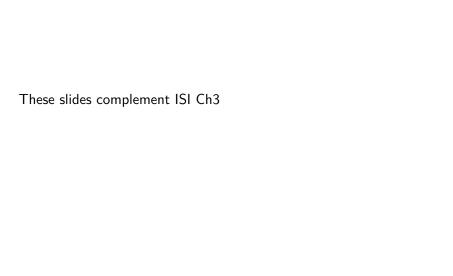
# Probability Part 1 STAT-S520

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## Probability

#### Different Interpretations:

- ► A long run frequency of occurrence ✓
- Expression of degrees of belief
- Other interpretations

## Probability Model

Think of performing an experiment that results in one of possibly many outcomes. A probability model (based on Kolmogorov's axioms) is defined by:

- ▶ The Sample Space, S₁ is the collection of all outcomes
- Events are subsets of outcomes
  - ▶ We use uppercase letters to denote events, e.g., A, D, E, etc.
  - The set of all relevant events is denoted by  $\Omega$
- A probability measure is a function that assigns probabilities to events

#### Some Conventions

- P(S) = 1
- $ightharpoonup \Omega$  is a sigma-algebra, a mathematical structure with some properties beyond the scope of this course.
  - lackbox For simplicity, we can think of  $\Omega$  as the set of all relevant events

### Example 1

Experiment: Toss a fair coin twice. Let's represent an outcome by the results in order of occurrence.

A = {HH

- $\triangleright$   $S = \{HH, HT, TH, TT\}$
- Some events:
  - $\blacktriangle = \{HH\}$
  - $\triangleright$  B = { first coin is heads } = {HH, HT}
  - $C = \{$  at least one tail $\}$
- Some probabilities
  - P(A) = 1/4 P(B) = 1/2 P(C) = 3/4

# Key Property: Finite Additivity

If events A and B are disjoint, then

$$P(A \cup B) = P(A) + P(B)$$

# Using finite additivity, we can derive the following results

a. For any event A, 
$$P(A^c) = 1 - P(A)$$

- a. For any event A,  $P(A^c) = 1 P(A)$ In particular, since P(S) = 1 then  $P(\emptyset) = 0$ b. If  $A \subset B$ , then  $P(A) \leq P(B)$ c.  $P(A \cup B) = P(A) + P(B) P(A \cap B)$