Probability Part 1

STAT-S520

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These slides complement ISI Ch3

Probability

Different Interpretations:

- A long run frequency of occurrence
- Expression of degrees of belief
- Other interpretations

Probability Model

Think of performing an experiment that results in one of possibly many outcomes. A probability model (based on Kolmogorov's axioms) is defined by:

- The Sample Space, S, is the collection of all outcomes
- Events are subsets of outcomes
  - We use uppercase letters to denote events, e.g., A, D, E, etc.
  - The set of all relevant events is denoted by Ω omega
- A probability measure is a function that assigns probabilities to events

$$P : \Omega \to [0, 1]$$

$A \in \Omega$

$S \in \Omega$

$A \subset S$

$s_1, s_2 \in S$

Some Conventions

- $P(S) = 1$
- Ω is a sigma-algebra, a mathematical structure with some properties beyond the scope of this course.
  - For simplicity, we can think of Ω as the set of all relevant events

Example 1

Experiment: Toss a fair coin twice. Let's represent an outcome by the results in order of occurrence.

- $S = \{HH, HT, TH, TT\}$
- Some events:
  - $A = \{HH\}$
  - $B = \{$ first coin is heads $\} = \{HH, HT\}$
  - $C = \{$ at least one tail$\}$
- Some probabilities
  - $P(A) = 1/4$
  - $P(B) = 1/2$
  - $P(C) = 3/4$

$A = \{HH\}$

Key Property: Finite Additivity

If events A and B are disjoint, then

$$P(A \cup B) = P(A) + P(B)$$

Using finite additivity, we can derive the following results

a. For any event A, $P(A^c) = 1 - P(A)$
   - In particular, since $P(S) = 1$ then $P(\emptyset) = 0$

b. If $A \subset B$, then $P(A) \leq P(B)$

c. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$