Economic of Networks/Microeconomics II
Exam 3-Spring 2023

NAME:

QUESTION 1 (40 Points)
In class we discuss the assignment game (P,Q,α). Accordingly, show the following:

1. Show that if x is an optimal assignment, then it is compatible with any stable payoff (u,v). (10 Points)

2. If $((u,v),x)$ is a stable outcome, then x is an optimal assignment. (10 Points)

3. Let $((u,v),x)$ and $((u',v'),x')$ be stable outcomes of the assignment game (P,Q,α). Show that if $x'_{ij}=1$ and $u'_i > u_i$ implies $v'_j < v_j$. (20 Points)

QUESTION 2 (60 Points)
In class we discuss the assignment game (P,Q,α). In this question we interpret P as a set of bidders and Q as a set of objects. Each object has a reservation price of c_j. The value of object j to bidder i is $\alpha_{ij} \geq 0$.

A feasible price vector p is a function from Q to $\mathbb{R}_+$ such that $p(j) = p_j$ satisfies $p_j \geq c_j$. We will assume that Q contains a null object O whose value is $\alpha_{iO} = 0$ for all bidders and whose price p_O is always zero. Then if a bidder is unmatched we will say that she or he is assigned the null object (note that more than one bidder may be assigned to the null object).

1. For a given price vector p, define the demand set $D_i(p)$ for bidder i. (15 Points)

2. A price vector p is called *quasi-competitive* if there is a matching μ from P to Q such that $\mu(i) = j$ then j is in $D_i(p)$, and if i is unmatched under μ then O is in $D_i(p)$. In other words, at a quasi-competitive prices p each buyer can be assigned to an object in his or her demand set. In this case, μ is said to be compatible with p. The pair (p,μ) is a *competitive equilibrium* if p is quasi-competitive, μ is compatible with p, and $p_j = c_j$ for all $j \notin \mu(P)$. In this case we denote (p,μ) is a competitive equilibrium and p is called an equilibrium price vector.

   Show that if (p,μ) is a competitive equilibrium then the corresponding payoffs are stable. (15 Points).

3. Describe an algorithm to compute an equilibrium price vector. (20 points)

4. In your previous answer: does the algorithm generate an equilibrium price vector that maximizes total happiness? (10 Points)