Question 3 :

A) if $a = b = c$

(i) $F(x) \longrightarrow$ positive / non negative

$\therefore$ for $0 \leq x \leq 1$
& as $a \geq 0$
therefore it is non negative.

As well as, $b \geq 0$ and $c \geq 0$
both are non-negative

- After satisfying given conditions
It is PDF of X and Value of $a = 1/3 = 0.333$

(ii) Area Under curve is must be 1.
- The integral of $f(x)$ over the entire real line must be equal to 1.

- For $F(x)$ to be PDF, It must statisfy both conditions

$$\int_{-\infty}^{\infty} F(x)\cdot dx = \int_{0}^{1} a\cdot dx + \int_{1}^{2} a\cdot dx + \int_{2}^{3} a\cdot dx = 1$$

solving of a, we get

$$a + a + a = 1$$
$$3a = 1$$
$$a = 1/3 = 0.333$$

B) if $a = 0.7$ and $b > c$

total Area:

$$\int_0^1 0.7\, dx + \int_1^2 b \cdot dx + \int_2^3 c \cdot dx = 1$$

$$\therefore \quad 0.7 + b + c = 1$$

$$\therefore \quad b + c = 0.3$$

as $c < b$,

as well as $c < \frac{0.3}{2}$

$$\therefore \quad c < 0.15$$

calculations after these

$c = 0.1$

$b = 0.2$

C. Given in question

$a = \frac{1}{6}$ | $b = \frac{1}{3}$ | $c = \frac{1}{2}$

Area =

$$= \int_0^1 a \cdot dx + \int_1^2 b \cdot dx + \int_2^3 c \cdot dx$$

$$= [ax]_0^1 + [bx]_1^2 + [cx]_2^3$$

$$= \frac{1}{6} + \frac{1}{3} + \frac{1}{2}$$

$$= \frac{6}{6} = \boxed{1}$$

here a, b, c all are positive values and Above 2 properties true for this to be PDF

$\therefore$ $\boxed{\text{" Yes, f be the PDF of X}}$

D. I) $P(1.5 < X < 2.5)$

$$= \int_{1.5}^{2} b \cdot dx + \int_{2}^{2.5} c \cdot dx$$

$$= [bx]_{1.5}^{2} + [cx]_{2}^{2.5}$$

$$= [2b - 1.5b] + [2.5c - 2c]$$

$$= 0.5b + 0.5c$$

$$= \frac{0.5}{3} + \frac{0.5}{2}$$

$$= \frac{2.5}{6}$$

$$= 0.4167$$

II) $P(X = 2)$

$$f(2) = c$$
$$= \frac{1}{2}$$

$$\boxed{f(2) = \frac{1}{2}}$$

III) IQR of X

quartiles 1 and quartile 3

q_1 (quartile 1) $= \frac{0.25}{a} = 1.5$

q_3 (quartiles 3) $= \frac{0.75}{c} = 3.5$

$\therefore$ IQR $= q_3 - q_1$

$= 3.5 - 1.5$

$= 2$

$$\boxed{IQR = 2}$$

IV) EX

$$EX = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$= \int_0^1 ax \cdot dx + \int_1^2 bx \cdot dx + \int_2^3 cx \cdot dx$$

$$= \left(\frac{1}{6} \times \frac{1}{2}\right) + \left(\frac{1}{3} \times \frac{3}{2}\right) + \left(\frac{1}{2} \times \frac{5}{2}\right)$$

$$EX = \frac{11}{6}$$

$$\boxed{EX = 1.833}$$

E) Values — a, b, c

$X \rightarrow$ random variable

— smallest Expected Value

— Smallest Variance ______ condition (1)

Var X

$a = b = c = \frac{1}{3}$

by condition (1)

$$EX = \int_0^3 x \cdot Fx \cdot dx$$

$$= \int_0^3 \frac{1}{3} \cdot x \cdot dx$$

$$= \left[\frac{x^2}{6}\right]_0^3$$

$$= \frac{9}{6}$$

$$\boxed{EX = 1.5}$$