PHN311 Assignment 4

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Contents

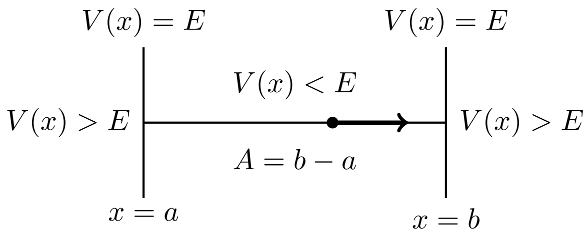
1 Occilation

Consider the one-dimensional motion of a particle of mass m in a time independent potential $V(x)$.Since the energy E is conserved, one can integrate the equation of motion and obtain a solution in a closed form.

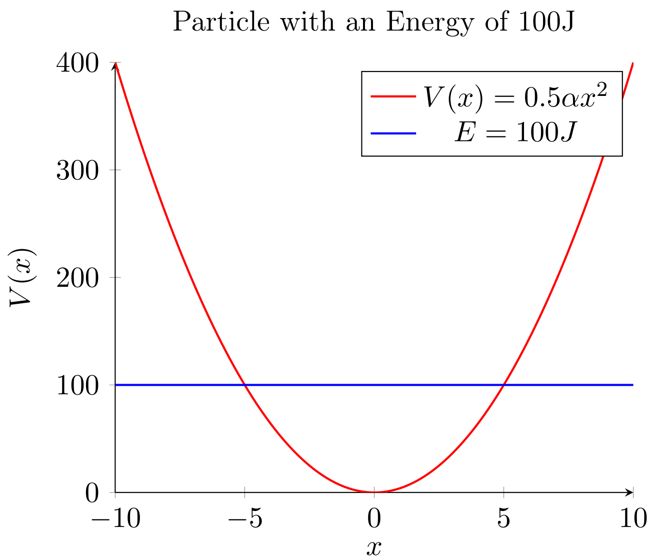
$$t - C = \sqrt{\frac{m}{2}} \int_{x_i}^{x} \frac{dx}{\sqrt{E - V(x)}} \tag{1}$$

For the choice $C = 0$, the particle is at the position x_i at the time $t = 0$ and x refers to its position at any arbitrary time t. Consider a particular case where the particle is in bound motion between twopoints a and b where $V(x) = E$ for $x = a, b$ and $V(x) < E$ for $a < x < b$. The time period of theoscillation T is given by,

$$T = 2\sqrt{\frac{m}{2}} \int_{a}^{b} \frac{dx}{\sqrt{E - V(x)}}$$



1.1 First consider a particle with $m = 1kg$ in the potential $V(x) = 0.5\alpha x^2$ with $\alpha = 4kgsec^2$. Numerically calculate the time period of oscillation and check this against the expected value. Verify that the frequency does not depend on the amplitude of oscillation.



From the above plot, we infer that the for a value of Energy E, the particle is bound for specifc values of a and b (decided by E).

$$E = V(a) = V(b) = 2x^2$$

$$\implies a = -\sqrt{\frac{E}{2}} = -\frac{A}{\sqrt{2}}, \quad b = +\sqrt{\frac{E}{2}} = +\frac{A}{\sqrt{2}}$$

Hence Amplitude of occilation is a function of Energy.

$$A(E) = b - a = \sqrt{E}$$

From the above equations, the alalytic solution,

$$T = \sqrt{2}\int_{-\frac{A}{\sqrt{2}}}^{+\frac{A}{\sqrt{2}}} \frac{dx}{\sqrt{A^2 - 2x^2}} = sin^{-1}\Big(\frac{\sqrt{2}x}{A}\Big)\Big|_{-\frac{A}{\sqrt{2}}}^{+\frac{A}{\sqrt{2}}} = \pi \quad (sec)$$

Using integral identities, T can be written as,

$$T = \sqrt{2}\int_{-\frac{A}{\sqrt{2}}}^{+\frac{A}{\sqrt{2}}} \frac{dx}{\sqrt{A^2 - 2x^2}}$$

Let us solve the above numerically for different Amplitudes,

```
program occilation1
  use mods
  real(dp) :: A(200), T(200), xi(6), wi(6), f, x
  f(x) = x**2
  call gauss_legendre(xi, wi)
  integ = sum(wi*f(xi))
  call linspace(-10._dp, 10._dp, A)
  print *, integ
end program occilation1
```

```
make occilation1
```

2 Modules

Module `mods` consists of all subroutines, variables and procedures that are used in this report. The codeblocks below are all part of the file `mods.f90`

```
module mods
  use, intrinsic :: iso_fortran_env, only: dp => real64
  use stdlib_quadrature, only: gauss_legendre
  implicit none
  contains
```

Linspace Subroutine for creating an array(sequence) with the equal step value.

```
subroutine linspace(from, to, array)
  real(dp), intent(in) :: from, to
  real(dp), intent(out) :: array(:)
  real(dp) :: range
  integer :: n, i
  n = size(array)
  range = to - from
  if (n == 0) return
  if (n == 1) then
     array(1) = from
     return
  end if
  do i=1, n
     array(i) = from + range * (i - 1) / (n - 1)
  end do
end subroutine linspace
```

N-point Gauss Legendre Quadrature

```
!working method using stdlib_quadrature

!integer, parameter :: N
!real(dp), dimension(N) :: x, w
!call gauss_legendre(x, w)
!integ = sum(w*f(x))
```

End of module `mods`

```
end module mods
```

3 Makefile