# Problem Set : 3

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**Q.1** ISI , sec 3.7 , Ex 8

$P(+|D) = 0.71$ , $P(-|D^c) = 0.88$ $P(D) = 0.03$

a) False positive test $(+|D^c)$

$P(+|D^c)$

$P(+|D^c) + P(-|D^c) = 1$

$\therefore P(+|D^c) = 1 - 0.88$

$\therefore \boxed{P(+|D^c) = 0.12}$

b) False Negative $P(-|D)$

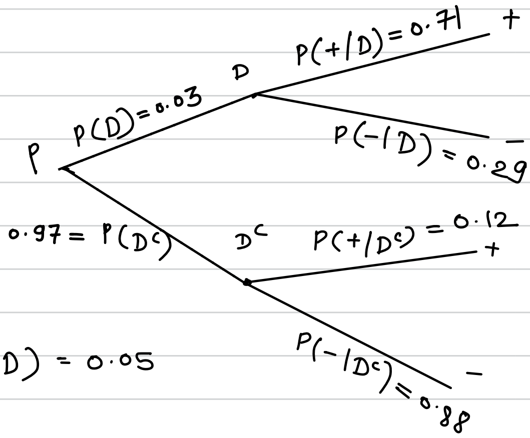
$P(-|D) = 1 - P(+|D)$

$= 1 - 0.71$

$= 0.29$

$\boxed{P(-|D) = 0.29}$

c)




here $P(D) = 0.05$

d) test will diagnose women having PE

$P(D) = P(+\cap D) + P(+\cap D^c)$

$= P(D)\cdot P(+|D) + P(D^c)\,P(+|D^c)$

$= 0.71 \times 0.003 + 0.12 \times 0.97$

$= 0.0213 + 0.1164$

$= \boxed{0.1377}$

e) $P(+|D) = \dfrac{P(+\cap D)}{P(D)} = \dfrac{P(+)\cdot P(+|D)}{P(D)}$

$= \dfrac{(0.03)\cdot(0.71)}{0.1377}$

$= \boxed{0.1546}$

Q.2

$\{1,1,1,1,1,3,3,3,7,7\}$

a) PMF (X)

$$f(y) = \begin{cases} P(X=y) & \text{IF } y \in X(s) \\ 0 & \text{otherwise} \end{cases}$$

$\underbrace{\{s \in S : X(s) = y\}}$

$X(S) = \{1, 3, 7\}$

$P(X=1) = 5/10 = 0.5$

$P(X=3) = 3/10 = 0.3$

$P(X=7) = 2/10 = 0.2$

$$F(x) = \begin{cases} 0.5, & x = 1 \\ 0.3, & x = 3 \\ 0.2, & x = 7 \\ 0, & \text{otherwise} \end{cases}$$

b) CDF (X)

$$F(y) = P(\{s \in S : X(s) \leq y\})$$

$$F(y) = \begin{cases} 0 & -\infty < y < 1 \\ 0.5 & 1 \leqslant y < 3 \\ 0.8 & 3 \leqslant y < 7 \\ 1.0 & 7 \leqslant y < \infty \end{cases}$$

c) Expected Value of X

$$EX = \sum_{x \in X(S)} X \cdot f(x)$$

$$= \sum_{x \in \{1,3,7\}} x \cdot f(x)$$

$$\mu = 1 \times 0.5 + 3 \times 0.3 + 7 \times 0.2$$

$$\boxed{\mu = 2.8}$$

d) Variance of X:

$$\operatorname{Var} X = E((X-\mu)^2 = \sum_{x \in X(S)} (x-\mu)^2 \cdot f(x)$$

$$\sigma^2 = \sum_{x \in \{1,3,7\}} = (1-2.8)^2 (0.5) + (3-2.8) \cdot (0.3) + (7-2.8) \cdot (0.2)$$

$$= 1.62 + 0.012 + 3.528$$

$$\boxed{\sigma^2 = 5.16}$$

e) Standard deviation

$$\sigma = \sqrt{\operatorname{Var} X} = \sqrt{5.16}$$

$$\boxed{\sigma = 2.2715}$$

Q.3

Toss → 3 coins, $X = (\text{No. of } T \times 10 - \text{No. of } H \times 5)$

| | | | |
|---|---|---|---|
| HHH = $-15$ | | HTT = | 15 |
| HHT = | 0 | THT = | 15 |
| HTH = | 0 | TTH = | 15 |
| THH = | 0 | TTT = | 30 |

a) $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

b) range of $X = X(S) = \{-15, 0, 15, 30\}$

c) The CDF of X

$$F(y) = \begin{cases} 0, & -\infty < y < -15 \\ 0.125, & -15 \leq y < 0 \\ 0.5, & 0 \leq y < 15 \\ 0.875, & 15 \leq y < 30 \\ 1, & 30 \leq y < \infty \end{cases}$$

d) PMF of X

$$F(x) = \begin{cases} 0.125 & x = -15 \\ 0.375 & x = 0 \\ 0.375 & x = 15 \\ 0.125 & x = 30 \\ 0 & \text{otherwise} \end{cases}$$

e) Expected Value of X

$$EX = \sum_{x \in X(S)} x \cdot F(x)$$

$$= \sum_{x \in \{-15, 0, 15, 30\}} x \cdot F(x)$$

$$= 0.125 \times (-15) + 0.375 \times 0 + 0.375 \times 15 + 0.125 \times 30$$

$$\boxed{EX = 7.5} = \mu$$

f) Variance & standard deviation.

$$\text{Var} X = E((X-\mu)^2 = \sum_{x \in X(S)} (x-\mu)^2 \cdot F(x)$$

$$= (-15-7.5)^2 \cdot (0.125) + (0-7.5)^2 \cdot (0.375) + (15-7.5)^2 \cdot (0.375) + (30-7.5)^2 \cdot (0.125)$$

$$= 63.28125 + 21.09375 + 21.09375 + 189.84375$$

$$\boxed{\sigma^2 = 295.312}$$

$$SD = \sqrt{\sigma^2}$$

$$\boxed{\sigma = 17.184}$$

Q. 4

52 cards $\Rightarrow$

Ace $= 1/13$ $\qquad$ Y= Random Variable

S: Sample Space

a> Here, one card is taken and replaced with another.
to Find 'ACE' with this method is uncountable.
because we can Find X in some countable outcome like 7 or 15
as well as It will take uncountable (or never be Found) attempts
to Find X $\Rightarrow$ cannot write all outcomes

i. (1 Hearts, 4 Diamond, 4 Club, Ace Hearts)
ii. (3 Heart, Ace club) —— possible outcomes

b> $Y(s) = \{1, 2, 3, \ldots\}$

c> i. $f(-4)$, $f(\pi)$, $f(4)$

$$f(-4) = 0$$
$$f(\pi) = 0$$
$$f(4) = \frac{48}{52} \times \frac{48}{52} \times \frac{48}{52} \times \frac{4}{52}$$
$$\boxed{f(4) = 0.06}$$

ii) $F(-2)$ and $F(2)$

$$F(-2) = 0$$
$$F(2) = \frac{4}{52} + \frac{48}{52} \cdot \frac{4}{52} = 0.146$$

d> $f(y) = \left\{ \frac{4}{52} \left(\frac{48}{52}\right)^{y-1} \right., \quad y \in Y(s)$

Q. 5 Expected Value ($\mu$) & Variance ($\sigma^2$)

a) $X \sim$ Bernoulli $(p)$

$$\text{PMF},\ f(x) = \begin{cases} p & , \quad x = 1 \\ 1-p & , \quad x = 0 \\ 0 & , \quad \text{Otherwise} \end{cases}$$

$$Ex = \mu = \sum_{x \in X(S)} x \cdot f(x)$$

$$= p \cdot 1 + (1-p) \cdot 0$$

$$\mu = p$$

$$Var X = \sum_{x \in X(S)} (x - \mu)^2 \cdot f(x)$$

$$\therefore Var X = (1-p)^2 \cdot p + (0-p)^2 (1-p)$$

$$= (1 + p^2 - 2p)(p) + (p^2)(1-p)$$

$$= p + p^3 - 2p^2 + p^2 - p^3$$

$$= p - p^2$$

$$\sigma^2 = Var X = p(1-p) \quad \text{———} \quad (1)$$

b) $Y \sim$ Binomial $(n, p)$

since Binomial distribution is multiple (n) independent Bernoulli's trials: and expected value of Bernoulli $= p$

$\therefore$ Ex of Bernoulli (Y) $= n \cdot p$

$\therefore$ Var Y = ——— from (1)

Var (Bernoulli's X $= p(1-p)$

$\therefore$ Var (Binomial Y) $= n \cdot p \cdot (1-p)$