CS B551, Fall 2016, In-class individual activity #1

Write your name and IU user id: ______________________________

On a power line sit five birds, each wearing a different number from 1 to 5. They start in a random order and their goal is to re-arrange themselves to be in order from 1 to 5, in as few steps as possible. In any one step, exactly one bird can exchange its place with exactly one of its neighboring birds. One can pose this as a search problem in which there is a set of states S corresponding to all possible permutations of the birds (i.e. $S = \{12345, 12354, 12453, ...\}$).

1. What is the goal state?
   12345

2. What is the value of the successor function for state 54321? In other words, what is succ(54321)?

$$\text{SUCC}(54321) = \{54312, 54231, 53421, 45321\}$$

3. To use A^* search, we need a heuristic function $h(s)$. Let $s[i]$ be the bird in position i in state s, (e.g., for $s = 53431$, $s[1] = 5$, $s[2] = 3$, etc.). For each of the following, say whether the heuristic is admissible, and give a brief justification (does not have to be a formal proof) why it is or is not.

   (a) $h(s) = s[1] + s[2] - 3$ Admissible. Notice that $s[1] - 1$ tells us how many spots away from the correct position is the bird in the first position; in other words, that bird has to move that many spots to the right. Similarly, if $s[2] \neq 1$, $s[2] - 2$ tells us how many spots the bird in spot 2 is from its correct position. Notice that any move (bird swap) moves one bird one spot to the right. So clearly we need at least $s[1] - 1 + s[2] - 2 = h(s)$ moves, so $h(s)$ is admissible in this case. If $s[2] = 1$, then we still need at least $s[1] - 1$ moves to the right, and $h(s) = s[1] + 1 - 3 = s[1] - 2$ which also never overestimates.

   (b) $h(s) = |\{i \in [1, 5] \,|\, s[i] \neq i\}|$ (i.e. number of birds that are not in the correct position)

Not admissible. For instance, 21345 is only 1 step away from goal but has 2 out of place birds.

(c) $h(s) = |\{i \in [1,4] \mid s[i+1] - s[i] \neq 1\}|$ (i.e. number of birds whose right neighbor is not one greater) Not admissible. For instance, 13245 is only 1 step away from goal but has 3 birds whose right neighbor is not one greater.

(d) $h(s)$ = Minimum number of steps required to reach either 12345 or 21345 Admissible, by definition can't exceed true number of steps to 12345.

(e) $h(s) = \begin{cases} 0 & \text{if } s[3] = 3 \\ 1 & \text{otherwise} \end{cases}$ Admissible. If 3 is out of place, we need at least 1 step to get to goal. Otherwise we need at least 0 steps.

(f) $h(s) = \max_{i \in [1,5]} |s[i] - i|$ Admissible. This simply measures how far away the most-out-of-place bird is. Each move brings any given tile just 1 position closer to the goal, so this never overestimates the optimal path length.

4. Of the above heuristic functions, which would you choose in an implementation? Why?
(b) and (c) are not admissible. (d) is the strongest bound but is going to be a lot of work to compute. (e) is a really weak bound (only 0 to 1). This leaves (a) and (f). There's no obvious choice because neither dominates the other; (a) has a wider range (0-6 instead of 0-5), but (f) is better on some states (e.g. when first two birds are 1 and 2 but rest are out of order.