)$

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$\omega \mapsto (x_1, w^0_{[1:T]}, w^1_{[1:T]}, w^2_{[1:T]}),$

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Literature overview

- Witsenhausen, Proc IEEE, 1971
- Sandell Athans, TAC 1974
- Yoshikawa, TAC 1975
- Varaiya Walrand, TAC 1978
- Nayyar Mahajan Teneketzis, TAC 2011
- Lamperski Doyle, arxiv 2012

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Common information approach

- Contruct a coordinator: $C_t = I^1_t \cap I^2_t = (y^{1,2}_{[1:t-d]}, u^{1,2}_{[1:t-d]})$

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- Coordinator chooses $(\varphi^1_t, \varphi^2_t) = \psi_t(C_t)$ where

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Example 2: Delayed sharing information structure*

$$\omega \mapsto (x_1, w^0_{[1:T]}, w^1_{[1:T]}, w^2_{[1:T]}), \qquad \ell(x_t, u^1_t, u^2_t)$$

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Common information approach

- Contruct a coordinator: $C_t = I^1_t \cap I^2_t = (y^{1,2}_{[1:t-d]}, u^{1,2}_{[1:t-d]})$
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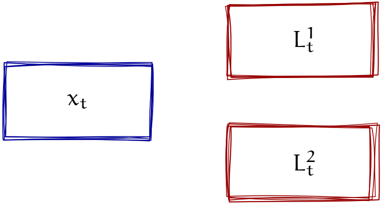

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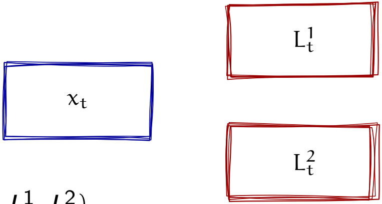

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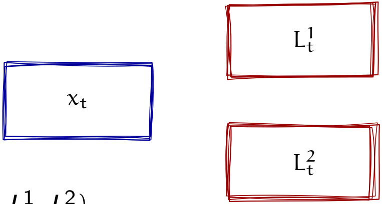

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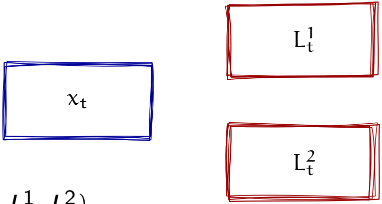

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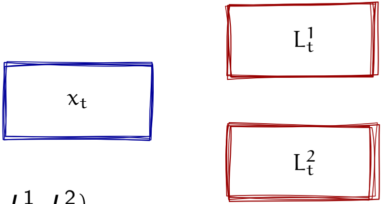

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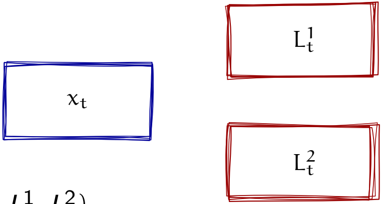

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- Also obtain a dynamic programming decomposition

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- Coordinator is equivalent to single DM

Outline of the talk

Dynamic programming for centralized control

Dynamic programming for decentralized control

Salient features of common information approach

- Combining with person-by-person approach
- Contructing common information

Using decentralized dynamic programming to specific applications

Conclusion

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- DP may be solvable when:
  - π_t is parameteric (e.g., Gaussian)
  - φ_t^i is parameteric (e.g., linear)

Outline of the talk

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- Look at the centralized subproblem of finding the best response strategy of DM i.
- If we identify a structure for the best response strategy of DM i that does not depend on the exact choice of the strategy of other DMs, then that structure is globally optimal.

Example 4: Coupled subsystems with control sharing*

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Directly using the common information approach not tractable

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- Use a person-by-person approach to show that, without loss of optimality

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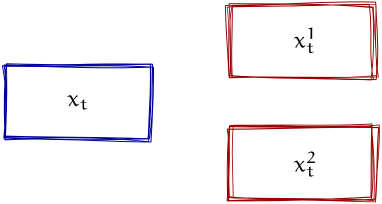

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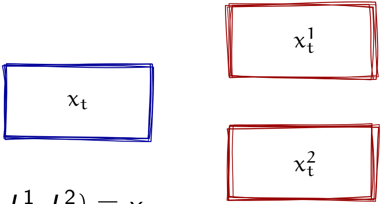

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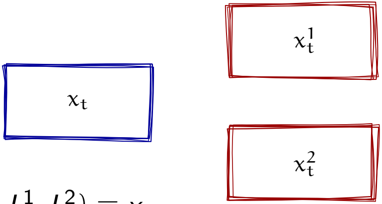

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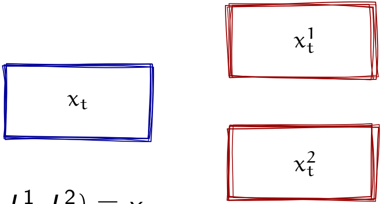
*Mahajan "Optimal decentralized control of coupled subsystems with control sharing," CDC 2011

Example 4:
Coupled subsystems with control sharing*



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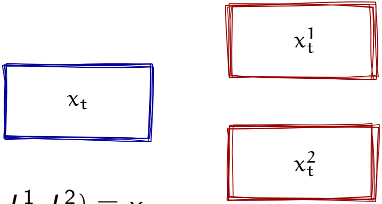

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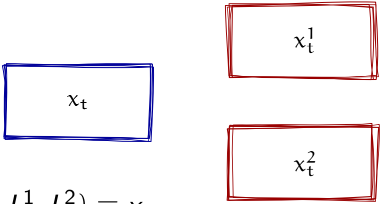

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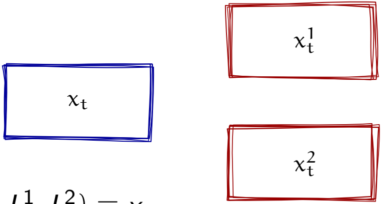

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- Also obtain a dynamic programming decomposition

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Outline of the talk

How to construct common information that is nested with time*

What if $I^1_t \cap I^2_t \not\subseteq I^1_{t+1} \cap I^2_{t+1}$?

*Nayyar "Sequential decision making in decentralized systems," PhD thesis 2011

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Coordinator's approach is same as Witsenhausen's standard form (or the designer's approach) (Witsenhausen, 1973).

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Outline of the talk

Dynamic programming for centralized control

Dynamic programming for decentralized control

Salient features of common information approach
- Combining with person-by-person approach
- Contructing common information

Using decentralized dynamic programming to specific applications

Conclusion

Using the dynamic programming approach to specific applications

- Real time communication
  - Witsenhausen 1978, Walrand Varaiya 1982, Teneketzis 2006, Mahajan Teneketzis 2010, Yüksel 2013 (to appear), Kaspi Merhav (arixv 2011)
- Networked control systems
  - Walrand Varaiya 1983, Imer Yüksel Başar 2006, Schenato, Sinopoli, Franceschetti, Poola, Sastry 2007, Mahajan Teneketzis 2009,
- Decentralized sequentail hypothesis testing
  - Teneketzis Ho 1987, LaVigna Makowski Baras 1986, Veeravalli Başar Poor 1993, Nayyar Teneketzis 2011
- Mobile cellular networks
  - Hajek Mitzel Yang 2008
- Sensor Networks
  - Imer Başar 2005, Lipsa Martins 2011, Nayyar Başar Teneketzis Veeravalli (arxiv 2012)

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Thank you