Chapter 5 #: 1, 2, 3, 4, 6, 7, 10, 15

Problem 1

```
fprintf('Problem 1\n')
```

```
Problem 1
```

```
clear all
A=[3, 0, -2; 0, 1, 2; 0, -2, 1]
```

```
A = 3×3
     3     0    -2
     0     1     2
     0    -2     1
```

```
lambda=eig(A) %calculates eigenvalues of A
```

```
lambda = 3×1 complex
   3.0000 + 0.0000i
   1.0000 + 2.0000i
   1.0000 - 2.0000i
```

```
[v,d]=eig(A) %calculates eigenvalues AND eigenvectors of A
```

```
v = 3×3 complex
   1.0000 + 0.0000i   0.3162 + 0.3162i   0.3162 - 0.3162i
   0.0000 + 0.0000i   0.0000 - 0.6325i   0.0000 + 0.6325i
   0.0000 + 0.0000i   0.6325 + 0.0000i   0.6325 + 0.0000i
d = 3×3 complex
   3.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   1.0000 + 2.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 - 2.0000i
```

```
%eigenvectors given as columns of v
%corresponding eigenvalues are on diagonal of d
lambda=eig(sym(A)) %uses sym to find and display eigenvalues
```

lambda =

$$\begin{pmatrix} 3 \\ 1-2\,\mathrm{i} \\ 1+2\,\mathrm{i} \end{pmatrix}$$

```
[v,d]=eig(sym(A))
```

v =

$$\begin{pmatrix} 1 & \frac{1}{2}-\frac{1}{2}\,\mathrm{i} & \frac{1}{2}+\frac{1}{2}\,\mathrm{i} \\ 0 & \mathrm{i} & -\mathrm{i} \\ 0 & 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1-2\,\mathrm{i} & 0 \\ 0 & 0 & 1+2\,\mathrm{i} \end{pmatrix}$$

```
d(1,1) %first eigenvalue of A
```

ans = 3

```
v(:,1) %corresponding first eigenvector of A
```

ans =

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

```
A*v(:,1)
```

ans =

$$\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

```
d(1,1)*v(:,1) %verifies that A*v=lambda*v for first
```

ans =

$$\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

```
% eigenvalue-eigenvector pair; the last two
%answers should be identical
A*v(:,3)
```

ans =

$$\begin{pmatrix} -\frac{1}{2}+\frac{3}{2}\,\mathrm{i} \\ 2-\mathrm{i} \\ 1+2\,\mathrm{i} \end{pmatrix}$$

```
d(3,3)*v(:,3) %verifies that A*v=lambda*v for third
```

ans =

$$\begin{pmatrix} -\frac{1}{2}+\frac{3}{2}\,\mathrm{i} \\ 2-\mathrm{i} \\ 1+2\,\mathrm{i} \end{pmatrix}$$

Problem 2

```
fprintf('Problem 2\n')
```

```
Problem 2
```

```
clear all
A=[-2,-1,-2; -4,-5,2; -5,-1,1]
```

```
A = 3×3
    -2    -1    -2
    -4    -5     2
    -5    -1     1
```

```
[v,d]=eig(sym(A))
```

v =

$$\begin{pmatrix} 1 & -\frac{1}{2} & 1 \\ -1 & \frac{1}{2} & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} -3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -6 \end{pmatrix}$$

```
syms t c1 c2 c3
%Type the line below on a single line, not with a <return>.
soln=[exp(d(1,1)*t)*v(:,1),exp(d(2,2)*t)*v(:,2),exp(d(3,3)*t)*v(:,3)]
```

soln =

$$\begin{pmatrix} e^{-3t} & -\frac{e^{3t}}{2} & e^{-6t} \\ -e^{-3t} & \frac{e^{3t}}{2} & 2\,e^{-6t} \\ e^{-3t} & e^{3t} & e^{-6t} \end{pmatrix}$$

```
%general soln is soln*[c1; c2; c3]
soln0=subs(soln,t,0)
```

soln0 =

$$\begin{pmatrix} 1 & -\frac{1}{2} & 1 \\ -1 & \frac{1}{2} & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

```
x0=[2; -1; -3]
```

```
x0 = 3×1
     2
```

```
    -1
    -3
```

```
cvals=soln0\x0
```

cvals =

$$\begin{pmatrix} 0 \\ -\frac{10}{3} \\ \frac{1}{3} \end{pmatrix}$$

```
v1n=v(:,1)*cvals(1)
```

v1n =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
v2n=v(:,2)*cvals(2)
```

v2n =

$$\begin{pmatrix} \frac{5}{3} \\ -\frac{5}{3} \\ -\frac{10}{3} \end{pmatrix}$$

```
v3n=v(:,3)*cvals(3)
```

v3n =

$$\begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$$

```
Vmat=[v1n,v2n,v3n]
```

Vmat =

$$\begin{pmatrix} 0 & \frac{5}{3} & \frac{1}{3} \\ 0 & -\frac{5}{3} & \frac{2}{3} \\ 0 & -\frac{10}{3} & \frac{1}{3} \end{pmatrix}$$

```
xvec=[exp(d(1,1)*t); exp(d(2,2)*t); exp(d(3,3)*t)]
```

```
xvec =
```

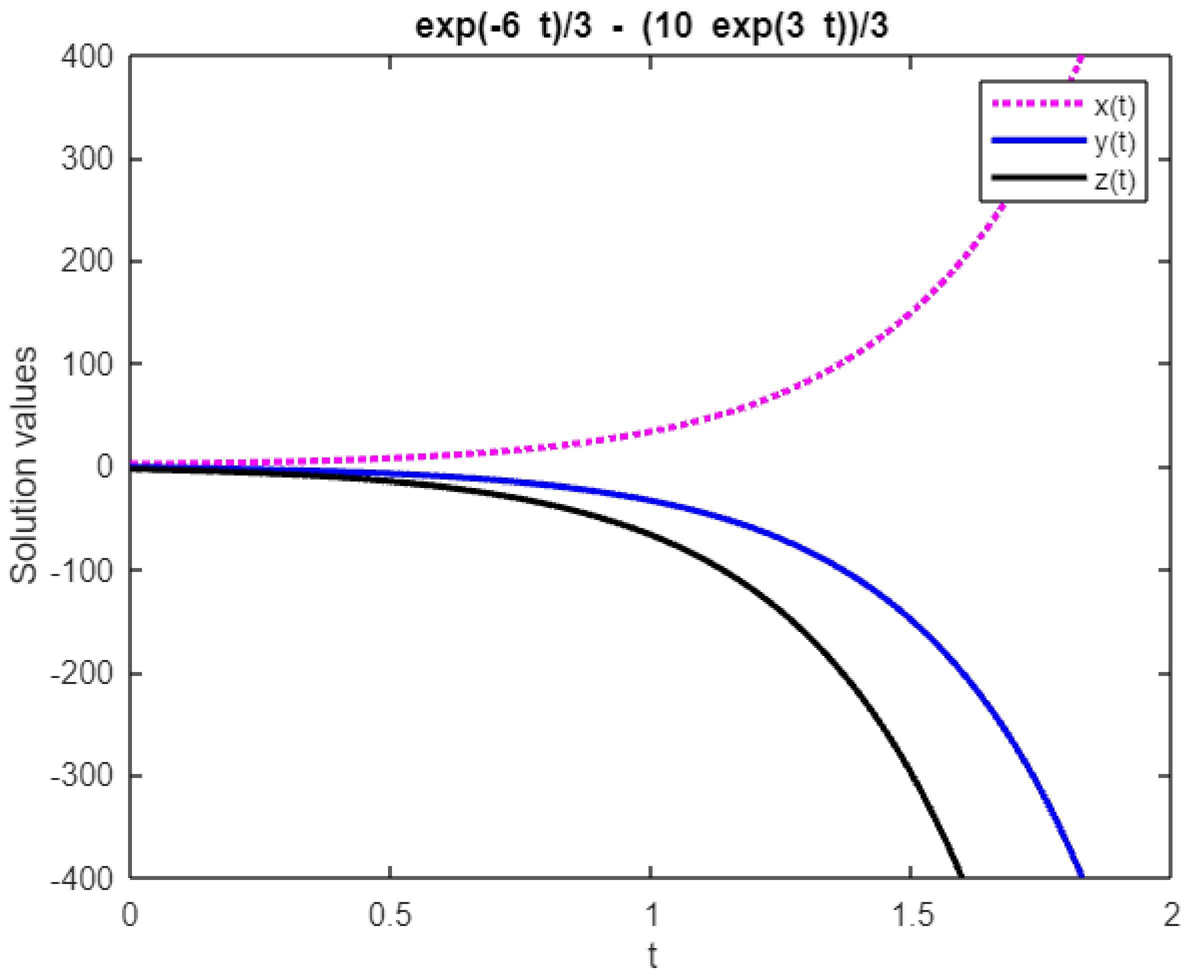
$$\begin{pmatrix} e^{-3t} \\ e^{3t} \\ e^{-6t} \end{pmatrix}$$

```
soln=Vmat*xvec %the solution to the IVP
```

```
soln =
```

$$\begin{pmatrix} \frac{5\,e^{3t}}{3} + \frac{e^{-6t}}{3} \\ \frac{2\,e^{-6t}}{3} - \frac{5\,e^{3t}}{3} \\ \frac{e^{-6t}}{3} - \frac{10\,e^{3t}}{3} \end{pmatrix}$$

```
h1=ezplot(soln(1),[0,2]);
set(h1,'Color','m','LineStyle',':','LineWidth',2)
hold on
h2=ezplot(soln(2),[0,2]);
set(h2,'Color','b','LineStyle','-','LineWidth',2)
h3=ezplot(soln(3),[0,2]);
set(h3,'Color','k','LineWidth',2)
axis([0 2 -400 400])
xlabel('t')
ylabel('Solution values')
legend('x(t)','y(t)','z(t)')
hold off
```



Problem 3

```
fprintf('Problem 3\n')
```

```
Problem 3
```

```
clear all
syms t c1 c2
A=[-1,-2; 2,-1]
```

```
A = 2×2
    -1    -2
     2    -1
```

```
[v,d]=eig(sym(A))
```

v =

$$\begin{pmatrix} -i & i \\ 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} -1-2\,\mathrm{i} & 0 \\ 0 & -1+2\,\mathrm{i} \end{pmatrix}$$

```
real(v(:,1))
```

ans =

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

```
a_vec=real(v(:,1))
```

a_vec =

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

```
b_vec=imag(v(:,1))
```

b_vec =

$$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

```
alpha=real(d(1,1))%real part of 1st eigenvalue
```

alpha = -1

```
beta=imag(d(1,1))%imaginary part of 1st eigenvalue
```

beta = -2

```
solnx1=exp(alpha*t)*(a_vec*cos(beta*t)-b_vec*sin(beta*t))
```

solnx1 =

$$\begin{pmatrix} -\sin(2\,t)\,\mathrm{e}^{-t} \\ \cos(2\,t)\,\mathrm{e}^{-t} \end{pmatrix}$$

```
solnx2=exp(alpha*t)*(a_vec*sin(beta*t)+b_vec*cos(beta*t))
```

solnx2 =

$$\begin{pmatrix} -\cos(2\,t)\,\mathrm{e}^{-t} \\ -\sin(2\,t)\,\mathrm{e}^{-t} \end{pmatrix}$$

```
%solnx1 and solnx2 are from Eq.(5.75)
soln=c1*solnx1+c2*solnx2
```

soln =

$$\begin{pmatrix} -c_2\cos(2\,t)\,\mathrm{e}^{-t} - c_1\sin(2\,t)\,\mathrm{e}^{-t} \\ c_1\cos(2\,t)\,\mathrm{e}^{-t} - c_2\sin(2\,t)\,\mathrm{e}^{-t} \end{pmatrix}$$

Problem 4

```
fprintf('Problem 4\n')
```

```
Problem 4
```

```
clear all
syms c1 c2 c3 t
A=sym([-1,2,-4; 0,-1,0; 0,0,-1])
```

A =

$$\begin{pmatrix} -1 & 2 & -4 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
[V,E]=eig(A)
```

V =

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}$$

E =

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
%output for MATLAB 7.0.1 was V(:,1)=[1,0; 0,2; 0,1]
eqA1=A-E(1,1)*eye(3,3)
```

eqA1 =

$$\begin{pmatrix} 0 & 2 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
equ_v1=eqA1\V(:,1) %soln to eqA1*equ v1=V(:,1)
```

```
Warning: Solution is not unique because the system is rank-deficient.
equ_v1 =
```

$$\begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

```
equ_v2=eqA1\V(:,2) %soln to eqA1*equ v2=V(:,2)
```

```
Warning: Solution does not exist because the system is inconsistent.
equ_v2 =
```

$$\begin{pmatrix} \infty \\ \infty \\ \infty \end{pmatrix}$$

```
%only V(:,1) yields an answer and so we work with equ v1
soln=c1*exp(E(1,1)*t)*V(:,2)+c2*exp(E(1,1)*t)*V(:,1)+c3*(t*exp(E(1,1)*t)*V(:,1))+exp(E(1,1)*t)*
```

soln =

$$\begin{pmatrix} c_2\,e^{-t} + c_3\,t\,e^{-t} \\ \frac{e^{-t}}{2} + 2\,c_1\,e^{-t} \\ c_1\,e^{-t} \end{pmatrix}$$

Problem 6

```
fprintf('Problem 6\n')
```

```
Problem 6
```

```
clear all
A=[3, 1, 0; 2, 4, 0; 3, -1, 1]
```

```
A = 3×3
     3     1     0
     2     4     0
     3    -1     1
```

```
lambda=eig(A) %calculates eigenvalues of A
```

```
lambda = 3×1
     1
     5
     2
```

```
[v,d]=eig(A) %calculates eigenvalues AND eigenvectors of A
```

```
v = 3×3
         0    0.4444    0.2357
         0    0.8889   -0.2357
    1.0000    0.1111    0.9428
d = 3×3
     1     0     0
     0     5     0
     0     0     2
```

```
%eigenvectors given as columns of v
%corresponding eigenvalues are on diagonal of d
lambda=eig(sym(A)) %uses sym to find and display eigenvalues
```

lambda =

$$\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$$

```
[v,d]=eig(sym(A))
```

v =

$$\begin{pmatrix} 4 & 0 & \frac{1}{4} \\ 8 & 0 & -\frac{1}{4} \\ 1 & 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

```
d(1,1) %first eigenvalue of A
```

ans = 5

```
v(:,1) %corresponding first eigenvector of A
```

ans =

$$\begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix}$$

```
A*v(:,1)
```

ans =

$$\begin{pmatrix} 20 \\ 40 \\ 5 \end{pmatrix}$$

```
d(1,1)*v(:,1) %verifies that A*v=lambda*v for first
```

ans =

$$\begin{pmatrix} 20 \\ 40 \\ 5 \end{pmatrix}$$

```
% eigenvalue-eigenvector pair; the last two
%answers should be identical
A*v(:,3)
```

ans =

$$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 2 \end{pmatrix}$$

```
d(3,3)*v(:,3) %verifies that A*v=lambda*v for third
```

```
ans =
```

$$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 2 \end{pmatrix}$$

Problem 7

```
fprintf('Problem 7A\n')
```

```
Problem 7A
```

```
clear all
A=[2, 1; 1, 2]
```

```
A = 2×2
     2     1
     1     2
```

```
lambda=eig(A) %calculates eigenvalues of A
```

```
lambda = 2×1
     1
     3
```

```
[v,d]=eig(A) %calculates eigenvalues AND eigenvectors of A
```

```
v = 2×2
   -0.7071    0.7071
    0.7071    0.7071
d = 2×2
     1     0
     0     3
```

```
%eigenvectors given as columns of v
%corresponding eigenvalues are on diagonal of d
lambda=eig(sym(A)) %uses sym to find and display eigenvalues
```

```
lambda =
```

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

```
[v,d]=eig(sym(A))
```

v =

$$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

```
d(1,1) %first eigenvalue of A
```

ans = 1

```
v(:,1) %corresponding first eigenvector of A
```

ans =

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

```
A*v(:,1)
```

ans =

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

```
d(1,1)*v(:,1) %verifies that A*v=lambda*v for first
```

ans =

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

```
% eigenvalue-eigenvector pair; the last two
%answers should be identical
A*v(:,2)
```

ans =

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

```
d(2,2)*v(:,2) %verifies that A*v=lambda*v for third
```

ans =

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

```


fprintf('Problem 7B\n')
```

Problem 7B

```
B=[1, 3; 3, 1]
```

```
B = 2×2
     1     3
     3     1
```

```
lambda=eig(B) %calculates eigenvalues of B
```

```
lambda = 2×1
    -2
     4
```

```
[v,d]=eig(B) %calculates eigenvalues AND eigenvectors of B
```

```
v = 2×2
   -0.7071    0.7071
    0.7071    0.7071
d = 2×2
    -2     0
     0     4
```

```
%eigenvectors given as columns of v
%corresponding eigenvalues are on diagonal of d
lambda=eig(sym(B)) %uses sym to find and display eigenvalues
```

lambda =

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

```
[v,d]=eig(sym(B))
```

v =

$$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$

d =

$$\begin{pmatrix} -2 & 0 \\ 0 & 4 \end{pmatrix}$$

```
d(1,1) %first eigenvalue of B
```

ans = -2

```
v(:,1) %corresponding first eigenvector of B
```

ans =

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

```
B*v(:,1)
```

ans =

$$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

```
d(1,1)*v(:,1) %verifies that B*v=lambda*v for first
```

ans =

$$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

```
% eigenvalue-eigenvector pair; the last two
%answers should be identical
B*v(:,2)
```

ans =

$$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

```
d(2,2)*v(:,2) %verifies that B*v=lambda*v for third
```

ans =

$$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

Problem 10

```
fprintf('Problem 10\n')
```

Problem 10

```
clear all
syms c1 c2 t
A=sym([-4,-1;6,1])
```

A =

$$\begin{pmatrix} -4 & -1 \\ 6 & 1 \end{pmatrix}$$

```
[V,E]=eig(A)
```

V =

$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{3} \\ 1 & 1 \end{pmatrix}$$

E =

$$\begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$$

```
%output for MATLAB 7.0.1 was V(:,1)=[1,0; 0,2; 0,1]
eqA1=A-E(1,1)*eye(2,2)
```

eqA1 =

$$\begin{pmatrix} -2 & -1 \\ 6 & 3 \end{pmatrix}$$

```
equ_v1=eqA1\V(:,1) %soln to eqA1*equ v1=V(:,1)
```

```
Warning: Solution does not exist because the system is inconsistent.
equ_v1 =
```

$$\begin{pmatrix} \infty \\ \infty \end{pmatrix}$$

```
equ_v2=eqA1\V(:,2) %soln to eqA1*equ v2=V(:,2)
```

```
Warning: Solution is not unique because the system is rank-deficient.
equ_v2 =
```

$$\begin{pmatrix} \frac{1}{6} \\ 0 \end{pmatrix}$$

```
%only V(:,2) yields an answer and so we work with equ v2
soln=c1*exp(E(1,1)*t)*V(:,2)+c2*exp(E(1,1)*t)*V(:,1)+exp(E(1,1)*t)*(equ_v2)
```

```
soln =
```

$$\begin{pmatrix} \frac{e^{-2t}}{6} - \frac{c_1 e^{-2t}}{3} - \frac{c_2 e^{-2t}}{2} \\ c_1 e^{-2t} + c_2 e^{-2t} \end{pmatrix}$$

Problem 15

```
fprintf('Problem 15\n')
```

```
Problem 15
```

```
clear all
syms t c1 c2
A=[-4,2; -10, 4]
```

```
A = 2×2
    -4     2
   -10     4
```

```
[v,d]=eig(sym(A))
```

```
v =
```

$$\begin{pmatrix} \frac{2}{5} + \frac{1}{5} i & \frac{2}{5} - \frac{1}{5} i \\ 1 & 1 \end{pmatrix}$$

```
d =
```

$$\begin{pmatrix} -2 i & 0 \\ 0 & 2 i \end{pmatrix}$$

```
real(v(:,1))
```

```
ans =
```

$$\begin{pmatrix} \frac{2}{5} \\ 1 \end{pmatrix}$$

```
a_vec=real(v(:,1))
```

a_vec =

$$\begin{pmatrix} \frac{2}{5} \\ 1 \end{pmatrix}$$

```
b_vec=imag(v(:,1))
```

b_vec =

$$\begin{pmatrix} \frac{1}{5} \\ 0 \end{pmatrix}$$

```
alpha=real(d(1,1))%real part of 1st eigenvalue
```

alpha = 0

```
beta=imag(d(1,1))%imaginary part of 1st eigenvalue
```

beta = -2

```
solnx1=exp(alpha*t)*(a_vec*cos(beta*t)-b_vec*sin(beta*t))
```

solnx1 =

$$\begin{pmatrix} \frac{2\cos(2\,t)}{5} + \frac{\sin(2\,t)}{5} \\ \cos(2\,t) \end{pmatrix}$$

```
solnx2=exp(alpha*t)*(a_vec*sin(beta*t)+b_vec*cos(beta*t))
```

solnx2 =

$$\begin{pmatrix} \frac{\cos(2\,t)}{5} - \frac{2\sin(2\,t)}{5} \\ -\sin(2\,t) \end{pmatrix}$$

```
%solnx1 and solnx2 are from Eq.(5.75)
soln=c1*solnx1+c2*solnx2
```

soln =

$$\begin{pmatrix} c_1\left(\frac{2\cos(2\,t)}{5} + \frac{\sin(2\,t)}{5}\right) + c_2\left(\frac{\cos(2\,t)}{5} - \frac{2\sin(2\,t)}{5}\right) \\ c_1\cos(2\,t) - c_2\sin(2\,t) \end{pmatrix}$$