Test 2

Problem 1

```
% a)
rng('shuffle')

C=randi([1,20],1,2)
```

```
C = 1×2
    10     7
```

```
m=C(1), k=C(2), b=min(C)
```

```
m = 10
k = 7
b = 7
```

```
% mx'  + bx' + kx = bsin(wt).
syms r omega t F0
omega=sym('omega','real')
```

omega = ω

```
omega=sym('omega','positive')
```

omega = ω

```
t=sym('t','real')
```

t = t

```
F0=b
```

```
F0 = 7
```

```
p(r)=1*r^2+6*r+25
```

p(r) = $r^2 + 6\,r + 25$

```
zpsol=F0*exp(i*omega*t)/p(i*omega)
```

zpsol =

$$\frac{7\,e^{\omega\,t\,\mathrm{i}}}{-\omega^2 + 6\,\omega\,\mathrm{i} + 25}$$

```
gabs=simplify(abs(zpsol))
```

gabs =

$$\frac{7}{\sqrt{(\omega^2-25)^2+36\,\omega^2}}$$

```
c1=imag(zpsol)
```

c1 =

$$-\frac{7\sin(\omega\,t)\,\left(\omega^2-25\right)}{(\omega^2-25)^2+36\,\omega^2}-\frac{42\,\omega\cos(\omega\,t)}{(\omega^2-25)^2+36\,\omega^2}$$

```
c2=real(zpsol)
```

c2 =

$$\frac{42\,\omega\sin(\omega\,t)}{(\omega^2-25)^2+36\,\omega^2}-\frac{7\cos(\omega\,t)\,\left(\omega^2-25\right)}{(\omega^2-25)^2+36\,\omega^2}$$

```
g=(sqrt(c1^2+c2^2))
```

g =

$$7\sqrt{\frac{\left(\frac{42\,\omega\sin(\omega\,t)}{\sigma_1}-\frac{7\cos(\omega\,t)\,\left(\omega^2-25\right)}{\sigma_1}\right)^2}{49}+\frac{\left(\frac{7\sin(\omega\,t)\,\left(\omega^2-25\right)}{\sigma_1}+\frac{42\,\omega\cos(\omega\,t)}{\sigma_1}\right)^2}{49}}$$

where

$$\sigma_1=(\omega^2-25)^2+36\,\omega^2$$

```
gain=simplify(g)
```

gain =

$$\frac{7}{\sqrt{\omega^4-14\,\omega^2+625}}$$

```
eq1=diff(g,omega)
```

eq1 =

$$7\left(\frac{2\,\sigma_2\left(\frac{42\sin(\omega\,t)}{\sigma_4}-\frac{14\,\omega\cos(\omega\,t)}{\sigma_4}-\frac{42\,\omega\sin(\omega\,t)\,\sigma_1}{{\sigma_4}^2}+\frac{7\,t\sin(\omega\,t)\,\left(\omega^2-25\right)}{\sigma_4}+\frac{42\,\omega\,t\cos(\omega\,t)}{\sigma_4}+\frac{7\cos(\omega}{}\right.}{49}\right.$$

where

$$\sigma_1 = 72\,\omega + 4\,\omega\,\left(\omega^2 - 25\right)$$

$$\sigma_2 = \frac{42\,\omega\sin(\omega\,t)}{\sigma_4} - \frac{7\cos(\omega\,t)\,\left(\omega^2-25\right)}{\sigma_4}$$

$$\sigma_3 = \frac{7\sin(\omega\,t)\,\left(\omega^2-25\right)}{\sigma_4} + \frac{42\,\omega\cos(\omega\,t)}{\sigma_4}$$

$$\sigma_4 = {\left(\omega^2-25\right)}^2 + 36\,\omega^2$$

```
solve(eq1,omega)
```

ans = $\sqrt{7}$

```
wres = sqrt(k/m-2*(b/(2*m))^2)
```

```
wres = 0.6745
```

```
fprintf("b) The resonance frequency is %4.4f%+4.4fi.",real(wres),imag(wres))
```

```
b) The resonance frequency is 0.6745+0.0000i.
```

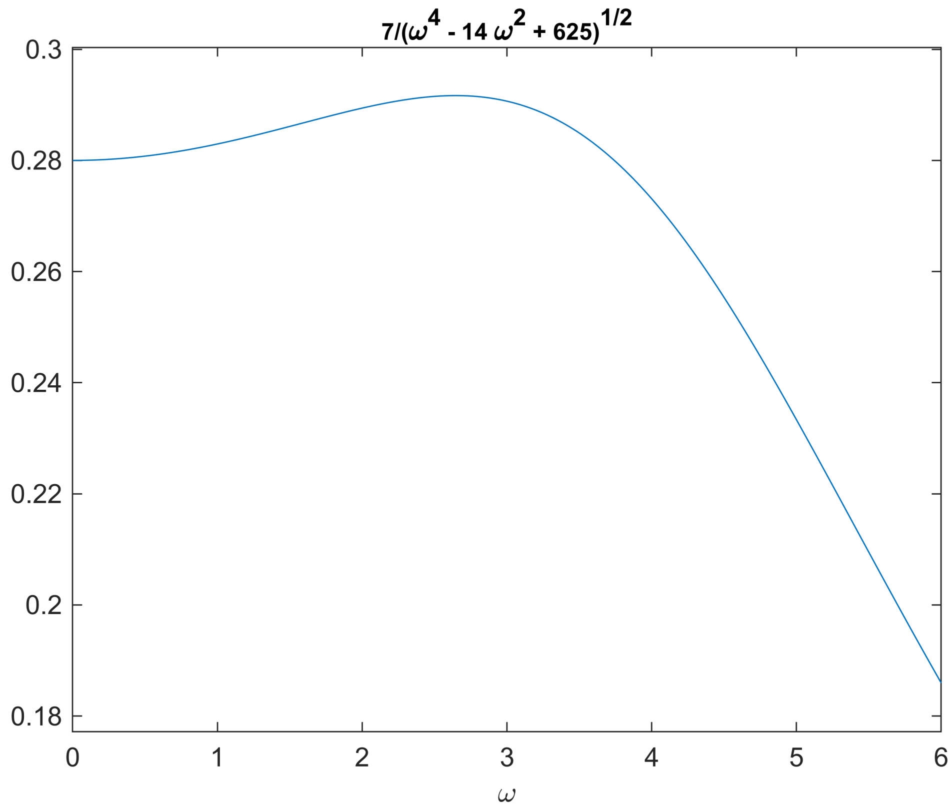
```
fprintf("c) The complex gain is %s \n and the absolute value of this gain is %s",zpsol,gabs)
```

```
c) The complex gain is (7*exp(omega*t*1i))/(omega*6i - omega^2 + 25)
 and the absolute value of this gain is 7/((omega^2 - 25)^2 + 36*omega^2)^(1/2)
```

```
fprintf("d) The gain frequency is %s ",gain)
```

```
d) The gain frequency is 7/(omega^4 - 14*omega^2 + 625)^(1/2)
```

```
ezplot(gain,[0,6]) %ploted as a function of w (omega)
```



Problem 2

```
% a)
clear all

rng('shuffle')

a=randi([-10,10])
```

```
a = 4
```

```
%x' =Ax
syms t c1 c2
A=[a,2; -2,a]
```

```
A = 2×2
     4     2
    -2     4
```

```
[v,d]=eig(sym(A))
```

v =

$$\begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} 4-2\,\mathrm{i} & 0 \\ 0 & 4+2\,\mathrm{i} \end{pmatrix}$$

```
real(v(:,1))
```

ans =

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

```
a_vec=real(v(:,1))
```

a_vec =

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

```
b_vec=imag(v(:,1))
```

b_vec =

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

```
alpha=real(d(1,1))%real part of 1st eigenvalue
```

alpha = 4

```
beta=imag(d(1,1))%imaginary part of 1st eigenvalue
```

beta = -2

```
solnx1=exp(alpha*t)*(a_vec*cos(beta*t)-b_vec*sin(beta*t))
```

solnx1 =

$$\begin{pmatrix} \sin(2\,t)\,\mathrm{e}^{4\,t} \\ \cos(2\,t)\,\mathrm{e}^{4\,t} \end{pmatrix}$$

```
solnx2=exp(alpha*t)*(a_vec*sin(beta*t)+b_vec*cos(beta*t))
```

solnx2 =

$$\begin{pmatrix} \cos(2\,t)\,\mathrm{e}^{4\,t} \\ -\sin(2\,t)\,\mathrm{e}^{4\,t} \end{pmatrix}$$

```
%solnx1 and solnx2 are from Eq.(5.75)
soln=c1*solnx1+c2*solnx2
```

soln =

$$\begin{pmatrix} c_2\cos(2\,t)\,\mathrm{e}^{4\,t}+c_1\sin(2\,t)\,\mathrm{e}^{4\,t} \\ c_1\cos(2\,t)\,\mathrm{e}^{4\,t}-c_2\sin(2\,t)\,\mathrm{e}^{4\,t} \end{pmatrix}$$

```
fprintf("b) The eigenvalues for the coefficient matrix A are alpha = %4i and beta = %4i", alpha
```

```
b) The eigenvalues for the coefficient matrix A are alpha =    4 and beta =   -2
```

```
fprintf("The eigenvectors for the coefficient matrix A is %s and %s for alpha in a colomn.\nThe
```

```
The eigenvectors for the coefficient matrix A is 0 and 1 for alpha in a colomn.
The eigenvectors for the coefficient matrix A is 1 and 0 for beta in a colomn.
```

```
fprintf("c) The formula is %s",soln)
```

```
c) The formula is c2*cos(2*t)*exp(4*t) + c1*sin(2*t)*exp(4*t)c) The formula is c1*cos(2*t)*exp(4*t) - c2*sin(2*t)*ex
```