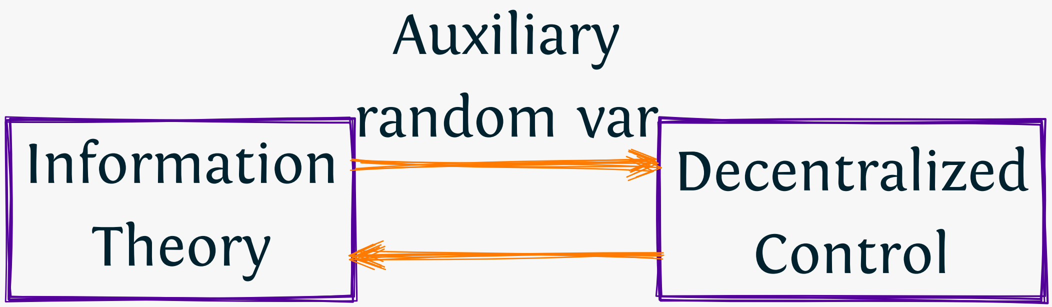

Aditya Mahajan
Yale

Acknowledgements: Demos Teneketzis (UMich), Ashutosh Nayyar (UMich), Sekhar Tatikonda (Yale), Ramji Venkataramanan (Stanford)

April 13, 2010, UC Berkeley

Stochastic control ⇒ Information theory

Converses in info theory

- Capacity of FSM channels
  Goldsmith-Varaiya-96
- Capacity of FSM channels with f/b
  Vishwanathan-99, Chen-Berger-05,
  Yang-Kavcic-Tatikonda-05,
  Tatikonda-Mitter-09,
  Kavcic-Mandic-Huang-Ma-09
- Error exponents of FSM channels with f/b
  Como-Yüksel-Tatikonda-09
- FS-MAC with partial state info
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Coding schemes

- Optimal coding schemes for channels with feedback
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Real-time communication

- Point-to-point communication
  Witsenhausen-79, Walrand-Varaiya-82, Teneketzis-06, Mahajan-Teneketzis-08,09
- Multi-terminal communication
  Nayyar-Teneketzis-09, Mahajan-09

Stochastic control $\Rightarrow$ Information theory

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This talk . . .

- Combination of all three

This talk . . .

Setup

- Discrete memoryless multiple access channel (MAC) with feedback
- Inner bounds on capacity region (achievable schemes) using block Markov superposition coding

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Setup

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- Inner bounds on capacity region (achievable schemes) using block Markov superposition coding

Approach

- Setup block Markov superposition codes as decentralized control problem with delayed sharing of information
- A systematic study of auxiliary random variables

This talk . . .

Setup

- Discrete memoryless multiple access channel (MAC) with feedback
- Inner bounds on capacity region (achievable schemes) using block Markov superposition coding

Approach

- Setup block Markov superposition codes as decentralized control problem with delayed sharing of information
- A systematic study of auxiliary random variables

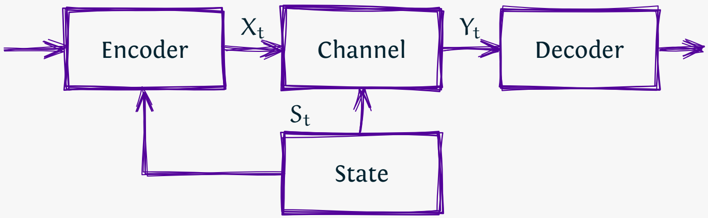
Main point

- Block Markov superposition coding schemes give rise to a specific information structure
- Stochastic control can provide insights into the design of coding schemes for a specific information structures

Are **auxiliary random variables** in info theory related to **information states** in stochastic control?

What are auxiliary random variables?

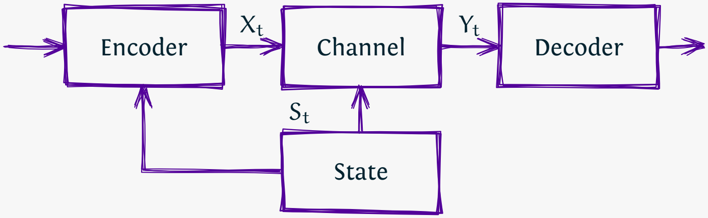
Channel coding with
causal side information



[Shannon-58]

What are auxiliary random variables?

Channel coding with causal side information



[Shannon-58]

Code functions

$$C = \max_{\substack{P_F \\ F:\mathcal{S}\to\mathcal{X}}} I(F \wedge Y)$$

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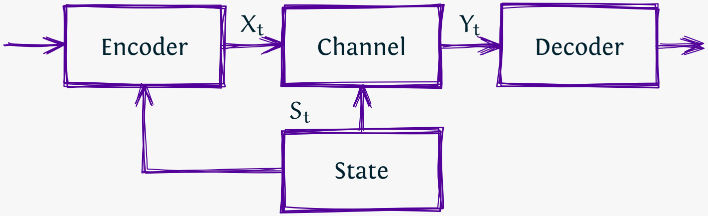

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$$C = \max_{\substack{P_F \\ F:\mathcal{S}\to\mathcal{X}}} I(F \wedge Y)$$

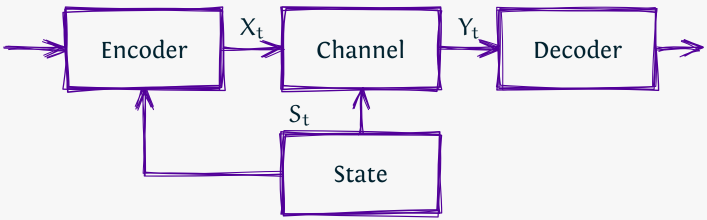
Auxiliary random variables

$$C = \max_{P_U, f:\mathcal{U}\times\mathcal{S}\to\mathcal{X}} I(U \wedge Y)$$

where $|\mathcal{U}| \leqslant |\mathcal{Y}|$

What are auxiliary random variables?

Channel coding with causal side information



[Shannon-58]

We can think of the auxiliary random variable indexing the set of all functions from $\mathcal{S}$ to $\mathcal{X}$.

[Keshet, Steinberg, Merhav, 2007]

Code functions

$$C = \max_{\substack{P_F \\ F:\mathcal{S}\to\mathcal{X}}} I(F \wedge Y)$$

Auxiliary random variables

$$C = \max_{P_U, f:\mathcal{U}\times\mathcal{S}\to\mathcal{X}} I(U \wedge Y)$$

where $|\mathcal{U}| \leqslant |\mathcal{Y}|$

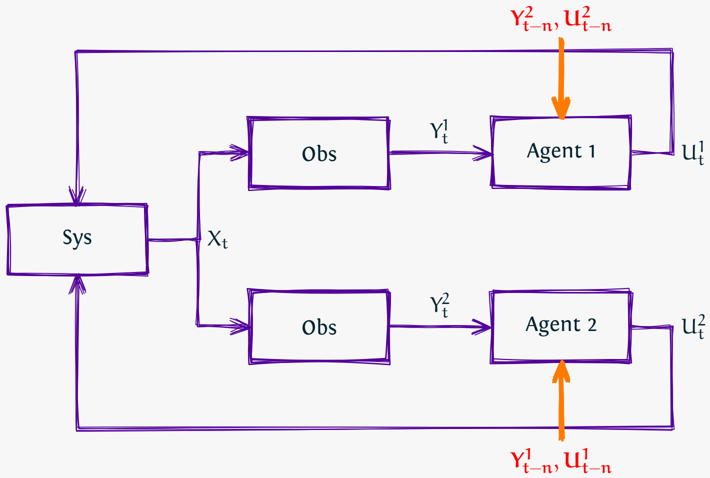
What are information states?

The (information) state should be
a **summary** ('compression') **of some data** (the 'past')
known to someone (an observer or a controller)
and **sufficient for some purposes**
(input-output map, optimization, dynamic programming).

[Witsenhausen-1976]

Understanding the relation

Delayed sharing information structure



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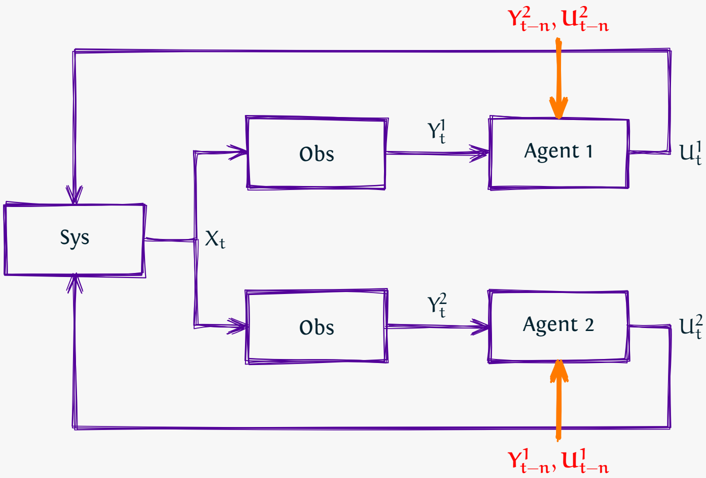
Delayed sharing information structure



History

- A bridge between classical ($n = 0$) and non-classical ($n = \infty$) info structures
- **Witsenhausen, 1971** proposed the n-DSIS and **asserted** a structure of optimal control policies
- **Varaiya and Walrand, 1979** proved that Witsenhausen's assertion is **true for $n = 1$** but **false of $n > 1$**

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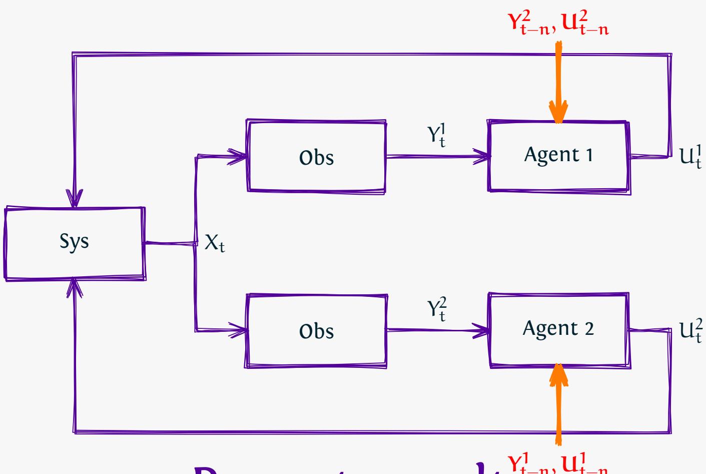

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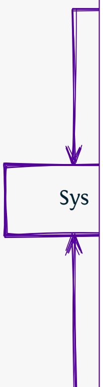
Recent results

Information state: [Nayyar-Mahajan-Teneketzis-10]

$\Pr(X_{t-n} \mid$ common info$)$ + n-partially evaluated past control functions

Understanding the relation

Delayed sharing information structure



History

- A bridge between classical ($n = 0$) and

Partially evaluated function

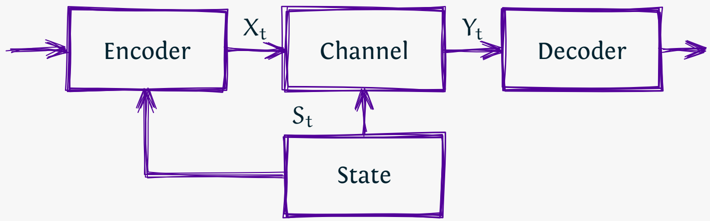
Function : $f : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$

Partial evaluation at x: $f(x, \cdot) : \mathcal{Y} \to \mathbb{R}$

$\mathbb{P}(X_{t-n} \mid$ common info$)$ + n partially evaluated past control functions

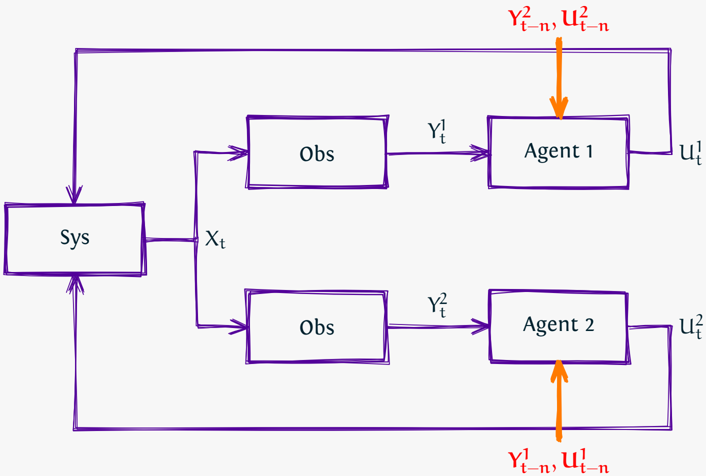
Understanding the relation

Channel coding with causal side information



Auxiliary random variable
is related to **functions**

Delayed sharing information structure

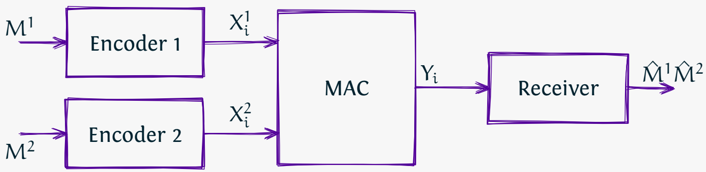


Information states are
related to **partial functions**

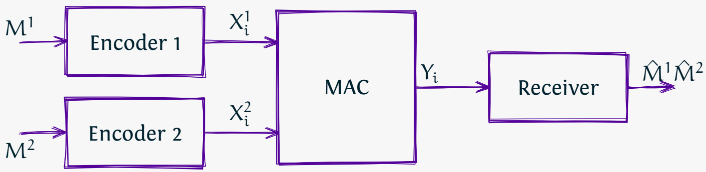
Outline

1. Overview of MAC with feedback
   - Cover-Leung scheme
   - Bross-Lapidoth scheme
   - Venkataramanan-Pradhan scheme
2. Formulation as a decentralized control problem with delayed sharing
   - delay = 1
   - delay = 2
3. Conclusion

Multiple access channel

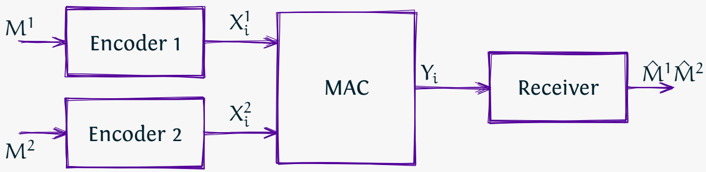


Multiple access channel



- Encoder
  $X^1_{[1:n]} = f^1(M^1), \quad X^2_{[1:n]} = f^2(M^2)$
- Channel
  $\mathbb{P}(Y_i \mid X^1_{[1:i]} X^2_{[1:i]}) = \mathbb{P}(Y_i \mid X^1_i X^2_i)$
- Decoder: $\hat{M}^1\hat{M}^2 = d(Y_{[1:n]})$
- Error: $\mathbb{P}(\hat{M}^1\hat{M}^2 \neq M^1M^2)$

Multiple access channel



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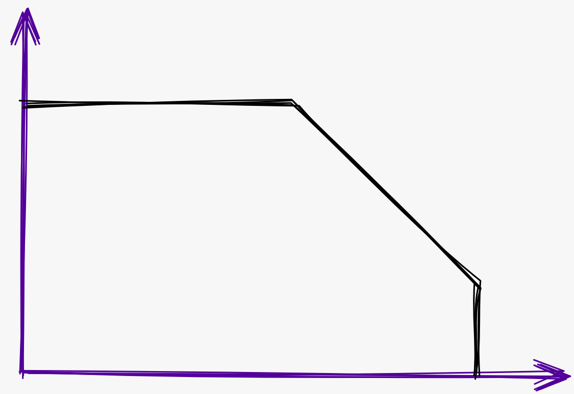
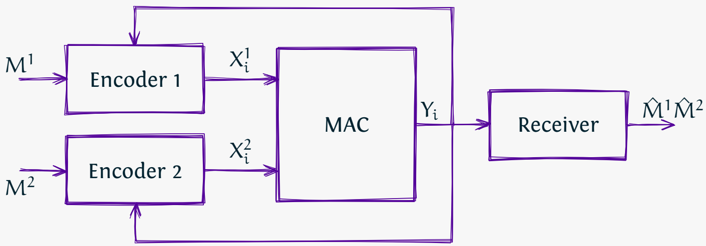
Capacity

$$C = \bigcup_{P_{X^1}P_{X^2}} \left\{ (R_1, R_2) : \begin{array}{r} R_1 \leqslant I(X^1 \wedge Y \mid X^2) \\ R_2 \leqslant I(X^2 \wedge Y \mid X^1) \\ R_1 + R_2 \leqslant I(X^1X^2 \wedge Y) \end{array} \right\}$$



Alshwede-71, Liao-72

MAC with feedback



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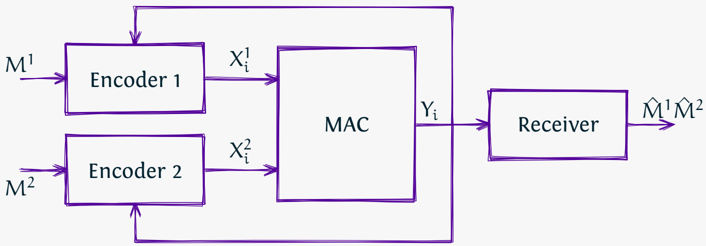

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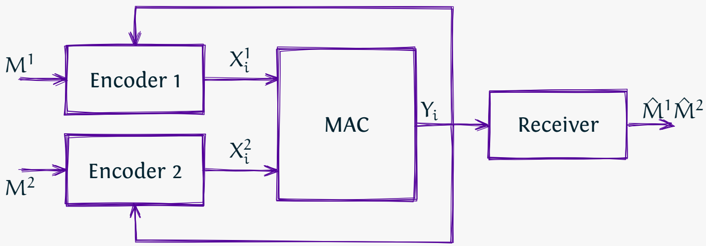

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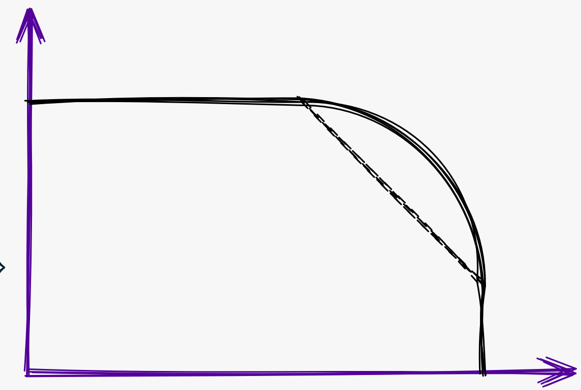
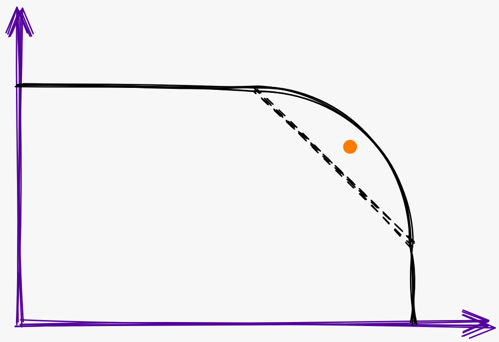


[Kramer-03]

Achievability scheme

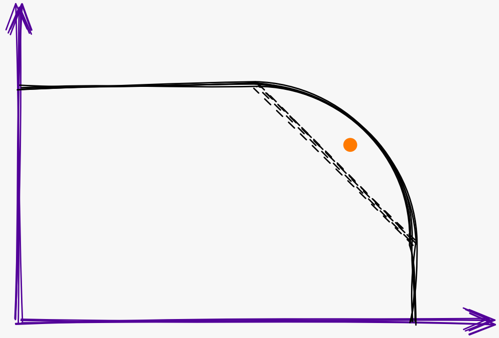


Achievability scheme

Main idea

Suppose we want to transmit at a rate point outside the no-feedback capacity region. Communicate in two phases

[Gaarder-Wolf-75]



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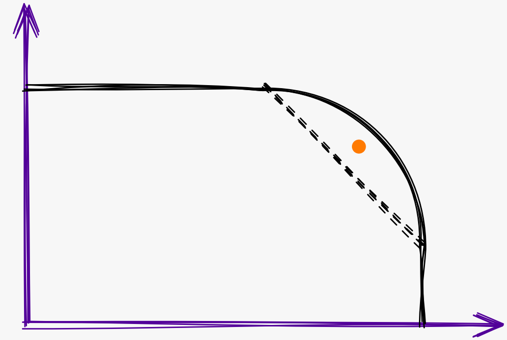

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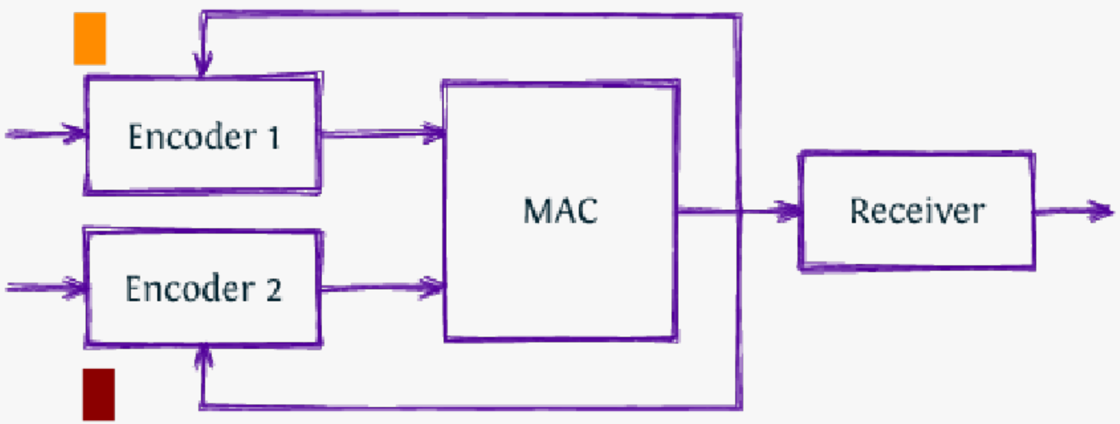

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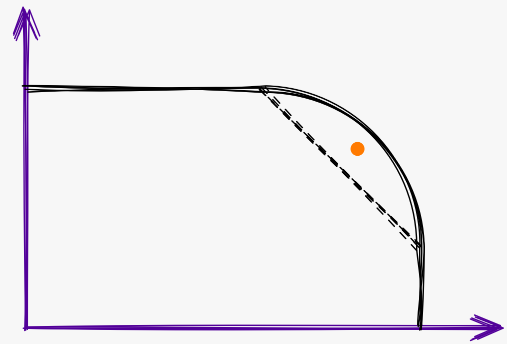

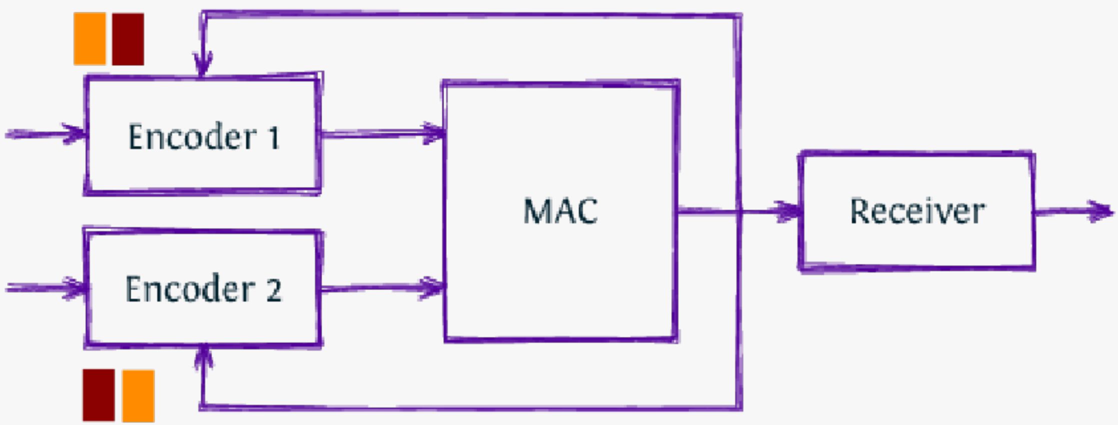

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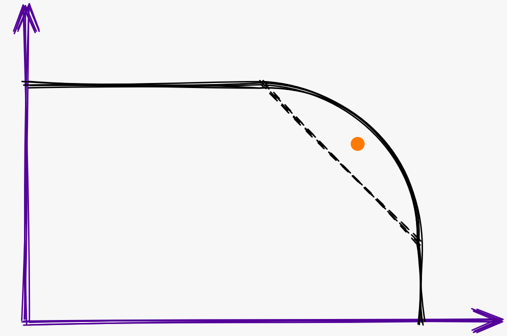

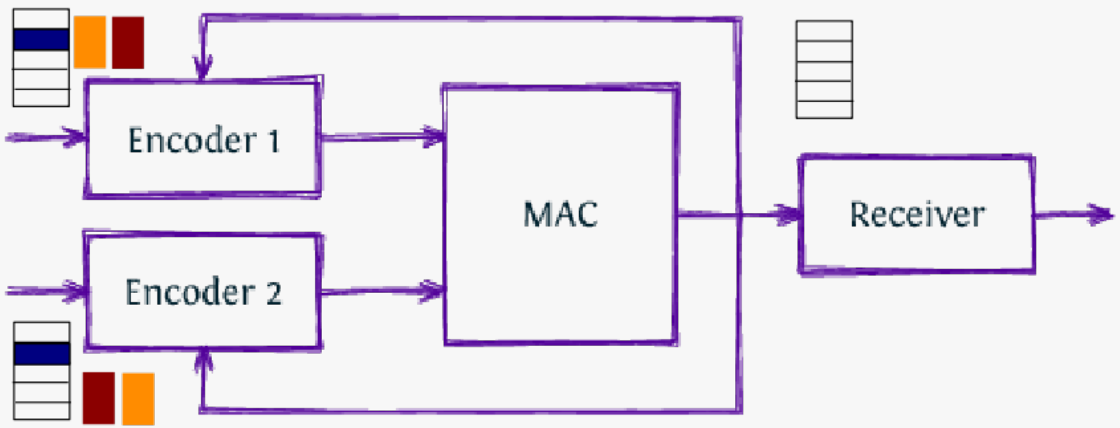

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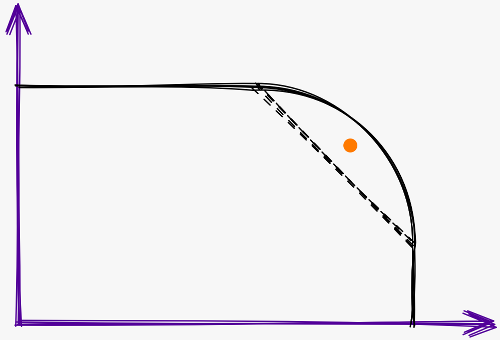
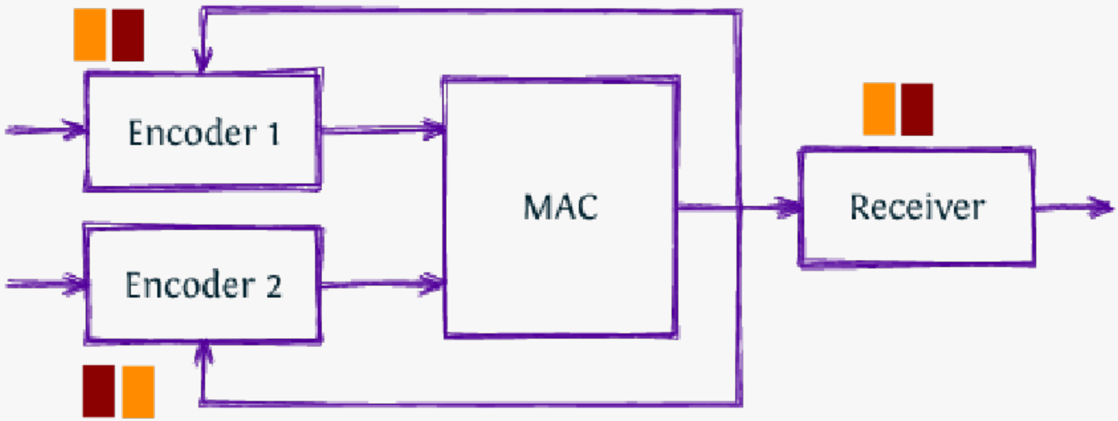
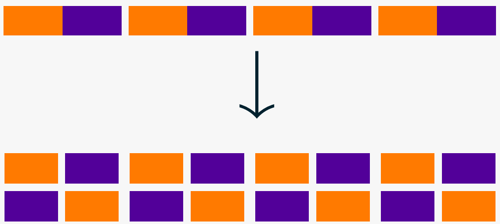




Block Markov superposition coding



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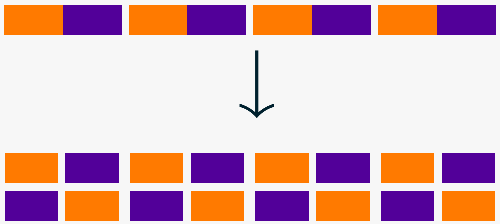
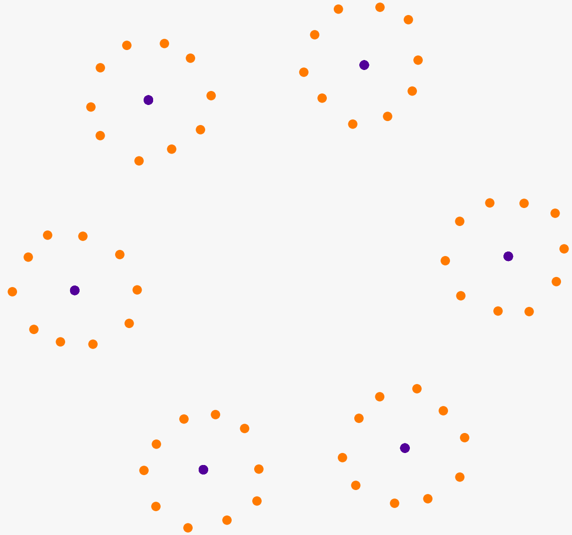


Superposition coding



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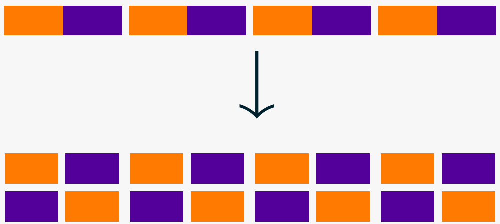

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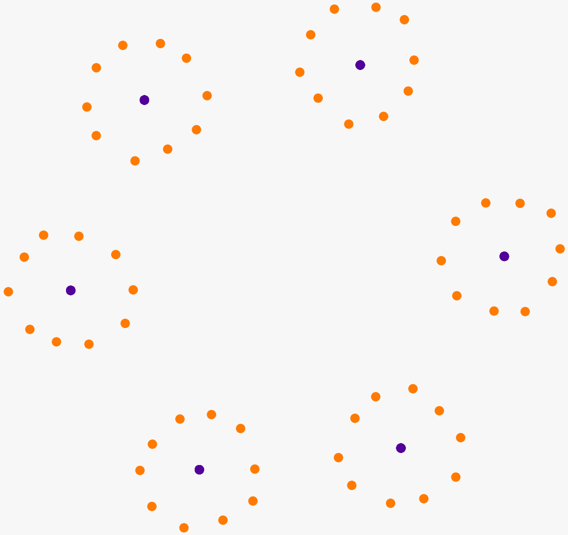
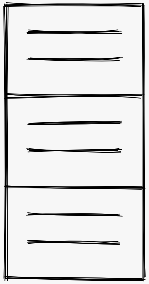


Binning



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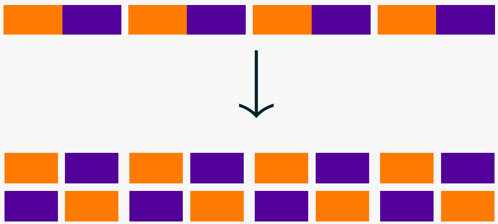

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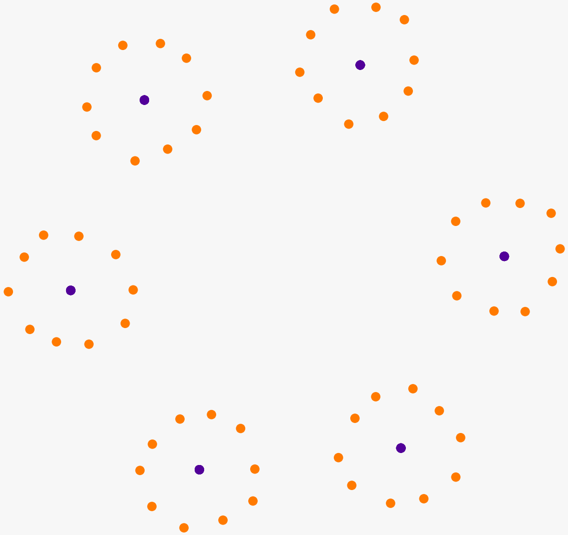
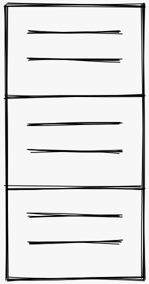


Cover-Leung region

$$R_{CL} = \bigcup_{P_U P_{X^1|U} P_{X^2|U}} \left\{ (R_1, R_2) : \begin{aligned} R_1 &\leqslant I(X^1 \wedge Y \mid UX^2) \\ R_2 &\leqslant I(X^2 \wedge Y \mid UX^1) \\ R_1 + R_2 &\leqslant I(X^1X^2 \wedge Y) \end{aligned} \right\}$$

[Cover-Leung-81]

Binning



Converse

Cover-Leung region

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[Cover-Leung-81]

Converse

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[Cover-Leung-81]

CL region is tight for . . .

- $H(X^1|YX^2) = 0$ [Willems-82]
- Partial feedback [Willems-83]
- Binary erasure MAC [Willems-84]

Converse

Cover-Leung region

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[Cover-Leung-81]

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- $H(X^1|YX^2) = 0$ [Willems-82]
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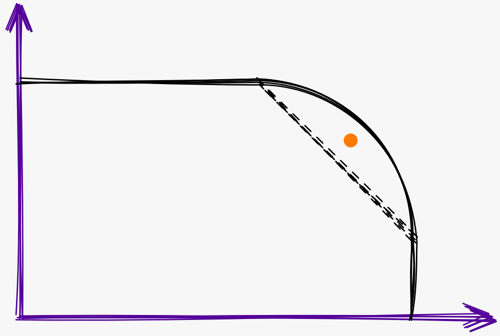
- Binary erasure MAC [Willems-84]

CL region is **not** tight for . . .

- Gaussian MAC [Ozarow-84]
- Poisson MAC [Bross-Lapidoth-05]
- Others . . . [Kramer-02, Bross-Lapidoth-05, Venkataramanan-Pradhan-09]

Adding two way communication

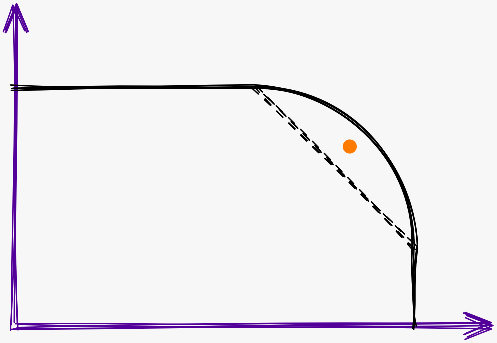


Adding two way communication

Main idea

Each block now consists of three phases

[Bross-Lapidoth-05]



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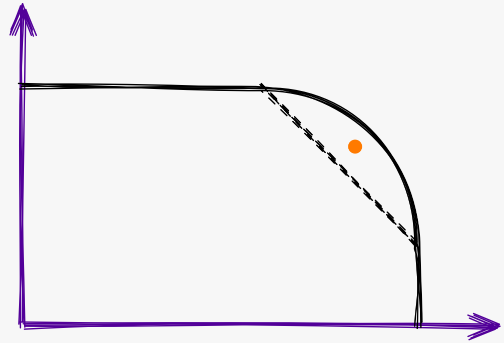

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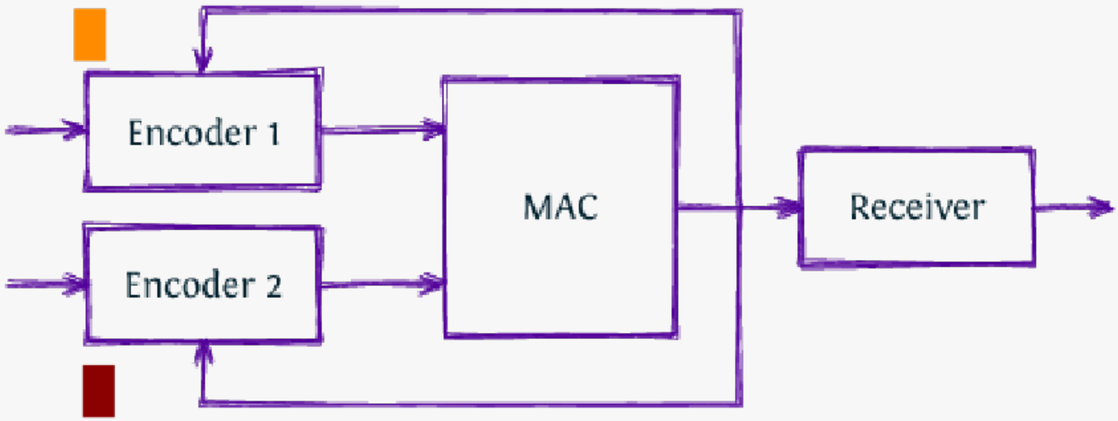

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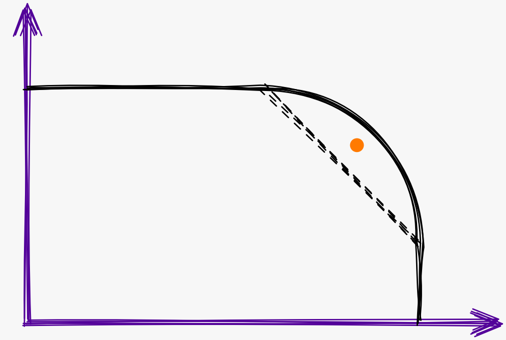

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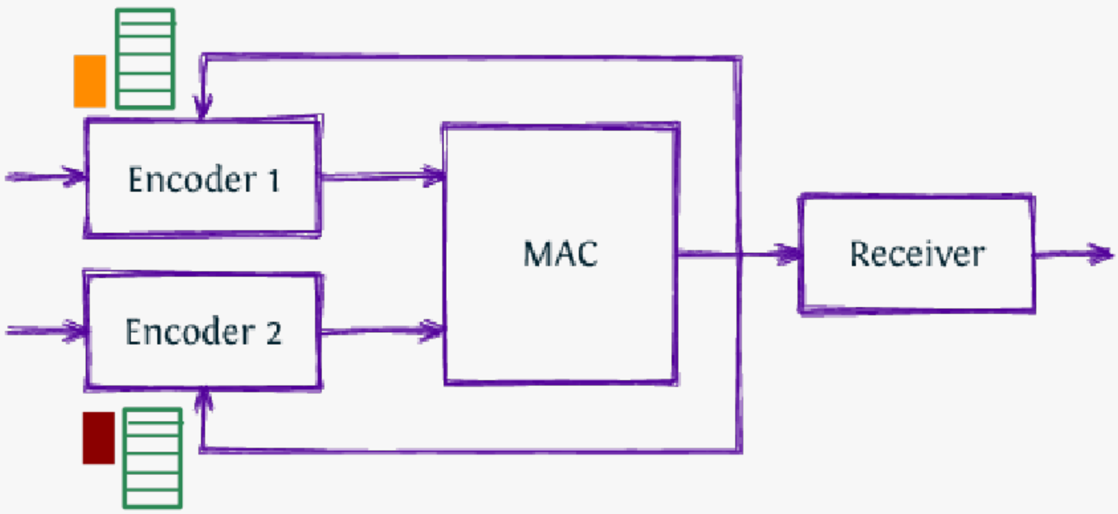

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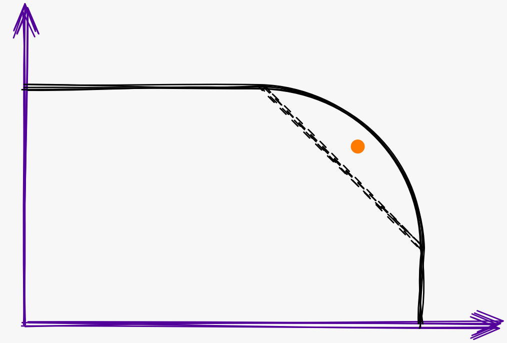

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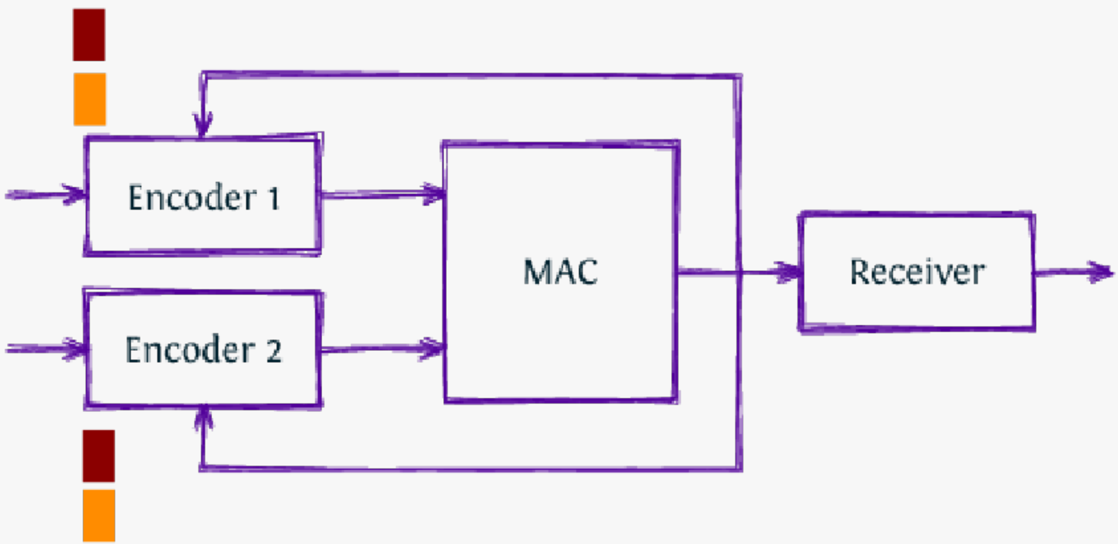

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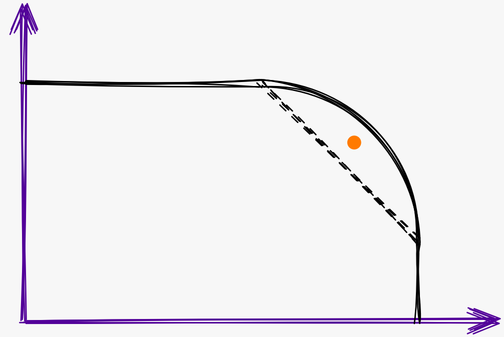

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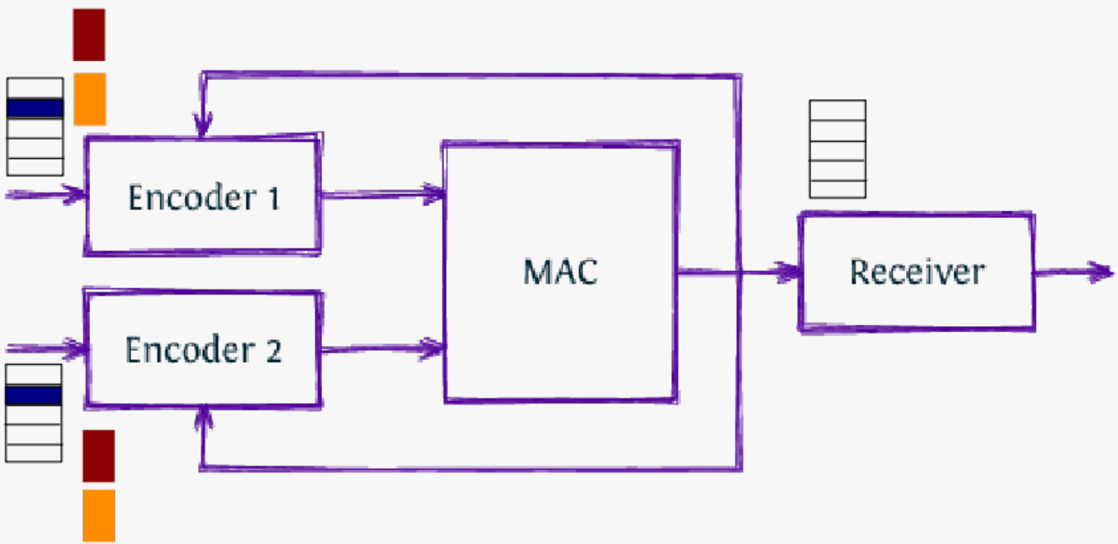
[Bross-Lapidoth-05]



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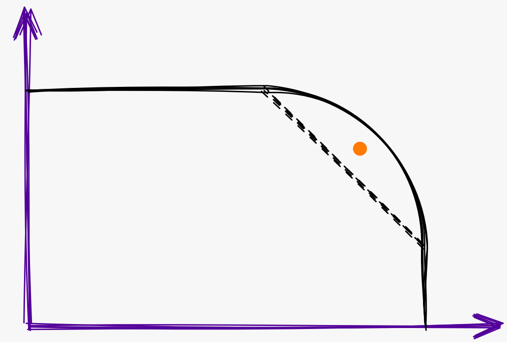

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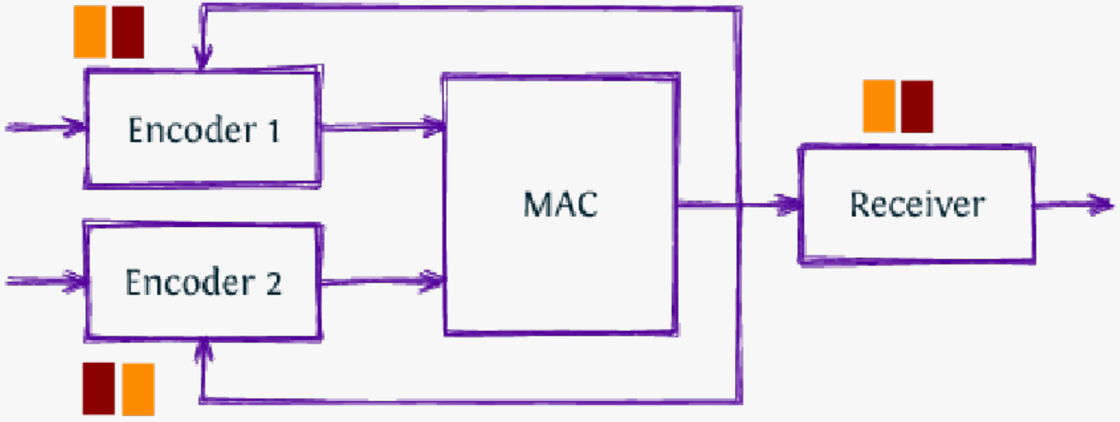
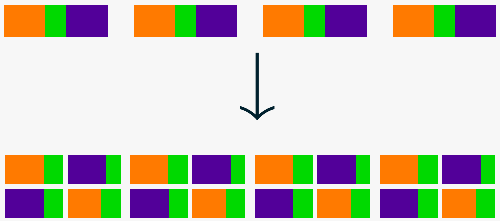


Bross-Lapidoth Region



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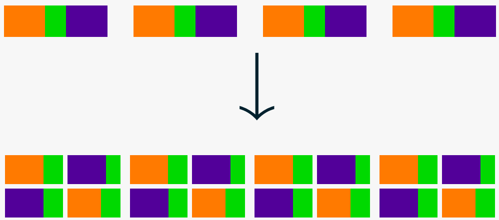

Capacity

$$R_{BL} = \bigcup_{\substack{P_U P_{X^1|U} P_{X^2|U} \\ g_1:\mathcal{X}^1\times\mathcal{Y}\to\mathcal{V}^1,\, g_2:\mathcal{X}^2\times\mathcal{Y}\to\mathcal{V}^2 \\ \text{stuff}}} \left\{ (R_1, R_2) : \begin{array}{rl} R_1 \leqslant & (1+\eta)^{-1} I(X^1 \wedge Y \mid U X^2 V^1) \\ R_2 \leqslant & (1+\eta)^{-1} I(X^2 \wedge Y \mid U X^1 V^2) \\ R_1 + R_2 \leqslant & (1+\eta)^{-1} I(X^1 X^2 \wedge Y \mid V^1 V^2) - R_L \\ R_L \leqslant & \text{complicated expression} \\ \eta \geqslant & \text{complicated expression} \end{array} \right\}$$

[Bross-Lapidoth-05]

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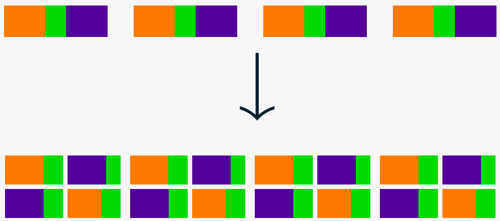

Comparison with CL

- BL region includes CL region
- In some cases the inclusion is strict

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[Bross-Lapidoth-05]

Block Markov superposition codes

Cover-Leung Scheme

- Decoder decodes after **one block**
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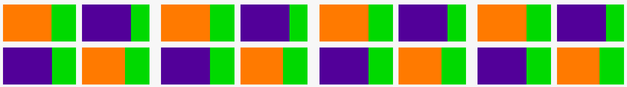
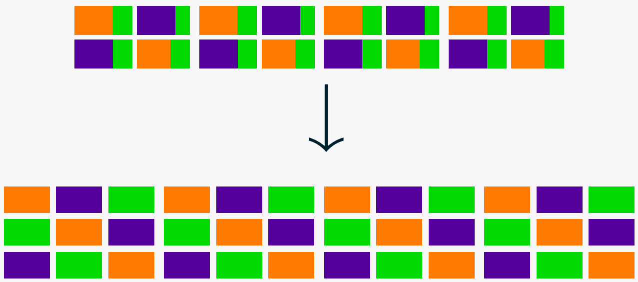


Questions

- Are these the best coding schemes that decode after one and two blocks?
- Can we improve the achievable region by using more auxiliary random variables?

Venkataramanan-Pradhan scheme



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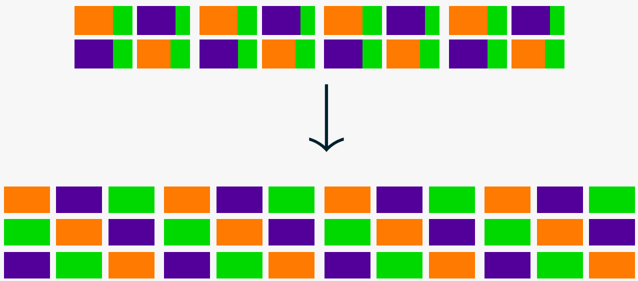
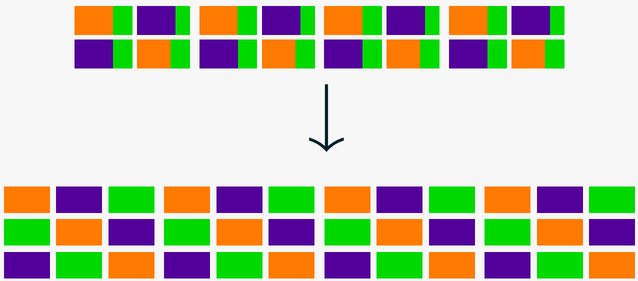


Capacity

$$R_{VP} = \bigcup \left\{ \begin{aligned} R_1 &\leqslant I(X^1 \wedge Y \mid UWX^2V^2) - [I(V^1 \wedge X^2 \mid YV^2WU) - I(U \wedge Y)]^+ \\ R_2 &\leqslant I(X^2 \wedge Y \mid UWX^1V^1) - [I(V^2 \wedge X^1 \mid YV^1WU) - I(U \wedge Y)]^+ \\ R_1 + R_2 &\leqslant I(X^1X^2 \wedge Y) - I(W \wedge Y \mid U) \end{aligned} \right\}$$

union over $P_U P_{V^1V^2} P_{X^1|UV^1} p_{X^2|UV^2} Q_{\tilde{V}^1|X^1UV^1V^2Y} Q_{\tilde{V}^1|X^1UV^1V^2Y}$,
$W = (U, V^1, V^2, Y)$, stuff [Venkataramanan-Pradhan-09]

Venkataramanan-Pradhan scheme



Comparison with earlier schemes

- VP region includes CL region
- In some cases the VP region strictly improves over BL region

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Questions

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  Given that we are decoding after n-blocks, can we improve performance if we use a more sophisticated coding scheme?

- **Nature of auxiliary random variables**

  Are the auxiliary random variables intrinsic to the problem or are they an artifact of our solution approach?

Outline

1. Overview of MAC with feedback
   - Cover-Leung scheme
   - Bross-Lapidoth scheme
   - Venkataramanan-Pradhan scheme
2. **Formulation as a decentralized control problem with delayed sharing**
   - **delay = 1**
   - **delay = 2**
3. Conclusion

Information structure at block level (n=1)

$\hat{M}_b = M_{b-1}^1 M_{b-1}^2$ with high probability

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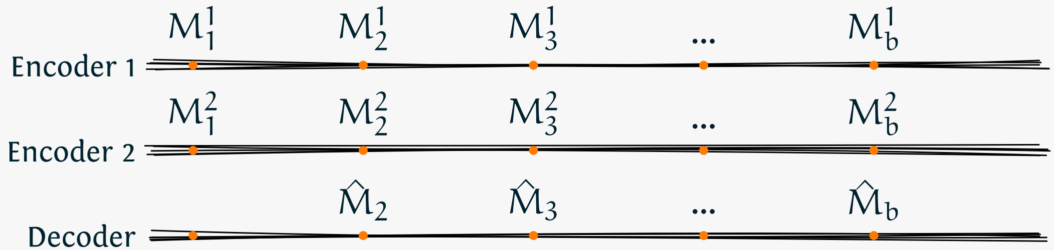

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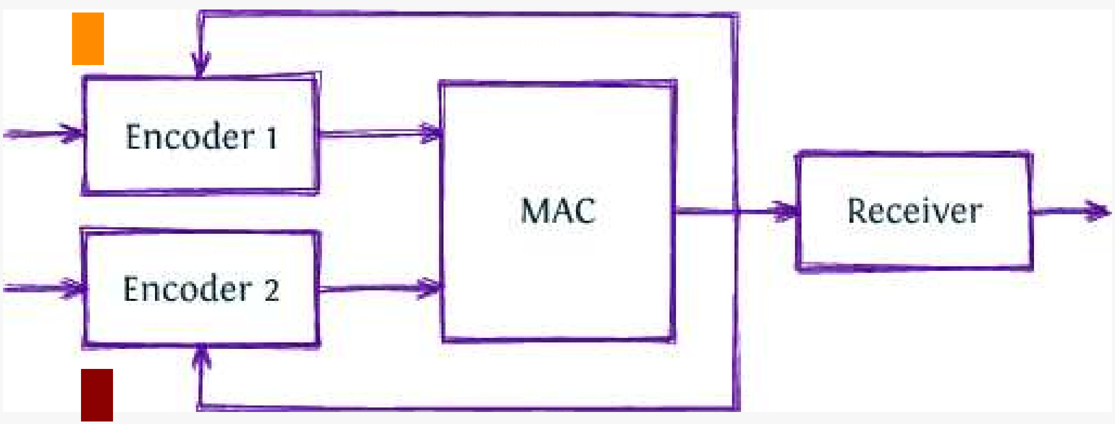

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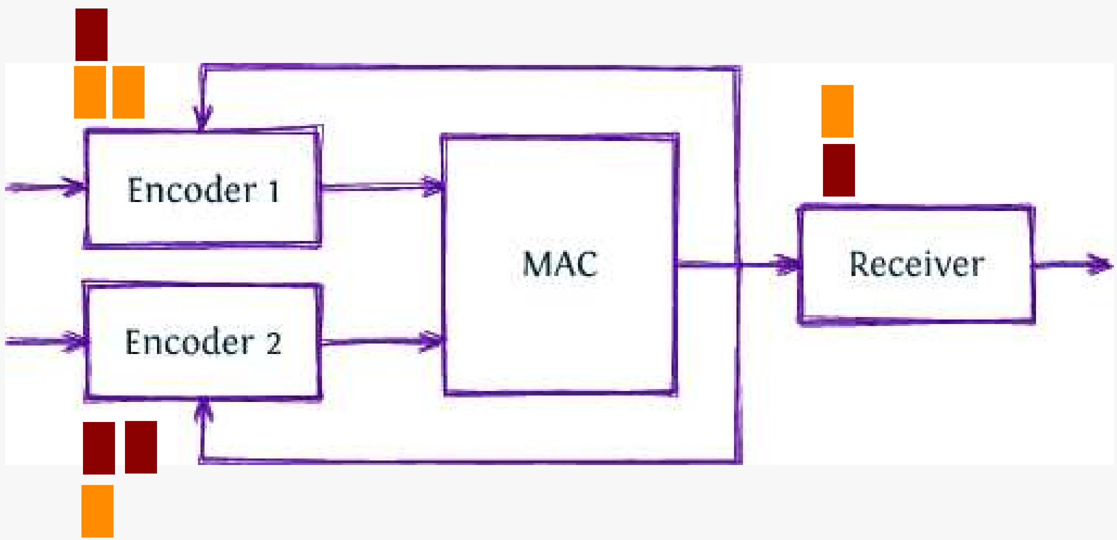

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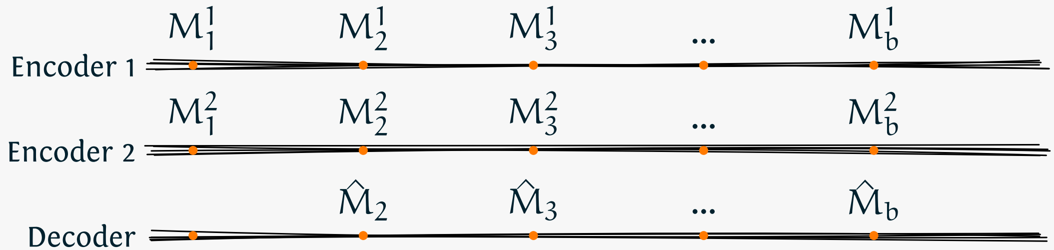

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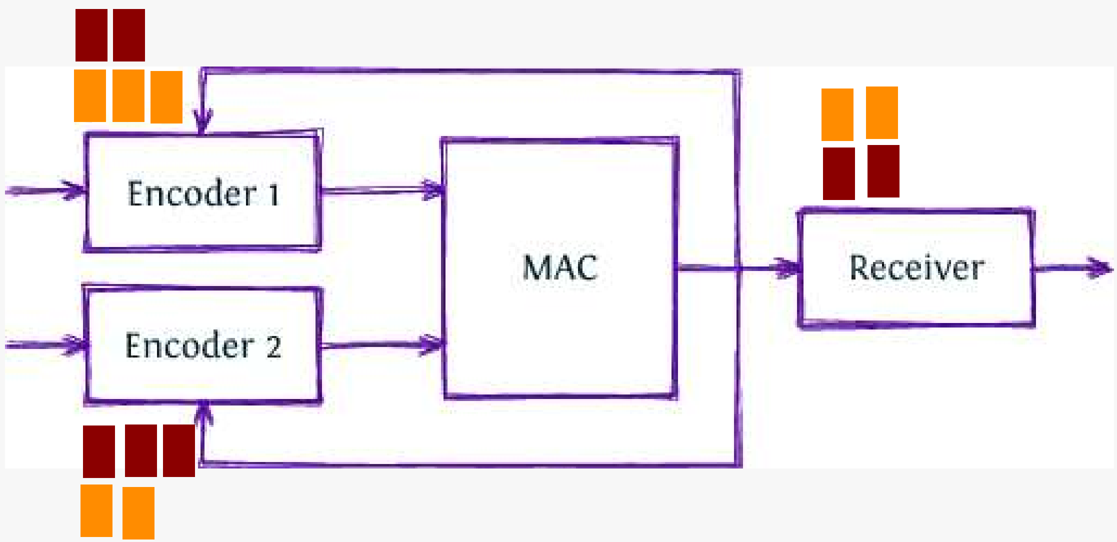

Information structure at block level (n=1)

Encoder 1

$\overline{M}_{[1:b-1]}, Y_{[1:b-1]},$

M_b^1

$\overline{M}_{[1:b-2]}, Y_{[1:b]}$

Decoder

$\overline{M}_{[1:b-1]}, Y_{[1:b-1]},$

M_b^2

Encoder 2

Info struc at block level (general n)

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$\overline{M}_{[1:b-n]}, Y_{[1:b-1]},$

$M^1_{[b-n+1:b]}$

$\overline{M}_{[1:b-n-1]}, Y_{[1:b]}$

Decoder

$\overline{M}_{[1:b-n]}, Y_{[1:b-1]},$

$M^2_{[b-n+1:b]}$

Encoder 2

Block Markov superposition coding at block level

Setup

- **i.i.d. Messages** at each encoders $\overline{M}_b = (M_b^1, M_b^2)$
- **Fixed delay decoding** $\hat{M}_b$ is trying to reproduce $\overline{M}_{b-n}$.

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Coding scheme

- **Encoder**: $X_b^i = f_b^i(\overline{M}_{[1:b-n]}, Y_{[1:b-1]}, M^i_{[b-n+1:b]})$
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- **Error**: $\rho(\overline{M}_{b-n}, \hat{M}_b)$

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- **Error**: $\rho(\overline{M}_{b-n}, \hat{M}_b)$
- **Objective**: Choose coding scheme to minimize $\sum_{b=n}^{B} \mathbb{E}[\rho(\overline{M}_{b-n}, \hat{M}_b)]$

Transforming information structures

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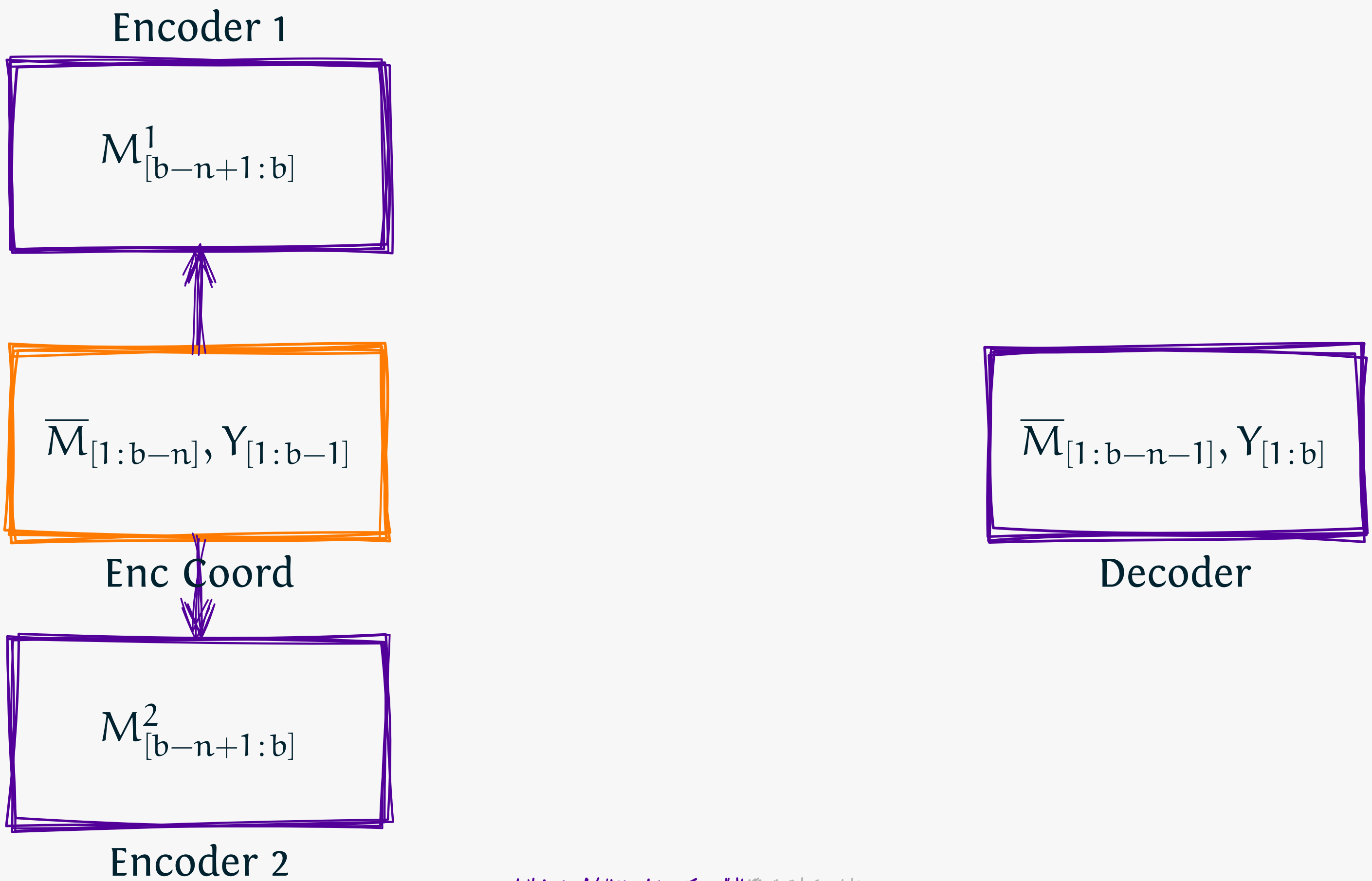

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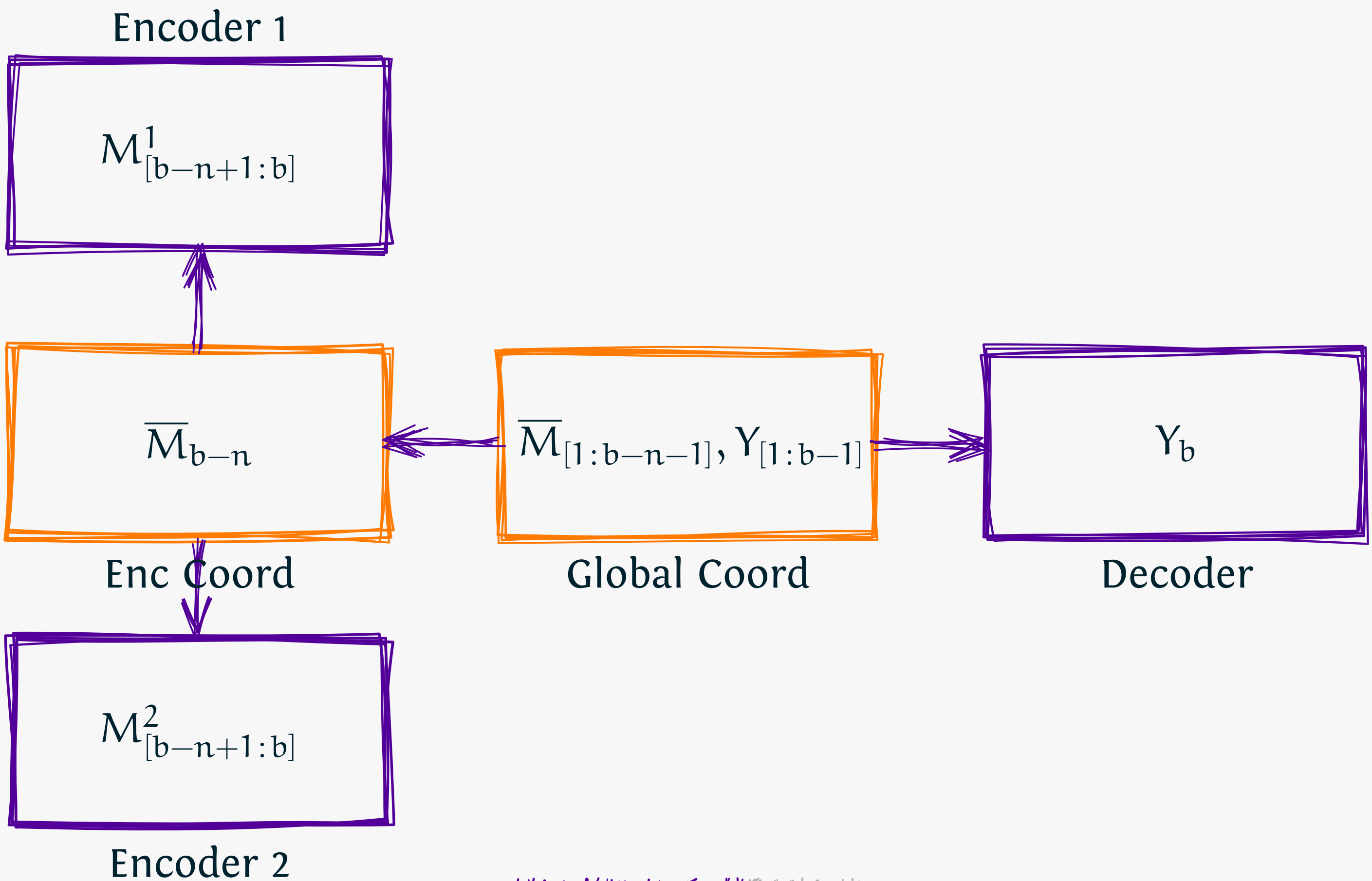
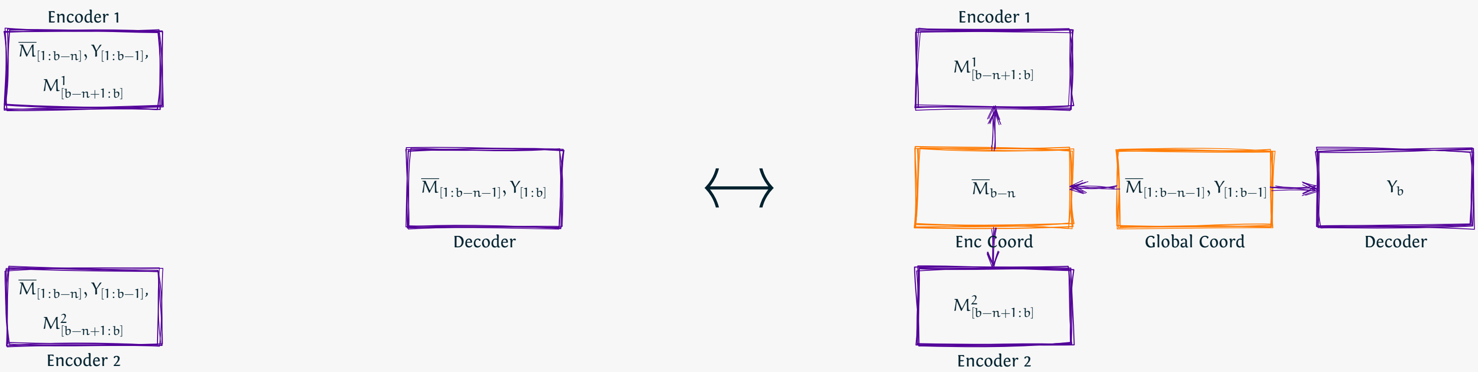


Centralized planning in decentralized systems



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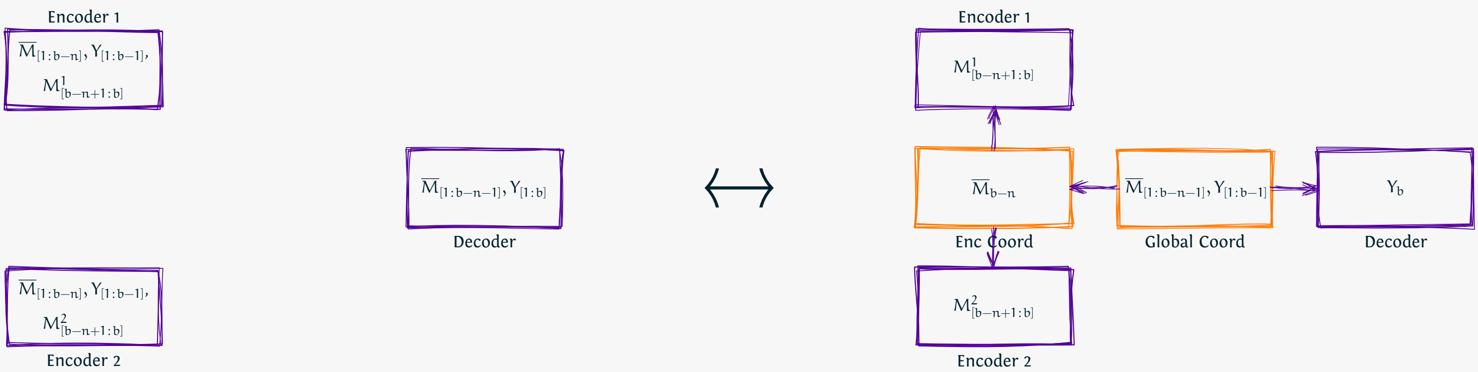

Hierarchical delayed sharing info structures

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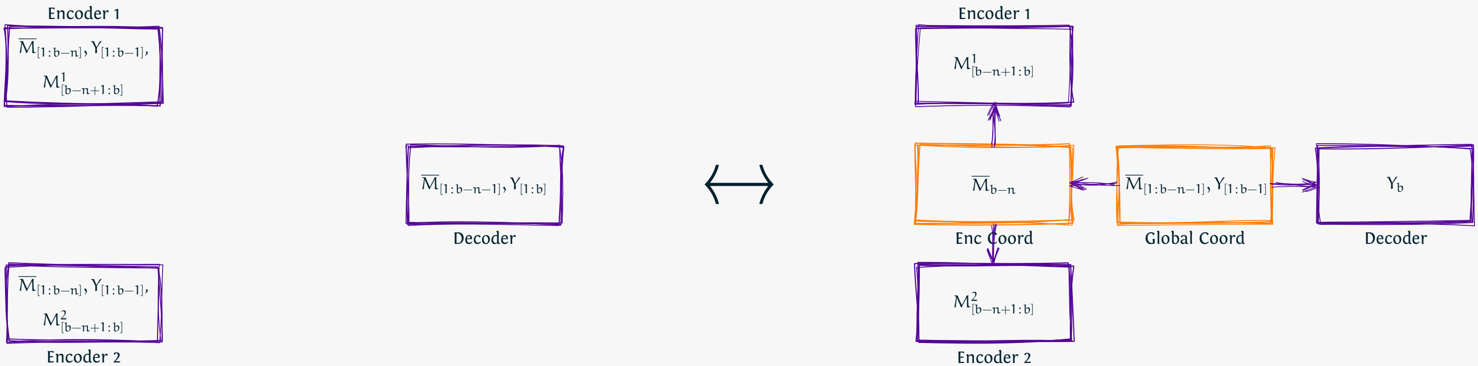
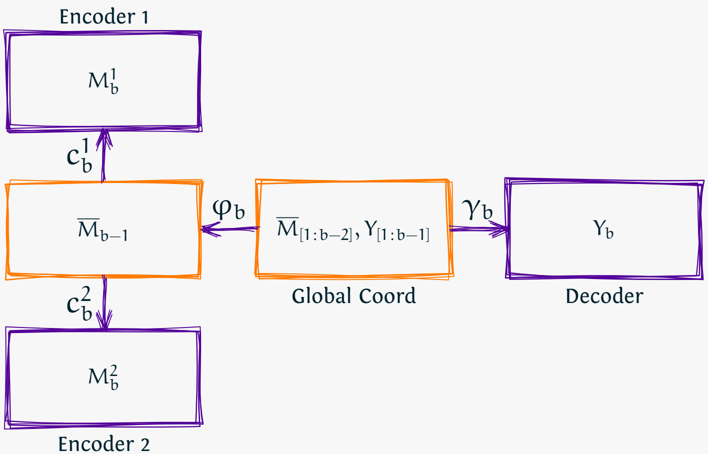


Hierarchical delayed sharing info structures

- Look at the system from global coordinator's point of view
- Show that the two optimization problems are **equivalent**
- Derive **structure** of optimal control policies **for the coordinator**

Delay n=1: Cover-Leung setup



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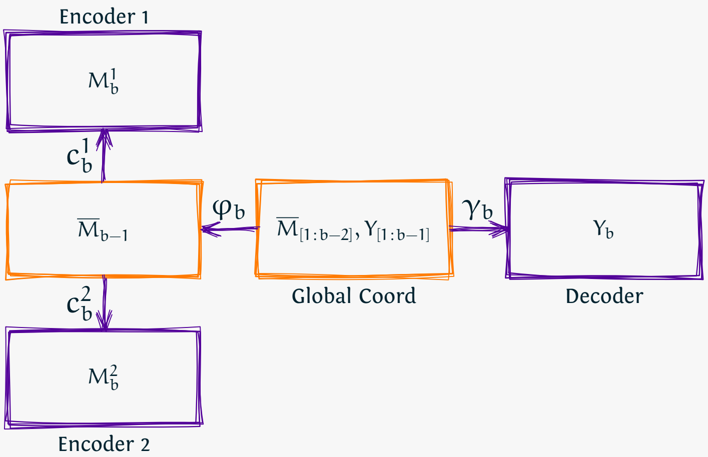

Global coordinator

$$(\varphi_b, \gamma_b) = \psi_b(\overline{M}_{[1:b-2]}, Y_{[1:b-1]})$$

where

$$\varphi_b : \overline{\mathcal{M}} \to \big((\mathcal{M}^1 \to \mathcal{X}^1), (\mathcal{M}^2 \to \mathcal{X}^2) \big)$$

$$\gamma_b : \mathcal{Y} \to \hat{\mathcal{M}}$$

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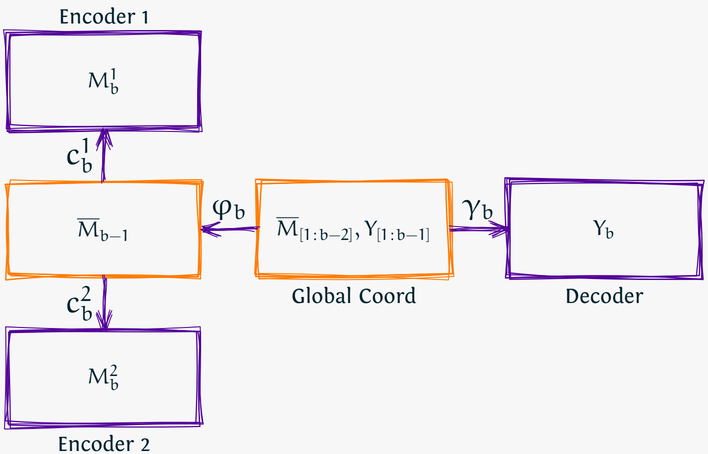

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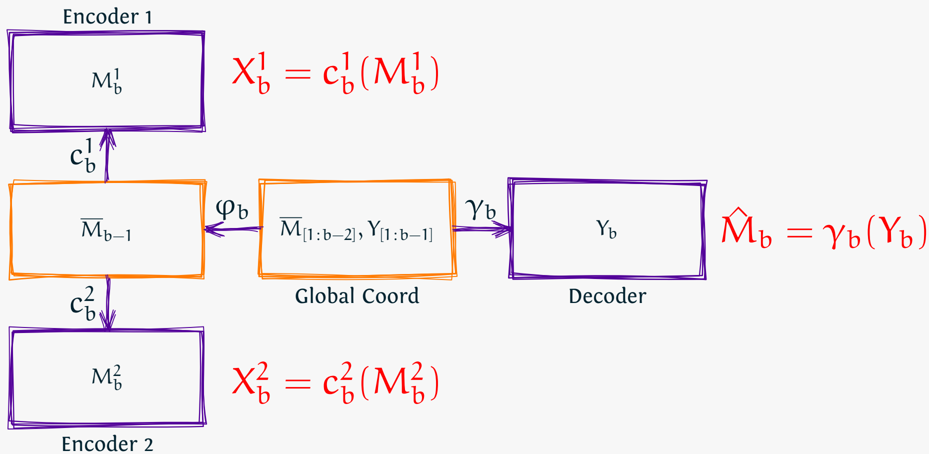

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Structure of optimal control policy

$$\left(\,\overline{M}_{[1:b-2]}, Y_{[1:b-1]}, \varphi_{[1:b-1]}, \gamma_{[1:b-1]}\,\right) \xrightarrow{\text{info state}} \left(Y_{b-1}, c^1_{b-1}, c^2_{b-1}\right)$$

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- Controlled Markov process

$$\begin{aligned}
&\mathbb{P}(Y_b, c^1_b, c^2_b \mid \overline{M}_{[1:b-2]}, Y_{[1:b-1]}, \varphi_{[1:b]}, \gamma_{[1:b]}) \\
&\quad = \sum_{\overline{M}_{b-1,b}} \mathbb{P}(Y_b \mid c^1_b(M^1_b) c^2_b(M^2_b)) \times \mathbb{1}[(c^1_b, c^2_b) = \varphi_b(\overline{M}_{b-1})] \\
&\qquad \times \frac{\mathbb{P}(Y_{b-1} \mid c^1_{b-1}(M^1_{b-1}) c^2_{b-1}(M^2_{b-1}))\mathbb{P}(\overline{M}_{b-1})}{\sum_{\underline{M}_{b-1}} \mathbb{P}(Y_{b-1} \mid c^1_{b-1}(\underline{M}^1_{b-1}) c^2_{b-1}(\underline{M}^2_{b-1}))\mathbb{P}(\underline{M}_{b-1})} \\
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\end{aligned}$$

- Expected cost per unit time

$$\begin{aligned}
&\mathbb{E}[\rho(\overline{M}_{b-1}, \hat{M}_b) \mid \overline{M}_{[1:b-2]}, Y_{[1:b-1]}, \varphi_{[1:b]}, \gamma_{[1:b]}] \\
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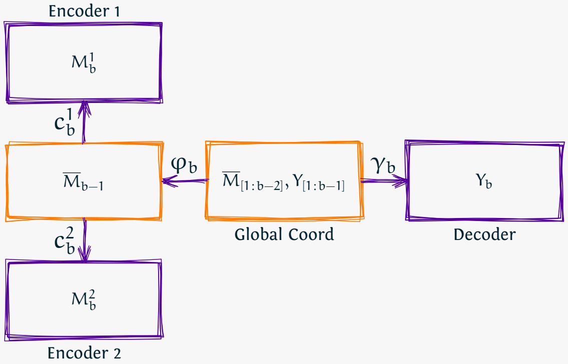

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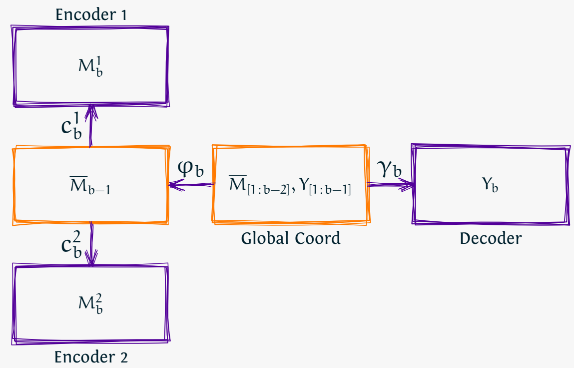

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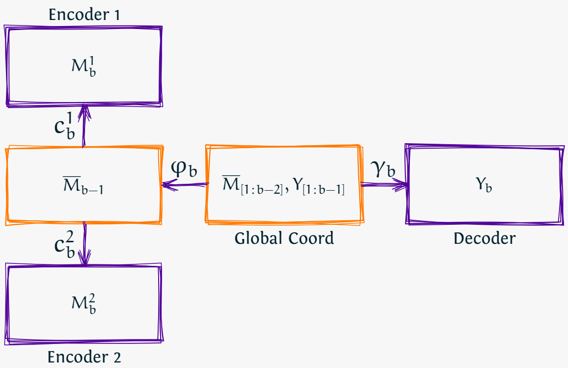

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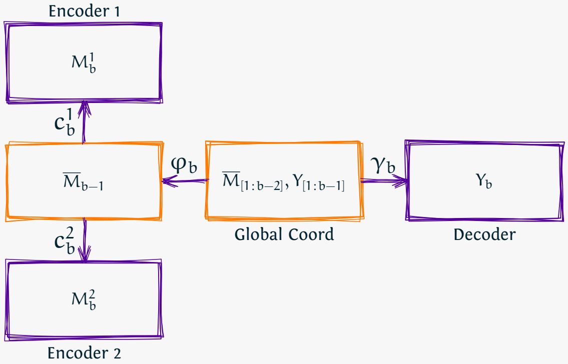

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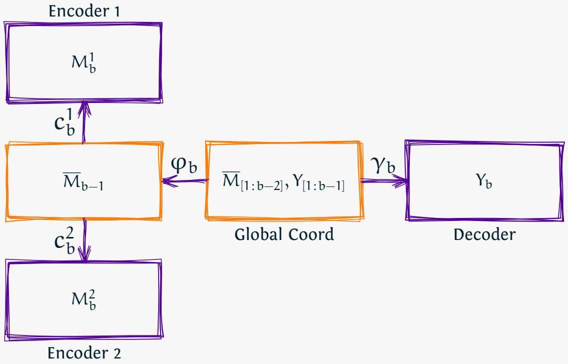

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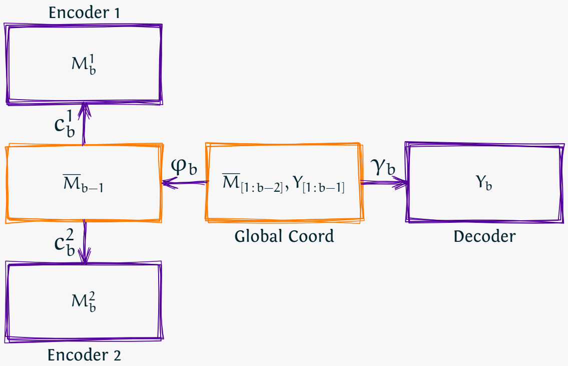

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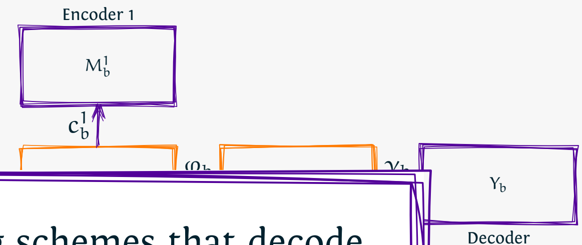

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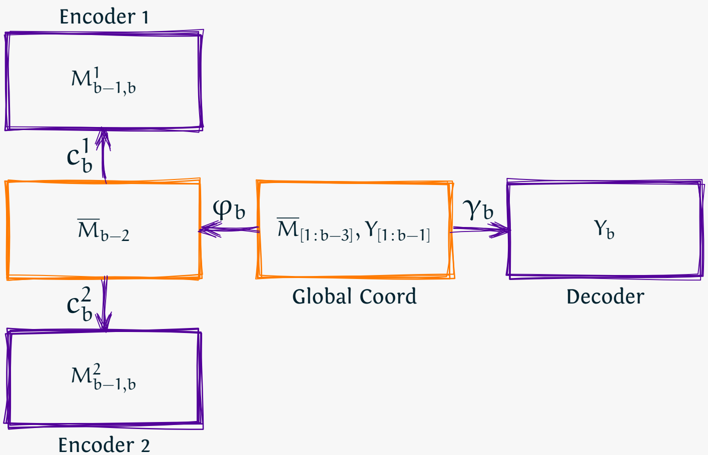
Delay n=2: Venkataramanan Pradhan setup



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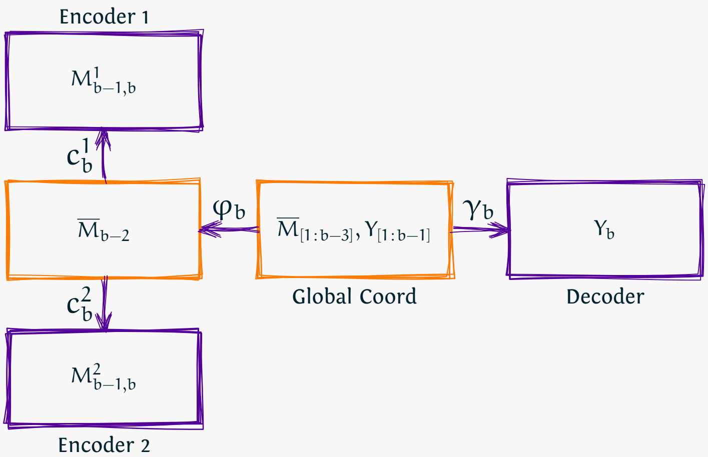

Global coordinator

$$(\varphi_b, \gamma_b) = \psi_b(\overline{M}_{[1:b-3]}, Y_{[1:b-1]})$$

where

$$\varphi_b : \overline{\mathcal{M}} \to \big((\mathcal{M}^1 \times \mathcal{M}^1 \to \mathcal{X}^1), (\mathcal{M}^2 \times \mathcal{M}^2 \to \mathcal{X}^2) \big)$$

$$\gamma_b : \mathcal{Y} \to \hat{\mathcal{M}}$$

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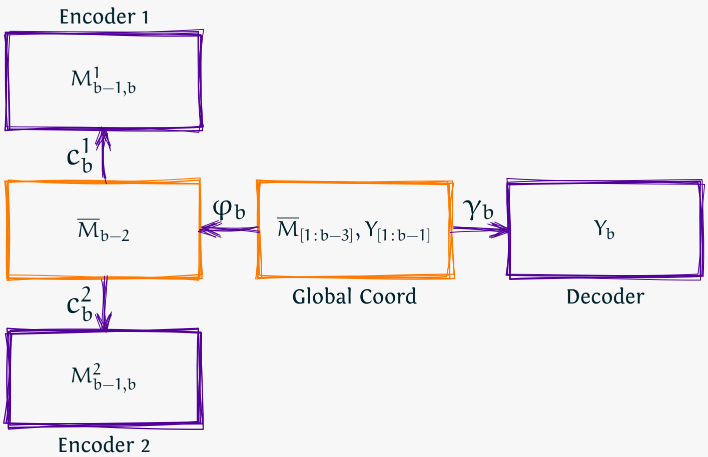

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Encoder coordinator

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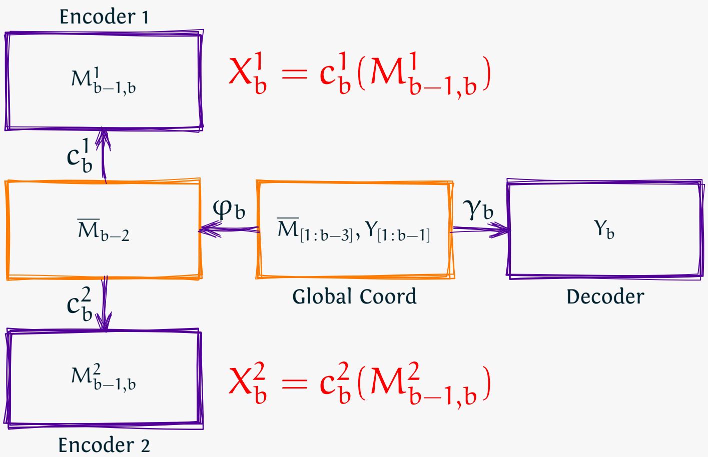

$$X^1_b = c^1_b(M^1_{b-1,b})$$

$$\hat{M}_b = \gamma_b(Y_b)$$

$$X^2_b = c^2_b(M^2_{b-1,b})$$

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Structure of optimal control policy

$$\left(\overline{M}_{[1:b-3]}, Y_{[1:b-1]}, \varphi_{[1:b-1]}, \gamma_{[1:b-1]} \right) \xrightarrow{\text{info state}} (Y_{b-2,b-1}, c^1_{b-1}, c^2_{b-1}, \hat{c}^1_{b-2}, \hat{c}^2_{b-2})$$

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Controlled Markov process

$$\begin{aligned}
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- Expected cost per unit time

$$\begin{aligned}&\mathbb{E}[\rho(\overline{M}_{b-2}, \hat{M}_b) \mid \overline{M}_{[1:b-3]}, Y_{[1:b-1]}, \varphi_{[1:b]}, \gamma_{[1:b]}] \\ &\quad = \mathbb{E}[\rho(\overline{M}_{b-2}, \hat{M}_b) \mid Y_{b-2,b-1}, c^1_{b-1}, c^2_{b-1}, \hat{c}^1_{b-2}, \hat{c}^2_{b-2}, \varphi_b, \gamma_b]\end{aligned}$$

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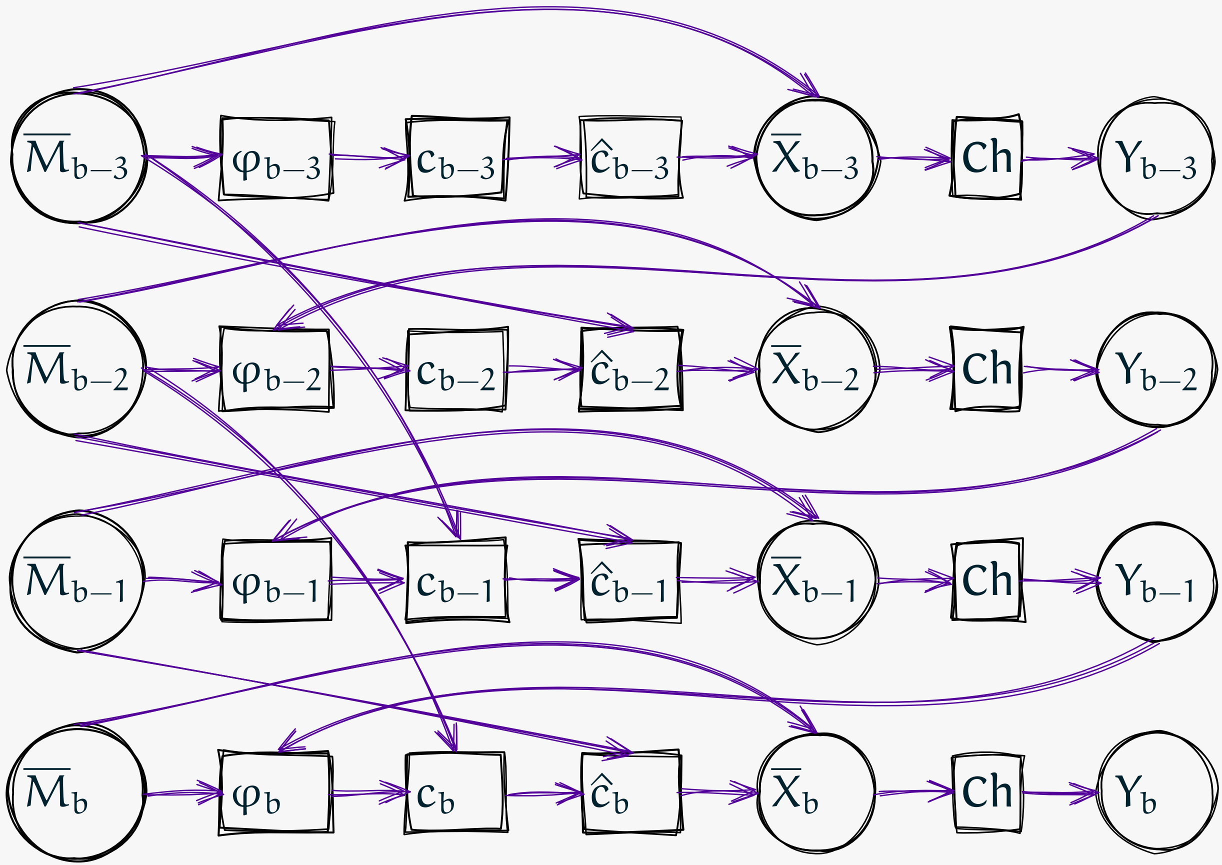

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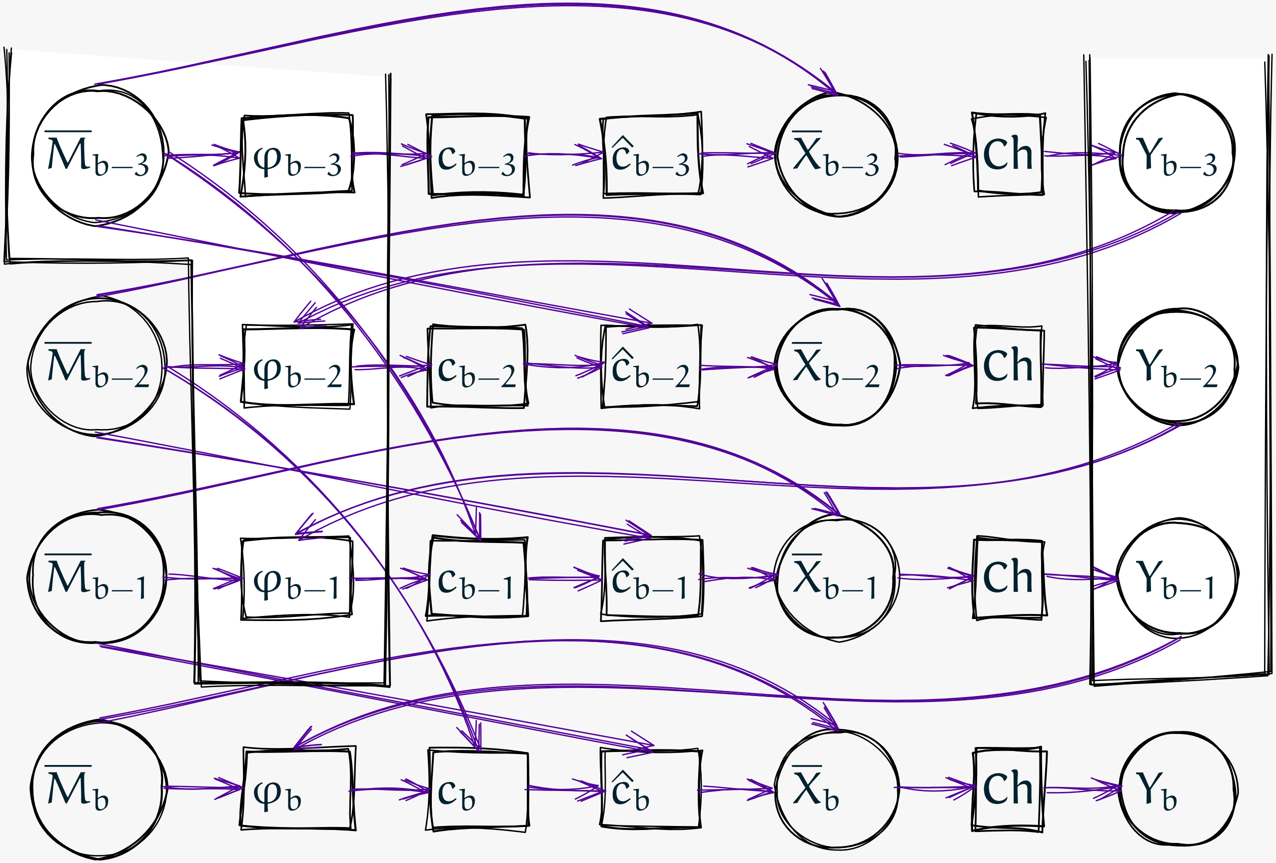

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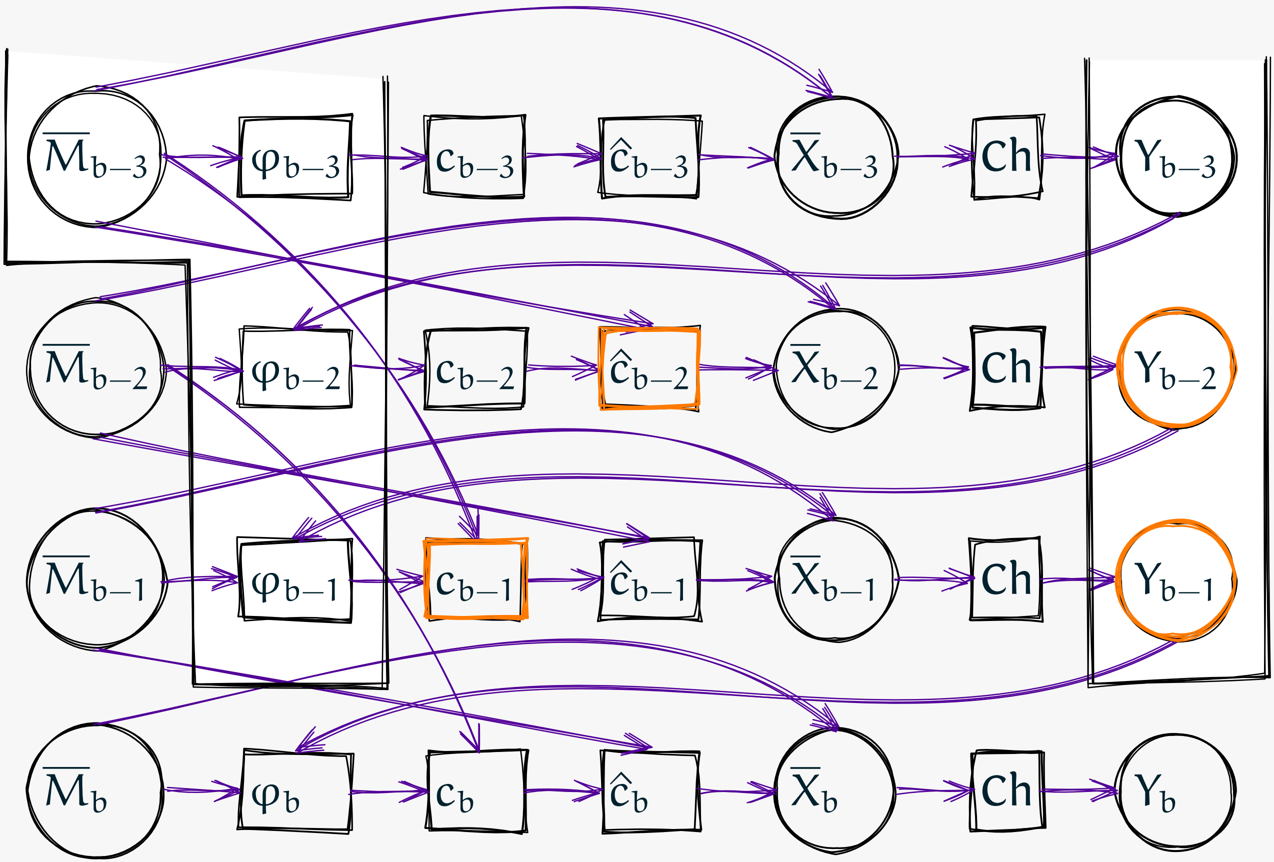

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- VP has optimal structure for delay-2

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Main point

- Auxiliary random variables are related to information states
- Systematic approaches to derive info states can be used for auxiliary random variables

Choice of info structures

- Block Markov superposition schemes enforce a specific information structure
- Stochastic control can identify whether an info structure is tractable (in many cases **without explicitly solving the system**)

Conclusion

Horses for courses

- Use **information theory** to determine the form of coding scheme **within blocks**
- Use **stochastic control** to determine the form of coding scheme **across blocks**

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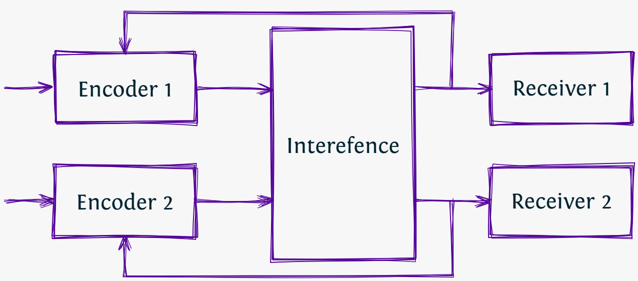
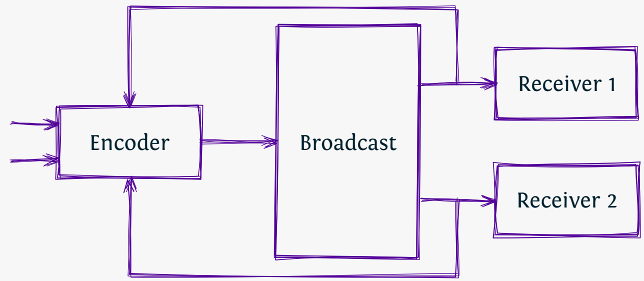
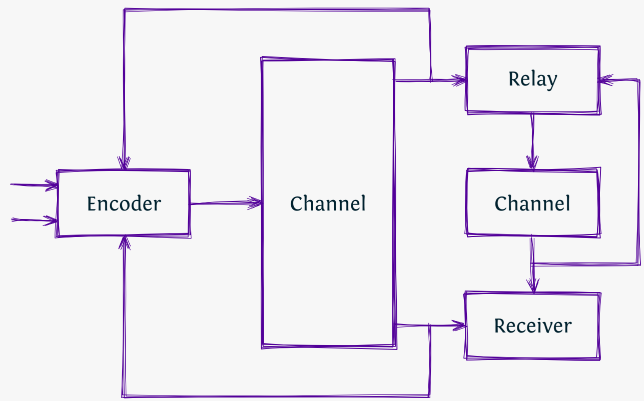


Interference

Future directions



Broadcast



Relay

Thank you