Lab Report 3

PHY 2010: Physics Lab 2

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1 Aim

To study the effects of electromagnetic damping on a rotating flywheel

2 Theoretical Background

According to Lenz's law, a change in the magnetic field in a conductor induces an electromotive force which drives a current opposing the motion of any existing current within the conductor. Thus, if a magnetic field varies through a conductor or if a conductor moves within a magnetic field, an opposing current is set up within the conductor. These currents flow in loops within the conductor and are known as eddy currents. These induced eddy current themselves induce a magnetic field which opposes the magnetic field which gave rise to the eddy currents. Thus, eddy currents have a retarding effect which means that the net force causing the motion of the conductor reduces right after the motion begins. This phenomenon is known as *electromagnetic damping*. Eddy currents while causing energy loss in transformers have a positive use also. Eddy currents can be used to melt metals in induction furnaces. They can be for braking purposes, i.e., in trains, roller coasters and in electrical saws for emergency shutoff. Eddy currents can also be used for magnetic levitation of Maglev trains.

In this experiement, electromagnetic damping is studied in a rotating aluminium disk. This disc is mounted on a horizontal axle around which a cord is wound. The other end of the cord is attached to a slotted mass which is allowed to fall causing the disk to rotate. On two supports on either side of the disk, two bar magnets are attached which cause the electromagnetic damping in the system.

The motion can be studied using Newton's Laws. For the configuration without magnets there is no electromagnetic damping. In the absence of an external magnetic field, the relation between the acceleration of the falling object and its mass is,

$$a = m\left(\frac{gr^2}{I}\right) - \left(\frac{\tau_f r}{I}\right)$$

Here I is the moment of inertia of the flywheel, τ_f is the frictional torque that opposes the motion of the flywheel, r is the radius of the axle and m is the mass of the falling object.

The effect of electromagnetic damping can be understood by studying the motion of an electron within the conducting flywheel. Electrons movign with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ experience the Lorentz force given by $\mathbf{F} = e\mathbf{v}\times\mathbf{B}$. In this case the magnetic field is perpendicular to the disk and the electrons move in the azimuthal direction. Thus the Lorentz force is directed radially. This causes the electrons to move radially. However charges piling up in one place would cause an electric field to be set up which grows in strength till it completely counteracts the effect of the magnetic field. After this point since the conductor is a finite object, the electrons return to their original places. Thus, closed current loops are set up within the conductor. Some of these eddy current flow through the region where the magnetic field is present. Thus while they move through this region these electrons again experience a Lorentz force. The electrons this time are moving radially in a magnetic field perpendicular to the disk. Thus time the force is experienced in the azimuthal direction. The force is experienced in a direction opposite to that of the motion of the disk since otherwise the force would add to the motion and the velocity of the rotating flywheel would become infinite. That is not what is observed. Thus, a 3 step process causes a Lorentz force in the opposite azimuthal direction which is the cause of a retarding magnetic torque. See *Appendix* for a quantitative explanation of the magnetic torque. When magnets are introduced into the setup, a net damping torque comes into the picture and causes the falling mass to attain a constant velocity after a while. This terminal velocity is related to the magnetic field and the mass of the falling object as,

$$v_t = \left(\frac{gr^2}{CB^2}\right)\left(m - \frac{\tau_f}{gr}\right)$$

Here, B is the magnetic field due to the bar magnets and C is an averaging factor that occurs while analytically accounting for the effect of the damping torque.

3 Experimental Setup

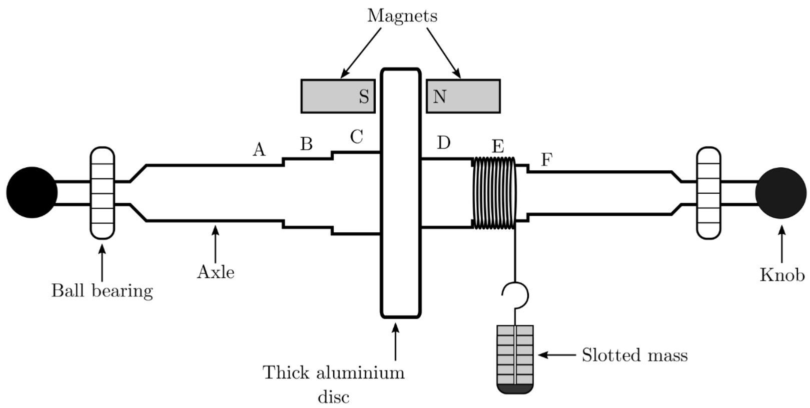

Figure 1: Schematic Diagram of the experimental setup. There are different axles of different radii [2]

4 Equipment

1. A flywheel disk assembly mounted on the wall
2. Two pairs of magnets with different pole strengths
3. A Gauss meter to measure the magnetic fields
4. A set of slotted masses (each weighing 50 grams). Different smaller masses of 10 and 20 grams.
5. Metre scales and tapes
6. A spirit level

A Gauss meter is used to measure the intensity of weaker magnetic fields in the unit gauss (gymbol G) which is equal to 10^{-4} teslas. It comprises of a gauss probe/sensor, the meter itself and a cable connecting both. The principle of working of the gauss meter is the Hall effect. When a current is a passed through a conductor placed in a perpendicular magnetic field, the magnetic field causes electrons to move to one side of the conductor. This sets up a measurable voltage inside the conductor which is directly proportional to the strength of the magnetic field and inversely proportional to the current density and conductor thickness. This phenomenon is called the Hall effect. In the gauss meter the sensor is almost completely flat to make use of this phenomenon. When the flat sensor is placed in a perpendicular magnetic field the Hall effect comes into play and is then the meter gives a reading. [1]

5 Procedure

5.1 Part A

In this part, the equation for the acceleration of the falling mass in the absence of a magnetic field was verified. The results were used to measure the moment of inertia of the flywheel and the frictional torque that opposed the motion of the falling mass.

1. The axle of diameter 31.2 mm was chosen for the experiment. A certain length of the cord was tied to the axle.
2. The other end of the cord was attached to a slotted mass and the wire was carefully wound around the axle.
3. The mass was allowed to fall and a video of it falling was taken. This video was analyzed using the *Tracker* application to obtain a position versus time graph of the falling mass.
4. The method of computing the acceleration from this graph is mentioned in the section titled *Graphs and Analysis*.
5. This procedure was repeated for more masses. A plot of the acceleration versus the mass was made.

5.2 Part B

In this part, a magnetic field was applied across the flywheel which introduced electromagnetic damping. The mass fell down and reached a terminal velocity.

1. A pair of magnets where placed in their holders and placed on the assembly.

2. The procedure in **Part A** was repeated. However this time the terminal velocity was measured from the position-time graph. The method to do this is explained in *Graphs and Analysis*.
3. A graph of terminal velocity versus the mass was plotted.

5.3 Part C

This part is the same as part B but this time the mass is kept constant and the magnetic field is varied.

1. A pair of magnets where placed in their holders and placed on the assembly.
2. First a gaussmeter was used to measure the magnetic field between the magnets at different separations. The tongue of the gaussmeter was placed approximately at the centre of the two magnets. Each time the separation was increased by a certain amount. Finally the data was used to plot a graph of the magnetic field versus the separation to get a relation between the magnetic field and the separation. This was used in the subsequent part of the experiment.
3. The procedure in **Part B** was repeated. But this time the separation between the magnets was changed to change the magnetic field. The mass used was 50 g and kept constant. THe terminal velocites were measured from the position-time graphs.
4. A graph of terminal velocity versus the magnetic field was plotted.

6 Data

6.1 Least Count of Instruments

1. Screw gauge: 0.01 mm
2. Vernier Callipers: 0.02 mm
3. Gauss meter: 0.01 G

6.2 Part A

6 masses were used in this part: 50, 100, 150, 200, 300 and 350 grams. The acceleration of each mass in the absence of a magnetic field is,

Mass (kg)	Acceleration (m/s^2)
0.05	0.00258
0.10	0.00603
0.15	0.00937
0.20	0.01226
0.30	0.01778
0.35	0.02128

Table 1: Acceleration for different slotted masses in the absence of a magnetic field

6.3 Part B

The constant magnetic field used was of strength 0.29 G.

Mass (kg)	Terminal Velocity (m/s)
0.0504	0.0351
0.0700	0.0447
0.1008	0.0664
0.1019	0.0734
0.1197	0.0835
0.1500	0.0982

Table 2: Terminal Velocity for different falling masses with a constant magnetic field applied across the flywheel

6.4 Part C

The constant mass used was 50.4 g.

Magnetic Field (G)	Terminal Velocity (m/s)
0.2756	0.0351
0.2618	0.0380
0.2484	0.0410
0.2355	0.0456
0.2230	0.0511
0.2110	0.0533

Table 3: Terminal Velocity for a falling mass of 50.4 g with different magnetic field strengths applied across the flywheel

7 Graphs and Analysis

7.1 Part A

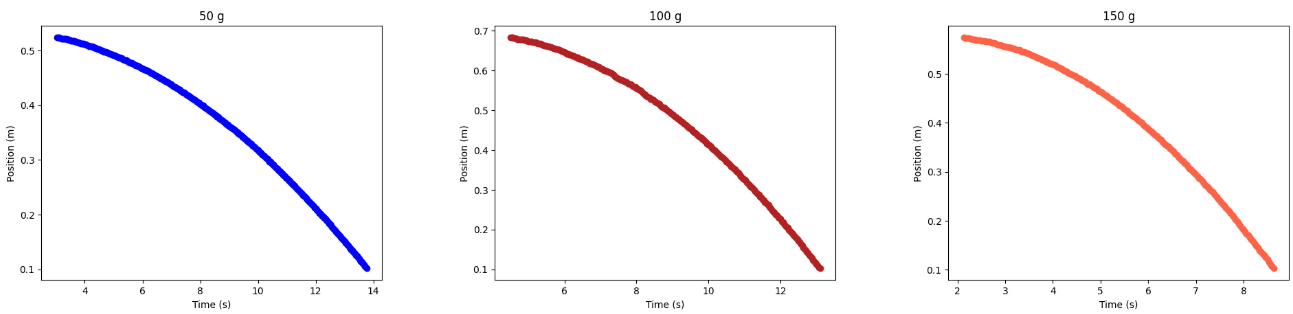

Figure 2: Position time graphs for 50, 100 and 150g falling masses

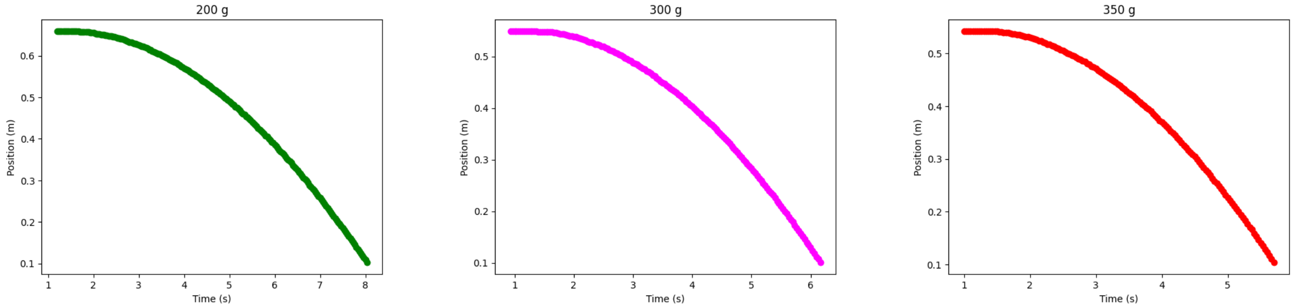

Figure 3: Position time graphs for 200, 300 and 350g falling masses

The graphs are all position-time graphs under uniform acceleration. Thus the equation for each graph is a quadratic one of the form $ax^2 + bx + c$. The acceleration is obtained by differentiating this graph twice which gives $2a$. Thus the acceleration is just equal to 2 times the parameter a obtained from using `numpy.polyfit` to fit a quadratic to the plot. Doing so the accelerations were obtained.

The plot of acceleration versus mass is,

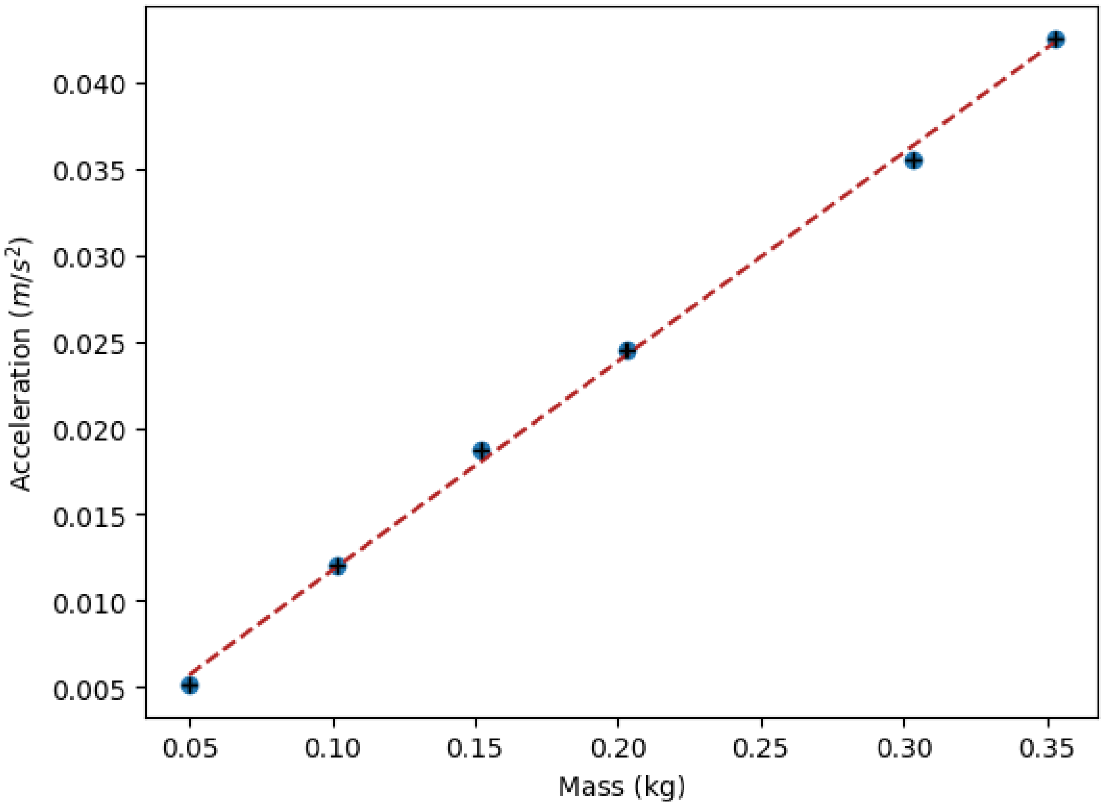

Figure 4: Acceleration versus mass graph for falling masses in the absence of a magnetic field

The slope and the intercept were used to calculate the moment of inertia and the frictional torque. The slope (m) of the graph is 0.12102 and the intercept (c) is -0.00035. The formulas used to measure the moment of inertia (I) and the frictional torque (τ_f) are,

$$I = \frac{g \times r^2}{\text{slope}}$$

$$\tau_f = \frac{\text{intercept} \times I}{r}$$

Here r is the radius of the axle. The values obtained are,

$$I = 0.01972 \ kgm^2$$

$$\tau_f = 0.00044 \ Nm$$

Error analysis is done in the next section

7.2 Part B

The terminal velocities were calculated by fitting a straight line to the last 50 points of the plot. The slope of that was the terminal velocity.

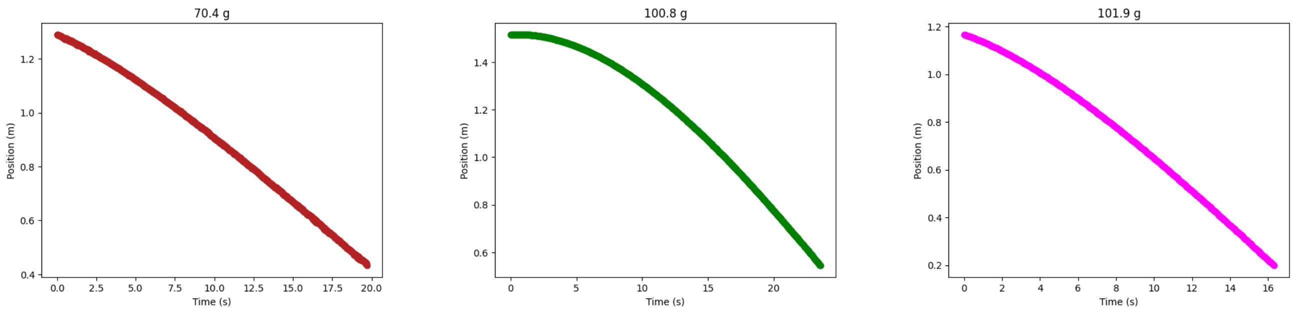

Figure 5: Position time graphs for 70.4, 100.8, 101.9 g falling masses with a constant magnetic field applied across the flywheel

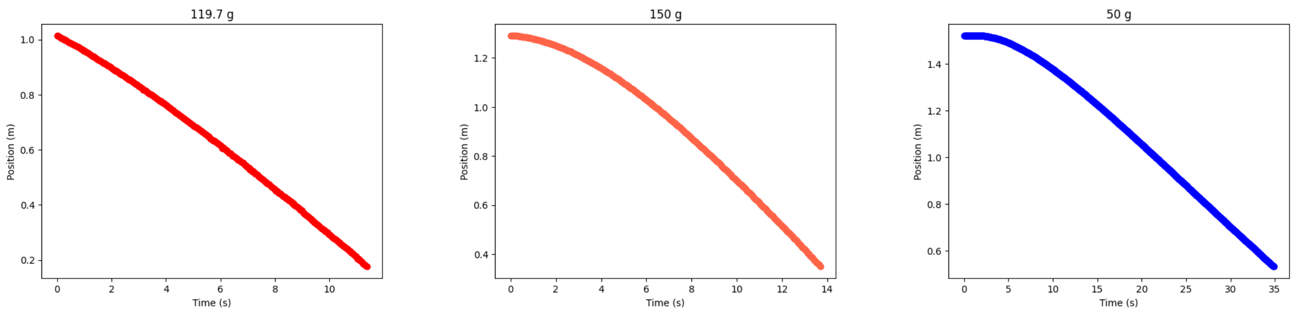

Figure 6: Position time graphs for 119.7, 150 and 50 g falling masses with a constant magnetic field applied across the flywheel

The graph of terminal velocity versus the mass was obtained as,

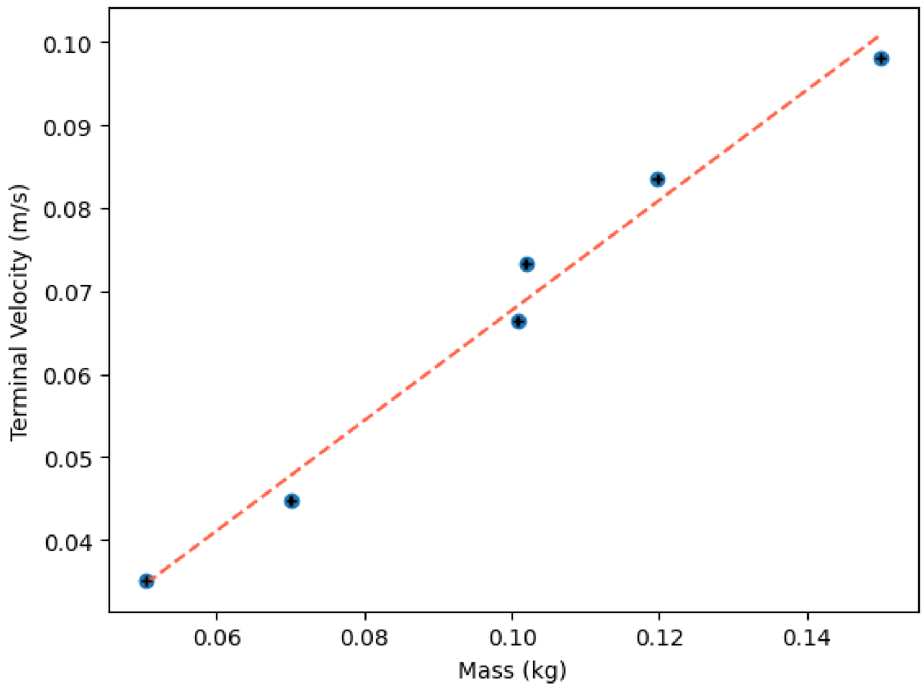

Figure 7: Plot of terminal velocity versus different masses.

The fit follows what is expected from the equation relating the terminal velocity and mass. The intercept is non-zero as is predicted by the equation. This plot is also used to find the value of the average factor C in the equation. From the above relation the value of C becomes,

$$C = \frac{gr^2}{\text{slope}}\left(m - \frac{\tau_f}{gr}\right)$$

Using the value of τ_f obtained in part A and different masses, the values of C come out to be,

Mass (kg)	Average Factor C
0.0504	0.00019
0.0700	0.00026
0.1008	0.00037
0.1019	0.00038
0.1197	0.00044
0.1500	0.00055

Table 4: Values of average factor C for different mass

7.3 Part C

The variation of the magnetic field with the separation between the two bar magnets was calculated as,

$$B(r) = 886.0322899111857r^2 - 66.19161219455441r + 1.289690631958771$$

The plot was,

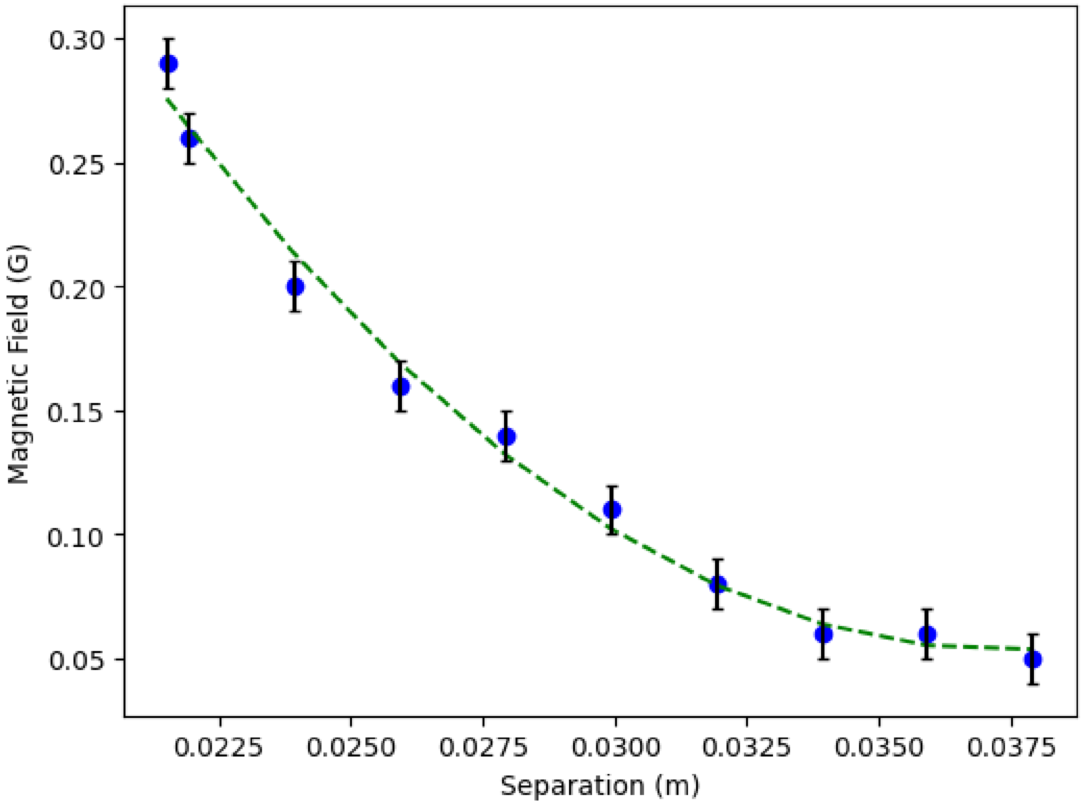

Figure 8: Magnetic field as a function of separation between the bar magnets.

Using this relation the magnetic field for different separations was calculated. The terminal velocities were measured as in part B above. The plots used to measure the terminal velocities were,

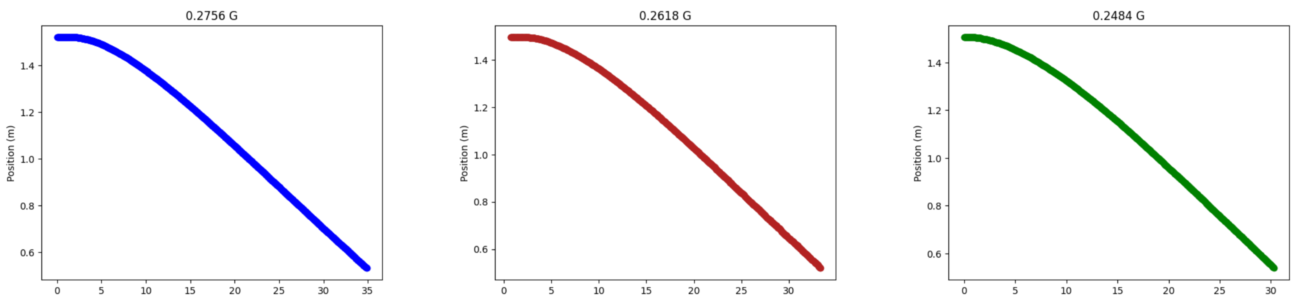

Figure 9: Position time graphs for 50.4 g mass with 0.2756, 0.2618, 0.2484 G magnetic field applied across the flywheel

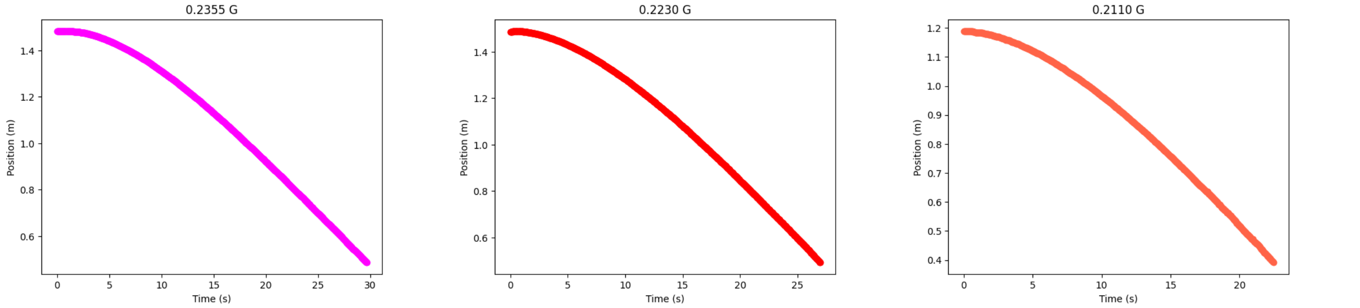

Figure 10: Position time graphs for 50.4 g mass with 0.2355, 0.2230, 0.2110 G magnetic field applied across the flywheel

The terminal velocities were plotted against the inverse of the square of the magnetic fields.

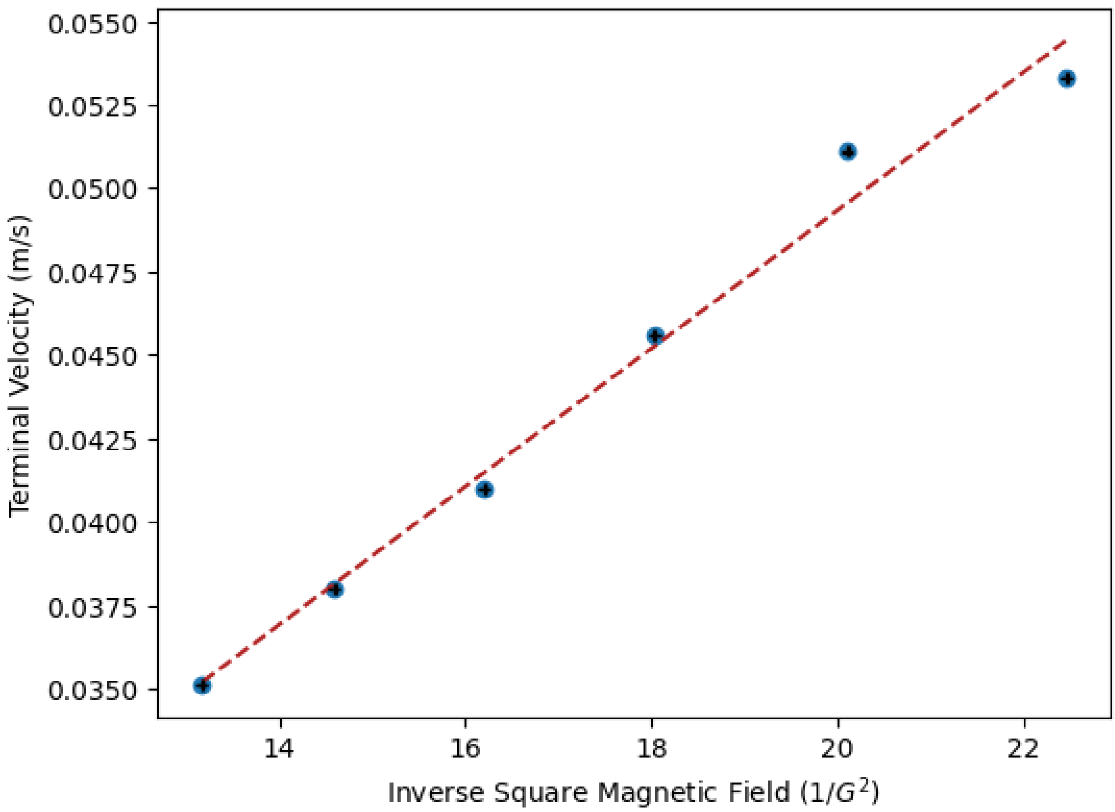

Figure 11: Plot of terminal velocity of 50.4 g falling mass versus the inverse of square of magnetic field.

8 Error Analysis

8.1 Part A

The relative errors in the parameters obtained for a fit using `numpy.polyfit` was measured by setting the `cov` argument for `polyfit` to true to get the covariance matrix. The relative errors are the square roots of the values along the diagonal of the covariance matrix. The errors in measuring the acceleration of the falling mass were calculated in the same way. For each mass the error in measuring the acceleratiosn were,

Mass (kg)	Acceleration (m/s^2)	Error in acceleration ($\times 10^{-8}\ m/s^2$)
0.05	0.00258	1.52559
0.10	0.00603	11.1127
0.15	0.00937	27.0489
0.20	0.01226	61.3286
0.30	0.01778	145.544
0.35	0.02128	202.908

Table 5: Caption

From the equations for I and τ_f the formula for measuring the relative error is are,

$$\frac{dI}{I} = \sqrt{\left(\frac{dm}{m}\right)^2 + 4 \cdot \left(\frac{dr}{r}\right)^2}$$

$$\frac{d\tau_f}{\tau_f} = \sqrt{\left(\frac{dI}{I}\right)^2 + \left(\frac{dc}{c}\right)^2 + \left(\frac{dr}{r}\right)^2}$$

Using these formulas the absolute errors in measuring the moment of inertia and the frictional torque are,

$$\Delta I = 6.86564 \times 10^{-5} \qquad \Delta\tau_f = 0.16624 \times 10^{-5}$$

8.2 Part B

The error in measuring the terminal velocity is calculated in the same way as above by using the covariance matrix for the parameters of the fit.

Mass (kg)	Terminal Velocity (m/s)	Error in Terminal Velocity ($\times 10^{6}$ m/s)
0.0504	0.0351	3.4736
0.0700	0.0447	3.4727
0.1008	0.0664	14.4733
0.1019	0.0734	14.965
0.1197	0.0835	15.2853
0.1500	0.0982	39.2004

Table 6: Error in measuring the terminal velocities for different masses.

8.3 Part C

The error in measuring the magnetic field is equal to the least count of the gauss meter which is 0.01 G. The error in measuring the terminal velocity was calculated using the same method as in part B.

Magnetic Field (G)	Terminal Velocity (m/s)	Error in Terminal Velocity ($\times 10^6$ m/s)
0.2756	0.0351	3.4736
0.2618	0.0380	4.3061
0.2484	0.0410	4.8356
0.2355	0.0456	6.504
0.2230	0.0511	8.1318
0.2110	0.0533	8.4421

Table 7: Error in measuring the terminal velocities for different magnetic fields

9 Results

9.1 Part A

The moment of inertia (I) of the flywheel is,

$$I = 0.01972 \ \pm 6.86564 \times 10^{-5} \ kgm^2$$

The frictional torque opposing the rotation of the flywheel is,

$$\tau_f = 0.00044 \pm 0.16624 \times 10^{-5} \ Nm$$

9.2 Part B

The terminal velocity and the mass of the falling object follow the established relationship for a constant magnetic field.

9.3 Part C

The magnetic field follows an inverse square law. The magnetic field at the centre of the disk is 1.2897 G. Furthermore the terminal velocity and the magnetic field follow the established relation for a constant mass.

10 Appendix

This section has been referred from [2].

To derive the relation between the acceleration of the falling object and the mass, Newton's law for rotational motion is considered including the frictional torque. The torque that causes the motion comes from the tension in the cord. Thus to find the tension,

$$\begin{aligned} mg - T &= ma \\ \implies T &= m(g-a) \end{aligned}$$

Thus,

$$
\begin{aligned}
I\alpha &= Tr - \tau_f \\
\implies I\frac{a}{r} &= mr(g-a) - \tau_f \\
\implies (I+mr^2)a &= mr^2 g - \tau_f r \\
\implies a &= m\left(\frac{gr^2}{I+mr^2}\right) - \left(\frac{\tau_f r}{I+mr^2}\right)
\end{aligned}
$$

Since the radius of the flywheel is much larger than the radius of the axle we can approximate $I + mr^2 \approx I$. Thus,

$$a = m\left(\frac{gr^2}{I}\right) - \left(\frac{\tau_f r}{I}\right)$$

For the magnetic torque τ_B,

$$I\alpha = Tr - \tau_f - \tau_B$$

The magnetic field is restricted to a small circular region, $r_m \ll R$ where r_m is the radius of the circular region and R is the radius of the flywheel. The magnetic field is uniform over this region. Consider a thin conducting path along the diameter of this circular region and is of length, $2r_m$. This conducting path moves at a velocity $v = \omega r_b$ where r_b is the distance from the centre of the small circular region to the centre of the flywheel. Thus the induced emf in this region is,

$$\mathcal{E}_b = r_b\omega \times B \times 2r_m$$

This induced emf gives rise to an eddy current i_b which moves along the path which has resistance R^* [1]. Thus,

$$i_b = \frac{\mathcal{E}_b}{R^*} = \frac{r_b\omega \times B \times 2r_m}{R^*}$$

The force experienced by the eddy current is,

$$F_b = B \times i_b \times 2r_m = \left(\frac{4r_m^2 r_b}{R^*}\right)\omega B^2$$

This force is responsible for the damping magnetic torque,

$$\tau_B = F_b r_b = \left(\frac{4r_m^2 r_b^2}{R^*}\right)\omega B^2$$

This can be written as $\tau_B = C'\omega\omega B^2$. Since the final magnetic torque will be an average of all the different torques due to different resistances and different eddy currents, an average factor C can be included to have,

$$\tau_B = C\omega B^2$$

Thus,

$$I\alpha = m(g-a)r - \tau_f - C\omega B^2$$

[1] This is a very simplified picture. Modelling eddy currents is not easy since they flow through different paths with different resistances.

At terminal velocity, $v_t = \omega_t r$ both $\alpha = a = 0$. Thus the equation can be re-written and rearranged as,

$$\begin{aligned} C\omega_t B^2 &= mgr - \tau_f \\ \implies \omega_t r &= \frac{1}{CB^2}\left(mgr^2 - \tau_f r\right) \\ \therefore v_t &= \left(\frac{gr^2}{CB^2}\right)\left(m - \frac{\tau_f}{gr}\right) \end{aligned}$$

References

[1] https://www.metravi.com/what-is-a-gauss-meter-or-magnetometer/

[2] Cherian, Philip, et al. "Experiment 4 Electromagnetic Damping", Ashoka University Physics Lab 2 Handouts, 2023, pp. 27-33