AMIT KUMAR
2K18/A1/054
14-Jan-2019.

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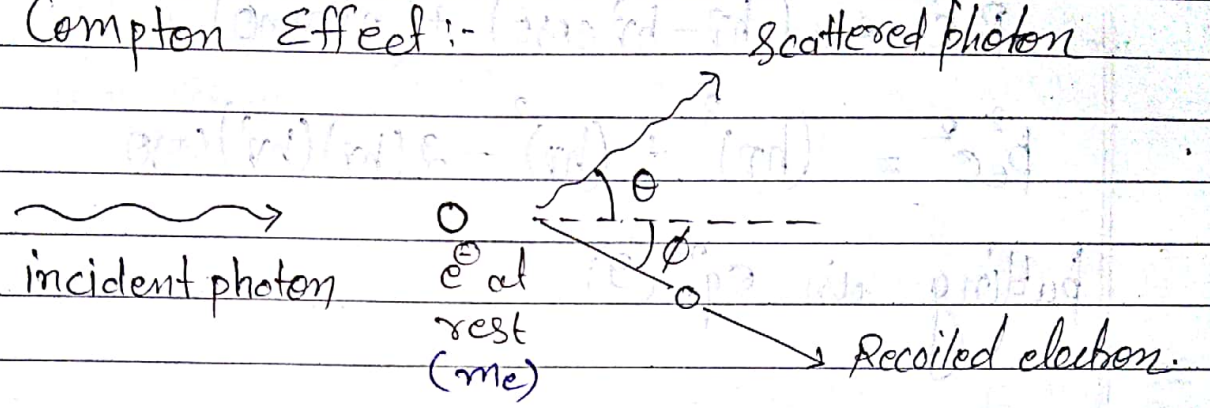
PHYSICS UNIT-1

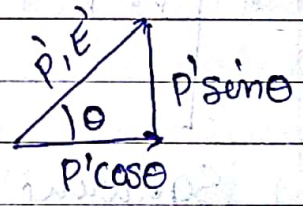
Compton Effect :-



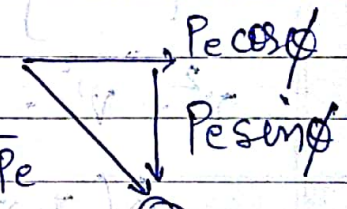
Conservation of momentum

Before = After

X : $\frac{h\nu}{c} + 0 = P'\cos\theta + P_e\cos\phi$



Y : $0 = P'\sin\theta - P_e\sin\phi$



$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + P_e\cos\phi \quad \text{——— (1)}$$

$$0 = \frac{h\nu'}{c}\sin\theta - P_e\sin\phi$$

$$\Rightarrow \frac{h\nu'}{c}\sin\theta = P_e\sin\phi \quad \text{——— (2)}$$

Conservation of energy.

Before = After.

$$h\nu + m_ec^2 = h\nu' + \sqrt{P_e^2c^2 + m_e^2c^4} \quad \text{——— (3)}$$

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Squaring & adding eq ① & eq ②

$$P_e^2 c^2 = (h\nu - h\nu' \cos\theta)^2 + (h\nu' \sin\theta)^2$$

$$P_e^2 c^2 = (h\nu)^2 + (h\nu)^2 - 2(h\nu)(h\nu')\cos\theta$$

putting in eq^n ③

$$h\nu + m_e c^2 = h\nu' + h\sqrt{\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta + \frac{m_e^2 c^4}{h^2}}$$

$$h\left[\nu - \nu' + \frac{m_e c^2}{h}\right] = h\sqrt{\nu^2 + \nu^2 - 2\nu\nu'\cos\theta + \frac{m_e^2 c^4}{h^2}}$$

Squaring both side

$$\nu^2 + \nu'^2 - 2\nu\nu^2 + 2(\nu - \nu')\frac{m_e c^2}{h} = \nu^2 + \nu'^2 - 2\nu\nu'\cos\theta$$

$$2(\nu - \nu')\frac{m_e c^2}{h} = 2\nu\nu' - 2\nu\nu'\cos\theta.$$

$$\cancel{2}(\nu - \nu')\frac{m_e c^2}{h} = \cancel{2}\nu\nu'(1 - \cos\theta)$$

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_e c^2}(1 - \cos\theta)$$

$$\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{m_e c^2}(1 - \cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Compton relation for e^-

$$\frac{h}{m_e c} = \lambda_c$$

$$\lambda_c = 2.4 \times 10^{-12}\, m$$

Compton wavelength.

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Question 1. High energy photons are scattered from $\bar{e}$ initially at rest assume that the photons are back scattered calculate the wavelength shift.

Sol^n

$\theta = 180^\circ$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos(180))$$

$$\lambda' - \lambda = 2.4 \times 10^{-12}(1 - (-1)) = 4.8 \times 10^{-12} m$$

Q2 Consider a photon that scattered from an e^{ns} at rest if the compton wavelength shift is observed 3 times the wavelen of photon & if photon scattered at 60° Calculate

(i) wavelength of scattered e^n

(ii) K. Energy of recoiling e^n.

(iii) angle at which e^n scattered.

Solution

(i) $\lambda' - \lambda = 3\lambda$

$\lambda' = 4\lambda$ $\qquad \theta = 60^\circ$

$$3\lambda = 2.4 \times 10^{-12}\left(1 - \frac{1}{2}\right)$$

$$3\lambda = 1.2 \times 10^{-12} m$$

$$\lambda = 0.4 \times 10^{-12} m$$

(ii) $KE = h\nu - h\nu' = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{4\lambda}\right) = \frac{3hc}{4\lambda}$$

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$$\frac{3}{4} \times \frac{2\times10^{-25}}{4\times0.4\times10^{-12}} = 2.3\,MeV$$

(c) $\frac{h}{\lambda}\cancel{\sin\theta} =$

$$\tan\phi = \left(\frac{\frac{h\nu'}{c}\sin\theta}{\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta}\right)$$

$$\tan\phi = \left(\frac{\nu'\sin\theta}{\nu - \nu'\cos\theta}\right)$$

$$\tan\phi = \left(\frac{\frac{1}{\lambda}\sin\theta}{\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\theta}\right) =$$

Q. Show that the max. K. energy transfer to ~~by an~~ ~~phton~~ to a proton when hit by an photon of energy $h\nu$. is ~~KE~~

$$KE = \frac{h\nu}{1 + \frac{m_p c^2}{2h\nu}}$$

$$K_p = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$\lambda' - \lambda = \frac{h}{m_p c}(1 - \cos\theta)$$

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$$\frac{c}{\lambda} = \nu$$

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$$K_p = hc\left(\frac{h}{m_p c}(1-\cos\theta)\right) \Big/ \frac{c^2}{\lambda_1 \lambda_2 c^2}$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_p c^2}(1-\cos\theta)$$

$$K_p = h\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right) = h(\nu - \nu')$$

$$= h\left(\nu - \frac{m_p c \nu}{m_p c^2 + \nu h(1-\cos\theta)}\right)$$

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_p c^2}(1-\cos\theta)$$

$$\nu' = \frac{m_p c^2 \nu}{m_p c^2 + \nu h(1-\cos\theta)}$$

$$= h\nu\left(1 - \frac{m_p c^2}{m_p c^2 + 2h\nu}\right)$$

$$= h\nu\left(\frac{\cancel{m_p c} + 2h\nu - \cancel{m_p c}}{m_p c^2 + 2h\nu}\right)$$

$$= \frac{2h^2\nu^2}{m_p c + 2h\nu} = \frac{2h\nu}{1 + \frac{m_p c^2}{2h\nu}}$$

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QUANTUM MECHANICS

Wave nature of radiation
particle Nature of Radiation
↳ photoelectric effect
↳ Compton effect

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

→ Dual Nature of Radiation / Light / photon

$$\bar{E} = \bar{P}c \qquad E = \frac{hc}{\lambda}$$

$\lambda = \frac{h}{P}$ for photons (rest mass = 0)
↓
De broglie relation

$\lambda = \frac{h}{P}$ { for all Material particle
Matter Waves"
↓ wave characterstic
→ particle charactic

ω, k $\begin{cases} \hbar\omega = E \\ \hbar k = P \end{cases}$

Wave function

$\psi(x,t)$

$\psi(\bar{r},t)$ Complex No.

$|\psi(\bar{r},t)|^2$ = probability Density

(1) $\int_{-\infty}^{\infty} |\psi(r,t)|^2 d^3r = 1$ Square Integrable

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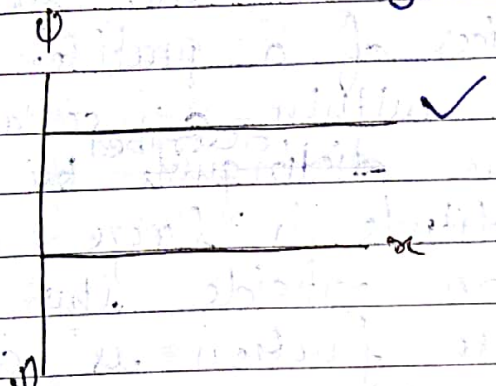
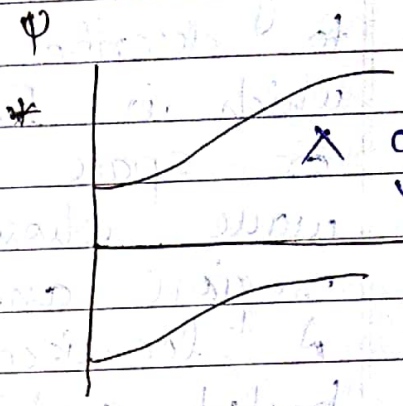
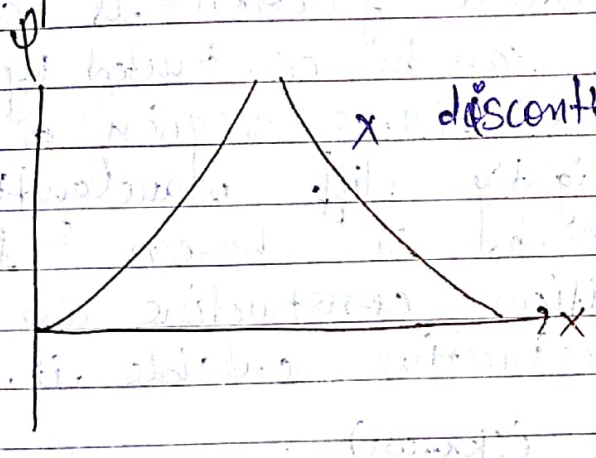
② Continuous & Single variable

③ $\frac{\partial \psi}{\partial x}$, $\frac{\partial \psi}{\partial y}$, $\frac{\partial \psi}{\partial t}$ continuous & single value.

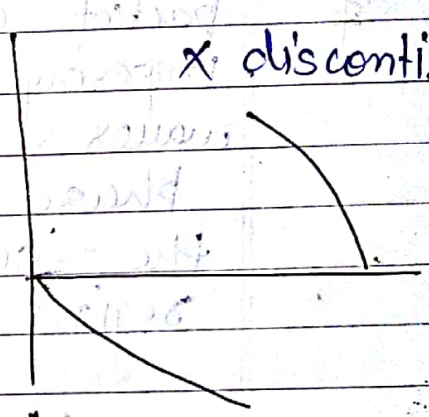
* 

* 

double value

* 

discontinous

* 

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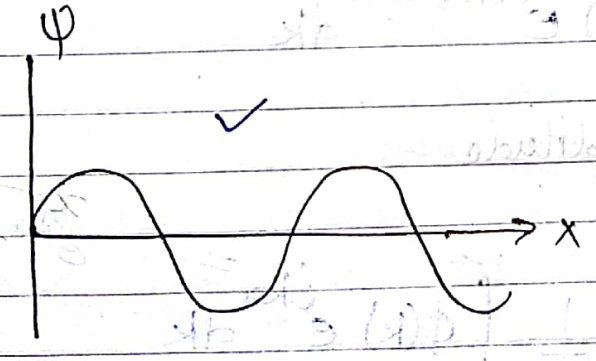
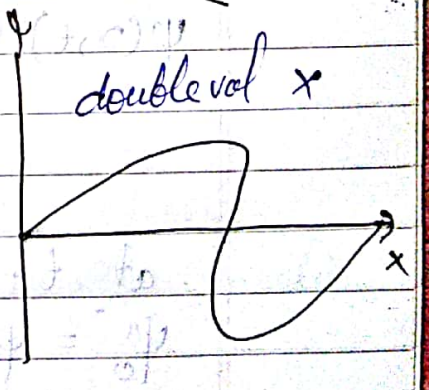
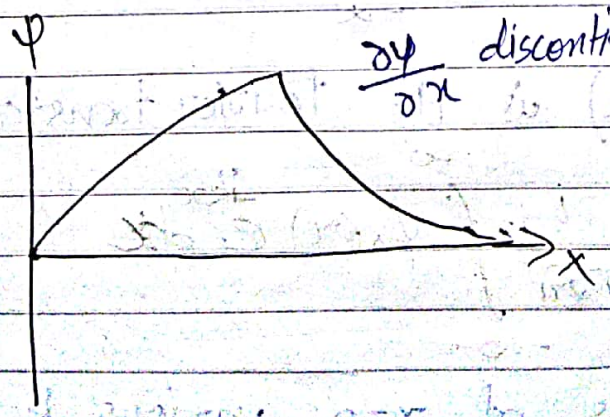
* 

* 

double val

* 

$\frac{\partial \psi}{\partial x}$ discontinuous

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Wave function for a particle

if ψ is made to vanish everywhere except the neighbourhood of particle it can then be used to describe dynamics of a particle. particle which is localised within a certain region or space can be ~~distinguish~~ described by a matter wave whose amplitude is large in that region and zero outside this region. A localised wave function is called wave packet and it can be constructed by superimposing in the same region or space waves of slightly diff. wavelength & but phase & Amp. so chosen to make the superimposition constructive in desired region & destructive outside it.

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{\phi(k)}_{\text{Amplitude}} e^{i(kx-\omega t)} dk$$

at $t=0$

$$\psi_0 = \psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{ikx} dk \Longrightarrow e^{ikx} \begin{cases} x \to 0 \\ \to \end{cases}$$

where $\phi(k)$ is the fourier transform of ψ_0

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi_0(x) e^{-ikx} dx$$

$\psi_0(x)$ peaks at $x=0$ various k integrations add constructively while far away from

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$x=0$ phase e^{ikx} goes through many periods of vidalcel oscillations that leads to destructive oscillation.

~~boris bords~~

In terms of probablstic interpretation it's a greater prob. of finding a particle near at $x=0$ & little prob. of finding away from $x=0$ hence particle is ~~not~~ represented not by a single D. broglie wave of well defined frequency but by a wave packet that is obtained by adding large no. of wave of different frequencies

Motion of Wave ~~Particle~~ packet

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(x) e^{i(kx-\omega t)} dk$$

propagation dictated by $\underline{\omega(k)}$

① $\omega \propto k$

$\omega = v_0 k$ v_0 dimen of velocity

↓ Non dispersive medium

For Non dispersive propagation medium transmits harmonic waves of different frequencies with same velocity.

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$V_g = \frac{d\omega(k)}{k}$ $\qquad$ $V_{ph} = \frac{\omega(k)}{k}$

group velocity $\qquad$ phase velocity

(velocity of wave packet)

$$V_{ph} = \frac{\omega(k)}{k} \Rightarrow \boxed{\omega = kV_{ph}} \quad \text{—— (1)}$$

$$\boxed{V_g = \frac{d\omega}{dk} = V_{ph} + k\left(\frac{dV_{ph}}{dk}\right)} \quad \text{—— (2)}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \frac{dV_{ph}}{dk} = \frac{dV_{ph}}{d\lambda} \cdot \frac{d\lambda}{dk}$$

$$k\frac{dV_{ph}}{dk} = -\frac{2\pi}{k^2} \cdot k\frac{dV_{ph}}{d\lambda}$$

$$= -\lambda\frac{dV_{ph}}{d\lambda}$$

$$V_g = \frac{d\omega}{dk} = V_{ph} + k\frac{dV_{ph}}{dk} = V_{ph} - \lambda\frac{dV_{ph}}{d\lambda} \quad \text{—— (3)}$$

$$k\frac{dV_{ph}}{dk} = \frac{p}{\hbar}\frac{dV_{ph}}{dp} \cdot \frac{dp}{dk} = p\frac{dV_{ph}}{dp}$$

$(p = \hbar k)$

$$V_g = V_{ph} + p\frac{dV_{ph}}{dp} \quad \text{—— (4)}$$

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if $p\frac{dV_{ph}}{dp} = 0$ $\quad V_g = V_{ph}$ $\quad$ Non dispersive Medium.
Eg. (inextensible string)

$p\frac{dV_{ph}}{dp} \neq 0$ $\quad V_g \neq V_{ph}$ $\quad$ Dispersive Medium

Total Energy = K.E + P.E

$$E(p) = \frac{p^2}{2m} + V \quad \text{——— (A)}$$

$E = \hbar\omega$
$p = \hbar k$

$$V_g = \frac{d\omega}{dk} = \frac{d(E/\hbar)}{d(p/\hbar)} = \frac{dE(p)}{dp}$$

using (A)

$$E(p) = \frac{p}{m} + 0$$

$$V_g = \frac{dE}{dp} = \frac{p}{m} = v \text{ (particle velocity)}$$

group velocity is thus equal to classical velocity of particle and hence we should view the centre of wavepacket as travelling like a classical particle that obeys classical laws and hence wave packet concept offers the clear connection b/w Classical description and Quantum description.

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Q. Consider an inextensible string of linear density μ if the string is subjected to a Tension 'T' the angular frequency of string wave is given by $\omega = k\sqrt{\frac{T}{\mu}}$ find the group phase & velocity.

$$V_g = \frac{d\omega}{dk} = \frac{d\left(k\sqrt{T/\mu}\right)}{dk} = \sqrt{\frac{T}{\mu}} = V_{ph}$$

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1) Material particle (eg e^-, proton, atom...) need wave description.

2) $\psi(x,t)$ wave fun^n / state of particle.

3) $|\psi(x,t)|^2 \rightarrow$ probability of finding the particle.

$|\psi(x,t)|^2 dx \rightarrow$ probability of finding the particle in range x to $x+dx$

4) Wavepacket $\rightarrow$ represent a general wave fun^n (localized wave)

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k)\, e^{i(kx-\omega t)}\, dk$$

How the wavefunction evolves in time

$\hookrightarrow$ wave equation.

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Schrodinger Wave Equation.

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + U(x,t)\psi(x,t)}$$

potential Energy

Consider free wave fun of particle

$$\psi(x,t) = A e^{i(kx-\omega t)} + E \left\{ = \frac{p^2}{2m} + U \right.$$

$$\frac{\partial \psi}{\partial x} = A e^{i(kx-\omega t)}(ik) + 0$$

$$\frac{\partial^2 \psi}{\partial x^2} = -Ak^2 e^{i(kx-\omega t)} + 0 = -k^2\psi$$

$$\frac{\partial \psi}{\partial t} = A e^{i(kx-\omega t)}(-\omega i) = -i\omega\,\psi(x,t)$$

$$k = \frac{p}{\hbar} \qquad \frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2}{\hbar^2}\psi \qquad \hbar = \frac{h}{2\pi}$$

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar}\psi$$

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$$E = \frac{-\hbar}{i} \frac{\partial \psi}{\partial t} \frac{1}{\psi}$$

$$p^2 = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \frac{1}{\psi}$$

$$E = \frac{p^2}{2m} + U$$

$$\frac{i\hbar}{\psi} \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \frac{1}{\psi} + U$$

1-D

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}(x,t) + U(x,t)\,\psi(x,t)$$

Time dependent Schordinger wave equation.

3-D

$$i\hbar \frac{\partial \psi}{\partial t}(\vec{r},t) = \frac{-\hbar^2}{2m}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right] + U(\vec{r},t)\,\psi(\vec{r},t)$$

$\Rightarrow$ If potential U does not depend on time $\Rightarrow$

$$U(\underset{\times}{x},t) \text{ ——— } U(x)$$

$$\psi(x,t) = e^{i(kx-\omega t)}$$

$$\psi(x,t) = \Psi(x)\, e^{-i\omega t}$$

Substituting in Schordinger equation.

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$$i\hbar \frac{\partial}{\partial t}\left(\Psi(x) e^{-i\omega t}\right) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left\{\Psi(x) e^{-i\omega t}\right\} + U(x)\Psi(x) e^{-i\omega t}$$

$$i\hbar \Psi(x)(-i\omega)\cancel{e^{-i\omega t}} = \frac{-\hbar^2}{2m}\cancel{e^{-i\omega t}}\frac{\partial^2}{\partial x^2}(\Psi(x)) + U(x)\Psi(x)\cancel{e^{-i\omega t}}$$

$$\Psi(x)(\omega\hbar) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}(\Psi(x)) + U(x)\Psi(x)$$

1-D

$$E\Psi(x) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}(\Psi(x)) + U(x)\Psi(x)$$

Time independent Schordinger wave eqn

3-D

$$\boxed{E\Psi(\bar{r}) = \frac{-\hbar^2}{2m}\nabla^2(\bar{r}) + U(\bar{r})\Psi(r)}$$

Schordinger eqn can't be derived from other basic principles of physics it is a basic principle in itself what we must do is postulate Schordinger eqn solve it for variety of physical situations & compare the result of calculation with experimtal result. If the both result agree, the postulate embodied in schordinger eqn is valid

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QUANTUM PHYSICS

Schrodinger Equation :-

(Time dependent)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + U(x,t)\,\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

(Time Independent)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x).$$

Application of Schrondinger eqn

{particle in a Box}.

$U(x) = 0 \quad 0 < x < L$

$\infty \quad x < 0$

$\infty \quad x > L$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

for region 2.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + 0\,\psi(x) = E\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} - E\psi(x) = 0$$

$$+\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + E\psi(x) = 0$$

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$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar}\psi = 0$$

let $\frac{2mE}{\hbar} = k^2$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2\psi = 0$$

$$\psi = A\sin kx + B\cos kx$$

at $x=0$ $\psi = 0$

$0 = A\sin k\cdot 0 + B\cos k\cdot 0$

$0 = 0 + B(1)$

$\boxed{B = 0}$

at $x = L$ $\psi(x) = 0$

$0 = A\sin kL + 0$

$\sin kL = 0$

$kL = n\pi$

$k = \frac{n\pi}{L}$

$n = 0, 1, 2, \ldots n$

$$\psi_n = A\sin\left(\frac{n\pi x}{L}\right)$$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mEL^2}$$

$$\int_0^L |\psi_n|^2 dx = 1 = A^2\int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx$$

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$$= \frac{A^2}{2}\left[\int_0^L dx - \int_0^L \cos\left(\frac{2n\pi x}{L}\right)dx\right] = 1$$

$$\frac{A^2}{2}\left[x - \left(\frac{L}{2n\pi}\right)\sin\frac{2n\pi x}{L}\right]_0^L = 1$$

$$\frac{A^2}{2} L = 1$$

$$A = \sqrt{\frac{2}{L}}$$

$$\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

Zero point Energy :-

If the particle has 0 energy it will be at rest inside the & this violats heisenberg's uncertanity rule by localising the particle to a limited region it will aquired finite momentum leading to minimum K.E. the ZPE therefore reflects the ~~min~~ nessecity of min motion of a particle due to localities ZPE occurs in all bound state potential.

Consequence:-

Without the ZPM atoms would not be stable as electrons would fall into nucleus also it the zero pt energy which prevent He to freeze at very low Temp

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Q. Calculate ZPE for a particle in a infinite potential well for following three case

a) 100gm ball confined on a 5m long li

b) an oxygen atom confined to a 2×10^{-10} m lattice

c) an e^{n} confined to a 10×10^{-10} atom.

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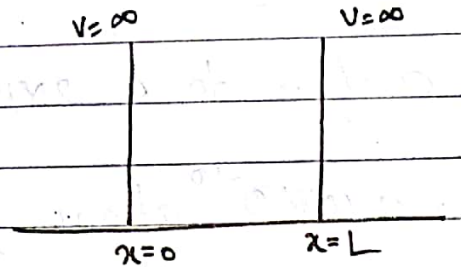
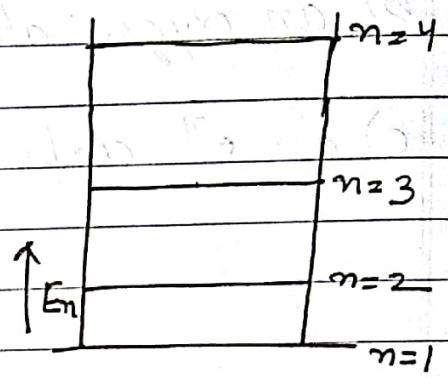
Schrodinger Equation :-

① Particle in a Infinite Potential

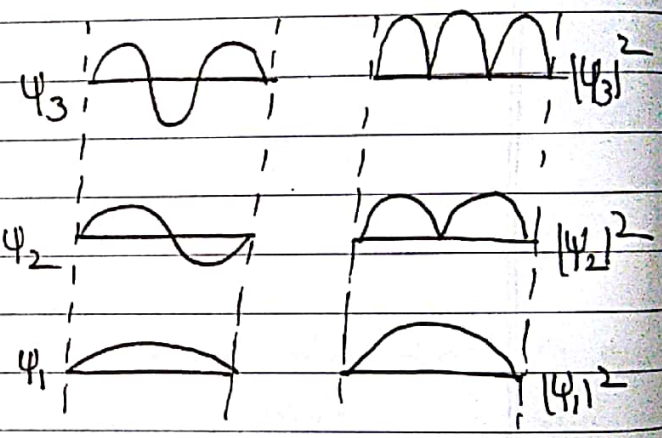




$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + E\psi(x) = 0$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$\psi_n = \sqrt{\frac{2}{L}}\,\sin\frac{n\pi x}{L}$$

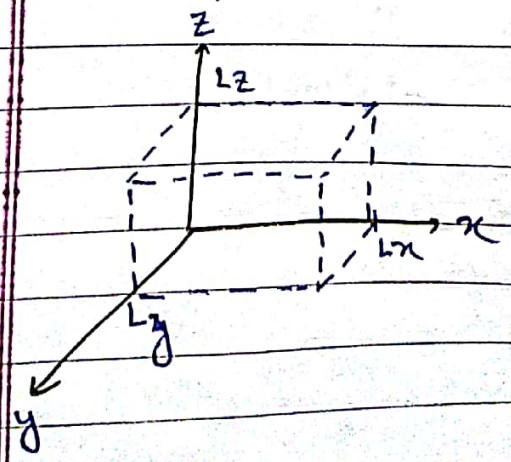


$$E_n/_{n=1} = \frac{\pi^2\hbar^2}{2mL^2}$$

Zero point Energy

$$\sqrt{\frac{2}{L}}\,\sin\frac{\pi x}{L} = \psi_1 \left(\text{Ground State}\right)$$

3-D Infinite Potential Box :-



$U = 0$ $\quad 0 < x < L_x$

$0 < y < L_y$

$0 < z < L_z$

∞ elsewhere

n_x, n_y, n_z

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$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+E\psi(x,y,z)=0$$

$$\psi_{n_x,n_y,n_z}=\left(\sqrt{\frac{2}{L_x}}\sin\frac{n_x\pi x}{L_x}\right)\left(\sqrt{\frac{2}{L_y}}\sin\frac{n_y\pi y}{L_y}\right)\left(\sqrt{\frac{2}{L_y}}\sin\frac{n_z\pi z}{L_z}\right)$$

↓ Energy Eigen function

$$E_{n_x,n_y,n_z}=\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right)\frac{\pi^2\hbar^2}{2m}$$

↳ Energy Eigenvalue.

for $L_x=L_y=L_z=L$ (cube)

$$\psi_{n_x,n_y,n_z}=\sqrt{\frac{8}{L^3}}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)\sin\left(\frac{n_z\pi z}{L}\right)$$

$$E_{n_x,n_y,n_z}=(n_x^2+n_y^2+n_z^2)\frac{\pi^2\hbar^2}{2mL^2}$$

n_x	n_y	n_z	ψ	E
1	1	1	$\psi_{111}=\sqrt{\frac{8}{L^3}}\sin\frac{\pi x}{L}\sin\frac{\pi y}{L}\sin\frac{\pi z}{L}$	$E_{111}=\frac{3\pi^2\hbar^2}{2mL^2}$
2	1	1		
1	2	1	$\psi_{211}\neq\psi_{121}\neq\psi_{112}$	$E_{211}=\frac{6\pi^2\hbar^2}{2mL^2}$
1	1	2		E_{121}, E_{112}

3 fold Degenerate

Energy degeneracy ← for same value of energy?

2	2	1	$\psi_{221}\neq\psi_{122}\neq\psi_{212}$	$E_{221}=E_{122}=E_{212}=\frac{9\pi^2\hbar^2}{2mL}$
1	2	2		
2	1	2		

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Q. find the probility that a particle traped in a box 'L' wide can be found b/w 0.45L & 0.55L for ground & first & Ist excited state

$$P = \int |\psi|^2 dx$$

$$P = \int_{0.45L}^{0.55L} \left(\sqrt{\frac{2}{L}} \sin^2 \frac{\pi x}{L}\right)^2 dx$$

$$\sqrt{\frac{2}{L}} \cancel{-\cos^2 \frac{\pi x}{L}} \quad L$$

$$\frac{2}{L} \int_{0.45L}^{0.55L} \sin^2 \frac{\pi x}{L} dx$$

$n=1$ $P=0.0198$
$n=2$ · $P=0.0065$

$$\text{Expectation Value } (x) = \int_{x_1}^{x_2} x|\psi|^2 dx$$

Harmonic Oscillator

$$\left\{ \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + (E-U)\psi = 0 \right.$$

Harmonic oscillator

$$U = \frac{1}{2} kx$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}\left(E - \frac{1}{2}kx^2\right)\psi = 0$$

put $y = \left(\frac{\sqrt{km}}{\hbar}\right)^{1/2} x = \sqrt{\frac{2\pi m \nu}{\hbar}}\, x$

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$$\alpha = \frac{2E}{\hbar}\sqrt{\frac{m}{K}} = \frac{2E}{h\nu} \qquad \nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

classical freq of oscillator

In terms of y & α, Schrodinger eqn becomes

$$\frac{d^2\psi}{dy^2} + (\alpha - y^2)\psi = 0$$

$$\frac{d^2\psi}{dy^2} + (\alpha - y^2)\psi = 0 \quad \text{——— (1)}$$

Solution ~~of~~ to this equation that are acceptable ~~here are~~ limited by the condition $\psi \to 0$ as $y \to \infty$ in order that

$$\int_{-\infty}^{\infty} |\psi|^2 dy = 1$$

Mathematical properties of (1) are such that this condition will be fullfilled only when

$$\alpha = 2n+1 \; ; \qquad n = 0, 1, 2, 3, \ldots$$

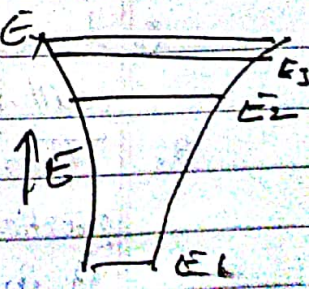
Since $\alpha = \frac{2E}{h\nu}$ Energy level of a Harmonic Oscillation is given by

$$E_n = \left(n + \frac{1}{2}\right) h\nu \; ; n = 0, 1, 2, \ldots$$

Zero point Energy $E_0 = \frac{1}{2} h\nu$

$E_n \propto \frac{1}{n^2}$

H-atom



Wave function

$$\psi_n = \left(\frac{2m\nu}{\hbar}\right)^{1/4} \left(\frac{n}{2^n n!}\right)^{-1/2} H_n(y)\, e^{-y^2/2}$$

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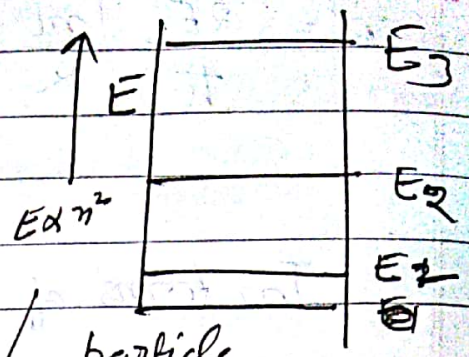
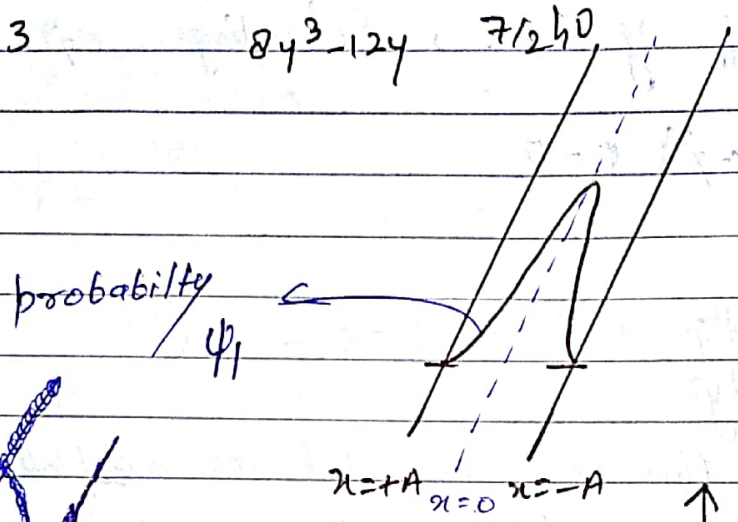
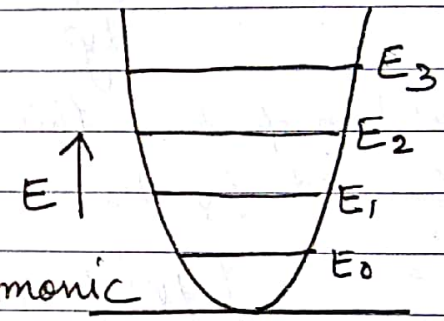
n	$H_n(y)$	E_n
0	1	$\frac{1}{2}h\nu$
1	$2y$	$\frac{3}{2}h\nu$
2	$4y^2-2$	$\frac{5}{2}h\nu$
3	$8y^3-12y$	$\frac{7}{2}h\nu$

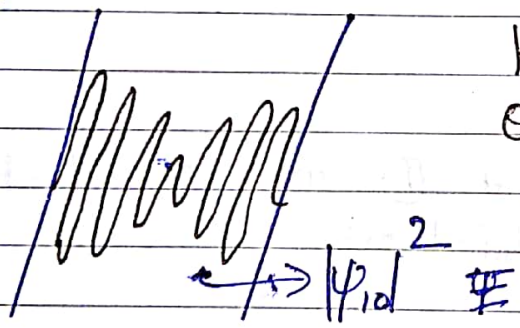


particle in a Box





Harmonic Oscillator.



Difference B/w Quantum Oscillation & Classical

1. The allowed Energies do not form a continuous spectrum but instead a discreate spectrum of certain specific values is obtained.
2. Lowest allowed energy will not be $E=0$ but will be some difinite minimum i.e $E=E_0=\frac{1}{2}h\nu$.
3. There will be a certain probability that a particle can penitrate the potential wall it is in & go beyonds the limits of $-A$ & $+A$.

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