Problem Set 5

STAT-S 520

Due on February 13th, 2023

Instructions:

- Submit your answers in Canvas.
- Your answers can be typed and/or handwritten, as long as your final submission is a single PDF file with answers in proper order.
- You are allowed to collaborate with your classmates as long as you write your own solutions.

Questions:

1. Buses go past my stop throughout the day. I arrive at the stop at a completely random time during the day. What is the expected length of time I will have to wait for a bus, if the schedule stops are the following

    a. Exactly 20 minutes apart.
    b. Exactly 20 minutes past the hour and 40 minutes past the hour (e.g. ~8:20, 8:40, 9:20, 9:40, etc.)
    c. Exactly at the top of the hour, 10 minutes past the hour, and 30 minutes past the hours (e.g. ~8:00, 8:10, 8:30, 9:00, 9:10, etc.)

2. ISI Section 5.6 Exercise 3

3. ISI Section 5.6 Exercise 7

4. Assume that, according to CDC data, the heights of adult men follow an approximately Normal distribution with mean 69.2 inches and SD 2.5 inches. The heights of adult women follow an approximately Normal distribution with mean 63.8 inches and SD 2.7 inches. Suppose we randomly select an adult man and an adult woman, independently. Let X_1 be the height of the random man and let X_2 be the height of the random woman.

    a. What is the probability that the man is over six feet?
    b. Let $Y = X_1 + X_2$. Is Y an (approximately) normal random variable? What is the expected value, and variance of Y?
    c. What is the probability that the sum of the man's and woman's height is over 12 feet?
    d. Let $D = X_1 - X_2$. Is D an (approximately) normal random variable? What are the expected value and variance of D?
    e. What is the probability that the random man is shorter than the random woman?

5. Let $Y \sim binomial(n, p)$.

    a. Obtain 10^5 random values from this distribution with $p = 0.01$ and start with $n = 10$ but try different sizes (of your choosing) for n and draw histograms. The goal is to determine what is the minimum size n for which the histogram looks very close to a normal distribution? Hint: n should probably be a large number.

b. Obtain 10^5 random values from this distribution with $n = 300$ and start with $p = 0.5$ but try also lower probabilities (of your choosing) and draw histograms. What is the lowest value of p for which the histogram still looks close enough to a normal distribution?

c. Using $p = 0.01$ and the chosen n value in part a, obtain the expected value and variance of Y and call them μ and σ^2, respectively. Now, let's construct $X \sim Normal(\mu, \sigma^2)$ with the obtained parameters. Obtain $P(\mu - \sigma < X \leq \mu + \sigma)$ using the theoretical curve and also the last vector used in part a (the vector that help you determine the appropriate n). Comment on your results.

Reading assignments

For Tuesday:

- Read ISI Sections 6.1 and 6.2 (pp. 141 - 148)

For Thursday:

- Read ISI Chapter 7.1 - 7.3 (p. 153 - 164)