TUTORIAL - IV

Q1. $T(n) = 3T(n/2) + n^2$

Ans. Here, $a = 3$, $b = 2$

$f(n) = n^2$

So, $n^{\log_b a} = n^{\log_2 3}$

Since, $n^{\log_2 3} < n^2$

So, According to master's theorem

$T(n) = \Theta(n^2)$.

Q2. $T(n) = 4T(n/2) + n^2$

Ans. Here $a = 4$, $b = 2$ and $f(n) = n^2$

So, $n^{\log_b a} = n^{\log_2 4} = n^{2\log_2 2} = n^2$

Since $n^{\log_b a} = f(n)$

According to Master's theorem

$T(n) = \Theta(n^2 \log n)$

Q3. $T(n) = T(n/2) + 2^n$

Ans. Here $a = 1$, $b = 2$ and $f(n) = 2^n$

So, $n^{\log_b a} = n^{\log_2 1} = n^{\log_2 2^0} = n^0 = 1$

Since, $1 < f(n)$

$\therefore$ According to Master's theorem

$T(n) = \Theta(2^n)$

Q4. $T(n) = 2^n T(n/2) + n^2$

Ans. Master's theorem is not applicable since a is a function

Q5. $T(n) = 16T(n/4) + n$

Ans. Here, $a = 16$, $b = 4$ & $f(n) = n$

So, $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^{2\log_4 4} = n^2$

Since $n^2 > n$

Therefore according to Master's theorem:

$T(n) = \Theta(n^2)$

Q6. $T(n) = 2T(n/2) + n\log n$

Ans. Here $a = 2$, $b = 2$ & $f(n) = n\log n$

So, $n^{\log_b a} = n^{\log_2 2} = n$

Since $n^{\log_b a} < f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n\log n)$

Q7. $T(n) = 2T(n/2) + n/\log n$

Ans. Here $a = 2$, $b = 2$ & $f(n) = n/\log n$

So, $n^{\log_b a} = n^{\log_2 2} = n$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n)$

Q8. $T(n) = 2T(n/4) + n^{0.51}$

Ans. Here $a = 2$, $b = 4$ and $f(n) = n^{0.51}$

So, $n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

Since $n^{\log_b a} < f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n^{0.51})$

Q9. $T(n) = 0.5T(n/2) + 1/n$

Ans. Master's theorem not applicable since $a < 1$

Q10. $T(n) = 16T(n/4) + n!$

Ans. Here $a = 16$, $b = 4$ & $f(n) = n!$

So, $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^2$

Since, $n^{\log_b a} < n!$

$\therefore$ According to Master's theorem

$T(n) = O(n!)$

Q11. $T(n) = 4T(n/2) + \log n$

Ans. Here $a = 4, b = 2$ & $f(n) = \log n$

So, $n^{\log_b a} = n^{\log_2 4} = n^2$

Since $n^{\log_b a} > f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n^2)$

Q12. $T(n) = sqrt(n) T(n/2) + \log n$

Ans. Since $a \neq$ constant

$\therefore$ Master's theorem not applicable

Q13. $T(n) = 3T(n/2) + n$

Ans. Here $a = 3, b = 2$ and $f(n) = n$

So, $n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

Since $n^{\log_b a} > f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n^{1.58})$

Q14. $T(n) = 3T(n/3) + \sqrt{n}$

Ans. Here $a = 3, b = 3$ and $f(n) = \sqrt{n}$

So, $n^{\log_b a} = n^{\log_3 3} = n$

Since $n^{\log_b a} > f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n)$

Q15. $T(n) = 4T(n/2) + cn$

Ans. Here $a = 4, b = 2$ and $f(n) = n$

So, $n^{\log_b a} = n^{\log_2 4} = n^2$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n^2)$

Q16. $T(n) = 3T(n/4) + n\log n$

Ans. Here $a=3, b=4$ & $f(n) = n\log n$

So, $n^{\log_b a} = n^{\log_4 3} = n^{0.787}$

Since, $n^{\log_b a} < f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n\log n)$

Q17. $T(n) = 3T(n/3) + n/2$

Ans. Here $a=3, b=3$ & $f(n) = n/2$

So, $n^{\log_b a} = n^{\log_3 3} = n$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n)$

Q18. $T(n) = 6T(n/3) + n^2\log n$

Ans. Here $a=6, b=3$ & $f(n) = n^2\log n$

So, $n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

Since, $n^{\log_b a} < n^2\log n$

$\therefore$ According to Master's theorem

$T(n) = O(n^2\log n)$

Q19. $T(n) = 4T(n/2) + n\log n$

Ans. Here $a=4, b=2$ & $f(n) = n\log n$

So, $n^{\log_b a} = n^{\log_2 4} = n^2$

Since, $n^{\log_b a} > f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n^2)$

Q20. $T(n) = 64T(n/8) - n^2\log n$

Ans. Here, $f(n)$ is not an increasing function

$\therefore$ Master's theorem is not applicable

Q21. $T(n) = 7T(n/3) + n^2$

Ans. Here $a=7$, $b=3$ & $f(n) = n^2$

So, $n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

Since, $n^{\log_b a} < f(n)$

$\therefore$ According to Master's theorem

$T(n) = O(n^2)$

Q22. $T(n) = T(n/2) + n(2-\cos n)$

Ans22. Here, Master's theorem is not applicable due to violation of regularity condition.

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