Mean-field teams

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What is team theory?

A brief overview of decision making

Decision making by a single agent

Static optimization

$$\min_{u \in \mathcal{C}} c(u)$$

- Linear programming
- Convex optimization
- Non-convex optimization

Bayesian optimization

$$\min_{g} \mathbb{E}[c(\omega, g(Y(\omega)))]$$

- Stochastic programming
- Stochastic approximation
- Markov Chain Monte Carlo

Dynamic optimization/ Stochastic control

$$\min_{(g_1, \dots, g_T)} \mathbb{E}\left[\sum_{t=1}^{T} c_t(x_t, u_t)\right]$$

where

$$x_{t+1} = f_t(x_t, u_t, W_t),$$
$$y_t = h_t(x_t, N_t),$$
$$u_t = g_t(y_{1:t}, u_{1:t-1})$$

- Dynamic programming
- Pontryagin maximum principle
- Multi-stage stochastic programming

Decision making by multiple agents

Game theory

Each agent has an individual objective. Agents compete to minimize individual costs.

- Static games
- Bayesian games
- Dynamic games with imperfect information

Decision making by multiple agents

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Team theory/ Decentralized stochastic control

All agents have a common objective. Agents cooperate to minimize team costs.

- Static (Bayesian) teams
- Dynamic teams or decentralized stochastic control

Research in team theory started in Economics in mid 50's in the context of organizational behaviour. It has been studied in Systems and Control since the late 60's and in Artificial Intelligence since late 90's.

Decision making by multiple agents

Game theory — Each agent has an individual objective. Agents compete to minimize individual costs.

The **motivation** of team theory/decentralized control is not that it is more powerful than centralized control;

Rather it is **necessary** in systems where centralized information is not available or is not practical.

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Common theme: multi-stage multi-agent
decision making under uncertainty

Conceptual difficulties in dynamic teams

Witsenhausen Counterexample

- A two step dynamical system with two controllers
- Linear dynamics, quadratic cost, and Gaussian dist.
- Non-linear controllers outperform linear control strategies: cannot use Kalman filtering + Riccati eqn

Whittle and Rudge Example

- Infinite horizon system with two symmetric controllers
- Linear dynamics, quadratic cost, and Gaussian dist.
- *A priori* restrict attention to linear controllers
- Best linear controllers **not** representable by recursions of finite order

Complexity analysis

- All random variables are finite valued
- Finite horizon setup
- The problem of finding the best control strategy is in **NEXP**

- Witsenhausen, "A counterexample in stochastic optimum control," SICON 1969.
- Whittle and Rudge, "The optimal linear solution of a symmetric team control problem," App. Prob. 1974.
- Bernstein, et al, "The complexity of decentralized control of Markov decision processes," MOR 2002.

Brief overview of research in team theory

Research theme

- Identify specific information structures that capture key features of applications but, at the same time, are amenable to analysis.
- Develop analytic and computation approaches to optimally design controllers for these information structures.

Solution Approaches

- The person-by-person approach
  Radnar (1962), Ho (1970's)
- The designer's approach
  Witsenhausen (1970's), Mahajan (2008)
- The common-information approach
  Nayyar (2011), Nayyar Mahajan Tenenketzis (2013)

Specific Information Structures

Delayed state sharing, delayed observation sharing, control sharing, periodic sharing, belief sharing, . . .

Mean-field teams

Decentralized multi-agent systems with mean-field coupled dynamics

Motivating examples: Systems with homogeneous agents

Homogeneous agents

- Population of homogeneous agents
- Influence the dynamics of each other through their mean-field (or empirical distribution)
- Equivalent to an interacting particle model
- Arbitrarily coupled cost

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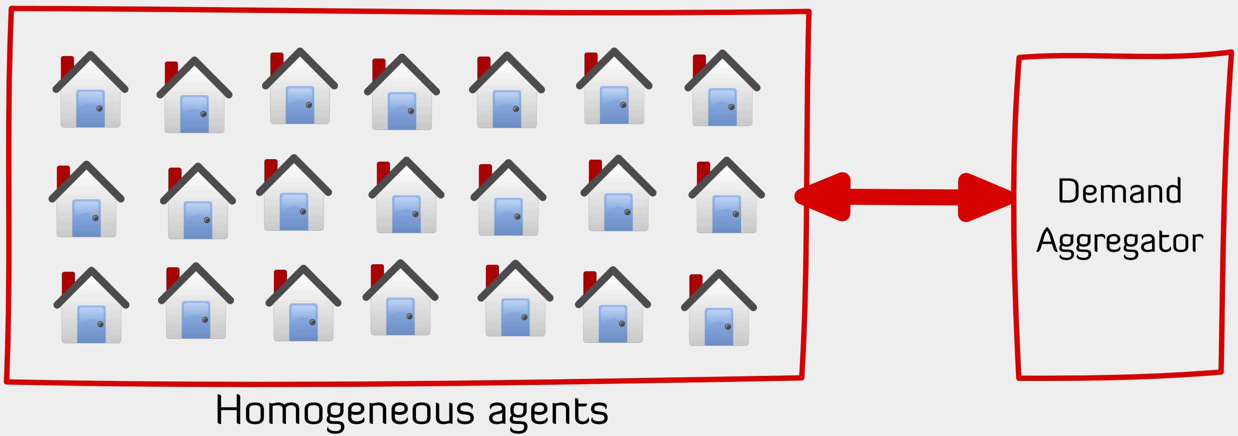

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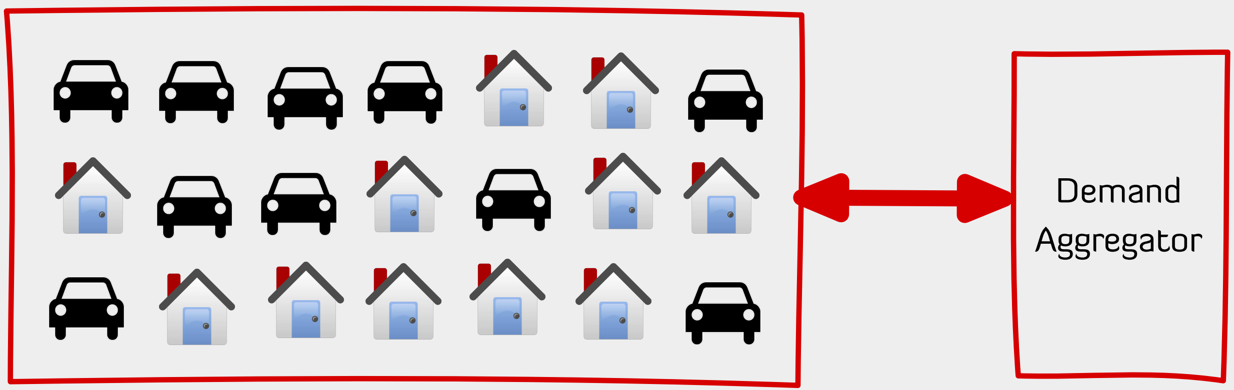

Homogeneous agents with multiple types

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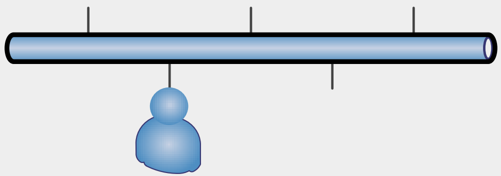

Major and minor homogeneous agents