The common-information approach to multi-agent teams

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Disco-Modesty Seminar
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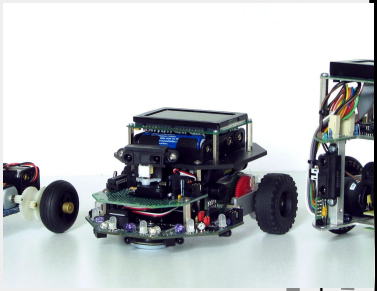
Common theme: multi-stage multi-agent decision making under uncertainty

Networked control systems



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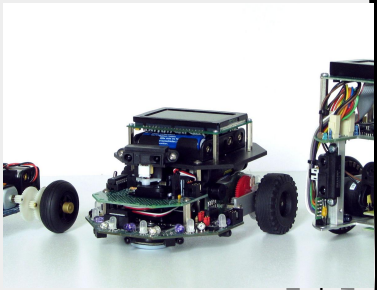

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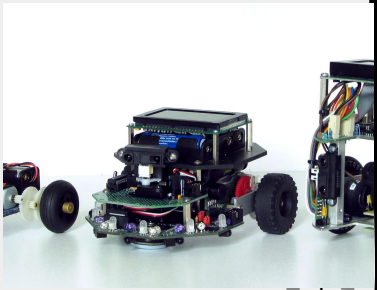

Challenges

- Signals sent over wireless channels (packet drops)

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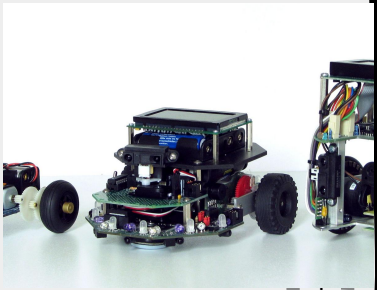

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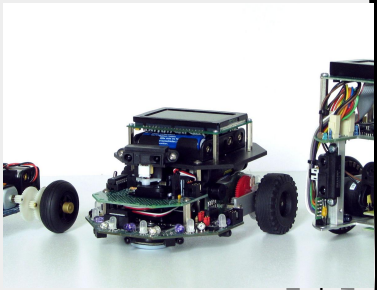

Challenges

- Signals sent over wireless channels (packet drops)
- Different vehicles have different information
  - Decentralized control
  - Decentralized estimation
  - Decentralized learning

Salient Features of Modern Engineering Systems

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Multiple agents

Agents have different partial information about the environment and each other

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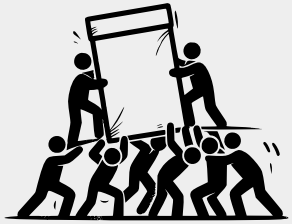
Teams versus Games

Teams vs Games



Teams

- All agents have **common objective**
- Agents **cooperate** to min team cost
- Agents are **not strategic**
- Solution concepts: person-by-person optimality, global optimality . . .

Games

- Each agent has **individual objective**
- Agents **compete** to minimize individual cost
- Agents are **strategic**
- Solution concepts: Nash equil, Bayesian Nash, Subgame perfect, Markov perfect, Bayesian perfect, . . .

Teams vs Games

Teams

- All agents have **common objective**

Games

- Each agent has **individual objective**

In many engineering problems, game theory is used as an algorithmic toolbox to provide distributed solutions to **static** problems.

We are interested in finding **globally optimal** solution to problems where **agents have decentralized information**.

Teams have a reputation of
being notoriously difficult . . .

Some historical context

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S&C until the 1960s

- About 300 years of knowledge in designing **LTI systems**
- Good "intuitive" understanding of **frequency domain methods**
  - Root locus
  - Bode plots
  - Nyquist plots
  - Loop shaping

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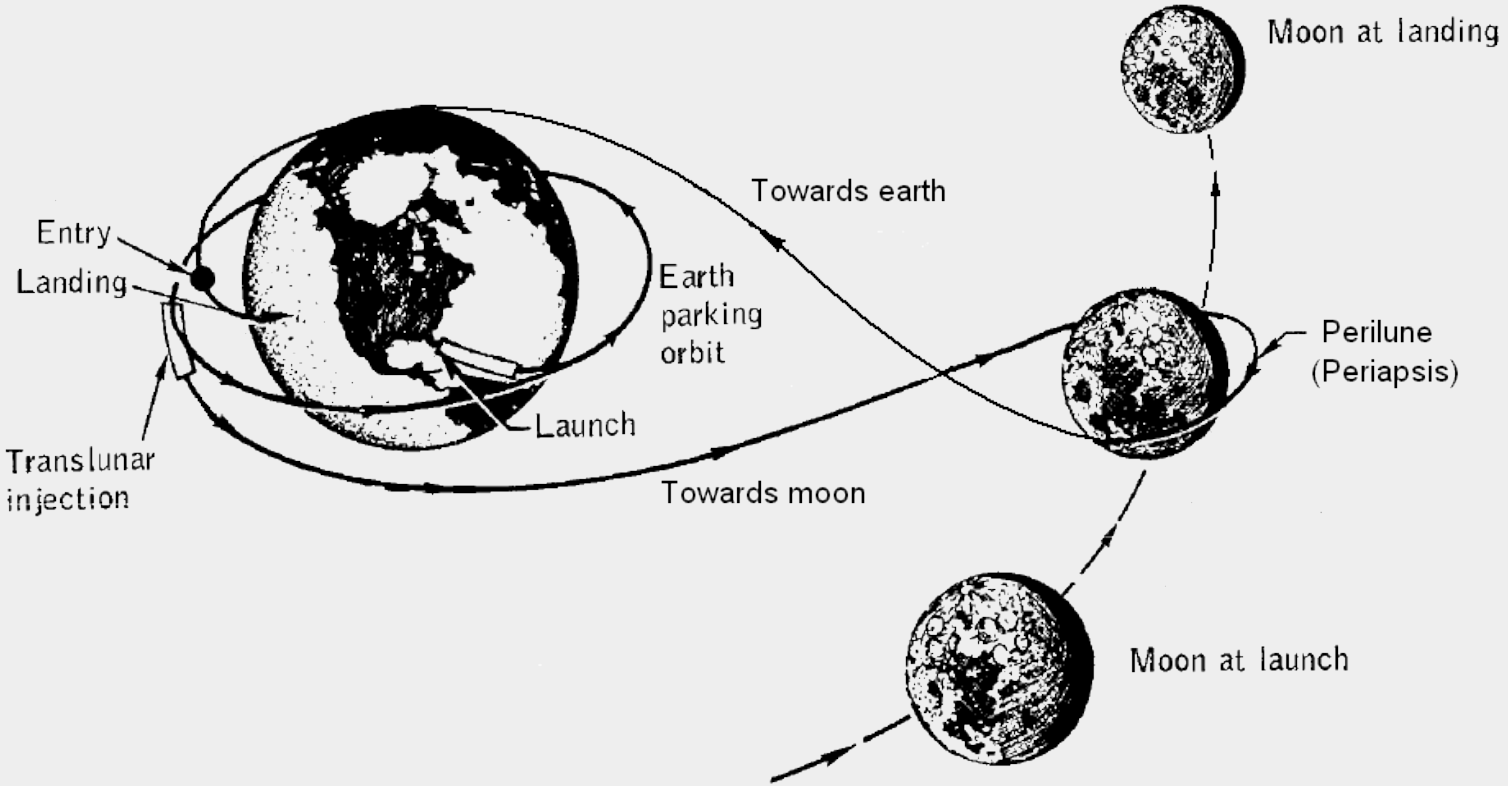
State Space Deisgn

- Linearize the system dynamics
- Design **optimal control** assuming full state feedback (LQR)
  $\text{control action}(t) = -\text{gain}(t) \cdot \text{state}(t)$
- Estimate the state using noisy measurements (Kalman filtering)
  $\text{state estimate}(t) = \text{Function}(\text{estimate}(t\text{-}1), \text{measurement}(t))$.
- **Optimal controller**:
  $\text{control action}(t) = -\text{gain}(t) \cdot \text{state estimate}(t)$


Moon at landing
Towards earth
Entry
Landing
Earth
parking
orbit
Perilune
(Periapsis)
Launch
Translunar
injection
Towards moon
Moon at launch

Conceptual difficulties in team problems

Witsenhausen Counterexample

- A two step dynamical system with two controllers
- Linear dynamics, quadratic cost, and Gaussian disturbance
- Non-linear controllers outperform linear control strategies . . .

. . . cannot use Kalman filtering + Riccati equations

Witsenhausen, "A counterexample in stochastic optimum control," SICON 1968.

Whittle and Rudge, "The optimal linear solution of a symmetric team control problem," App. Prob. 1974.

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Complexity analysis

- All random variables are finite valued
- Finite horizon setup
- The problem of finding the best control strategy is in NEXP

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Why are team problems hard?

Why are team problems hard?

Why are single agent problems easy?

Static stochastic optimization problems

$$\min_{g:\mathcal{Y}\to\mathcal{U}} \mathbb{E}[c(X, g(Y))]$$

	$X=0$	$X=1$	$X=2$	$X=3$
$U=0$	0.5	0.2	1.2	0.5
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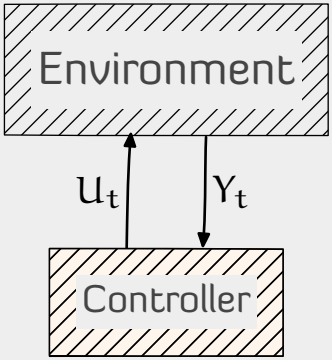
for each y, $\min_{u\in\mathcal{U}} \mathbb{E}[c(X, u) \mid Y = y]$

- Each sub-problem is a **parameter optimization** problem.
- Search complexity $|\mathcal{U}| \cdot |\mathcal{Y}|$.

Dynamic stochastic optimization problems

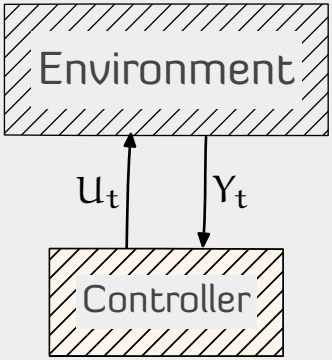


Dyanmics $X_{t+1} = f_t(X_t, U_t, W_t)$

Observations $Y_t = h_t^i(X_t, N_t)$

Control law $U_t = g_t(Y_{1:t}, U_{1:t-1})$

Dynamic stochastic optimization problems



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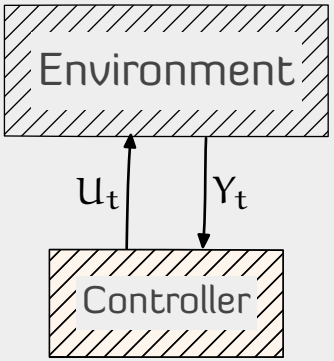
Objective

Choose control strategy $g = (g_1, \dots, g_T)$ to minimize

$$J(g) = \mathbb{E}\left[\sum_{t=1}^{T} c_t(X_t, U_t)\right]$$

Dynamic stochastic optimization problems



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Objective	Choose control strategy $g = (g_1, \dots, g_T)$ to minimize $\mathbb{E}^g\left[\sum_{t=1}^{T} c_t(X_t, U_t)\right]$
Dynamic programming solution	▷ Define **belief state** $b_t = P(X_t \mid Y_{1:t}, U_{1:t-1})$. ▷ Write a DP in terms of the belief state b_t. ▷ Solution complexity: $T \cdot \lvert\mathcal{U}\rvert \cdot \lvert\mathcal{Z}\rvert$.

Why don't these simplifications work for teams?

Static team problem

$$\min_{g^1, g^2} \mathbb{E}[c(X, g^1(Y^1), g^2(Y^2))]$$

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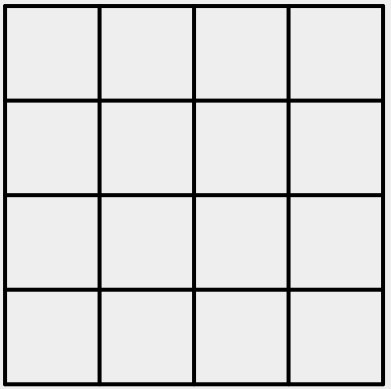

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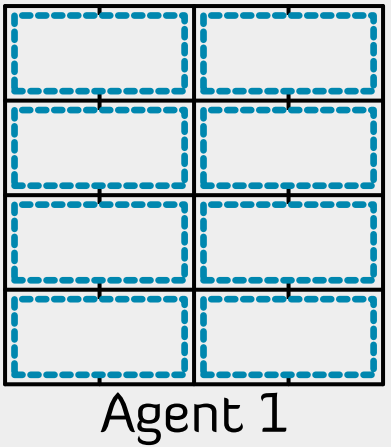

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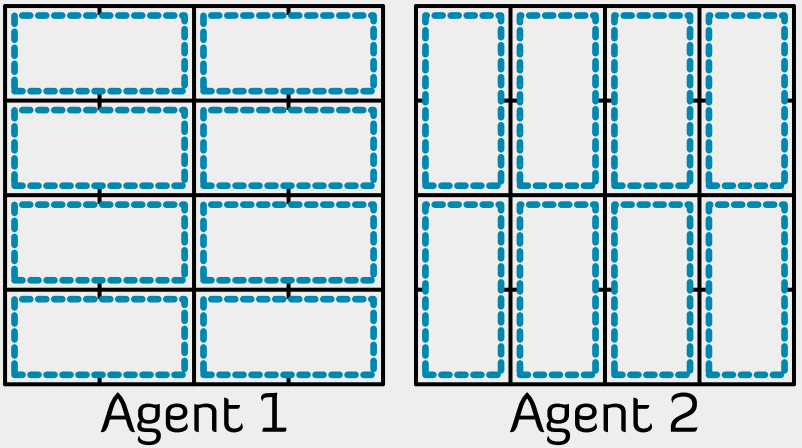

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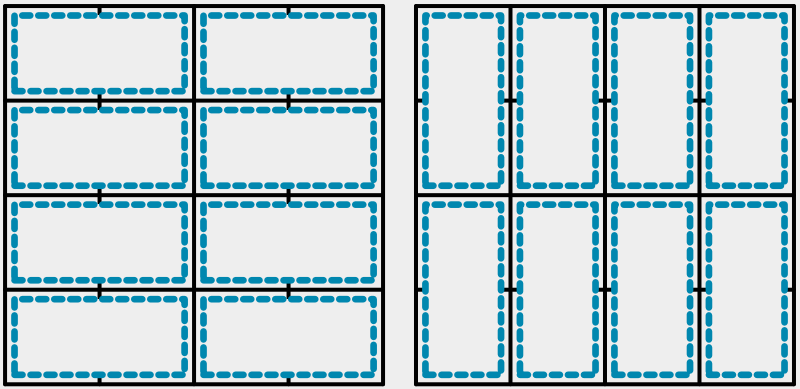

Previous idea of

$$\text{for all } y^1, \quad \min_{u^1} \mathbb{E}[c(X, u^1, g^2(Y^2)) \mid Y^1 = y^1]$$

leads to person-by-person optimal solution (not globally opt.)

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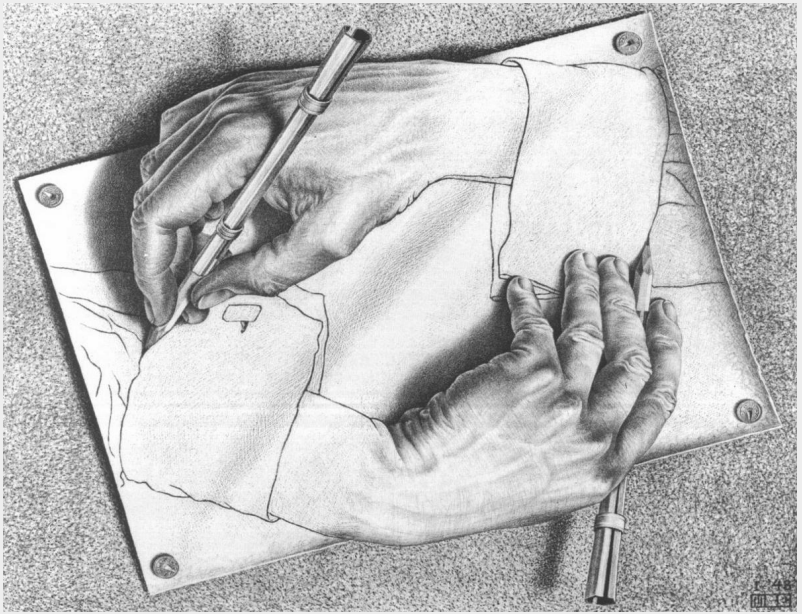

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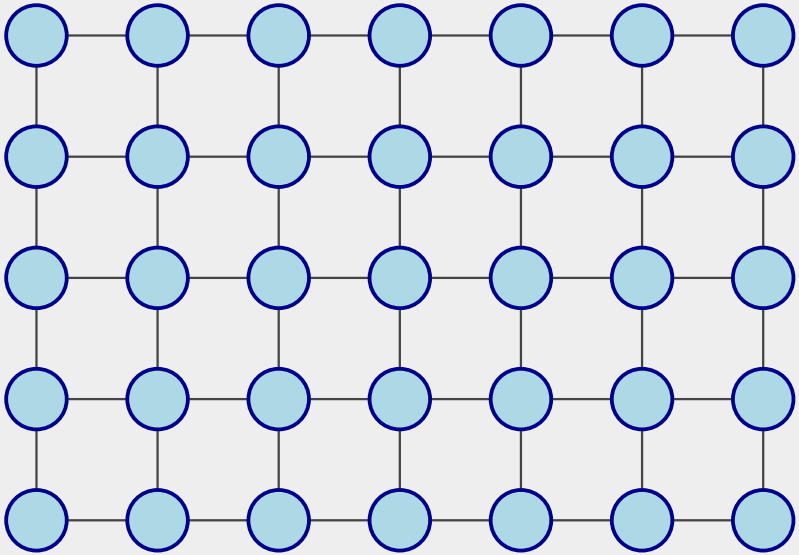
There are additional challenges
in dynamic problems

k-step delayed sharing information structure

- Consider a network with coupled dynamics.
- Information exchange between nodes with unit delay.



k-step delayed sharing information structure

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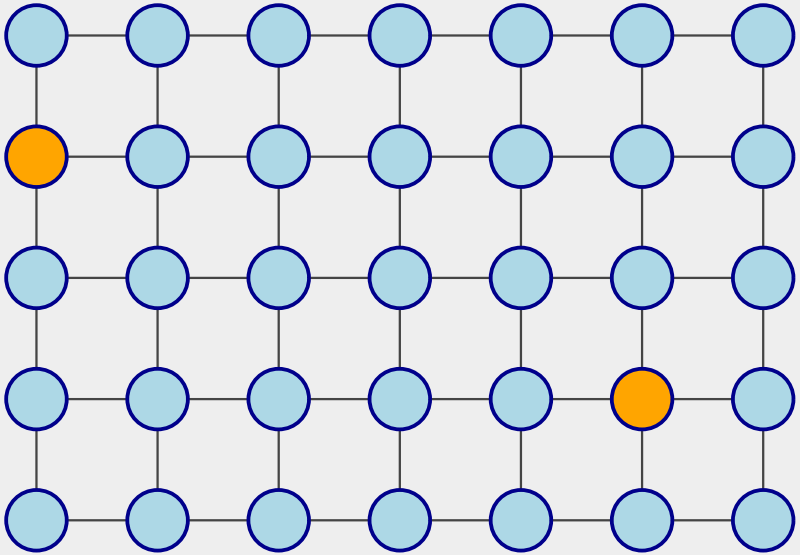
- Fix the strategy of all but two subsystems which are k-hop apart. What is the best response strategy at these two nodes?



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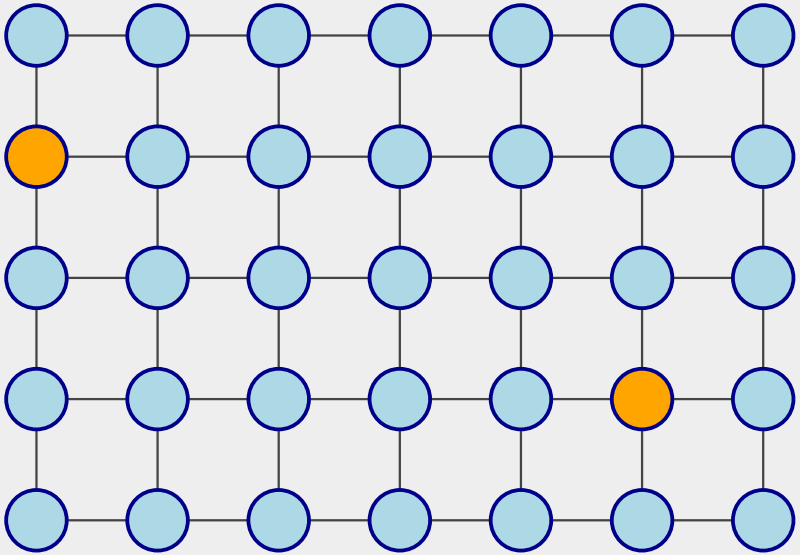

- Fix the strategy of all but two subsystems which are k-hop apart. What is the best response strategy at these two nodes?

- Proposed by Witsenhausen in a seminal paper.
- Allows to smoothly transition between centralized ($k = 0$) and completely decentralized ($k = \infty$).

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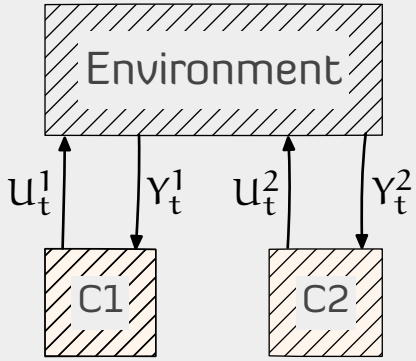
System Model



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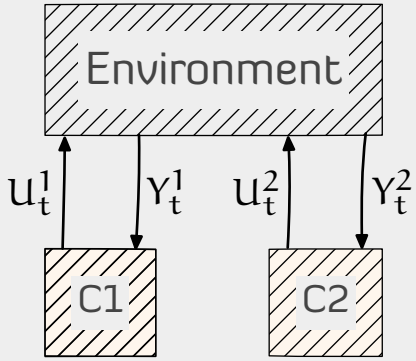

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Information Structure	$I_t^i = \{Y_{1:t}^i, U_{1:t-1}^i Y_{1:t-k}^{-i}, U_{1:t-k}^{-i}\}$
Control law	$U_t^i = g_t^i(I_t^i)$

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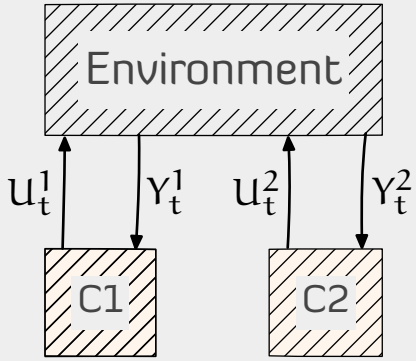

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The data I_t^i available at each controller is increasing with time.
How to find a sufficient statistic or an information state?

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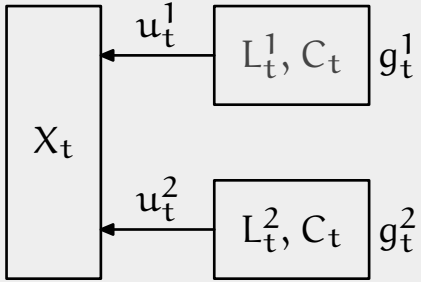

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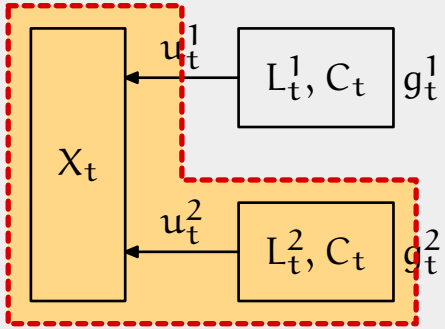

- Unobserved state from the p.o.v. of ctrl 1: X_t, L_t^2, C_t.
  Information state $\pi_t^1 = \mathbb{P}(X_t, L_t^2, C_t | L_t^1, C_t)$.

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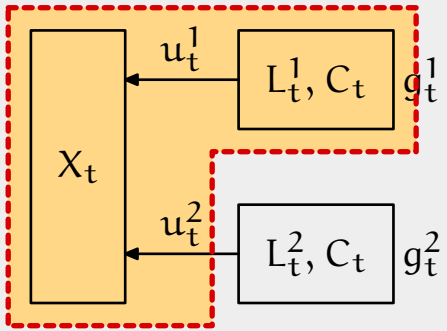

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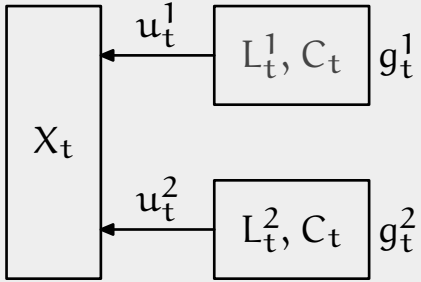

- Unobserved state from the p.o.v. of ctrl 1: X_t, L_t^2, C_t.
  Information state $\pi_t^1 = \mathbb{P}(X_t, L_t^2, C_t | L_t^1, C_t)$.
- Unobserved state from the p.o.v. of ctrl 2: X_t, π_t^1.
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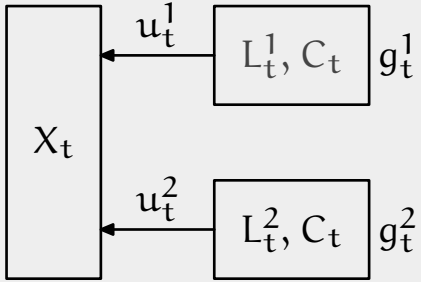
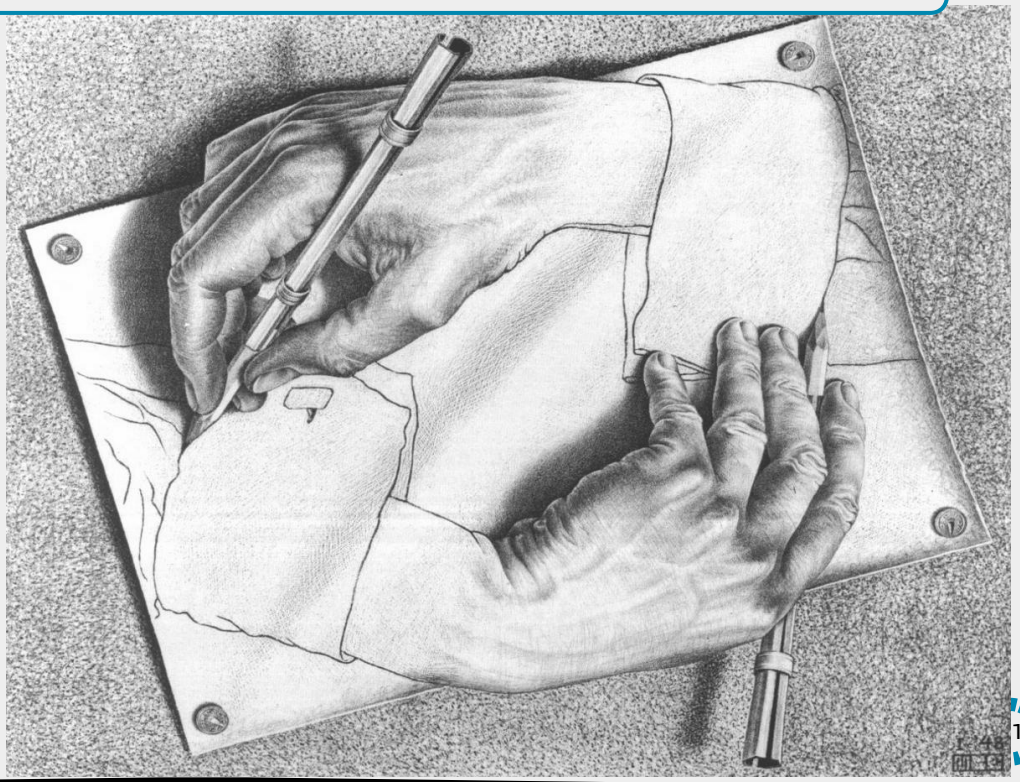
- Unobserved state from the p.o.v. of ctrl 1: X_t, π_t^2.
  Information state $\pi_t^{1,2} = \mathbb{P}(X_t, \pi_t^2 | L_t^2, C_t)$.
- . . . infinite regress . . .

Conceptual difficulty

The data I_t^i available at each controller is increasing with time.
How to find a sufficient statistic or an information state?





History of the problem

Witsenhausen's Assertion

Let $C_t = \{Y_{1:t-k}, U_{1:t-k}\}$ and $L_t^i = \{Y_{t-k+1:t}^i, u_{t-k+1:t-1}^i\}$.

Then $\mathbb{P}(X_{t-k} \mid C_t)$ is a sufficient statistic for C_t.

Rationale: $\mathbb{P}(X_{t-k}|Y_{1:t-k}, U_{1:t-k})$ is policy independent.

Witsenhausen, "Separation of Esitmation and Control for Discrete-Time Systems," Proc. IEEE, 1971.

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Follow-up Literature

- **Assertion true for $k=1$**
  [Sandell, Athans, 1974], [Kurtaran, 1976]
- **Assertion false for $k>1$**
  [Varaiya, Walrand 1979], [Yoshikawa, Kobayashi, 1978]
- **No subsequent positive result!**

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Are there sufficient statistics or information states for C_t?

Witsenhausen, "Separation of Esitmation and Control for Discrete-Time Systems," Proc. IEEE, 1971.

Importance of the problem

Applications (of one-step delay sharing)	
	▷ **Power systems**: Altman et al, 2009
	▷ **Queueing theory**: Kuri and Kumar, 1995
	▷ **Communication networks**: Grizzle et al, 1982
	▷ **Stochastic games**: Papavassilopoulos, 1982; Chang and Cruz, '83
	▷ **Economics**: Li and Wu, 1991.

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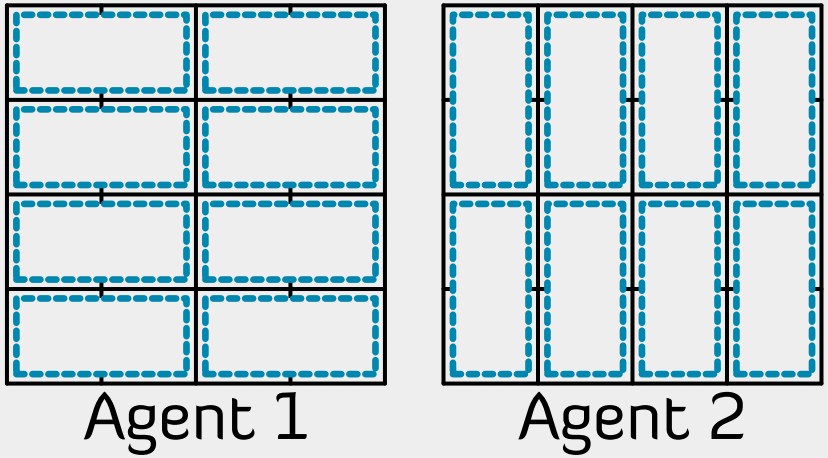
Conceptual Significance	▷ Understanding the **design of networked control systems** ▷ **Bridge** between centralized and decentralized systems ▷ **Insights** for the design of general decentralized systems.

Common information approach for teams [Nayyar, Mahajan, Teneketzis (TAC 2011, 2013)]

Key idea: exploit common knowledge



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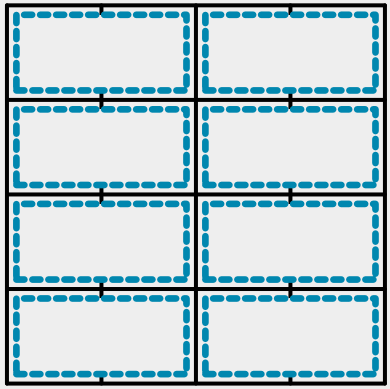
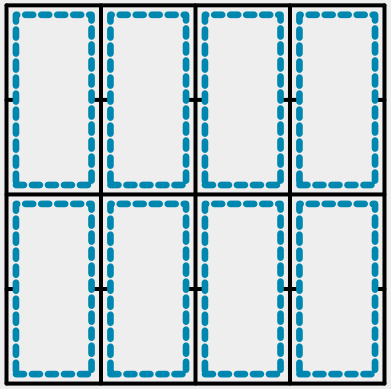
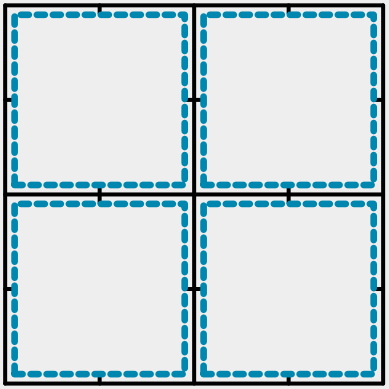


Agent 1



Agent 2



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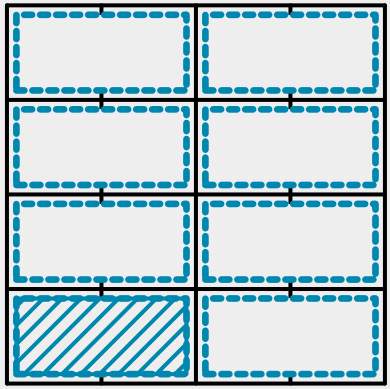

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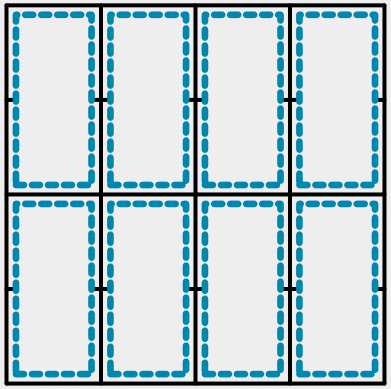
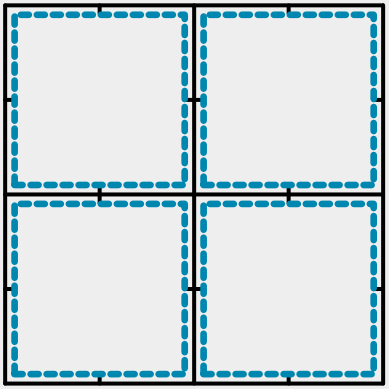


Agent 2



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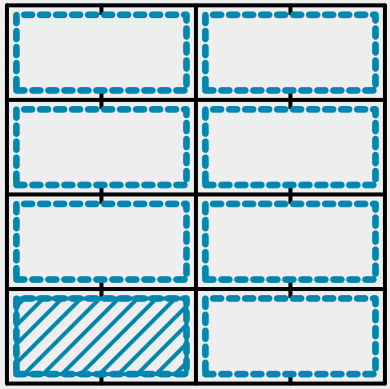
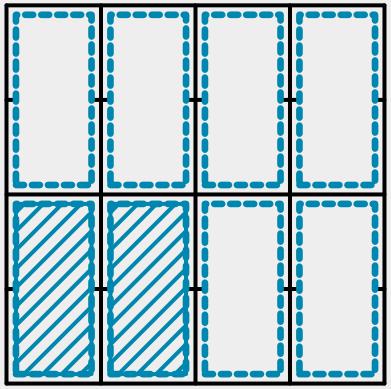
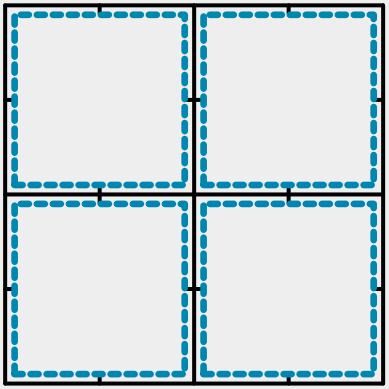


Agent 1



Agent 2



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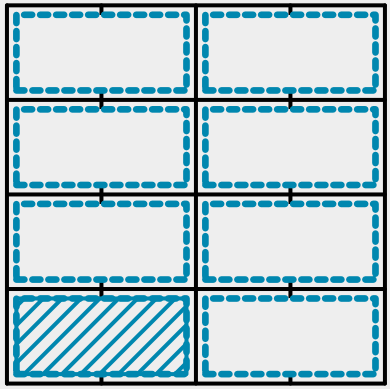

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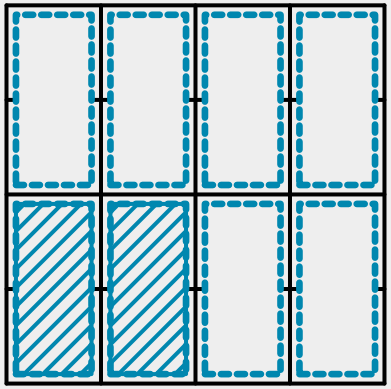
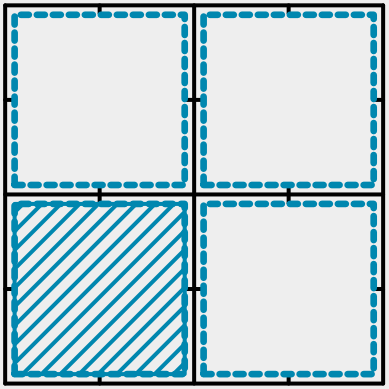


Agent 2



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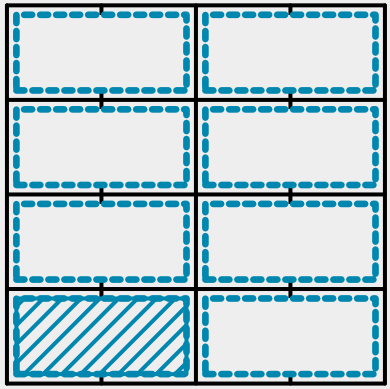

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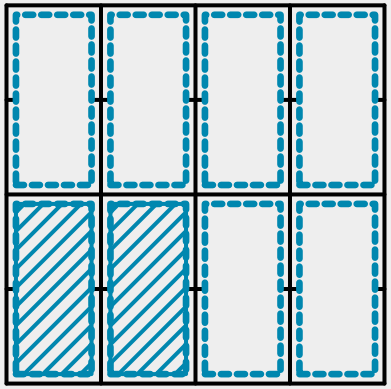
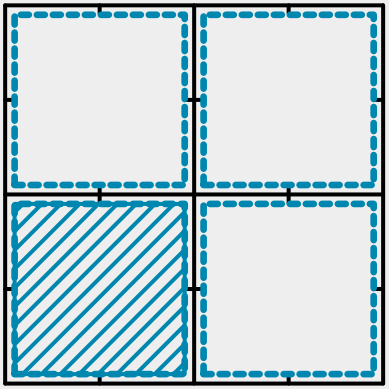


Agent 2



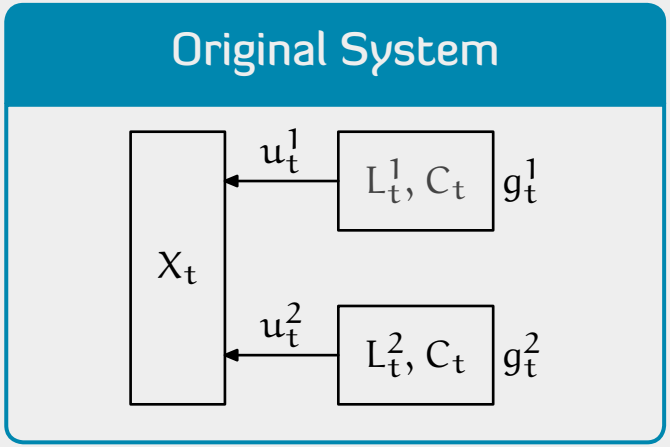
Common knowledge

Split $Y^1 = (L^1, C)$ and $Y^2 = (L^2, C)$.

for all c, $\min_{\gamma^1, \gamma^2} \mathbb{E}[c(X, \gamma^1(L^1), \gamma^2(L^2))) \mid C = c]$

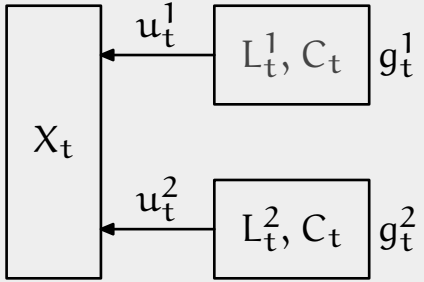
Reduction in complexity: $|\mathcal{U}|^8 \cdot |\mathcal{U}|^8$ to $4|\mathcal{U}|^2 \cdot |\mathcal{U}|^2$

Common-info approach for k-step delay sharing

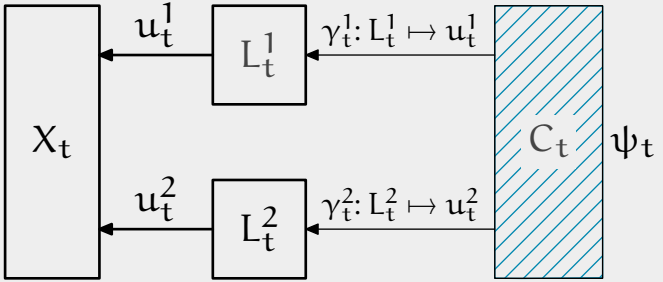


Common-info approach for k-step delay sharing

Original System



Virtual Coordinated System



Common-info approach for k-step delay sharing

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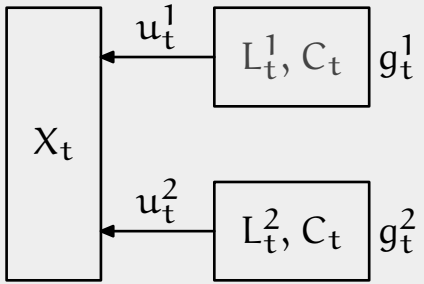

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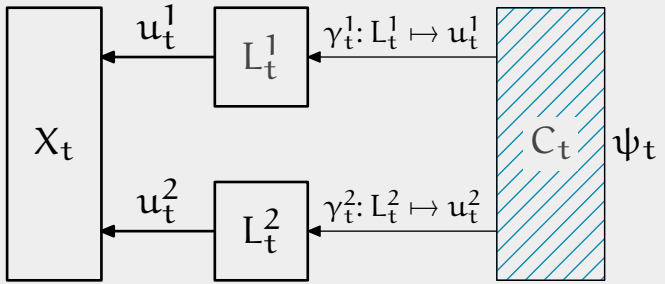

Information split

- Common information: $C_t = I_t^1 \cap I_t^2 = \{Y_{1:t-k}, U_{1:t-k}\}$
- Local information: $L_t^i = I_t^i \setminus C_t = \{Y_{t-k+1:t}^i, u_{t-k+1:t-1}^i\}$.
- Prescription: $\gamma_t^i : L_t^i \mapsto u_t^i$.

Common-info approach for k-step delay sharing

Main Result

- The virtual coordinator is a single agent stochastic ctrl problem.
- **Information state**: for C_t: $b_t = \mathbb{P}(X_t, L_t^1, L_t^2 \mid C_t, \gamma_{1:t-1}^1, \gamma_{1:t-1}^2)$.
- **Dynamic program**: $V_{T+1}(b) = 0$ and

$$V_t(b_t) = \min_{\gamma_t^1, \gamma_t^2} \{\mathbb{E}[c_t(X_t, u_t^1, u_t^2) + V_{t+1}(B_+) \mid b_t, \gamma_t^1, \gamma_t^2]\}.$$

- Each step of the DP is a **functional** optimization problem.

Common-info approach for k-step delay sharing

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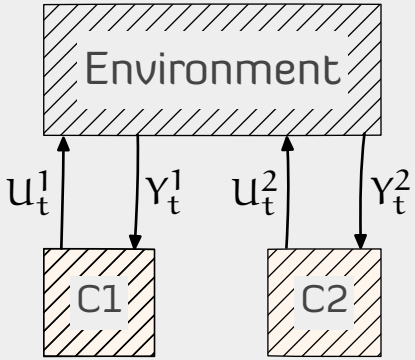
Salient Features

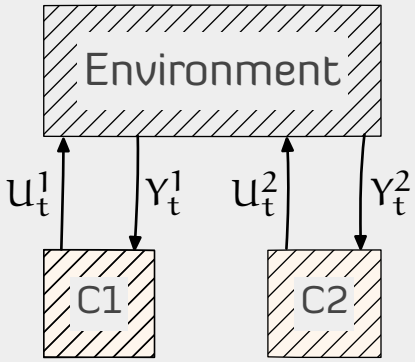
- The virtual coordinator is purely for conceptual clarity as it allows us to view the original problem from the p.o.v. of a "higher authority". The presence of the coordinator is not necessary.
- The common information is known to both controllers and therefore both of them can carry out the calculations to solve the DP on their own.

The general common-info approach



- n controllers with general info structure $\{I_t^i\}_{i=1}^n$.

The general common-info approach



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$$C_t = \bigcap_{s \geqslant t} \bigcap_{i=1}^{n} I_s^i.$$
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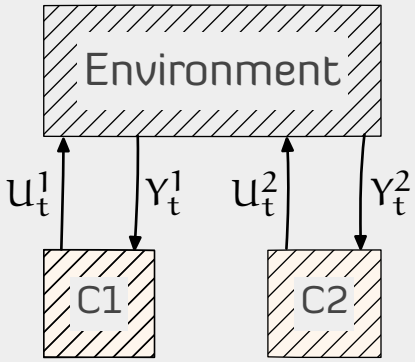
The general common-info approach



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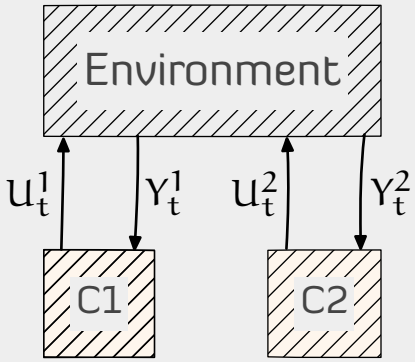
Partial history sharing

- $|L_t^i|$ is uniformly bounded.
- $\mathbb{P}^{\psi}(C_{t+1} \setminus C_t \mid C_t, \gamma_t^1, \gamma_t^2)$ doesn't depend on ψ.

The general common-info approach



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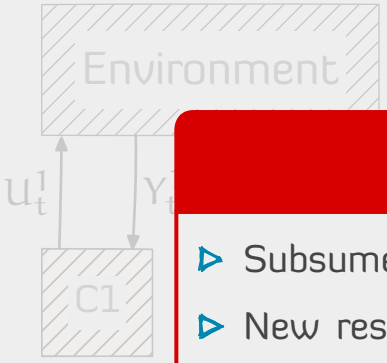
The general common-info approach



- n controllers with general info structure $\{I_t^i\}_{i=1}^n$.

Implications and impact

- Subsumes many existing results (. . .)
- New results on sufficient statistics and DP for specific models (control sharing, mean-field sharing, NCS, and others)
- Common-information based refinements of Nash equilibrium in dynamic games with asymmetric information

C_t.

doesn't depend on ψ.

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Some examples

Control sharing information structure

Dynamics $X^i_{t+1} = f^i(X^i_t, \overrightarrow{U}_t, W^i_t)$

Info structure $I^i_t = \{X^i_{1:t}, U_{1:t}\}$

Sandell and Athans, "Solution of some non-classical LQG decision problems," TAC 1974.

Mahajan, "Optimal decentralized control of coupled subsystems with control sharing," TAC 2013.

Control sharing information structure

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Step 1: Using person-by-person approach

- Show that: $X^1_{1:t} \perp X^2_{1:t} \perp \cdots \perp X^n_{1:t} \mid U_{1:t}$
- Implies no loss of optimality in shedding $X^i_{1:t-1}$ at agent i.

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Step 2: Use common information approach

- Common-info based belief simplies due to the conditional independence (see step 1)

  Suff statistic for $U_{1:t} = (\mathbb{P}(X^1_t \mid U_{1:t}), \dots, \mathbb{P}(X^n_t \mid U_{1:t}))$

Sandell and Athans, "Solution of some non-classical LQG decision problems," TAC 1974.

Mahajan, "Optimal decentralized control of coupled subsystems with control sharing," TAC 2013.

Mean-field teams

Dynamics $X^i_{t+1} = f^i(X^i_t, U^i_t, Z_t, W^i_t)$

Info structure $I^i_t = \{X^i_t, Z_{1:t}\}$

Mean-field $Z_t = \frac{1}{n} \sum_{i=1}^{n} \delta_{X^i_t}$

Arabneydi and Mahajan, "Team optimal control of coupled subsystems with mean field sharing," CDC 2013.

Mean-field teams

Dynamics	$X^i_{t+1} = f^i(X^i_t, U^i_t, Z_t, W^i_t)$

Info structure	$I^i_t = \{X^i_t, Z_{1:t}\}$

Mean-field	$Z_t = \frac{1}{n} \sum_{i=1}^{n} \delta_{X^i_t}$

- Interesting model for applications with large population of a few types of agents
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Use common information approach

- Using ideas from exchangeable Markov chains show that

  Suff statistic for $Z_{1:t} = Z_t$

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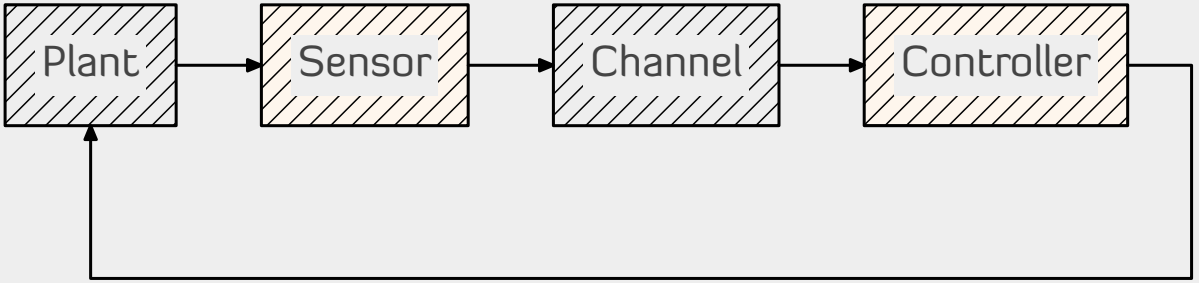
Networked control over wireless channels



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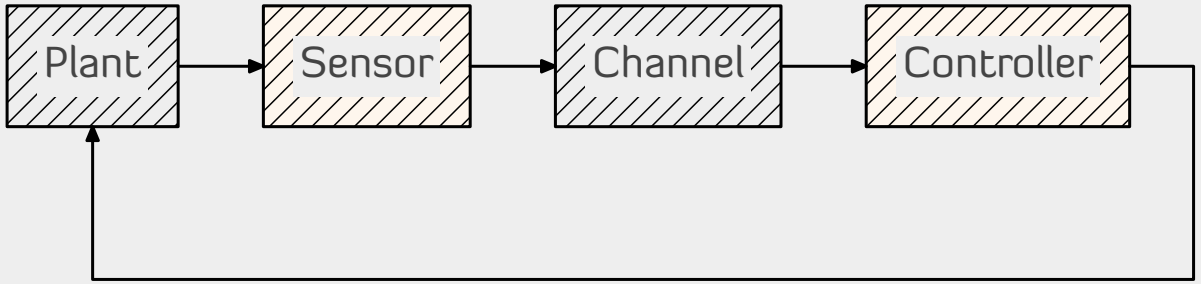

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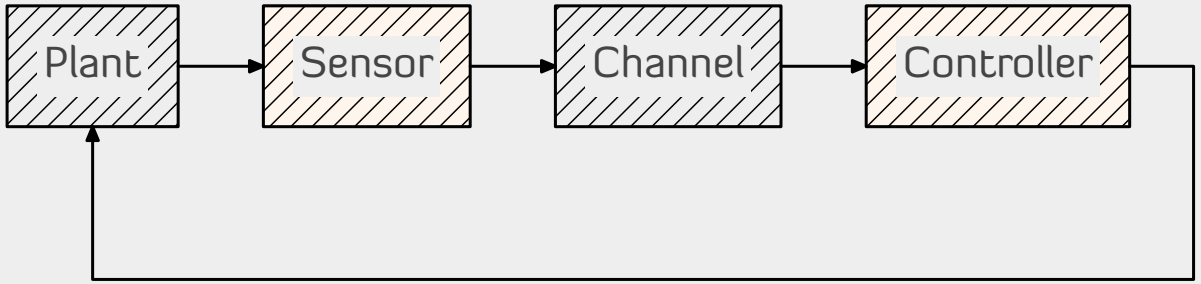

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Wireless Channel

- Sensor sends a packet to the controller using power level $p_t \in \mathcal{P}$.
- Packet is dropped with probability $q(p_t)$, which is decreasing in p_t.
- TCP-like transport layer protocol, so sensor knows when packet is dropped.

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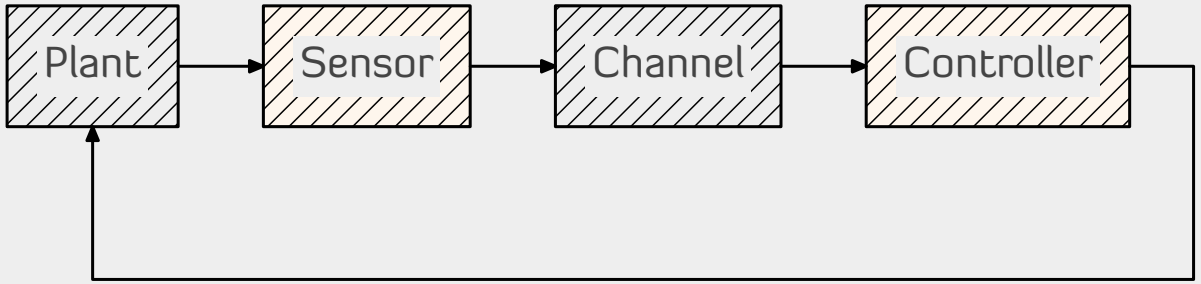

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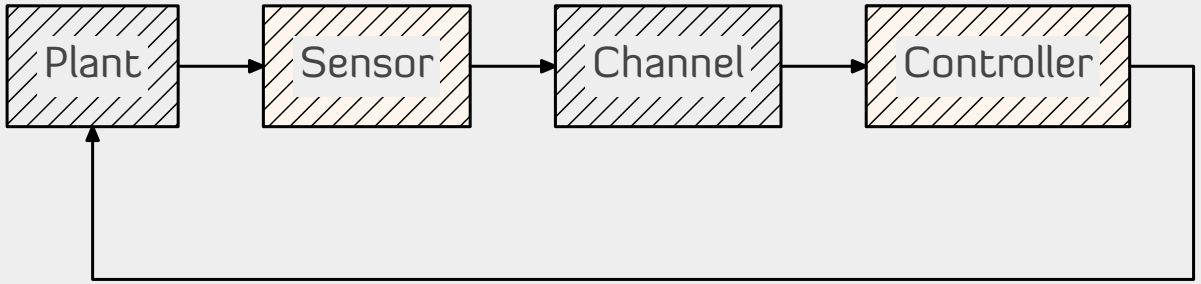

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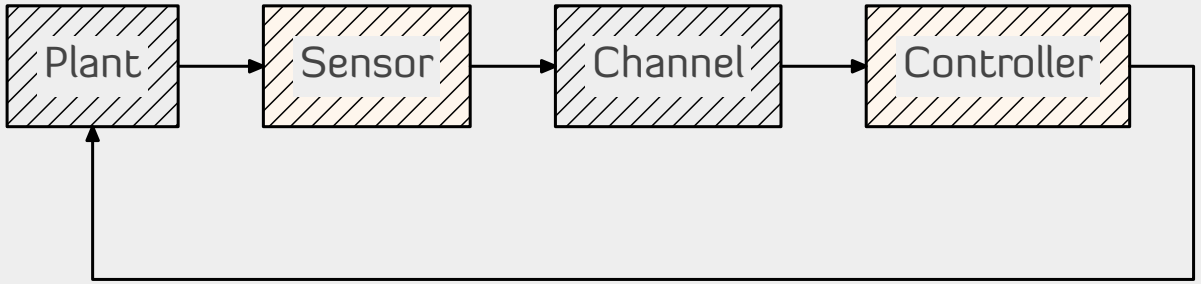

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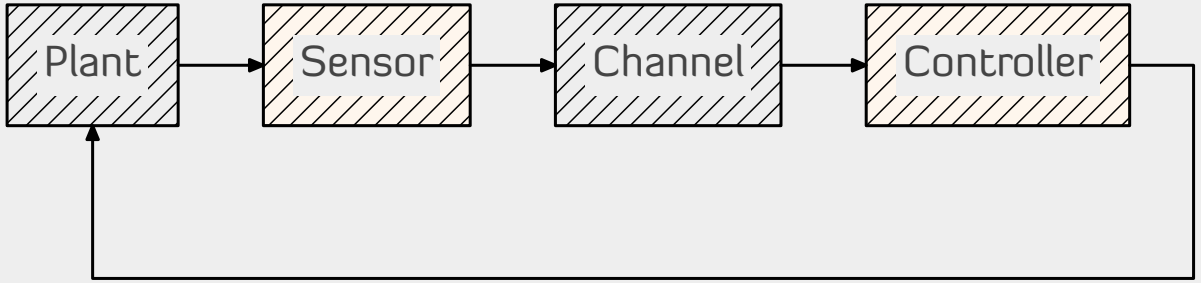

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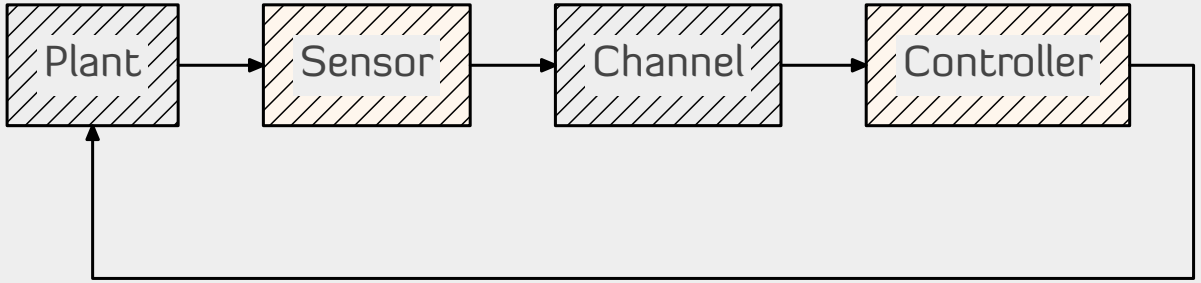

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Objective $J(f, g) = \mathbb{E}\left[\sum_{t=1}^{T} c(x_t, u_t, p_t)\right]$

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Sensor can use power-levels to signal information

- As an example, suppose $\mathcal{P} = \{0, 1\}$, with $q(0) = 1$ and $q(1) = 0$. If the controller doesn't receive a packet, it knows that the state lied in the set where the transmitter chooses $p = 0$.
- Related to real-time communication (a notoriously difficult problem).

Common-info based solutions to NCS

Large literature on these models

- Using the common-info based dynamic program, prove that there are optimal transmission strategies that don't depend on the control strategy.
- Highly non-trivial because the state space of the DP is belief valued; the action space is function valued.
- Implication: there is **no dual effect** and there is separation of estimation and control.
- Note that there is no contradiction. Under an arbitrary policy, control has a dual effect; under the optimal policy it doesn't.

Rabi, Moustakides, and Baras, "Adaptive sampling for linear state estimation," SICON 2012.
Lipsa and Martins, "Remote state estimation with communication costs for first order LTI systems," TAC 2011.
Molin and Hirsche, "Event triggered state estimation: An iterative algorithm and optimality properties," TAC 2017.
Chakravorty and Mahajan, "Fundamental limits of remote estimation of autoregressive Markov processes under communication constraints," TAC 2017.

Common information resolves conceptual difficulties in decentralized control

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But, funding agencies want to hear about learning

Learning in dynamic teams

Implications of common-info approach

- Converts planning in multi-agent teams to a POMDP
- In the learning setting, use your favorite RL algo for POMDP at the coordinator (offline traning) or each agent's local copy of the coordinator (online training)
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But no theory! How do we develop RL theory MARL?

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Next Steps	▷ Credit assignment (among agents) ▷ Agents helping each other to learning