Cross-layer communication over fading channels with adaptive decision feedback

Borna Sayedana[1] Aditya Mahajan[1] Edmund Yeh[2]

[1]McGill University

[2]Northeastern University

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Motivation

Physical Layer Design Objective

1. Reliable Communication
2. Efficiency in Rate and Power

Network Layer Design Objective

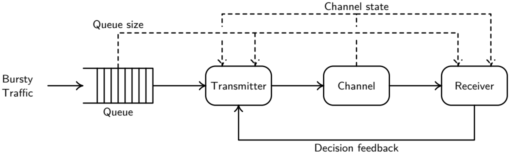
1. Quality of Service(delay, etc.)
2. System Stability

Cross Layer Design Objective

1. Physical and Network Layer consideration Reliable Communication
2. Minimizing both Power and Delay in Reliable Communication

Features of Cross Layer Design in Wireless systems

1. Rate Adaptation in wireless systems:
   - Traffic Load in Network
   - Channel State
2. Optimization Objectives:
   - Block Error Probability
   - End to End Packet Delays
   - Transmission Power



Motivating Question

Effect of Feedback on System Performance

Effect of an ACK/NACK feedback channel

- It can improve decoding error performance
- It can also increase queuing delay due to re-transmission

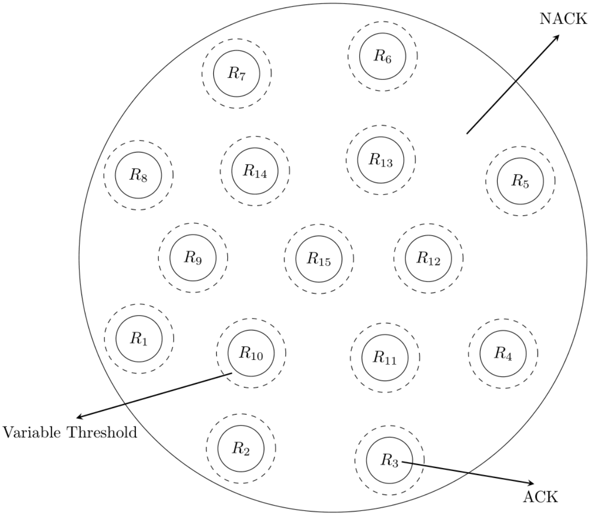
What controls the expected number of retransmissions?

1. Channel State (exogenous process)
2. The decoding threshold at the receiver.

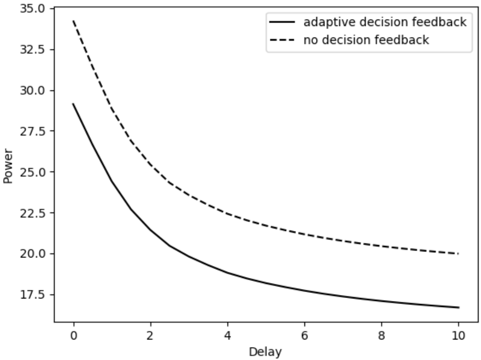
Important Question

- How can we control decoding threshold?

Our main idea: exploit adaptive decision feedback

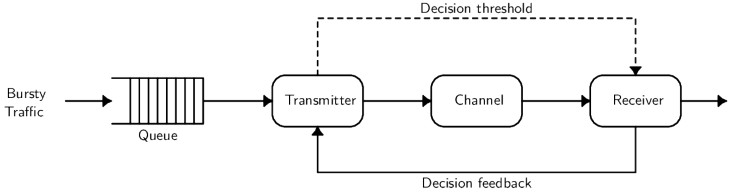


Adaptive Decision feedback implies significantly better power-delay trade-off



- $\mathrm{P}(\alpha) = \min_g \{\text{power of policy (g)} \mid \text{Delay of policy (g)} \leq \alpha\}$

Communication System



System Variables

- Number of new packets: A_k
- Queue length: Q_k
- Channel state: S_k
- Number of transmitter packets : U_k
- Decoding threshold : T_k
- Fading gain : $H(S_k)$

Processes and Queue Dynamic

- A_k is an i.i.d process with decreasing and convex pdf
- S_k is an i.i.d process
- $Q_{k+1} = \begin{cases} Q_k - U_k + A_k, & \text{if} \quad \mathfrak{E}_k = 1 \\ Q_k + A_k, & \text{if} \quad \mathfrak{E}_k = 0 \end{cases}$

Solution Approach

Ensuring Reliable Communication

Performance Metrics

- Queuing delay
- Probability of Error
- Transmission power

Our Solution

- Buffering delay is $d(Q_{k+1} - A_k)$
- We upper bound P_e by ε
- How to find transmission power?

Probability of error and re-transmission

By [Forney 1968], we know error exponent for channels with feedback

$$P_e \leq \exp\Big(-\frac{\rho}{2} M \log\big(1 + \frac{\pi H(s)}{(1+\rho)}\big) + \rho N u + \frac{1}{1+\rho} M t\Big)$$

and

$$P_r = \exp(Mt) P_e$$

Solution Approach

Ensuring Reliable Communication

Finding Power needed to ensure desired probability of error

1. Power Function is $\phi(u,t)h(s)$, where

$$\phi(u,t) = (1+\rho)\left[\exp\left(-\frac{2\log\varepsilon}{\rho M} + 2\frac{N}{M}u - \frac{2}{1+\rho}t\right) - 1\right]$$

2. probability of re-transmission $p(t)$

$$p_r(U_k, T_k, \Pi_k, S_k) = \exp(MT_k)\varepsilon =: p(T_k)$$

Imposing power constraint

- Maximum power constraint: $\phi(u,t)h(s) \leq \Pi_{\max}$

Solution Approach

Assumptions on the model

Communication cost at each time

$$c(q, s, u, t) = \lambda_\pi \phi(u, t) h(s) + \lambda_d \left[p(t) d(q) + (1 - p(t)) d(q - u) \right]$$

Assumptions on the Model

- $d(\cdot)$ increasing convex
- $\phi(\cdot, \cdot)$ increasing in u_k
- $\phi(\cdot, \cdot)$ decreasing in t_k
- $h(\cdot)$ increasing, convex
- $p(t)$ increasing in t_k

Solution Approach

MDP Formulation

Prolem(1)

State Space:

(Q_k, S_k)

Action Space:

(U_k, T_k)

communication rule:

$(U_k, T_k) = g_k(Q_k, S_k)$

communication policy :

$g = (g_1, g_2, \ldots, g_N)$

Find policy g which minimizes:

$$J(g) = \mathbb{E}\Big[\sum_{k=1}^{K} c(Q_k, S_k, U_k, T_k)\Big]$$

Question: What are some qualitative properties of policy g?

- Power-delay trade-off only with rate adaption. [Berry and Gallager 2002],[Uysal-Biyikoglu,Prabhakar, and El Gamal 2002],[Zafer and Modiano 2009],[Bettesh and Shamai 2006],[Goyal, Kumar, and Sharma 2008],[Fu, Modiano, and Tsitsiklis 2006]
    - Dynamic Programming Decomposition/structure of optimal policy in 1 dimension.
- [Cao and Yeh 2008]: Investigated *Adaptive Decision Feedback*, and 2 dimensional structural Properties
    - Only for binary choice for rate and threshold

Key Contribution of This paper

Investigating qualitative structure of multi-dimensional optimal policies

Question

Question: Does the optimal policy have *monotonicity Properties*?

Why this question is important?

1. Understanding the physical system
2. Can be exploited in online-learning algorithms

Why it's hard to answer?

1. Both state and action spaces are multi-dimensional
2. Most of previous results in cross layer design are for 1 dimensional systems.

Solution and Main Results

Dynamic Programming Decomposition

Defining value functions and action-value functions

Proposition 1

For any q and s

$$V_{K+1}(q, s) = 0$$

and for $k \in \{K, K-1, \dots, 1\}$,

$$W_k(q, s, u, t) = c_k(q, s, u, t) + \sum_{(s,a)} P_S(s) P_A(a) \big[(1 - p(t)) V_{k+1}(q - u + a, s) + p(t) V_{k+1}(q + a, s) \big]$$

- Value function : $V_k(q, s) = \min_{u,t} W_k(q, s, u, t)$.
- Then: $g_k(q, s) \to \arg\min$ at stage $k \Rightarrow g = (g_1, \dots, g_K)$ is optimal for Problem.

Theorem 1

For any time slot k, the value function $V_k(q, s)$ satisfies the following properties:

1. For any $s \in \mathcal{S}$, $V_k(q, s)$ is weakly increasing in q.
2. For any $q \in \mathcal{Q}$, $V_k(q, s)$ is weakly increasing in s.

- $1 \Rightarrow \uparrow$ number of packets $\Rightarrow \uparrow$ cost to go.
- $2 \Rightarrow$ channel deteriorates $\Rightarrow \uparrow$ cost to go.

Theorem 2 (Informal Representation)

- $\uparrow$ number of packets $\Rightarrow$ (Rate $\uparrow$) Or (Threshold $\downarrow$).
- channel deteriorates $\Rightarrow$ (Rate $\downarrow$) Or (Threshold $\uparrow$).

Theorem 3

Suppose the cost function satisfies the following property:

(P) for any $(q, s) \in \mathcal{Q} \times \mathcal{S}$, and any $u_1, u_2 \in \mathcal{U}$ and $t_1, t_2 \in \mathcal{T}$ such that $u_1 \leq u_2$ and $t_1 \leq t_2$, we have

$$c(q, s, u_1, t_2) + c(q, s, u_2, t_1) \leq c(q, s, u_2, t_2) + c(q, s, u_1, t_1).$$

Then, the "or" in Theorem 2 can be replaced by "and".

- Verify property (P) $\rightarrow$ system parameters $(\lambda_\pi, \lambda_d, \Pi_{\max}, M)$
- 1 $\Rightarrow \uparrow$ number of packets $\Rightarrow$ (Rate $\uparrow$) And (Threshold $\downarrow$).
- 2 $\Rightarrow$ channel deteriorates $\Rightarrow$ (Rate $\downarrow$) And (Threshold $\uparrow$).

Key Idea of the Proof

Partial Order on Action Space

Definition

$\preceq_{\mathcal{A}}$: a partial order on $\mathcal{U} \times \mathcal{T}$

- we say $(u_1, t_1) \preceq_{\mathcal{A}} (u_2, t_2)$ if $u_1 \leq u_2$ and $t_1 \geq t_2$

1. Fixed $s \in \mathcal{S}$, $W_k(q, s, u, t)$ has decreasing differences on $\mathcal{Q} \times (\mathcal{U} \times \mathcal{T})$.
2. Fixed $q \in \mathcal{Q}$, $W_k(q, s, u, t)$ has increasing differences on $\mathcal{S} \times (\mathcal{U} \times \mathcal{T})$.
3. Given the sub-modularity on $(\mathcal{U} \times \mathcal{T}) \Rightarrow$ Theorem 3.

Conclusion and Future Directions

Conclusion

1. We investigate the impact of adaptive decision feedback on power-delay curve.
2. We establish *monotonicity* property of the optimal policy and value function.

Future Directions

1. Extend these results to *infinite horizon setup*
2. Develop online learning algorithms to utilize such properties.

Thank you!

Questions

borna.sayedana@mail.mcgill.ca