Lecture 8: Basic models of strategic interaction in networks

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- Social networks are important in many facets of our lives.
- Most decisions that people make, from which products to buy to whom to vote for, are infuenced by the choices of their friends and acquaintances.
- For example the decision of an individual of whether to adopt a new technology, attend a conference, find a job is often influenced by the choices of his or her friends and acquaintances.

- The emerging empirical evidence on these issues motivates the theoretical study of network effects.[1]
- In this course we provide an overview on the analysis of the interaction of individuals who are connected via a network and whose behaviors are influenced by those around them.
- Such interactions are natural ones to model using *game theory*, as the payoffs that an individual receives from various choices *depends on the behaviors of his or her neighbors*.

[1] Diifferent from the IO network effect approach.

- This particular view of games on networks, where an agent chooses an action and then the payoffs of each player is determined by those of his or her neighbors is a special perspective, but one that applies to many different context including the peer effect mentioned in previous slides.

- There are also other applications that involve strategic decision making and networks of relationships, such as exchange or trade on networks (networked markets).

- We can view these settings as special cases of game theory more generally, and so some results from the general literature directly apply: for example, existence of various forms of equilibria can be deduced from standard results.
- Our interest, therefore is instead in whether there is anything we can deduce that holds systematically regarding how play in a game depends on the network structure of interactions.

- For example, if individuals only wish to buy a new product if a sufficient fraction of their friends do, can we say something about how segregation patterns in the network of friendships affects the purchase of the product?
- Can we say anything about who is the most infuential individual in a network where people look to their peers in choosing an effort level in education?
- This course focuses on models relating network characteristics to behavior.

- The main challenge that faced in studying strategic interaction in social settings is the inherent complexity of networks.
- Without focusing in on specific structures in terms of the games, it is hard to draw any conclusions.

- The literature has primarily taken three approaches to this challenge, and form the basis for our discussion.

- One involves looking at games of *strategic complements* and *strategic substitutes*, where the interaction in payoffs between players satisfies some natural and useful monotonicity properties.

- With strategic complementarities, a player's incentives to take an action (or a "*higher*" action) are increasing in the number of his or her friends who take the (higher) action; and with strategic substitutes the opposite incentives are in place.

- A second approach relies on looking at quite tractable "*linear-quadratic*" setting where agents choose a continuous level of activity.
- That simple parametric specification permits an explicit solution for equilibrium behavior as a function of a network, and thus leads to interesting comparative statics and other results that are useful in empirical work.

- A third approach considers settings with an uncertain pattern of interactions, where players make choices (such as learning a language) without being certain about with whom they will interact.

- The uncertainty actually simplifies the problem since behavior depends on anticipated rates of interaction, rather than complex realizations of interactions.

- Together all of these various approaches and models make a number of predictions about behavior, relating levels of actions to network density, relating players' behaviors to their position in the network, and relating behavior to things like the degree distribution and cost of taking given actions.
- The theory thus makes predictions both about how a player's behavior relates to his/her position in a network, as well as what overall behavior patterns to expect as a function of the network structure.

- Let N be the set of players, where $N = \{1, \ldots, n\}$.
- For all $i \in N$, let $\mathbf{X}_i$ be player i's strategy set.
- Let $\mathbf{X}_{-i} = \prod_{j \neq i} X_j$ and let $\mathbf{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$.
- Let $\mathbf{L}$ denote the set of links.
- Let $G = (N, \mathbf{L})$ be the network representing players relationships, where $g_{ij} = 1$ whenever i and j are connected.

- Player i's payoff function is denoted

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- A Nash equilibrium is a profile $\mathbf{x}^*$ such that:

$$u_i(x_i^*; \mathbf{x}_{-i}^*; \mathbf{G}) \geq u_i(x_i, \mathbf{x}_{-i}^*; \mathbf{G}) \quad \text{for all } i,\ x_i \in \mathbf{X}_i.$$

- More formally, player i's payoff depends only on x_i and $\{x_j\}_{j \in N_i(\mathbf{G})}$, so that for any i, x_i and $\mathbf{G}$:

$$u_i(x_i, \mathbf{x}_{-i}; \mathbf{G}) = u_i(x_i, \mathbf{x}'_{-i}; \mathbf{G}),$$

whenever $x_j = x'_j$ for all $j \in N_i(\mathbf{G})$.

The Majority game

- Players actions space are $X_i = \{0, 1\}$.
- In this particular game, if more than one half of i's neighbors choose action 1, then it is best for player i to choose 1.
- if fewer than one half of i's neighbors choose action 1 then it is best for player i to choose action 0.
- Specifically, the payoff to a player from taking action 1 compared to action 0 depends on the fraction of neighbors who choose action 1, such that:

$$u_i(1, \mathbf{x}_{-i}; \mathbf{G}) > u_i(0, \mathbf{x}_{-i}; \mathbf{G}) \quad if \quad \frac{\sum_{j=1}^{n} g_{ij} x_j}{|N_i(\mathbf{G})|} > \frac{1}{2},$$

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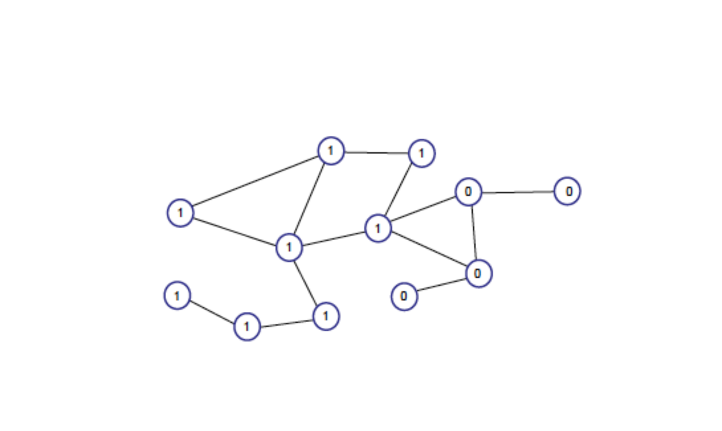
This game has multiple equilibria.
For example, all players taking action 0 (or 1) is an equilibrium.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Best-Shot Public Goods Games (Hirshleifer, 1983)

- For instance, the action might be learning how to do something.
- The information is easily communicated,
- For instance, buying a book or other product that is easily lent from one player to another.
- Taking the action 1 is costly and if any of a player's neighbors takes the action then the player is better off not taking the action.
- Taking the action and paying the cost is better than having nobody in a player's neighborhood take the action.

$$u_i(x_i, \mathbf{x}_{-i}; \mathbf{G}) = \begin{cases} 1 - c & \text{if } x_i = 1 \text{ and } 0 < c < 1 \\ 1 & \text{if } x_i = 0 \text{ and } x_j = 1 \text{ for some } (i,j) \in \mathbf{L} \\ 0 & \text{if } x_i = 0 \text{ and } x_j = 0 \text{ for all } (i,j) \in \mathbf{L} \end{cases}$$

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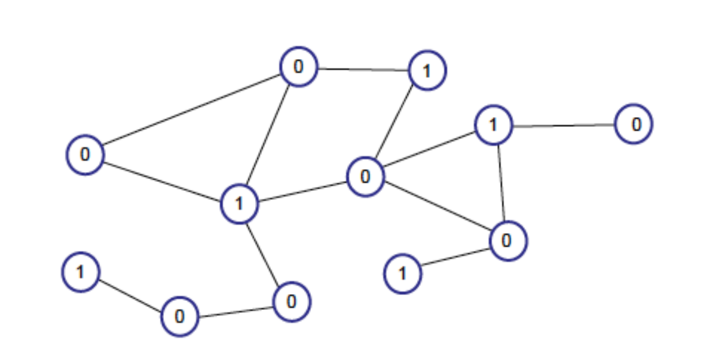
There are many possible equilibria in this game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- The equilibria in this game correspond exactly to having the set of players who choose action 1 form a maximal independent set of nodes in the network (Bramoulle and Kranton, JET (2007))
- The equilibria form a maximal set of nodes that have no links to each other in the network

Public goods in networks, (Bramoulle, Kranton 2007)

- There are n agents, and the set of agents is $N = \{1, \ldots, n\}$.
- Let $x_i \in \mathbf{X}_i = [0, \infty)$ denote denote agent i's level of effort.
- For instance, or x_i could be the amount of time a consumer spends researching a new product.
- We assume that the individual marginal cost of effort is constant and equal to $c > 0$.
- Let $\mathbf{x} = (x_1, \ldots, x_n)$ denote an effort profile of all agents.

- Agents are arranged in an undirected network $\mathbf{G}$.
- Since agent i knows the results of his own effort, we may set $g_{ii} = 1$.
- Two important assumptions:
  1. First, an agent's effort is a substitute of the efforts of her neighbors, but not of individuals further away in the network.
  2. Second, a neighbor effort is a perfect substitute with one's own.

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- Two important assumptions:
  1. First, an agent's effort is a substitute of the efforts of her neighbors, but not of individuals further away in the network.
  2. Second, a neighbor effort is a perfect substitute with one's own.
- With these assumptions an agent i derives benefits from the total of his own and his neighbors efforts.

- Agent i benefit function is given by twice-differentiable and strictlty concave function $b(\mathbf{x}, \mathbf{G})$ where $b(0) = 0$, $b' > 0$ and $b'' < 0$.
- In particular, $b(x_i + \sum_{j \in N_i(\mathbf{G})} x_j)$.
- Then agent i's payoff from profile $\mathbf{x}$ in network $\mathbf{G}$ is then:

$$u_i(x_i, \mathbf{x}_{-i}; \mathbf{G}) = b\left(x_i + \sum_{j \in N_i(\mathbf{G})} x_j\right) - cx_i.$$

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- Given a structure $\mathbf{G}$, agents simultaneously choose effort levels.

The shape of equilibrium outcomes

- Let x^* denote the effort level at which, to an individual agent, the marginal benefit equals its marginal cost; $b'(x^*) = c$.
- Let $\bar{\mathbf{x}}_{-i} = \sum_{j \in N_i(\mathbf{G})} x_j$ be the total effort of i's neighbors.
- A profile $\mathbf{x}$ is a Nash equilibrium if and only if for every agent i either
  (i) $\bar{\mathbf{x}}_{-i} \geq x^*$ and $x_i = 0$, or
  (ii) $\bar{\mathbf{x}}_{-i} \leq x^*$ and $x_i = x^* - \bar{\mathbf{x}}_{-i}$.

- Agents want to exert effort as long as their total benefits are less than $b(x^*)$.
- If the benefits they acquire from their neighbors are more than $b(x^*)$ they exert no effort.
- If the benefits are less than $b(x^*)$, they exert effort up to the point where their benefits equal $b(x^*)$.

Definition

We say a profile $\mathbf{x}$ is specialized when every agent either exerts the maximum amount of effort x^* or exerts no effort; for all agents i either $x_i = 0$ or $x_i = x^*$. We call an agent who exerts x^* a *specialist*.

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Hybrid equilibria fall between these two extremes.

Definition

An independent set $\mathbf{S}$ of a network $\mathbf{G}$ is a set of agents such that no two agents who belong to $\mathbf{S}$ are linked; i.e., $\forall i, j \in \mathbf{S}$ such that $i \neq j$, $g_{ij} = 0$. An independent set is *maximal* when it is not a proper subset of any other independent set.

- Given a maximal independent set $\mathbf{S}$, every agent either belongs to $\mathbf{S}$ or is connected to an agent who belongs to $\mathbf{S}$.
- Thus, we can partition the population into two disjoint sets of agents: those who belong to maximal independent set $\mathbf{S}$, and those who are linked to an agent in $\mathbf{S}$.
- For any agent i, there exists a maximal independent set $\mathbf{S}$ of the graph $\mathbf{G}$ such that i belongs to $\mathbf{S}$.

Theorem (Bramoullé and Kranton, 2007)

A specialized profile is a Nash equilibrium if and only if its set of specialists is a maximal independent set of the structure $\mathbf{G}$. Since for every $\mathbf{G}$ there exists a maximal independent set, there always exists a specialized Nash equilibrium.

- In games of strategic complements, an increase in the actions of other players leads a given player's higher actions to have relatively higher payoffs compared to that player's lower actions.
- Examples of such games include situations like the adoption of a technology, human capital decisions, and criminal efforts.

- Games of strategic substitutes are such that the opposite is true: an increase in other players' actions leads to relatively lower payoffs to higher actions of a given player.
- Applications of strategic substitutes include, for example, local public good provision and information gathering.

- This section: Games with Strategic Substitutes, Continuous Action Spaces and Linear Best-Replies.

- Games with strategic substitutes are difficult to characterize and multiple equilibria rather than unique equilibrium are the rule.

Ballester, Calvo- Armengol, and Zenou (2006)

- Players' payoffs functions are given by:

$$u_i(x_i, \mathbf{x}_{-i}; \mathbf{G}) = x_i - \frac{1}{2}x_i^2 - \delta \sum_{j=1}^{n} g_{ij} x_j x_i \quad \text{for all } i \in N,$$

with $\delta \geq 0$ and $x_i \in \mathbf{X}_i = \mathbb{R}_+$ for all $i \in N = \{1, \ldots, n\}$.

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- The first order condition is

$$\frac{\partial u_i(x_i, \mathbf{x}_{-i}; \mathbf{G})}{\partial x_i} = 1 - x_i - \delta \sum_{j=1}^{n} g_{ij} x_j \leq 0.$$

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- It is straightforward to check:

$$\frac{\partial^2 u_i(x_i, \mathbf{x}_{-i}; \mathbf{G})}{\partial x_j \partial x_i} = -1 \quad \text{and} \quad \frac{\partial^2 u_i(x_i, \mathbf{x}_{-i}; \mathbf{G})}{\partial x_j \partial x_i} = -\delta g_{ij}.$$

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- A Nash equilibrium is given by Bonacich centrality measure.

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- For this particular case, the potential function is given by:

$$P(\mathbf{x};\delta,\mathbf{G}) = \mathbf{x}^T\mathbf{1} - \frac{1}{2}\mathbf{x}^T[\mathbf{I}+\delta\mathbf{G}]\mathbf{x}.$$

Proposition

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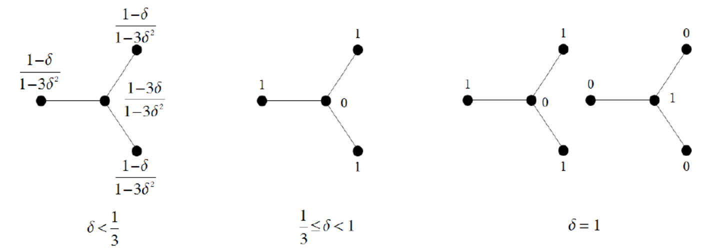
The results depend on the sparsity of the network $\mathbf{G}$, which is captured by $\lambda_{min}(\mathbf{G}) < 0$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$\delta = 1$

Corollary

For regular graphs, there is a unique equilibrium if and only if $|\lambda_{min}(\mathbf{G})| < \frac{1}{\delta}$ and this equilibrium is interior. If $|\lambda_{min}(\mathbf{G})| < \frac{1}{\delta}$ there are both interior and corner equilibria.

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Main insight from this approach: Connection with spectral analysis and the lowest eigenvalue of a network

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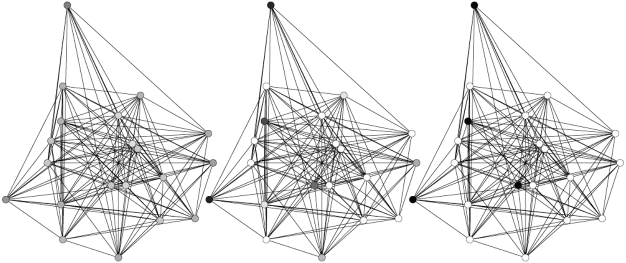
Complex networks can be analyzed



(a) $\delta = 0.15$, unique equilibrium (b) $\delta = 0.35$, 3 equilibria (c) $\delta = 0.55$, 31 equilibria

Comparative Statics

- In this section we ask how changes in the graph or the payoff impact parameter δ affect equilibrium play.
- Strategic substitutes pose a challenge for comparative statics, especially in networks.
- Consider a new link between an agent i and j.
- The new link lowers the incentives of the newly connected agents and thus increases the incentives of their neighbors, and so on.

- The direct effects and the indirect effects can pull in opposite directions.
- The potential function $P(\mathbf{x};\delta,\mathbf{G})$ help us to overcome these problems.
- We focus on changes on the aggregate play $\sum_{i\in N} x_i(\delta,\mathbf{G})$.
- For a given δ and $\mathbf{G}$, let $\mathbf{x}^*(\delta,\mathbf{G})$ be an equilibrium with the highest aggregate play.
- This equilibrium is also a global maximum of the potential, which follows from

$$P(\mathbf{x}^*(\delta,\mathbf{G});\delta,\mathbf{G}) = \frac{1}{2}\mathbf{x}^{*T}(\delta,\mathbf{G})\cdot\mathbf{1}.$$

Proposition

Consider a δ and $\mathbf{G}$ and a highest-aggregate-play equilibrium $\mathbf{x}^*(\delta, \mathbf{G})$. Consider a δ' and $\mathbf{G}'$ where $\delta' \geq \delta$ and $\mathbf{G}$ is a subgraph of $\mathbf{G}'$ and any equilibrium vector $\mathbf{x}(\delta', \mathbf{G}')$. Then

$$\sum_{i \in N} x_i(\delta', \mathbf{G}') \leq \sum_{i \in N} x_i^*(\delta, \mathbf{G}).$$

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- Agent i's returns from x_i are higher when total crime levels are lower, capturing the possibility that criminals compete for victims or territory.
- Agent i's costs are lower when i's friends engage in more crime, capturing peer effects.
- Together, player i's payoffs are given by:

$$u_i(x_i, \mathbf{x}_{-i}; \alpha, \varphi, \mathbf{G}) = x_i(1 - \sum_{j=1}^{n} x_j) - cx_i(1 - \varphi \sum_{j=1}^{n} g_{ij}x_j)$$

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- $0 \leq c < 1$ is a cost parameter.
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- Maximizing u_i subject to $x_i \geq 0$, we get

$$x_i = \max\{x_0 - \sum_{j=1}^{n} h_{ij} g_{ij}, 0\}$$

where $x_0 = (1-c)/2\alpha$ is individually optimal when no other agent engages in crime and $h_{ij} = \frac{1}{2}(1 - \frac{c\beta}{\alpha} g_{ij})$.

- Assume $\alpha = \beta$. Then the matrix $\mathbf{H} = \frac{1}{2}\mathbf{C} - c\mathbf{G}$ is key.

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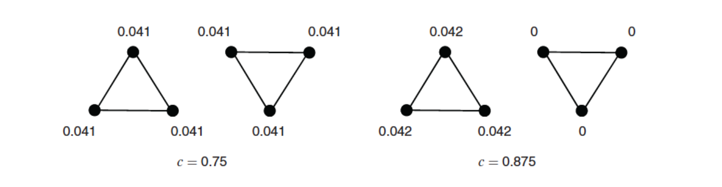
- The index $\lambda_{min}(\mathbf{H})$ shapes the equilibrium set.
- Studying the friendships, $\mathbf{G}$, and costs, c, as they feed into $\lambda_{min}(\mathbf{H})$ yields predictions as to levels and patterns of crime.


0.041
0.041
0.041
0.041
0.041
0.041
c = 0.75
0.042
0.042
0.042
0
0
0
c = 0.875

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