Lecture 5: Applications

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Problem 1

- A shortest path between two nodes is a path of the minimum possible length. We say that a node X is pivotal for a pair of distinct nodes Y and Z if X lies on every shortest path between Y and Z (and X is not equal to either Y or Z).

- For example, in the graph in Figure 2.13, node B is pivotal for two pairs: the pair consisting of A and C, and the pair consisting of A and D. (Notice that B is not pivotal for the pair consisting of D and E since there are two different shortest paths connecting D and E, one of which (using C and F) doesn't pass through B. So B is not on every shortest path between D and E.) On the other hand, node D is not pivotal for any pairs.

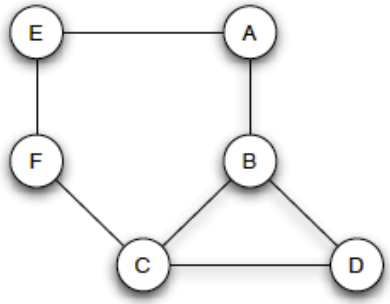

Figure 2.13: In this example, node B is pivotal for two pairs: the pair consisting of A and C, and the pair consisting of A and D. On the other hand, node D is not pivotal for any pairs.

- Give an example of a graph in which every node is pivotal for at least one pair of nodes.
- Give an example of a graph having at least four nodes in which there is a single node X that is pivotal for every pair of nodes (not counting pairs that include X).

Problem 2

- Consider the graph in Figure 3.21, in which each edge-except the edge connecting b and c-is labeled as a strong tie (S) or a weak tie (W).
- According to the theory of strong and weak ties, with the strong triadic closure assumption, how would you expect the edge connecting b and c to be labeled? Give a brief (1-3 sentence) explanation for your answer.

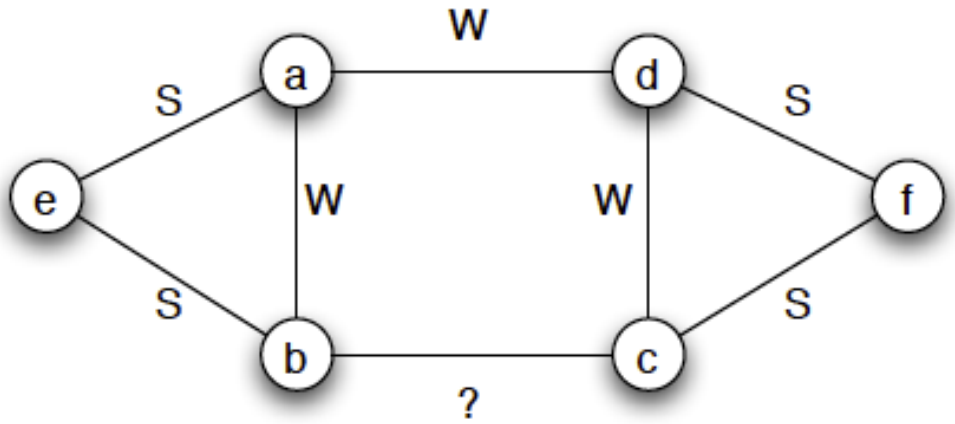

Figure 3.21:

Problem 3

- In the social network depicted in Figure 3.22, with each edge labeled as either a strong or weak tie, which nodes satisfy the Strong Triadic Closure Property from Chapter 3, and which do not? Provide an explanation for your answer.

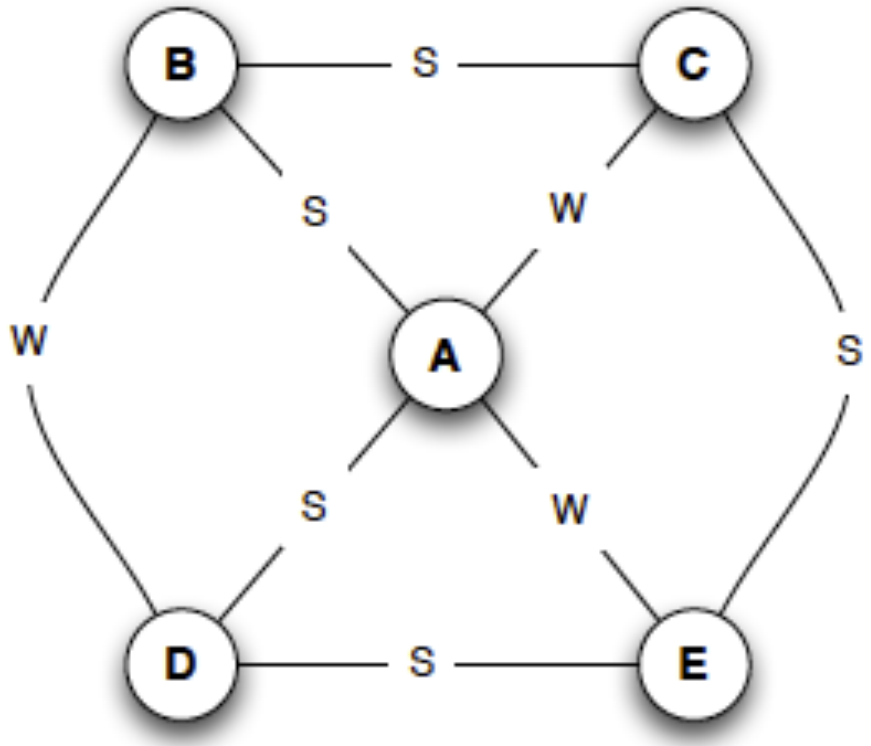

Figure 3.22:

Problem 4

- In the social network depicted in Figure 3.23 with each edge labeled as either a strong or weak tie, which two nodes violate the Strong Triadic Closure Property? Provide an explanation for your answer.

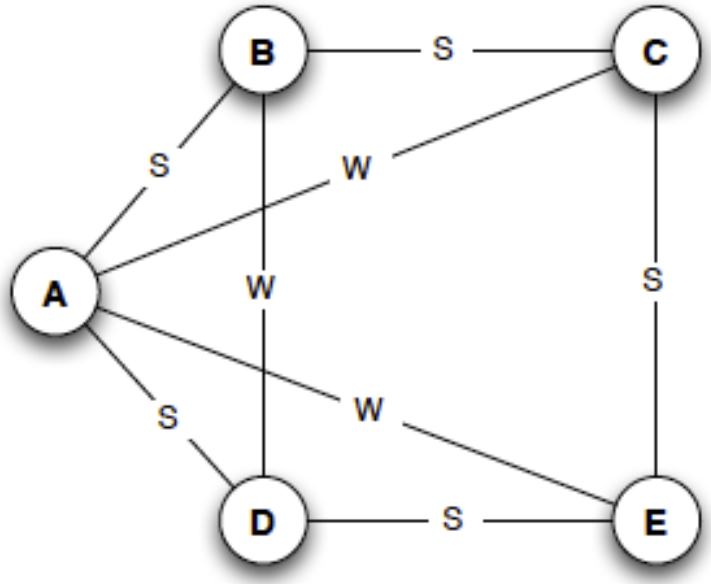

Figure 3.23: A graph with a strong/weak labeling.

Problem 5

- In the social network depicted in Figure 3.24, with each edge labeled as either a strong or weak tie, which nodes satisfy the Strong Triadic Closure Property from Chapter 3, and which do not? Provide an explanation for your answer.

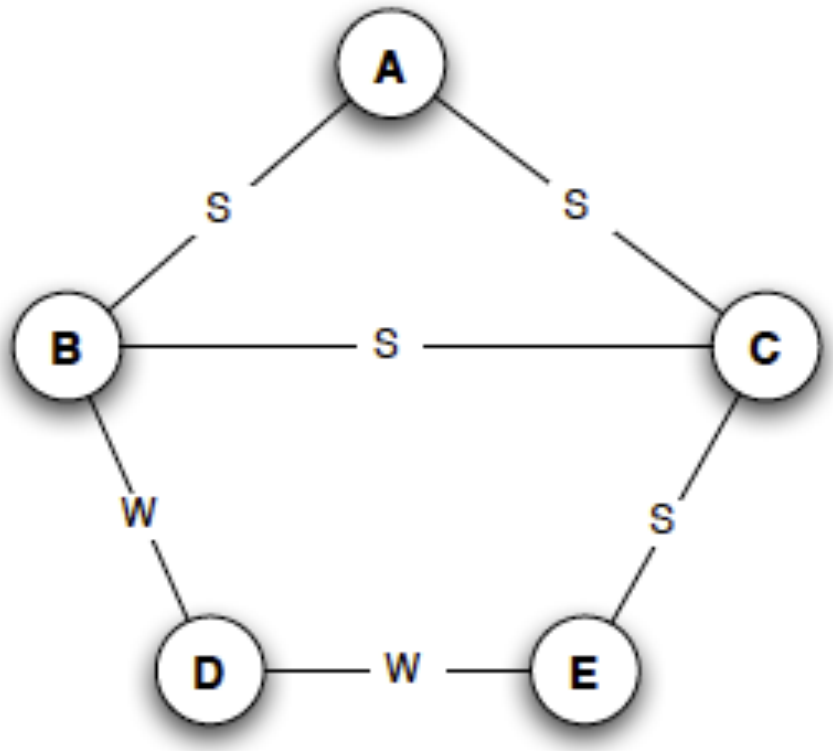

Figure 3.24: