Continuous Random Variables 2

STAT-S520

Arturo Valdivia

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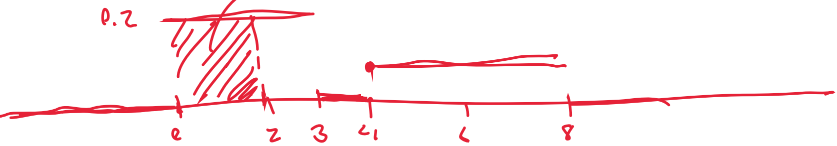
Exercise 1:

Let X be a continuous random variables and the function $f()$ given by:

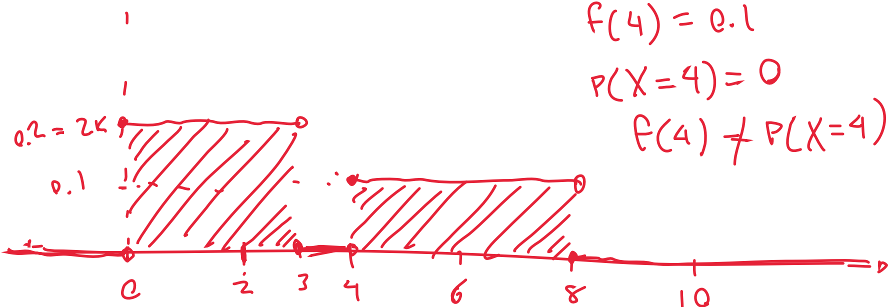
$$f(x) = \begin{cases} 2k & 0 \leq x < 3 \\ k & 4 \leq x < 8 \\ 0 & otherwise \end{cases}$$

1. Find k so $f()$ is a PDF of X ✓ $K = 0{,}01$
2. $P(X < 2)$ $= \int_{-\infty}^{2} f(x)\,dx = 0.4$
3. $P(3 < X \leq 4) = 0$
4. EX

$(2-0).0.2 = 0.4$



Exercise 1: (continued)



$f(4) = 0.1$

$P(X=4) = 0$

$f(4) \neq P(X=4)$

Aside $P(X=2) = 0$

$P(1.999 < X < 2.001) > 0$

$(3-0)\cdot 2K + (8-4)\cdot K = 1$

$6K + 4K = 1$

$\Rightarrow K = 1/10 = 0.1$

$EX = 0.6 \cdot 1.5 + 0.4 \cdot 6$

$= 0.9 + 2.4 = 3.3$

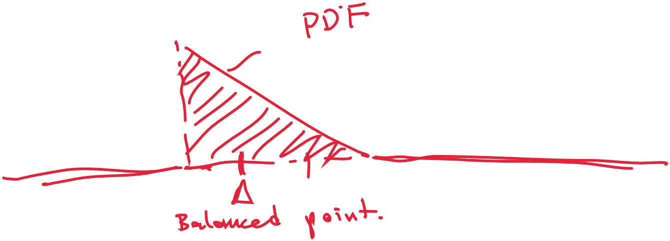
$\boxed{EX = 3.3}$

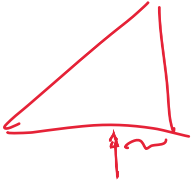
Discrete binomial $P(X \le 7) = P(X < 8)$

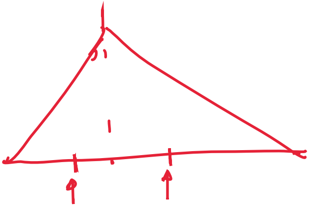
Continuous $P(X = 7) = 0$

$P(X \le 7) = P(X < 7)$

~~Exercise 2: ISI Section 5.6. Exercise 5~~

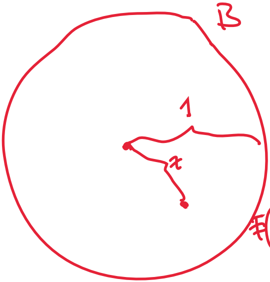






Exercise 2 (continued)

Exercise 3: ISI Section 5.6. Exercise 4

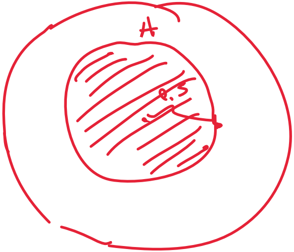


$$X(S) = [0, 1]$$

$$\text{Area}(B) = \pi$$

$$F(0.5) = \underline{P(X \leq 0.5)} = P(A) = 0.25$$

$$P(A) = \frac{\text{Area}(A)}{\pi} = \frac{\cancel{\pi}\, 0.5^2}{\cancel{\pi}} = 0.25$$



$$P(0.5 < X \leq 0.7)$$

$$= P(X \leq 0.7) - P(X \leq 0.5)$$

$$F(0.7) - F(0.5)$$

$$= \frac{0.7^2 \cancel{\pi}}{\cancel{\pi}} - \frac{0.5^2 \cancel{\pi}}{\cancel{\pi}} = 0.49 - 0.25 = 0.24$$

Exercise 3: (continued)

Recall that $F(y) = P(X \le y)$ for all $y \in \mathbb{R}$

$$F(y) = \begin{cases} 0 & y < 0 \\ y^2 & 0 \le y < 1 \\ 1 & y \ge 1 \end{cases}$$

$\frac{dF(y)}{dy} = f(y) = 2y$

$0 \le y < 1$

$f(x) = 2x \quad 0 \le x < 1$

Normal distribution

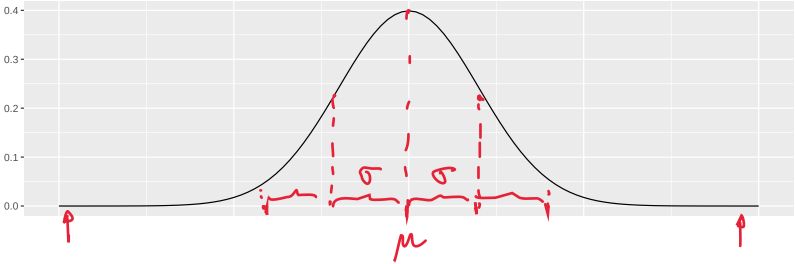
We say that X follows the Normal distribution if its PDF is given by

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}}$$

for $x \in \mathbb{R}$ where $\mu = EX$ and $\sigma^2 = VarX$. We then write

$$X \sim \text{Normal}(\mu, \sigma^2)$$

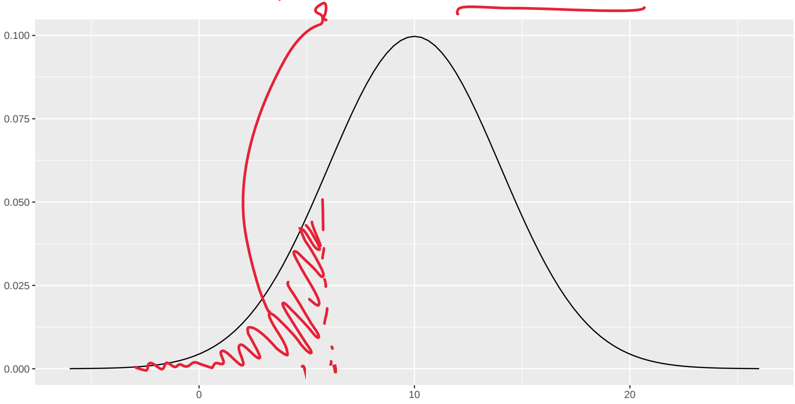
The graph of the PDF is:



CDF for the Normal

- Recall that the CDF and PDF are related by $F(y) = \int_{-\infty}^{y} f(x)dx$. This is just the area under the curve for $(-\infty, y]$.
- Let $X \sim \text{Normal}(10, 16)$, let's graph

$$F(6) = P(X \leq 6) = \int_{-\infty}^{6} f(x)dx$$



Standard normal

- Let $X \sim \text{Normal}(\mu, \sigma^2)$ and
$$Z = \frac{X - \mu}{\sigma}$$
then $Z \sim \text{Normal}(0, 1)$ is called a standard normal random variable.
- *Exercise:* Use properties of the expected value and variance to show that $EZ = 0$ and $VarZ = 1$.

Sums and differences of Normals

If $X \sim \text{Normal}(\mu_x, \sigma_x^2)$ and $Y \sim \text{Normal}(\mu_y, \sigma_y^2)$ are two independent normal random variables and let $S = X + Y$ and $D = X - Y$ then

$$S \sim \text{Normal}(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2)$$

and

$$D \sim \text{Normal}(\mu_x - \mu_y, \sigma_x^2 + \sigma_y^2)$$