Economics of networks/Micro Theory II
Final Exam-Spring 2023
Due Date: April 30th by 11:59pm

NAME:

QUESTION 1 (20 Points)
Based on the paper *An Invitation to Market Design*, by Kominers et al. (2017), describe an application of how market can be used to solve real world problems.[1]

QUESTION 2 (80 Points)

1. Consider a course allocation problem where $S = \{s_1, s_2, s_3, s_4\}$ and $C = \{c_1, c_2, c_3, c_4\}$. Assume that $q_c = 1$ for all $c \in C$. The course priorities are described by:

$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$	$\succ_{c_4}$
s_4	s_2	s_3	s_1
s_1	s_3	s_4	$\vdots$
s_2	$\vdots$	$\vdots$	$\vdots$

Students preferences are represented by:

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
c_1	c_1	c_2	c_3
c_4	$\underline{c_2}$	c_3	c_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$

   (a) Find the SOSM. (20 Points)

   (b) Describe all the steps used in the algorithm used in part (a). (10 Points)

   (c) Now, suppose that a college administrator observe the course priorities and students preferences. The administrator argues it is possible to achieve efficiency, strategy proofnes, and stability all at the same time. (10 Points)

2. Apart from two-sided matching markets, one-sided matching problems (aka assignment problems), where only one side of the market has preferences, can also often be found. In course assignment problems students often need to be assigned to tutor groups at different times of the week. The material discussed in these tutor groups is identical within a week and tutors do not have preferences, but students do have preferences for different times in their weekly schedule. This is an example of a one-sided one-to-many matching problem. In order to solve this type of one-sided problem, we can implement the following algorithm:
   STEP 0: Each agent i reports her preferences $\succ_i$ on objects. All agents are marked active.

[1] Your example must be different from the ones discussed in classes.

STEP 1: Each agent i points to her most preferred object (possibly her own); each object points back to its owner.
STEP 2: This creates a directed graph. In this graph, cycles are identified. Each agent has an out-degree (i.e., the number of directed edges departing from a vertex) equal to 1 , so that there exists at least one directed cycle in the graph. With strict preferences each agent is in at most one cycle.
STEP 3: Each agent in a cycle is given the object she points at, and she is then removed from the market with her assigned house.
STEP 4: If unmatched agents remain, jump to step 1 and iterate.

(a) Consider a set of agents $A = \{1, 2, 3, 4\}$, a set of objects $O = \{a, b, c, d\}$, and the preference profile

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
c	d	a	c
b	a	d	b
d	b	c	a
a	c	b	d

Applying the algorithm described above shows that the unique matching is $\mu = \{(1, c), (2, d), (3, a), (4, b)\}$. (20 Points)

3. Is the one-sided matching algorithm strategy proof? (20 Points).