Stat-S 520 Problem Set 1

Q.1)

$S = \{x : x \in R \text{ and } x \leqslant 20\}$

$S = \{-\infty, \ldots, -1, 0, 1, 2 \ldots 20\}$

$P = \{2, 3, 5, 7, 11, 13, 17, 19\}$ = Prime Nos

$Q = \{0, 1, 4, 9, 16\}$

$F = \{0, 1, 2, 3, 5, 8, 13\}$

a) $(F \cap P) \cup (Q \cap F)$

$F \cap P = \{2, 3, 5, 13\}$

$Q \cap F = \{0, 1\}$

$\therefore (F \cap P) \cup (Q \cap F) = \{0, 1, 2, 3, 5, 13\}$

b) $(F \cup P^c) \cap (F \cup Q^c)$

here $R \in (-\infty, \infty)$

$P^c = \{-\infty, \ldots -1, 0, 1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$

$Q^c = \{-\infty, \ldots, -1, 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$(F \cup P^c) = \{-\infty, \ldots -1, 0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 14, 15, 16, 18, 20\}$

$(F \cup Q^c) = \{-\infty, \ldots, -1, 0, 1, 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20\}$

$(F \cup P^c) \cap (F \cup Q^c) = \{-\infty, \ldots, -1, 0, 1, 2, 3, 5, 6, 8, 10, 12, 13, 14, 15, 18, 20\}$

c) Show $(P \cup Q)^c = P^c \cap Q^c$

$P^c = \{-\infty, \ldots, -1, 0, 1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$

$Q^c = \{-\infty, \ldots, -1, 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$(P \cup Q) = \{0, 1, 2, 3, 4, 5, 7, 9, 11, 13, 16, 17, 19\}$

$(P \cup Q)^c = \{-\infty, \ldots, -1, 6, 8, 10, 12, 14, 15, 18, 20\}$

$(P^c \cap Q^c) = \{-\infty, \ldots, -1, 6, 8, 10, 12, 14, 15, 18, 20\}$

$\therefore \; (P \cup Q)^c = (P^c \cap Q^c)$

Q. 2

Win 4 Games – Win the tournament.

$\therefore$ min – 4 Games

max – 7 Games

Let's assume National team wins $\longrightarrow$

① 4 matches held

$\rightarrow$ 1 possible outcomes

② 5 matches held

— — — — —

$= {}^5C_4 - 1 = 4$

$=$ 4 matches played

③ 6 matches are held

— — — — — —

${}^6C_2 - 5 = \frac{6!}{4! \times 2!} = 10$ matches played

④ 7 matches held

— — — — — — —

${}^7C_4 - 15 = \frac{7!}{4!\,3!} \cdot 20$ $\therefore$ 20 matches played

$\therefore$ total possible combinations are : 35 $\longrightarrow$ National team to win

$\therefore$ Total possible games $= 35 \times 2$

$= 70$

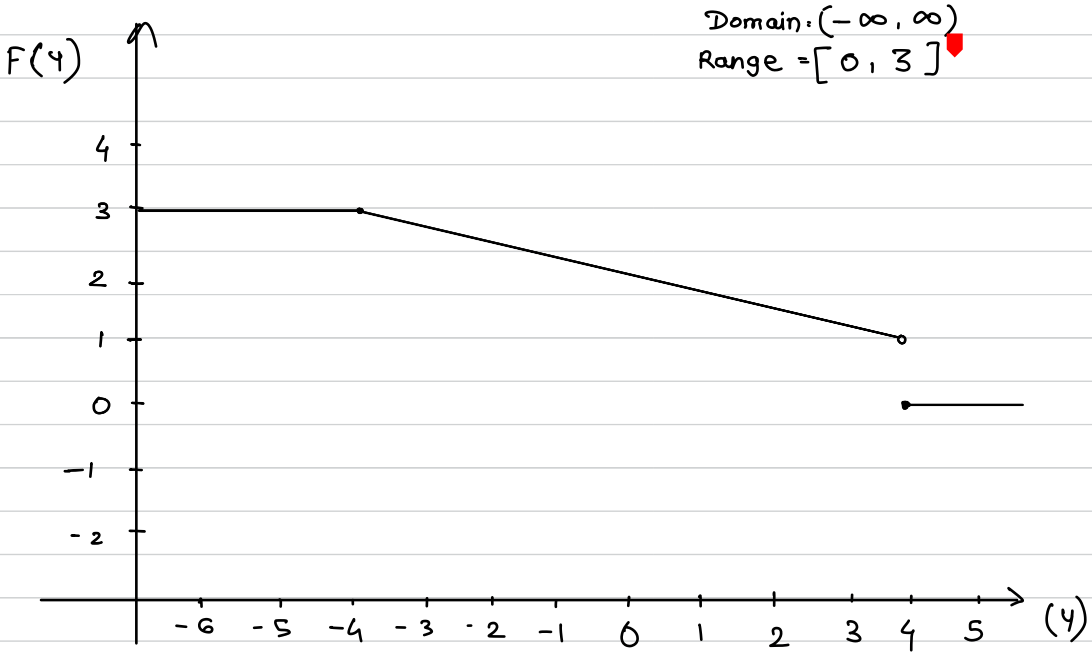
70 matches can be played.

Q. 3

$$F(y) = \begin{cases} 3 & y < -4 \\ 2 - y/4 & -4 \leq y < 4 \\ 0 & y \geq 4 \end{cases}$$

a)

Domain: $(-\infty, \infty)$
Range $= [0, 3]$



b) Formal Mathematical Expression

$$f(y) = \begin{cases} 0 & , -\infty < y < 2 \\ 0.5 & , 2 \leq y < 3 \\ 1.0 & , 3 \leq y < \infty \end{cases}$$

$y = 3 = 2 - \frac{3}{4}$
$= 1.25$

$y = -4 = 2 - (-1)$
$= 3$

$$F(y) = \begin{cases} 1.0 & 3 \leq y < \infty \\ 0.5 & 2 \leq y < 3 \\ 0.0 & -\infty < y < 2 \end{cases}$$

Q.4) $\phi(x) = 4^{x}$

a) $\phi(6) = 4^{6} = 4096$

b) $\phi(-3) = 4^{(-3)} = \frac{1}{4^{3}} = 0.0156$

c) $\phi(R) = (0, \infty)$

$R \in (-\infty \text{ to } \infty)$

$4^{(-\infty, \infty)} = (0, \infty)$

d) $\phi^{-1}(16) = 4^{(x)}$

$4^{2} = 4^{x} \quad \therefore (x = 2)$

e) $\phi^{-1}(1/4) = 4^{(x)}$

$4^{(-1)} = 4^{(x)} \quad \therefore (x = -1)$

f) $\phi^{-1}([2, 32]) = 4^{(x)}$

$(4^{1/2}) = 4^{(x)}$ | $(4^{5/2}) = 4^{(x)}$

$= 1/2$ | $= 5/2$

$= \left[\frac{1}{2}, \frac{5}{2}\right]$

Q.5 coin tossed – 8 times

a) possible outcomes

$\Rightarrow 2^{8} = 256$ ways

b) Exactly 5 heads

${}^{8}C_{5} = 56$ ways

c) atleast 1 heads

$= 2^{8} - 1$

(all possible outcomes – all tails)

$= 255$ ways