Chapter 3 #: 2, 3, 4, 5, 9, 10, 12a, 12c, 16, 20

```
% Problem 2
fprintf("Problem 2")
```

Problem 2

```
clear all
syms x
f1=exp(x)
```

f1 = e^x

```
f2=exp(-x)
```

f2 = e^{-x}

```
f3=exp(2*x)
```

f3 = e^{2x}

```
diff([f1 f2 f3],x) %2nd row of Wronskian
```

ans = $\begin{pmatrix} e^x & -e^{-x} & 2\,e^{2x} \end{pmatrix}$

```
A=[f1,f2,f3; diff([f1,f2,f3],x); diff([f1,f2,f3],x,2)]
```

A =

$$\begin{pmatrix} e^x & e^{-x} & e^{2x} \\ e^x & -e^{-x} & 2\,e^{2x} \\ e^x & e^{-x} & 4\,e^{2x} \end{pmatrix}$$

```
Wronsk=det(A)
```

Wronsk = $-6\,e^{2x}$

```
subs(Wronsk,x,0) %This calculates W(0).
```

ans = -6

Problem 3

```
fprintf("Problem 3")
```

Problem 3

```
clear all
syms r1 r2 z x A1 A2
r1=2+3*1i
```

r1 = 2.0000 + 3.0000i

```
real(r1)
```

```
ans = 2
```

```
imag(r1)
```

```
ans = 3
```

```
conj(r1)
```

```
ans = 2.0000 - 3.0000i
```

```
r2=1/(4+5*1i)
```

```
r2 = 0.0976 - 0.1220i
```

```
exp(3+4*1i)
```

```
ans = -13.1288 - 15.2008i
```

```
exp(pi*1i)
```

```
ans = -1.0000 + 0.0000i
```

```
z=1-3*1i
```

```
z = 1.0000 - 3.0000i
```

```
R=abs(z)
```

```
R = 3.1623
```

```
theta=angle(z)
```

```
theta = -1.2490
```

```
R.*exp(1i*theta)
```

```
ans = 1.0000 - 3.0000i
```

```
f(x)=x^4
```

f(x) = x^4

```
solve(f(x)-pi,x)
```

ans =

$$\begin{pmatrix} -\pi^{1/4} \\ -\pi^{1/4}\,\mathrm{i} \\ \pi^{1/4}\,\mathrm{i} \\ \pi^{1/4} \end{pmatrix}$$

```
p(x)=x^2+6*x+25
```

p(x) = $x^2 + 6\,x + 25$

```
solve(p(x),x)
```

ans =

$$\begin{pmatrix} -3-4\,\mathrm{i} \\ -3+4\,\mathrm{i} \end{pmatrix}$$

```
subs(p(x),x,1i)
```

ans = $24+6\,\mathrm{i}$

```
subs(p(x),x,2+1i)
```

ans = $40+10\,\mathrm{i}$

```
[solA1, solA2]=solve((2+1i)*A1==3, A1+3*A2*1i==0)
```

solA1 =

$$\frac{6}{5}-\frac{3}{5}\,\mathrm{i}$$

solA2 =

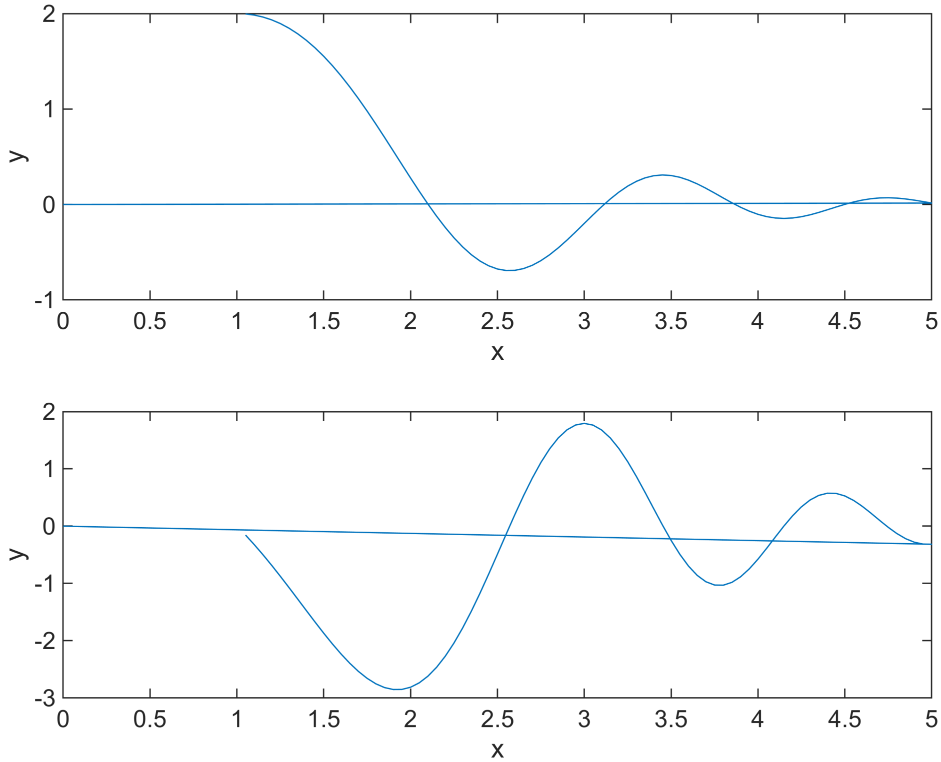
$$\frac{1}{5}+\frac{2}{5}\,\mathrm{i}$$

Problem 4

```
fprintf("Problem 4")
```

Problem 4

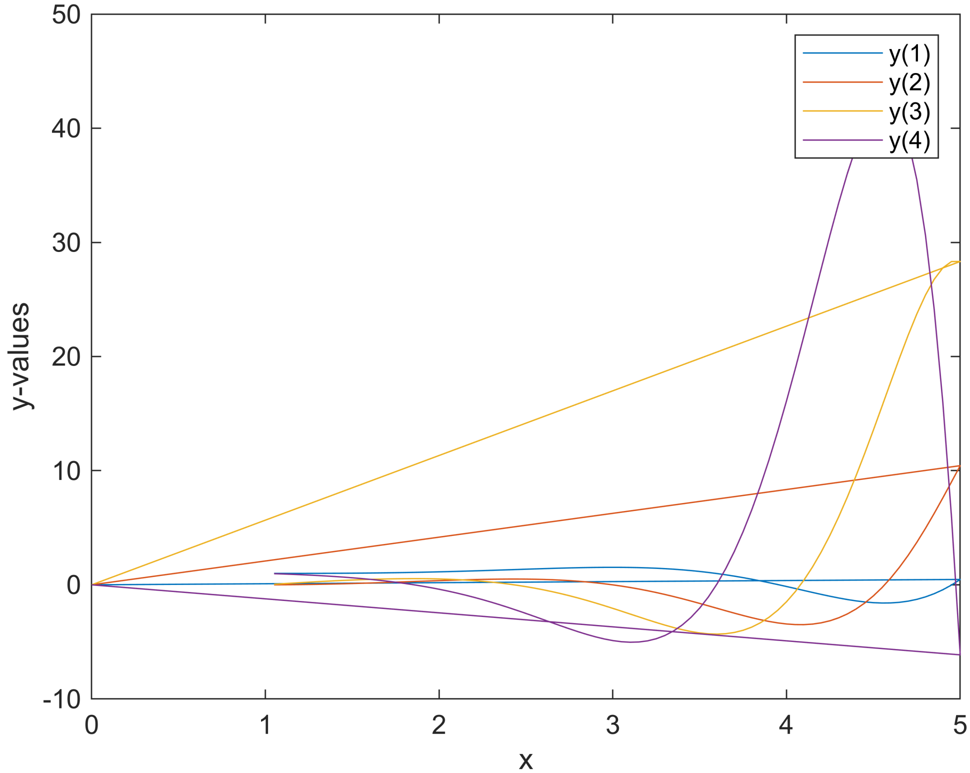
```
clear all
x0=1; xf=5;
y0=[2,0];
[x1,y1]=RK4(@Ch3NumExample1,[x0,xf],y0,.05);
subplot(2,1,1),plot(x1,y1(:,1))
xlabel('x'); ylabel('y')
subplot(2,1,2),plot(x1,y1(:,2))
xlabel('x'); ylabel('y')
```




```
[x1(end), y1(end,:)] %This shows the last entry of vector x1 and matrix y1. Note that we set xf
```

```
ans = 1×3
     0     0     0
```

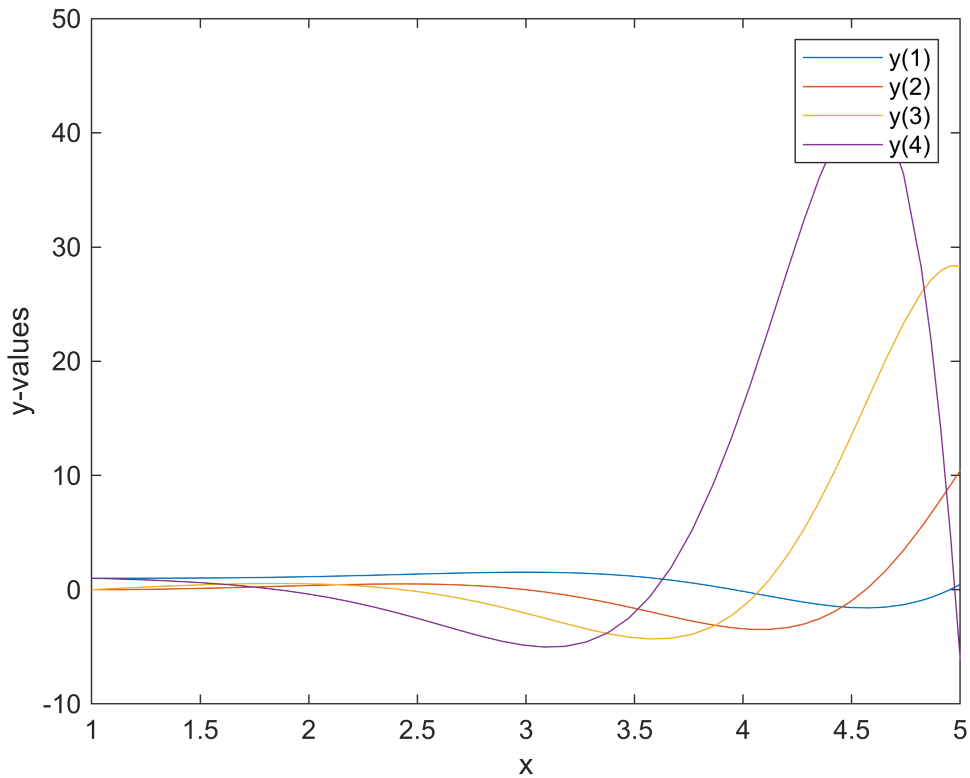
```
figure %calls a new blank figure window
x0=1; xf=5;
y0=[1,0,0,1];
[x2,y2]=RK4(@Ch3NumExample2,[x0,xf],y0,.05);
plot(x2,y2)
legend('y(1)','y(2)','y(3)','y(4)')
xlabel('x'); ylabel('y-values')
```




```
[x2(end), y2(end,:)]
```

```
ans = 1×5
     0     0     0     0     0
```

```
figure
[x2,y2]=ode45(@Ch3NumExample2,[x0,xf],y0); %Or use ode45.
plot(x2,y2)
legend('y(1)','y(2)','y(3)','y(4)')
xlabel('x'); ylabel('y-values')
```




```
[x2(end), y2(end,:)]
```

```
ans = 1×5
    5.0000    0.4679   10.4357   28.3371   -6.0848
```

Problem 5

```
fprintf("Problem 5")
```

```
Problem 5
```

```
clear all
%Next three lines do NOT require the Symbolic Math Toolbox
p=[1 0 3 0 1] %these are the coefficients in (v)
```

```
p = 1×5
     1     0     3     0     1
```

```
%if a coefficient is 0, you must put 0 in its position
roots(p)
```

```
ans = 4×1 complex
  -0.0000 + 1.6180i
  -0.0000 - 1.6180i
  -0.0000 + 0.6180i
  -0.0000 - 0.6180i
```

```
%Now we use the Symbolic Math Toolbox:
syms x r
```

```
f(r)=r^2+3*r-4
```

f(r) = $r^2+3\,r-4$

```
solve(f(r),r)
```

ans =

$$\begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

```
g(r)=r^4-4*r^3+6*r^2-4*r-15
```

g(r) = $r^4-4\,r^3+6\,r^2-4\,r-15$

```
solve(g(r),r)
```

ans =

$$\begin{pmatrix} -1 \\ 3 \\ 1-2\,\mathrm{i} \\ 1+2\,\mathrm{i} \end{pmatrix}$$

```
h(r)=r^4+r^3-2*r-1
```

h(r) = $r^4+r^3-2\,r-1$

```
solve(h(r),r)
```

ans =

$$\begin{pmatrix} \mathrm{root}(z^4+z^3-2\,z-1, z, 1) \\ \mathrm{root}(z^4+z^3-2\,z-1, z, 2) \\ \mathrm{root}(z^4+z^3-2\,z-1, z, 3) \\ \mathrm{root}(z^4+z^3-2\,z-1, z, 4) \end{pmatrix}$$

```
F(r)=r^3+r-3
```

F(r) = r^3+r-3

```
Fsoln=solve(F(r),r)
```

Fsoln =

$$\begin{pmatrix} \mathrm{root}(z^3+z-3, z, 1) \\ \mathrm{root}(z^3+z-3, z, 2) \\ \mathrm{root}(z^3+z-3, z, 3) \end{pmatrix}$$

```
Fsoln(1) % This is the first root
```

ans = $\mathrm{root}(z^3+z-3, z, 1)$

```
double(Fsoln(1))
```

```
ans = 1.2134
```

```
double(Fsoln)
```

```
ans = 3×1 complex
   1.2134 + 0.0000i
  -0.6067 - 1.4506i
  -0.6067 + 1.4506i
```

```
G(r)=r^5+3*r^2-1
```

G(r) = $r^5 + 3\,r^2 - 1$

```
Gsoln=solve(G(r),r)
```

Gsoln =

$$\begin{pmatrix} \mathrm{root}(z^5 + 3\,z^2 - 1, z, 1) \\ \mathrm{root}(z^5 + 3\,z^2 - 1, z, 2) \\ \mathrm{root}(z^5 + 3\,z^2 - 1, z, 3) \\ \mathrm{root}(z^5 + 3\,z^2 - 1, z, 4) \\ \mathrm{root}(z^5 + 3\,z^2 - 1, z, 5) \end{pmatrix}$$

Problem 9

9a

```
fprintf("Problem 9a")
```

```
Problem 9a
```

```
clear all
syms x % {e^x, xe^x, x^2}
f1=exp(x)
```

f1 = e^x

```
f2=x*exp(x)
```

f2 = $x\,\mathrm{e}^x$

```
f3=x.^2
```

f3 = x^2

```
diff([f1 f2 f3],x) %2nd row of Wronskian
```

ans = $\begin{pmatrix} \mathrm{e}^x & \mathrm{e}^x + x\,\mathrm{e}^x & 2\,x \end{pmatrix}$

```
A=[f1,f2,f3; diff([f1,f2,f3],x); diff([f1,f2,f3],x,2)]
```

```
A =
```

$$\begin{pmatrix} e^x & x\,e^x & x^2 \\ e^x & e^x + x\,e^x & 2\,x \\ e^x & 2\,e^x + x\,e^x & 2 \end{pmatrix}$$

```
Wronsk=det(A)
```

Wronsk = $2\,e^{2\,x} - 4\,x\,e^{2\,x} + x^2\,e^{2\,x}$

```
subs(Wronsk,x,0) %This calculates W(0).
```

ans = 2

9b

```
fprintf("Problem 9b")
```

```
Problem 9b
```

```
clear all
syms x %{sin(4x) + cos(4x), cos(4x) - sin(4x)}
f1=sin(4*x) + cos(4*x)
```

f1 = $\cos(4\,x) + \sin(4\,x)$

```
f2=cos(4*x) - sin(4*x)
```

f2 = $\cos(4\,x) - \sin(4\,x)$

```

diff([f1 f2],x) %2nd row of Wronskian
```

ans = $\left(4\cos(4\,x) - 4\sin(4\,x) \quad -4\cos(4\,x) - 4\sin(4\,x)\right)$

```
A=[f1,f2; diff([f1,f2],x)]
```

```
A =
```

$$\begin{pmatrix} \cos(4\,x) + \sin(4\,x) & \cos(4\,x) - \sin(4\,x) \\ 4\cos(4\,x) - 4\sin(4\,x) & -4\cos(4\,x) - 4\sin(4\,x) \end{pmatrix}$$

```
Wronsk=det(A)
```

Wronsk = $-8\cos(4\,x)^2 - 8\sin(4\,x)^2$

```
subs(Wronsk,x,0) %This calculates W(0).
```

ans = -8

Problem 10

10a

```
fprintf("Problem 10a")
```

```
Problem 10a
```

```
clear all
syms z
%(a) 2 - 3i (b) 2i (c) -1 - 5i (d) 3 + i
z=2-3i
```

```
z = 2.0000 - 3.0000i
```

```
R=abs(z)
```

```
R = 3.6056
```

```
theta=angle(z)
```

```
theta = -0.9828
```

```
R.*exp(1i*theta) %output/check
```

```
ans = 2.0000 - 3.0000i
```

10b

```
fprintf("Problem 10b")
```

```
Problem 10b
```

```
clear all
syms z
%(a) 2 - 3i (b) 2i (c) -1 - 5i (d) 3 + i
z=2i
```

```
z = 0.0000 + 2.0000i
```

```
R=abs(z)
```

```
R = 2
```

```
theta=angle(z)
```

```
theta = 1.5708
```

```
R.*exp(1i*theta) %output/check
```

```
ans = 0.0000 + 2.0000i
```

10c

```
fprintf("Problem 10c")
```

```
Problem 10c
```

```
clear all
syms z
%(a) 2 - 3i (b) 2i (c) -1 - 5i (d) 3 + i
```

```
z=-1-5i
```

```
z = -1.0000 - 5.0000i
```

```
R=abs(z)
```

```
R = 5.0990
```

```
theta=angle(z)
```

```
theta = -1.7682
```

```
R.*exp(1i*theta) %output/check
```

```
ans = -1.0000 - 5.0000i
```

10d

```
fprintf("Problem 10d")
```

```
Problem 10d
```

```
clear all
syms z
%(a) 2 – 3i (b) 2i (c) –1 – 5i (d) 3 + i
z=3+1*1i
```

```
z = 3.0000 + 1.0000i
```

```
R=abs(z)
```

```
R = 3.1623
```

```
theta=angle(z)
```

```
theta = 0.3218
```

```
R.*exp(1i*theta) %output/check
```

```
ans = 3.0000 + 1.0000i
```

Problem 12a

```
fprintf("Problem 12a")
```

```
Problem 12a
```

```
syms z
p(z)=z.^2+4*z+8
```

p(z) = $z^2 + 4\,z + 8$

```
subs(p(z),z,2i)
```

ans = $4 + 8\,\mathrm{i}$

Problem 12c

```
fprintf("Problem 12c")
```

```
Problem 12c
```

```
syms z
p(z)=z.^2+4*z+8
```

p(z) = $z^2 + 4z + 8$

```
subs(p(z),z,-1+4i)
```

ans = $-11 + 8\,\mathrm{i}$

Problem 16

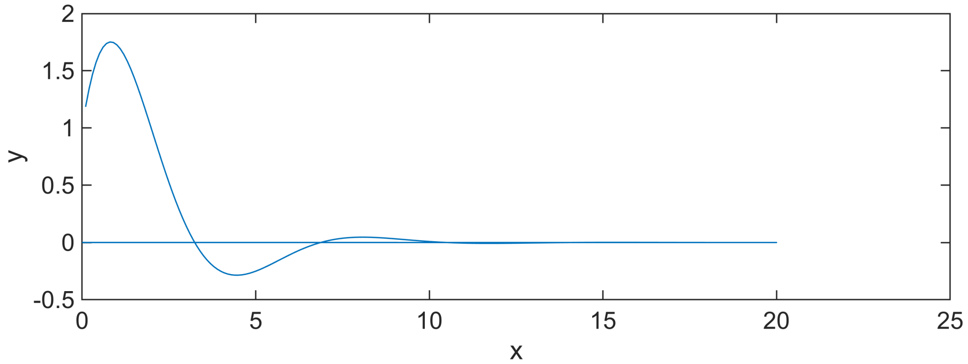
```
fprintf("Problem 16")
```

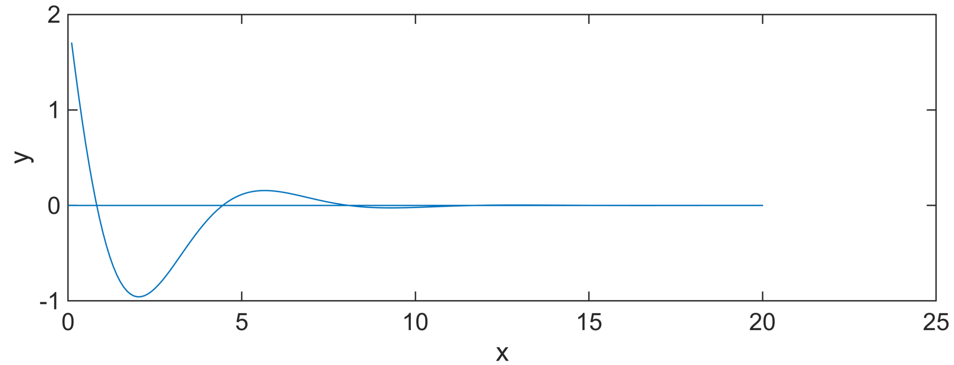
```
Problem 16
```

```
clear all
x0=0; xf=20;
y0=[1,2]; %in dy order
h=0.1
```

```
h = 0.1000
```

```
[x1,y1]=RK4(@Ch3NumExample16,[x0,xf],y0,h);
subplot(2,1,1),plot(x1,y1(:,1))
xlabel('x'); ylabel('y')
subplot(2,1,2),plot(x1,y1(:,2))
xlabel('x'); ylabel('y')
```






```
[x1(end), y1(end,:)] %This shows the last entry of vector x1 and matrix y1. Note that we set xf
```

```
ans = 1×3
     0     0     0
```

Problem 20

20a

```
fprintf("Problem 20a")
```

```
Problem 20a
```

```
clear all
% (a)y''+y'+y = 0 (b) y'''+8y''+37y'+50y = 0 (c) y^(4)-4y^(3)-2y''+36y'-63y = 0
syms x r
f(r)=r^2+r+1
```

f(r) = $r^2 + r + 1$

```
solve(f(r),r)
```

```
ans =
```

$$\begin{pmatrix} -\frac{1}{2}-\frac{\sqrt{3}\,\mathrm{i}}{2} \\ -\frac{1}{2}+\frac{\sqrt{3}\,\mathrm{i}}{2} \end{pmatrix}$$

20b

```
fprintf("Problem 20b")
```

```
Problem 20b
```

```
clear all
% (a)y''+y'+y = 0 (b) y'''+8y''+37y'+50y = 0 (c) y^(4)-4y^(3)-2y''+36y'-63y = 0
syms x r
g(r)=r^3+8*r^2+37*r+50
```

g(r) = $r^3+8\,r^2+37\,r+50$

```
solve(g(r),r)
```

ans =

$$\begin{pmatrix} -2 \\ -3-4\,\mathrm{i} \\ -3+4\,\mathrm{i} \end{pmatrix}$$

20c

```
fprintf("Problem 20c")
```

```
Problem 20c
```

```
clear all
% (a)y''+y'+y = 0 (b) y'''+8y''+37y'+50y = 0 (c) y^(4)-4y^(3)-2y''+36y'-63y = 0
syms x r
h(r)=r^4-4*r^3-2*r^2+36*r-63
```

h(r) = $r^4-4\,r^3-2\,r^2+36\,r-63$

```
solve(h(r),r)
```

ans =

$$\begin{pmatrix} -3 \\ 3 \\ 2-\sqrt{3}\,\mathrm{i} \\ 2+\sqrt{3}\,\mathrm{i} \end{pmatrix}$$

```

```

```
function dy= Ch3NumExample1(x,y)
```

```
%
%The original ode is 2*x*y”+x^2*y’+3*x^3*y=0
%The system of first-order equations is
%u1'=u2
%u2'=(-x/2)*u2-((3*x^2)/2)*u1
%
%We let y(1)=u1, y(2)=u2
%
dy=zeros(2,1); %dy is a column vector!
dy(1) = y(2);
dy(2) = (-1/2)*x*y(2)-(3/2)*x^2*y(1);
end
%end of function Ch3NumExample1.m

function dy= Ch3NumExample2(x,y)
%
%The original ode is y^(4)+x^2*y’+y=cos(x)
%The system of first-order equations is
%u1'=u2
%u2'=u3
%u3'=u4
%u4'=-x^2*u2-u1+cos(x)
%
%We let y(1)=u1, y(2)=u2, y(3)=u3, y(4)=u4
%
dy=zeros(4,1); %dy is a column vector!
dy(1) = y(2);
dy(2) = y(3);
dy(3) = y(4);
dy(4) = -x^2*y(2)-y(1)+cos(x);
end
%end of function Ch3NumExample2.m

function dy= Ch3NumExample16(x,y)
%
%The original ode is y''+y'+y=0

dy=zeros(2,1); %dy is a column vector!
dy(1) = y(2);
dy(2) = -y(2)-y(1);
end
```