

Fig. 3.17 Density of states functions, Fermi-Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is below the intrinsic Fermi energy

$$n_0 = N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right]$$

and

$$p_0 = N_v \exp \left[\frac{-(E_F - E_v)}{kT} \right]$$

As we just discussed, the Fermi energy may vary through the bandgap energy, which will then change the values of n_0 and p_0 .

Example 3.10

Objective ✓ To calculate the thermal equilibrium concentrations of electrons and holes for a given Fermi energy.

Consider silicon at $T = 300$ K so that $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$ and $N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$. Assume that the Fermi energy is 0.25 eV below the conduction band. If we assume that the bandgap energy of silicon is 1.12 eV, then the Fermi energy will be 0.87 eV above the valence band.

Using Eq. (3.16), we have

$$n_0 = (2.8 \times 10^{19}) \exp\left(\frac{-0.25}{0.0259}\right) = 1.8 \times 10^{15} \text{ cm}^{-3}$$

From Eq. (3.24), we can write

$$p_0 = (1.04 \times 10^{19}) \exp\left(\frac{-0.87}{0.0259}\right) = 2.7 \times 10^4 \text{ cm}^{-3}$$

Comment The change in the Fermi level is actually a function of the donor or acceptor impurity concentrations that are added to the semiconductor. However, this example shows that electron and hole concentrations change by orders of magnitude from the intrinsic carrier concentration as the Fermi energy changes by a few tenths of an electron-volt.

In this example, since $n_0 > p_0$, the semiconductor is *n* type. In an *n*-type semiconductor, electrons are referred to as the majority carrier and holes as the minority carrier. By comparing the relative values of n_0 and p_0 in the example, it is easy to see how this designation came about. Similarly, in a *p*-type semiconductor where $p_0 > n_0$, holes are the majority carrier and electrons are the minority carrier.

We may derive another form of the equations for the thermal-equilibrium concentrations of electrons and holes. If we add and subtract an intrinsic Fermi energy in the exponent of Eq. (3.16), we can write

$$n_0 = N_c \exp\left[\frac{-(E_c - E_{Fi}) + (E_F - E_{Fi})}{kT}\right] \quad (3.43a)$$

$$n_0 = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \exp\left[\frac{(E_F - E_{Fi})}{kT}\right] \quad (3.43b)$$

The intrinsic carrier concentration is given by Eq. (3.25) as

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \quad (3.44)$$

Similarly, if we add and subtract an intrinsic Fermi energy in the exponent of Eq. (3.24), we will obtain

$$p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right] \quad (3.45)$$

As we will see, the Fermi level changes when donors and acceptors are added, but Eqs (3.44) and (3.45) show that, as the Fermi level changes from the intrinsic Fermi level, n_0 and p_0 change from the n_i value. If $E_F > E_{Fi}$, then we will have $n_0 > n_i$ and $p_0 < n_i$. One characteristic of an *n*-type semiconductor is that $E_F > E_{Fi}$ so that $n_0 > p_0$. Similarly, in a *p*-type semiconductor, $E_F < E_{Fi}$ so that $p_0 > n_i$ and $n_0 < n_i$; thus $p_0 > n_0$.

We can see the functional dependence of n_0 and p_0 with E_F in Fig. 3.16 and 3.17. As E_F moves above or below E_{Fi} , the overlapping probability function with the density of states functions in the conduction band and valence band changes. As E_F moves above E_{Fi} , the probability function in the conduction band increases,

while the probability, $1 - f_F(E)$, of an empty state (hole) in the valence band decreases. As E_F moves below E_{F0} , the opposite occurs.

3.4.2 The $n_0 p_0$ Product

We may take the product of the general expressions for n_0 and p_0 as given in Eq. (3.16) and (3.24), respectively,

The result is

$$n_0 p_0 = N_c N_v \exp\left[\frac{-(E_c - E_F)}{kT}\right] \exp\left[\frac{-(E_F - E_v)}{kT}\right] \quad (3.46)$$

which may be written as

$$n_0 p_0 = N_c N_v \exp\left[\frac{-E_g}{kT}\right] \quad (3.47)$$

As Eq. (3.47) was derived for a general value of Fermi energy, the values of n_0 and p_0 are not necessarily equal. However, Eq. (3.47) is exactly the same as Eq. (3.28), which we derived for the case of an intrinsic semiconductor. We then have that, for the semiconductor in thermal equilibrium,

$$n_0 p_0 = n_i^2 \quad (3.48)$$

Equation (3.48) states that the product of n_0 and p_0 is always a constant for a given semiconductor material at a given temperature. Although this equation seems very simple, it is one of the fundamental principles of semiconductors in thermal equilibrium. The significance of this relation will become more apparent in the chapters that follow. It is important to keep in mind that Eq. (3.48) was derived using the Boltzmann approximation. If the Boltzmann approximation is not valid, then likewise, Eq. (3.48) is not valid.

- An extrinsic semiconductor in thermal equilibrium does not, strictly speaking, contain an intrinsic carrier concentration, although some thermally generated carriers are present. The intrinsic electron and hole carrier concentrations are modified by the donor or acceptor impurities. However, we may think of the intrinsic concentration n_i in Eq. (3.48) simply as a parameter of the semiconductor material.

3.4.3 The Fermi-Dirac Integral

In the derivation of the Eqs (3.16) and (3.24) for the thermal-equilibrium electron and hole concentrations, we assumed that the Boltzmann approximation was valid. If the Boltzmann approximation does not hold, the thermal-equilibrium electron concentration is written from Eq. (3.8) as

$$n_0 = \frac{4\pi}{h^3} (2m_n^*)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_c)^{1/2} dE}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \quad (3.49)$$

If we again make a change of variable and let

$$\eta = \frac{E - E_c}{kT}$$

$$\eta_F = \frac{E_F - E_c}{kT} \quad (3.50a)$$

$$(3.50b)$$

then we can rewrite Eq. (3.49) as

$$n_0 = 4\pi \left(\frac{2m_e kT}{\hbar^2} \right)^{3/2} \int_0^\infty \frac{\eta^{5/2} d\eta}{1 + \exp(\eta - \eta_F)} \quad (3.51)$$

The integral is defined as

$$F_{1/2}(\eta_F) = \int_0^\infty \frac{\eta^{5/2} d\eta}{1 + \exp(\eta - \eta_F)} \quad (3.52)$$

This function, called the Fermi-Dirac integral, is a tabulated function of the variable η_F . Figure 3.18 is a plot of the Fermi-Dirac integral. Note that if $\eta_F > 0$, then $E_F > E_c$; thus the Fermi energy is actually in the conduction band.

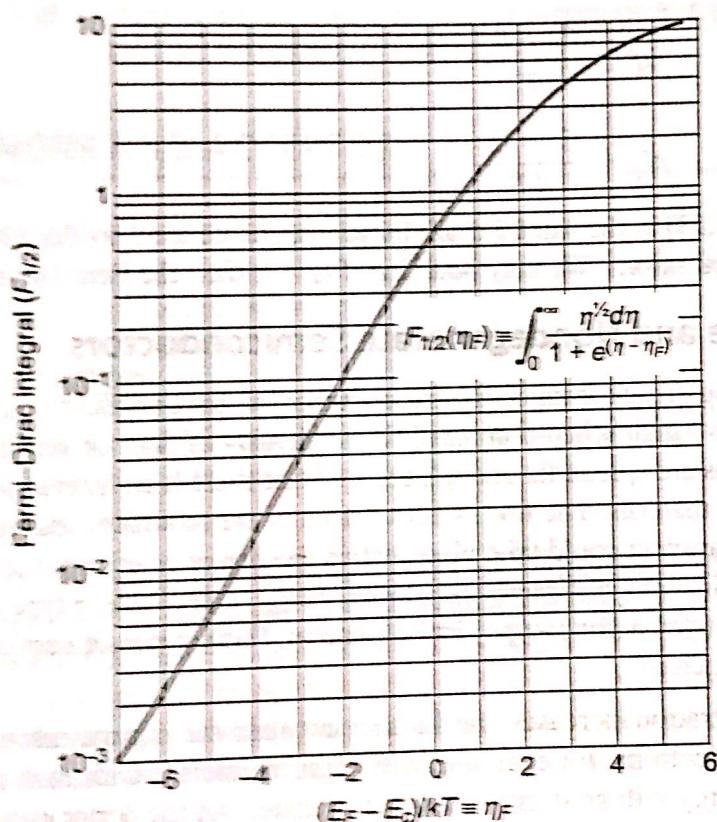


Fig. 3.18 The Fermi-Dirac integral $F_{1/2}$ as a function of the Fermi energy (From Sze [13])

Example 3.11

Objective To calculate the electron concentration using the Fermi-Dirac integral.

Let $\eta_F = 2$ so that the Fermi energy is above the conduction band by approximately 52 meV at $T = 300$ K.

Equation (3.51) can be written as

$$n_0 = \frac{2}{\sqrt{\pi}} N_c F_{1/2}(\eta_F)$$

For silicon at 300 K, $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$ and, from Fig. 3.18, the Fermi-Dirac integral has a value $F_{1/2}(2) = 2.3$. Then

$$n_0 = \frac{2}{\sqrt{\pi}} (2.8 \times 10^{19}) (2.3) = 7.27 \times 10^{19} \text{ cm}^{-3}$$

Comment Note that if we had used Eq. (3.16), the thermal equilibrium value of n_0 would be $n_0 = 2.8 \times 10^{20} \text{ cm}^{-3}$, which is incorrect since the Boltzmann approximation is not valid for this case.

We may use the same general method to calculate the thermal equilibrium concentration of holes. We ob-

$$p_0 = 4\pi \left(\frac{2m_p^* kT}{h^2} \right)^{3/2} \int_0^\infty \frac{(\eta')^{1/2} d\eta'}{1 + \exp(\eta' - \eta'_F)}$$

where

and

$$\eta' = \frac{E_v - E}{kT} \quad (3.53)$$

$$\eta'_F = \frac{E_v - E_F}{kT} \quad (3.54)$$

The integral in Eq. (3.53) is the same Fermi-Dirac integral defined by Eq. (3.52), although the variables have slightly different definitions. We may note that if $\eta'_F > 0$, then the Fermi level is in the valence band.

3.4.4 Degenerate and Nondegenerate Semiconductors

In our discussion of adding dopant atoms to a semiconductor, we have implicitly assumed that the concentration of dopant atoms added is small when compared to the density of host or semiconductor atoms. The small number of impurity atoms are spread far enough apart so that there is no interaction between donor electrons, for example, in an *n*-type material. The few impurity atoms were so widely spaced throughout the sample, we can say that no charge transport could take place within the donor or acceptor levels. We have assumed that the impurities introduce discrete, noninteracting donor energy states in the *n*-type semiconductor and discrete noninteracting acceptor states in the *p*-type semiconductor. These types of semiconductors are referred to as nondegenerate semiconductors.

If the impurity concentration increases, the distance between the impurity atoms decreases and a point will be reached when donor electrons, for example, will begin to interact with each other. When this occurs, the single discrete donor energy will split into a band of energies. As the donor concentration further increases, the band of donor states widens and may overlap the bottom of the conduction band. This overlap occurs when the donor concentration becomes comparable with the effective density of states. When the concentration of electrons in the conduction band exceeds the density of states N_c , the Fermi energy lies within the conduction band. This type of semiconductor is called a degenerate *n*-type semiconductor.

In a similar way, as the acceptor doping concentration increases in a *p*-type semiconductor, the discrete acceptor energy states will split into a band of energies and may overlap the top of the valence band. The Fermi energy will lie in the valence band when the concentration of holes exceeds the density of states N_A . This type of semiconductor is called a degenerate *p*-type semiconductor.

Schematic models of the energy-band diagrams for a degenerate *n*-type and degenerate *p*-type semiconductor are shown in Fig. 3.19. The energy states below E_F are mostly filled with electrons and the energy states above E_F are mostly empty. In the degenerate *n*-type semiconductor, the states between E_F and E_c are mostly filled.