Approximate information states for partially observable systems

Aditya Mahajan
McGill University

Joint work with: Jayakumar Subramanian, Amit Sinha,
Raihan Seraj, Erfan Seyedsalehi

DCL Seminar, UIUC
8 Feb 2023

Recent successes of RL

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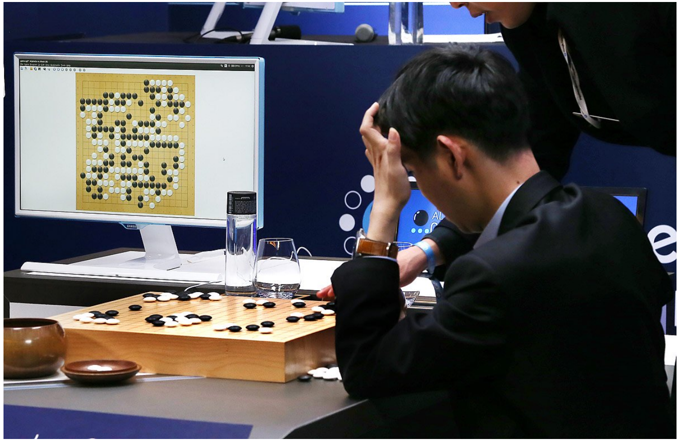

Alpha Go

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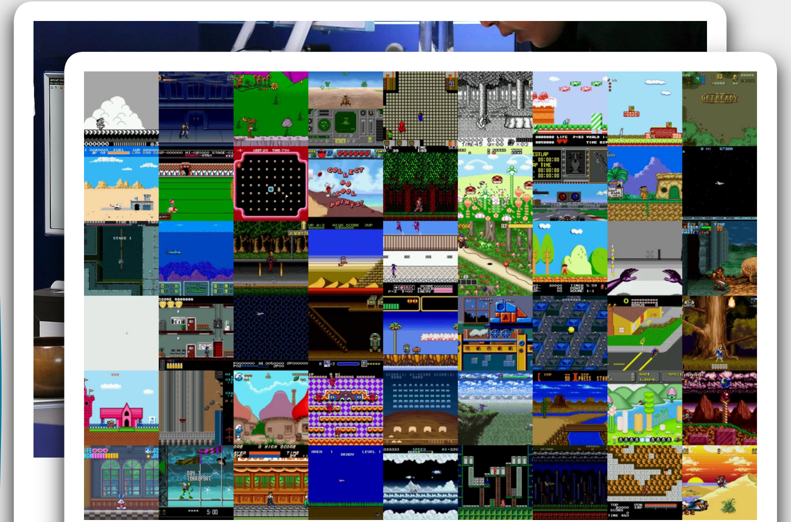
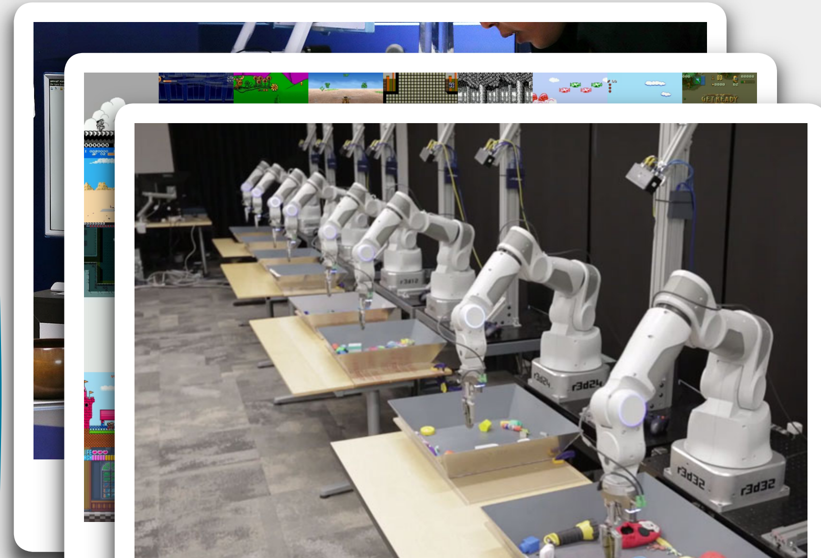


Arcade games

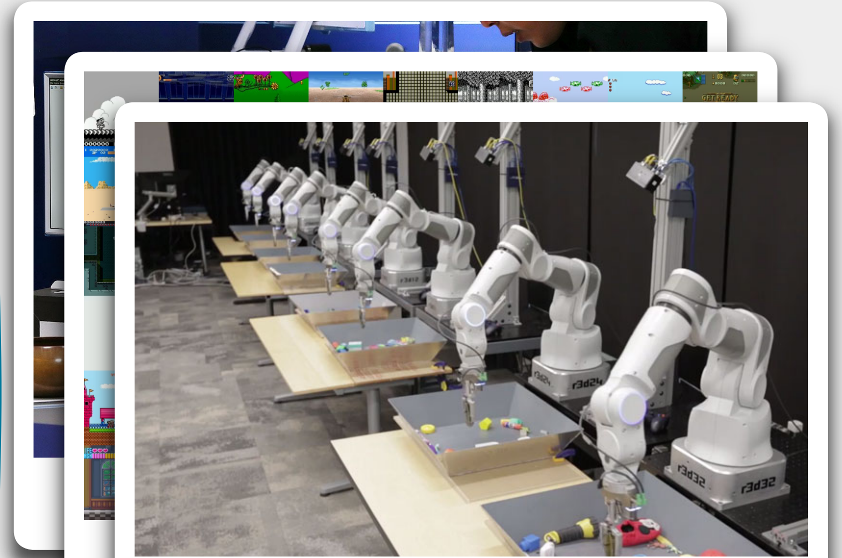
Recent successes of RL



Robotic grasping

Recent successes of RL

- Algorithms based on comprehensive theory



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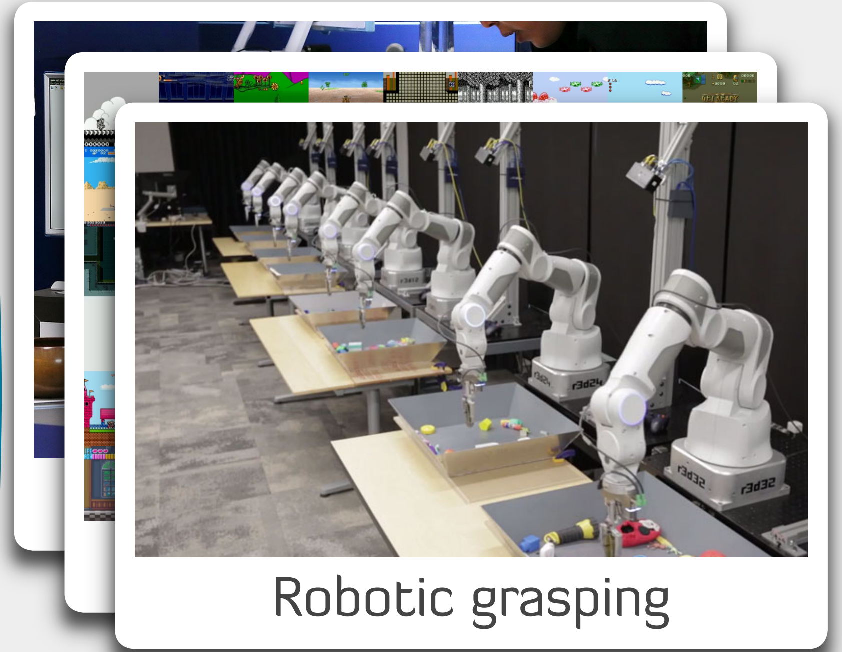
- The theory is restricted almost exclusively to systems with perfect state observations.



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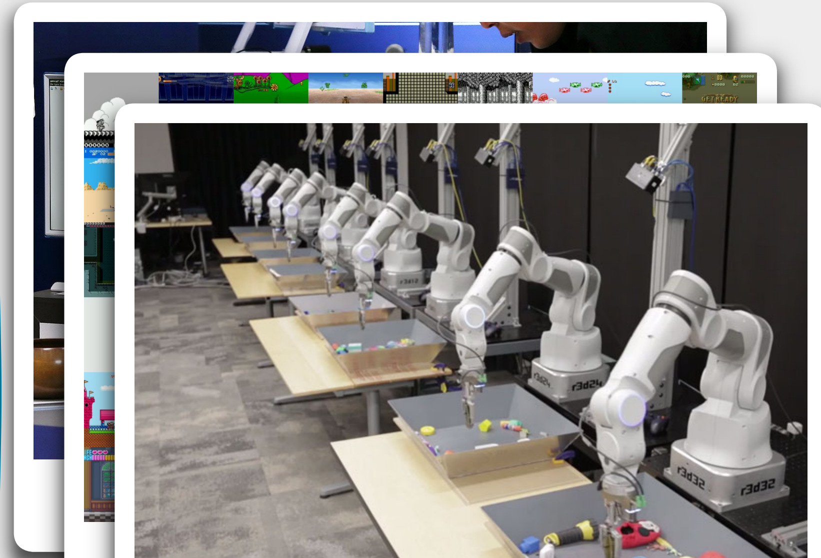

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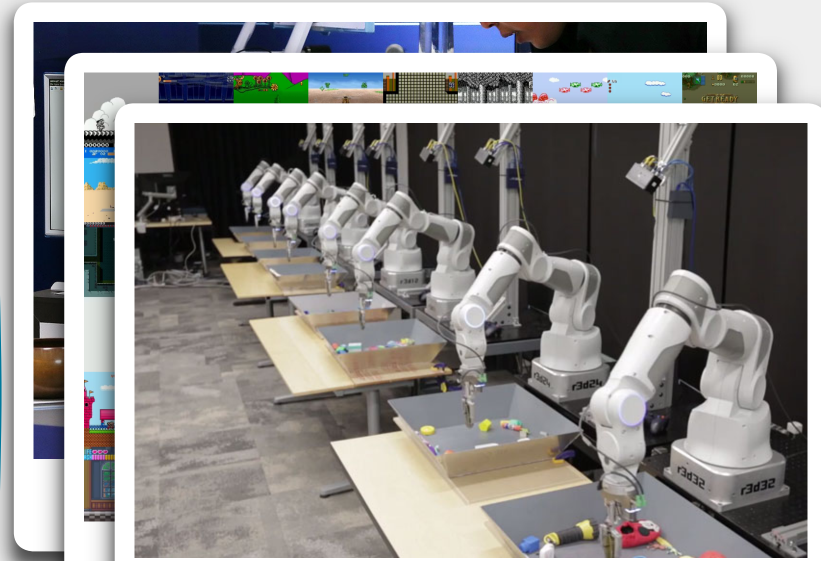
Many real-world applications are partially observed

- Healthcare
- Autonomous driving
- Finance (portfolio management)
- Retail and marketing



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How do we develop a theory for RL for partially observed systems?

Outline

Background

- Review of MDPs and RL
- Review of POMDPs
- Why is RL for POMDPs difficult?

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Approximate Planning for POMDPs	
	▷ Preliminaries on information state ▷ Approximate information state ▷ Approximation bounds

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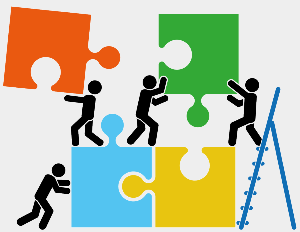

RL for POMDPs

- From approximation bounds to RL
- Numerical experiments

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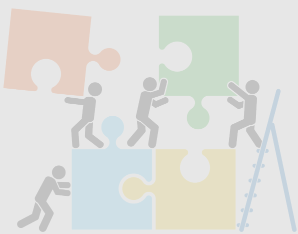

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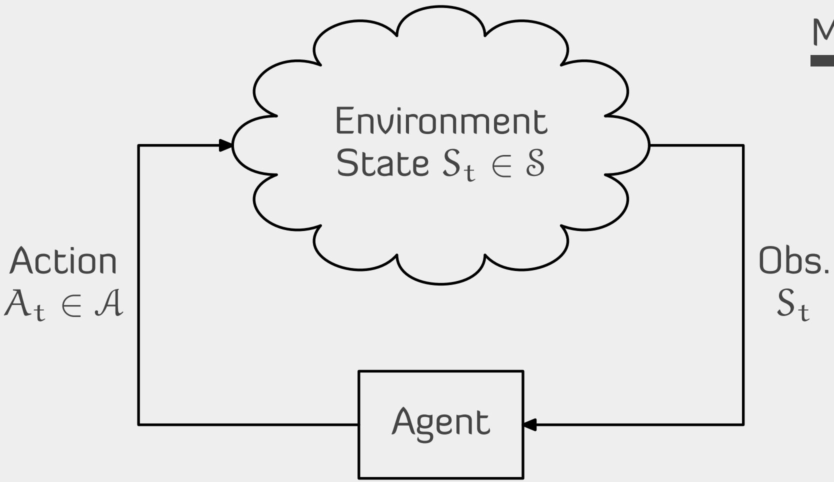
Review: Markov decision processes (MDPs)



MDP: MARKOV DECISION PROCESS

Dynamics: $\mathbb{P}(S_{t+1} \mid S_t, A_t)$

Observations: S_t

Reward $R_t = r(S_t, A_t)$.

Action: $A_t \sim \pi_t(S_{1:t}, A_{1:t-1})$.

$\pi = (\pi_t)_{t \geqslant 1}$ is called a policy.

The objective is to choose a policy π to maximize:

$$J(\pi) := \mathbb{E}^{\pi}\Big[\sum_{t=1}^{\infty} \gamma^{t-1} R_t\Big]$$

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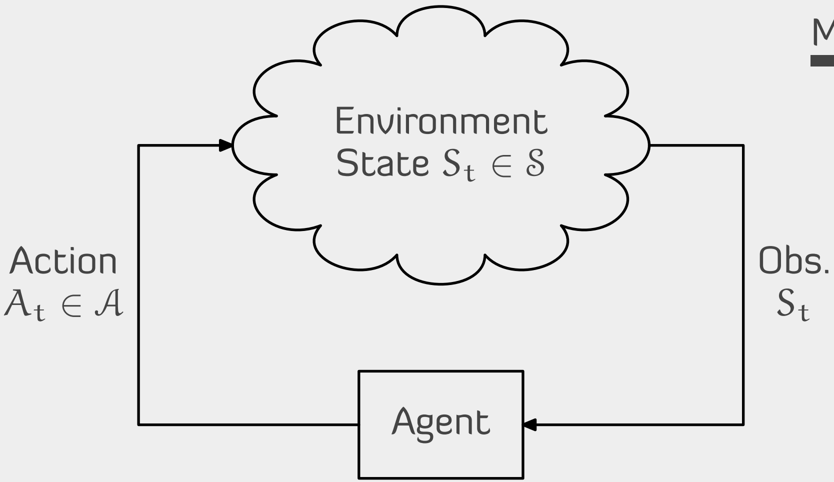

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Conceptual challenge

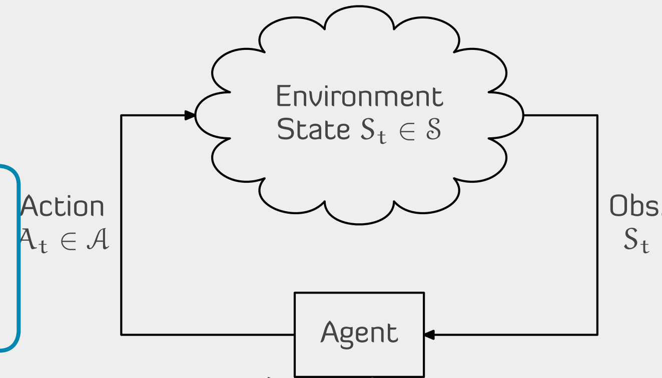
- Brute force search has an exponential complexity in time horizon.
- How to efficiently search an optimal policy?

Review: Markov decision processes (MDPs)

Key simplifying ideas

Principle of Irrelevant Information

Structure of optimal policy	There is no loss of optimality in choosing the action A_t as a function of the current state S_t



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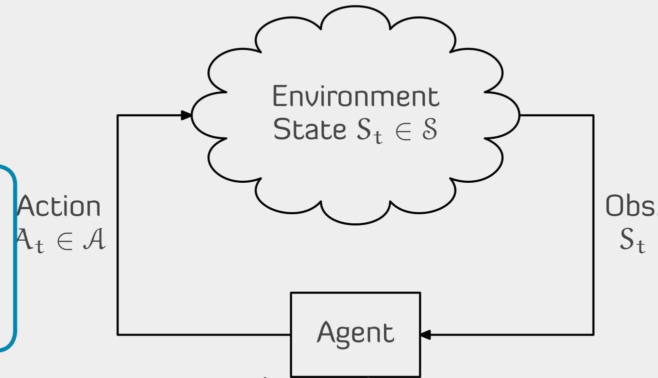

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Principle of Optimality

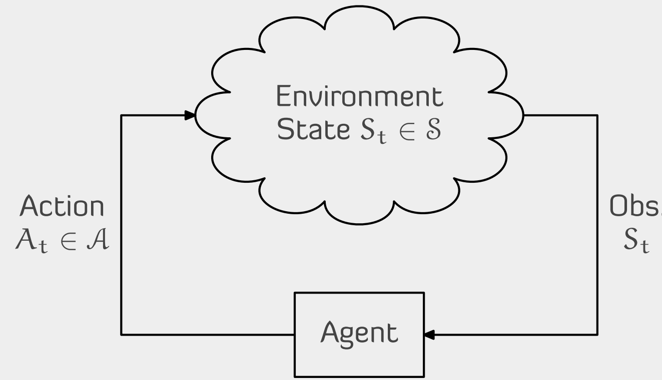
Dynamic Program	The optimal control policy is given a DP with state S_t: $V(s) = \max_{a \in \mathcal{A}} \Big\{ r(s,a) + \gamma \int V(s')P(ds'\|s,a) \Big\}$

Bellman, "Dynamic Programming," 1957.

Review: Reinforcement Learning (RL)

The (online) RL setting

- Dynamics and reward functions are unknown.
- Agent can interact with the environment and observe states and rewards.
- Design an algorithm that asymptotically identifies an optimal policy.



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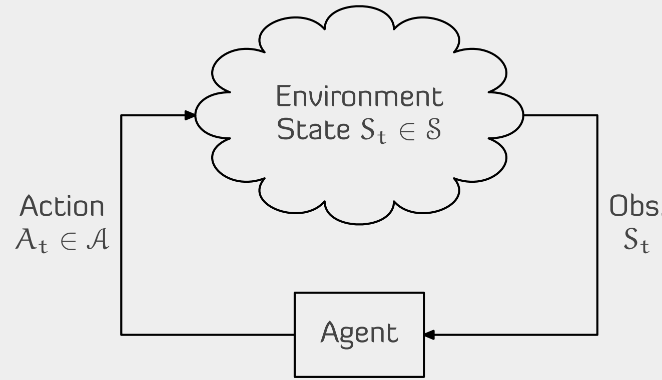
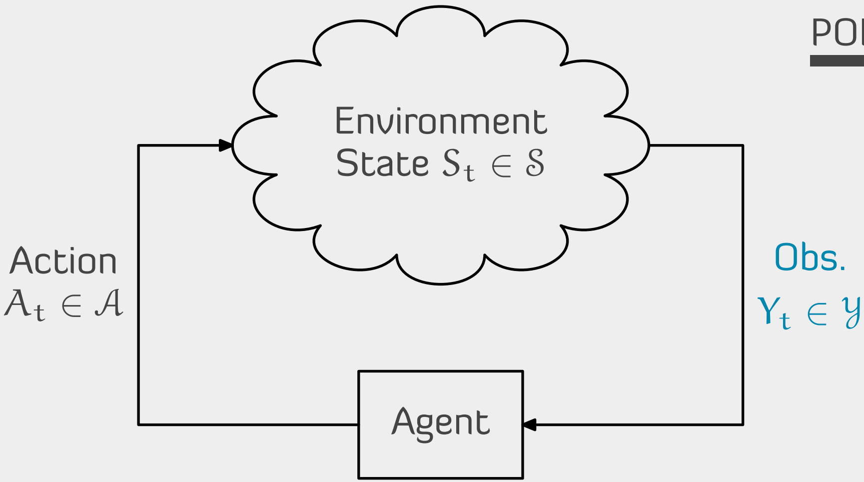


Value based methods	Estimate the Q-function $Q(s, a) = r(s, a) + \gamma \int V(s')P(ds'\|s, a)$ using temporal difference learning (i.e., stochastic approximation). [Watkins and Dayan, 1992; Tsitsiklis, 1994]
Policy-based methods	Use parameterized policies π_θ. Estimate $\nabla_\theta V_\theta(s)$ using single trajectory gradient estimates (i.e., infitesimal perturbation analysis). [Sutton 2000, Marback and Tsitsiklis 2001], [Cao, 1985; Ho, 1987]

Why is learning difficult in partially observable environments?

Review: Planning in partially observable environments



POMDP: PARTIALLY OBSERVABLE MARKOV DECISION PROCESS

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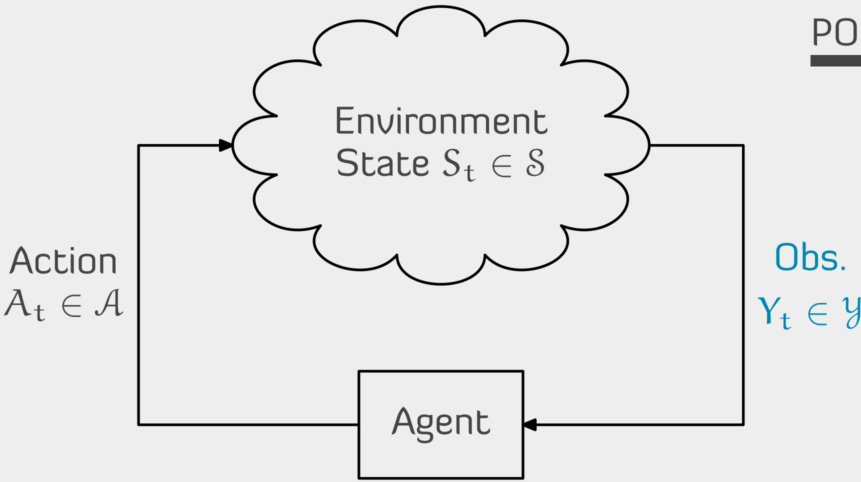

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The objective is to choose a policy π to maximize:

Conceptual challenge

- Action is a function of the history of observations and actions.
- The history is increasing in time. So, the search complexity increases exponentially in time.

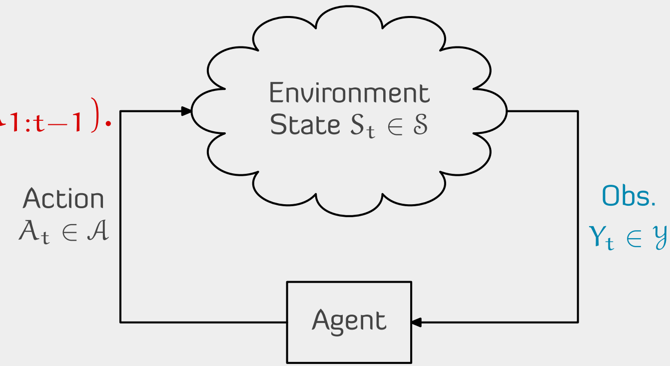
Review: Planning in partially observable environments

Key simplifying idea

Define **belief state** $B_t \in \Delta(\mathcal{S})$ as $B_t(s) = \mathbb{P}(S_t = s \mid Y_{1:t}, A_{1:t-1})$.

- Belief state updates in a state-like manner
$$B_{t+1} = \text{function}(B_t, Y_{t+1}, A_t).$$
- Belief state is sufficient to evaluate rewards
$$\mathbb{E}[R_t \mid Y_{1:t}, A_{1:t}] = \hat{r}(B_t, A_t).$$

Thus, $\{B_t\}_{t \geq 1}$ is a perfectly observed controlled Markov process.



Astrom, "Optimal control of Markov processes with incomplete information," JMAA 1965.

Stratonovich, "Conditional Markov Processes," TVP 1960.

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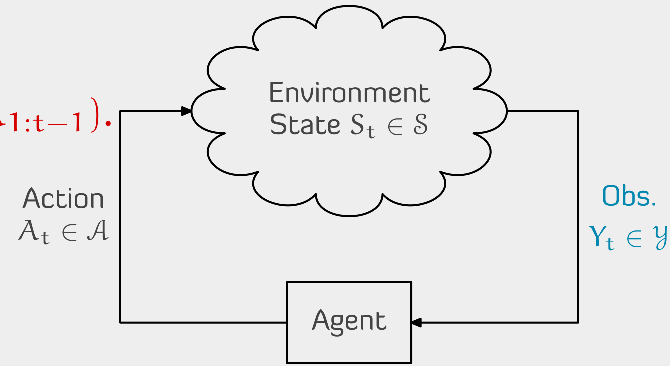

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Thus, $\{B_t\}_{t \geqslant 1}$ is a perfectly observed controlled Markov process. Therefore:

Structure of optimal policy	There is no loss of optimality in choosing the action A_t as a function of the belief state B_t
Dynamic Program	The optimal control policy is given a DP with belief B_t as state.

Implications of the POMDP modeling framework

Implications for planning

- Allows the use of the MDP machinery for partially observed systems.
- Various exact and approximate algorithms to efficiently solve the DP.

  Exact: incremental pruning, witness algorithm, linear support algo

  Approximate: QMDP, point based methods, SARSOP, DESPOT, . . .

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Implications for learning

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- So, when the system model is unknown, we cannot construct the belief state and therefore cannot use standard RL algorithms.

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- **On the theoretical side:**
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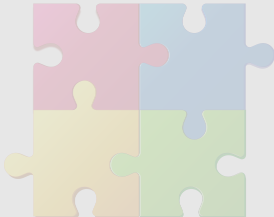
  - Good theoretical guarantees, but difficult to scale.
- **On the practical side:**
  - Simply stack the previous k observations and treat it as a "state".
  - Instead of a CNN, use an RNN to model policy and action-value fn.
  - Can be made to work but lose theoretical guarantees and insights.

Our result: A theoretically grounded method for RL in partially observable models which has strong empirical performance for high-dimensional environments.

- co-authors: J. Subramanian, A. Sinha, and R. Seraj.
- paper: JMLR, Feb 2022
- code: https://github.com/info-structures/ais

Outline



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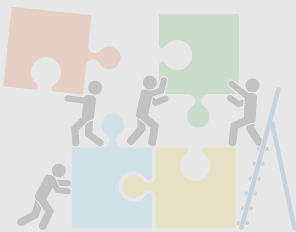

RL for POMDPs

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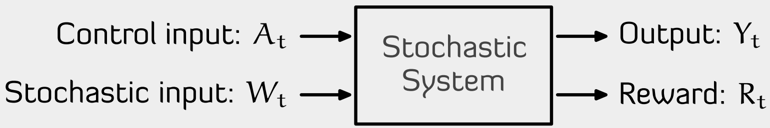
System model

- In RL, unobserved state space may not be known
- So, we work directly with input-output model

System model



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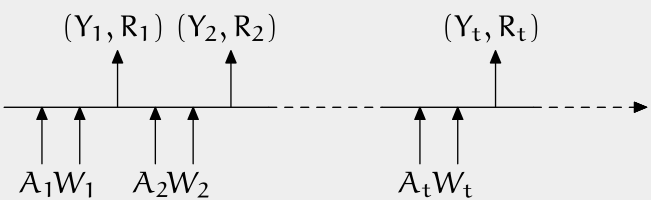

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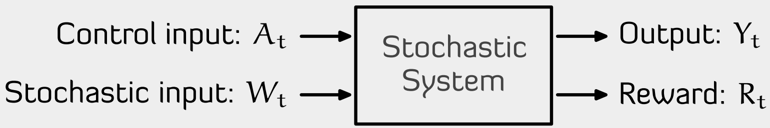

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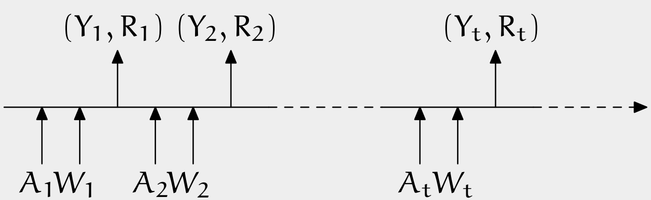

- $H_t = (Y_{1:t-1}, A_{1:t-1})$ denotes the history of all data available to the agent at time t.
- Agent chooses an $A_t \sim \pi_t(H_t)$.
- $\pi = (\pi_1, \pi_2, \dots)$ denotes the control policy.

The objective is to choose a policy π to maximize:

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Key solution concept: Information state

Informally, an information state is a compression of information which is sufficient for performance evaluation and predicting itself.

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Historical overview

- Old concept. May be viewed as as generalization of the notion of state (Nerode, 1958).
- Informal definitions given in Kwakernaak (1965), Bohlin (1970), Davis and Varaiya (1972), Kumar and Varaiya (1986) but no formal analysis.
- Related to but different from concepts such bisimulation, predictive state representations (PSR), and ε-machines.

Information state: Definition

Given a state space $\mathcal{Z}$, an INFORMATION STATE GENERATOR is a tuple of

- history compression functions $\{\sigma_t \colon \mathcal{H}_t \to \mathcal{Z}\}_{t \geqslant 1}$
- reward function $\hat{r} : \mathcal{Z} \times \mathcal{A} \to \mathbb{R}$
- transition kernel $\widehat{P} : \mathcal{Z} \times \mathcal{A} \to \Delta(\mathcal{Z})$

which satisfies two properties:

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(P1) The reward function $\hat{r}$ is sufficient for performance evaluation:

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(P2) The transition kernel $\widehat{P}$ is sufficient for predicting the info state:

$$\mathbb{P}(Z_{t+1} \in B \mid H_t = h_t, A_t = a_t) = \widehat{P}(B \mid \sigma_t(h_t), a_t).$$

Information state: Key result

An information state **always** leads to a dynamic programming decomposition.

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Let $\{Z_t\}_{t \geqslant 1}$ be any information state process. Let $\hat{V}$ be the fixed point of:

$$\hat{V}(z) = \max_{a \in \mathcal{A}} \Big\{ \hat{r}(z, a) + \gamma \int_{\mathcal{Z}} \hat{V}(z_+) \hat{P}(dz_+ | z, a) \Big\}$$

Let $\pi^*(z)$ denote the arg max of the RHS. **Then, the policy $\pi = (\pi_t)_{t \geqslant 1}$ given by $\pi_t = \pi^* \circ \sigma_t$ is optimal.**

Examples of information state

Markov decision processes (MDP)

Current state S_t is an info state

POMDP

Belief state is an info state

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MDP with delayed observations

$(S_{t-\delta+1}, A_{t-\delta+1:t-1})$ is an info state

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Belief state is an info state

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$(\mathbb{P}(S_{t-\delta} | Y_{1:t-\delta}, A_{1:t-\delta}), A_{t-\delta+1:t-1})$
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Linear Quadratic Gaussian (LQG)

The state estimate $\mathbb{E}[S_t | H_t]$ is an info state

Machine Maintenance

(τ, S_τ^+) is info state,
where τ is the time of last maintenance

And now to Approximate Information States . . .

Main idea

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Main idea

- Info state is defined in terms of two properties (P1) & (P2).
- An AIS is a process which satisfies these approximately

- Show that AIS always leads to approx. DP
- Recover (and improve up on) many existing results

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$$\left| \mathbb{E}[R_t \mid H_t = h_t, A_t = a_t] - \hat{r}(\sigma_t(h_t), a_t) \right| \leqslant \varepsilon$$

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(AP2) $\hat{P}$ **is sufficient for** approximately **predicting next AIS:**

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Results depend on the choice of metric on probability spaces

AIS based approximation bounds

Let V denote the optimal value and $\hat{V}$ denote the fixed point of the following equations:

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$$\hat{V}(z) = \max_{a \in \mathcal{A}} \left\{ \hat{r}(z, a) + \gamma \int_{\mathcal{Z}} \hat{V}(z_+) \hat{P}(dz_+ | z, a) \right\}$$

Value function approximation

The value function $\hat{V}$ is approximately optimal, i.e.,

$$|V_t(h_t) - \hat{V}(\sigma_t(h_t))| \leqslant \alpha := \frac{\varepsilon + \gamma \rho_{\mathfrak{F}}(\hat{V}) \delta}{1 - \gamma}.$$

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Policy approximation

Let $\hat{\pi}^*\colon \mathcal{Z} \to \Delta(\mathcal{A})$ be an optimal policy for $\hat{V}$.

Then, the policy $\pi = (\pi_1, \pi_2, \dots)$ where $\pi_t = \hat{\pi}^* \circ \sigma_t$ is approx. optimal:

$$V_t(h_t) - V_t^{\pi}(h_t) \leqslant 2\alpha.$$

Some remarks on AIS

- Two ways to interpret the results:
  - Given the information state space $\mathcal{Z}$, find the best compression $\sigma_t \colon \mathcal{H}_t \to \mathcal{Z}$
  - Given any compression function $\sigma_t \colon \mathcal{H}_t \to \mathcal{Z}$, find the approximation error.

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- Most of the existing literature on approximate DPs focuses on the first interpretation
- The second interpretation allows us to develop AIS-based RL algorithms

- Results depend on the choice of metric on probability spaces.
- The bounds use what are known as integral probability metrics (IPM), which include many commonly used metrics:
  - Total variation
  - Wasserstein distance
  - Maximum mean discrepancy (MMD)

Examples of AIS

Example 1: Robustness to model mismatch in MDPs

Real-world model (P, r)

Simulation model $(\hat{P}, \hat{r})$

What is the loss in performance if we choose a policy using the simulation model and use it in the real world?

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Model mismatch as an AIS

- (Identity, $\hat{P}$, $\hat{r}$) is an (ε, δ)-AIS with $\varepsilon = \sup_{s,a} \left| r(s,a) - \hat{r}(s,a) \right|$ and $\delta_{\mathfrak{F}} = \sup_{s,a} d_{\mathfrak{F}}(P(\cdot|s,a), \hat{P}(\cdot|s,a))$.

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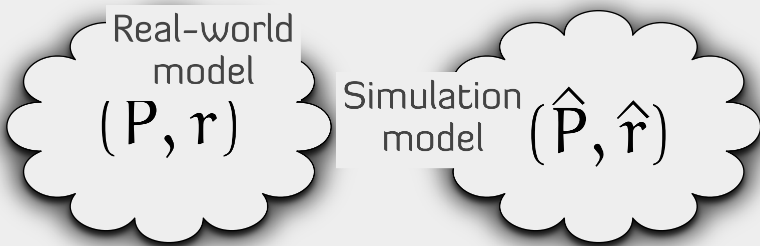

Müller, "How does the value function of a Markov decision process depend on the transition probabilities?" MOR 1997.

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$d_{\mathfrak{F}}$ is total variation

$$V(s) - V^{\pi}(s) \leqslant \frac{2\varepsilon}{1-\gamma} + \frac{\gamma\delta\,\text{span}(r)}{(1-\gamma)^2}$$

Recover bounds of Müller (1997).

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Asadi, Misra, Littman, "Lipscitz continuity in model-based reinfocement learning," ICML 2018.

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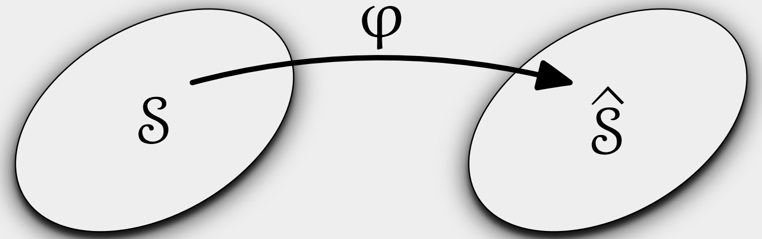
$$V(s) - V^{\pi}(s) \leqslant \frac{2\varepsilon}{1-\gamma} + \frac{2\gamma\delta L_r}{(1-\gamma)(1-\gamma L_p)}$$

Recover bounds of Asadi, Misra, Littman (2018).

Example 2: Feature abstraction in MDPs



$(\widehat{P}, \hat{r})$ is determined from (P, r) using φ

What is the loss in performance if we choose a policy using the abstract model and use it in the original model?

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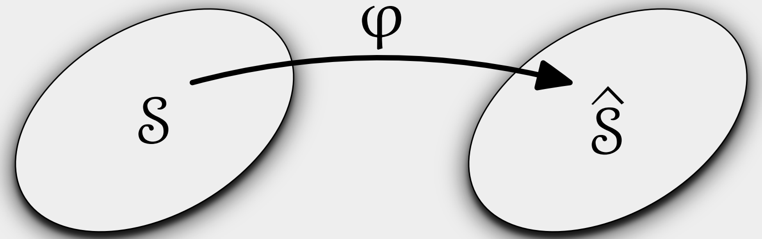

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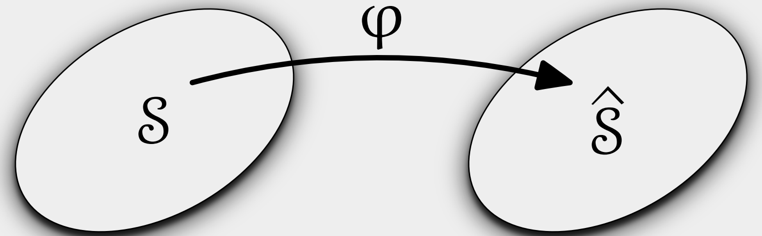

Abel, Hershkowitz, Littman, "Near optimal behavior via approximate state abstraction," ICML 2016.

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Improve bounds of Abel et al. (2016)

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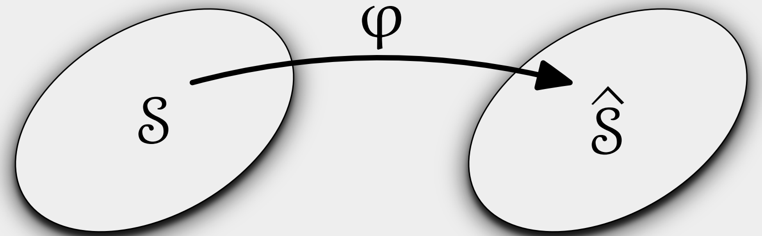

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Abel, Hershkowitz, Littman, "Near optimal behavior via approximate state abstraction," ICML 2016.

Gelada, Kumar, Buckman, Nachum, Bellemare, "DeepMDP: Learning continuous latent space models for representation learning," ICML 2019.

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$d_{\mathfrak{F}}$ is Wasserstein distance

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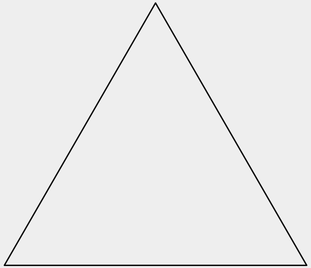
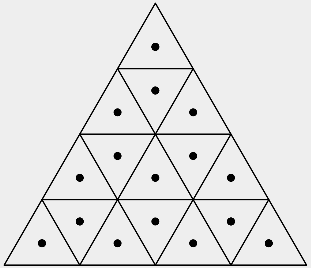
Recover bounds of Gelada et al. (2019).

Example 3: Belief approximation in POMDPs



Belief space



Quantized beliefs

What is the loss in performance if we choose a policy using the approximate beliefs and use it in the original model?

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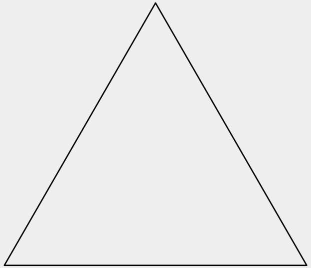

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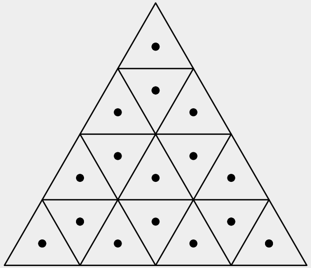

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Belief approximation in POMDPs

- Quantized cells of radius ε (in terms of total variation) are $(\varepsilon\|r\|_\infty, 3\varepsilon)$-AIS.

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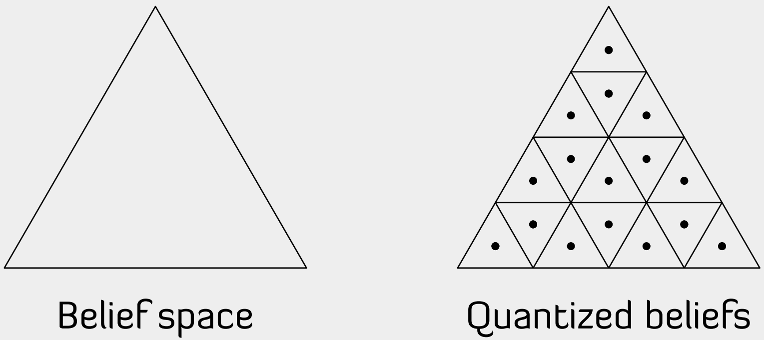


Francois-Lavet, Rabusseau, Pineau, Ernst, Fonteneau, "On overfitting and asymptotic bias in batch reinforcement learning with partial observability," JAIR 2019.

Belief approximation in POMDPs

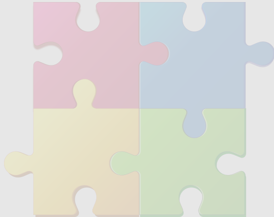
- Quantized cells of radius ε (in terms of total variation) are $(\varepsilon\|r\|_\infty, 3\varepsilon)$-AIS.

$$V(s) - V^\pi(s) \leqslant \frac{2\varepsilon\|r\|_\infty}{1-\gamma} + \frac{6\gamma\varepsilon\|r\|_\infty}{(1-\gamma)^2}$$

Improve bounds of Francois Lavet et al. (2019) by a factor of $1/(1-\gamma)$.

Thus, the notion of AIS unifies many of the approximation results in the literature, both for MDPs and POMDPs.

Outline

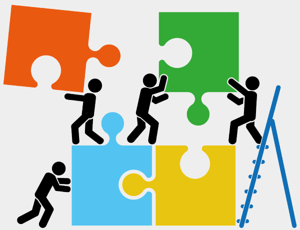


Background

- Review of MDPs and RL
- Review of POMDPs
- Why is RL for POMDPs difficult?

Approximate Planning for POMDPs

- Preliminaries on information state
- Approximate information state
- Approximation bounds



RL for POMDPs

- From approximation bounds to RL
- Numerical experiments

From approximation bounds to reinforcement learning. . .

Main idea

- AIS is defined in terms of two losses ε and δ.
- Minimizing ε and δ will minimize the AIS approximation loss.

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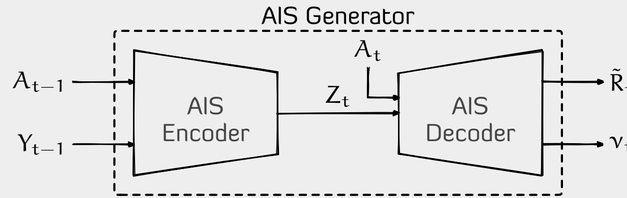
- AIS is defined in terms of two losses ε and δ.
- Minimizing ε and δ will minimize the AIS approximation loss.

- Use $\lambda\varepsilon^2 + (1-\lambda)\delta^2$ as surrogate loss for the AIS generator
- . . . and combine it with standard actor-critic algorithm using multi-timescale stochastic approximation.

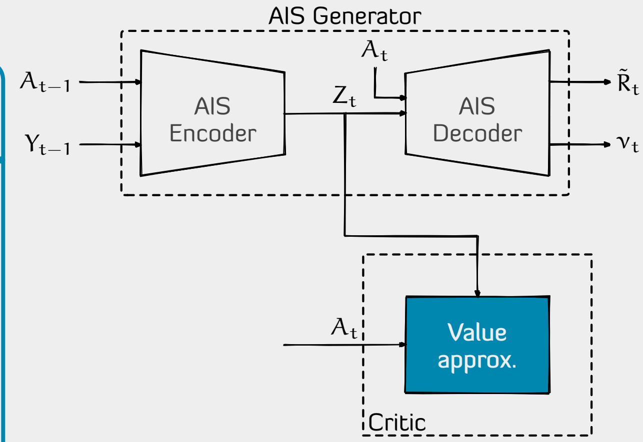
Reinforcement learning setup



AIS Generator

- Use LSTM for $\sigma_t\colon \mathcal{H}_t \to \mathcal{Z}$ and a NN for functions $\hat{r}$ and $\hat{P}$.
- Use $\lambda(\tilde{R}_t - R_t)^2 + (1-\lambda) d_{\mathfrak{F}}(\mu_t, \nu_t)^2$ as surrogate loss.
- We show that $\nabla d_{\mathfrak{F}}(\mu_t, \nu_t)^2$ can be computed efficiently for Wasserstein distance and MMD.

Reinforcement learning setup



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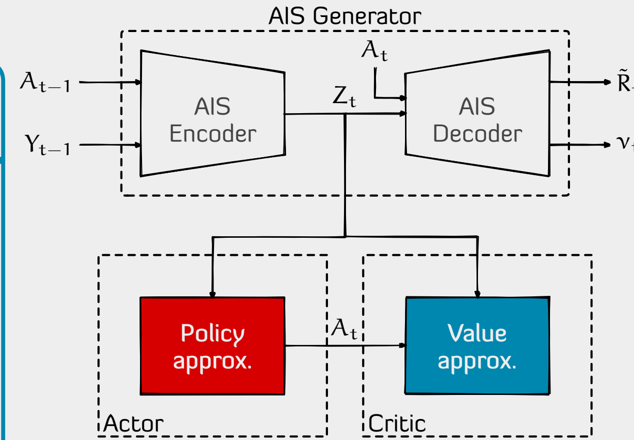
Value approximator

- Use a NN to approx. action-value function $Q : \mathcal{Z} \times \mathcal{A} \to \mathbb{R}$.
- Update the parameters to minimize temporal difference loss

Reinforcement learning setup



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Policy approximator

- Use a NN to approx. policy $\pi \colon \mathcal{Z} \to \Delta(\mathcal{A})$.
- Use policy gradient theorem to efficiently compute $\nabla J(\pi)$.

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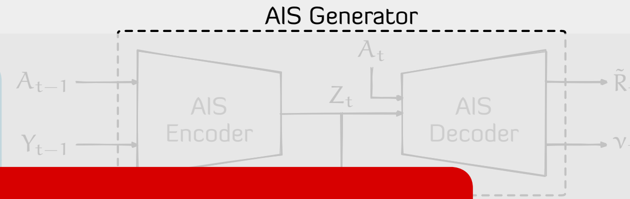

AIS Generator

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- Use $\lambda(\tilde{R}_t - R$
- We show th ciently for W

Convergence Guarantees

- Use multi-timescale stochastic approximation to simultaneously learn AIS generator, action-value function, and policy.
- Under appropriate technical assumptions, converges to the stationary point corresponding to the choice of function approximators.

Poli

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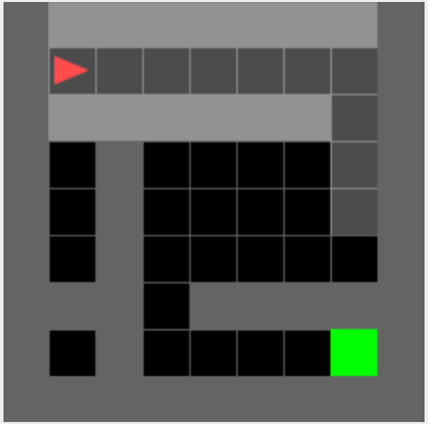
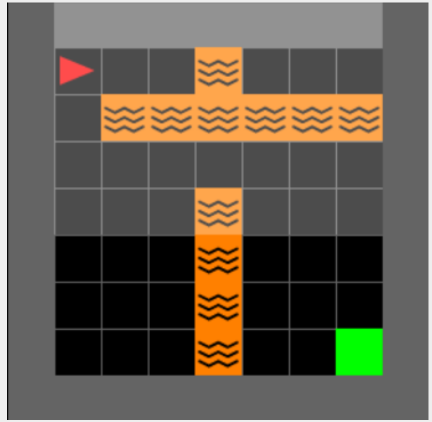
Numerical Experiments

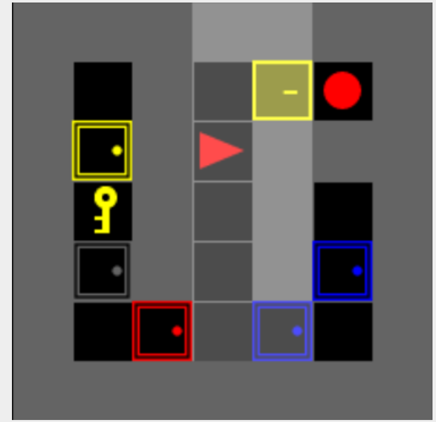
MiniGrid Environments



Simple Crossing



Lava Crossing

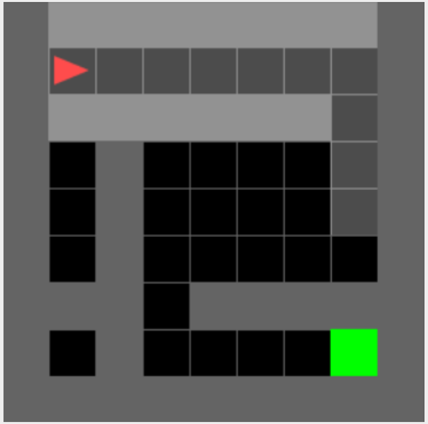


Key Corridor

Features

- Partially observable 2D grids. Agent has a view of a 7×7 field in front of it. Observations are obstructed by walls.
- Multiple entities (agents, walls, lava, boxes, doors, and keys)
- Multiple actions (Move Forward, Turn Left, Turn Right, Open Door/Box, . . .)

MiniGrid Environments



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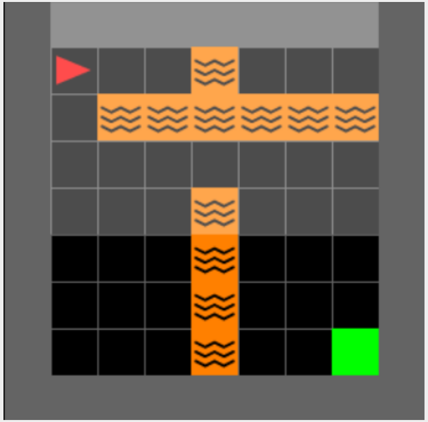

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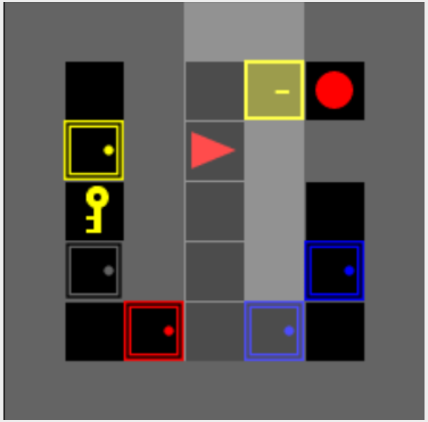


Key Corridor

Algorithms

AIS + MMD

AIS with MMD as IPM

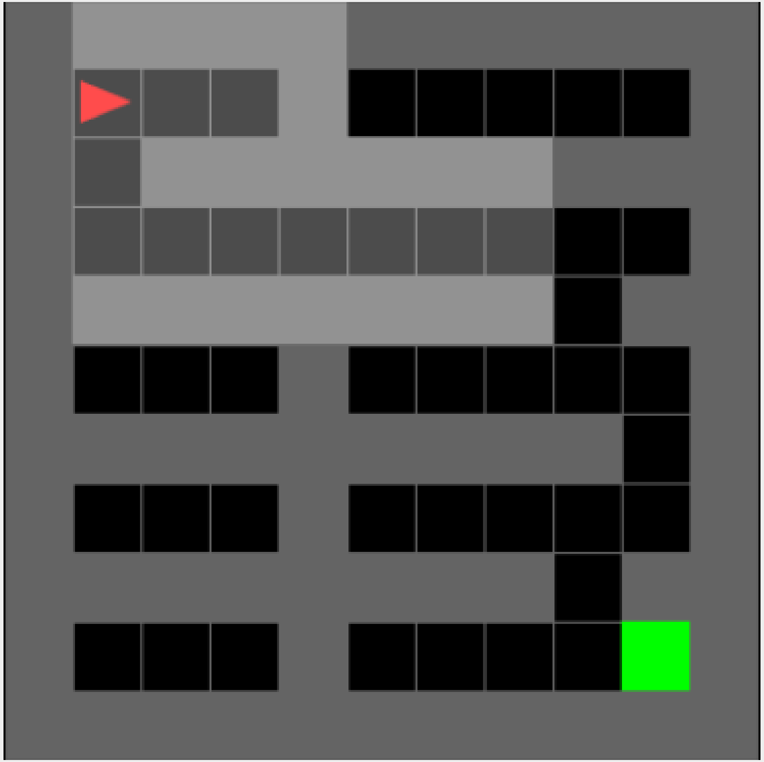
AIS + KL

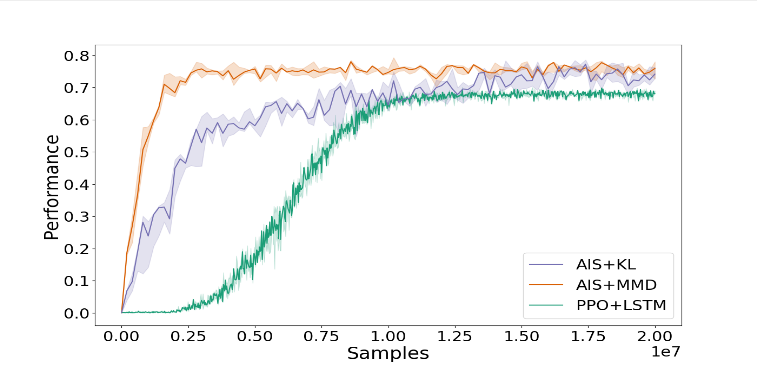
AIS with KL as upper bound of Wasserstein distance

PPO + LSTM

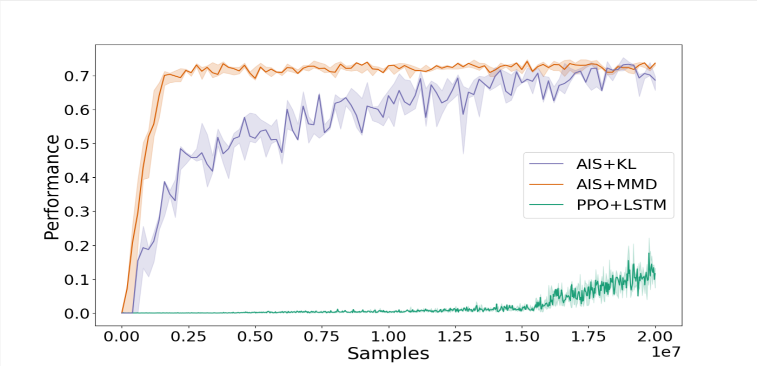
Baseline proposed in paper introducing minigrid envs

Simple Crossing



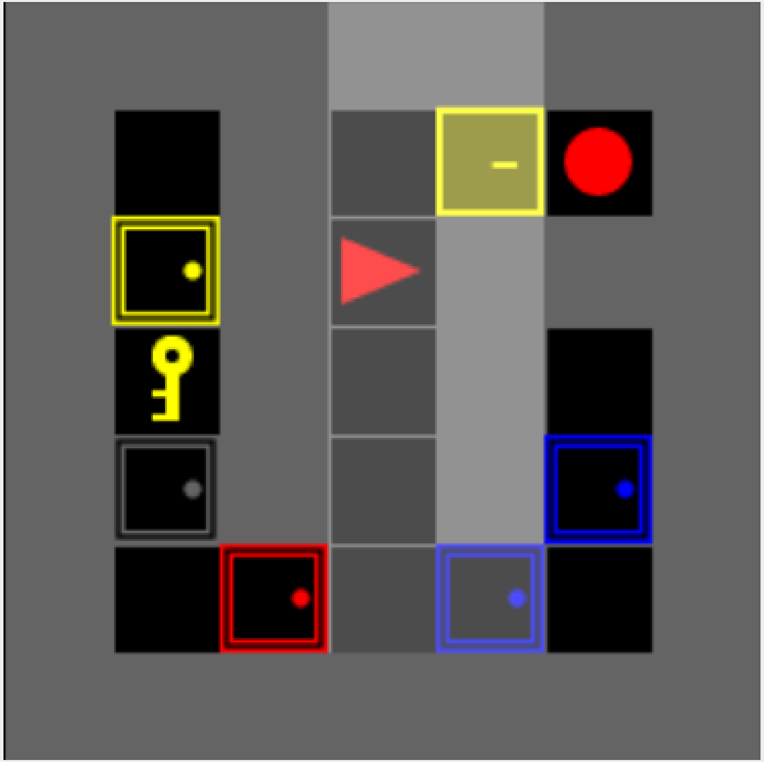


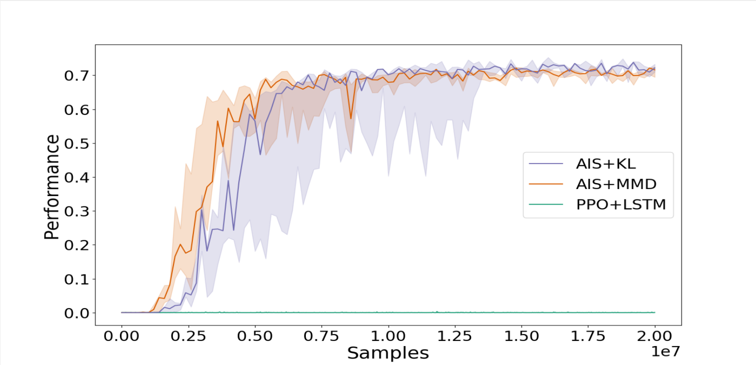
Simple Crossing S9N3



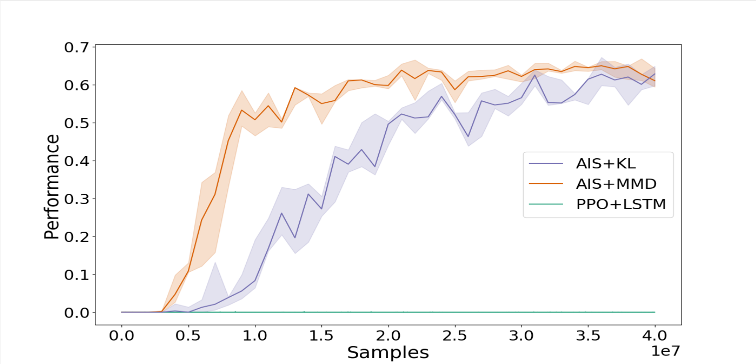
Simple Crossing S11N5

Key Corridor



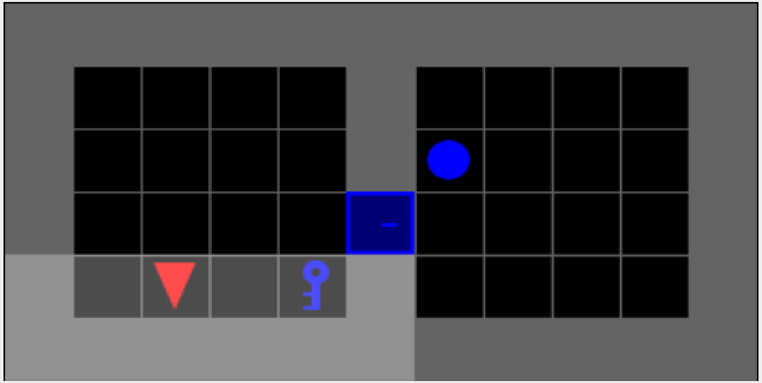


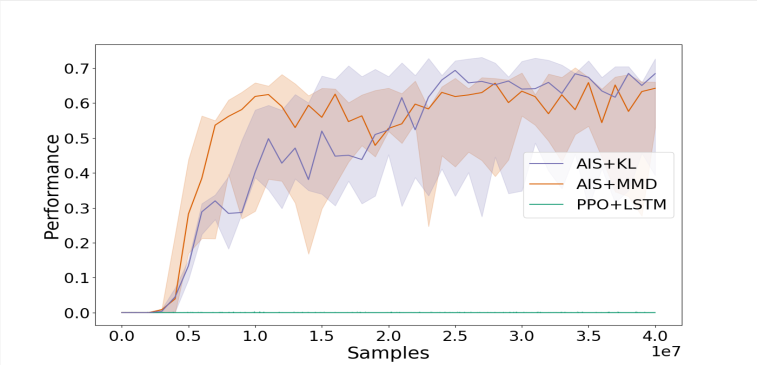
Key Corridor S3R2



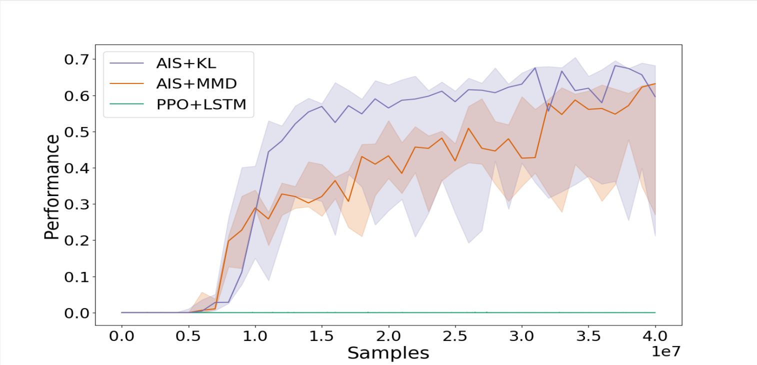
Key Corridor S3R3

Obstructed Maze





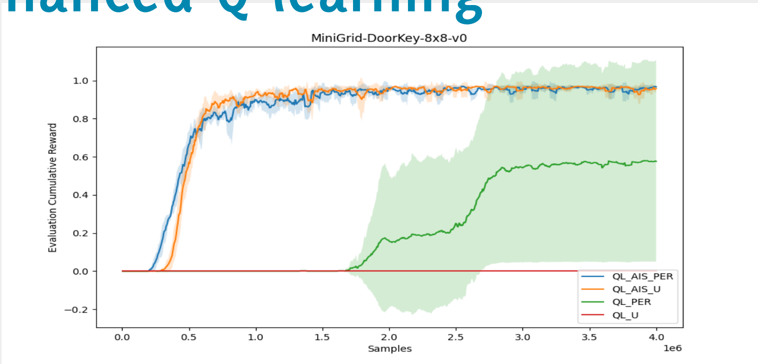
Obstructed Maze 1Dl



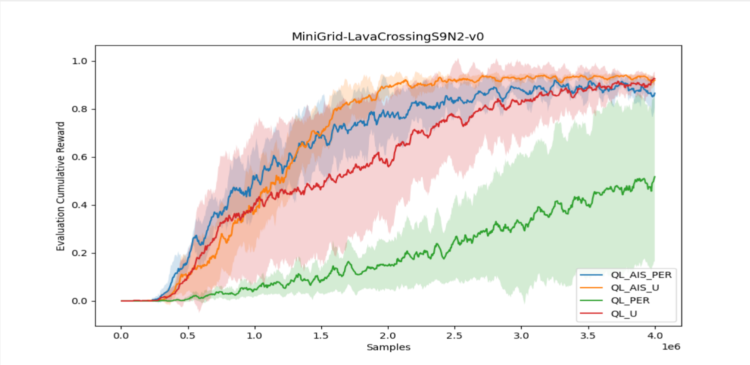
Obstructed Maze 1Dlh

Similar improvements for AIS-enhanced Q-learning

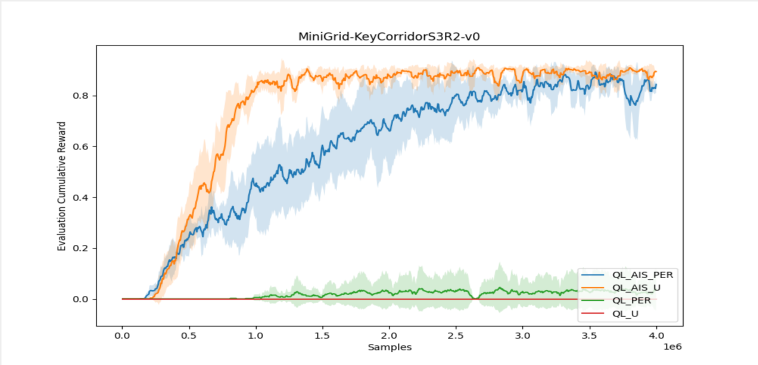
Simple Crossing S9N3



Doorkey 8x8



Lava Crossing S9N2



Key Corridor S3R2

Summary

A conceptually clean framework for approximate DP and online RL in partially observed systems

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Approximation results generalize to

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- action quantization
- lifelong learning
- multi-agent teams
- Markov games

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- Markov games

Ongoing work

- Other RL settings such as model based RL, offline RL, inverse RL.
- A building block for multi-agent RL.
- . . .

- email: aditya.mahajan@mcgill.ca
- web: http://cim.mcgill.ca/~adityam

Thank you

Funding: NSERC, DND

- paper: JMLR, Feb 2022
- code: https://github.com/info-structures/ais