Optimal Control of Teams with Exchangeable Agents: A Design Methodology for Demand Response

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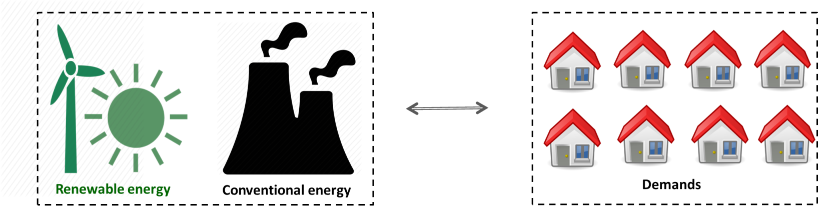
Outline

- Motivation and connection with mean field teams.
- Mean field teams — Markov Chain.
- Mean field teams — Linear Quadratic.
- Summary and Conclusion.

Motivation

- Demand response: It manages the power consumption of demands in order to decrease the volatility of power grids.



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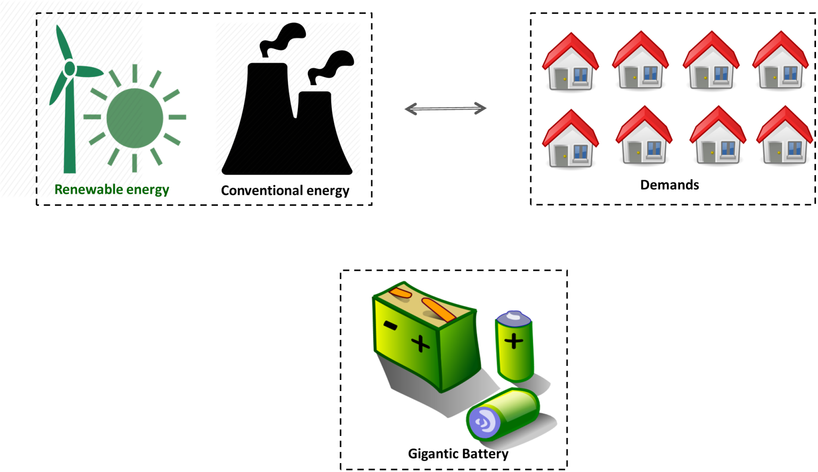

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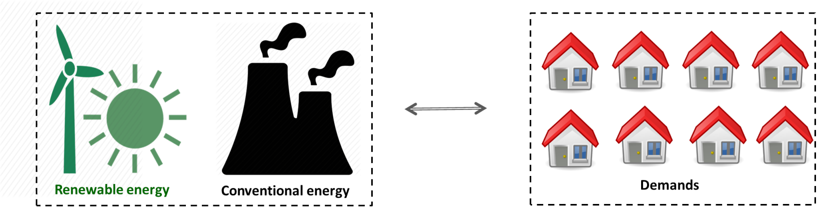
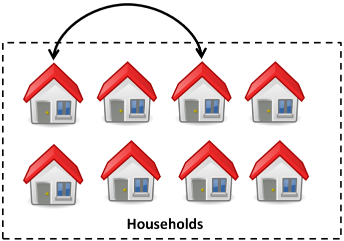


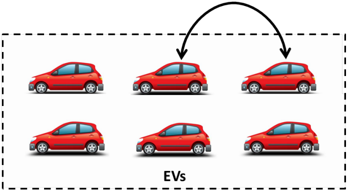
Demand Response acts like Virtual Battery!

Main Challenges

We are dealing with large scale systems.

- Decentralized information at demands $\Longrightarrow$ cooperation is difficult!
- Communication $\Longrightarrow$ costly and may not be feasible!
- Computational complexity $\Longrightarrow$ exponential in number of demands!

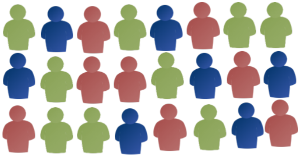




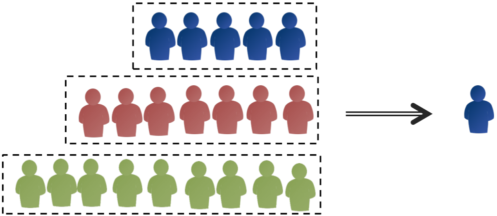
We prove exchangeable systems are equivalent to mean-field (aggregate-behaviour) coupled systems.

Exchangeability

Consider a heterogeneous population with partial exchangeable agents.

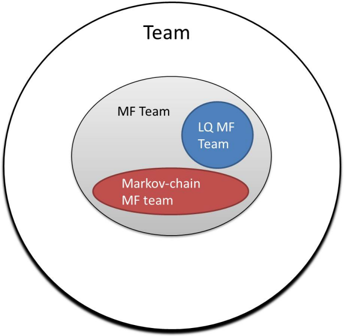


Since there is no dependence on the index of agents, agents are only influenced by aggregate behaviour of other agents, i.e., mean field.

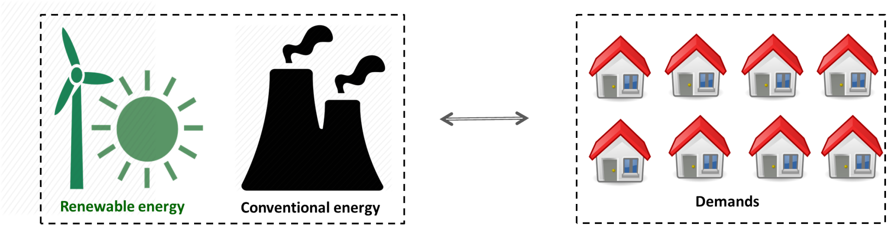


Mean Field Teams

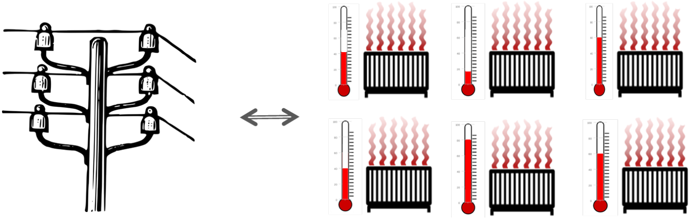
- Key feature of mean-field teams is that the solution is tractable. In particular,
  - Markov chain mean-field team (J. Arabneydi and A. Mahajan, CDC 2014):
    - mean-field: empirical distribution
  - Linear quadratic mean-field team (J. Arabneydi and A. Mahajan, CDC 2015):
    - mean-field: empirical mean



Motivating Example 1: Markov Chain



Objective: Keep the distribution of demands close to a desired reference trajectory with minimum force.



Objective: Control the average temperature with minimum forcing of space heaters.

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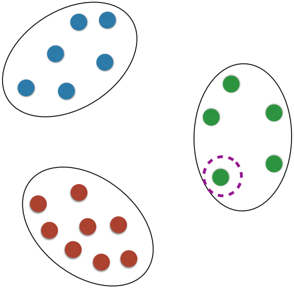
Notation

- N : number of heterogeneous agents (entire population)
- K : number of types (sub-populations)



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- N : number of heterogeneous agents (entire population)
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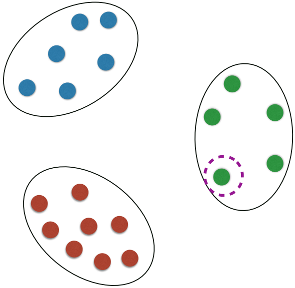
For agent $i \in \mathcal{N}^k$ **of type** $k \in \mathcal{K}$

- $X_t^i \in \mathcal{X}^k$: state of agent i
- $U_t^i \in \mathcal{U}^k$: action of agent i



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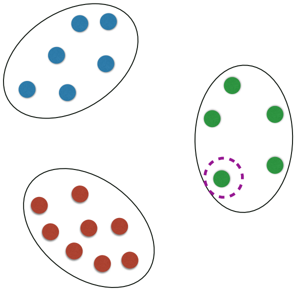
For sup-population of type $k \in \mathcal{K} = \{1, \dots, K\}$

- $\mathcal{N}^k$: entire sub-population of type k
- $\bar{X}_t^k = \frac{1}{|\mathcal{N}^k|} \sum_{i \in \mathcal{N}^k} \delta_{X_t^i}$: mean-field of states
- $\bar{U}_t^k = \frac{1}{|\mathcal{N}^k|} \sum_{i \in \mathcal{N}^k} \delta_{U_t^i}$: mean-field of actions



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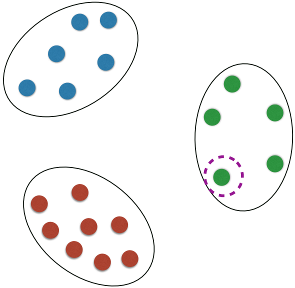

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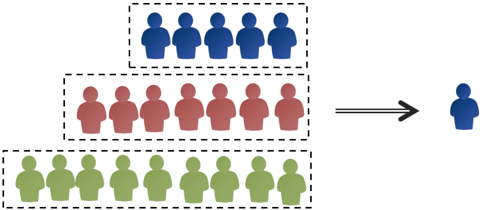
For entire population

- $\mathcal{N} = \mathcal{N}^1 \bigcup \ldots \bigcup \mathcal{N}^K$: entire population
- $\mathbf{X}_t = (X_t^i)_{i \in \mathcal{N}}$: joint state of entire population at time t
- $\mathbf{U}_t = (U_t^i)_{i \in \mathcal{N}}$: joint action of entire population at time t
- $\bar{\mathbf{X}}_t = \mathrm{vec}(\bar{X}_t^1, \ldots, \bar{X}_t^K)$: mean-field of states of entire population at time t
- $\bar{\mathbf{U}}_t = \mathrm{vec}(\bar{U}_t^1, \ldots, \bar{U}_t^K)$: mean-field of actions of entire population at time t

Problem Formulation: Markov Chain

- Dynamics of agent $i \in \mathcal{N}^k$ with type $k \in \{1, \dots, K\}$:

$$X^i_{t+1} = f_t(X^i_t, U^i_t, W^i_t, \bar{\mathbf{X}}_t, \bar{\mathbf{U}}_t), \quad i \in \{1, \dots, N\}. \tag{1}$$



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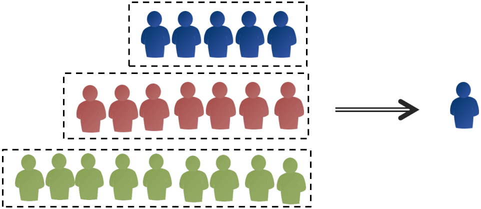

- **Mean-field sharing** Information structure: $U^i_t = g^i_t(\bar{\mathbf{X}}_{1:t}, X^i_t)$, where g^i_t is called control law of subsystem i at time t.

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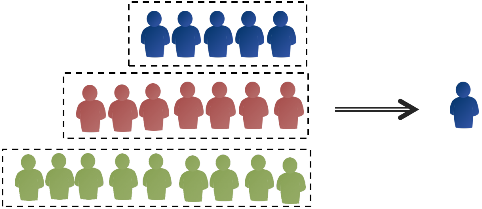

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- Optimization problem: We are interested in finding a strategy $\mathbf{g}$ that minimizes

$$J(\mathbf{g}) = \mathbb{E}^{\mathbf{g}} \left[\sum_{t=1}^{T} \ell_t(\mathbf{X}_t, \mathbf{U}_t) \right]. \tag{2}$$

Key Assumption:

- All agents use **identical control laws**.

$$g_t^i(\cdot) = g_t^j(\cdot) = g_t, \quad \forall i, j \in \mathcal{N}^k$$

If agents $i, j \in \mathcal{N}^k$ have the same status, then

$$x_t^i = x_t^j = x \longrightarrow u_t^i = g_t^i(\bar{\mathbf{x}}_{1:t}, x) = g_t^j(\bar{\mathbf{x}}_{1:t}, x) = u_t^j, \quad \forall i, j \in \mathcal{N}^k$$

- In general, above assumption leads to a loss in performance. However, it is a standard assumption in the literature on large scale systems for reasons of **simplicity**, **fairness**, and **robustness**.

Main Results: Markov Chain

We identify a dynamic program to compute an optimal strategy. In particular,

Theorem 1:

Let ψ_t^* be a solution to the following dynamic program: at time t for every $\bar{\mathbf{x}}_t$

$$V_t(\bar{\mathbf{x}}_t) = \min_{\gamma_t} \mathbb{E}\big[\ell_t(\mathbf{X}_t, \mathbf{U}_t) + V_{t+1}(\bar{\mathbf{X}}_{t+1}) | \bar{\mathbf{X}}_t = \bar{\mathbf{x}}_t, \Gamma_t = \gamma_t\big]$$

where $\gamma_t = (\gamma_t^1, \dots, \gamma_t^K), \gamma_t^k : \mathcal{X}^k \to \mathcal{U}^k$, and $\gamma_t = \psi_t(\bar{\mathbf{x}}_t)$. Then, optimal solution is

$$g_t^{*,k}(\bar{\mathbf{x}}, x) := \psi_t^{*,k}(\bar{\mathbf{x}})(x), \quad \forall \bar{\mathbf{x}}, \forall x \in \mathcal{X}^k, k \in \mathcal{K}.$$

Agent i of sub-population k at time t:

- Upon observing mean-field $\bar{\mathbf{x}}_t$, it solves the above dynamic program and **computes the optimal strategy in a decentralized manner** i.e. $g_t^{*,k}(\bar{\mathbf{x}}_t, \cdot)$.
- Upon observing local state x_t^i, it chooses local control action

$$u_t^i = g_t^{*,k}(\bar{\mathbf{x}}_t, x_t^i).$$

Salient feature of the model:

- Very few assumptions on the model.
- Allow for mean-field coupled dynamics.
- Allow for arbitrary coupled cost. (We do not assume cost to be weakly coupled.)
- Mean-field of the system can be computed and communicated easily.

Main Results: Markov Chain

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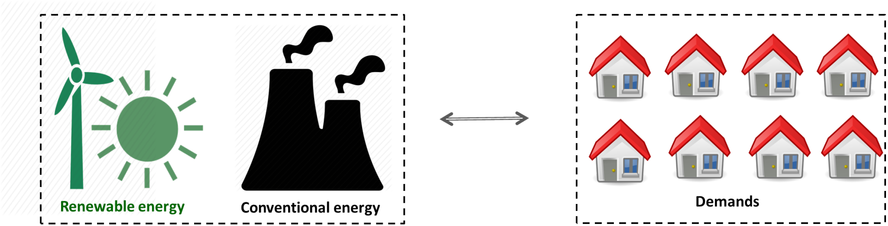
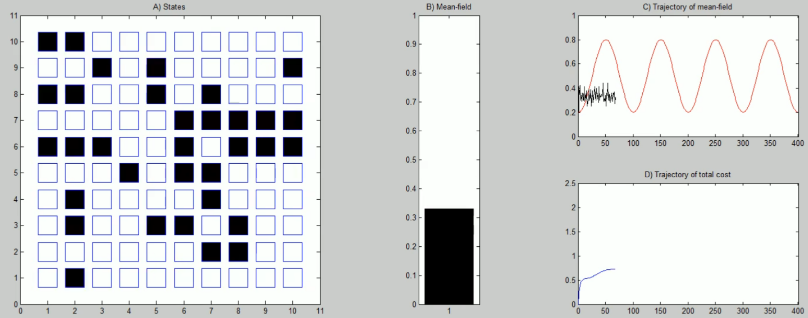
Salient feature of the results:

- Computing **globally optimal** solution.
- Solution approach works for **arbitrary number of agents**.
- Computational complexity of solution increases **polynomially** (rather than exponentially) w.r.t. the number of agents.
- The results extend to **infinite horizon**, **noisy observation**, **Major Minor**, **infinite population**, and **randomized strategies**.



- $X_t^i \in \mathcal{X} = \{OFF, ON\}$, $\quad \bar{X}_t = \frac{1}{n}\sum_{i=1}^{n} 1(X_t^i = OFF)$
- Dynamics: $\mathbb{P}(X_{t+1}^i | X_t^i, U_t^i) =: [P(u_t^i)]_{x_t^i x_{t+1}^i}$
- Actions: $U_t^i \in \mathcal{U} = \{FREE, OFF, ON\}$, $\quad$ Cost of action: $C(U_t^i)$
- Objective: $\min_{\mathbf{g}} \mathbb{E}^{\mathbf{g}} \left[\sum_{t=1}^{\infty} \beta^t \left(\frac{1}{n}\sum_{i=1}^{n} C(U_t^i) + D(\bar{X}_t \parallel \zeta_t)\right)\right].$

Motivating Example 1: Markov Chain



Uncontrolled case

Mean-field tracking case

Optimal case

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Problem Formulation: Linear Quadratic (LQ)

- Dynamics of agent $i \in \mathcal{N}^k$ with type $k \in \{1, \dots, K\}$:

$$x_{t+1}^i = A_t^k x_t^i + B_t^k u_t^i + D_t^k \bar{\mathbf{x}}_t + E_t^k \bar{\mathbf{u}}_t + w_t^i, \tag{3}$$

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- Per-step cost: for $t = 1, \dots, T-1$,

$$c_t(\mathbf{x}_t, \mathbf{u}_t, \bar{\mathbf{x}}_t, \bar{\mathbf{u}}_t) = \bar{\mathbf{x}}_t{}^\intercal P^x_t \bar{\mathbf{x}}_t + \bar{\mathbf{u}}_t{}^\intercal P^u_t \bar{\mathbf{u}}_t + \sum_{k \in \mathcal{K}} \left[\frac{1}{|\mathcal{N}^k|} \sum_{i \in \mathcal{N}^k} \left[x^i_t{}^\intercal Q^k_t x^i_t + u^i_t{}^\intercal R^k_t u^i_t \right] \right] \tag{4}$$

and $t = T$,

$$c_T(\mathbf{x}_T, \bar{\mathbf{x}}_T) = \bar{\mathbf{x}}_T{}^\intercal P^x_T \bar{\mathbf{x}}_T + \sum_{k \in \mathcal{K}} \left[\frac{1}{|\mathcal{N}^k|} \sum_{i \in \mathcal{N}^k} x^i_T{}^\intercal Q^k_T x^i_T \right], \tag{5}$$

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where P^x_t, P^u_t, Q^k_t, and R^k_t are matrices of appropriate dimension; above matrices are symmetric and satisfy the following condition:

$$\begin{aligned} Q^k_t \geq 0 \ \ \forall k \in \mathcal{K}, \quad \operatorname{diag}\{Q^1_t, \dots, Q^K_t\} + P^x_t \geq 0, \\ R^k_t > 0 \ \ \forall k \in \mathcal{K}, \quad \operatorname{diag}\{R^1_t, \dots, R^K_t\} + P^u_t > 0. \end{aligned} \tag{6}$$

Problem Formulation: Linear Quadratic (LQ)

- **Mean-field sharing** Information structure: $u_t^i = g_t^i(\bar{\mathbf{x}}_{1:t}, x_t^i)$, where g_t^i is called control law of agent i at time t.

- Optimization problem: We are interested in finding a strategy $\mathbf{g}$ that minimizes

$$J(\mathbf{g}) = \mathbb{E}^{\mathbf{g}}[\sum_{t=1}^{T-1} c_t(\mathbf{x}_t, \mathbf{u}_t, \bar{\mathbf{x}}_t, \bar{\mathbf{u}}_t) + c_T(\mathbf{x}_T, \bar{\mathbf{x}}_T)], \tag{7}$$

  where the expectation is with respect to the measure induced on all system variables by the choice of strategy $\mathbf{g}$.

Theorem 2:

Let

$$\bar{A}_t := \operatorname{diag}\{A_t^1, \dots, A_t^K\} + \operatorname{vec}(D_t^1, \dots, D_t^K), \quad \bar{Q}_t := \operatorname{diag}\{Q_t^1, \dots, Q_t^K\}$$
$$\bar{B}_t := \operatorname{diag}\{B_t^1, \dots, B_t^K\} + \operatorname{vec}(E_t^1, \dots, E_t^K), \quad \bar{R}_t := \operatorname{diag}\{R_t^1, \dots, R^K\}.$$

Under (A1) and (A2), we have the following results.

1. *Structure of optimal strategy:* The optimal strategy is unique and is linear in local state and the mean-field of the system. In particular,

$$u_t^i = \breve{L}_t^k (x_t^i - \bar{x}_t^k) + \bar{L}_t^k \bar{\mathbf{x}}_t \tag{8}$$

where the above gains are obtained by the solution of $K + 1$ Riccati equations: one for computing each $\breve{L}_t^k$, $k \in \mathcal{K}$, and one for $\bar{L}_t := \operatorname{vec}(\bar{L}_t^1, \dots, \bar{L}_t^K)$.

Theorem 2:

2 *Riccati equations:* For $t \in \{1, \dots, T-1\}$,

$$\breve{L}_t^k = -\left(B_t^{k\intercal} \breve{M}_{t+1}^k B_t^k + R_t^k\right)^{-1} B_t^{k\intercal} \breve{M}_{t+1}^k A_t^k \tag{9}$$

and

$$\bar{L}_t = -\left(\bar{B}_t^\intercal \bar{M}_{t+1} \bar{B}_t + \bar{R}_t + P_t^u\right)^{-1} \bar{B}_t^\intercal \bar{M}_{t+1} \bar{A}_t, \tag{10}$$

where $\breve{M}_{1:T}^k$ and $\bar{M}_{1:T}$ are the solutions of following Riccati equations:

$$\breve{M}_{1:T}^k = \text{Riccati}(A_{1:T}^k, B_{1:T}^k, Q_{1:T}^k, R_{1:T}^k)$$

$$\bar{M}_{1:T}^k = \text{Riccati}(\bar{A}_{1:T}, \bar{B}_{1:T}, \bar{Q}_{1:T} + P_{1:T}^x, \bar{R}_{1:T}^k + P_{1:T}^u)$$

Agent i of sub-population k:

- It computes $\breve{L}_{1:T}^k$ and $\bar{L}_{1:T}$ by solving above two Riccati equations.
- Upon observing local state x_t^i and global mean-field $\bar{\mathbf{x}}_t$, it chooses local control action

$$u_t^i = \breve{L}_t^k (x_t^i - \bar{x}_t^k) + \bar{L}_t^k \bar{\mathbf{x}}_t.$$

Salient feature of the results

1 We show that the obtained optimal control laws perform as well as the optimal centralized control laws.

2 The solution and the solution complexity depend on the number of types but **not on the number of agents of each type**.

3 Each agent can independently solve the appropriate Riccati equations and **compute the optimal strategy in a decentralized manner**.

4 The results extend to **weighted mean field**, **infinite horizon**, **Infinite population**, **Tracking problem**, **major-minor**, and **Noisy observation**.

5 When population is infinite, mean-field becomes deterministic and computable.

Tracking problem

- Per-step cost

$$c_t(\mathbf{x}_t, \mathbf{u}_t, \bar{\mathbf{x}}_t, \bar{\mathbf{u}}_t) = (\bar{\mathbf{x}}_t - \mathbf{s}_t)^\intercal P_t^x (\bar{\mathbf{x}}_t - \mathbf{s}_t) + \bar{\mathbf{u}}_t^\intercal P_t^u \bar{\mathbf{u}}_t$$
$$+ \sum_{k \in \mathcal{K}} \left[\frac{1}{|\mathcal{N}^k|} \sum_{i \in \mathcal{N}^k} \left[(x_t^i - r_t^i)^\intercal Q_t^k (x_t^i - r_t^i) + u_t^{i\,\intercal} R_t^k u_t^i \right] \right].$$

- Everything else remains the same as in the basic model.

Theorem 2:

Under (A1) and (A2), we have

1 *Structure of optimal strategy:*

$$u_t^i = \breve{L}_t^k(x_t^i - \bar{x}_t^k) + \bar{L}_t^k \bar{\mathbf{x}}_t + \breve{F}_t^k v_t^i + \bar{F}_t^k \bar{v}_t, \tag{11}$$

where gains $\{\breve{L}_t^k, \bar{L}_t^k\}_{t=1}^{T-1}$ are the same as in Theorem 1.

2 *Riccati equations:* Let $\{\breve{M}_t^k\}_{t=1}^T$ and $\{\bar{M}_t\}_{t=1}^T$ be the solution of (K+1) Riccati equations defined in Theorem 1. For $t = 1, \dots, T-1$:

$$\breve{F}_t^k = \left(B_t^{k\intercal} \breve{M}_{t+1}^k B_t^k + R_t^k\right)^{-1} B_t^{k\intercal} \quad \text{and} \quad \bar{F}_t = \left(\bar{B}_t^\intercal \bar{M}_{t+1} \bar{B}_t + \bar{R}_t + P_t^u\right)^{-1} \bar{B}_t^\intercal, \tag{12}$$

where $\bar{F}_t =: \mathrm{vec}(\bar{F}_t^1, \dots, \bar{F}_t^K)$. For $t = T$,

$$v_T^i = Q_T^k r_T^i, \quad \bar{v}_T = \bar{Q}_T \bar{\mathbf{r}}_T + P_T^x \mathbf{s}_T \tag{13}$$

and for $t = T-1, \dots, 1$,

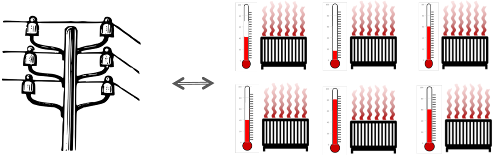
$$v_t^i = (A_t^k - B_t^k \breve{L}_t^k)^\intercal v_{t+1}^i + Q_t^k r_t^i \quad \text{and} \quad \bar{v}_t = (\bar{A}_t - \bar{B}_t \bar{L}_t)^\intercal \bar{v}_{t+1} + \bar{Q}_t \bar{\mathbf{r}}_t + P_t^x \mathbf{s}_t. \tag{14}$$

Agent i of sub-population k:

- It computes $\bar{L}_{1:T}$ and $\bar{F}_{1:T}$ by solving one Riccati equation of types. In addition, given the mean-field of local reference trajectories $\bar{\mathbf{r}}_{1:T}$ and mean-field reference trajectory $s_{1:T}$, it computes the global correction trajectory $\bar{v}_{1:T}$.
- It computes $\breve{L}^k_{1:T}$ and $\breve{F}^k_{1:T}$ by solving one Riccati equation of type k. In addition, given the local reference trajectories $r^i_{1:T}$, it computes the local correction trajectory $v^i_{1:T}$.
- Upon observing local state x^i_t and global mean-field $\bar{\mathbf{x}}_t$, it chooses local control action

$$u^i_t = \breve{L}^k_t(x^i_t - \bar{x}^k_t) + \bar{L}^k_t \bar{\mathbf{x}}_t + \breve{F}^k_t v^i_t + \bar{F}^k_t \bar{v}_t.$$

Motivating Example 2: Linear Quadratic



- Space heater i:

$$x^i_{t+1} = a(x^i_t - x_{nominal}) + b(u^i_t + u_{nominal}) + w^i_t$$

- Objective:

$$\mathbb{E}\left[\frac{1}{N}\sum_{t=1}^{T} q_t(x^i_t - r^i_t)^2 + p_t(\bar{x}_t - s_t)^2 + m_t(x^i_t - x_{nominal})^2 + r_t {u^i_t}^2\right]$$

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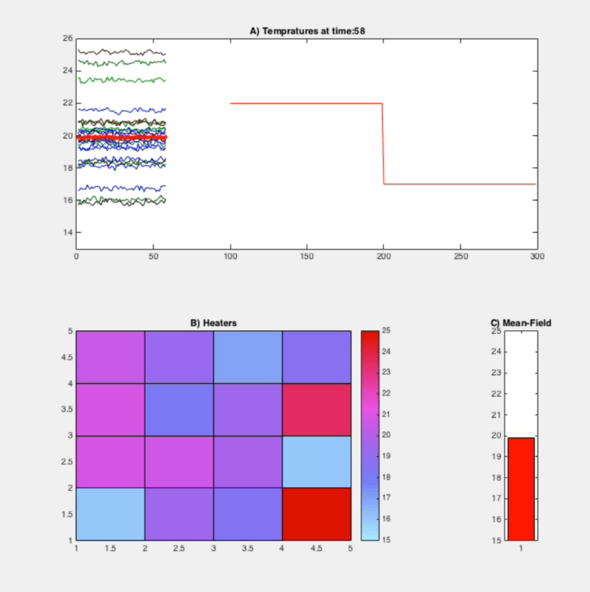

mean field

local+ mean-field + moderate-temperature local+ mean-field 1

local$+$ mean-field 2

Mean Field Team

- Every exchangeable system may be presented as mean field teams.
- Each agent can independently compute optimal strategy in a decentralized manner.
- Mean-field can be computed and communicated easily (via consensus algorithms).

Summary & Conclusion

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- The size of the information state of the dynamic program increases polynomially with the number of agents rather than exponentially.
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Linear Quadratic Mean Field Team

- We showed that optimal strategy is unique, identical across sub-populations, and linear in local state and (global) mean-field. To compute the optimal gains, we obtained $K+1$ standard Ricatti equations.
- The computational complexity of our solution is independent of the number of agents in each type and polynomial in number of types.
- When population is infinite, mean-field becomes deterministic and computable.

Thank You

- J. Arabneydi and Aditya Mahajan, "Team Optimal Solution of Finite Number of Mean-Field Coupled LQG Subsystems", CDC 2015.

- J. Arabneydi and Aditya Mahajan, "Team Optimal Control of Coupled Major-Minor Subsystems with Mean-Field Sharing", ICC 2015.

- J. Arabneydi and Aditya Mahajan, "Team Optimal Control of Coupled Subsystems with Mean-Field Sharing", CDC 2014.