Test 2

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Question 1

Best Response Function:

$$BR_i(a_{-i}) = \alpha \sum_j w_{ij} a_j + b_i$$

Here $\alpha > 0$, $W = (W_{ij})_{i,j \in N}$ and $(b_i)_{i \in N}$ are constant parameters &
— do not depend upon $a = (a_i)_{i \in N}$

1) The previous game represents Strategic complements
Reasons
1. The best response function of player i is increasing in the actions of other player j. That is if player j increases their action, then the player i's best response is to increase their own action as well.
2. The players action has positive effect on each other's playoff. Which helps to mutual benefit. The positive coefficient α is the best response formula implies that if one player increases their action, it will increase the marginal benefit of other player's actions as well. This creates a situation where players come a tendency to co-ordinate.

2) Characterize the unique equilibrium of the game:
- 1 Solve Nash equilibrium
  where player has incentive to deviate from their choosen strategy.

$a = \begin{pmatrix} a_1 \\ a_n \end{pmatrix}$ $\quad b = \begin{pmatrix} b_1 \\ b_m \end{pmatrix}$ $\quad W$

$a = b + \alpha W a$ ______ solving for a,
$a - \alpha W a = b$
$b = [I - \alpha W] a$
$= [I - \alpha W]^{-1}$

The unique equilibrium of the game

$a^* = (a_1, a_2, a_3, \ldots, a_n)$ where
$a_i = (\alpha / u)$

3> 3. How is your answer to question 2 related to the network topology?

→ My answer to question 2 is related to the network topology because the equilibrium actions of the players depend on the structure of the interaction network represented by the matrix W.

In particular, the equilibrium actions are proportional to the weighted sum of the actions of the other players, where the weights are given by the entries of the matrix W. The weight wij captures the influence that player j's action has on player i's best-response. The more connected player i is to player j (i.e., the higher the value of wij), the greater the influence of player j's action on player i's best-response.

Therefore, the equilibrium actions depend on the pattern of connections in the network represented by the matrix W. If two players are more tightly connected in the network (i.e., they have a higher value of wij), their actions will tend to be more closely aligned in the equilibrium, as each player's best-response will depend more on the other player's action. Conversely, if two players are less connected in the network (i.e., they have a lower value of wij), their actions will tend to be less aligned in the equilibrium, as each player's best-response will depend less on the other player's action.

Thus, the network topology plays an important role in determining the equilibrium actions and the degree of coordination among the players. A more tightly connected network will tend to produce greater coordination among the players, while a less connected network will tend to produce less coordination.

4) Now, assume that α < 0. How does your answer to question 1 change?

→ If we assume that α < 0, then the sign of each player's best-response function changes, since each term in the sum becomes negative. Specifically, we have:

$$BR_i\ (a-i) = \alpha \sum_j W_{ij}\ a_j + b_i < b_i$$

This means that each player's best-response is to select the minimum possible action, since smaller actions result in smaller negative values. Therefore, in this case, the players' actions are strategic complements rather than substitutes. That is, each player's optimal action decreases as the other players' actions also decrease.

Intuitively, when α is negative, each player has a negative incentive to increase their own action, as this increases the actions of other players through the sum in the best-response function. Therefore, the players are motivated to coordinate on low actions, which leads to a complementarity between their actions.

In conclusion, if α < 0, the game represents strategic complements rather than substitutes, and the players are motivated to coordinate on low actions.

- as well as since $\alpha < 0$,
  the team $\alpha \sum_j W_{ij}\ a_j$ is negative

- which means that each agent's best response is strictly decreasing in the action of other agent.

5)

Yes, we can use the concept of social welfare to measure the gap between a socially optimal outcome and the Nash equilibrium of the game. Social welfare is a measure of the total utility generated by the players' actions in the game, and a socially optimal outcome corresponds to the action profile that maximizes social welfare.

Inefficiency = (W(a*) - W(s*)) / W(s*)

This measure captures the percentage loss in social welfare due to the players' failure to coordinate on the socially optimal outcome. If the Nash equilibrium achieves the socially optimal outcome, then the inefficiency is zero. If the Nash equilibrium achieves no social welfare (i.e., W(a*) = 0), then the inefficiency is undefined.

One measure to quality in efficiency in Nash equilibrium of the game is the price of Anorchy which is POA.

POA = ratio of worst case social well fare (in Nash Equilibrium) to the Maximum Social Welfare Achievable

S → set of all action profiles

$W(a)$ → Social welfare Function

socially optimal outcome : $a^* = \text{argmax}\_a \in S \; W(a)$

a_N → Nash Equilibrium

POA → $\max\_a \in S \, (w(a) / W(a_N))$

If

POA $= 1$, $a_N = a^*$

POA > 1, a_N → Socially inefficient and the gap between a_N and a^* is larger.

∴ POA can be measure to qualify inefficiency.

Question 2:

1) town A $\longrightarrow$ town B

there can be 2 possible ways to travel

- Paths : ADB or ACB

senario 1:

500 cars travelling ACB and
500 cars travelling ADB

This is Nash Equilibrium which yield by equal balance.

therefore,

Travel time $\rightarrow$ $(500/100) + 12$
$= 17 \longrightarrow$ for drivers on ADB & ACB

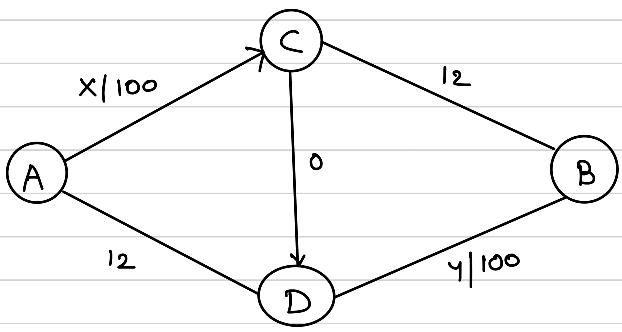
senario 2:

If X & Y $\neq$ 500 then this senario will have unequal travel time
as well as drivers can change the routes any time.
there fore, X & Y $\neq$ 500 For any value
and it won't be Nash Equilibrium

there fore,

Nash Equilibrium Values of X & Y are : X = 500 and Y = 500.

2)



- connection of Road C & D

- Nash Equilibrium for New Network:

① Path ADB $\rightarrow$

cost of travel $= 12 + 1000/100 = \boxed{22}$

- here driver is using path DB

② Path ACDB

- To reduce travel time choose path ACDB

cost $= (x + 1000 + (1000/100)) < 22$

③ At point C :

Driver at point C → won't select path CB

because CDB = 1000|100 = 10

(10 < 12).

Driver will select ACB and will reach B with path DB.

After considering all the senarios,

Drives will choose ACDB and

Nash Equilibrium for x & y = 1000

→ Total cost of travel

for New roads

1. Path AC = 1000| 100 = 10
2. Path C = 0
3. Path DB = 1000| 100 = 10

There fore,

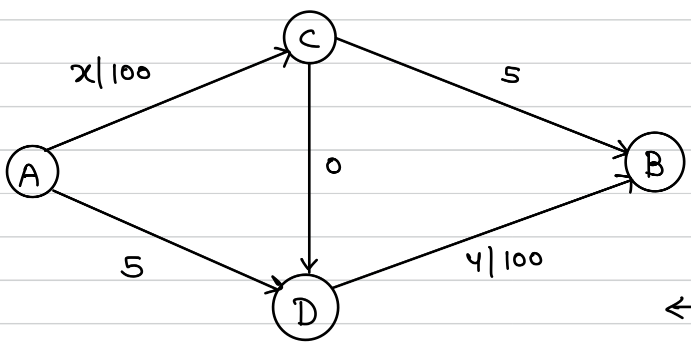
Total cost of travel = 1000 | 100 + 0 + 1000| 100

= 20

And Total cost of Network = 20 × 1000

= 20,000

3>



← Network

Nash Equilibrium :

Senario 1

ADB = 500 cars

ACD = 500 cars

Travel time -

For both paths = 500 | 100 + 5

= 10

there fore it is Nash Equilibrium,

here any driver changes the route the trave time will be > 10.
& it is not Good Value

here,
Total cost of travel = 10×1000
= 10,000

Because no one chooses the road CD.
- Total cost of travel (considering government closes the road) From C to D will remain same.