Lab Report 2

PHY 2010: Physics Lab 2

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1 Aim

To verify Newton's laws of motion using an Air track.

2 Theoretical Background

The basis of this experiment are Newton's laws of motion. The first law states that in the absence of an external and unbalanced force an object's state of motion or rest remains unchanged in the same direction with the same speed. The first law is a qualitative definition of what a force is. The second law provides a quantitative definition of a force. This law relates the force applied on a mass to its acceleration through the famous formula,

$$\boldsymbol{F} = m\boldsymbol{a}$$

The first law can be derived from the 2nd law. In the absence of a force, the acceleration of the object is 0 which means that the velocity remains constant. The third law of motion states that every action has an equal and opposite reaction. In mathematical terms it can be written as,

$$F_{12} = -F_{21}$$

Where F_{12} is the force exerted by the 1st body on the 2nd and F_{21} is the force exerted by the 2nd body on the first and the 3rd law implies that it is opposite in direction and equal in magnitude. A consequence of Newton's 3rd law is the principle of conservation of momentum. Newton's second law can also be written as the rate of change of momentum, i.e.,

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}$$

In the absence of an external force this implies that, $p = m \times v$ is constant. In the case of a collision between two objects of masses, m_1 and m_2 moving with velocites, u_1 and u_2, Newton's 3rd law relates the force between the two objects. If we consider the forces to be acting over a small time period dt we get the change in momenta to be equal, i.e.,

$$m_1 u_1 = -m_2 u_2$$

Thus in collisions, momentum is transferred from one object to the other. There are 2 types of collisions: elastic and inelastic. Collisions in real life are usually a combination of both types. In perfectly elastic collisions, both momentum and kinetic energies are conserved, i.e., if the two objects mentioned above after collision move away with velocities [1], v_1 and v_2 respectively then conservation of momentum gives us,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Since the collision is elastic kinetic energy is also conserved and thus,

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Perfectly inelastic collisions are where the momentum is conserved but the initial and final kinetic energies are not the same. This situation may occur when the colliding bodies stick together after collision. Consider the above situation but after collision the bodies stick together and move with a velocity, V. In inelastic collisions we get,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$$

There is a loss in kinetic energy in perfectly inelastic collisions. This loss can be quantified using the relation,

$$\Delta E = \frac{1}{2} \mu (u_2 - u_1)^2$$

Here μ is the reduced mass of entire system and is equal to $m_1 m_2 / (m_1 + m_2)$.

The most important assumption in these equations is that there is no friction involved which would oppose the motion of an object. To simulate an almost frictionless surface a device called the air track is used which creates a cushion of air between the object that slides on it and the surface of the air track itself.

With a such a frictionless setup, one can use the second law of motion to measure the value of acceleration due to gravity. A pulley with a certain mass, m attached to one end of a string with the other end attached to the rider of mass, M (which is the object that slides on the air track) can be used to measure the value of g. The equation that is used is,

$$g = a \left(\frac{M + m}{m} \right)$$

Here a is the acceleration of the rider due to the force of the falling mass.

3 Experimental Setup

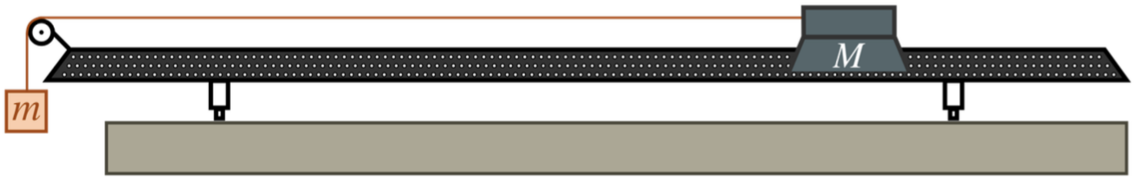

Figure 1: Schemtic Diagram of the Air track with the rider attached [1]. This diagram represents the situation for part B of the experiment. Furthermore, there are two Vernier photogates placed along the length of the air track that will help measure the velocity of

4 Equipment

A **2 metre long air track** to simulate a frictionless surface. The air track has a lot of small holes through which air is pumped thus creating an almost frictionless surface to verify Newton's laws of motion.

Riders of masses 200 and 400 grams. The riders are the objects that slide on the air track. **Rubber band buffers** that serve as end attachments to the rider and help simulate elastic collisions. **Brass buffers** are also end attachments but help to simulate inelastic collisions. **Plastic counterweights** that are also end attachments but act as counterweights to prevent the rider from moving to one side due the downward force of a buffer attached to it.

Two **Vernier photogates** to measure the velocites of the riders. The photogates when switched on have small opening through which a thin line of radio waves pass through. Interrupting this is equivalent to recording data. The interruptor cards attached onto the riders when passing through the photogate interrupt the signal.

LabQuest Mini is the interface connecting the photogate to the computer. This interface records the exact time at which the signal was first interrupted till when the signal came back on.

LoggerPro 3 is the software that was used to collect data and record the time stamps.

Retort stands to position the photogates.

5 Procedure

5.1 Part A

In this part, verification of Newton's 1st law of motion was attempted.

1. The air track was set up as shown in the figure 1. Instead of the pulley, a metal spring was attached to the side and two photogates were also set up over the air track at two different points.

2. The rider was pushed against the spring on one side and let go.

3. The rider used was of mass 200 grams (the interruptor card added to the mass but the mass is irrelevant for this part). The spring was compressed to different lengths to give different

initial velocities to the rider. The data from the photogates was then used to measure the velocities at two different points along the air track.

4. Furthermore the compression in the spring is proportional to the velocity of the rider. This fact was used to plot the velocities at the two photogates against the compression. The compression was calculated by subtracting the length of the rider when the string was compressed from the length of the rider when the string was uncompressed.

5.2 Part B

In this part, verification of Newton's 2nd law of motion was attempted.

1. On one end of the air track, the steel spring was removed and a pulley was attached.
2. A mass-less and inextensible string was tied to end of the rider. The other end of the string was tied to different masses and taken over the pulley.
3. The rider was allowed to accelerate through the photogates. The data from the photogates was used to determine the acceleration of rider due to the mass. This data was used to plot the equation and obtain the value of acceleration due to gravity from the slope. This process was repeated for different masses.

5.3 Part C

In this part, Newtons's 3rd law of motion of was verified. Furthermore it also served as a study of elastic and inelastic collisions.

1. Two riders were used for this part. For elastic collisions, one side of the rider was fitted with a buffer while the other side was fitted with a counterweight. The side with the buffer was the part that would cause the collision.
2. The two photogates were set up a little distance away from each other and the two riders were pushed a little bit from both ends of the air track.
3. The riders were allowed to collide and the initial and final time stamps from the photogates were noted down for measuring the velocity.
4. These velocities were used to measure the initial and final momenta and kinetic energies.

In all parts of the experiment the velocity at each photogate was measured by looking at the time stamps given by the photogate. To measure the time interval for which the rider interrupted the signal, the difference in the time stamps for each photogate was noted. To get the velocity, the length of interruptor card was divided by the difference between these times. The time stamps for each photogate can be differentiated and the method for distinction is mentioned below in the section on *Precautions*. This gives the velocity at each photogate. To get the acceleration at each photogate the difference in velocities was divided by the total duration for which the rider was in between the two photogates. Thus, the very first time stamp and the very last time stamp record when the rider first entered the area of the first photogate and when it left the last photogate respectively. This gives the total duration for which the photogate was accelerated. Thus dividing the difference in velocity by this time duration gave the acceleration of the rider.

5.4 Precautions

1. Using the right air pressure for the air track is crucial. If the air pressure is too high it will cause the rider to wobble which would give faulty readings. On the other hand, low air pressure would cause the rider to scratch the surface of the air track which again would give faulty readings.
2. The air track should be level. If it is not then the rider would automatically move to one side. The air track can be made level by using a spirit level.
3. Since 2 photogates are being used it is important to be able to distinguish between them when noting down the data captured by them. In the LoggerPro 3 application being used, there are multiple columns besides the one logging the time for the photogate. On the column just to the right of the time column the data for one of the photogates shows 1 or 0 (called the state of the photogate) while for the other one it does not show anything in this column. This can be used to distinguish between the data of the photogates.
4. While calculating the momenta of the riders in part C, it should be noted that velocity is a vector quanitity. Thus, the signs of the velocities of the two riders will be opposite when measuring the initial and final momenta of the system.

6 Data

6.1 Least Count of Instruments

1. Weighing Scale: 1 g
2. Air Track: 1 mm
3. Photogate: 0.000001 s

6.2 Part A

The compression of the spring and the velocities at each photogate are listed below,

Compression (cm)	Velocity at Photogate 1 (cm/s)	Velocity at Photogate 2 (cm/s)
1.200	11.158	9.129
1.700	16.162	14.498
2.200	20.983	19.821
2.700	25.724	24.796
3.200	31.352	30.622
3.700	42.042	41.439
3.900	44.401	43.775

Table 1: Data for verifying Newton's first law

6.3 Part B

The rider used was of mass 223.9 grams. The acceleration and the mass tied to the pulley are listed below

Mass (g)	Acceleration (cm/s^2)
1.9	7.19
3.4	13.02
4.5	17.37
5.6	21.08
5.7	21.69
8.6	32.92
10.0	38.01
12.4	47.96
15.1	57.41
20.1	75.58

Table 2: Acceleration of the rider for each mass tied to the pulley. The plot was made by converting the masses to kilograms and the acceleration to m/s^2.

6.4 Part C

Two sets of data for elastic and inelastic collisions each were noted down. The data for elastic collisions are as follows,

Collision 1		
	Mass 1 = 249.1 g	Mass 2 = 444.3 g
Initial Velocity (cm/s)	46.578	65.265
Final Velocity (cm/s)	91.924	11.714
Initial Momentum (gcm/s)	16928.714	
Final Momentum (gcm/s)	18612.795	
Initial Kinetic Energy (erg)	1227295.604	
Final Kinetic Energy (erg)	1125173.655	
Collision 2		
	Mass 1 = 249.1 g	Mass 2 = 444.3 g
Initial Velocity (cm/s)	44.130	41.772
Final Velocity (cm/s)	15.290	64.100
Initial Momentum (gcm/s)	8783.714	
Final Momentum (gcm/s)	9814.813	
Initial Kinetic Energy (erg)	658683.682	
Final Kinetic Energy (erg)	584233.206	

Table 3: Data for elastic collisions

Collision 1		
	Mass 1 = 429.3 g	Mass 2 = 447.6 g
Initial Velocity (cm/s)	39.298	52.672
Final Velocity (cm/s)	26.757	9.097
Initial Momentum (gcm/s)	6705.284	
Final Momentum (gcm/s)	7414.793	
Initial Kinetic Energy (erg)	952381.557	
Final Kinetic Energy (erg)	172192.184	
Collision 2		
	Mass 1 = 429.3 g	Mass 2 = 447.6 g
Initial Velocity (cm/s)	46.572	21.387
Final Velocity (cm/s)	11.273	30.564
Initial Momentum (gcm/s)	-10420.661	
Final Momentum (gcm/s)	-8841.303	
Initial Kinetic Energy (erg)	567923.496	
Final Kinetic Energy (erg)	236347.637	

Table 4: Data for inelastic collisions.

7 Graphs and Analysis

7.1 Part A

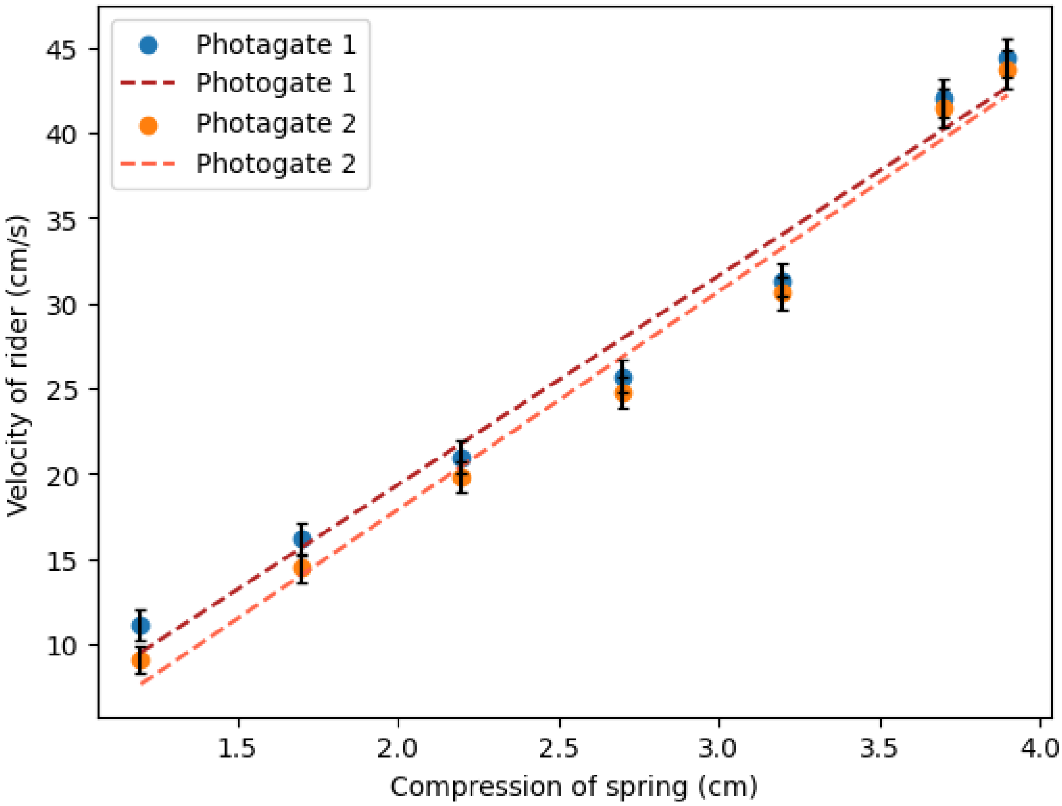

Figure 2: Plot of velocities at the two photogates versus the compression in the spring.

The slope of the line for photogate 1 is equal to 12.276 and for photogate 2 is 12.802. The slope of this graph is the natural frequency of the metal spring used. The two lines are not perfectly parallel. If the lines were parallel that would have meant that the velocites at the two photogates are the same. However the data shows that they are not. The slopes being different does not immediately imply that Newton's first law is not valid. It only means that there is either a problem in recording the data or there is some inherent error in the experimental setup. The latter is more likely. This is so since the velocities were measured using the data of the LabQuest Mini Interface. Thus the error in recording of the data would be minimal. The possibilty of inherent errors is discussed in the section on *Error Analysis* below.

7.2 Part B

The formula as mentioned in theory above was used to plot and get the value of g. The value of g is the slope in a graph of a vs $m/(M+m)$. Thus, the graph was plotted as,

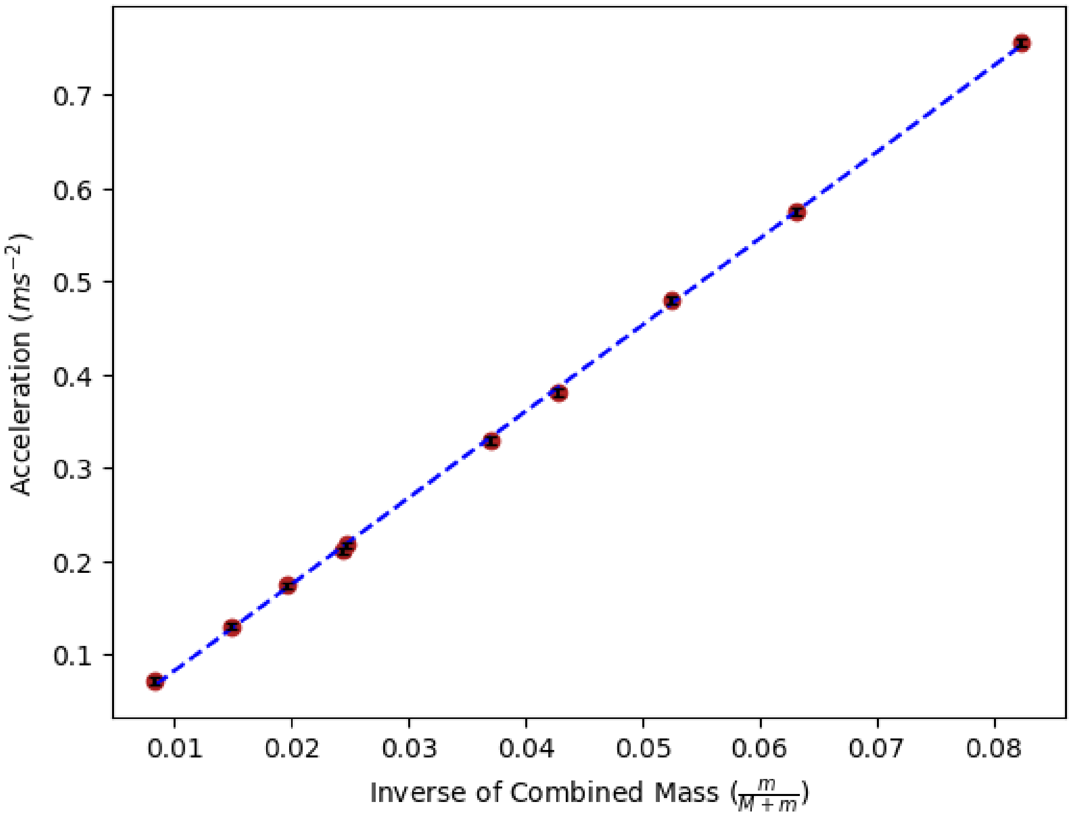

Figure 3: Plot of the acceleration of the rider versus the inverse of the combined mass function as shown in the equation in the theory above

The value of the slope was calculated as 9.272. Thus from the graph the value of acceleration due to gravity is 9.272 ms^2.

The possible reasons for why the actual value was not obtained are discussed in the section on *Error Analysis* below.

7.3 Part C

From the data above it seems like even in elastic collisions momentum and kinetic energies are not conserved. The change in the momentum and kinetic energies are not too much indicating that given ideal conditions we would be able to get much better results. However as in part A the changes can be attributed to an inherent error in the setup. More is said in *Error Analysis* below.

For the inelastic collisions the change in kinetic energies are expected. The faster the initial velocities of the riders are the bigger the loss in energy is. Since they riders are moving in opposite directions, on colliding they exert a force on each other in opposite directions. Thus, due to this fact and due to it being an inelastic collision there is a mutual reduction in the energies of the riders. This was observed as the riders moved much slower compared to their initial velocities on colliding inelastically.

8 Error Analysis

The first major source of error is the presence of a very small amount of friction. It is near impossible to get a surface that is completely frictionless. Thus while the film of air does reduce friction, it does not get rid of it completely. This shows up in the data for each part of the experiment. There is some some loss of energy of the rider which makes it slow down a little bit. There are also other factors. The riders are first held by hand and then left or pushed. This pushing itself can cause the rider to wobble a little bit and cause a loss in velocity.

The relative changes in the velocities at the two points in part A are as follows,

Velocity 1 (cm/s)	Velocity 2 (cm/s)	Relative Change (%)
11.158	9.129	18.187
16.162	14.498	10.296
20.983	19.821	5.536
25.724	24.796	3.606
31.352	30.622	2.326
42.042	41.439	1.434
44.401	43.775	1.411

Table 5: Relative changes in the velocity at two points along the air track in part A of the experiment.

As can be seen, the relative change in the velocity is lesser for higher velocities. This is a consequence of the formula for measuring the relative error in a certain quantity. Furthermore the plot of the relative change versus the compression (which is the independent variable in this experiment) is a parabola opening upwards. The relative change in the slopes of the two graphs is approximately 4.285%.

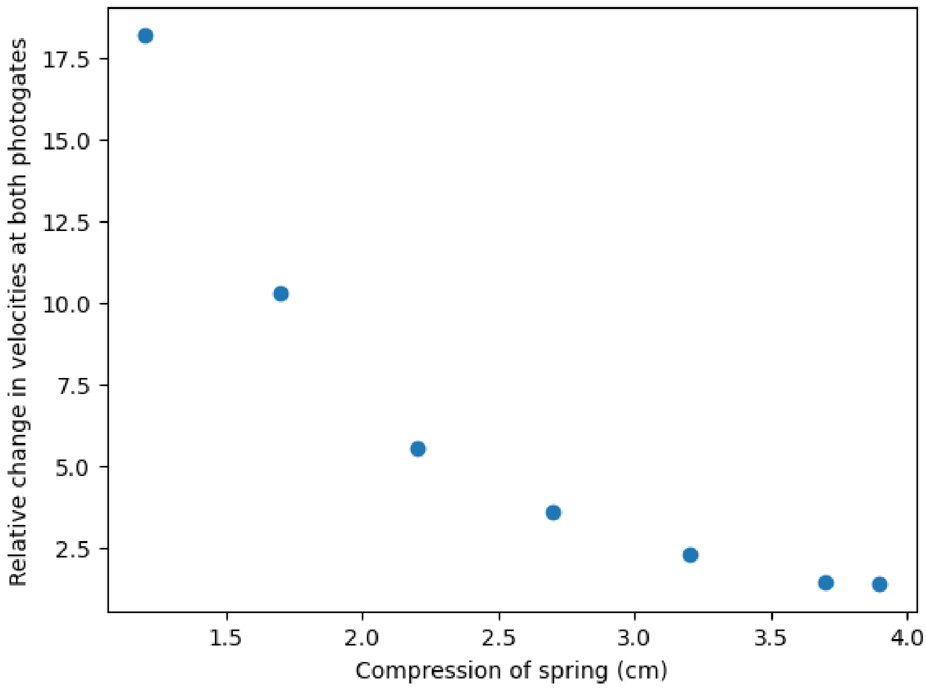

Figure 4: Plot of relative change in velocities at each photogate versus the compression in the spring.

The value of acceleration due to gravity obtained is lesser because this friction was not accounted for in the formula. The friction involved in part B can be calculated now. Using the actual value of $g = 9.807$ we get the frictional forces for each mass to be,

Mass (kg)	Friction (N)
0.0019	0.0024
0.0034	0.0037
0.0045	0.0045
0.0056	0.0065
0.0057	0.0061
0.0086	0.0078
0.01	0.0092
0.0124	0.0083
0.0151	0.0109
0.0201	0.0127

Table 6: Frictional force that acts on the rider due to each mass on the pulley.

The relative error in measuring the value of g is,

$$\frac{\Delta g}{g} = \quad \text{mod}\left(\frac{9.272 - 9.807}{9.272}\right)$$
$$\therefore \frac{\Delta g}{g} = 5.761\%$$

The relative changes in initial and final momenta and kinetic energies in part C of the experiment are as follows,

	Relative Change in Momentum (%)	Relative Change in Kinetic Energy (%)
	Elastic Collisions	
Collision 1	-9.948	8.321
Collision 2	-11.739	11.303
	Inelastic Collisions	
Collision 1	-10.581	81.920
Collision 2	15.156	58.384

Table 7: Relative changes in the initial and final momenta and kinetic energies in part C of the experiment

The changes in the kinetic energies in inelastic collisions are expected. However the changes in momenta in both collision and in kinetic energies in elastic collisions can be attributed to the presence of a slight frictional force on the air track.

9 Results

9.1 Part A

The velocity of the rider remains approximately constant in the absence of an external force. The slight changes in the velocities are only due to the presence of a very small amount of friction on the air track. The graphs of the velocities versus the compression of the spring also show that these two quantities are directly proportional to each other. Thus, Newton's 1st Law holds.

9.2 Part B

The value of the acceleration due to gravity is measured to be $9.272 \pm 0.534\ m/s^2$. The lower-than-actual value is due to the presence of a small amount of friction which has been quantified and listed above. Furthermore the graph shows that the acceleration in an object is directly proportional to the inverse of the mass proving Newton's 2nd law.

9.3 Part C

For the elastic collisions momentum and kinetic energies are conserved to a certain extent. The data for inelastic collisions were also expected. The losses are relatively small and thus we can consider Newton's 3rd law to holds as well.

10 Appendix

The principle of conservation of momentum can be derived as follows, Consider two bodies of masses m_1 and m_2 initially moving with velocities, u_1 and u_2 collide. Also consider the force exerted by each of the objects on each other to act over a small time period Δt. Then by Newton's 3rd law,

$$\begin{aligned} F_{12} &= -F_{21} \\ \implies F_{12}\Delta t &= -F_{21}\Delta t \\ \implies \Delta p_1 &= -\Delta p_2 \\ \implies m_1\Delta u_1 &= -m_2\Delta u_2 \end{aligned}$$

Now if the final velocities are v_1 and v_2, then we get,

$$m_1(u_1 - v_1) = -m_2(u_2 - v_2)$$
$$\therefore m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

This is the equation for the principle of conservation of momentum.

The equation for the loss in kinetic energy in perfectly inelastic collisions is derived as follows,

$$\begin{aligned}
m_1u_1 + m_2u_2 &= (m_1 + m_2)V \\
\implies V &= \frac{m_1u_1 + m_2u_2}{m_1 + m_2} \\
\Delta E &= \frac{1}{2}[m_1u_1^2 + m_2u_2^2 - (m_1 + m_2)V^2] \\
&= \frac{1}{2}\left[m_1u_1^2 + m_2u_2^2 - \frac{(m_1u_1 + m_2u_2)^2}{m_1 + m_2}\right] \\
&= \frac{1}{2}\left[\frac{m_1m_2u_1^2 + m_2m_1u_2^2 - 2m_1m_2u_2u_1}{m_1 + m_2}\right] \\
\therefore \Delta E &= \frac{1}{2}\mu(u_2 - u_1)^2
\end{aligned}$$

Where $\mu = \frac{m_1m_2}{m_1+m_2}$

In part A it is mentioned that the compression in the spring is proportional to the velocity of the rider. This is consequence of the principle of conservation of energy where the potential energy stored in the spring due to its compression is completely converted into the kinetic energy of the rider (this is of course in a scenario where there is no frictional force). If the compression in the spring (of spring constant k) is x, and the rider of mass, m, moves with a velocity, v, then conservation of energy gives us,

$$\begin{aligned}
\frac{1}{2}kx^2 &= \frac{1}{2}mv^2 \\
\implies \left(\frac{v}{x}\right)^2 &= \frac{k}{m} \\
\implies \frac{v}{x} &= \omega_0 \\
\therefore v &\propto x
\end{aligned}$$

The term 'ω_0' is the natural frequency of the spring.

The equation used in part B was,

$$g = a\left(\frac{M + m}{m}\right)$$

Where a is the acceleration of the rider, m is the mass tied to the pulley that causes acceleration of the rider and M is the mass of the rider.

From the free body diagrams,

$$Ma = T$$

and,

$$ma = mg - T$$

Replacing T from the equation above gives,

$$\begin{aligned} ma &= mg - Ma \\ \implies (m+M)a &= mg \\ \therefore \left(\frac{M+m}{m}\right)a &= g \end{aligned}$$

To measure the frictional force involved in part B, we can include a force f in the equations above. Thus,

$$Ma - f = T$$

$$ma = mg - T$$

These two equations give the frictional force to be,

$$f = (M+m)a - mg$$

References

[1] Cherian, Philip, et al. "Experiment 2 Air Track", *Ashoka University Physics Lab 2 Handouts*, 2023, pp. 9-19.