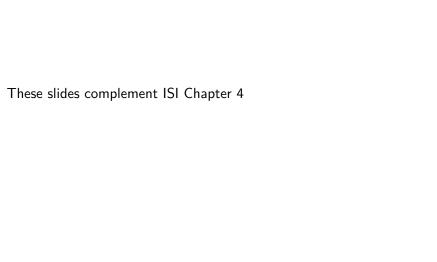
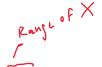
Discrete Random Variables 1 STAT-S520

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Discrete random variable



- Let X be a random variable. X is discrete if X(S) is countable
 - ► A set is countable if it is finite or denumerable
 - \blacktriangleright A set is denumerable if there is a one-to-one correspondence with $\Bbb N$

To set of radural numbers

- Let $X(S) = \{0, 1, 2\}$, then X is .. discrete
- Let $Y(S) = \mathbb{N}$ (naturals), then Y is ...disorder
- Let $X_1(S) = \mathbb{R}$ (reals), then X_1 is ... Continuous.
- ▶ Let $X_2(S) = \mathbb{Z}$ (integers), then X_2 is ... discrete
- ▶ Let $X_3(S) = \mathbb{Q}$ (rationals), then X_3 is ... discrete
- Let Z(S) = [0,1], then Z is ... continues

Probability Mass Function

Let X be a discrete random variable. The probability mass function (PMF) of X is a function f, with $f: \mathbb{R} \to [0,1]$, such that, for any $y \in \mathbb{R}$: $f(y) = \begin{cases} P(X = y) & \text{if } y \in X(S) \\ 0 & \text{otherwise} \end{cases}$

We toss a fair coin twice and X is the number of heads. What is

- ightharpoonup f(2) = 0.25
- $f(\pi) = 0$
- f(10) = 0

CDF and PMF

If X is a discrete random variable with range X(S) and for any given $y \in \mathbb{R}$ the relationship between the PMF and CDF is given by:

$$F(y) = \sum_{x: x \in L(y)} f(x)$$

where L(y) is the set of numbers in the range of X that are less than or equal to y, or in math notation: $L(y) = X(S) \cap (\infty, y]$,

We toss a fair coin twice and X is the number of heads. Let's find $F(\sqrt{2})$ using the PMF

= 40,15

$$\begin{array}{lll}
X(S) = \{0, 1, 2\} \\
F(\sqrt{2}) = \sum_{\alpha: \alpha \in L(\sqrt{2})} F(\alpha) & L(\sqrt{2}) = \{0, 1, 2\} \land \{-\alpha, \sqrt{2}\} \\
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Bernoulli trials

- A Bernoulli trial is a random variable where $X(S) = \{0, 1\}$.
 - It is customary to call X = 1 a success and X = 0 a failure.
- ► There is a family of Bernoulli trials with parameter p where p = P(X = 1) and we write

$$X \sim \text{Bernoulli}(p)$$

PMF of a Bernoulli trial

▶ If $X \sim \text{Bernoulli}(p)$, the PMF of X is given by

$$f(x) = \begin{cases} \frac{p}{1-p} & \text{if } x = 1\\ \frac{1-p}{0} & \text{otherwise} \end{cases}$$

We can simplify this notation by writing

$$f(x) = p^x (1-p)^{1-x}$$

for $x \in \{0,1\}$ and f(x) = 0 otherwise.

Binomial distribution

- Let X_1, X_2, \ldots, X_n independent Bernoulli trials with parameter
- ► We construct a random variable as follows:

$$Y = \sum_{i=1}^{n} X_i = X_1 + \dots + X_n$$

We then say that Y follows a binomial distribution with parameters n and p and write

$$Y \sim binomial(n, p)$$

- ▶ You invite 50 friends to your birthday party. We assume that
 - Friends don't influence each other
 - ► Each friend has the same chance to attend
- ▶ There is a 0.8 chance that any given friend attends the party.

Build ever vormables:

$$X_1, X_2, ..., X_{50}$$
 all follow $X_i \sim \text{Bernoulli}(0.8)$
 $Y = \sum_{i=1}^{50} X_i$ -v $Y \sim B$ ino unfal $(50, 0.8)$
Chance that exoutly 30 friends attend 7
Chance that exoutly 30 friends attend 7

PMF of a Binomial distribution

If $Y \sim binomial(n, p)$, the PMF of Y is given by

$$f(x) = P(Y = x) = \binom{n}{x} p^{x} (1 - p)^{n-x}$$

for $x \in Y(S)$ and f(x) = 0 otherwise.

Expected value

Greek latter

Let X be a discrete random variable. The expected value of X is given by $EX = \sum_{x \in S} x \cdot f(x)$

The expected value can be understood as the long-run average of values that X assigns when the experiment is performed many times

Variance

► Let X be a discrete random variable. The variance of X is given by 5.

$$VarX = E\left((X - \mu)^2\right) = \sum_{x \in X(S)} (x - \mu)^2 \cdot f(x)$$

▶ The standard deviation of *X* is the square root of the variance

Properties of the expected value and variance

Let X, Y be random variables and a, b scalars (constant values).

- \triangleright E(a+X)=a+EX
- \triangleright E(bX) = bEX
- \triangleright E(X + Y) = EX + EY

If in addition, X and Y are independent:

- ightharpoonup Var(a+X) = VarX
- $ightharpoonup Var(bX) = b^2 \cdot VarX$
- ightharpoonup Var(X+Y)=VarX+VarY

Let X,Y be independent random variables with EX=1, EY=2, VarX=4, VarY=9. Find the expected value and variance of 2X+1 and X-Y