Problem Set : 4

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Q.1) Dormitory Space : 95
Applicant : 1000
Accepted (n) : 225
— out of accepted 36% accepts the admission

$\therefore n = 225 \qquad p = (0.36)$

a) Freshman – Accomodate
Expected Value : $n \cdot p$
$= 225 \times 0.36$
$E = \boxed{81}$

b) Only if more than 95 students accepts, there will be off-campus arrangement

$P(Y > 95) = 1 - P(Y \leq 95)$
$P(Y > 95) = 1 - F(95)$
$P(Y > 95) = 1 - \text{pbionom}(95, 225, 0.36)$
$= \boxed{0.0229}$

* All the calculations in R provided at the end.

Q.2

plus	Square	Star	Circle	Wave
1/5	1/5	1/5	1/5	1/5

$P(\text{correct symbol}) = 1/5 = 0.2$

This process followed for 25 times $\therefore n = 25$
25 - independent binomial trials

a) Expected Value $= n \cdot p$
$= 25 \times 0.2$
$= \boxed{5}$

b) ESP score > 7
$\therefore \quad P(Y > 7) = 1 - P(Y \leq 7)$
$= 1 - \text{pbinom}(7, 25, 0.2)$ ——— (R program)
$= \boxed{0.1091}$

c) Atleast one of 20 receivers will attain score of ESP
ESP, P(receiver indicating ESP) $= 0.1091$
$\therefore P(Y \geq 1)$
$\therefore P(Y \geq 1) = 1 - P(Y = 0)$

$P(Y = 0) = {}^{20}C_0 \, (0.1091)^0 \, (1 - 0.1091)^{20}$
——— Binomial Distribution

$\therefore P(Y \geq 1) = 1 - \left[\frac{20}{20} \times 1 \times (0.8909)^{20} \right]$
$= \boxed{0.90}$

Q. 3

2 section of stats
- for 35 years
- 1500 total students

1500 X 89 $\longrightarrow$ & counts head

$p(\text{heads}) = 0.3$

$p(\text{tails}) = 0.7 = p(\text{no heads})$

$n = 89$

$X \sim \text{Binomial}(n = 89, p = 0.3)$

p (No more than 2 heads)

$= P(X=0) + P(X=1) + P(X=2)$

$= {}^{89}C_0 (0.3)^0 (0.7)^{89} + {}^{89}C_1 (0.3)^1 (0.7)^{88} + {}^{88}C_2 (0.3)^2 (0.7)^{87}$

$= (0.7)^{89} + 89(0.3)(0.7)^{88}$

$= \frac{89 \times 88}{2} (0.3)^2 (0.7)^{87}$

= dbinom (x = 0 , size = 89 , prob = 0.3) +
dbinom (x = 1 , size = 89 , prob = 0.3) +
dbinom (x = 2 , size = 89 , prob = 0.3)

$= 1.24 \times 10^{-11}$

p (atleast 1 observing no. more than 2 heads)

= 1 − p ('0' students observing ≥ 2 heads)

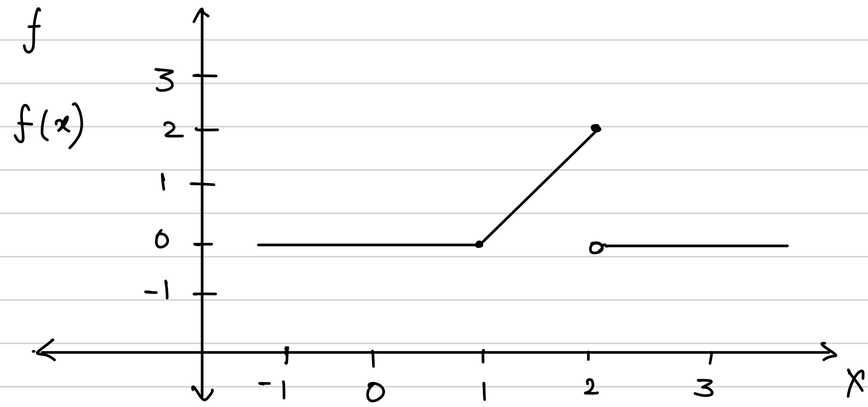
$= 1 - (1 - (1.24 \times 10^{-11}))^{1500}$

Probability = $\boxed{1.8608 \times 10^{-8}}$

Q.4 X = contious RV

$$f(x) = \begin{cases} 0 & \text{if } x < 1 \\ 2(x-1) & \text{if } 1 \le x \le 2 \\ 0 & \text{if } x > 2 \end{cases}$$

a) Graph f



b) Verify that f is PDF.

PDF is a Function of $f: R \to R$ such that

1. $f(x) \ge 0$ for every $x \in R$
2. $\text{Area}_{(-\infty, \infty)}(f) = \int_{-\infty}^{\infty} f(x)\,dx = 1$

here

$f(x) \ge 0$ For every $x \in R$

$$\text{Area}_{[1,2]}(f) = \int_1^2 F(x) \cdot dx$$

$$= \int_1^2 (2(x-1))\,dx$$

$$= \left(\frac{2x^2}{2}\right)_1^2 - \left(2x\right)_1^2$$

$$= 3 - 2$$

$$= \boxed{1} \quad \therefore f \text{ is PDF}$$

c) Compute $P(1.50 < X < 1.75)$

$$= P(X \in (1.50, 1.75))$$

$$= \text{Area}_{(1.50, 1.75)}(f)$$

$$= \int_{1.50}^{1.75} f(x) \cdot dx$$

$$= \left[x^2\right]_{1.50}^{1.75} - \left[2x\right]_{1.50}^{1.75}$$

$$= \left[(1.75)^2 - (1.50)^2\right] - \left[2(1.75) - 2(1.50)\right]$$

$$= \boxed{0.3125}$$

Question 5

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```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library(ggplot2)
library(fivethirtyeight)

?bechdel
```

Mean and Standard deviation

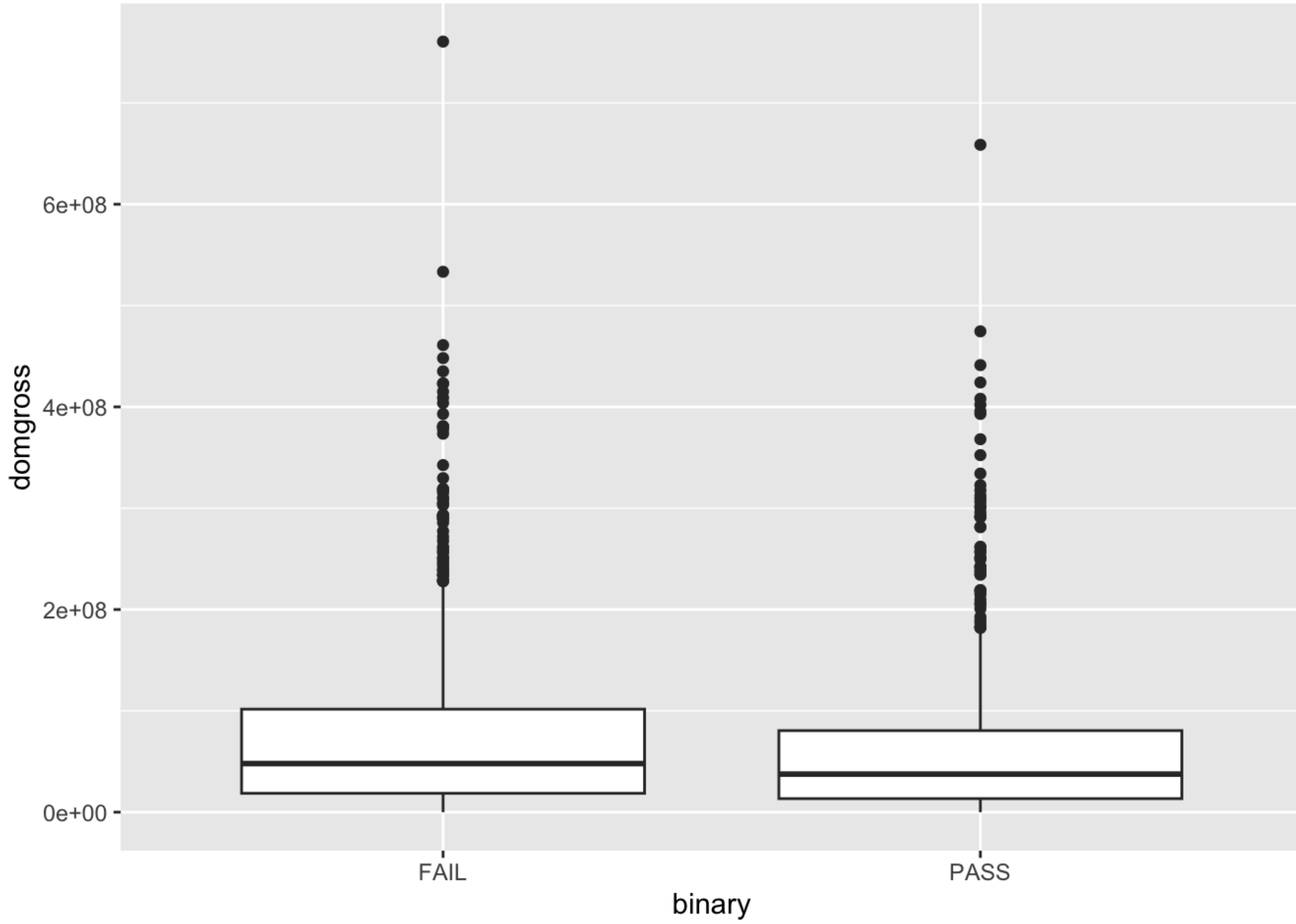
```
std <- bechdel %>%
  group_by(binary) %>%
  summarize(mean = mean(domgross, na.rm = TRUE),
    std_dev = sd(domgross, na.rm = TRUE))
std
```

```
## # A tibble: 2 × 3
##   binary      mean   std_dev
##   <chr>      <dbl>     <dbl>
## 1 FAIL   74985189. 83484962.
## 2 PASS   61885653. 75758965.
```

Box plots

```
ggplot(data = bechdel, mapping = aes(x= binary, y = domgross)) + geom_boxplot()
```

```
## Warning: Removed 17 rows containing non-finite values (`stat_boxplot()`).
```



Total number of films for different periods

```
count <- bechdel %>%
  group_by(period_code) %>%
  summarize(count = n())
count
```

```
## # A tibble: 6 × 2
##   period_code count
##         <int> <int>
## 1           1   438
## 2           2   488
## 3           3   352
## 4           4   247
## 5           5    90
## 6          NA   179
```