Continuous Random Variables 1

STAT-S520

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These slides complement ISI Chapter 5

Probability density function (PDF)

A probability density function (PDF) is a function $f : \mathbb{R} \to \mathbb{R}$ such that

1. $f(x) \geq 0$ for every $x \in \mathbb{R}$
2. $Area_{(-\infty,\infty)}(f) = \int_{-\infty}^{\infty} f(x)dx = 1$

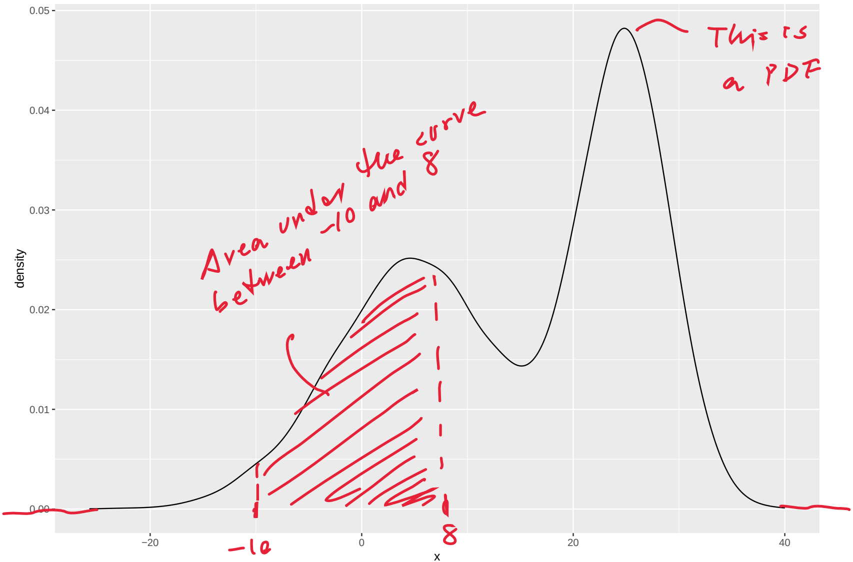
Note: The function f here is different than the PMF (also f) that we used for discrete random variables.

If X is a continuous random variable.

$P(X = 3) = 0$ $\qquad$ $P(X = y) = 0$

$P(X = 10) = 0$ $\qquad$ for any $y \in \mathbb{R}$

PDF Example


0.05
0.04
0.03
0.02
0.01
0.00
density
−20
0
20
40
x
This is a PDF
Area under the curve between -10 and 8
-10
8

Continuous random variable

A random variable X is continuous if there exists a probability density function f such that

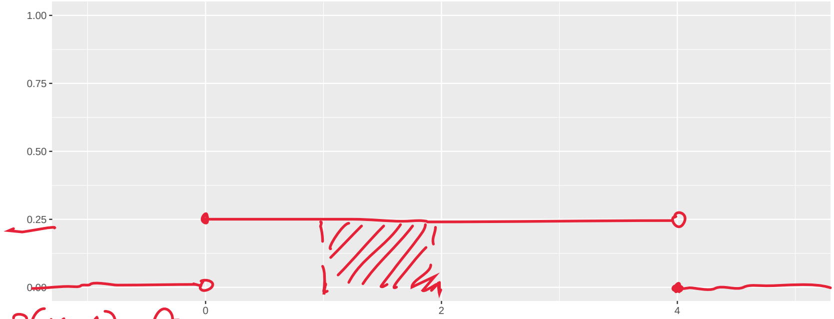
$$P(X \in [a, b]) = \underbrace{\int_a^b f(x)dx}_{\text{Area}(a,b)(f)}$$

Example 1

Let X be a random variable with PDF given by

$$f(x) = \begin{cases} 0 & x \in (-\infty, 0) \\ 1/4 & x \in [0, 4) \\ 0 & x \in [4, \infty) \end{cases}$$

Uniform $(0,4)$



$P(X=1)=0$

What is $P(X \in (1,2))$?

$(2-1)\frac{1}{4} = \frac{1}{4} = 0.25$

Continuous random variable and CDF

The CDF of a continuous random variable X is defined as before:

$$F(y) = P(X \leq y)$$

Based on the definition of a continuous random variable observe that:

$$F(y) = P(X \leq y) = P(X \in (-\infty, y]) = \int_{-\infty}^{y} f(x)dx$$