Lab Report 5

PHY 2010: Physics Lab 2

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1 Aim

To study the properties of electric fields in two dimensions by drawing equipotential curves for different shapes of conductors.

2 Theoretical Background

One of the most fundamental quantities in electrostatics/electrodynamics is the electric field due to a charge distribution. Electric fields are produced by a single or collection of charges. For a static point charge in vacuum the electric field can be written as,

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

The net electric field due to a collection of charges in vacuum is the vector sum of all the individual charges. However analyzing electric fields is not easy due to its vector nature. To make analysis easier, the concept of a scalar potential, V is introduced.

Maxwell's equations for electrostatics are complete definition of the electric field,

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\boldsymbol{\nabla} \times \mathbf{E} = 0$$

The fact that the curl of the electric field is 0 allows us to define a scalar potential, V, such that,

$$\mathbf{E} = -\boldsymbol{\nabla} V$$

In a region where there is no net charge we arrive at Laplace's Equation for electrostatics,

$$\nabla^2 V = 0$$

The solution to this equation gives us all the information required about the system since we can get the electric field from the gradient of the potential.

Equipotential curves are lines that join points around a charge distribution with the same value of the potential in a 2 dimensional distribution of charge. For a 3 dimensional distribution it is an equipotential surface. The spacing of equipotential curves gives information about the potential and the electric field due to a charge distribution. If the curves are closely spaced it means that the potential changes rapidly while for loosely spaced lines the potential change is slower. The electric field magnitude is larger in the first case and smaller in the second case. Electric field lines are always perpendicular to the equipotential curves. This is so because electric field being the gradient of the potential points in the direction where the potential is changing the most. Thus a direction perpendicular to the direction of greatest increase of potential is a direction in which the potential is not increasing at all and that is the equipotential curve. Thus, locally electric field lines meet the equipotential curves at right angles.

Conductors can be used to generate equipotential curves. A very important feature of conductors is that the potential inside the surface of the conductor is uniform throughout even when kept in an external electric field. This occurs because when a conductor is kept in an external field, electrons within the conductor move and set up an opposing field within the conductor. This process occurs till the field inside becomes equal to the field outside re-establishing static equilibrium. It also makes the potential within uniform. Thus the surface of a conductor is an equipotential surface. Since conductors are equipotential surfaces we can keep two conductors and find the potential in the region between the conductors by solving Laplace's equation. For an arbitrary surface this is very difficult but for simple symmetric systems it is easier to do.

For this experiment in each setup one of the electrodes is kept grounded while the other is ungrounded. For two infinite parallel bars separated by a distance, d, (in practice the bars wont be infinite and edge effects would have an effect on the equipotential curves) the potential $V(y)$ at a distance y from the grounded bar is,

$$V(y) = \left(\frac{V_0}{d}\right)y \tag{1}$$

Here V_0 is the potential of the ungrounded bar.

The other simple setup is two concentric circular conductors. The radii are r_0 for the smaller one and r_1 for the larger one. In this setup the larger one is kept grounded (consider it at some potential V_1 for calculation purposes) while the smaller one is at some potential V_0. Thus the potential at some radius r is,

$$V(r) = \left(\frac{V_1 - V_0}{\ln\left(\frac{r_1}{r_0}\right)}\right)\ln\left(\frac{r}{r_0}\right) + V_0 \tag{2}$$

3 Experimental Setup

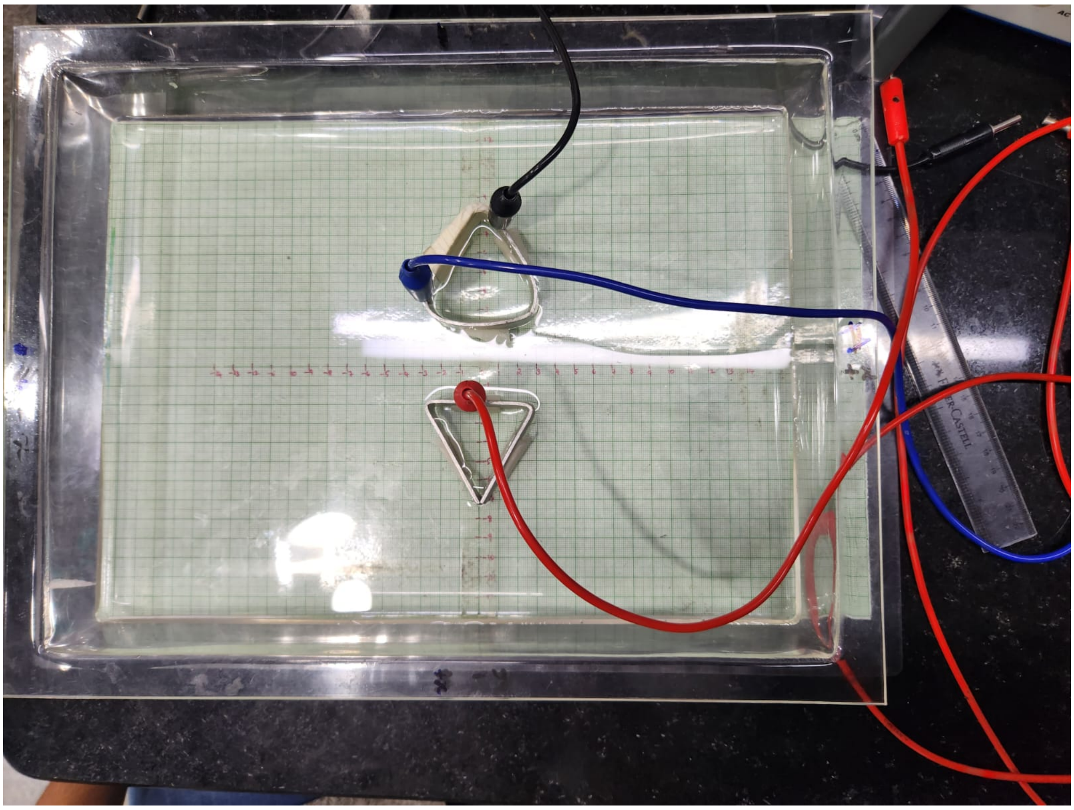

Figure 1: General setup for the experiment. The electrodes are different for different parts of the experiment.

4 Equipment

1. An acrylic tray with water as the electrolytic medium
2. An AC power supply
3. A set of metal electrodes: straight, circular, triangular and so on
4. A digital multimeter
5. A point probe attached to an XY-stage
6. A set of connecting cords
7. A set of measuring scales
8. Graph papers

5 Procedure

For this experiment a coordinate system was drawn on a graph paper and it was stuck at the bottom acrylic tray.

5.1 Part A

In this part equipotential curves for a long and straight metallic electrode.

1. The two electrodes were stuck to the acrylic tray with tape at a distance of 8 cm. Then water was filled into the tray upto three-quarters of the height of the electrodes.
2. An AC power supply was connected across the electrodes an a voltage of 2.66 V was supplied. After measuring the supply voltage, one of the electrodes was kept grounded thus setting the 0 of the potential along the length of that electrode.
3. Following this the point probe was put into the water such that its tip just grazed the surface of the water. The non-grounded end of the multimeter was connected to the probe.
4. The voltage at a certain distance along the y-axis from the grounded electrode was measured at the point where $x = 0$.
5. Further points along the x-axis with the potential were found and their coordinates were noted down. These x-y coordinates were plotted to obtain the first equipotential curve. The points were considered till edge effects became apparent in the curve.
6. This process was repeated for a total of 5 times to obtain 5 equipotential curves.

5.2 Part B

1. Two cylindrical electrodes were taken and placed concentric to each other with both their centres at the origin.
2. Following this, the same steps as in part A were followed to obtain equipotential curves and the plot for the relation between the potential and distance from the origin.

5.3 Part C

1. One of the two triangles for the *Kaju Barfi* configuration were made by using a flat aluminium bar.
2. The two triangles were placed with one inverted with their bases parallel equidistant from the origin.
3. Finally equipotential curves for this configuration were plotted and compared with the expected configuration.

6 Data

6.1 Least Count of Instruments

1. Graph Paper: 0.1 cm
2. Multimeter: 0.01 V
3. AC Power Supply: 0.01 V

6.2 Part A

	0.56 V	0.87 V	1.34 V	1.87 V	2.16 V
x (cm)	y (cm)	y (cm)	y (cm)	y (cm)	y (cm)
-14	4.5	2.6	0.3	-2.0	-3.8
-13	3.7	2.3	0.2	-1.9	-3.3
-12	3.2	2.1	0.2	-1.8	-3.0
-11	3.0	2.0	0.1	-1.7	-2.8
-10	2.8	2.0	0.1	-1.5	-2.6
-9	2.7	1.9	0.1	-1.4	-2.6
-8	2.6	1.8	0.1	-1.4	-2.6
-7	2.5	1.7	0.1	-1.4	-2.6
-6	2.5	1.5	0.1	-1.4	-2.6
-5	2.5	1.5	0.1	-1.4	-2.6
-4	2.5	1.5	0.1	-1.4	-2.6
-3	2.5	1.5	0.1	-1.4	-2.6
-2	2.5	1.5	0.1	-1.4	-2.6
-1	2.5	1.6	0.1	-1.4	-2.6
0	2.5	1.5	0.1	-1.4	-2.6
1	2.5	1.5	0.1	-1.4	-2.6
2	2.5	1.5	0.1	-1.4	-2.6
3	2.5	1.5	0.1	-1.4	-2.6
4	2.5	1.5	0.1	-1.4	-2.6
5	2.5	1.5	0.1	-1.4	-2.6
6	2.5	1.5	0.1	-1.4	-2.6
7	2.5	1.5	0.1	-1.4	-2.6
8	2.5	1.5	0.1	-1.4	-2.6
9	2.6	1.6	0.1	-1.4	-2.6
10	2.7	1.7	0.1	-1.6	-2.8
11	3.0	1.8	0.1	-1.7	-3.1
12	3.5	2.0	0.1	-1.8	-3.7
13	4.5	2.3	0.1	-2.1	-4.7

Table 1: X and Y coordinates for different points on the equipotential surfaces for 5 different voltages

The voltage at different distances from the ungrounded electrode are,

Distance (cm)	Voltage(V)
6.5	0.56
5.5	0.87
4.1	1.34
2.6	1.85
1.4	2.16

Table 2: Voltage at different distances from teh ungrounded eletrode

6.3 Part B

3.42 V		2.84 V		2.06 V		1.40 V		0.86 V	
x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)
2.6	0.0	3.0	0.0	4.0	0.0	5.1	0.0	0.0	-6.1
2.4	-0.7	2.8	-1.2	3.7	-1.3	4.7	-1.5	4.8	-3.6
2.2	-1.2	2.4	-1.7	3.1	-2.5	3.9	-3.1	2.8	-5.4
2.1	-1.5	1.6	-2.7	1.9	-3.6	2.5	-4.4	-4.7	-3.6
1.8	-1.8	0.0	-3.0	0.0	-4.1	0.0	-5.2	-5.6	-1.8
1.2	-2.3	-1.6	-2.7	-1.9	-3.6	-2.1	-4.7	5.6	-1.8
0.0	-2.7	-2.9	-1.4	-2.8	-3.0	-3.8	-3.6	-2.8	-5.4
-0.8	-2.5	-3.1	0.0	-3.7	-2.0	-4.7	-2.4	-6.0	0.0
-1.9	-1.9	-3.1	1.0	-4.2	0.0	-5.1	0.0	6.1	0.0
-2.3	-1.4	-2.7	2.0	-4.0	1.3	-4.9	2.0	0.0	6.1
-2.5	-1.0	-1.9	2.6	-3.5	2.5	-4.0	3.5	-4.8	3.6
-2.7	0.0	0.0	3.1	-2.0	3.8	-2.5	4.7	-2.8	5.4
-2.6	0.6	1.4	2.7	0.0	4.1	0.0	5.1	4.7	4.6
-2.3	1.4	2.4	2.0	1.8	3.7	2.4	4.5	5.6	1.8
-1.6	2.1	2.9	1.3	3.1	2.4	3.6	3.5	-5.6	1.8
0.0	2.7			3.8	1.2	4.9	0.9	2.8	5.4
1.3	2.2								
1.9	1.7								
2.2	1.3								

Table 3: X and Y coordinates for equipotential surfaces for concentric cylinders with smaller one of radius 2 cm and larger one of radius 8 cm.

There were various different values of radii for the equipotential surfaces that were obtained. To check whether the relation for the equipotential surface and distance is satisfied the mean of all the values were taken. Thus, the mean radii of the equipotential curves were,

Potential (V)	Radius of equipotential curve (cm)
3.42	2.62
2.84	3.12
2.06	4.10
1.40	5.14
0.86	6.04

Table 4: Radii of equipotential curves for different potentials

6.4 Part C

2.2 V		3.18 V		4.54 V		1.4 V		0.34 V	
x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)	x (cm)	y (cm)
-0.1	0.0	0	-1.0	0.0	-8.0	0	1.0	0.0	9.0
1.0	0.0	-1	-1.0	-1.0	-7.5	-2	1.0	-1.5	8.0
1.5	0.0	-2	-1.0	-1.8	-6.5	-1	1.0	-2.3	7.0
1.8	0.0	-3	-1.3	-2.1	-5.5	0	1.0	-2.8	6.0
2.4	0.0	-4	-1.6	-2.4	-4.0	1	1.0	-3.4	5.0
3.7	0.0	-5	-2.2	-2.5	-3.0	2	1.0	-3.1	4.0
5.0	0.2	-6	-2.9	1.3	-7.4	3	1.0	-2.8	3.0
6.5	0.2	1	-1.0	1.9	-6.6	4	1.0	2.1	8.1
7.9	0.4	2	-1.1	2.1	-5.3	5	2.0	2.9	7.0
-1.0	0.0	3	-1.3	2.4	-4.1	6	2.7	3.2	6.0
-2.0	0.0	4	-1.6			-3	1.2	3.5	5.0
-3.0	0.0	5	-2.2			-4	1.6	3.6	4.0
-4.0	0.1	6	-2.9			-5	2.4	3.3	3.0
-5.0	0.1					-6	3.1		
-6.0	0.4								
-7.0	0.4								

Table 5: X and Y coordinates for various points on equipotential curves for the Kaju Barfi electrode configuration

7 Graphs and Analysis

The expected equipotential curves are included in *Discussion* below.

7.1 Part A

The equipotential curves for two long straight electrodes are,

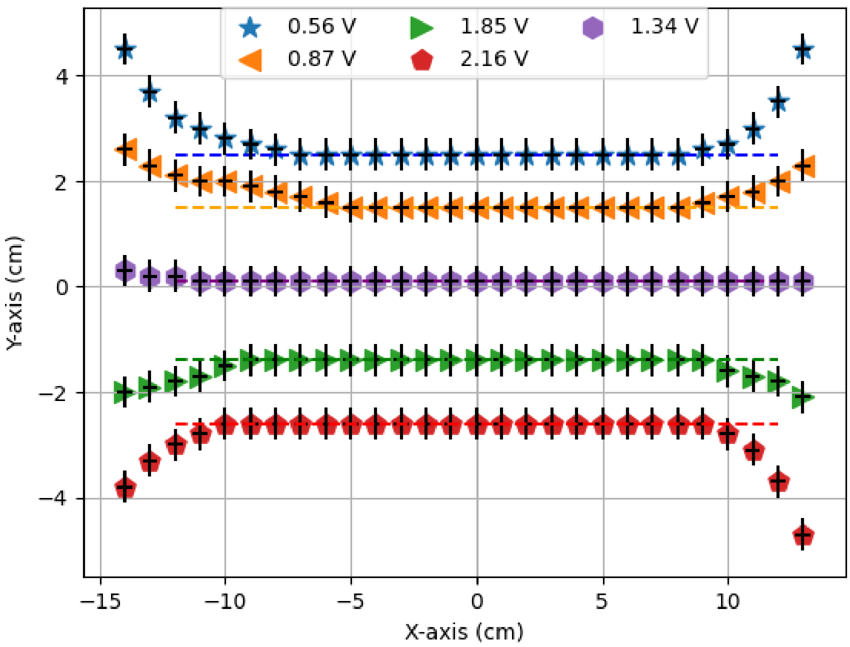

Figure 2: Equipotential Curves for two long straight electrodes

The dotted lines represent the equipotential curves for an ideal situation where the electrodes are infinitely long. However since that is practically not possible, edge effects are seen as the curves start diverging as the edges are reached.

The plot for the potential versus distance from the grounded electrode is,

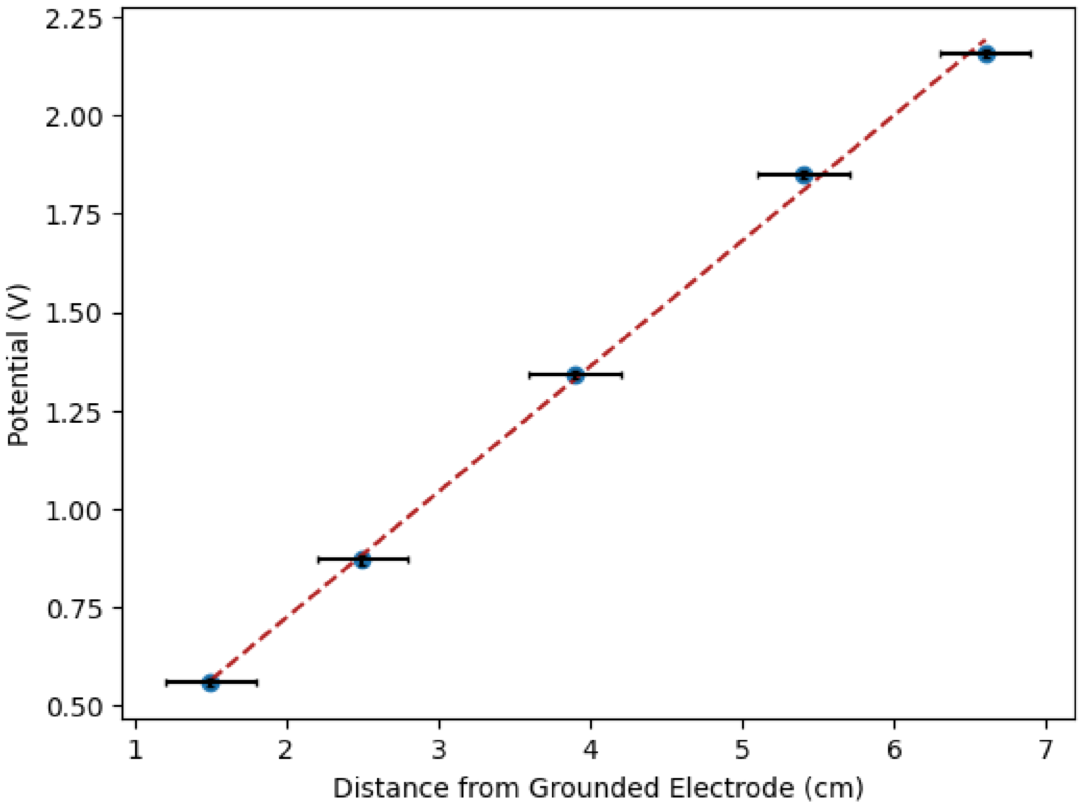

Figure 3: Plot of for potential versus distance of equipotential curve from origin.

Plot Parameters from regression analysis using `numpy.polyfit` and `numpy.coerrcof`:

1. Slope: 0.3194
2. Intercept: 0.0847
3. R^2 value: 0.9983

The magnitude of the slope of this graph gives the electric field that is present in between the two electrodes before edge effects become apparent. The electric field is constant for the area in between the two electrodes and its magnitude is 31.94 V/m.

7.2 Part B

Equipotential Curve for concentric circular electrodes are,

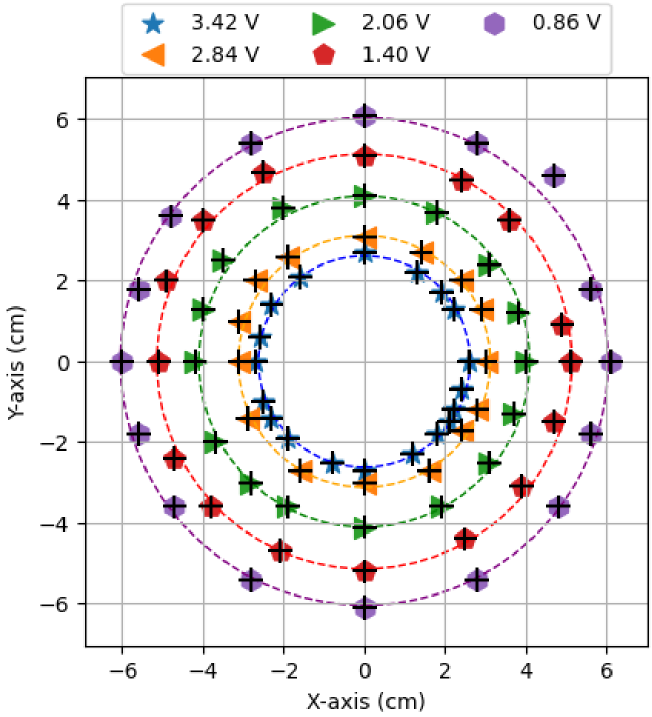

Figure 4: Equipotential Curve for two concentric circular electrodes

The equipotential curves are obtained as expected. The radius of each circle is the mean of the radii obtained from each x and y coordinate for a given equipotential curve. The relation between the natural log of the radii and the potential follows the expected linear relation.

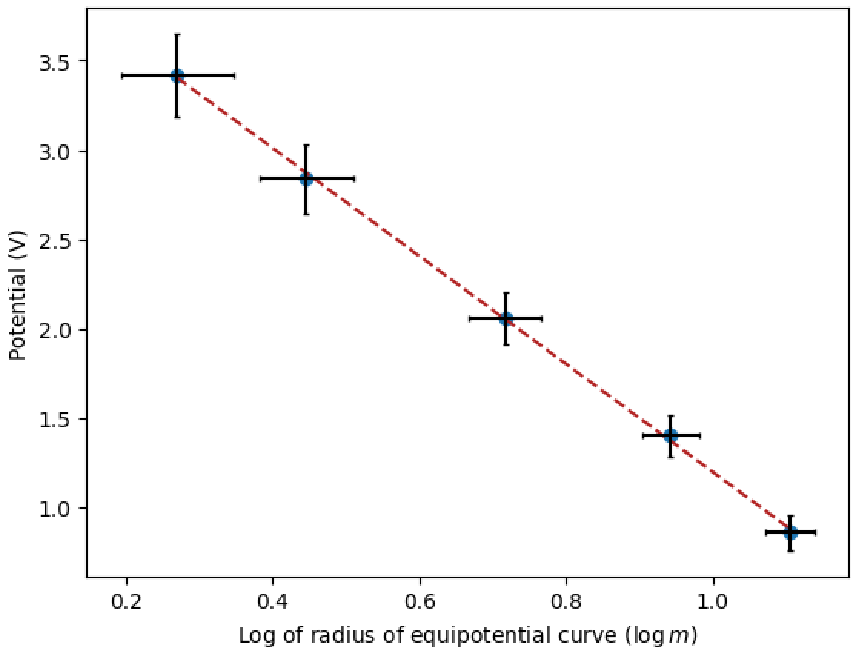

Figure 5: Plot of potential versus the log of the radius of the equipotential curve.

Plot Parameters from regression analysis using `numpy.polyfit` and `numpy.coerrcoef`:

1. Slope: -3.021
2. Intercept: 4.22
3. R^2 value: 0.9994

The electric field at radius equal to the radii of the equipotential curves can be calculated from the equation for the potential for concentric circular electrodes. Since $\mathbf{E} = -\boldsymbol{\nabla} V$ the electric field from the graph comes out to be,

$$E = -\frac{\text{slope}}{r^2}$$

Thus the electric field values are,

Potential (V)	Radius (m)	Electric Field (V/m)
3.42	0.0262	4399.5707
2.84	0.0312	3094.346
2.06	0.041	1798.2169
1.40	0.0514	1145.2834
0.86	0.0604	827.7031

Table 6: Electric field at different distances from the origin of the coordinate system

7.3 Part C

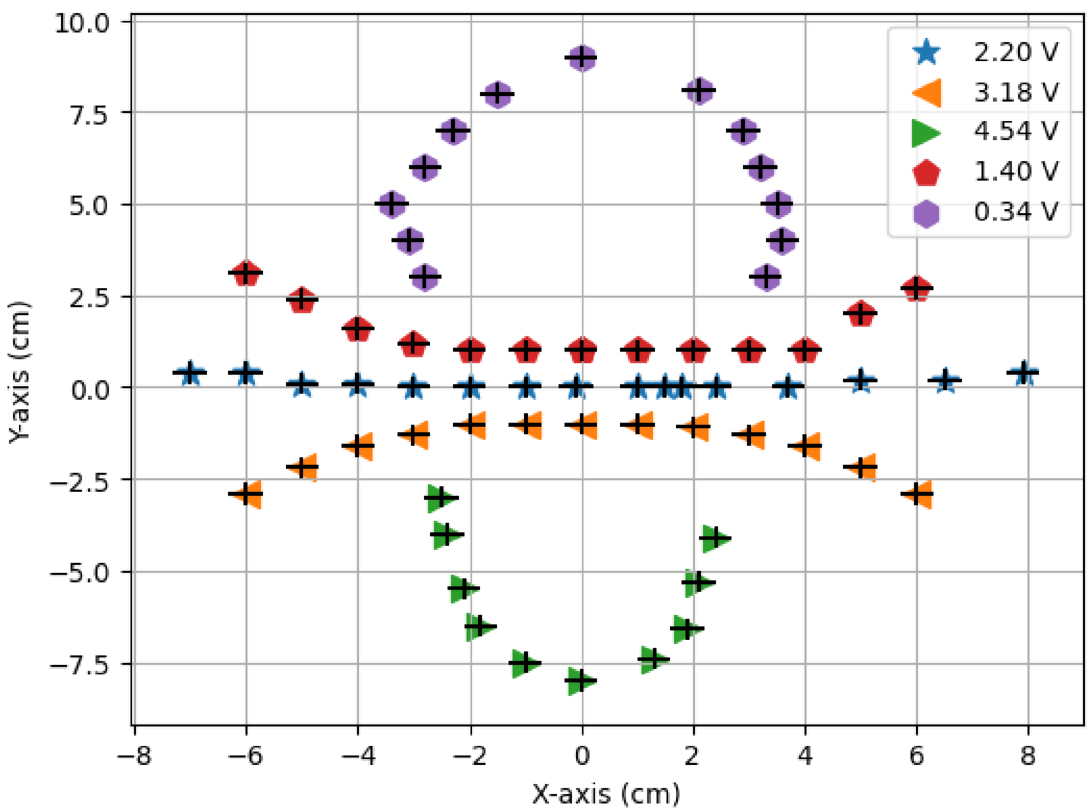

Figure 6: Equipotential curves for the *Kaju Barfi* configuration

The top and bottom are not completely symmetrical since the top electrode was bent and made by hand. The electrode was thus not a proper triangle. However, the equipotential curves do follow the expected trend where the line which is equidistant from the grounded and ungrounded electrode is straight. The lines just below and above diverge once the edges of the triangles are reached. This behaviour is similar to that of two straight electrodes in *Part A*. The other two lines are around the triangles themselves. As expected, the curves more or less trace the shape from the vertex that is on the vertical axis towards the two other vertices.

8 Error Analysis

The errors for measuring the x and y coordinates are the same. The error arises from the least count of the graph paper and also the blurring of the point where the probe needle is due to refraction. Both of these have a combined effect. The first one is equal to 0.1 cm while the second one is estimated to be 0.2 cm. Thus the total error in measuring the coordinates are 0.3 cm on both axes.

8.1 Part A

The error in measuring the slope was calculated by using the covariance matrix obtained from the `numpy.polyfit` function. This gave the relative error in measuring the slope as 0.76%. Thus the absolute error in measuring the slope, i.e., the electric field is 0.24 V/m.

8.2 Part B

Apart from the error in measuring the coordinates errors in computing the potential and the radius are also there.

The absolute error for measuring the voltage is,

$$\delta V = \sqrt{\left(\text{slope} \times \frac{\delta r}{r}\right)^2 + (\text{intercept})^2}$$

Using this the different values of δV and the error in r on the log scale are,

$\log r$ ($\log cm$)	Error in r in log scale ($\log cm$)	Potential (V)	δV
0.270	0.076	3.42	0.269
0.446	0.064	2.84	0.238
0.717	0.049	2.06	0.202
0.943	0.039	1.40	0.181
1.105	0.033	0.86	0.17

Table 7: The errors in measuring r and V

8.3 Part C

The only errors are the errors in measuring the coordinates which are the same as mentioned above.

9 Discussion

9.1 Part A

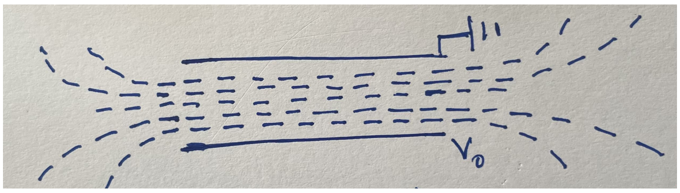

Figure 7: Expected equipotential curves for configuration in Part A

Figure 7 shows how the equipotential curves should be for parallel electrodes taking into account edge effects. Experimental observations match the expectation.

The slope of the graph in figure 3 is negative indicating that the electric field is in a direction opposite to the coordinate system setup for the equipotential curves. Theoretically the electric field points in the direction opposite to the greatest increase of the potential ($\because \mathbf{E} = -\boldsymbol{\nabla} V$). This is confirmed experimentally since the electric field is pointing in a direction opposite to the direction in which the potential is increasing, i.e., from the grounded to the ungrounded electrode. Furthermore, it is expected that the electric field will point in the direction away from the source as is seen experimentally in *Part A*.

9.2 Part B

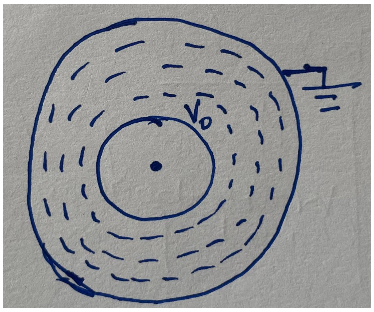

Figure 8: Expected equipotential curves for concentric cylindrical electrodes

The equipotential curves are as expected since field lines for a spherical/circular charge distribution diverges out from a single point.

Furthermore, the relation between the potential and the distance from the origin gives the correct relation for the electric field for a circular/charge distribution, i.e., a $1/r^2$ dependence. Theoretically the electric field at points $r > R$, for a spherical conductor of radius R also varies as $1/r^2$ (derived from Gauss Law). The direction of the electric field, i.e., radially outwards, also matches the theoretical prediction.

9.3 Part C

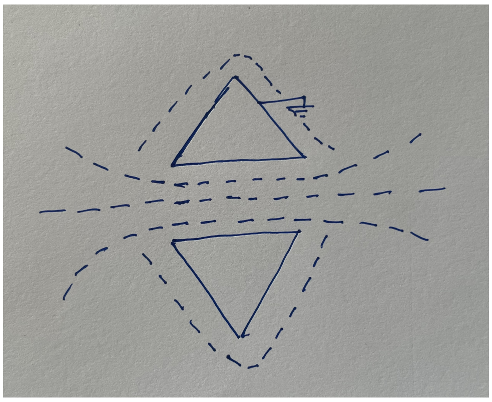

Figure 9: Expected equipotential curves for the *Kaju Barfi* electrode configuration

The equipotential curve almost completely match the expected figure. However since one of the electrodes was made by hand, it was not a perfect triangle and that is why the experimentally obtained shape is a little different from the theoretical observation.

10 Results

10.1 Part A

The equipotential curves match theoretical predictions. The electric field in between the two electrodes is,

$$\mathbf{E} = -(31.94 \pm 0.24)\hat{\mathbf{y}}\ V/m$$

10.2 Part B

The equipotential curves match theoretical predictions. The electric field also varies as theoretically predicted,

$$\mathbf{E} \propto -\frac{\hat{\mathbf{r}}}{r^2}$$

10.3 Part C

The equipotential curves approximately match the theoretical prediction. It does not fully match due to human error while making triangular electrodes.

11 Appendix

The fact that a curl-less vector can be written in terms of the gradient of a scalar potential follows from Helmholtz theorem for vector fields. Mathematically it states that an arbitrary vector, $\mathbf{F}(\mathbf{r})$ can be written as,

$$\mathbf{F}(\mathbf{r}) = -\boldsymbol{\nabla}\phi(\mathbf{r}) + \boldsymbol{\nabla} \times \mathbf{A}(\mathbf{r})$$

For a conservative vector field (like the electric field, $\mathbf{E}$), Applying Stokes's Theorem for $\mathbf{F}$ over a closed loop gives,

$$\oint_C \mathbf{F} \cdot d\mathbf{l} = \int_S \boldsymbol{\nabla} \times \mathbf{F} \cdot d\mathbf{S} = 0 \implies \boldsymbol{\nabla} \times \mathbf{F} = 0$$

Now we can define a scalar function $\phi(\mathbf{r})$ such that,

$$\phi(\mathbf{r}) = \int \mathbf{F} \cdot d\mathbf{l}$$

Thus, using the basic principles of partial derivatives we can vary the scalar function along any direction and finally get that,

$$\mathbf{F} = \boldsymbol{\nabla}\phi$$

Thus, $\boldsymbol{\nabla} \times \mathbf{E} = 0 \implies \mathbf{E} = -\boldsymbol{\nabla}V$

For equation 1 Laplace's equation is of the form,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Theoretically for infinitely long electrodes and for the region along the length of the parallel electrodes in this experiment there is no variation in the potential in the x-direction. Thus by symmetry the PDE reduces to,

$$\frac{\mathrm{d}^2 V}{\mathrm{d}y^2} = 0$$

Thus this implies,

$$\frac{\mathrm{d}V}{\mathrm{d}y} = K$$

$$\implies V(y) = Ky + C$$

Initial conditions: $V(0) = 0 \implies C = 0$ and $V(d) = V_0 \implies K = \frac{V_0}{d}$ Thus finally,

$$V(y) = \frac{V_0}{d}y$$

For equation 2 Laplace's equations in cylindrical coordinates with no change in the potential in the z-direction is,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0$$

By symmetry it can be inferred that V does not depend on θ since rotating the setup does not change the potential. Thus Laplace's equation reduces to,

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}V}{\mathrm{d}r}\right) = 0$$

This implies that,

$$r\frac{\mathrm{d}V}{\mathrm{d}r} = K$$

Let a new variable be defined as, $r \equiv \frac{r'}{r_0} \implies dr = \frac{dr'}{r_0}$. Thus the ODE above becomes,

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}r'} &= \frac{K}{r'} \\ \implies V(r') &= K \ln r' + C \\ &= K \ln\left(\frac{r}{r_0}\right) + C \end{aligned}$$

Initial conditions: $V(r_0) = V_0 \implies C = V_0$. Also, $V(r_1) = V_1 \implies V_1 = K \ln\left(\frac{r_1}{r_0}\right) + V_0 \implies K = \frac{V_1 - V_0}{\ln\left(\frac{r_1}{r_0}\right)}$. This finally gives,

$$V(r) = \left(\frac{V_1 - V_0}{\ln\left(\frac{r_1}{r_0}\right)}\right) \ln\left(\frac{r}{r_0}\right) + V_0$$