Problem Set 3

STAT-S 520

Due on January 30th, 2023

Instructions:

- Submit your answers in Canvas.
- Your answers can be typed and/or handwritten, as long as your final submission is a single PDF file with answers in proper order.
- You are allowed to collaborate with your classmates as long as you write your own solutions.

Questions:

1. ISI Section 3.7 Exercise 8, but use instead $P(+|D) = 0.71$, $P(-|D^c) = 0.88$, and $P(D) = 0.03$.
2. ISI Section 4.5 Exercise 3, but use instead the urn

$$\{1, 1, 1, 1, 1, 3, 3, 3, 7, 7\}$$

3. Toss three fair coins. Let X be the random variable that uses the rule of assignment: $(10 \times \text{number of tails}) - 5 \times \text{number of heads}$. For example if the outcome is TTH then $X(TTH) = 10 \times 2 - 5 = 15$. Determine each of the following:

   a. The sample space, S.
   b. The range of X?
   c. The CDF of X.
   d. The PMF of X.
   e. The expected value of X.
   f. The variance and standard deviation of X.

4. Let's use a 52 card traditional deck with four suits. Draw a random card, with replacement, until an ace appears. Let Y be a random variable that counts the number of draws needed.

   a. Describe S and at least two possible outcomes. Can you write down all the outcomes? Explain why or why not.
   b. What is $Y(S)$?
   c. As usual, use F and f as the CDF and PMF of Y, respectively, and obtain
      i. $f(-4), f(\pi)$, and $f(4)$
      ii. $F(-2)$ and $F(2)$
   d. Write down $f(y)$ as a single formula in terms of $y \in Y(S)$,

5. Determine the expected value and variance of

   a. $X \sim \text{Bernoulli}(p)$. (Hint: Write your solution in terms of p)
   b. $Y \sim \text{Binomial}(n, p)$. (Hint: Write your solution in terms of n and p)

Reading assignments

For Tuesday:

- ISI selected topics of Ch4 (pp. 103 - 108)
- ISI Chapter 5, Sections 5.1 - 5.3 (pp. 117 - 127)