Wein's Displacement Law

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1 Aim

1. To verify Wein's displacement for the given blackbody and to determine the Wein's constant.

2. To evaluate the value of Planck's constant, given the value of Boltzmann-constant.

2 Theory

"On the Theory of the Energy Distribution Law of the Normal Spectrum"- Max Planck's paper, marked the beginning of a revolution in physics. The date of its presentation, 14 Dec. 1900, is widely considered to be the birthday of quantum physics. Just as relativity extends the range of application of physical laws to the region of high velocities, quantum physics extends that range to smaller dimensions. And just like the universal constant: velocity of light (c) characterises relativity; the quantum physics also has a universal constant of fundamental significance called the Planck's constant, h. It was while trying to explain the observed properties of thermal radiation that Planck introduced this constant in his 1900 paper. Let us now examine thermal radiation ourselves.

2.1 Thermal Radiation

The radiation emitted by a body as a result of its temperature is called thermal radiation. All bodies emit such radiation to their surroundings and absorb such radiation from them. If a body is at first hotter than its surroundings, it will cool off because its rate of emitting energy exceeds its rate of absorbing energy. When thermal equilibrium is reached the rates of emission and absorption are equal.
Matter in a condensed state (i.e., solid or liquid) emits a continuous spectrum of radiation. The details of the spectrum are almost independent of the particular material of which a body is composed, but they depend strongly the temperature. At ordinary temperatures most bodies are visible to us not by their emitted light but by the light they reflect. If no light shines on them we cannot see them. At very high temperatures, however, bodies are self-luminous. We can see them glow in a darkened room: but even at temperatures as high as several thousand degrees Kelvin well over 90 per-cent of the emitted thermal radiation is invisible to us, being in the infrared part of the electromagnetic spectrum. Therefore, self-luminous bodies are quite hot.

Consider, for example, heating an iron poker to higher and higher temperatures in a fire, periodically withdrawing the poker from the fire long enough to observe its properties. When the poker is still at a relatively low temperature it radiates heat, but it is not visibly hot. With increasing temperature the amount of radiation that the poker emits increases very rapidly and visible effects are noted. The poker assumes a dull red color, then a bright red color, and, at very high temperatures, an intense blue-white color. That is, with increasing temperature the body emits more therm radiation and the frequency of the most intense radiation becomes higher.

The relation between the temperature of a body and the frequency spectrum of the emitted radiation is used in a device called an optical pyrometer. This is essentially a rudimentary spectrometer that allows the operator to estimate the temperature of a hot body, such as a star, by observing the color, or frequency composition, of the thermal radiation that it emits. There is a continuous spectrum of radiation emitted, the eye seeing chiefly the color corresponding to the most intense emission in the visible region. Familiar examples of objects which emit visible radiation include hot coals, lamp filaments, and the sun.

Generally speaking, the detailed form of the spectrum of the thermal radiation emitted by a hot body depends somewhat upon the composition of the body. However, experiment shows that there is one class of hot bodies that emits thermal spectra of a universal character. These are called *blackbodies*, that is, bodies that have surfaces which absorb all the thermal radiation incident upon them. The name is appropriate because such bodies do not reflect light and appear black when their temperatures are low enough that they are not self-luminous. One example of a (nearly) blackbody would be *any* object coated with a diffuse layer of black pigment, such as lamp black or bismuth black. Another, quite different, example will be described shortly. Independent of the details of their composition, it is found that *all* black bodies at the same temperature emit thermal radiation with the same spectrum. This general fact can be understood on the basis of classical arguments involving thermodynamic equilibrium. The specific form of the spectrum, however, cannot be obtained from thermodynamic arguments alone. The universal properties of the radiation emitted by blackbodies make them of particular theoretical interest and physicists sought to explain the specific features of their spectrum.

The spectral attribution of blackbody radiation is specified by the quantity R-(v), called the $spectralradiancy$, which is defined so that $R_T(v)dv$ is equal to the energy emitted per unit time in radiation of frequency in the interval v to $v + dv$ from a unit area of the surface at absolute temperature T. The earliest accurate measurements of this quantity were made by Lummer and Pringsheim in 1899. They used an instrument essentially similar to the prism spectrometers used in measuring optical spectra, except that special materials were required for the lenses, prisms, etc., so that they would be transparent to the relatively low frequency thermal radiation. The experimentally observed dependence of $R_T(v)$ on v and T is shown in Figure 1-1.

The integral of the spectral radiancy $R_T(v)$ over all v is the total energy emitted per unit time per unit area from a blackbody at temperature T. It is called the radiancy R_T. That is,

$$R_T = \int_0^\infty R_T(v)dv \tag{1}$$

As we have seen in the preceding discussion of Figure 1-1, R_T, increases rapidly with increasing temperature. In fact, this result is called $Stefan'slaw$, and it was first stated in 1879 in the form of an empirical equation

$$R_T = \sigma T^4 \tag{2}$$

where σ = 5.67 x 10^{-8} W/m^2-$^\circ K^4$ is called the $Stefan$- $Boltzmann\ constant$. Figure 1-1 also shows us that the spectrum shifts towards higher frequencies as T increases. This result is called the **Wein's Displacement Law**

$$v_{max} \propto T \tag{3}$$

where v_{max} is the frequency v at which $R_T(v)$ has its maximum value for a particular T. Since, λv = c, the constant velocity of light, Wein's Displacement Law can also be written as

$$\lambda_{max} T = constant \tag{4}$$

where λ_{max} is the wavelength at which the spectral radiancy has its maximum value for a particular T.

As T increases, v_{max} is displaced towards higher frequencies. All these results are in agreement with the familiar experiences discussed earlier, namely that the amount of thermal radiation emitted increases rapidly (the poker radiates much more heat energy at higher temperatures), and the principal frequency of the radiation becomes higher (the poker changes color from dull red to blue-white), with increasing temperature.

2.2 Blackbody in your Lab!

Another example of blackbody, which we shall see to be particularly important, can be found by considering an object containing a cavity which is connected to the outside by a small hole, as in Figure 1-2. Radiation incident upon the hole from the outside enters the cavity and is reflected back and forth by the walls of the cavity, eventually being absorbed on these walls. If the area of the hole is very small compared to the area of the inner surface of the cavity, a negligible amount of the incident radiation will be reflected back through the hole. Essentially all the radiation incident upon the hole is absorbed; therefore, the *hole* must have the properties of the surface of a black body. Most blackbodies used in laboratory experiments, like the one in your laboratory, are constructed along these lines.

Now assume that the walls of the cavity are uniformly heated to a temperature T. Then the walls will emit thermal radiation which will fill the cavity. The small fraction of this radiation incident from the inside upon the hole will pass through the hole. Thus the hole will act as an emitter of thermal radiation. Since the hole must have the properties of the surface of a blackbody, the radiation emitted by the hole must have a blackbody spectrum; but since the hole is merely sampling the thermal radiation present inside the cavity, it is clear that the radiation in the cavity must also have a blackbody spectrum. In fact, it will have a blackbody spectrum characteristic of the temperature T on the walls, since this is the only temperature defined for the system. The spectrum emitted by the hole in the cavity is specified in terms of the energy flux $R_T(v)$. It is more useful, however, to specify the spectrum of radiation inside the cavity, called *cavity radiation*, in terms of an *energy density*, $p_T(v)$, which is defined as the energy contained in a unit volume of the cavity at temperature T in the frequency interval v to $v + dv$. It is evident that these quantities are proportional to one another; that is

$$p_T(v) \propto R_T(v) \tag{5}$$

Hence the radiation inside a cavity whose walls are at temnerature T has the has the same character as the radiation emitted by the surface of a blackbody at temper ature T. It is convenient experimentally to produce a blackbody spectrum by means of a cavity in a heated body with a hole to the outside, and it is convenient in theoretical work to study blackbody radiation by analyzing the cavity radiation because it is possible to apply very general arguments to predict the properties of cavity radiation.

Exercise:

For the Sun, λ_{max} = 5100 Angstrom, whereas for the North star, λ_{max} = 3500 Angstrom. Find the surface temperature of these stars, given the Wein's constant = 2.898 x 10^{-3} m-°K.

2.3 Fitting the Energy Denisty curve of a Blackbody

I'll add more here.

3 Apparatus Required

1. Light Sensor by PASCO
2. Laptop with PASCO-software installed in it.
3. Optical Spectrometer without Telescope
4. Blackbody (Tungsten filament in a cavity)
5. Prism
6. Screen with a mount
7. Magnifying Glass
8. Torch

4 Formula

- The prism equation

$$\text{Refractive index, } n = \frac{\sin(A + D/2)}{\sin(A/2)} \tag{6}$$

where,
A is the angle of the prism
D is the angle of minimum deviation

- Temperature of the bulb

$$T = T_0 + \frac{((R - R_0) - 1)}{\alpha} \tag{7}$$

where,
T_0 is the room temperature
R is the resistance at temperature T
R_0 is the resistance at room temperature
α is the temperature coefficient of resistance of tungsten filament

- From Planck's law, we obtain

$$\text{Planck's constant, } h = \ln\left(\frac{I_1}{I_2}\right)\lambda\left(\frac{T_1 T_2}{T_1 - T_2}\right)\frac{k_B}{c} \tag{8}$$

where,
T_1 is a sample temperature
T_2 is a different sample temperature
λ is a suitable chosen wavelength of light to analyze
I_1 is the intensity corresponding to temperature T_1 of the filament for fixed wavelength
I_2 is the intensity corresponding to temperature T_2 of the filament for fixed wavelength

5 Procedure

5.1 Initial setup

1. The source of light is placed in line with a collimator. The sensor with its reference screen is then placed directly in line with the collimator. The light is switched on and the direct reading is noted.

2. The prism is placed on the prism table such that the light from the collimator falls on one of the polished surfaces. The dispersed light emanating from the other polished surface is observed by moving the reference screen.

3. Once the spectrum is obtained on the screen, the prism table is rotated in either clockwise or anti-clockwise direction slowly. The dispersed spectra on the screen is seen to bounce (reverse its direction of movement) at a certain instant. This position of the prism is called the minimum deviation position and the prism is set as such for the rest of the experiment.

5.2 Part 1 - Determining Wien's constant

1. The voltage supplied to the bulb is fixed at maximum (220V) and the corresponding current reading is noted. The temperature of the filament is calculated from the same.

2. PASCO Capstone software is opened with Manual Sampling setting and the light sensor is connected to the computer via USB cable. The sensor is them tared to complete darkness so as to obtain a zero reading. Using the Preview option, a suitable range of angles is studied and the angle corresponding to maximum intensity reading is noted.

3. This angle is then translated into corresponding dispersed wavelength using the prism equation (Sellmeier equation) provided by the prism manufacturer (F2 50 mm prism by Thor Labs).

4. Thus, knowing λ_{max} and T (Temperature), Wien's constant is deduced. The above steps are repeated for various temperatures, set by controlling the voltage (Eg: 220V, 190V, and 160V).

5.3 Part 2 (a)- Determining Planck's constant

1. A fixed voltage is supplied and suitable range of angles is chosen, as in Part 1. Using the Manual Sampling option in PASCO Capstone, the relative intensity corresponding to various angles (in increments of twenty minutes or 0.33 degree) are recorded using the Keep Sample button. The corresponding wavelength-intensity curve is obtained, once again utilizing the Sellmeier equation.

2. The same is repeated for various temperatures (various voltages) and the corresponding plots, showing the displacement of the peak wavelength, are obtained.

3. For a fixed value of the wavelength, the ratio of the intensities corresponding to two different temperatures is noted. Using this data in tandem with the given formula, Planck's constant is deduced.

4. This analysis is repeated for various wavelengths and different pairs of curves.

5.4 Part 2 (b)- Alternative method of determining Planck's constant

1. After fixing the sensor at a suitable angle (fixed wavelength), the voltages and thus the temperatures are changed. The readings of relative intensities corresponding to the temperatures are noted and the given formula is used to obtain Planck's constant.

2. The same procedure is repeated for various wavelengths.