Lab Report 4

PHY 2010: Physics Lab 2

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1 Aim

1. To determine the acceleration due to gravity experienced by a freely falling object.
2. To study the dependence of terminal velocity on mass.

2 Theoretical Background

The equation governing the motion of the body is Newton's Second Law of Motion for this experiment. In the absence of any kind of non-conservative force on the system the equation in differential form can be written as,

$$m\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} = \mathbf{F}$$

This differential equation can be solved given initial conditions to obtain the position as a function of time, i.e., $\mathbf{x}(t)$ which can then be used to compute other parameters of the motion like velocity or energy.

When the damping effect of a non-conservative force is included the problem can be in general expressed as,

$$\mathbf{F} = m\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} + \mathbf{F}_d$$

The damping force, $\mathbf{F}_d$ always depends on the velocity of the object. At some point in the trajectory of the object, the damping effect becomes equal to the positive force and the velocity reaches a terminal value. This velocity is known as the terminal velocity, v_t. For the motion of an object falling under gravity the damping force is proportional to the square of the velocity, i.e., $F_d \propto v^2$. An object of mass m with cross sectional area A and velocity v has the damping force,

$$F_d = -C\rho_{air}Av^2$$

This can also be written as $F_d = -\alpha v^2$. Here C is a dimensionless constant which depends on the shape of the object falling. The terminal velocity of this object obtained from the equation of motion is,

$$v_t = \sqrt{\frac{mg}{C\rho_{air}A}}$$

By solving the differential equation the relation between the velocity at any time and the terminal velocity is,

$$v(t) = v_t \tanh\left(\frac{g}{v_t}t\right)$$

3 Experimental Setup

The experimental setup consists of nothing but a ladder. One person has to get up on the ladder and let the a small sphere or a cupcake liner from a height while the other person takes the video of the motion.

4 Equipment

1. A phone camera that takes videos at 60 fps.
2. Small spheres of different masses
3. A set of cupcake liners each of mass 0.15 g.
4. A set of small magnets each of mass 0.1g to increase the mass of the cupcake liners.
5. *Tracker Video Analysis* software.

5 Procedure

5.1 Part A

In this part, the value of acceleration due to gravity was measured by studying the motion of a freely falling sphere of mass 0.24 g.

1. The sphere was of sufficient mass so that the effect of air drag can be ignored.
2. The sphere was dropped from a certain height and a video of its fall was taken.
3. This video was analyzed using *Tracker* to obtain a position time graph for its motion.
4. From the position time graph the acceleration was computed. This was done by fitting a quadratic function of the form $ax^2 + bx + c$ to the plot. The value of the acceleration from this plot is $2a$. This is one value of acceleration due to gravity.
5. This was repeated 14 times and a histogram of different values was made.

5.2 Part B

In this part the dependence of terminal velocity on the mass of the falling object was studied.

1. Cupcake liner of mass 0.15 g were used for this since we require the falling object to attain terminal velocity before hitting the ground. The mass of the liner was measured by measuring the total mass of 20 cupcake liners and then divding the mass of those by 20. Here it was assumed that each liner is approximately of the same mass.
2. A cupcake liner was dropped from a certain height and a video of its fall was taken. This was repeated by increasing the mass of the liner by attaching small magnets each of mass 0.1 g (something that was calculated in the same way as for the liners).
3. Each video was analyzed using *Tracker* and a position-time and velocity-time graphs were drawn using the data from *Tracker*.
4. The terminal velocity of the liner was measured by fitting a straight line to the last few data points in the position-time graph. The slope of this fit was the terminal velocity of the liner.
5. The theoretical equation for the velocity in terms of time was also drawn over the velocity-time graph and the goodness of the fit was measured using the coefficient of determination, R^2.
6. A graph of the terminal velocities versus the mass of the object was also plotted and $\log - \log$ graph was drawn to check for what power law was followed for this relation. $v_t \propto m^p$, the value of p was obtained.

5.3 Precautions

1. While taking the videos a clear contrasting point of some colour should be present on the sphere or the cupcake liner to allow *Tracker* to track the motion properly using the `Point Mass` function on *Tracker*.
2. For the cupcake liners, they should not flip or wobble too much while falling down since that would make tracking very difficult and would give faulty results.
3. The cupcake liners should not rotate too much while falling as that would make tracking difficult.
4. The videos should not be started after the object starts falling as that would give the velocity at time $t = 0$ as some non-zero value which is not correct. Thus the videos should start a moment before the object should start falling.

6 Data

6.1 Least Count of Instruments

1. Weighing Scale: 0.1g
2. Tracker: 0.017 s (60 fps)

6.2 Part A

Sl No.	Acceleration (m/s^2)
1	9.686
2	9.55
3	9.817
4	9.876
5	9.57
6	9.612
7	9.721
8	9.694
9	9.531
10	9.753
11	10.511
12	10.594
13	10.259
14	10.144

Table 1: Different values of acceleration due to gravity obtained for the same mass.

6.3 Part B

The terminal velocities for different masses are,

Mass (g)	Terminal Velocity (m/s)
0.35	2.338
0.45	2.563
0.55	2.826
0.65	3.250

Table 2: Terminal velocities for different masses falling under gravity

7 Graphs and Analysis

7.1 Part A

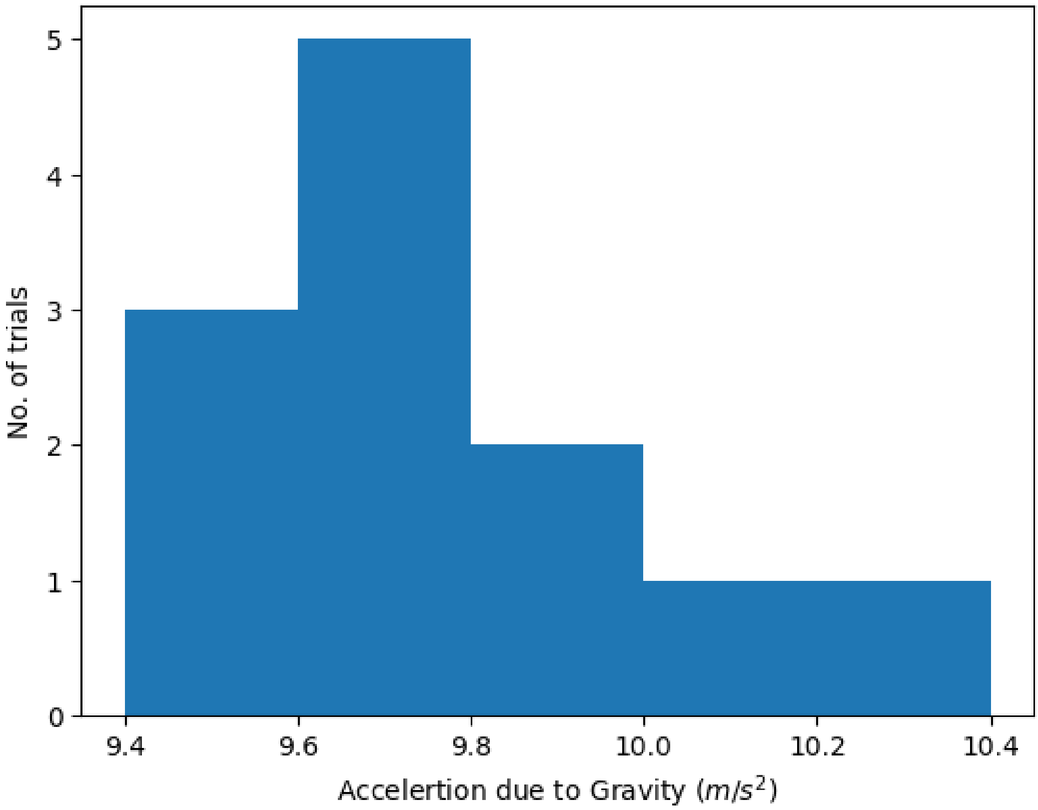

Figure 1: Histogram of values obtained for acceleration due to gravity for 24 trials.

The mean value was calculated as 9.879 and the standard deviation was 0.355. The median value of this data is 9.737 while the mode is 9.686. This data set is slightly skewed but the skewness is not too much and there aren't any outliers in the data. Thus, the mean is a reasonably good representation for measure of central tendency of the entire data set. Thus, the value of acceleration due to gravity is the mean of this data, i.e., 9.879.

7.2 Part B

The position time graphs for the 4 different masses were obtained as,

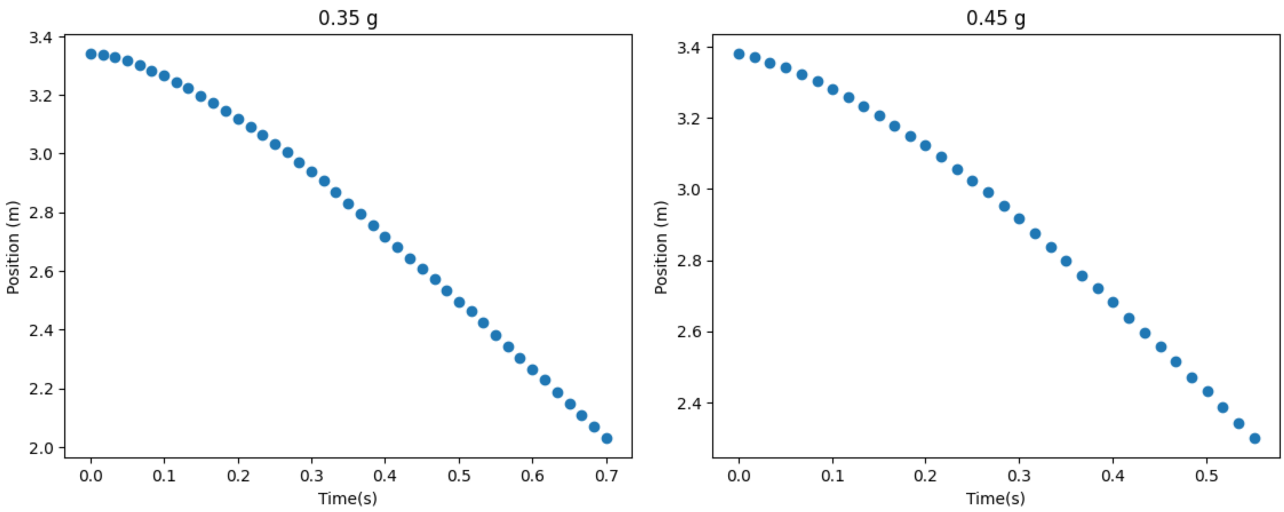

Figure 2: Position time graphs for 0.35 and 0.45 g falling masses.

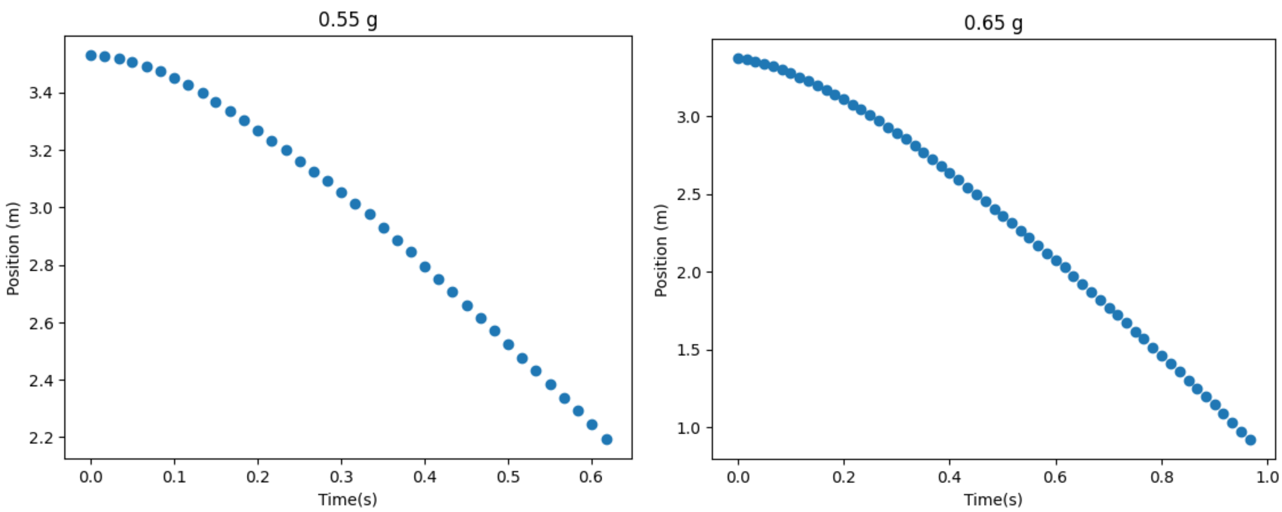

Figure 3: Position time graphs for 0.55 and 0.65 g falling masses.

The terminal velocity was measured for each falling mass by fitting a straight line to the last few points in the scatter plot. The slope of that linear fit was the terminal velocity for each falling mass.

The velocity time graph for the data obtained from *Tracker* was obtained as,

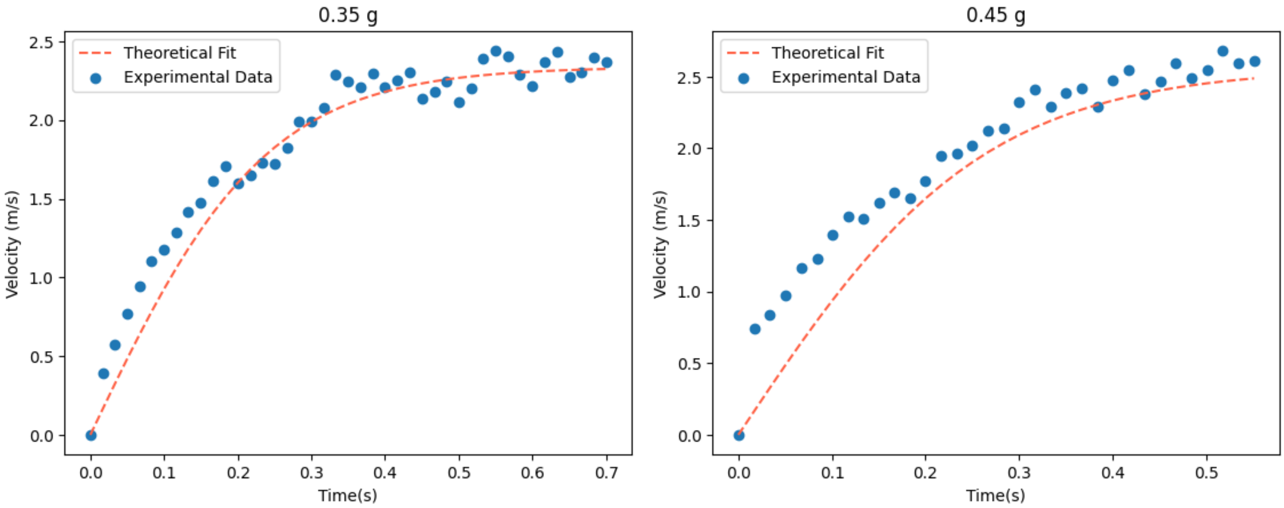

Figure 4: Velocity time graphs for 0.35 and 0.45 g falling masses. The theoretical relation is also plotted.

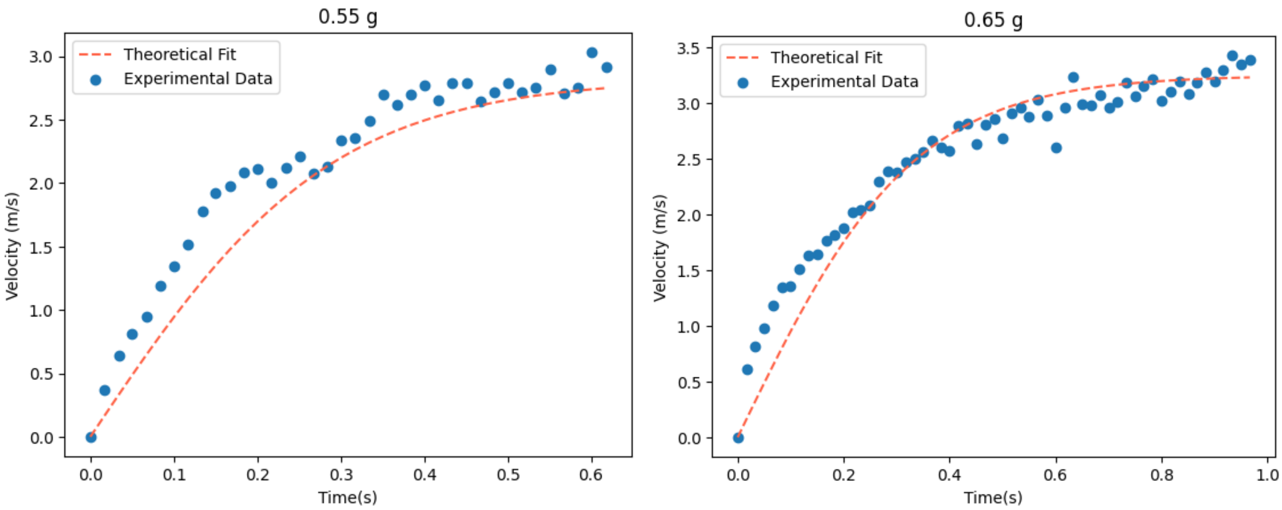

Figure 5: Position time graphs for 0.55 and 0.65 g falling masses. The theoretical relation is also plotted.

The theoretical and experimental data fit approximately well. This is confirmed by calculating the R^2 value for each data set. This was done by using the `numpy.corrcoef` function that returns correlation matrix between two sets of data. The correlation between the x-values and the y-values is in the [0,1] position of the 2×2 correlation matrix. The coefficient of determination is just the square of this value, i.e., R^2 and is a measure of how well the fit is to the data. A value of $R^2 \geq 0.9$ means that the fit is good. Doing this process the R^2 values obtained were,

Mass (g)	R^2 value
0.35	0.973
0.45	0.968
0.55	0.965
0.65	0.970

Table 3: R^2 value for the velocity time graphs for each mass.

A log − log graph drawn for terminal velocity versus the mass was obtained as,

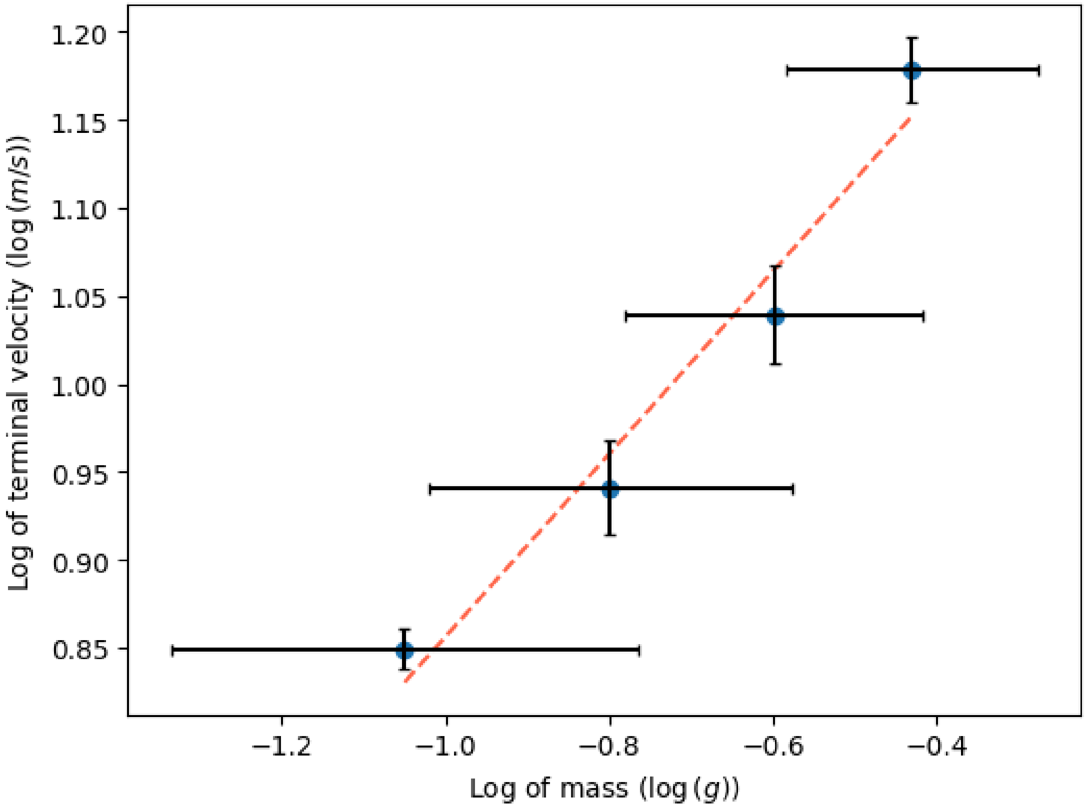

Figure 6: log − log graph for terminal velocity versus the mass to obtain the power law relation between the two.

On fitting a straight line the slope is obtained as 0.5. This implies that,

$$v_t \propto m^{0.5}$$

8 Error Analysis

8.1 Part A

The errors in measuring individual values of the accelerations due to gravity are equal to $2\Delta a$, i.e, twice the error in measuring the parameter a of the quadratic fit. The error in a is obtained by using the covariance matrix obtained from the `numpy.polyfit` function in python. Thus the errors are,

Measured value of g (m/s^2)	Error
9.686	0.358
9.550	0.460
9.817	0.562
9.876	0.390
9.570	0.446
9.612	0.467
9.721	0.501
9.694	0.414
9.531	0.361
9.753	0.328
10.511	0.276
10.594	0.380
10.259	0.390
10.144	0.421

Table 4: Error in measuring the value of acceleration due to gravity for each trial

8.2 Part B

The error in terminal velocity depends on the mass of the object. The equation relating terminal velocity and mass can be written as,

$$v_t = km^{0.5}$$
$$\implies \log v_t = 0.5 \log m + \log k$$

This can be rewritten as

$$Y = pX + C$$

Thus the errors,

$$\delta Y = p\delta X$$

In terms of log

$$\delta Y = \frac{\delta v_t}{v_t} \quad \delta X = \frac{\delta m}{m}$$

The relative error in measuring the terminal velocity is obtained from the covariance matrix for the linear fits on the end part of the position time graphs obtained from *Tracker*. The error in measuring the mass is just the least count of the instrument. Thus the errors on the logarithmic scale are,

Mass ($\log g$)	Error in mass on log scale	Terminal Velocity ($\log m/s$)	Error in terminal velocity on log scale
-1.05	0.288	0.849	0.011
-0.799	0.222	0.941	0.026
-0.598	0.182	1.039	0.028
-0.431	0.154	1.179	0.019

Table 5: Errors in the log scale for the graph of terminal velocity versus the mass

9 Results

9.1 Part A

The value of the acceleration due to gravity is obtained as,

$$g = 9.879 \pm 0.355$$

9.2 Part B

The theoretical prediction for the relation between the velocity and time account for a quadratic damping law holds true. Furthermore the power law relationship between the mass of a falling object and its terminal velocity is,

$$v_t \propto m^{0.5}$$

10 Appendix

Assume that the object falling has a cross-sectional area, A. In a small time interval Δt is exerts a force on mass of air equal to $m_{air} = \rho_{air} A v \Delta t$ The momentum of the air moving with the same velocity as the object and having mass m_{air} is equal to $m_{air} v$. This mass of air undergoes an increase in momentum of $\rho_{air} A v^2 \Delta t$. By the conservation of linear momentum it means that the object has lost momentum equal to this value. Taking into account the shape of the object, a dimensionless constant, C is introduced into the equation and is known as the *drag coefficient*. The force is the rate of change of momentum. Thus, the drag force is,

$$F_d = -C \rho_{air} A v^2$$

The object reaches terminal velocity when the drag force becomes equal to the gravitational force. Thus,

$$\begin{aligned} F_d &= mg \\ \implies C \rho_{air} A v_t^2 &= mg \\ \therefore v_t &= \sqrt{\frac{mg}{C \rho_{air} A}} \end{aligned}$$

The differential equation for the motion of the object with damping is,

$$\begin{aligned} m \frac{\mathrm{d}v}{\mathrm{d}t} &= mg - \alpha v^2 \\ \implies \frac{\mathrm{d}v}{\mathrm{d}t} &= g \left(1 - \frac{\alpha}{mg} v^2 \right) \end{aligned}$$

The terminal velocity from this equation comes out to be

$$v_t = \sqrt{\frac{mg}{\alpha}}$$

Thus the differential equation becomes

$$\begin{aligned}
\frac{\mathrm{d}v}{\mathrm{d}t} &= g\left(1 - \left(\frac{v}{v_t}\right)^2\right) \\
\implies \int_0^v \frac{dv'}{(1 - (\frac{v'}{v_t})^2)} &= g\int_0^t dt' \\
\implies v_t^2 \int_0^v \frac{dv'}{v_t^2 - v'^2} &= gt
\end{aligned}$$

Comparing with

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$$

The differential equation is

$$\begin{aligned}
v_t \tanh^{-1}\left(\frac{v}{v_t}\right) = gt \\
\therefore v(t) = v_t \tanh\left(\frac{g}{v_t}t\right)
\end{aligned}$$