Value of Common Information in Static Teams

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Decentralized decision making involves multiple agents that act based on local information. It is an important area of research with applications in networked control systems, sensor networks, traffic management, vehicle coordination, and economics. When all agents have a common cost (or utility) function, the problem is called a *team*; when agents have individual cost (or utility), the problem is called a *game*. A team is called *static* if agents do not affect observations of others; it is called *dynamic* if agents affect observations of others.

Consider a static team with n agents. Agent i observes y_i and chooses action u_i according to decision rule g_i, i.e., $u_i = g_i(y_i)$. The performance is measured by the following cost function $c(x, u_1, \dots, u_n)$, where x is an unobserved random variable. The objective is to choose decision rules to minimize $J(g) = \mathbb{E}^g[c(x, u_1, \dots, u_n)]$.

Radner [1] investigated the static team problem when $(x, y_1, \dots, y_n)$ are jointly Gaussian and the cost function is quadratic and of the form

$$c(x, u_1, \dots, u_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} u_i^\intercal R_{ij} u_j + 2 \sum_{i=1}^{n} u_i^\intercal P_i x,$$

and showed that the optimal decision rules are given by

$$u_i = K_i(y_i - \mathbb{E}[y_i]) + H_i \mathbb{E}[x]$$

where K_i and H_i are computed by solving a system of linear equations.

In this paper, we address the following question. Suppose it is possible to provide an additional observation z to all agents. *What is the value of such information?* In particular, let z denote the additional common observation. Thus, agent i observes (y_i, z). To differentiate from the previous model, we denote the action of agent i by $\tilde{u}_i$ and the decision rule by $\tilde{g}_i$; thus, $\tilde{u}_i = \tilde{g}_i(y_i, z)$. The cost function $c(\cdot, \cdot)$ is the same as before. In this case, the value of information is defined as the reduction in the expected cost due to the availability of z.

We present two approaches to identify the optimal decision rules. In the first approach, all agents form a common posterior on $(x, y_1, \dots, y_n)$ given z. Such a conditional system is equivalent to the standard static team problem and we show that the optimal decision rules are given by

$$\tilde{u}_i = \tilde{K}_i(y_i - \mathbb{E}[y_i | z]) + H_i \mathbb{E}[x | z],$$

where H_i are same as before and $\tilde{K}_i$ are computed by solving a system of linear equations. Using the above expression for the decision rules, we derive a formula for the value of information.

In the second approach, we assume a hierarchical structure. Agents compute *local* control actions

$$\hat{u}_i = \tilde{K}_i(y_i - \mathbb{E}[y_i]) + H_i \mathbb{E}[x],$$

that do not depend on the common information. A global coordinator then computes a coorective bias $[L_1 \ \dots \ L_n](z - \mathbb{E}[z])$ and sends the i-th component to agent i. Agent i then chooses the action

$$\tilde{u}_i = \hat{u}_i + L_i(z - \mathbb{E}[z]) = \tilde{K}_i(y_i - \mathbb{E}[y_i]) + H_i \mathbb{E}[x] + L_i(z - \mathbb{E}[z]).$$

Using the above expression, we derive another formula for the value of information.

References

[1] R. Radner, "Team decision problems," *Annals Math. Stats.*, vol. 33, pp. 857–881, 1962.