```
clear all
syms r zeta wn omega F0 m t
zeta=sym('zeta','real')
```

zeta = ζ

```
zeta=sym('zeta','positive')
```

zeta = ζ

```
wn=sym('wn','real')
```

wn = wn

```
wn=sym('wn','positive')
```

wn = wn

```
omega=sym('omega','real')
```

omega = ω

```
omega=sym('omega','positive')
```

omega = ω

```
F0=sym('F0','real')
```

F0 = F_0

```
F0=sym('F0','positive')
```

F0 = F_0

```
m=sym('m','real')
```

m = m

```
m=sym('m','positive')
```

m = m

```
t=sym('t','real')
```

t = t

```
p(r)=r^2+2*zeta*wn*r+wn^2
```

p(r) = $r^2 + 2\,\zeta\,r\,\mathrm{wn} + \mathrm{wn}^2$

```
zpsol=(F0/m)*exp(i*omega*t)/p(i*omega)
```

zpsol =

$$\frac{F_0\, e^{\omega t\, i}}{m\,(-\omega^2 + 2\, i\, \zeta\, \omega\, wn + wn^2)}$$

```
gabs=simplify(abs(zpsol))
```

gabs =

$$\frac{F_0}{m\,|-\omega^2 + 2\, i\, \zeta\, \omega\, wn + wn^2|}$$

```
c1=imag(zpsol)
```

c1 =

$$-\frac{F_0 \sin(\omega t)\,(\omega^2 - wn^2)}{m\,\left((\omega^2 - wn^2)^2 + 4\,\omega^2\, wn^2\, \zeta^2\right)} - \frac{2\, F_0\, \omega\, wn\, \zeta \cos(\omega t)}{m\,\left((\omega^2 - wn^2)^2 + 4\,\omega^2\, wn^2\, \zeta^2\right)}$$

```
c2=real(zpsol)
```

c2 =

$$\frac{2\, F_0\, \omega\, wn\, \zeta \sin(\omega t)}{m\,\left((\omega^2 - wn^2)^2 + 4\,\omega^2\, wn^2\, \zeta^2\right)} - \frac{F_0 \cos(\omega t)\,(\omega^2 - wn^2)}{m\,\left((\omega^2 - wn^2)^2 + 4\,\omega^2\, wn^2\, \zeta^2\right)}$$

```
g=(sqrt(c1^2+c2^2))
```

g =

$$\sqrt{\left(\frac{F_0 \cos(\omega t)\,(\omega^2 - wn^2)}{\sigma_1} - \frac{2\, F_0\, \omega\, wn\, \zeta\, \sin(\omega t)}{\sigma_1}\right)^2 + \left(\frac{F_0 \sin(\omega t)\,(\omega^2 - wn^2)}{\sigma_1} + \frac{2\, F_0\, \omega\, wn\, \zeta\, \cos(\omega t)}{\sigma_1}\right)^2}$$

where

$$\sigma_1 = m\,\left((\omega^2 - wn^2)^2 + 4\,\omega^2\, wn^2\, \zeta^2\right)$$

```
gain=simplify(g)
```

gain =

$$\frac{F_0\,\sqrt{\omega^4 + 4\,\omega^2\, wn^2\, \zeta^2 - 2\,\omega^2\, wn^2 + wn^4}}{m\,|\omega^4 + 4\,\omega^2\, wn^2\, \zeta^2 - 2\,\omega^2\, wn^2 + wn^4|}$$

```
eq1=diff(g,omega)
```

eq1 =

$$-\frac{2\,\sigma_2\left(\frac{F_0\,t\,\sin(\omega\,t)\,\sigma_5}{m\,\sigma_4}-\frac{2\,F_0\,\omega\,\cos(\omega\,t)}{m\,\sigma_4}+\frac{2\,F_0\,\mathrm{wn}\,\zeta\,\sin(\omega\,t)}{m\,\sigma_4}+\frac{F_0\,\cos(\omega\,t)\,\sigma_1\,\sigma_5}{m\,{\sigma_4}^2}+\frac{2\,F_0\,\omega\,t\,\mathrm{wn}\,\zeta\,\cos(\omega\,t)}{m\,\sigma_4}-\right.}{}$$

where

$$\sigma_1 = 4\,\omega\,\sigma_5 + 8\,\omega\,\mathrm{wn}^2\,\zeta^2$$

$$\sigma_2 = \frac{F_0\,\cos(\omega\,t)\,\sigma_5}{m\,\sigma_4} - \frac{2\,F_0\,\omega\,\mathrm{wn}\,\zeta\,\sin(\omega\,t)}{m\,\sigma_4}$$

$$\sigma_3 = \frac{F_0\,\sin(\omega\,t)\,\sigma_5}{m\,\sigma_4} + \frac{2\,F_0\,\omega\,\mathrm{wn}\,\zeta\,\cos(\omega\,t)}{m\,\sigma_4}$$

$$\sigma_4 = {\sigma_5}^2 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2$$

$$\sigma_5 = \omega^2 - \mathrm{wn}^2$$

```
sol = solve(eq1,omega)
```

```
Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the
solution, specify the 'ReturnConditions' value as 'true'.
```

sol = $\mathrm{wn}\,\sqrt{1-2\,\zeta^2}$

Problem 2

```
fprintf('Problem 2')
```

```
Problem 2
```

```
clear all
syms r omega t F0
omega=sym('omega','real')
```

omega = ω

```
omega=sym('omega','positive')
```

omega = ω

```
t=sym('t','real')
```

t = t

```
% x'' + 3x' + 4x = 4 sin ωt
F0=4
```

```
F0 = 4
```

```
p(r)=1*r^2+3*r+4
```

p(r) = $r^2 + 3\,r + 4$

```
zpsol=F0*exp(i*omega*t)/p(i*omega)
```

zpsol =

$$\frac{4\,\mathrm{e}^{\omega\,t\,\mathrm{i}}}{-\omega^2 + 3\,\omega\,\mathrm{i} + 4}$$

```
gabs=simplify(abs(zpsol))
```

gabs =

$$\frac{4}{\sqrt{\left(\omega^2 - 4\right)^2 + 9\,\omega^2}}$$

```
c1=imag(zpsol)
```

c1 =

$$-\frac{4\sin(\omega\,t)\,\left(\omega^2 - 4\right)}{\left(\omega^2 - 4\right)^2 + 9\,\omega^2} - \frac{12\,\omega\cos(\omega\,t)}{\left(\omega^2 - 4\right)^2 + 9\,\omega^2}$$

```
c2=real(zpsol)
```

c2 =

$$\frac{12\,\omega\sin(\omega\,t)}{\left(\omega^2 - 4\right)^2 + 9\,\omega^2} - \frac{4\cos(\omega\,t)\,\left(\omega^2 - 4\right)}{\left(\omega^2 - 4\right)^2 + 9\,\omega^2}$$

```
g=(sqrt(c1^2+c2^2))
```

g =

$$4\sqrt{\frac{\left(\frac{12\,\omega\sin(\omega\,t)}{\sigma_1} - \frac{4\cos(\omega\,t)\,\left(\omega^2 - 4\right)}{\sigma_1}\right)^2}{16} + \frac{\left(\frac{4\sin(\omega\,t)\,\left(\omega^2 - 4\right)}{\sigma_1} + \frac{12\,\omega\cos(\omega\,t)}{\sigma_1}\right)^2}{16}}$$

where

$$\sigma_1 = \left(\omega^2 - 4\right)^2 + 9\,\omega^2$$

```
gain=simplify(g)
```

gain =

$$\frac{4}{\sqrt{\omega^4 + \omega^2 + 16}}$$

```
eq1=diff(g,omega)
```

eq1 =

$$2\left(\frac{\sigma_2\left(\frac{12\sin(\omega t)}{\sigma_4}-\frac{8\,\omega\cos(\omega t)}{\sigma_4}-\frac{12\,\omega\sin(\omega t)\,\sigma_1}{{\sigma_4}^2}+\frac{4\,t\sin(\omega t)\,(\omega^2-4)}{\sigma_4}+\frac{12\,\omega\,t\cos(\omega t)}{\sigma_4}+\frac{4\cos(\omega t)\,\sigma}{\sigma_4}\right.\right.}{8}$$

where

$$\sigma_1 = 18\,\omega + 4\,\omega\,(\omega^2-4)$$

$$\sigma_2 = \frac{12\,\omega\sin(\omega t)}{\sigma_4} - \frac{4\cos(\omega t)\,(\omega^2-4)}{\sigma_4}$$

$$\sigma_3 = \frac{4\sin(\omega t)\,(\omega^2-4)}{\sigma_4} + \frac{12\,\omega\cos(\omega t)}{\sigma_4}$$

$$\sigma_4 = (\omega^2-4)^2 + 9\,\omega^2$$

```
solve(eq1,omega)
```

ans =

Empty sym: 0-by-1

```
m = 1
```

m = 1

```
b = 3
```

b = 3

```
k = 4
```

k = 4

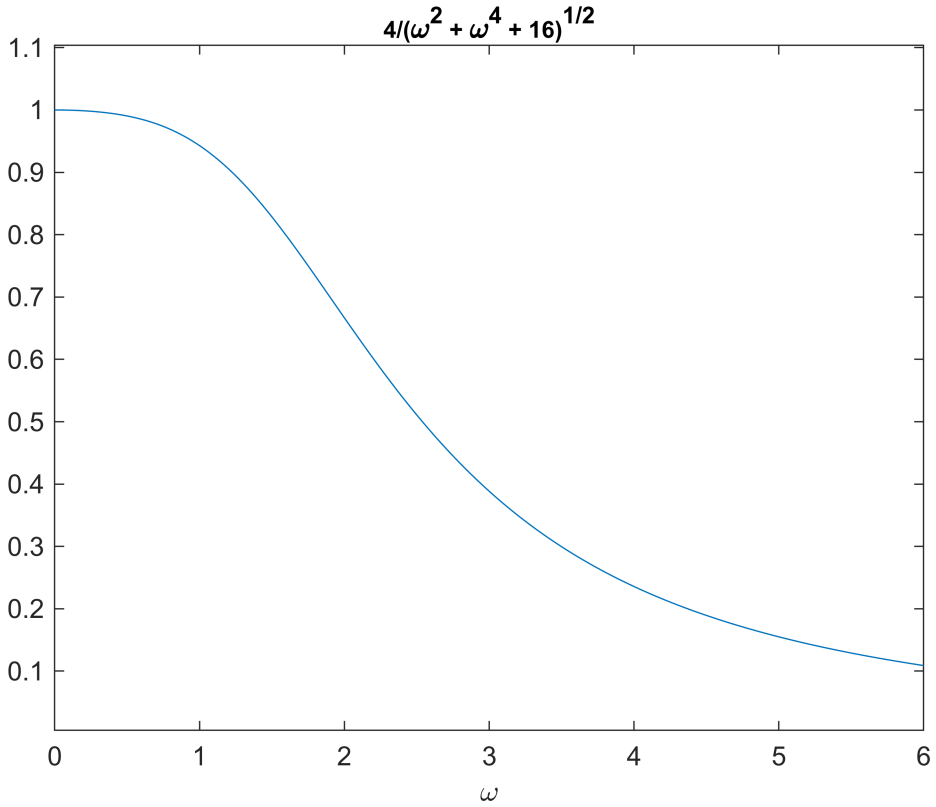
```
wres = sqrt(k/m-2*(b/(2*m))^2)
```

wres = 0.0000 + 0.7071i

```
fprintf("The gain frequency is %s and the resonance frequency is %4.4f%+4.4fi.",gain,real(wres)
```

The gain frequency is 4/(omega^2 + omega^4 + 16)^(1/2) and the resonance frequency is 0.0000+0.7071i.

```
ezplot(gain,[0,6])
```



Problem 4

```
fprintf('Problem 4')
```

```
Problem 4
```

```
clear all
syms r omega t F0
omega=sym('omega','real')
```

```
omega = ω
```

```
omega=sym('omega','positive')
```

```
omega = ω
```

```
t=sym('t','real')
```

```
t = t
```

```
% x'' + 6x' + 25x = 3 sin ωt
F0=3
```

```
F0 = 3
```

```
p(r)=1*r^2+6*r+25
```

p(r) = $r^2 + 6\,r + 25$

```
zpsol=F0*exp(i*omega*t)/p(i*omega)
```

zpsol =

$$\frac{3\,\mathrm{e}^{\omega\,t\,\mathrm{i}}}{-\omega^2 + 6\,\omega\,\mathrm{i} + 25}$$

```
gabs=simplify(abs(zpsol))
```

gabs =

$$\frac{3}{\sqrt{(\omega^2 - 25)^2 + 36\,\omega^2}}$$

```
c1=imag(zpsol)
```

c1 =

$$-\frac{3\sin(\omega\,t)\ (\omega^2 - 25)}{(\omega^2 - 25)^2 + 36\,\omega^2} - \frac{18\,\omega\cos(\omega\,t)}{(\omega^2 - 25)^2 + 36\,\omega^2}$$

```
c2=real(zpsol)
```

c2 =

$$\frac{18\,\omega\sin(\omega\,t)}{(\omega^2 - 25)^2 + 36\,\omega^2} - \frac{3\cos(\omega\,t)\ (\omega^2 - 25)}{(\omega^2 - 25)^2 + 36\,\omega^2}$$

```
g=(sqrt(c1^2+c2^2))
```

g =

$$3\sqrt{\frac{\left(\frac{18\,\omega\sin(\omega\,t)}{\sigma_1} - \frac{3\cos(\omega\,t)\ (\omega^2 - 25)}{\sigma_1}\right)^2}{9} + \frac{\left(\frac{3\sin(\omega\,t)\ (\omega^2 - 25)}{\sigma_1} + \frac{18\,\omega\cos(\omega\,t)}{\sigma_1}\right)^2}{9}}$$

where

$$\sigma_1 = (\omega^2 - 25)^2 + 36\,\omega^2$$

```
gain=simplify(g)
```

gain =

$$\frac{3}{\sqrt{\omega^4 - 14\,\omega^2 + 625}}$$

```
eq1=diff(g,omega)
```

eq1 =

$$3\left(\frac{2\,\sigma_2\left(\frac{18\sin(\omega\,t)}{\sigma_4}-\frac{6\,\omega\cos(\omega\,t)}{\sigma_4}-\frac{18\,\omega\sin(\omega\,t)\,\sigma_1}{{\sigma_4}^2}+\frac{3\,t\sin(\omega\,t)\,(\omega^2-25)}{\sigma_4}+\frac{18\,\omega\,t\cos(\omega\,t)}{\sigma_4}+\frac{3\cos(\omega\,t}{9}\right.\right.$$

where

$$\sigma_1 = 72\,\omega + 4\,\omega\,(\omega^2-25)$$

$$\sigma_2 = \frac{18\,\omega\sin(\omega\,t)}{\sigma_4} - \frac{3\cos(\omega\,t)\,(\omega^2-25)}{\sigma_4}$$

$$\sigma_3 = \frac{3\sin(\omega\,t)\,(\omega^2-25)}{\sigma_4} + \frac{18\,\omega\cos(\omega\,t)}{\sigma_4}$$

$$\sigma_4 = (\omega^2-25)^2 + 36\,\omega^2$$

```
solve(eq1,omega)
```

ans = $\sqrt{7}$

```
m = 1
```

```
m = 1
```

```
b = 6
```

```
b = 6
```

```
k = 25
```

```
k = 25
```

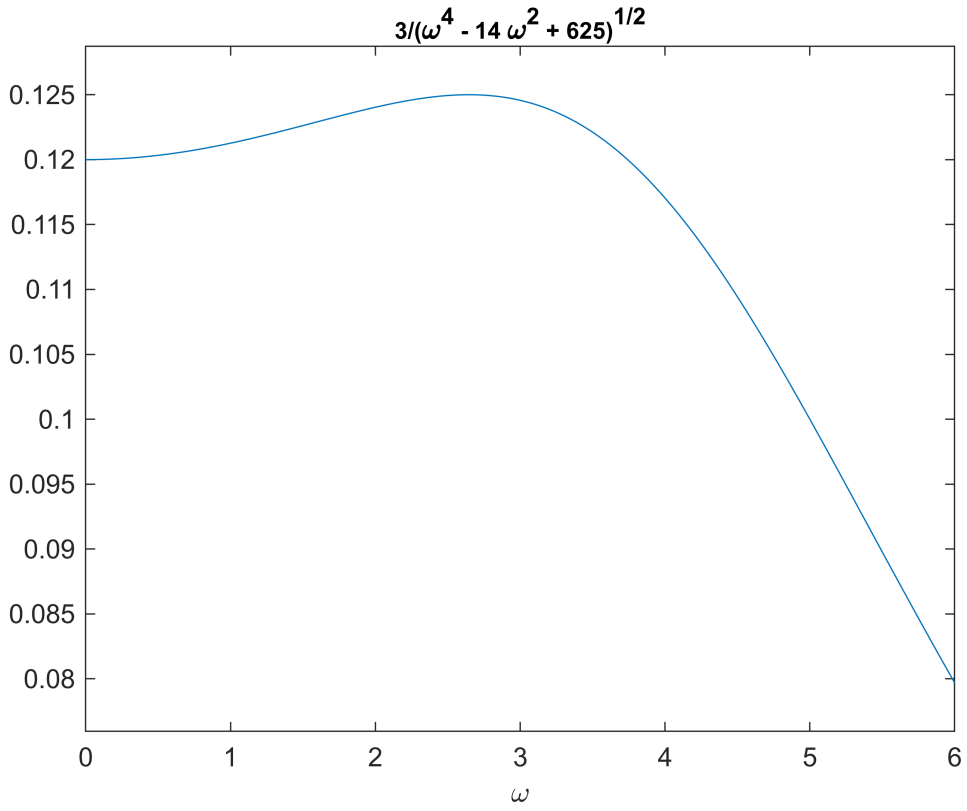
```
wres = sqrt(k/m-2*(b/(2*m))^2)
```

```
wres = 2.6458
```

```
fprintf("The gain frequency is %s and the resonance frequency is %4.4f%+4.4fi.",gain,real(wres)
```

```
The gain frequency is 3/(omega^4 - 14*omega^2 + 625)^(1/2) and the resonance frequency is 2.6458+0.0000i.
```

```
ezplot(gain,[0,6])
```


3/(ω^4 - 14 ω^2 + 625)^1/2
0.125
0.12
0.115
0.11
0.105
0.1
0.095
0.09
0.085
0.08
0
1
2
3
4
5
6
ω