Auctions

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- An auction is a kind of economic activity that has been brought into many people's everyday lives by the Internet, through sites such as eBay
- But auctions also have a long history that spans many different domains. For example, the U.S. government uses auctions to sell Treasury bills and timber and oil leases
- Christie's and Sotheby's use them to sell art
- Morrell & Co. and the Chicago Wine Company use them to sell wine.

- The underlying assumption we make when modeling auctions is that each bidder has an intrinsic value for the item being auctioned
- The buyer is willing to purchase the item for a price up to this value, but not for any higher price
- We refer to this intrinsic value as the bidder's true value for the item

There are four main types of auctions when a single item is being sold

- Ascending-bid auctions, also called English auctions
- Descending-bid auctions, also called Dutch auctions
- First-price sealed-bid auctions
- Second-price sealed-bid auctions, also called Vickrey auctions

- These auctions are carried out interactively in real time, with bidders present either physically or electronically
- The seller gradually raises the price, bidders drop out until finally only one bidder remains, and that bidder wins the object at this final price
- Oral auctions in which bidders shout out prices, or submit them electronically, are forms of ascending-bid auctions

- This is also an interactive auction format, in which the seller gradually lowers the price from some high initial value until the first moment when some bidder accepts and pays the current price
- These auctions are called Dutch auctions because flowers have long been sold in the Netherlands using this procedure

- In this kind of auction, bidders submit simultaneous "*sealed bids*" to the seller
- The terminology comes from the original format for such auctions, in which bids were written down and provided in sealed envelopes to the seller, who would then open them all together
- The highest bidder wins the object and pays the value of her bid

- Bidders submit simultaneous sealed bids to the sellers
- The highest bidder wins the object and pays the value of the second-highest bid
- These auctions are called Vickrey auctions in honor of William Vickrey, who wrote the first game-theoretic analysis of auctions (including the second-price auction)
- Vickery won the Nobel Memorial Prize in Economics in 1996 for this body of work

- First, consider a descending-bid auction.
- Here, as the seller is lowering the price from its high initial starting point, no bidder says anything until finally someone actually accepts the bid and pays the current price
- Bidders therefore learn nothing while the auction is running, other than the fact that no one has yet accepted the current price.
- For each bidder i, there's a first price b_i at which she will be willing to break the silence and accept the item at price b_i

So with this view, the process is equivalent to a sealed-bid first-price auction: this price b_i plays the role of bidder i's bid; the item goes to the bidder with the highest bid value; and this bidder pays the value of her bid in exchange for the item

- Now let's think about an ascending-bid auction, in which bidders gradually drop out as the seller steadily raises the price
- The winner of the auction is the last bidder remaining, and she pays the price at which the second-to-last bidder drops out
- Suppose that you're a bidder in such an auction; let's consider how long you should stay in the auction before dropping out
- First, does it ever make sense to stay in the auction after the price reaches your true value?

- No: by staying in, you either lose and get nothing, or else you win and have to pay more than your value for the item
- Second, does it ever make sense to drop out before the price reaches your true value for the item?

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- Second, does it ever make sense to drop out before the price reaches your true value for the item?
- Again, no: if you drop out early (before your true value is reached), then you get nothing, when by staying in you might win the item at a price below your true value

- So this informal argument indicates that you should stay in an ascending-bid auction up to the exact moment at which the price reaches your true value
- If we think of each bidder i's "drop-out price" as her bid β_i, this says that people should use their true values as their bids
- With this definition of bids, the rule for determining the outcome of an ascending bid auction can be reformulated as follows
- The person with the highest bid is the one who stays in the longest, thus winning the item, and she pays the price at which the second-to-last person dropped out-in other words, she pays the bid of this second-to-last person

- Auctions with common values introduce new sources of complexity
- To see this, let's start by supposing that an item with a common value is sold using a second-price auction
- Each bidder i may have some private information about the common value, leading to a signal v_i of this value
- Individual bidder estimates will typically be slightly wrong, and they will also typically not be independent of each other
- One possible model for such estimates is to suppose that the true value is v, and that each bidder i's signal v_i is defined by $v_i = v + x_i$, where x_i is a random number with a mean of 0, representing the error in i's signal

- Is it still optimal for bidder i to bid v_i?
- In fact, it is not
- To get a sense for why this is, we can use the model with random errors $v + x_i$
- Suppose there are many bidders, and that each bids her estimate of the true value
- Then from the result of the auction, the winning bidder not only receives the object, she also learns something about her estimate of the common value- that it was the highest of all the estimates

- So in particular, her estimate is more likely to be an *over-estimate* of the common value than an *under-estimate*
- Moreover, with many bidders, the second-place bid- which is what she paid- is also likely to be an over-estimate
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- This is known as the winner's curse, and it is a phenomenon that has a rich history in the study of auctions