

Electronic Circuits

Lab Report 1

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1 Aim

1. To study the response of an RC circuit as a low-pass Filter and a high-pass Filter.
2. To study the response of an LCR circuit in different configurations.

2 Equipment

1. Set of resistors, capacitors and inductors.
2. Breadboard
3. Function Generator
4. Digital Storage Oscilloscope
5. BNC T-connector
6. BNC cables

3 Experimental Setup

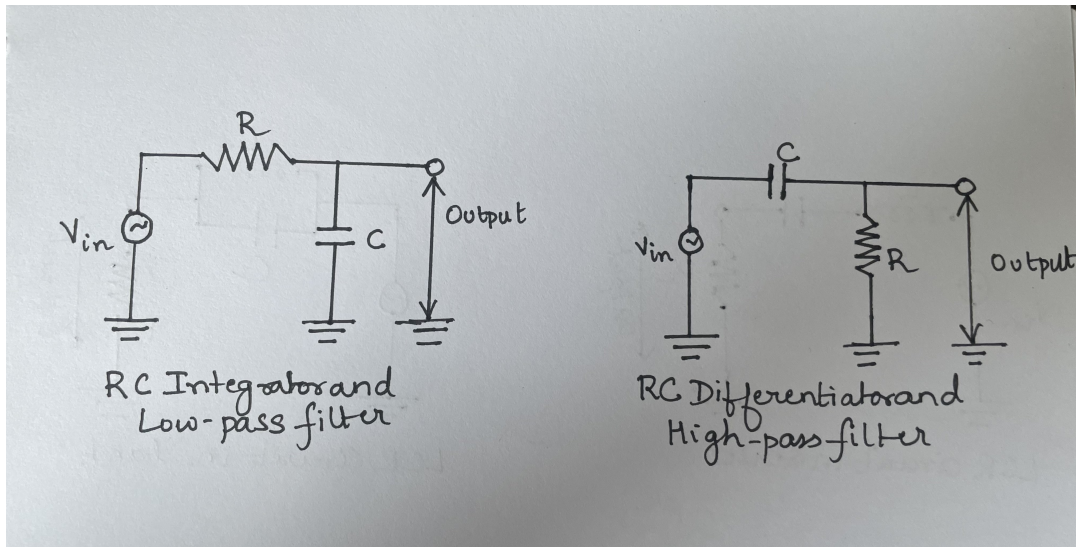


Figure 1: RC integrator and differentiator circuits

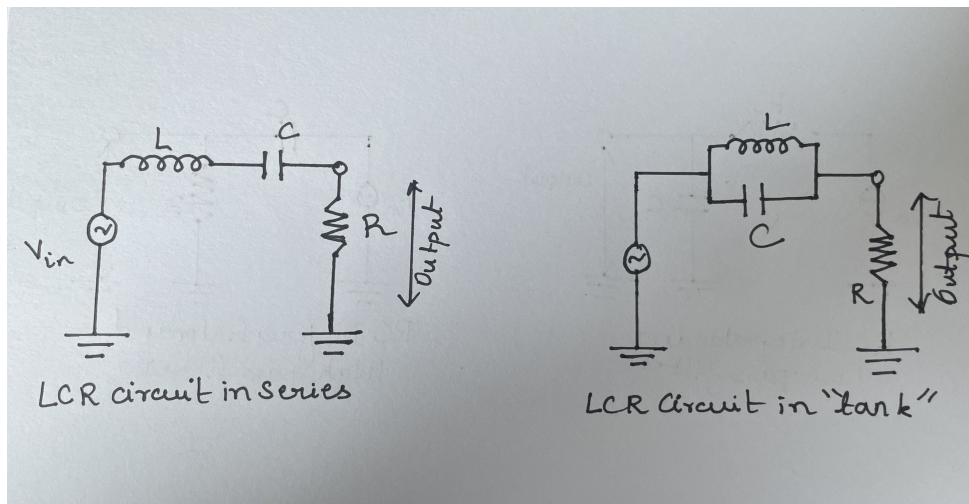


Figure 2: LCR circuit in series and tank configuration

4 Theoretical Background

Resistors are electrical components that impede the flow of current in a circuit. In a circuit with a resistor the voltage applied is proportional to the current flowing through it with a proportionality constant which is known as the resistance of the circuit (Ohm's Law),

$$V = IR$$

Capacitors are components which consist of two conducting plates connected to conducting wires. As a potential difference is applied charge builds up on both plates since they are not in contact. The voltage across the plates is proportional to the charge buildup with a proportion-

ality constant known as the capacitance,

$$V = \frac{Q}{C}$$

Inductors are components which produce a magnetic field which is stored within the coils of the loop (in an ideal inductor) when a voltage is applied across it. Since the voltage applied is varying, the varying current in the coils of the inductor causes variation in the magnetic field. This changing magnetic field produces an electromotive force (Lenz's Law) which induces a voltage in the circuit which has opposite polarity. The induced voltage is proportional to the variation of the current and the constant of proportionality is known as the inductance of the coil,

$$V = L \frac{dI}{dt}$$

The purpose of this experiment is to study the response of resistors, capacitors and inductors to alternating current. To study such responses sinusoidally varying voltages are used that can be expressed in terms of a complex exponent as:

$$V = \tilde{V} e^{i\omega t}$$

Where \tilde{V} (which can be expressed as $V_0 e^{i\phi}$ where ϕ is the phase difference) is a complex number independent of time. Complex exponents are used since it is easier to perform mathematical operations. An actual time-varying voltage is given by just the real part of the complex exponent.

In a time varying sinusoidal voltage, the current flowing through each component described above can be expressed as,

$$\begin{aligned} I_R &= \frac{V_0}{R} \cos \omega t \\ I_C &= -\omega C V_0 \sin \omega t \\ I_L &= \frac{V_0}{\omega L} \sin \omega t \end{aligned}$$

From these 3 relations it is evident that in a resistor the current and voltage are in the same phase. In a capacitor, the current leads the voltage by a phase of $\pi/2$ while in an inductor, the current lags behind the voltage by a phase of $\pi/2$.

4.1 RC Circuit

An RC circuit is one where a resistor and a capacitor are connected either in series or in parallel. When an AC voltage is applied across such a circuit, it can be used as an integrator or a differentiator. In other words, specific configurations of the circuit can be used to visualize the mathematical process of integration and differentiation. There are two voltages in the circuit, the input and the output,

$$V_{in} = \tilde{V}_{in} e^{i\omega t} \quad V_{out} = \tilde{V}_{out} e^{i\omega t}$$

4.1.1 RC Circuit as Integrator and Differentiator

An RC circuit works as an integrator for high frequency currents, i.e.,

$$\omega \gg \frac{1}{RC}$$

Over this very short time scale the capacitor cannot build up sufficient charge and thus the voltage drop is almost completely across the resistor, i.e., $V_{in} \approx V_R$. Since the circuit is in series the current and thus,

$$V_{in} \approx RC \frac{dV_C}{dt} \implies V_C \approx \frac{1}{RC} \int_0^t V_{in}(t') dt'$$

Thus in this configuration measuring the voltage across the capacitor can be used to visualize as the integral of the waveform of the input voltage. For this, voltage of a low frequency is chosen, i.e.,

$$\omega \ll \frac{1}{RC}$$

For this circuit the capacitor has enough time to build up voltage and thus the input voltage is almost completely across the capacitor, i.e., $V_{in} \approx V_C$. The circuit is in series and thus the current through the capacitor runs through the entire circuit. Thus,

$$V_R = IR \implies V_R \approx RC \frac{dV_{in}}{dt}$$

Hence the output voltage, i.e. the voltage across the resistor can be visualized as the derivative of the waveform of the input voltage.

4.1.2 RC Circuit as high and low pass filter

In the integrator configuration the voltage in the circuit is almost completely across the resistor at a high frequency, i.e., the output voltage is almost 0 across the capacitor. This means that such a circuit blocks the passage of high frequency currents and allows low frequencies to pass. In such a configuration the circuit is said to be a low-pass filter.

Similarly in the differentiator configuration the voltage in the circuit is almost completely across the capacitor at a low frequency. Such a configuration of the RC circuit blocks the passage of low frequency currents and allows high frequency currents to pass. This configuration is known as a high-pass filter.

4.1.3 Gain and Bode Plots

To study responses to time varying voltages, a quantity known as the gain is defined. It is the logarithm base 10 of the ratio of the power of the output to the input signal. The unit is known as "decibel",

$$\text{gain in dB} = 10 \log_{10} \left(\frac{P_{out}}{P_{in}} \right)$$

Since power is proportional to the square of the voltage the equation becomes,

$$\text{gain in dB} = 20 \log_{10} \left(\frac{V_{out}}{V_{in}} \right)$$

Gain in a high pass and low pass filter can be graphically analyzed with Bode plots. Bode plots are graphs of the gain (dB) versus the frequency in a logarithmic scale. The formula of gain in

terms of the frequency for low-pass and high-pass filter respectively are,

$$\text{gain}_{\text{low-pass}} = \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}}$$

$$\text{gain}_{\text{high-pass}} = \frac{(\omega/\omega_c)^2}{\sqrt{1 + (\omega/\omega_c)^2}}$$

The Bode plots are,

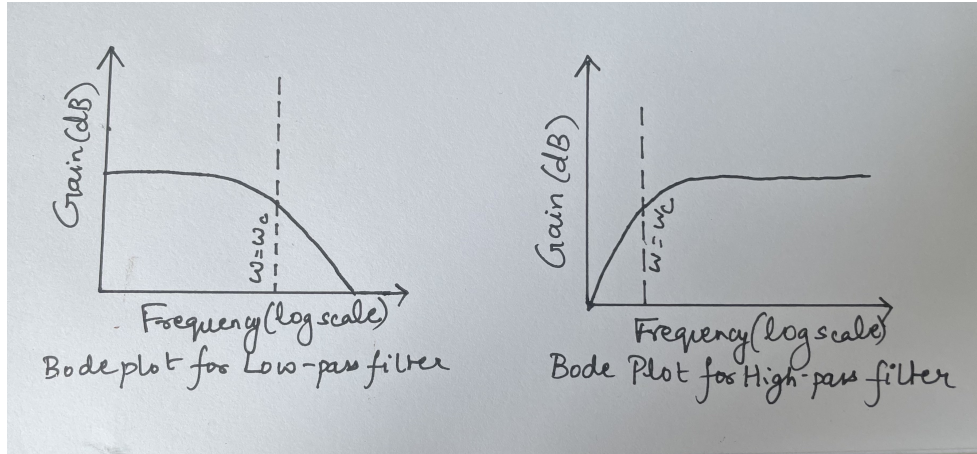


Figure 3: Bode plots for low-pass and high-pass filters. In both plots the vertical line $\omega = \omega_c$ marks the point beyond which there is a change in the graph. For a low-pass filter the gain is constant till the cutoff frequency and then reduces sharply. The opposite happens for a high-pass filter.

4.2 LCR Circuit

The LCR circuit is another kind of electronic circuit with resistors, inductors and capacitors connected and an external voltage applied. The differential equation governing the current flowing through the circuit is same as the equation of a damped harmonic oscillator where the natural frequency (ω_0) is equal to $1/\sqrt{LC}$ and the damping constant is (γ) is equal to R/L .

When the resistor, inductor and capacitor are all connected in series the current in the circuit is expressed as,

$$I = \frac{V_{in}}{R + i\left(\omega L - \frac{1}{\omega C}\right)}$$

It is evident that the highest current in the circuit will be drawn when $\omega L = 1/\omega C$. The frequency at which this occurs is known as the resonant frequency and the phenomenon is called resonance. Thus,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Another possible combination is where the inductor and the capacitor is connected in parallel and this is connected in series with the resistor. This circuit is also known as a tank circuit. The current in the circuit can be expressed as,

$$I = \frac{V_{in}}{R + \left(\frac{i}{\omega C - \frac{1}{\omega L}}\right)}$$

It is evident that at resonance the impedance of the circuit is almost infinite. The current in the circuit at resonance will be almost 0 which means the gain in the circuit will also be almost 0 or even negative.

Note: All the equations written above are derived in the Appendix at the end

5 Procedure

5.1 Part A: RC Circuit as integrator and differentiator

1. The circuit was connected as shown in the circuit diagram in figure 1. This configuration works as an integrator. The values of the components used were, $R = 4.7 \times 10^5 \Omega$ and $C = 0.001 \mu\text{F}$.
2. A high frequency alternating voltage was applied across the circuit. The input wave was a square wave.
3. The output waveform was viewed on the oscilloscope.
4. For the differentiator, the circuit was made as in the circuit diagram
5. A low frequency alternating voltage was applied across the circuit and the output was viewed on the oscilloscope.

5.2 Part B: Studying frequency response of low-pass and high-pass filter circuits

1. A low-pass filter circuit was made as shown in the circuit diagram in figure 1. The components used were the same as in Part A of the experiment.
2. Alternating voltage with frequency $\omega \ll \omega_c$ was applied across the circuit.
3. The voltage was gradually increased and the peak to peak voltage of both the input and the output signal were noted. The voltage was increased far beyond the cutoff voltage, i.e., $\omega \gg \omega_c$.
4. The gain in dB was calculated and plotted against log of the frequency of the input signal to obtain the first Bode plot.
5. The same process was followed for a high pass filter to obtain another Bode plot.

5.3 Part C: Studying frequency response in an LCR circuit

1. The circuit was made as shown in the circuit diagram in figure 2. The values of the components in the circuit were, $L = 30 \text{ mH}$, $R = 330 \Omega$, and $C = 0.01 \mu\text{F}$.
2. Sinusoidally varying current was passed into the circuit. The initial frequency was low and then increased gradually to past the natural frequency of the circuit.
3. The peak to peak voltage of both the input and output signal were noted from the oscilloscope and used to calculate the gain at each point.
4. A Bode plot for this circuit was made to study the frequency response.
5. The same process was repeated for the “tank” configuration of the LCR circuit.

6 Data

6.1 Part A: RC Circuit as an integrator and differentiator

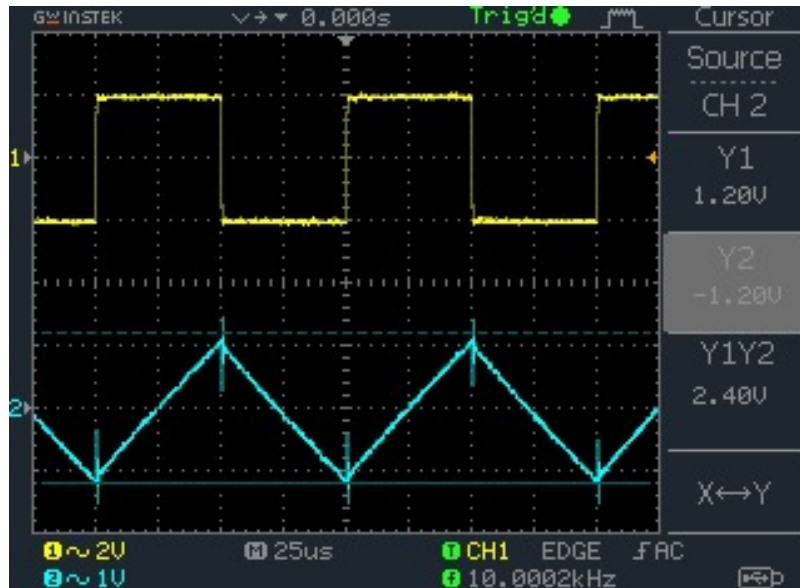


Figure 4: RC Circuit as integrator. Here the input signal is a square wave and the output is in blue

6.2 Part B: Studying frequency response of low-pass and high-pass filter circuits

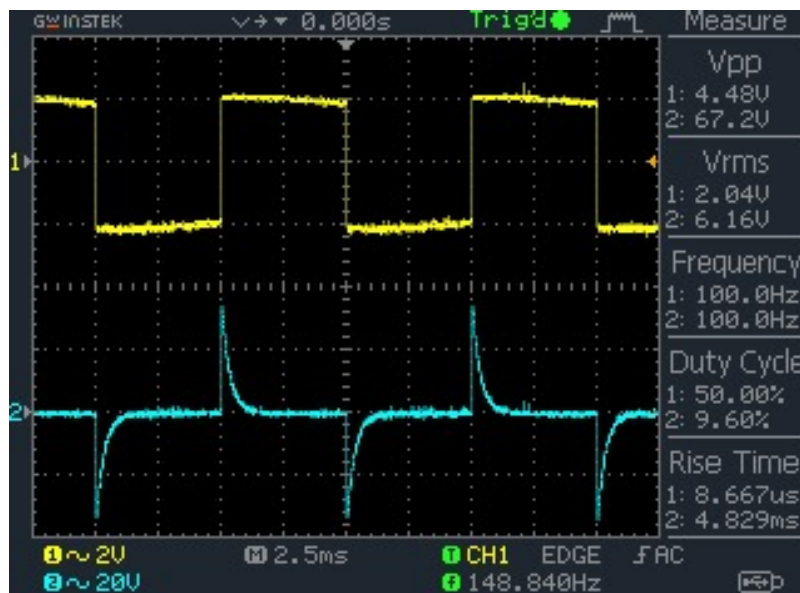


Figure 5: RC Circuit as a differentiator. Here the input signal is a square wave and the output is in blue

Input Frequency (Hz)	Gain (dB)
100	19.82
200	19.66
350	19.82
500	19.65
700	19.28
1000	18.89
1200	18.49
1600	17.15
1750	16.90
2000	16.39
2500	15.56
3000	13.98
4000	12.04
5000	10.10
6000	8.94
7000	7.60
9500	6.02
11000	4.08
14000	1.58
17000	1.58

Table 1: Gain for different frequencies of voltage passed through the low-pass RC configuration

Input Frequency (Hz)	Gain (dB)
100	-4.82
200	0.95
350	5.67
300	4.27
450	7.62
500	8.63
600	9.93
750	11.65
1000	13.63
1200	14.81
1400	15.71
1600	16.26
2000	17.38
2500	18.06
3000	18.49
4000	18.99
5000	19.18
6000	19.37
7000	19.55
8000	19.55
9500	19.55
11000	19.55
14000	19.91
17000	19.82
20000	19.71
22500	19.71
35000	19.83
50000	19.83

Table 2: Gain for different frequencies of voltage passed through the high-pass RC configuration

6.3 Part C: Studying frequency response in an LCR circuit

Input Frequency (Hz)	Gain (dB)
500	0.60
700	4.25
1000	7.04
1250	8.98
1500	11.35
2000	14.88
2250	16.32
2500	17.57
2750	18.26
2900	18.26
3200	17.76
3500	17.08
4000	15.84
4500	14.51
5000	13.42
6000	11.24
7500	8.31
9000	6.44
11000	4.61
14000	2.92
17000	0.83
20000	0.00

Table 3: Gain for different frequencies of voltage passed through an LCR circuit in series configuration

Input Frequency (Hz)	Gain (dB)
200	18.37
500	18.30
750	17.82
1000	16.98
1250	16.33
1500	15.19
1750	13.60
2000	11.90
2250	9.12
2500	5.28
2750	1.18
3000	-1.94
3250	2.28
3500	6.62
4000	10.00
4500	13.04
5000	14.67
5500	15.95
6000	16.39
6500	16.93
7000	17.50
7500	17.75
8750	18.42
9500	18.53
12500	18.98
15000	19.39
20000	19.39
25000	19.60
30000	19.70
40000	19.90

Table 4: Gain for different frequencies of voltage passed through an LCR circuit in “tank” configuration

7 Graphs and Analysis

7.1 Part A

A square wave can be divided into three parts- two lines parallel to the y-axis and a line parallel to the x-axis. Thus, the equations for the three sections can be written as,

$$x = \pm\lambda$$

For the lines parallel to the y-axis where λ is a constant and as

$$y = \mu$$

For the line parallel to the x-axis where μ is another constant.

Integrating these functions would give a straight line with slope equal to ± 1 for the part of the input which is parallel to the x-axis as can be seen in figure 4.

In the output of the differentiator circuit in figure 5 it can be seen that there are spikes at exactly the points where the square wave rises or falls. This is so because the slope (hence

the derivative) of a sharp rise or fall itself is a sharp rise or fall. For the part parallel to the x-axis, the derivative is 0 and thus the output is almost 0 at those points. However there is a little smooth fall and rise from the spike in the output since the capacitor takes some time to discharge.

7.2 Part B

The Bode plots obtained for the low-pass and high-pass circuits are shown below. The vertical line in each graph marks the cutoff frequency. The line was made at the point where the gain is equal to its root mean square value, i.e., $\text{gain}/\sqrt{2}$. The calculated cutoff frequency, i.e., the cutoff frequency measured using the values of the resistance and capacitance was around 9.90 kHz.

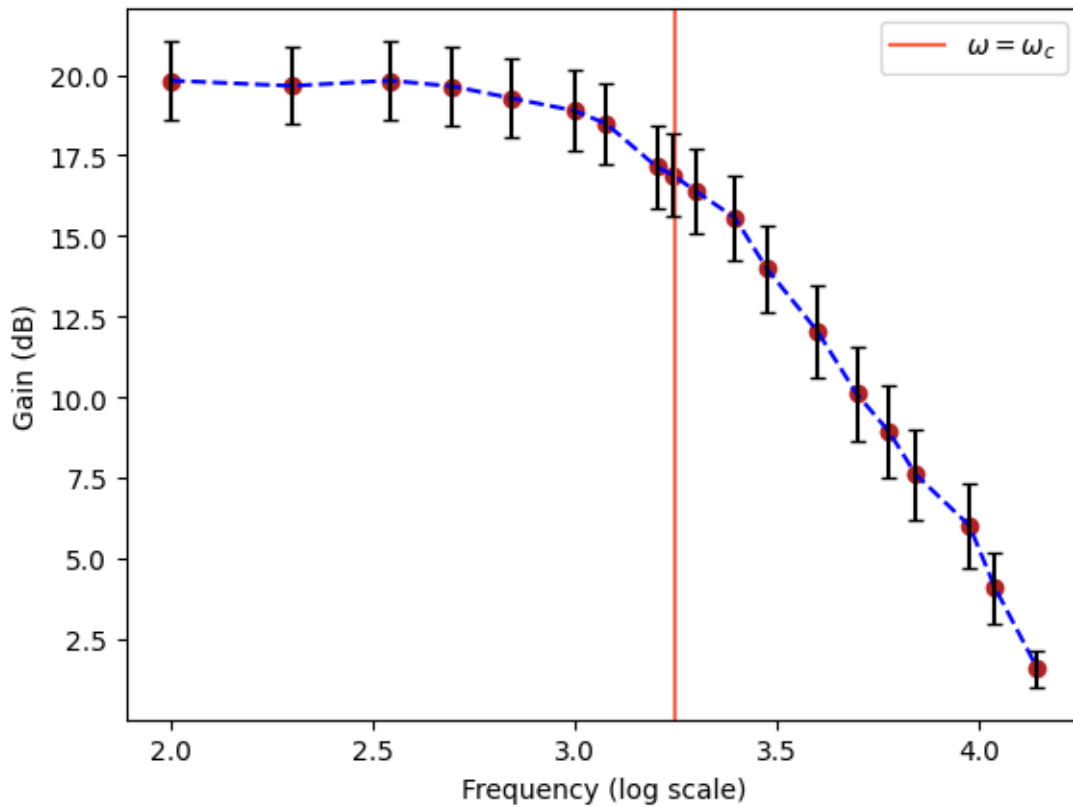


Figure 6: Bode plot for low-pass RC circuit

The vertical line in the graph is at 3.25. This gives the cutoff frequency from the graph as,

$$\omega_c = 2\pi \times 10^{3.25} = 11.17 \text{ kHz}$$

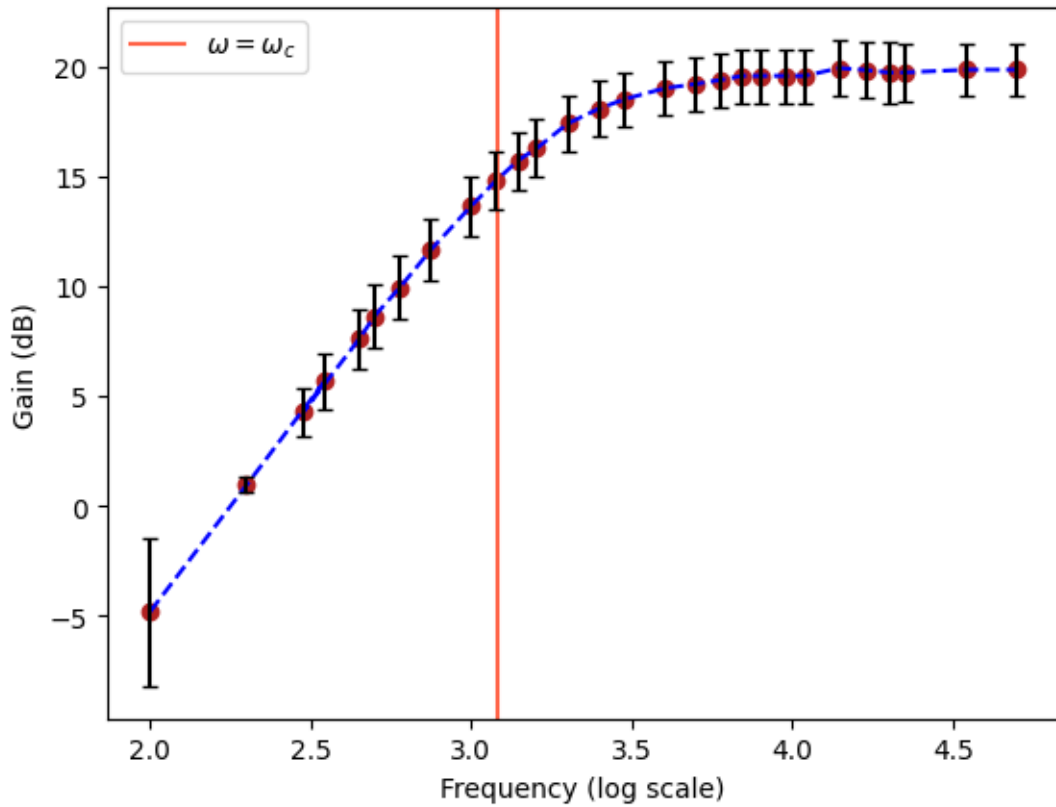


Figure 7: Bode plot for high-pass RC circuit

The vertical line in the graph is at 3.08. This gives the cutoff frequency from the graph as,

$$\omega_c = 2\pi \times 10^{3.01} = 7.55 \text{ kHz}$$

The shape of the two graphs is as expected. In the low-pass filter, voltages of higher frequencies have a lower gain, i.e., the signal is cutoff a lot by the circuit than at lower frequencies which is the function of a low-pass filter. It is just the opposite for a high-pass filter where gain in voltage of lower frequencies is lower than at higher frequencies.

7.3 Part C

The Bode plots for two different LCR circuits are shown below. The vertical line marks the natural frequency of the circuit. It is equal to 18.28 kHz as calculated from the values of the components used.

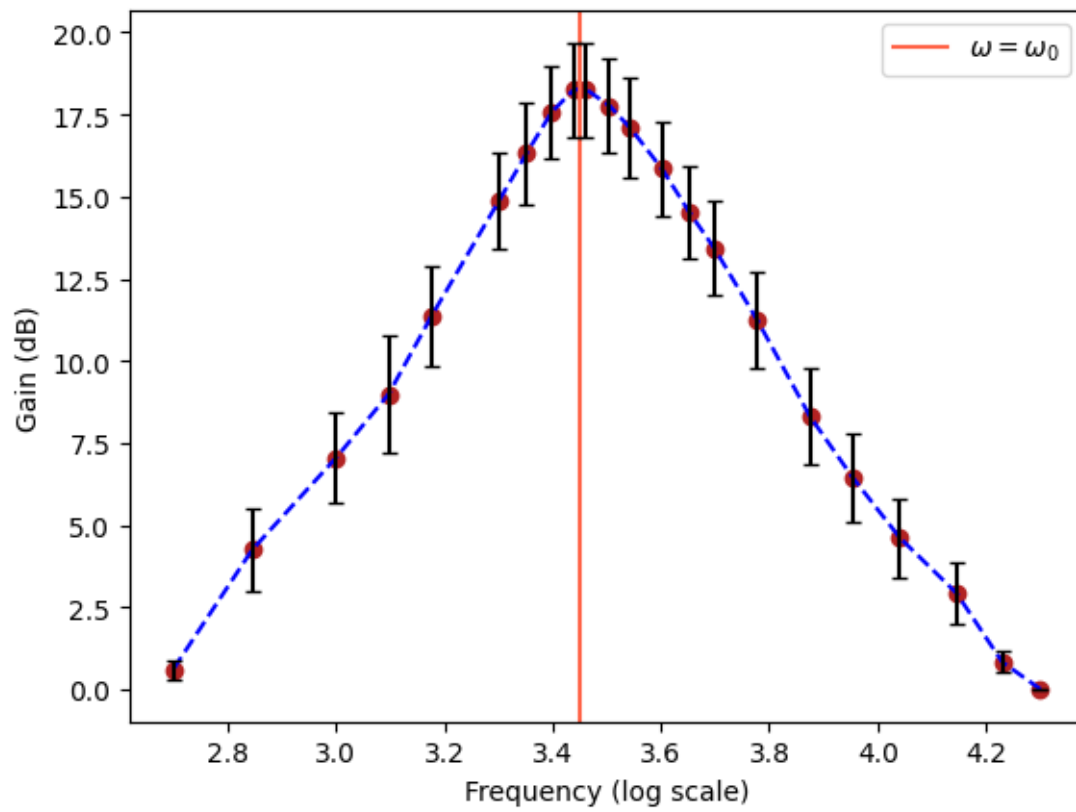


Figure 8: Bode plot for an LCR circuit in series

The vertical line is at 3.48 which gives the resonant frequency from the plot as,

$$\omega_0 = 2\pi \times 10^{3.48} = 18.97 \text{ kHz}$$

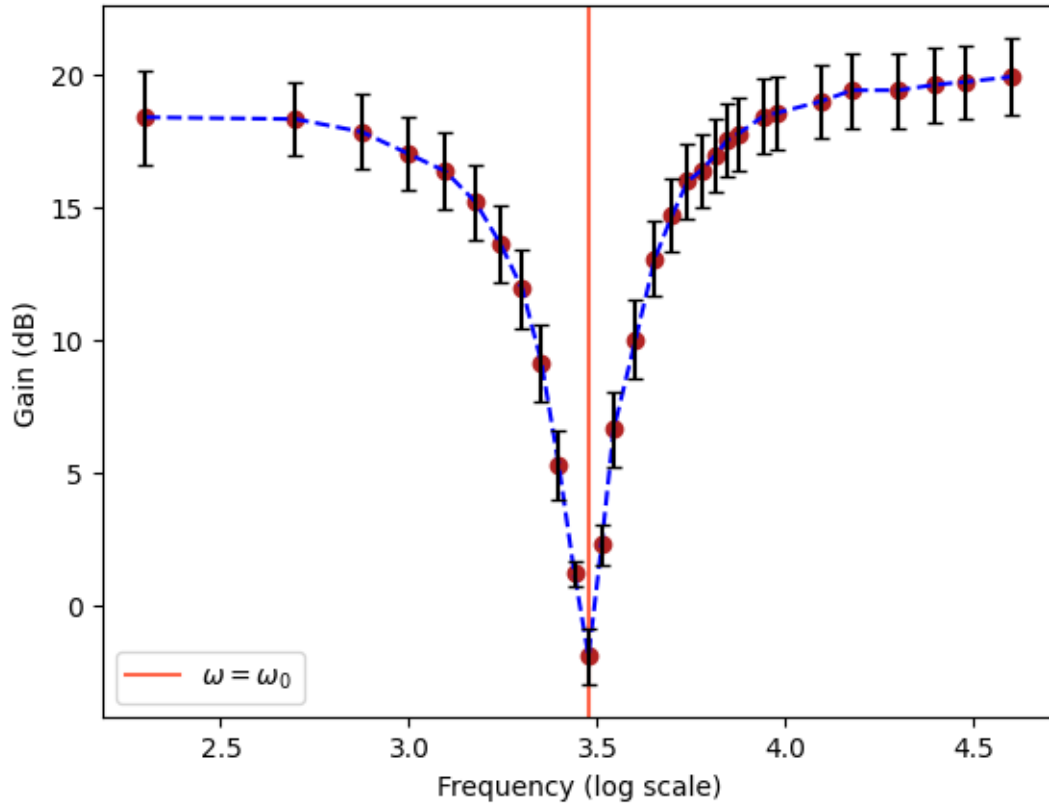


Figure 9: Bode plot for an LCR circuit in series

The vertical line in this plot is at 3.45 which gives the resonant frequency as,

$$\omega_0 = 2\pi \times 10^{3.45} = 17.71 \text{ kHz}$$

8 Error Analysis

The formula of gain in both part B and C is,

$$\text{gain} = 20 \log_{10} \left(\frac{V_{out}}{V_{in}} \right)$$

The general formula for measuring relative error for a function $q(x_1, x_2, \dots, x_n)$ is,

$$\frac{\delta q}{q} = \sqrt{\sum_{i=1}^n \left(\frac{\partial q}{\partial x_i} \delta x_i \right)^2}$$

Using this formula the relative error in measuring the values of gain in each part was obtained as follows,

Gain (dB)	Relative error in measuring gain (%)
19.82	6.2
19.66	6.02
19.82	6.2
19.65	6.27
19.28	6.41
18.89	6.57
18.49	6.75
17.15	7.43
16.9	7.57
16.39	7.88
15.56	8.44
13.98	9.71
12.04	11.69
10.1	14.25
8.94	16.11
7.6	18.61
6.02	22.14
4.08	27.49
1.58	36.45

Table 5: Relative error in measuring gain for different values for the low-pass filter. The error bars in graph 6 were plotted using these values of relative error

Gain (dB)	Relative error in measuring gain (%)
-4.82	70.16
0.95	37.67
5.67	22.56
4.27	25.89
7.62	17.87
8.63	16.66
9.93	14.22
11.65	11.92
13.63	9.83
14.81	9.01
15.71	8.34
16.26	7.97
17.38	7.3
18.06	6.95
18.49	6.75
18.99	6.53
19.18	6.45
19.37	6.38
19.55	6.31
19.55	6.31
19.55	6.31
19.55	6.31
19.91	6.43
19.82	6.47
19.71	6.94
19.71	6.79
19.83	5.96
19.83	5.96

Table 6: Relative error in measuring gain for different values for high-pass filter. The error bars in graph 7 were plotted using these values of relative error

Gain (dB)	Relative error in measuring gain (%)
0.60	48.53
4.25	29.34
7.04	19.78
8.98	20.05
11.35	13.62
14.88	9.74
16.32	9.44
17.57	8.0
18.26	7.79
18.26	7.79
17.76	8.07
17.08	8.9
15.84	8.96
14.51	9.64
13.42	10.66
11.24	13.21
8.31	17.95
6.44	21.13
4.61	25.91
2.92	31.32
0.83	39.72
0.00	43.65

Table 7: Relative error in measuring gain for different values for LCR circuit in series configuration. The error bars in graph 8 were plotted using these values of relative error

Gain (dB)	Relative error in measuring gain (%)
18.37	9.72
18.3	7.6
17.82	7.86
16.98	8.18
16.33	8.8
15.19	9.28
13.6	10.7
11.9	12.61
9.12	16.13
5.28	24.53
1.18	39.74
-1.94	54.46
2.28	33.69
6.62	21.15
10.0	14.69
13.04	10.82
14.67	9.5
15.95	8.69
16.39	8.39
16.93	8.04
17.5	7.87
17.75	7.73
18.42	7.54
18.53	7.48
18.98	7.26
19.39	7.24
19.39	7.24
19.6	7.15
19.7	7.1
19.9	7.36

Table 8: Relative error in measuring gain for different values for LCR circuit in “tank” configuration. The error bars in graph 9 were plotted using these values of relative error

An interesting thing is noticed when the relative error is plotted against the log of frequency for each of the data tables shown above. The plots looked like a reflection of the plots of the actual data. For example in case of the Bode plot of the LCR circuit in tank configuration, the plot of error versus log of frequency gave a graph with a spike at the same point but in the positive y-direction. This graphically shows that the relative error is increasing for a smaller input and output voltage. This also becomes clear from the the formula for the error in measuring gain where both the voltages are in the denominator. The graphs are attached in the Appendix below.

The error in measuring the cutoff and the resonant frequencies are measured by comparing the calculated and experimentally measured values. For the low-pass filter, the relative error is,

$$\frac{\Delta\omega_c}{\omega_c} = \left| \frac{11.17 - 9.99}{9.99} \right| \times 100 = 11.81\%$$

For the high-pass filter,

$$\frac{\Delta\omega_c}{\omega_c} = \left| \frac{7.55 - 9.99}{9.99} \right| \times 100 = 24.42\%$$

For the LCR series circuit the error in measuring the resonant frequency is,

$$\frac{\Delta\omega_0}{\omega_0} = \left| \frac{18.97 - 18.28}{18.28} \right| \times 100 = 1.57\%$$

For the tank configuration of the LCR circuit,

$$\frac{\Delta\omega_0}{\omega_0} = \left| \frac{17.71 - 18.28}{18.28} \right| \times 100 = 3.12\%$$

9 Appendix

The first set of equations that are to be derived are,

$$\begin{aligned} I_R &= \frac{V_0}{R} \cos \omega t \\ I_C &= -\omega C V_0 \sin \omega t \\ I_L &= \frac{V_0}{\omega L} \sin \omega t \end{aligned}$$

For this the impedance in purely resistive, capacitive and inductive circuits are derived.

From Ohm's Law $\implies R = V/I$. Thus,

$$\begin{aligned} I_R &= Re\left(\frac{V}{I}\right) \\ &= Re\left(\frac{V_0 e^{i\omega t}}{I}\right) \\ \therefore I_R &= \frac{V_0}{R} \cos(\omega t) \end{aligned}$$

For a capacitor the voltage and charge in the circuit are related as, $Q = VC = V_0 e^{i\omega t}$. Differentiating this we get,

$$\begin{aligned} I_C &= Re(i\omega C V_0 e^{i\omega t}) \\ &= Re[\omega C V_0 (i \cos \omega t - \sin \omega t)] \\ \therefore I_C &= -\omega C V_0 \sin(\omega t) \end{aligned}$$

For an inductor the differential equation is, $V = L \frac{dI}{dt}$. Taking integrals on both sides we get,

$$\begin{aligned} \int_0^{I_L} dI' &= \int_0^t \frac{V_0}{L} e^{i\omega t'} dt' \\ \implies I_L &= Re\left[\frac{V_0}{i\omega L} e^{i\omega t}\right] \\ &= Re\left[\frac{V_0}{\omega L} (-ie^{i\omega t})\right] \\ \therefore I_L &= \frac{V_0}{\omega L} \sin(\omega t) \end{aligned}$$

From the 3 relations above it is evident that the impedances of a resistor, capacitor and inductor respectively are,

$$\begin{aligned} z_R &= R \\ z_C &= \frac{1}{i\omega C} \\ z_L &= i\omega L \end{aligned}$$

In the RC integrator, the input voltage is almost completely across the resistor, i.e., $V_{in} \approx V_R$. Furthermore, the net impedance of an RC circuit is,

$$z = R + \frac{1}{i\omega C}$$

Hence the current in this circuit is,

$$I = \frac{V_{in}}{R + 1/i\omega C}$$

For the RC integrator the voltage is in the high frequency regime, i.e. $\omega \gg 1/RC$. The above relation can also be written as,

$$I = \frac{V_{in}}{R(1 + 1/i\omega RC)}$$

From this it is obvious that,

$$I \approx \frac{V_{in}(t)}{R}$$

Since the circuit is in series it means that the same current flows through the capacitor as well. The voltage in the capacitor is,

$$\begin{aligned} V_C &= \frac{Q}{C} \\ \Rightarrow I_C &= C \frac{dV_C}{dt} \\ \Rightarrow V_{in}(t) &\approx RC \frac{dV_C}{dt} \\ \therefore V_C(t) &\approx \frac{1}{RC} \int_0^t V_{in}(t') dt' \end{aligned}$$

For a differentiator the voltage is almost completely across the capacitor and thus the relation for the current can be written as,

$$I = \frac{V_{in}}{1/\omega C(\omega RC + 1/i)}$$

Since $\omega \ll 1/RC \Rightarrow \omega RC \ll 1$. Thus,

$$I \approx \frac{V_{in}(t)}{i\omega C}$$

The same current flows in the entire circuit thus,

$$\begin{aligned} V_R &= IR \\ \Rightarrow V_R &= \frac{dQ}{dt} R \\ \therefore V_R(t) &\approx RC \frac{dV_{in}(t)}{dt} \end{aligned}$$

For an LCR circuit connected in series, the net impedance in the circuit can be expressed as,

$$\begin{aligned}
 z &= z_R + z_C + z_L \\
 &= R + \frac{1}{i\omega C} + i\omega L \\
 &= R - \frac{i}{\omega C} + i\omega L \\
 &= R + i\left(\omega L - \frac{1}{\omega C}\right)
 \end{aligned}$$

Thus the current in the circuit can be expressed as,

$$I = \frac{V_{in}}{R + i\left(\omega L - \frac{1}{\omega C}\right)}$$

From this relation it can be seen that the current in the circuit will be maximum when the impedance is purely real, i.e, when $\omega L = 1/\omega C$. The frequency that this happens at is known as the resonant frequency, $\omega_0 = 1/\sqrt{LC}$

For the tank configuration the impedance of the inductor and capacitor is,

$$\begin{aligned}
 \frac{1}{z_{LC}} &= \frac{1}{i\omega L} + i\omega C \\
 &= i\left(\omega C - \frac{1}{\omega L}\right) \\
 \Rightarrow z_{LC} &= \frac{i}{\left(\frac{1}{\omega L} - \omega C\right)}
 \end{aligned}$$

Thus the net impedance in the circuit is,

$$z = R + \frac{i}{\left(\frac{1}{\omega L} - \omega C\right)}$$

Thus the current in the circuit is,

$$I = \frac{V_{in}}{R + \left(\frac{i}{\omega C - \frac{1}{\omega L}}\right)}$$

To visualize the relative error in measuring the gain for different frequencies, the graphs of error versus log of frequency are obtained as,

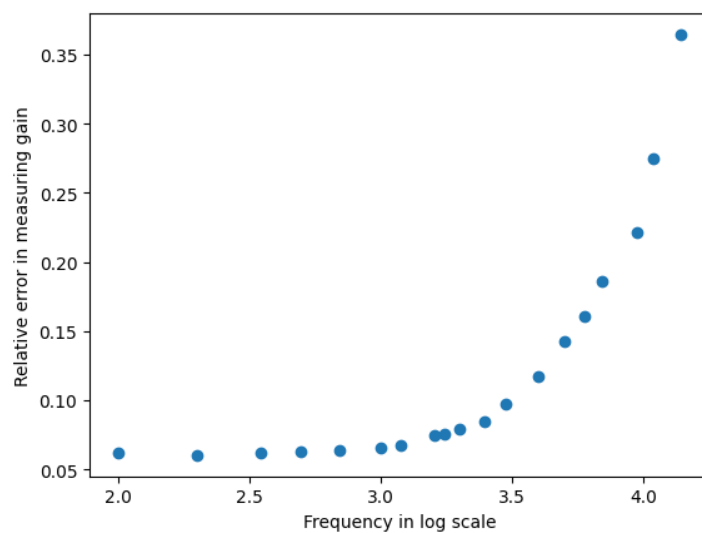


Figure 10: Relative error versus log of frequency for a low-pass filter

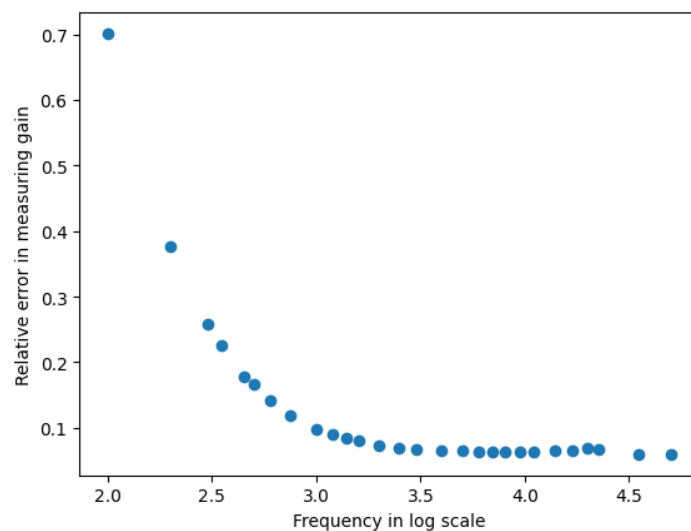


Figure 11: Relative error versus log of frequency for a high-pass filter

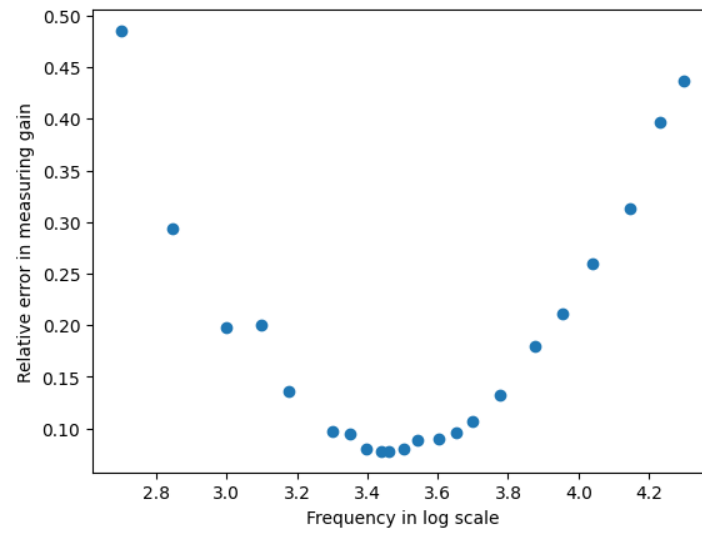


Figure 12: Relative error versus log of frequency for an LCR circuit in series

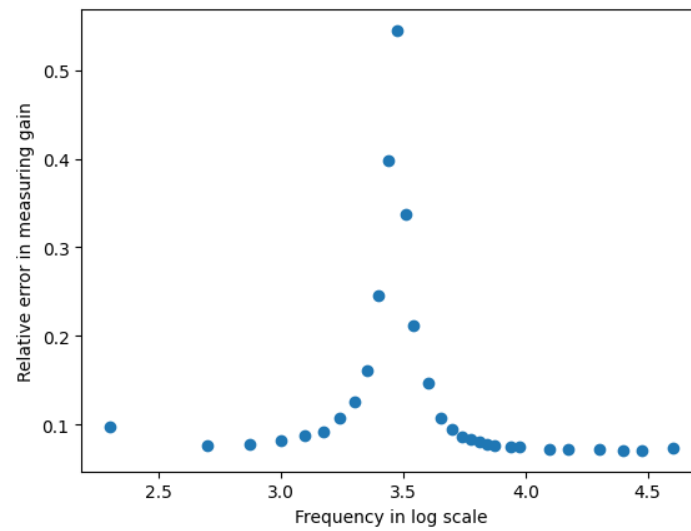


Figure 13: Relative error versus log of frequency for an LCR circuit in tank configuration