Microeconomics Theory II
Test 4 - Final Exam

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Question 1:

Based on the paper An Invitation to Market Design, by Kominers et al.(2017), describe an application of how market can be used to solve real world problems

The market is a powerful mechanism for allocating resources and solving real-world problems. One example of this is the allocation of radio spectrum, which is used for communication between devices such as cell phones, TV, and radio. In the United States, the Federal Communications Commission (FCC) is responsible for managing the allocation of radio spectrum. Traditionally, the FCC has used a command-and-control approach to allocate spectrum, where the government decides which frequencies can be used for which purposes. This approach has been criticized for being slow and inflexible.

To address these concerns, the FCC has adopted a market-based approach to spectrum allocation called the spectrum auction. In a spectrum auction, the government sells the right to use a particular frequency band to the highest bidder. This approach has several advantages over the traditional command-and-control approach. First, it is more efficient because it allows market forces to determine the value of different frequency bands. Second, it is more flexible because it allows the government to adapt to changing demand for spectrum. Third, it is more transparent because it provides a clear price signal for the value of spectrum.

One of the most successful examples of a spectrum auction is the Advanced Wireless Services (AWS) auction held by the FCC in 2006. In this auction, the FCC sold the rights to use 90 MHz of spectrum in the 1710-1755 MHz and 2110-2155 MHz bands. The auction generated $13.7 billion in revenue for the US government, making it the largest spectrum auction in history at the time.

The AWS auction was successful for several reasons. First, it was designed to be fair and transparent. The FCC established clear rules for the auction and made sure that all bidders had equal access to information about the spectrum being auctioned.

Second, it was designed to be efficient. The auction used a simultaneous multiple-round format, which allowed bidders to bid on multiple spectrum licenses at the same time. This format encouraged bidders to reveal their true valuations of the spectrum, which increased the efficiency of the auction. Third, it was designed to be flexible. The FCC allowed bidders to bid on partial licenses, which allowed them to tailor their bids to their specific needs.

In conclusion, the market can be used to solve real-world problems such as the allocation of radio spectrum. Spectrum auctions have several advantages over traditional command-and-control approaches, including increased efficiency, flexibility, and transparency. The success of the AWS auction demonstrates the power of the market to allocate resources and solve complex problems.

Question 2 - Part 1

course allocation Problem

$S = \{s_1, s_2, s_3, s_4\}$ and $C = \{c_1, c_2, c_3, c_4\}$

assume $q_c = 1$ for all $c \in C$

course priorities :

$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$	$\succ_{c_4}$
s_4	s_2	s_3	s_1
s_1	s_3	s_4	$\vdots$
s_2	$\vdots$	$\vdots$	$\vdots$

Student Preferences

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
c_1	c_1	c_2	c_3
c_4	c_2	c_3	c_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$

a)

- Resulting Matching is

$$\mu_2 = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_4 & s_2 & s_3 & s_1 \end{pmatrix}$$

and corresponding entries are underlined in table.

b) Explain steps

i. Initialize the algorithm by having each student propose to their top rank course

ii. Each course review its proposals and creates a ranked list of its preferred students. Courses rejects all but their top ranked students, and any students who are rejected from temperory group.

iii. Each unmatch student proposes to their next course.

iv. Each reviews its proposals & recreates new list of preferred students.

v. Algorithm terminates since no more students are unmatched.

→ The student-optimal stable mechanism is widespread because it satisfies a number of desirable properties such as strategy proofness for the students and stability c)

Efficiency, strategy-proofness, and stability are important criteria for evaluating the performance of a matching algorithm. In the context of the course allocation problem, efficiency means that every student is assigned to a course they prefer over any other available course, and every course is filled to its capacity with students who prefer it to any other available course. Strategy-proofness means that every student is incentivized to truthfully report their preferences, without having to worry about other students manipulating the system to their advantage. Stability means that no student and course prefer each other over their current assignments, and there is no incentive for any of them to break their current match.

The administrator argues that a matching algorithm called the Deferred Acceptance Algorithm (DAA) can achieve all three criteria simultaneously. The DAA works by having each student propose to their most preferred course, and each course accept or reject the proposal based on their own preferences. Any rejected students are added to a temporary pool of unmatched students, who then propose to their next preferred course. This process continues until all students are matched.

The DAA is known to be efficient because it ensures that every student is assigned to their most preferred course that has an available seat, and every course is filled with the most preferred students who have ranked it as their first choice. It is also strategy-proof, as it is always optimal for each student to truthfully report their preferences. No student can benefit from lying about their preferences, since any untruthful proposal will be rejected by the course and the student will end up in the temporary pool of unmatched students. Additionally, the DAA ensures that the matching is stable, meaning that no unmatched student and course both prefer each other over their current matches.

Therefore, the college administrator is correct that the Deferred Acceptance Algorithm can achieve efficiency, strategy-proofness, and stability simultaneously. This algorithm is widely used in many real-world matching problems, including school choice, kidney exchange, and labor markets, because it has been shown to perform well in practice and to satisfy important economic and ethical criteria.

Question 2 Part 2

Agents $A = \{1, 2, 3, 4\}$
object $O = \{a, b, c, d\}$

Initial ownership can be written as
$a \to 1$, $b \to 2$, $c \to 3$, $d \to 4$

preference Profile $\longrightarrow$

$>_1$	$>_2$	$>_3$	$>_4$
c	d	a	c
b	a	d	b
d	b	c	a
a	c	b	d

According to the given algorithm we start the 1st iteration we get,

$1 \to c$ $\quad$ $a \to 1$
$2 \to d$ $\quad$ $b \to 2$
$3 \to a$ $\quad$ $c \to 3$
$4 \to c$ $\quad$ $d \to 4$

we find the following cycle:

$$1 \longrightarrow c \longrightarrow 3 \longrightarrow a \longrightarrow 1$$

we can assign $(1, c)$ & $(3, a)$
we start 2nd iteration

$1 \to c$ $\quad$ $a \to 3$
$2 \to d$ $\quad$ $b \to 2$
$3 \to a$ $\quad$ $c \to 1$
$4 \to b$ $\quad$ $d \to 4$

we get following cycle

$$2 \longrightarrow d \longrightarrow 4 \longrightarrow b \longrightarrow 2$$

we assign $(2, d)$ & $(4, b)$
Using the algorithm we get a unique matching of

$$\{(1, c), (2, d), (3, a), (4, b)\}$$

<u>Question 3.</u> Is the one-sided matching algorithm strategy proof?

A one-sided matching algorithm is a mechanism used to match agents in a market where one side of the market (for example, students) have preferences over the other side (for example, courses), but the other side does not have preferences or constraints. In such a market, it is important to design a mechanism that is incentive compatible, meaning that agents have no incentive to lie about their preferences or take actions that would lead to a more favorable outcome for themselves.

The question of whether a one-sided matching algorithm is strategy-proof depends on the specific algorithm being used. There are many different algorithms that can be used for one-sided matching, and some are strategy-proof while others are not.

One-sided matching algorithms that are not strategy-proof include the Boston mechanism and the Top Trading Cycles algorithm. These algorithms allow agents to manipulate the market by misreporting their preferences. For example, a student may lie about their true preferences in order to get assigned to a more popular course, or a course may strategically rank students lower in order to increase its bargaining power.

On the other hand, the Deferred Acceptance Algorithm (DAA) is a one-sided matching algorithm that is strategy-proof. The DAA works by having one side (in our example, the students) propose to their most preferred option on the other side (the courses). The courses then accept or reject each proposal based on their own preferences. Any rejected students are added to a temporary pool of unmatched students, who then propose to their next preferred course. This process continues until all students are matched.

The DAA is strategy-proof because it is always optimal for students to truthfully report their preferences. Specifically, no student can benefit from misreporting their preferences, since any untruthful proposal will be rejected by the course and the student will end up in the temporary pool of unmatched students. Additionally, the DAA ensures that the matching is stable, meaning that no unmatched student and course both prefer each other over their current matches.

Other strategy-proof one-sided matching algorithms include the Immediate Acceptance Algorithm (IAA), the Random Serial Dictatorship (RSD) algorithm, and the Probabilistic Serial algorithm.

In conclusion, the question of whether a one-sided matching algorithm is strategy-proof depends on the specific algorithm being used. The Deferred Acceptance Algorithm and other strategy-proof algorithms ensure that agents have no incentive to lie about their preferences or take actions that would lead to a more favorable outcome for themselves.