Problem Set 1

STAT-S 520

Due on January 16th, 2023 at 11:59 PM

Note: "Minor typos were updated on 12/01/23 at 20:09 PM"

1

Let $S = \{x : x \in \mathbb{R} \text{ and } x \leq 20\}$ and P, Q and F subsets of S, where $P = \{2, 3, \ldots\ldots\}$ is the set of prime numbers, Q the set of square numbers (including zero), and $F = \{0, 1, 2, 3, 5, 8, 13, \ldots\}$ the set of Fibonacci numbers.

a. Obtain $(F \cap P) \cup (Q \cap F)$
b. Obtain $(F \cup P^c) \cap (F \cup Q^c)$
c. Show that $(P \cup Q)^c$ is the same as $P^c \cap Q^c$. Show your work.

2.

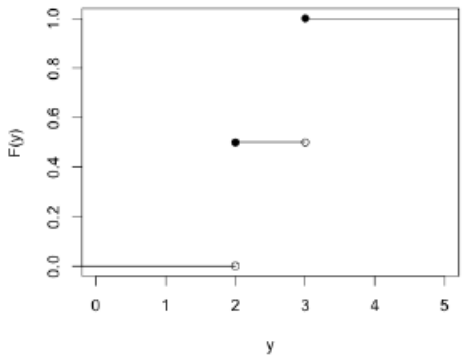
ISI 2.5 Exercises, problem 12 (p. 39)

3.

a. Draw a graph and determine what are the Domain and Image of the function $F(y)$:

$$F(y) = \begin{cases} 3 & y < -4 \\ 2 - \frac{y}{4} & -4 \leq y < 4 \\ 0 & y \geq 4 \end{cases}$$

b. Write down a formal mathematical expression for the piece-wise function $F(y)$ pictured in the graph below:



4.

Consider the function defined by $\phi(x) = 4^x$.

a. What is $\phi(6)$?
b. What is $\phi(-3)$?
c. What is $\phi(\mathbb{R})$?
d. What is $\phi^{-1}(16)$?
e. What is $\phi^{-1}(1/4)$?
f. What is $\phi^{-1}([2, 32])$?

5.

An experiment consist on tossing a (fair) coin 8 times. Assume we are interested in ordered sequences of tosses.

a. What is the number of possible outcomes? (Assume you keep the coins separated)
b. What is the number of possible ways of getting exactly 5 heads?
c. What is the number of possible ways of getting at least 1 head?

Reading Assignment

Read ISI pp.50 - 68 (from Theorem 3.1 to Example 3.15)