```
%% Computer Lab 2 Chapter 2 #: 1, 2, 3, 5b, 5d, 6b, 6d, 9
%The "Example 1", "Example 2", and "Example 3" referred to in #1,2,3 are on pp. 115-118 of the
% On #6, you may use RK4 or ode45 and choose three initial conditions of your choice.  On #9, p
% (as shown in the class lecture on 2.5 Numerical Methods) that compares the outputs;
% you must use Euler.m and RK4.m that you wrote for the previous problems.
```

```
%% Chapter 2 problems
fprintf('Chapter 2 \n')
```

```
Chapter 2
```

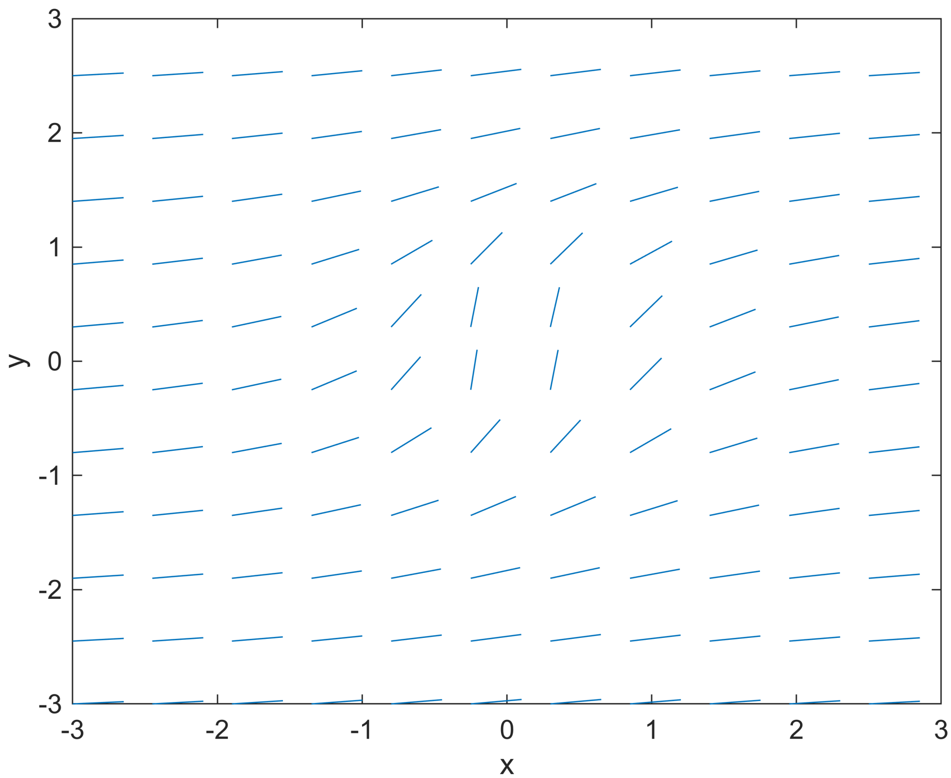
```
```

1

```
fprintf('Problem 1 \n')
```

```
Problem 1
```

```
clear all;
[X,Y]=meshgrid(-3:.55:3,-3:.55:3); %Note the CAPITAL letters
DY=1./(X.^2+Y.^2); %DY is rhs of original equation
DX=ones(size(DY));
DW=sqrt(DX.^2+DY.^2); %the length of each vector
quiver(X,Y,DX./DW,DY./DW,.5,'.'); %plots normalized vectors
xlabel('x');
ylabel('y');
```

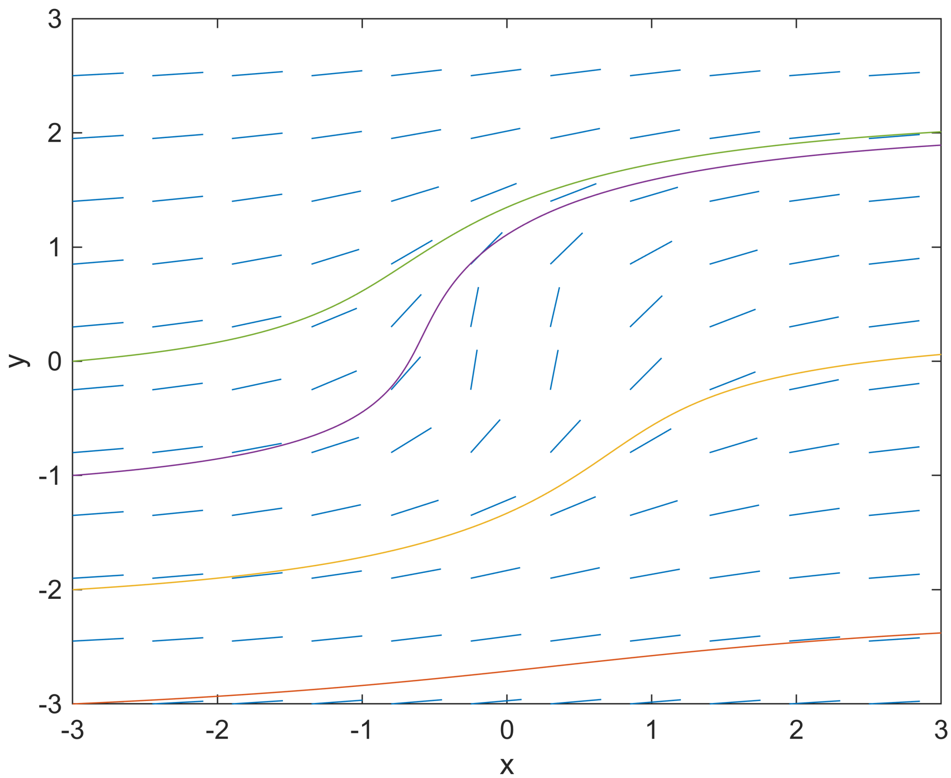


2

```
fprintf('Problem 2 \n')
```

```
Problem 2
```

```
x0=-3; xf=3; y0=-3;
options=odeset('refine',10,'AbsTol',1e-15);
[x,y]=ode45(@Ch2Example,[x0,xf],y0,options);
x0=-3; xf=3; y0=-2;
[x1,y1]=ode45(@Ch2Example,[x0,xf],y0,options);
x0=-3; xf=3; y0=-1;
[x2,y2]=ode45(@Ch2Example,[x0,xf],y0,options);
x0=-3; xf=3; y0=0;
[x3,y3]=ode45(@Ch2Example,[x0,xf],y0,options);
hold on %keeps direction field plot open
plot(x,y) %superimpose solutions
axis([-3 3 -3 3])
plot(x1,y1)
plot(x2,y2)
plot(x3,y3)
hold off
```



3

```
fprintf('Problem 3 \n')
```

```
Problem 3
```

```
clear all
close all
x0=0; xf=3.1; y0=1; h=.1;
[x,y]=Euler(@Ch2NumExample,[x0, xf],y0,h);
```

```
dy =
     0
dy =
       0.110363832351433
dy =
       0.218305022796984
dy =
       0.320386435389078
dy =
       0.413712507686657
dy =
       0.496182403080285
dy =
        0.56659622439268
dy =
       0.624616574788414
```

```
dy =
    0.67062339244843
dy =
   0.705515268177255
dy =
   0.730505968463867
dy =
   0.746949055862323
dy =
   0.756205720253907
dy =
   0.759557169346483
dy =
   0.758154997377913
dy =
   0.752999993009861
dy =
   0.744940046283041
dy =
   0.734679505767135
dy =
   0.722794407057919
dy =
   0.709749862320008
dy =
   0.695917354343739
dy =
    0.68159070491022
dy =
   0.667000158404602
dy =
   0.652324429961379
dy =
   0.637700796000314
dy =
   0.623233418076801
dy =
   0.609000134048658
dy =
   0.595057953950923
dy =
   0.581447480836889
dy =
    0.56819645074062
dy =
   0.555322557442728
```

```
[x y]
```

```
ans = 32×2
                          0                         1
                        0.1                         1
                        0.2          1.01103638323514
                        0.3          1.03286688551484
                        0.4          1.06490552905375
                        0.5          1.10627677982241
                        0.6          1.15589502013044
                        0.7          1.21255464256971
                        0.8          1.27501630004855
                        0.9           1.3420786392934
      ⋮
```

```
[x1,y1]=RK4(@Ch2NumExample,[x0 xf],y0,h);
```

```
dy =
     0
dy =
        0.0551819161757164
dy =
        0.0550298738282303
dy =
          0.10975816958021
dy =
         0.109758164994845
dy =
         0.163736208055812
dy =
          0.16329489584847
dy =
         0.215960848966649
dy =
         0.215960806326338
dy =
          0.26705174737768
dy =
         0.266370421733253
dy =
         0.315426283513581
dy =
         0.315426236737642
dy =
         0.362239003509169
dy =
          0.36139212451519
dy =
         0.405640669275633
dy =
         0.405640828726448
dy =
         0.447183535351887
dy =
         0.446255638648175
dy =
         0.484921051464039
dy =
          0.48492180797136
dy =
         0.520636315255032
dy =
         0.519707431393938
dy =
         0.552436486883488
dy =
         0.552438317294688
dy =
         0.582170043524813
dy =
         0.581305240468597
dy =
         0.608113741943226
dy =
          0.60811707457364
dy =
         0.632041109085162
dy =
         0.631285512432947
```

```
dy =
        0.652473327962298
dy =
        0.652478432814243
dy =
        0.671006474306501
dy =
        0.670385140363589
dy =
        0.686442598577256
dy =
         0.68644953833478
dy =
        0.700138001880765
dy =
        0.699658975152167
dy =
        0.711181260657514
dy =
        0.711189902722213
dy =
        0.720661943489736
dy =
        0.720320717333399
dy =
        0.727939248351342
dy =
        0.727949316200274
dy =
        0.733836111192877
dy =
        0.733620145840792
dy =
        0.737953480141151
dy =
        0.737964617968111
dy =
        0.740865904950134
dy =
        0.740758439514751
dy =
        0.742381115491112
dy =
        0.742392946847644
dy =
        0.742853878808345
dy =
        0.742836758750881
dy =
        0.742262517102998
dy =
        0.742274688000072
dy =
        0.740775017830785
dy =
         0.74083056582317
dy =
        0.738505972314031
dy =
        0.738518177054638
dy =
        0.735469910631506
dy =
        0.735582014586047
```

```
dy =
        0.731886930007409
dy =
        0.731898922439185
dy =
        0.727649250166382
dy =
        0.727803880136106
dy =
        0.723055985482835
dy =
        0.723067579926623
dy =
        0.717904712045072
dy =
        0.718090058325993
dy =
        0.712551433811034
dy =
        0.712562499767003
dy =
        0.706722776438722
dy =
        0.706929159842014
dy =
        0.700813590697719
dy =
        0.700824044531996
dy =
        0.694499285454127
dy =
        0.694718947218002
dy =
        0.688199143505235
dy =
        0.688208939286017
dy =
        0.681553429745799
dy =
        0.681780271755127
dy =
        0.674994551943565
dy =
        0.675003672714938
dy =
        0.668140476567974
dy =
        0.668369794869357
dy =
        0.661428022469466
dy =
          0.6614364726099
dy =
        0.654462950101984
dy =
        0.654691185495444
dy =
        0.647679896621494
dy =
        0.647687695517517
dy =
        0.640680215883264
dy =
         0.64090473289167
```

```
dy =
        0.633891478320276
dy =
        0.633898655343215
dy =
        0.626916555518669
dy =
        0.627135453424236
dy =
        0.620172423144134
dy =
         0.62017901379462
dy =
        0.613267883709314
dy =
        0.613479839034548
dy =
        0.606606856005583
dy =
        0.606612899042257
dy =
        0.599807283551412
dy =
        0.600011421168519
dy =
        0.593258394590105
dy =
        0.593263929953449
dy =
        0.586589535979294
dy =
        0.586785325130037
dy =
        0.580174248834421
dy =
        0.580179316192947
dy =
        0.573654806434465
dy =
        0.573841977781966
dy =
        0.567388550702289
dy =
        0.567393188473588
dy =
        0.561031633387632
dy =
        0.561210113453435
dy =
        0.554925049026633
dy =
        0.554929293752784
dy =
        0.548739343302254
dy =
        0.548909203053749
dy =
         0.54279928434245
dy =
        0.542803170324425
```

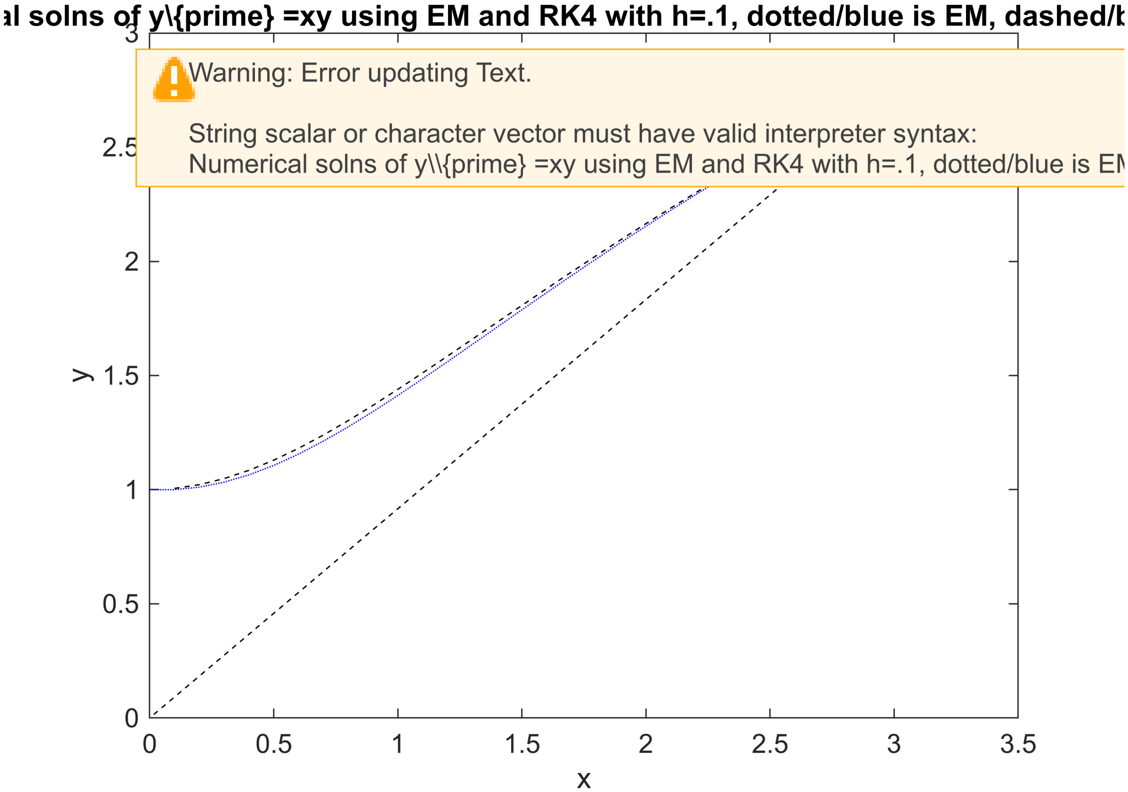
```
[x1 y1]
```

```
ans = 32×2
                   0.1     1.0055030291598
                   0.2     1.0218327161893
                   0.3      1.048469906657
                   0.4     1.0846087260247
                   0.5    1.12923272982788
                   0.6    1.18120015963042
                   0.7     1.2393252034175
                   0.8     1.3024459308437
                   0.9    1.36947433518923
                     1    1.43942808107353
      ⋮
```

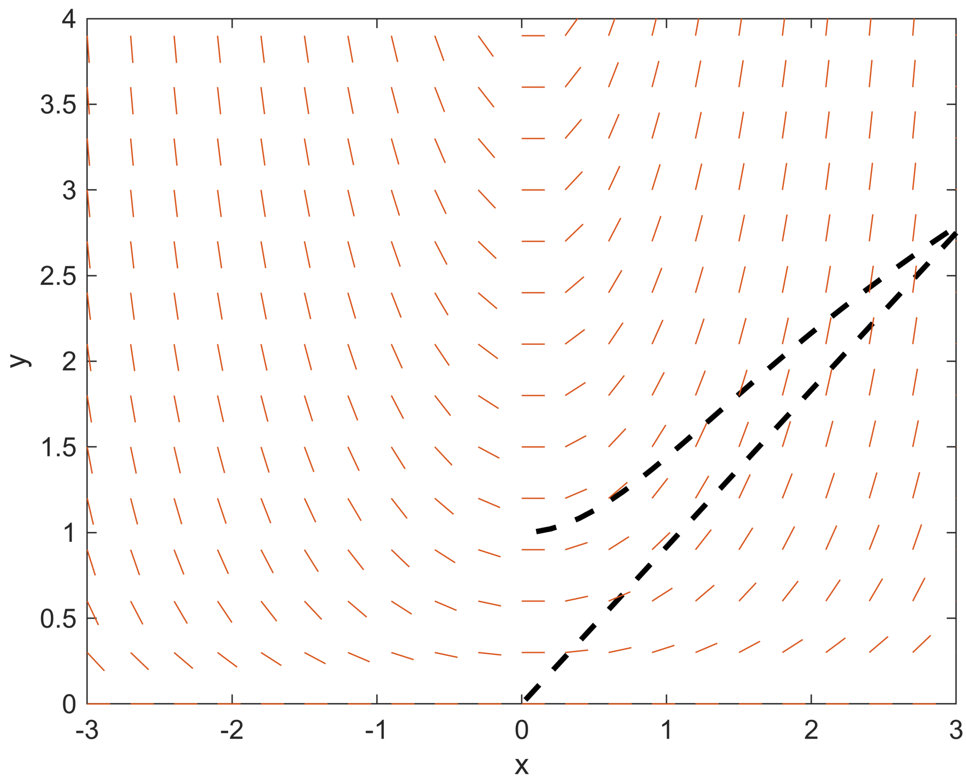
```
x2=x0:h:xf;
y2= exp(x2.^2/2); %this is the analytical solution
[x2' y2']
```

```
ans = 32×2
                     0                   1
                   0.1     1.0050125208594
                   0.2    1.02020134002676
                   0.3    1.04602785990872
                   0.4    1.08328706767496
                   0.5    1.13314845306683
                   0.6    1.19721736312181
                   0.7    1.27762131320489
                   0.8    1.37712776433596
                   0.9    1.49930250005677
      ⋮
```

```
plot(x,y,'b:') %Now graphically compare Euler with RK4
hold on
plot(x1,y1,'k--')
xlabel('x')
ylabel('y')
title('Numerical solns of y\{prime} =xy using EM and RK4 with h=.1, dotted/blue is EM, dashed/b
hold off
```




```
plot(x1,y1,'k--','LineWidth',2)
hold on
[X,Y]=meshgrid(-3:.3:3,0:.3:4);
DY=X.*Y; %DY is rhs of original equation
DX=ones(size(DY));
DW=sqrt(DX.^2+DY.^2);
quiver(X,Y,DX./DW,DY./DW,.4,'.');
xlabel('x');
ylabel('y');
axis([-3 3 0 4])
hold off
```



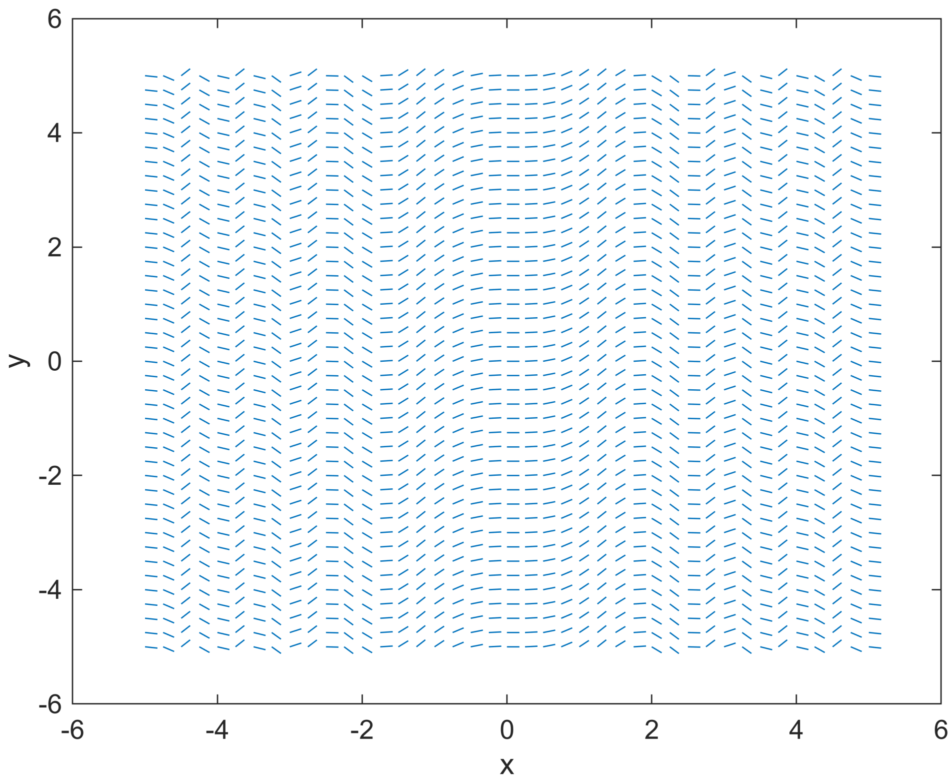

```
%End of Example 3.
```

5b

```
fprintf('Problem 5b \n')
```

```
Problem 5b
```

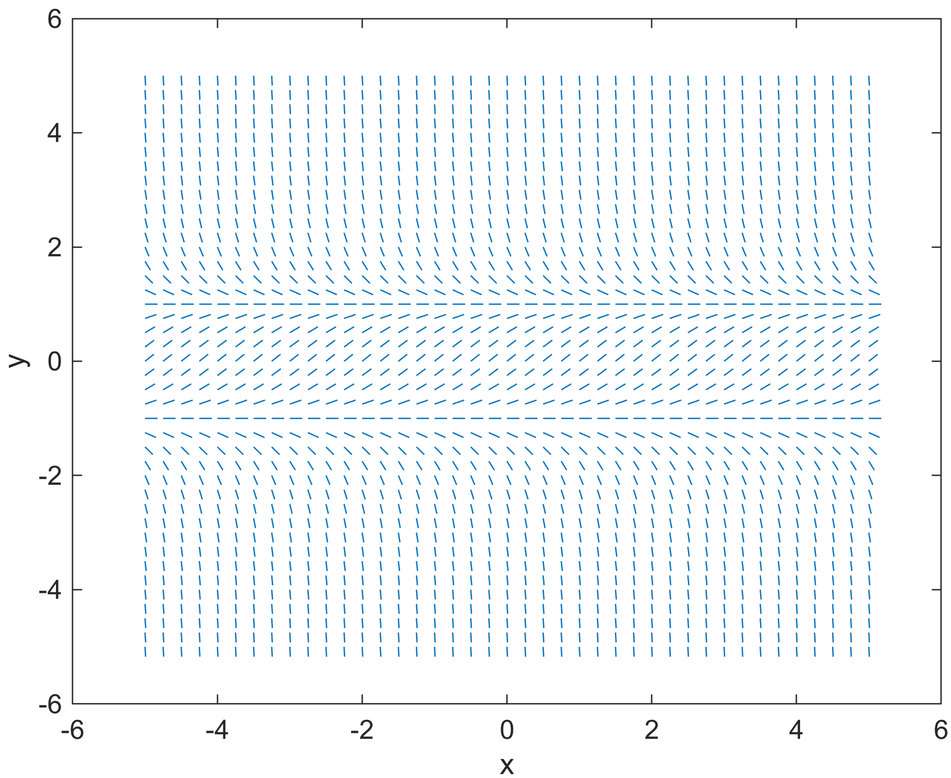
```
clear all;
[X,Y]=meshgrid(-5:.25:5,-5:.25:5); %Note the CAPITAL letters
DY=sin(X.^2); %DY is rhs of original equation
DX=ones(size(DY));
DW=sqrt(DX.^2+DY.^2); %the length of each vector
quiver(X,Y,DX./DW,DY./DW,.5,'.'); %plots normalized vectors
xlabel('x');
ylabel('y');
```




```
%% 5d
fprintf('Problem 5d \n')
```

```
Problem 5d
```

```
clear all;
[X,Y]=meshgrid(-5:.25:5,-5:.25:5); %Note the CAPITAL letters
DY=1-Y.^2; %DY is rhs of original equation
DX=ones(size(DY));
DW=sqrt(DX.^2+DY.^2); %the length of each vector
quiver(X,Y,DX./DW,DY./DW,.5,'.'); %plots normalized vectors
xlabel('x');
ylabel('y');
```



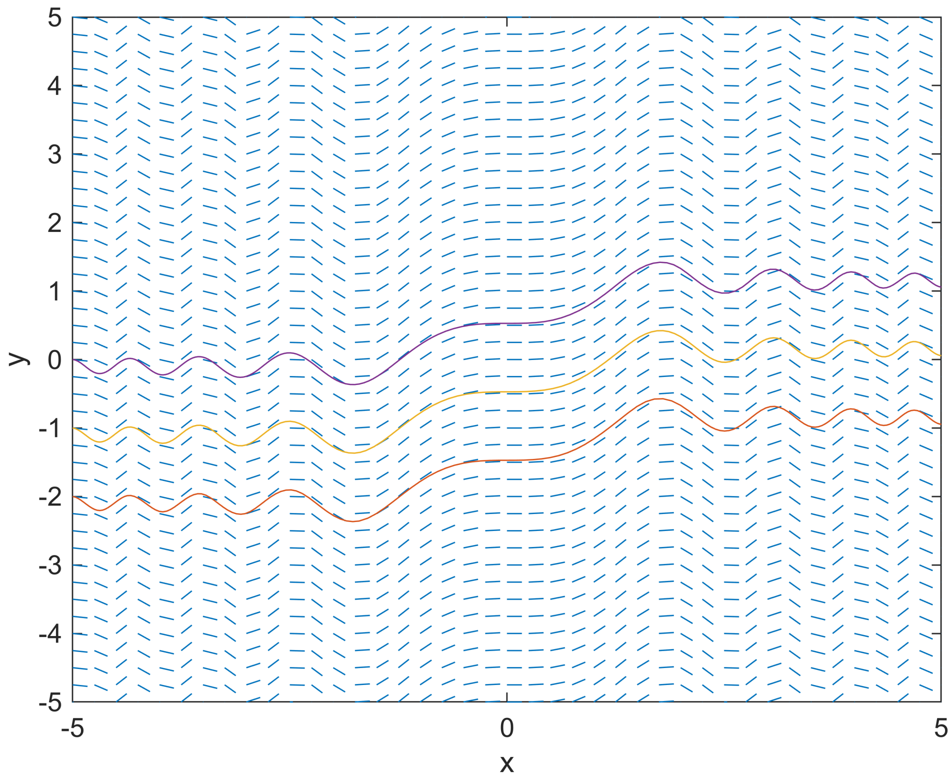
6b

```
fprintf('Problem 6b \n')
```

```
Problem 6b
```

```
clear all;
[X,Y]=meshgrid(-5:.25:5,-5:.25:5); %Note the CAPITAL letters
DY=sin(X.^2); %DY is rhs of original equation
DX=ones(size(DY));
DW=sqrt(DX.^2+DY.^2); %the length of each vector
quiver(X,Y,DX./DW,DY./DW,.5,'.'); %plots normalized vectors
xlabel('x');
ylabel('y');

options=odeset('refine',10,'AbsTol',1e-15);

x0=-5; xf=5; y0=-2;
[x1,y1]=ode45(@Ch5bNumExample,[x0,xf],y0,options);

x0=-5; xf=5; y0=-1;
[x2,y2]=ode45(@Ch5bNumExample,[x0,xf],y0,options);
```

```
x0=-5; xf=5; y0=0;
[x3,y3]=ode45(@Ch5bNumExample,[x0,xf],y0,options);

hold on %keeps direction field plot open

axis([-5 5 -5 5])
plot(x1,y1)%superimpose solutions
plot(x2,y2)
plot(x3,y3)
hold off
```



6d

```
fprintf('Problem 6d \n')
```

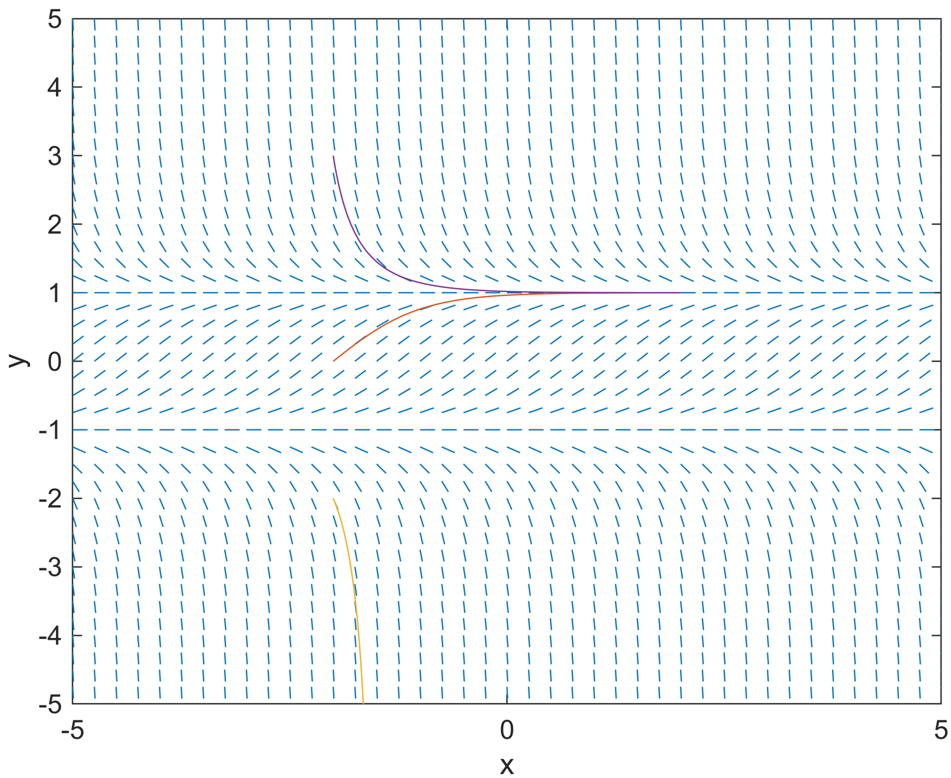
```
Problem 6d
```

```
clear all;
[X,Y]=meshgrid(-5:.25:5,-5:.25:5); %Note the CAPITAL letters
DY=1-Y.^2; %DY is rhs of original equation
DX=ones(size(DY));
DW=sqrt(DX.^2+DY.^2); %the length of each vector
```

```
quiver(X,Y,DX./DW,DY./DW,.5,'.'); %plots normalized vectors
xlabel('x');
ylabel('y');

options=odeset('refine',10,'AbsTol',1e-15);

x0=-2; xf=2; y0=0;
[x1,y1]=ode45(@Ch5dNumExample,[x0,xf],y0,options);

x0=-2; xf=2; y0=-2;
[x2,y2]=ode45(@Ch5dNumExample,[x0,xf],y0,options);
```

```
Warning: Failure at t=-1.450699e+00.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (3.552714e-15) at time t.
```

```
x0=-2; xf=2; y0=3;
[x3,y3]=ode45(@Ch5dNumExample,[x0,xf],y0,options);

hold on %keeps direction field plot open
%superimpose solutions
axis([-5 5 -5 5])
plot(x1,y1)
plot(x2,y2)
plot(x3,y3)
hold off
```



9

```
fprintf('Problem 9 \n')
```

```
Problem 9
```

```
clear all;
close all;
x0=0; xf=5; y0=-1; h=.1;
[x,y]=Euler(@Ch9NumExample,[x0, xf],y0,h);
[x y]
```

ans = 51×2

0	-1
0.1	-1.1
0.2	-1.209
0.3	-1.3259
0.4	-1.44949
0.5	-1.578439
0.6	-1.7112829
0.7	-1.84641119
0.8	-1.982052309
0.9	-2.1162575399

⋮

```
[x1,y1]=RK4(@Ch9NumExample,[x0 xf],y0,h);
[x1 y1]
```

ans = 51×2

0.1	-1.10482895833333
0.2	-1.21859699057205
0.3	-1.34014080990134
0.4	-1.46817478566267
0.5	-1.6012780763498
0.6	-1.73788040937124
0.7	-1.87624636525849
0.8	-2.01445800903136
0.9	-2.15039569488953
1	-2.28171685211747

⋮

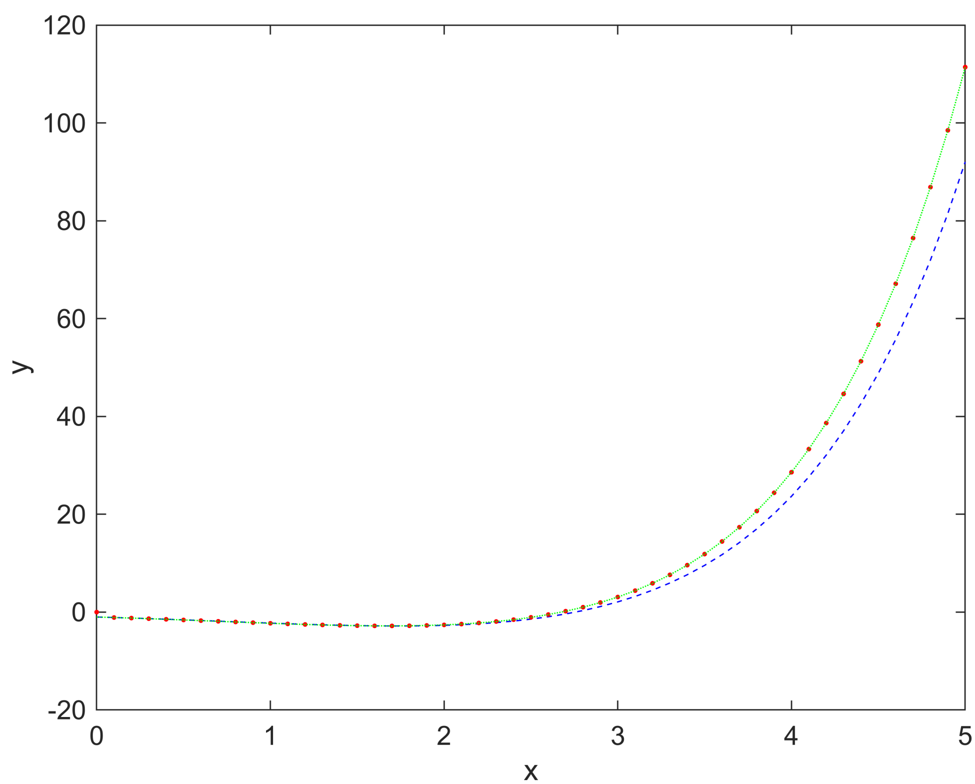
```
syms X Y(X)
eqn=diff(Y,X)==Y(X)+X.^2;
cond=Y(0)==-1;
ysol(X)=dsolve(eqn,cond);
ysol(X)
```

ans = $e^X - 2\,X - X^2 - 2$

```
x2=x0:h:xf;
y2=exp(x)-2*x-x.^2-2 %this is the analytical solution
```

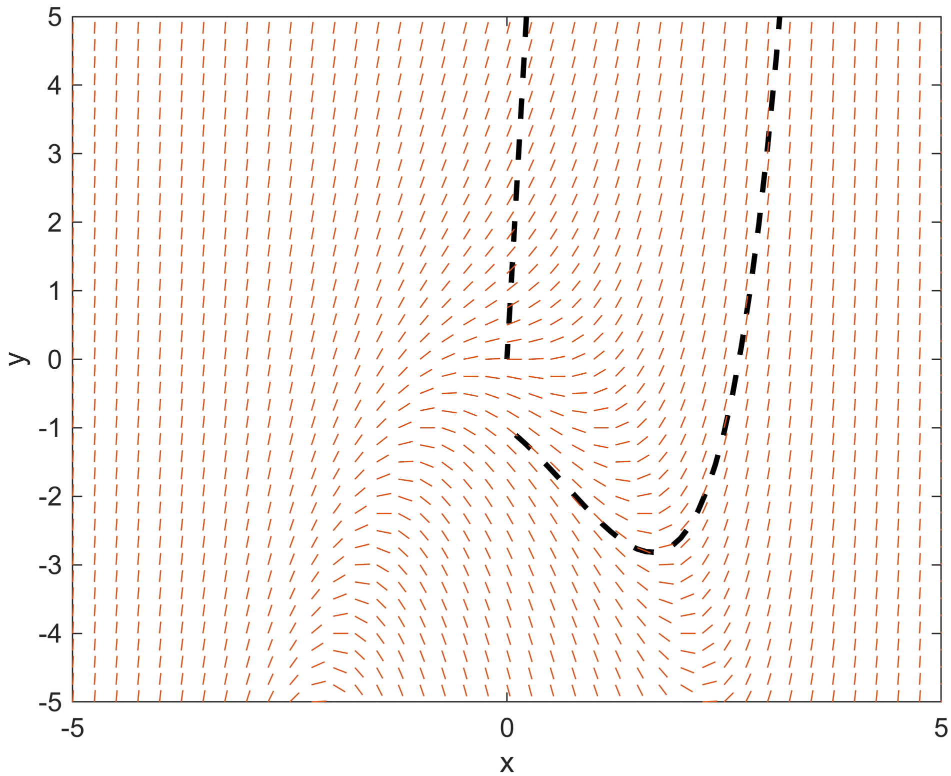
```
y2 = 51×1
                            -1
             -1.10482908192435
             -1.21859724183983
               -1.340141192424
             -1.46817530235873
             -1.60127872929987
             -1.73788119960949
             -1.87624729252952
             -2.01445907150753
             -2.15039688884305
        ⋮
```

```
%[x2' y2']
plot(x,y,'b--') %Now graphically compare Euler with RK4
hold on
plot(x1,y1,'r.')
plot(x2,y2,'g:')
xlabel('x')
ylabel('y')
title('')
hold off
```




```

plot(x1,y1,'k--','LineWidth',2)
hold on
[X,Y]=meshgrid(-5:.25:5,-5:.25:5);
DY=Y+X.^2; %DY is rhs of original equation
```

```
DX=ones(size(DY));
DW=sqrt(DX.^2+DY.^2);
quiver(X,Y,DX./DW,DY./DW,.5,'.');
xlabel('x');
ylabel('y');
axis([-5 5 -5 5])
hold off
```




```
format longg
syms X Y(X)
eqn=diff(Y,X)==Y(X)+X.^2;
cond=Y(0)==-1;
ysol(X)=dsolve(eqn,cond);
ysol(X)
```

ans = $e^X - 2\,X - X^2 - 2$

```
[x1,y1]=Euler(@Ch9NumExample,[0,5],-1,0.1);
[x1, y1, double(ysol(x1))]
```

ans = 51×3

0	-1	-1
0.1	-1.1	-1.10482908192435
0.2	-1.209	-1.21859724183983
0.3	-1.3259	-1.340141192424
0.4	-1.44949	-1.46817530235873

0.5	-1.578439	-1.60127872929987
0.6	-1.7112829	-1.73788119960949
0.7	-1.84641119	-1.87624729252952
0.8	-1.982052309	-2.01445907150753
0.9	-2.1162575399	-2.15039688884305

⋮

```
[x2,y2]=RK4(@Ch9NumExample,[0,5],-1,0.1);
[x2, y2, double(ysol(x2))]
```

ans = 51×3

0.1	-1.10482895833333	-1.10482908192435
0.2	-1.21859699057205	-1.21859724183983
0.3	-1.34014080990134	-1.340141192424
0.4	-1.46817478566267	-1.46817530235873
0.5	-1.6012780763498	-1.60127872929987
0.6	-1.73788040937124	-1.73788119960949
0.7	-1.87624636525849	-1.87624729252952
0.8	-2.01445800903136	-2.01445907150753
0.9	-2.15039569488953	-2.15039688884305
1	-2.28171685211747	-2.28171817154095

⋮

```
xeulerr4kanalyticalans=[0 -1 0 -1;
        0.1 -1.1 -1.10482895833333 -1.10482908192435;
        1 -2.24688329389 -2.28171685211747 -2.28171817154095;
        5 92.029938167665 111.412882112512 111.413159102577]
```

xeulerr4kanalyticalans = 4×4

0	-1	0 ⋯
0.1	-1.1	-1.10482895833333
1	-2.24688329389	-2.28171685211747
5	92.029938167665	111.412882112512

```
difference=[xeulerr4kanalyticalans(:,1) abs(xeulerr4kanalyticalans(:,2)-xeulerr4kanalyticalans(
```

difference = 4×3

0	0	1
0.1	0.00482908192434994	1.23591020084746e-07
1	0.0348348776509502	1.3194234798064e-06
5	19.383220934912	0.000276990065003702

```
fprintf("The RK4 is more accurate than the Euler method from the analytical solution.")
```

The RK4 is more accurate than the Euler method from the analytical solution.