Chapter 4 #: 1, 2, 4, 5, 6, 7, 8

Problem 1

```
fprintf('Problem 1')
```

```
Problem 1
```

```
clear all
syms x A B C
eqyp=A*x*exp(x)+B*cos(x)+C*sin(x)
```

eqyp = $B\cos(x) + C\sin(x) + A\,x\,e^x$

```
eq1=diff(eqyp,x,2)+2*diff(eqyp,x)-3*eqyp -4*exp(x)+sin(x)
```

eq1 = $\sin(x) - 4\,e^x - 4\,B\cos(x) + 2\,C\cos(x) + 4\,A\,e^x - 2\,B\sin(x) - 4\,C\sin(x)$

```
%The above eq1 is the original ode written as lhs-rhs=0,
%where the unwritten ‘=0' at the end is understood by
%matlab. We now type the relationships of the
%coeffs by inspection of the calculated ode eq1
eq2a=4*A-4 %coefficient of exp(x)
```

eq2a = $4\,A - 4$

```
eq2b=2*C-4*B %coefficient of cos(x)
```

eq2b = $2\,C - 4\,B$

```
eq2c=-4*C-2*B+1 %coefficient of sin(x)
```

eq2c = $1 - 4\,C - 2\,B$

```
[A,B,C]=solve(eq2a,eq2b,eq2c)
```

A = 1

B =

$$\frac{1}{10}$$

C =

$$\frac{1}{5}$$

```
yp=subs(eqyp)
```

yp =

$$\frac{\cos(x)}{10} + \frac{\sin(x)}{5} + x\,e^x$$

Problem 2

```
fprintf('Problem 2')
```

```
Problem 2
```

```
clear all
syms x y(x) A1 A2 A3 B1 B2
eqODE=diff(y(x),x,4)+diff(y(x),x,2)-3*x^2-4*sin(x)+2*cos(x)
```

eqODE =

$$2\cos(x) - 4\sin(x) - 3x^2 + \frac{\partial^2}{\partial x^2}\, y(x) + \frac{\partial^4}{\partial x^4}\, y(x)$$

```
%The above eqODE is the original ode written as lhs-rhs=0,
%where the unwritten =0 at the end is understood by MATLAB
eqyp=A1*x^4+A2*x^3+A3*x^2+B1*x*sin(x)+B2*x*cos(x)
```

eqyp = $A_1\, x^4 + A_2\, x^3 + A_3\, x^2 + B_1\, x \sin(x) + B_2\, x \cos(x)$

```
eq1=subs(eqODE,y(x),eqyp)
```

eq1 = $24\, A_1 + 2\, A_3 + 2\cos(x) - 4\sin(x) + 6\, A_2\, x + 12\, A_1\, x^2 - 2\, B_1 \cos(x) + 2\, B_2 \sin(x) - 3\, x^2$

```
[c1,t1]=coeffs(eq1,x)
```

c1 = $\begin{pmatrix} 12\, A_1 - 3 & 6\, A_2 & 24\, A_1 + 2\, A_3 + 2\cos(x) - 4\sin(x) - 2\, B_1 \cos(x) + 2\, B_2 \sin(x) \end{pmatrix}$

t1 = $\begin{pmatrix} x^2 & x & 1 \end{pmatrix}$

```
eq2a=c1(1) %coefficient of x^2
```

eq2a = $12\, A_1 - 3$

```
eq2b=c1(2) %coefficient of x
```

eq2b = $6\, A_2$

```
c1(3) %this does NOT give the constant terms
```

ans = $24\, A_1 + 2\, A_3 + 2\cos(x) - 4\sin(x) - 2\, B_1 \cos(x) + 2\, B_2 \sin(x)$

```
%Now we need to sub sin(x)=0,cos(x)=0 to get constant terms
eq2c=subs(c1(3),{sin(x),cos(x)},{0,0}) %the const terms
```

eq2c = $24\, A_1 + 2\, A_3$

```
[c2,t2]=coeffs(eq1,cos(x))
```

c2 = $\begin{pmatrix} 2 - 2\, B_1 & 24\, A_1 + 2\, A_3 - 4\sin(x) + 6\, A_2\, x + 12\, A_1\, x^2 + 2\, B_2 \sin(x) - 3\, x^2 \end{pmatrix}$

t2 = $\begin{pmatrix} \cos(x) & 1 \end{pmatrix}$

```
eq2d=c2(1) %coefficient of cos(x)
```

eq2d = $2-2\,B_1$

```
[c3,t3]=coeffs(eq1,sin(x))
```

c3 = $\left(2\,B_2-4 \quad 24\,A_1+2\,A_3+2\cos(x)+6\,A_2\,x+12\,A_1\,x^2-2\,B_1\cos(x)-3\,x^2\right)$

t3 = $\left(\sin(x) \quad 1\right)$

```
eq2e=c3(1) %coefficient of sin(x)
```

eq2e = $2\,B_2-4$

```
[A1,A2,A3,B1,B2]=solve(eq2a,eq2b,eq2c,eq2d,eq2e)
```

A1 =

$$\frac{1}{4}$$

A2 = 0

A3 = -3

B1 = 1

B2 = 2

```
yp=subs(eqyp)
```

yp =

$$2\,x\cos(x)+x\sin(x)-3\,x^2+\frac{x^4}{4}$$

Problem 4

```
fprintf('Problem 4')
```

Problem 4

```
clear all
syms r zeta wn omega F0 m t
zeta=sym('zeta','real')
```

zeta = ζ

```
zeta=sym('zeta','positive')
```

zeta = ζ

```
wn=sym('wn','real')
```

wn = wn

```
wn=sym('wn','positive')
```

wn = wn

```
omega=sym('omega','real')
```

omega = ω

```
omega=sym('omega','positive')
```

omega = ω

```
F0=sym('F0','real')
```

F0 = F_0

```
F0=sym('F0','positive')
```

F0 = F_0

```
m=sym('m','real')
```

m = m

```
m=sym('m','positive')
```

m = m

```
t=sym('t','real')
```

t = t

```
p(r)=r^2+2*zeta*wn*r+wn^2
```

p(r) = $r^2 + 2\,\zeta\, r\,\mathrm{wn} + \mathrm{wn}^2$

```
zpsol=(F0/m)*exp(i*omega*t)/p(i*omega) %complex gain
```

zpsol =

$$\frac{F_0\, \mathrm{e}^{\omega\, t\, \mathrm{i}}}{m\, (-\omega^2 + 2\,\mathrm{i}\,\zeta\,\omega\,\mathrm{wn} + \mathrm{wn}^2)}$$

```
zpsol1=simplify(zpsol*conj(zpsol))/conj(zpsol)
```

zpsol1 =

$$-\frac{F_0\, \mathrm{e}^{\omega\, t\, \mathrm{i}}\, (\omega^2 + 2\,\mathrm{i}\,\zeta\,\omega\,\mathrm{wn} - \mathrm{wn}^2)}{m\, (\omega^4 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2 - 2\,\omega^2\,\mathrm{wn}^2 + \mathrm{wn}^4)}$$

```
g1=abs(zpsol1) %real gain but not nice looking
```

g1 =

$$\frac{F_0\, |\omega^2 + 2\,\mathrm{i}\,\zeta\,\omega\,\mathrm{wn} - \mathrm{wn}^2|}{m\, |\omega^4 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2 - 2\,\omega^2\,\mathrm{wn}^2 + \mathrm{wn}^4|}$$

```
c1=imag(zpsol)
```

```
c1 =
```

$$-\frac{F_0 \sin(\omega\, t)\ (\omega^2 - \mathrm{wn}^2)}{m\ \left((\omega^2 - \mathrm{wn}^2)^2 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2\right)} - \frac{2\,F_0\,\omega\,\mathrm{wn}\,\zeta\ \cos(\omega\, t)}{m\ \left((\omega^2 - \mathrm{wn}^2)^2 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2\right)}$$

```
c2=real(zpsol)
```

```
c2 =
```

$$\frac{2\,F_0\,\omega\,\mathrm{wn}\,\zeta\ \sin(\omega\, t)}{m\ \left((\omega^2 - \mathrm{wn}^2)^2 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2\right)} - \frac{F_0 \cos(\omega\, t)\ (\omega^2 - \mathrm{wn}^2)}{m\ \left((\omega^2 - \mathrm{wn}^2)^2 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2\right)}$$

```
g=simplify(sqrt(c1^2+c2^2)) %also real gain
```

```
g =
```

$$\frac{F_0\ \sqrt{\omega^4 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2 - 2\,\omega^2\,\mathrm{wn}^2 + \mathrm{wn}^4}}{m\,|\omega^4 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2 - 2\,\omega^2\,\mathrm{wn}^2 + \mathrm{wn}^4|}$$

```
eq1=diff(g,omega)
```

```
eq1 =
```

$$\frac{F_0\ (4\,\omega^3 + 8\,\omega\,\mathrm{wn}^2\,\zeta^2 - 4\,\omega\,\mathrm{wn}^2)}{2\,m\,|\sigma_1|\ \sqrt{\sigma_1}} - \frac{F_0\,\mathrm{sign}(\sigma_1)\ (4\,\omega^3 + 8\,\omega\,\mathrm{wn}^2\,\zeta^2 - 4\,\omega\,\mathrm{wn}^2)\ \sqrt{\sigma_1}}{m\,|\sigma_1|^2}$$

where

$$\sigma_1 = \omega^4 + 4\,\omega^2\,\mathrm{wn}^2\,\zeta^2 - 2\,\omega^2\,\mathrm{wn}^2 + \mathrm{wn}^4$$

```
solve(eq1,omega)
```

Problem 5

```
fprintf('Problem 5')
```

```
Problem 5
```

```
clear all
syms x A B C
eqyp=A*x*exp(x)+B*cos(x)+C*sin(x)
```

eqyp = $B\cos(x) + C\,\sin(x) + A\,x\,\mathrm{e}^x$

```
%y''+2y'-3y=4e^x-sinx
eq1=diff(eqyp,x,2)+2*diff(eqyp,x)-3*eqyp -4*exp(x)+sin(x)
```

eq1 = $\sin(x) - 4\,\mathrm{e}^x - 4\,B\cos(x) + 2\,C\,\cos(x) + 4\,A\,\mathrm{e}^x - 2\,B\sin(x) - 4\,C\,\sin(x)$

```
%The above eq1 is the original ode written as lhs-rhs=0,
```

```
%where the unwritten '=0' at the end is understood by
%matlab. We now type the relationships of the
%coeffs by inspection of the calculated ode eq1
eq2a=4*A-4 %coefficient of exp(x)
```

eq2a = $4\,A-4$

```
eq2b=2*C-4*B %coefficient of cos(x)
```

eq2b = $2\,C-4\,B$

```
eq2c=-4*C-2*B+1 %coefficient of sin(x)
```

eq2c = $1-4\,C-2\,B$

```
[A,B,C]=solve(eq2a,eq2b,eq2c)
```

A = 1

B =

$$\frac{1}{10}$$

C =

$$\frac{1}{5}$$

```
yp=subs(eqyp)
```

yp =

$$\frac{\cos(x)}{10}+\frac{\sin(x)}{5}+x\,e^{x}$$

Problem 6

```
fprintf('Problem 6')
```

Problem 6

```
clear all
syms x A B C
eqyp=A*exp(2*x)+B*exp(-3*x)+C
```

eqyp = $C+A\,e^{2\,x}+B\,e^{-3\,x}$

```
%y''-2y'-8y=4e^2x-21e^-3x
eq1=diff(eqyp,x,2)-2*diff(eqyp,x)-8*eqyp -4*exp(2*x)+21*exp(-3*x)
```

eq1 = $21\,e^{-3\,x}-4\,e^{2\,x}-8\,C-8\,A\,e^{2\,x}+7\,B\,e^{-3\,x}$

```

eq2a=-8*A-4 %coefficient of exp(2x)
```

eq2a = $-8\,A-4$

```
eq2b=21+7*B %coefficient of exp(-3x)
```

eq2b = $7\,B+21$

```
eq2c=-8*C %coefficient of 1
```

eq2c = $-8\,C$

```
[A,B,C]=solve(eq2a,eq2b,eq2c)
```

A =

$$-\frac{1}{2}$$

B = -3

C = 0

```
yp=subs(eqyp)
```

yp =

$$-\frac{e^{2\,x}}{2}-3\,e^{-3\,x}$$

Problem 7

```
fprintf('Problem 7')
```

Problem 7

```
clear all
syms x A B C D
eqyp=A*x*cos(x)+B*x*sin(x)+C*cos(x)+D*sin(x)
```

eqyp = $C\,\cos(x)+\mathrm{D}\,\sin(x)+B\,x\,\sin(x)+A\,x\,\cos(x)$

```
%y''-2y'+2y=xcosx
eq1=diff(eqyp,x,2)-2*diff(eqyp,x)+2*eqyp -x*cos(x)
```

eq1 = $2\,B\cos(x)-2\,A\cos(x)+C\,\cos(x)-2\,\mathrm{D}\cos(x)-2\,A\sin(x)-2\,B\sin(x)+2\,C\,\sin(x)+\mathrm{D}\sin(x)-x\cos(x$

```


eq2a=-1+A-2*B %coefficient of x*cos(x)
```

eq2a = $A-2\,B-1$

```
eq2b=2*A+B %coefficient of x*sin(x)
```

eq2b = $2\,A+B$

```
eq2c=2*B-2*A+C-2*D %coefficient of cos(x)
```

eq2c = $2\,B-2\,A+C-2\,\mathrm{D}$

```
eq2d=-2*B+2*C+D %coefficient of sin(x)
```

eq2d = $2\,C - 2\,B + \mathrm{D}$

```
[A,B,C,D]=solve(eq2a,eq2b,eq2c,eq2d)
```

A =

$\frac{1}{5}$

B =

$-\frac{2}{5}$

C =

$-\frac{2}{25}$

D =

$-\frac{16}{25}$

```
yp=subs(eqyp)
```

yp =

$\frac{x\cos(x)}{5} - \frac{16\sin(x)}{25} - \frac{2\cos(x)}{25} - \frac{2\,x\sin(x)}{5}$

Problem 8

```
fprintf('Problem 8')
```

Problem 8

```
clear all
syms x A B
eqyp=A*exp(x)*x*cos(2*x)+B*exp(2*x)*sin(2*x)
```

eqyp = $B\sin(2\,x)\,\mathrm{e}^{2\,x} + A\,x\cos(2\,x)\,\mathrm{e}^{x}$

```
%y''-2y'+ 5y = e^x sin 2x.
eq1=diff(eqyp,x,2)-2*diff(eqyp,x)+5*eqyp -exp(x)*sin(2*x)
```

eq1 = $4\,B\cos(2\,x)\,\mathrm{e}^{2\,x} - \sin(2\,x)\,\mathrm{e}^{x} + B\sin(2\,x)\,\mathrm{e}^{2\,x} - 4\,A\sin(2\,x)\,\mathrm{e}^{x}$

```

eq2a=4*B %coefficient of exp(x)*cos(2x)
```

eq2a = $4\,B$

```
eq2b=-1+B-4*A %coefficient of exp(x)*sin(2x)
```

eq2b = $B - 4\,A - 1$

```
[A,B]=solve(eq2a,eq2b)
```

A =

$$-\frac{1}{4}$$

B = 0

```
yp=subs(eqyp)
```

yp =

$$-\frac{x\cos(2\,x)\,e^x}{4}$$