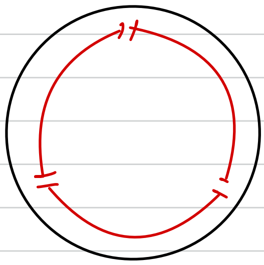
Problem Set . 5

Aditya Mhaske

Q.1)

a) Exactly 20 mins apart.



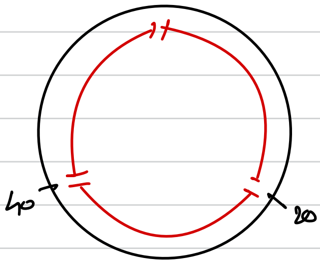
→ 3 cycles of 20 mins

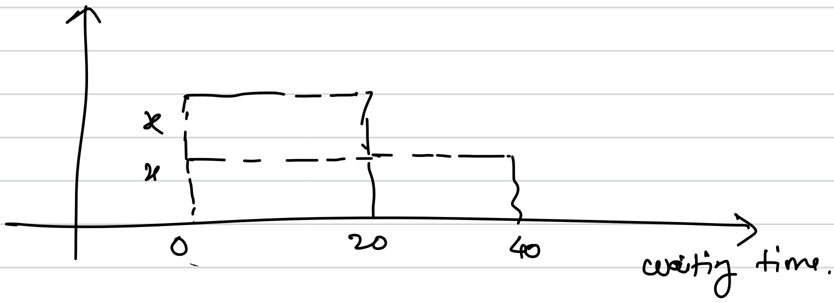
– Expected wait time could be 10 mins

– balance point $= \frac{20-0}{2} = 10$

∴ Expected waiting time is $\boxed{10 \text{ mins}}$

b) Exactly 20 mins past the hour & 40 mins past the hour

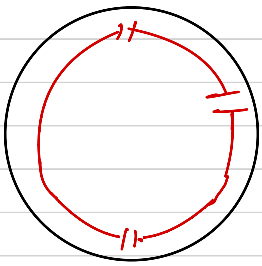


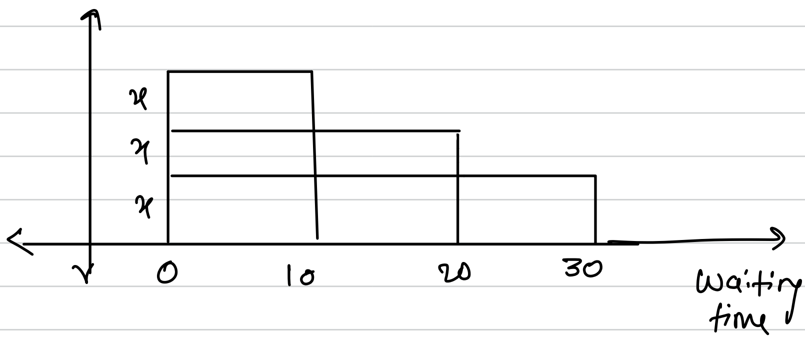


balancing point $= \dfrac{\frac{40}{20} + \frac{20}{2}}{2} = 15$

∴ Expected time for waiting is $\boxed{15 \text{ mins}}$

c) Exactly at the top of hour, 10 mins past the hour & 30 mins past the hour.





balance point : $\dfrac{\frac{10}{20} + \frac{20}{2} + \frac{30}{2}}{3} = 10$

∴ Expected Waiting point is $\boxed{10 \text{ mins}}$

Q. 2) $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} 0 & x < 0 \\ cx & 0 < x < 1.5 \\ c(3-x) & 1.5 < x < 3 \\ 0 & x > 3 \end{cases}$$

a) $c \to$ PDF

Area Under curve

$$\int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{1.5} cx \cdot dx + \int_{1.5}^{3} (3c - cx) \cdot dx \int_{3}^{\infty} 0 \cdot dx = 1$$

$$\left[c \cdot \frac{x^2}{2} \right]_0^{1.5} + \left[3cx - \frac{cx^2}{2} \right]_{1.5}^{3} = 1$$

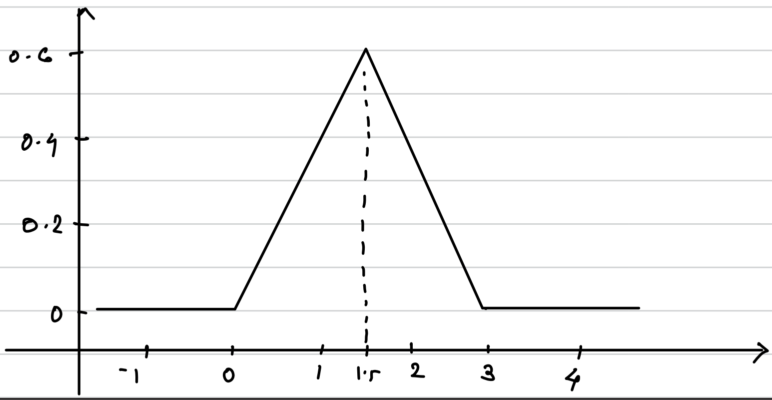
$$\frac{2.25}{2} \times c + \left[\left(9c - \frac{9c}{2} \right) - \left(4.5c - \frac{2.25c}{2} \right) \right] = 1$$

$$\left(\frac{2.25}{2} c + \frac{2.25}{2} \right) c = 1$$

$$c = \frac{2}{4.5} \approx \boxed{4/9}$$

b) $EX =$

$$F(x) = \begin{cases} 0 & , x < 0 \\ 4/9 \cdot x & , 0 < x < 1.5 \\ 4/9 \cdot (3-x) & , 1.5 < x < 3 \\ 0 & , x > 3 \end{cases}$$



– Given Graph is equilateral triangle & Expected Value is balance point $\to$ base/2

$$EX = \frac{3-0}{2}$$

$$\boxed{EX = 1.5}$$

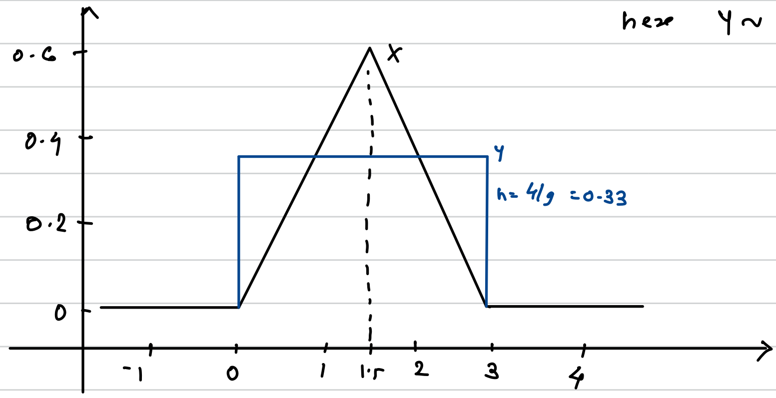
c) Compute $P(X>2)$

Area of triangle between 2 & 3 $= \frac{1}{2} \times (3-2) \times 4/9$

$F(2) = \frac{4}{9} \times (3-x) = \frac{4}{9}$

$$\boxed{= \frac{2}{9}}$$

d) here $Y \sim \text{uniform}(0,3)$



$EX = 1.5$ | $EY = \text{Base}/2 = 1.5$

- Value of Y are distributed uniformly through $Y=0$ to 3

e) Graph CDF

→ $F(Y) = 1$

For $0 \leq Y \leq 1.5$, area $= F(Y)$

$$F(Y) = \frac{1}{2} \cdot Y \cdot \frac{4}{9} \cdot Y$$

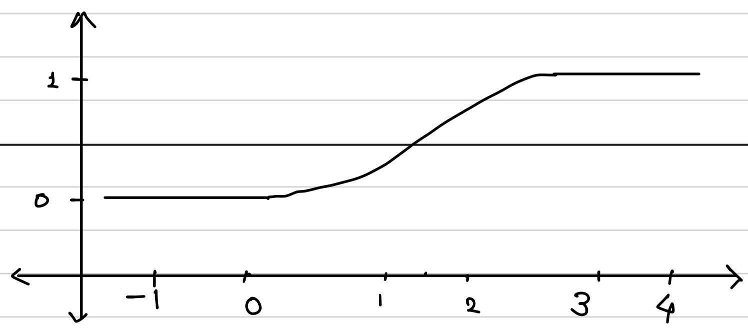
$$= \frac{2}{9} Y^2$$

- For $1.5 \leq Y \leq 3$.

$$F(Y) = 1 - \left[\frac{1}{2}(3-Y) \times \frac{4}{9}(3-Y)\right]$$

$$= 1 - \frac{2}{9}(3-Y)^2$$

$$\therefore F(Y) = \begin{cases} 0, & Y < 0 \\ \frac{2Y^2}{9}, & 0 \leq Y < 1.5 \\ 1 - 2/9(3-Y)^2, & 1.5 \leq Y < 3 \\ 1, & 3 \leq Y \end{cases}$$



Q.4

height of men - Normal Distribution,

(mean) $\mu = 69.2$

(SD) $\sigma = 2.5$

height of women - Normal Distribution

$\mu = 63.8$

$\sigma = 2.7$

$X_1 \sim \text{Normal}(69.2, 6.25)$

$X_2 \sim \text{Normal}(63.8, 7.29)$

a) $P(\text{man} > 6\text{fts}) \mid (\text{man} > 72 \text{ inch})$

$P(X_1 > 72) = 1 - P(X_1 \leq 72)$

$= 1 - F_1(72)$

$= 1 - \text{pnorm}(72, 69.2, 2.5)$

$P(X_1 > 72) = \underline{0.1313}$

b) $Y = X_1 + X_2$

$Y \rightarrow$ Normal Distribution

$EY = EX_1 + EX_2$

$EY = 69.2 + 63.8$

$EY = 133.0$

$\text{Var } Y = \text{Var} X_1 + \text{Var } X_2$

$= 6.25 + 7.29$

$\text{Var } Y = \underline{13.54}$

c) Sum is over 12 feet i.e. 144 inches

Using Y from above example,

$P(Y > 144) = 1 - P(Y \leq 144)$

$= 1 - \text{pnorm}(144, 133, \text{Sqrt}(13.54))$

$= \underline{0.0013}$

d) $D = X_1 - X_2$

D is a random Variable $\longrightarrow$ Normal distribution

$ED = E(X_1 + (-X_2))$

$= EX_1 - EX_2$

$= 69.2 - 63.8$

$ED = \underline{5.4}$

$\longrightarrow$ continue

$$\begin{aligned}
\mathrm{Var}\,D &= \mathrm{Var}\,(X_1 + (-X_2)) \\
&= \mathrm{Var}\,X_1 + \mathrm{Var}\,X_2 \\
&= (2.5)^2 + (2.7)^2 \\
&= 6.25 + 7.29 \\
\mathrm{Var}\,D &= \underline{13.54}
\end{aligned}$$

4) $$\begin{aligned}
P(X_1 < X_2) &= P(X_1 - X_2 < 0) \\
&= P(D \leqslant 0) \\
&= \mathrm{pnorm}(0,\ 5.4.\ \mathrm{sqrt}(13.54)) \\
&= \underline{0.071171}
\end{aligned}$$

PS05: R

Aditya Sanjay Mhaske

2/13/2023

Question 3

X ~ Normal(-5, 10) (a) P(X < 0)

```
pnorm(0,-5, 10)
```

```
## [1] 0.6914625
```

b. P(X > 5)

```
pnorm(5, -5, 10)
```

```
## [1] 0.8413447
```

c. P(-3 < X < 7)

```
pnorm(7, -5, 10) - pnorm(-3, -5, 10)
```

```
## [1] 0.3056706
```

d. P(|X+5| < 10) = P(-15 < x < 5)

```
pnorm(5, -5, 10) - pnorm(-15, -5, 10)
```

```
## [1] 0.6826895
```

e. P(|X-5| > 2) = P(x > 5) + P(x < 1)

```
(1 - pnorm(5,-5,10)) + pnorm(1,-5,10)
```

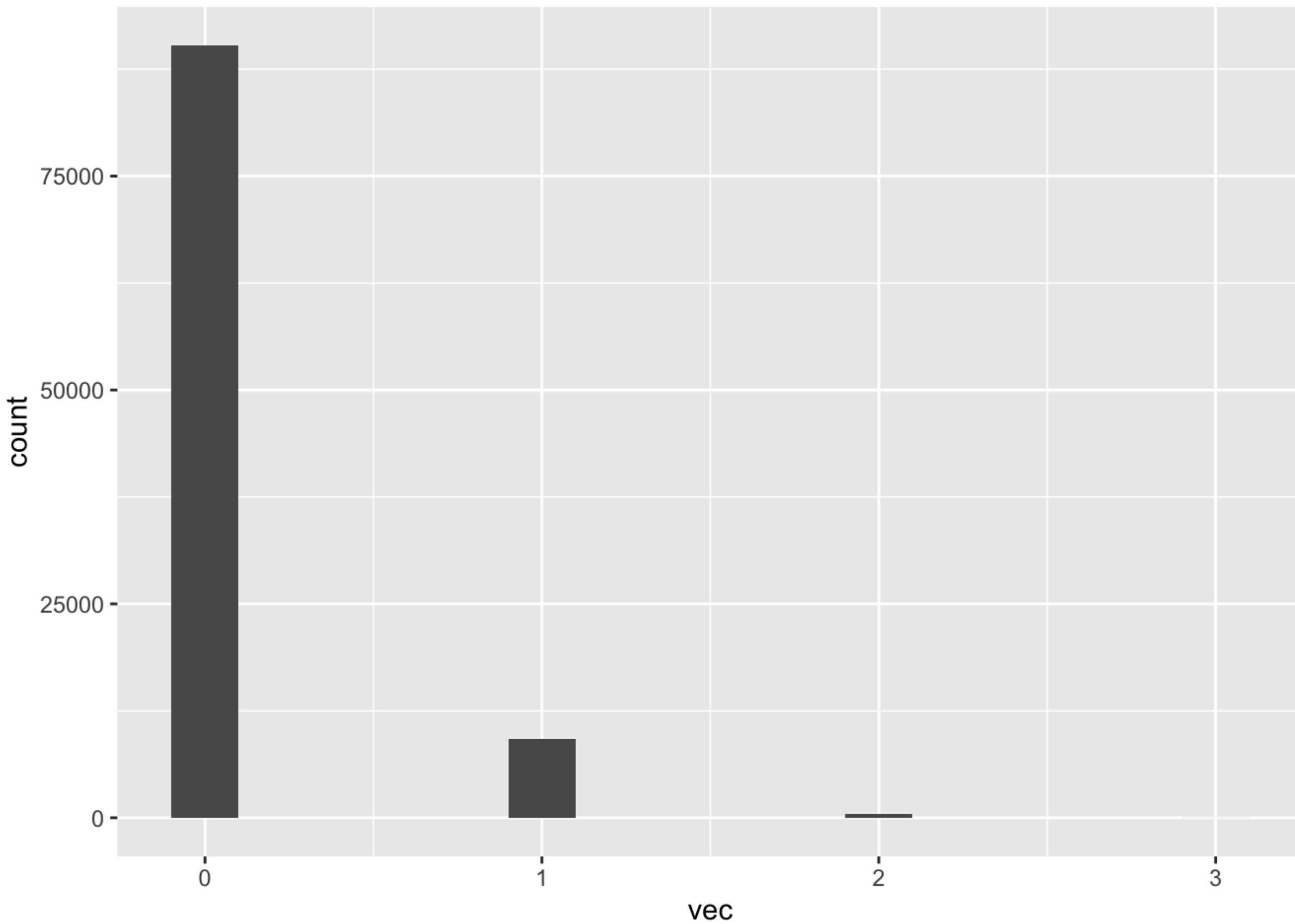
```
## [1] 0.8844021
```

Question 5

Y~Binomial(n, p)

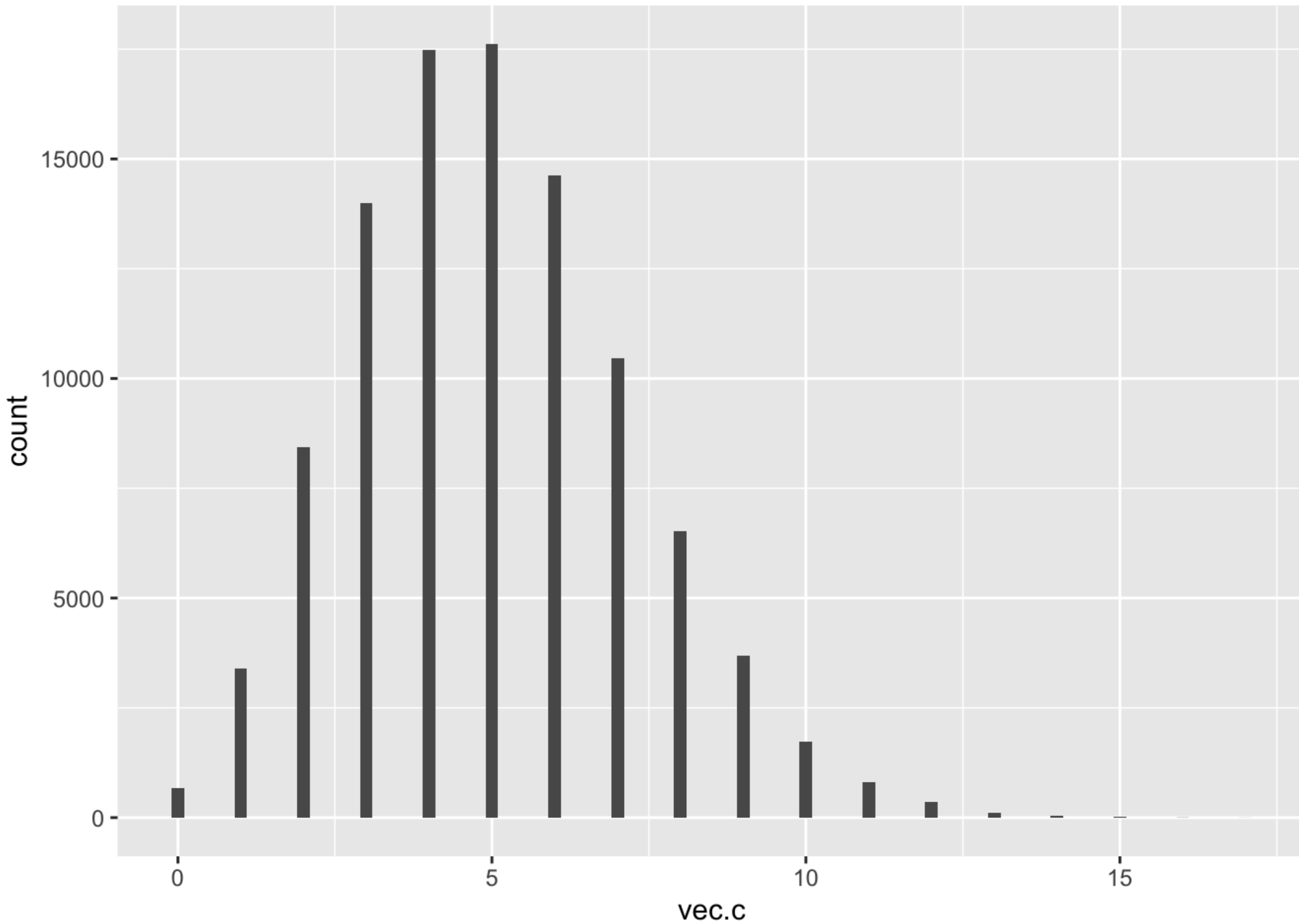
a. n = 10, p = 0.01

```r
vec = rbinom(10^5, 10, 0.01)
#vec
df = data.frame(vec)
library(ggplot2)
ggplot(data = df, mapping = aes(x = vec))+geom_histogram(binwidth = 0.2)
```



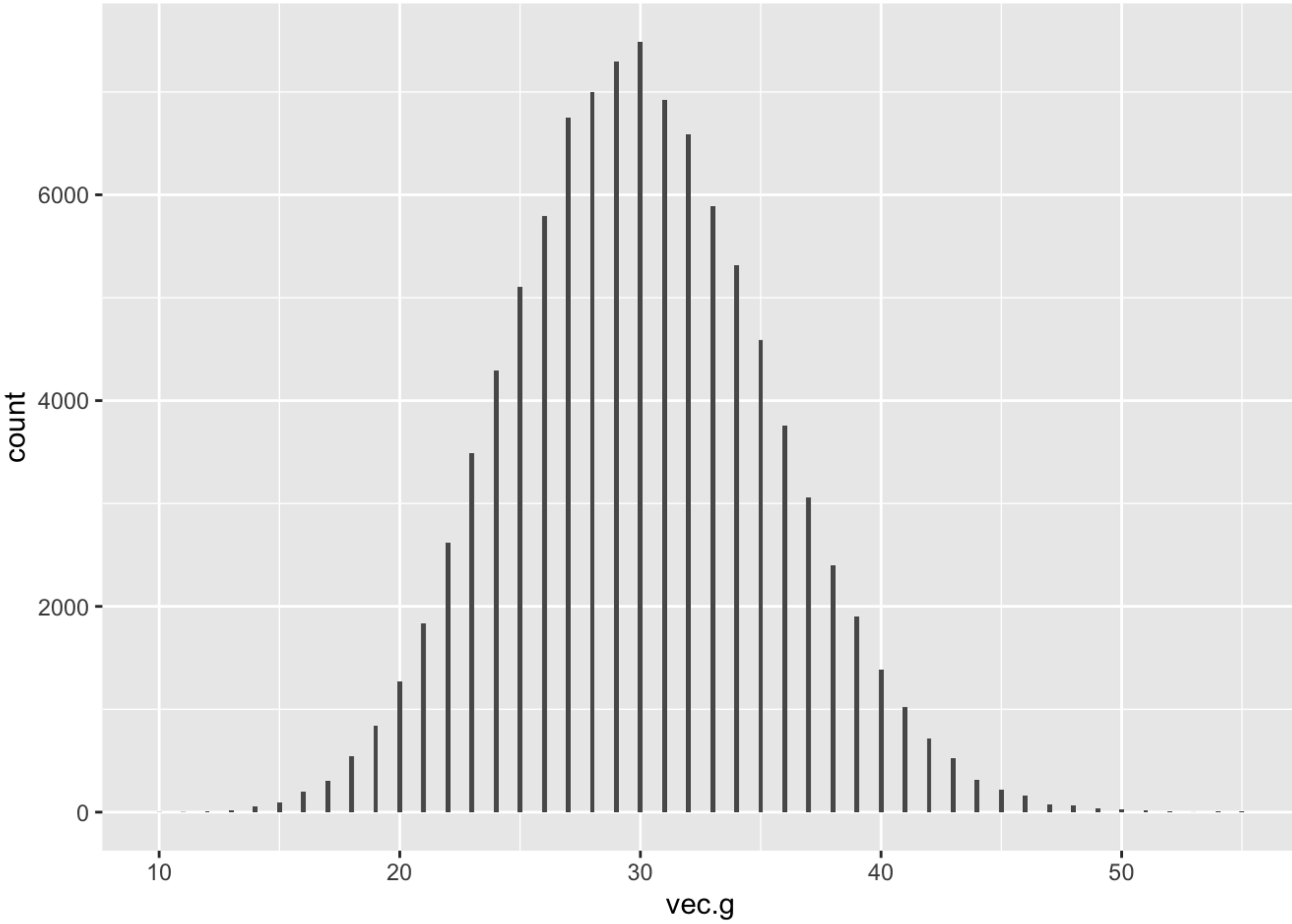

```r
vec.c = rbinom(10^5, 500, 0.01)
#vec.c
df3 = data.frame(vec.c)

ggplot(data = df3, mapping = aes(x = vec.c))+geom_histogram(binwidth = 0.2)
```



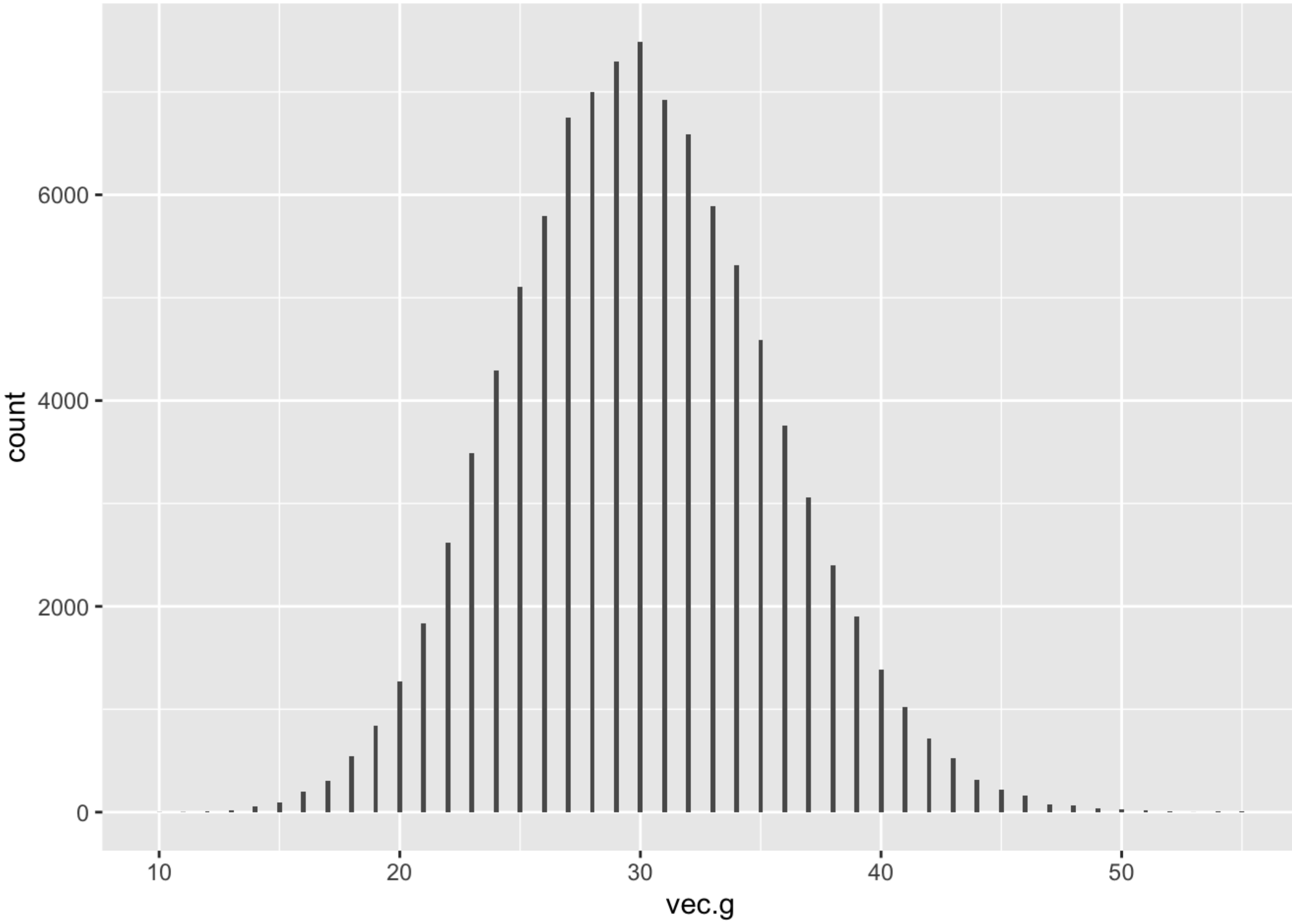

```
vec.g = rbinom(10^5, 3000, 0.01)
#vec.g
df7 = data.frame(vec.g)

ggplot(data = df7, mapping = aes(x = vec.g))+geom_histogram(binwidth = 0.2)
```




```
vec.h = rbinom(10^5, 3500, 0.01)
#vec.g
df8 = data.frame(vec.h)

ggplot(data = df8, mapping = aes(x = vec.g))+geom_histogram(binwidth = 0.2)
```



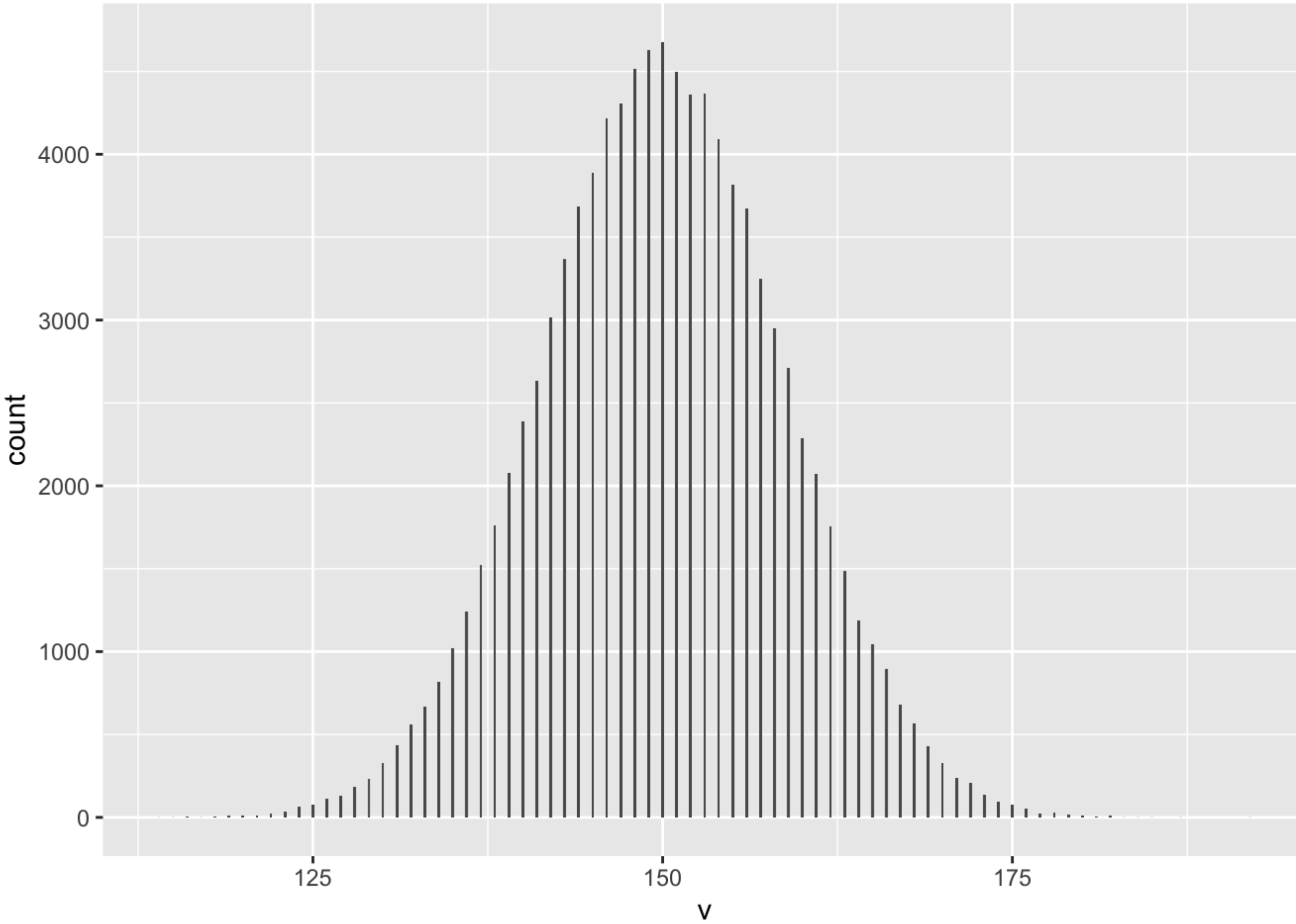
n = 3000 illustrates a histogram which looks close to a normal distribution

```
#n = 3500 illustrates an even better histogram which looks close to a normal distributio
n
```

b. n = 300, p = 0.5

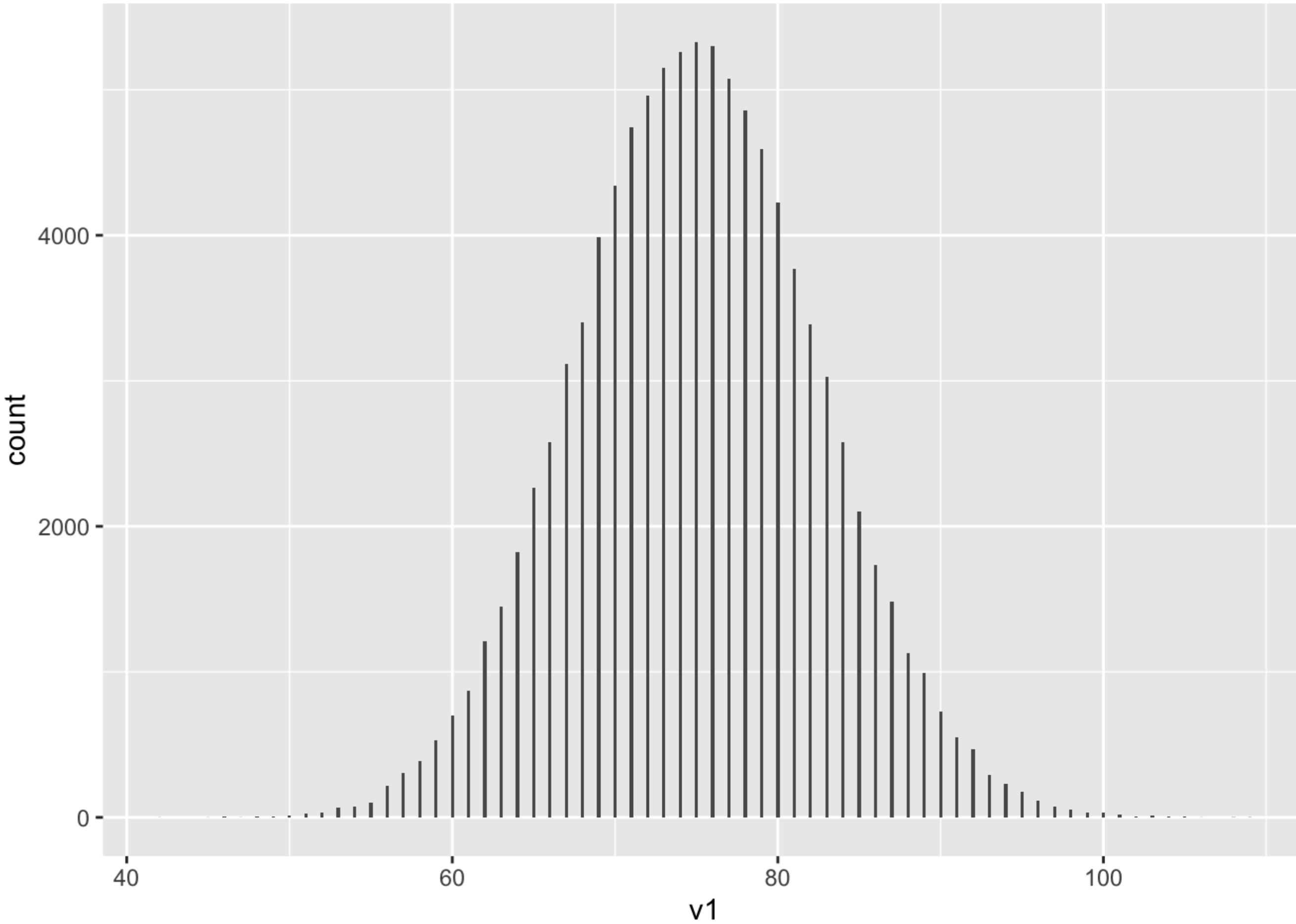
```
v = rbinom(10^5, 300, 0.5)
#v
d = data.frame(v)

ggplot(data = d, mapping = aes(x = v))+geom_histogram(binwidth = 0.2)
```



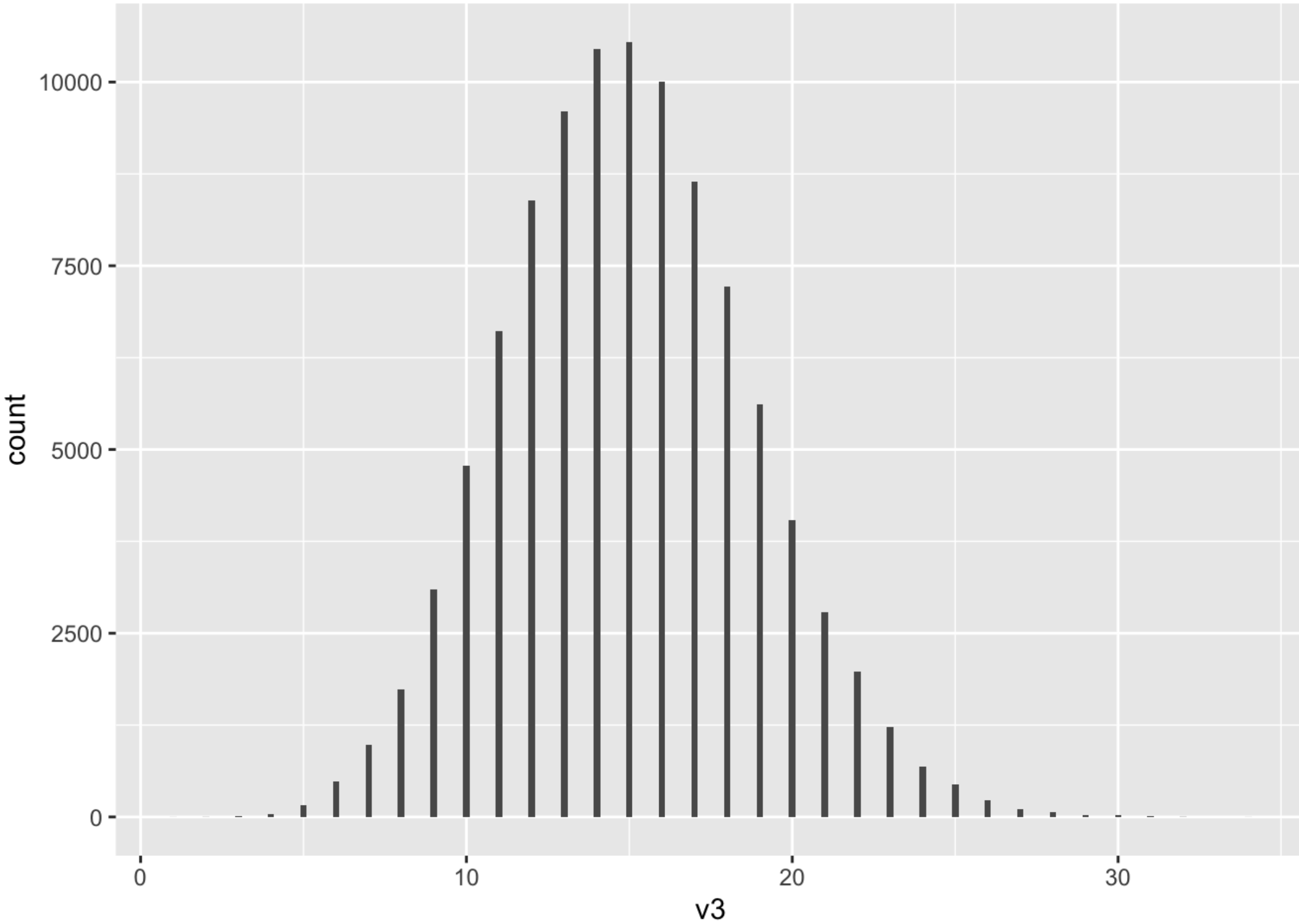

```
v1 = rbinom(10^5, 300, 0.25)
#v1
d1 = data.frame(v1)

ggplot(data = d1, mapping = aes(x = v1))+geom_histogram(binwidth = 0.2)
```




```
v3 = rbinom(10^5, 300, 0.05)
#v3
d3 = data.frame(v3)

ggplot(data = d3, mapping = aes(x = v3))+geom_histogram(binwidth = 0.2)
```



the lowest value of p, where the histogram still looks close enough to a normal distribution is p = 0.05

c. Y ~ Binomial(3500, 0.01)

E(Y) = np = 3500 * 0.01 = 35

Var(Y) = np(1-p) = 35*(1 - 0.01) = 34.65

Y ~ Normal (35, 34.65)

P(μ – σ < X <= μ + σ)

P(29.12 < X <= 40.88) = P(X <= 40.88) - P(X <= 29.12) = F(40.88) - F(29.12)

```
pnorm(40.88, 35, 5.88) - pnorm(29.12, 35, 5.88)
```

```
## [1] 0.6826895
```