ECON 390
Exam 2-Spring 2023

NAME:
DUE DATE: MARCH 8th BY MIDNIGHT

QUESTION 1 (50 Points)
Consider a complete-information game where each player (also called agent) $i \in N = \{1, 2, \ldots, n\}$ simultaneously selects a real-valued action $a_i \geq 0$ and receives a real-valued payoff $u_i(a_1, a_2, \ldots, a_n)$ that depends on everyone's action. Suppose that each agent i 's best-response function is given by

$$\mathrm{BR}_i(\mathbf{a}_{-i}) = \alpha \sum_j w_{ij} a_j + b_i, \tag{1}$$

Here $\alpha > 0$, $\mathbf{W} = (w_{ij})_{i,j \in N}$, and $(b_i)_{i \in N}$ are constants-parameters of the model that do not depend on $\mathbf{a} = (a_i)_{i \in N}$. The matrix $\mathbf{W}$ is irreducible, with $W_{ii} = 0$ for every i, and all its entries are nonnegative. All the b_i are positive.[1]

1. The previous game represents strategic of complements or substitutes? (10 Points)
2. Characterize the unique equilibrium of the game. (10 Points)
3. How is your answer to question 2 related to the network topology? (10 Points)
4. Now, assume that $\alpha < 0$. How does your answer to question 1 change? (10 Points)
5. Assume $\alpha > 0$ and suppose that you want to measure the gap between a socially optimal outcome and the Nash equilibrium of the game. Could you provide a measure to quantify inefficiency? (10 Points)

[1] We call a matrix irreducible if the corresponding weighted, directed graph is strongly connected. A 1-by-1 nonnegative matrix is said to be irreducible if its sole entry is positive.

QUESTION 2 (50 Points)

There are 1000 cars which must travel from town A to town B. There are two possible routes that each car can take: the upper route through town C or the lower route through town D. Let x be the number of cars traveling on the edge AC and let y be the number of cars traveling on the edge DB. The directed graph in Figure 8.8 indicates that travel time per car on edge AC is $x/100$ if x cars use edge AC, and similarly the travel time per car on edge DB is $y/100$ if y cars use edge DB. The travel time per car on each of edges CB and AD is 12 regardless of the number of cars on these edges. Each driver wants to select a route to minimize his travel time. The drivers make simultaneous choices.

1. Find Nash equilibrium values of x and y. (20 Points)

2. Now the government builds a new (one-way) road from town C to town D. The new road adds the path $ACDB$ to the network. This new road from C to D has a travel time of 0 per car regardless of the number of cars that use it. Find a Nash equilibrium for the game played on the new network. What are the equilibrium values of x and y ? What happens to total cost-of-travel (the sum of total travel times for the 1000 cars) as a result of the availability of the new road? (15 Points)

3. Suppose now that conditions on edges CB and AD are improved so that the travel times on each edge are reduced to 5. The road from C to D that was constructed in part 2 is still available. Find a Nash equilibrium for the game played on the network with the smaller travel times for CB and AD. What are the equilibrium values of x and y ? What is the total cost-of-travel? What would happen to the total cost-of-travel if the government closed the road from C to D ? (15 Points)

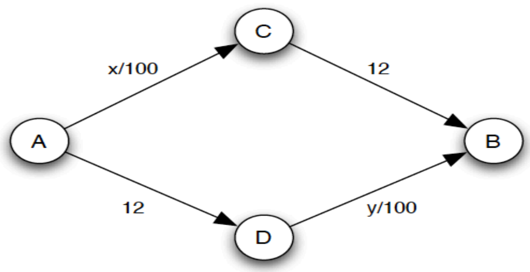

Figure 8.8: Traffic Network.