Measurement of Three-Phase Power

29.1 INTRODUCTION

The power in a 3-phase circuit is equal to the sum of the powers in three phases. For a balanced system

$$\text{total power} = 3 \times \text{power in one phase}$$

The power in any one phase is measured by a wattmeter. The total power is obtained by multiplying the wattmeter reading by three.

29.2 ONE-WATTMETER METHOD

Figure 29.1 shows a circuit for the measurement of power in one phase of a 3-phase balanced circuit where the star point is available. The current coil is connected in one phase and the voltage coil is connected between the same phase and star point. The reading on the wattmeter gives the power per phase. The total 3-phase power is given by three times the wattmeter reading.

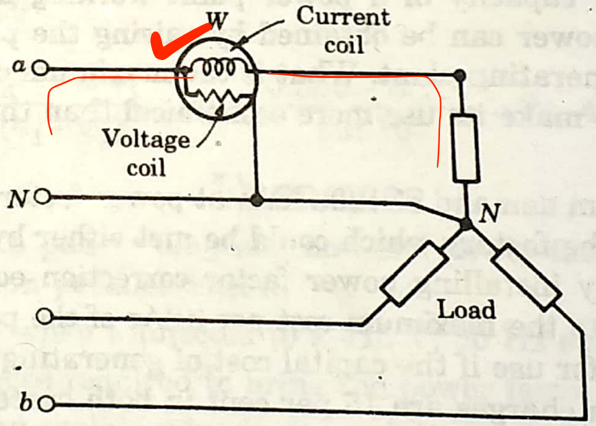

Fig. 29.1. One wattmeter method of power measurement where star point is accessible.

If the neutral point is not available an artificial neutral point N' is created by connecting two resistors R, R as shown in Fig. 29.2.

The total power is given by three times the wattmeter reading.

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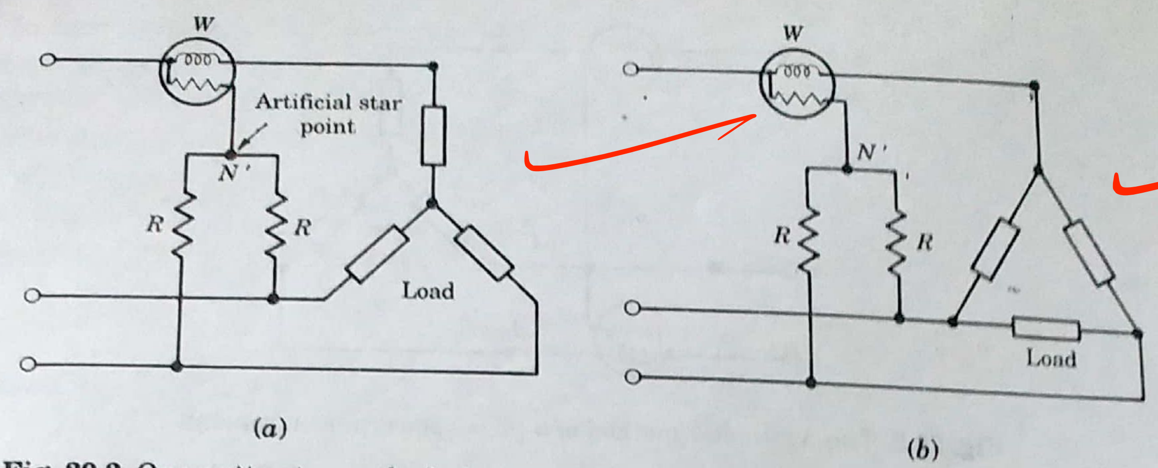

Fig. 29.2. One wattmeter method of power measurement in a 3-phase balanced system where neutral point is not available (*a*) star connected system (*b*) delta connected system

29.3 TWO-WATTMETER METHOD

For a 3-phase 3-wire system, whether balanced or unbalanced, star or delta connected, only two wattmeters are required to measure the three-phase power. This is the most popular method of measuring power. Fig. 29.3 shows the connections of the two wattmeters for a star-connected load. The wattmeter connections are the same for a delta connected load also. The current coils of the two wattmeters are connected to any two of the lines while their voltage coils are connected between the corresponding lines and the third line.

Total instantaneous power supplied to the load is

$$p = v_a\, i_a + v_b\, i_b + v_c\, i_c \tag{29.3.1}$$

In a 3-wire system $\quad i_a + i_b + i_c = 0 \qquad (29.3.2)$

Since i_c is not flowing in either of the wattmeters we eliminate i_c from Eqs. (29.3.1) and (29.3.2)

$$i_c = -(i_a + i_b)$$

$$p = v_a\, i_a + v_b\, i_b - v_c\,(i_a + i_b)$$

or

$$p = (v_a - v_c)\, i_a + (v_b - v_c)\, i_b \tag{29.3.3}$$

Now $(v_a - v_c)\, i_a$ = (instantaneous voltage across the voltage coil of wattmeters W_1) × (current in the current coil of wattmeter W_1)

$$= \text{instantaneous power recorded by } W_1 = p_1$$

and $(v_b - v_c)\, i_b$ = (instantaneous voltage across the voltage coil of wattmeter W_2) × (current in the current coil of wattmeter W_2)

$$= \text{instantaneous power recorded by } W_2 = p_2$$

Therefore $\quad p = p_1 + p_2 \qquad (29.3.4)$

Average value of p

$$= \text{average value of } p_1 + \text{average value of } p_2$$

$$P = \text{reading of } W_1 + \text{reading of } W_2 \tag{29.3.5}$$

$$P = P_1 + P_2$$

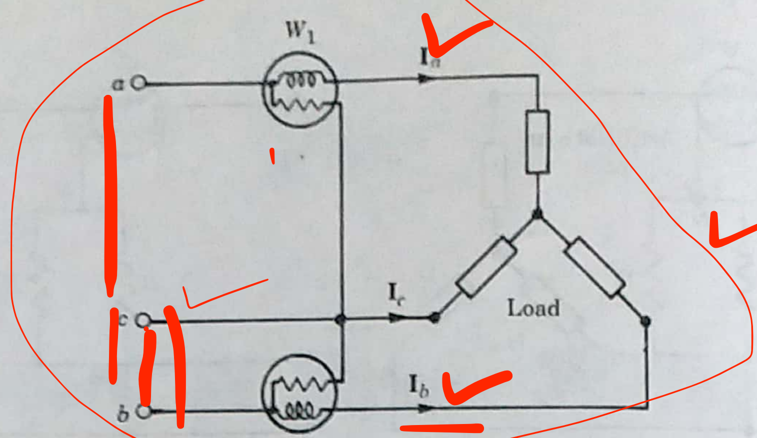

Fig. 29.3. Two-wattmeter method of 3-phase power measurement.

Thus, the sum of the two wattmeter readings gives the total 3–phase power in a 3–wire system (star or delta connected) under all conditions of load whether balanced or unbalanced, star or delta connected. Also, no assumption has been made with regard to phase sequence or waveform.

Under certain conditions one of the wattmeters may read backwards giving a negative reading. To take this negative reading, the connections of *either* the current coil *or* the voltage coils (but *not* both) are reversed, and treating the reading as though it were a negative value.

In case the supply is 3–phase 4–wire, the two wattmeter method cannot be used as there is a current in the neutral conductor also. For a 3–phase, 4–wire system three wattmeters are used to measure the total 3–phase power. Each wattmeter is connected with its current coil in series with the line and the voltage coil connected between phase wire and neutral as shown in Fig. 29.4.

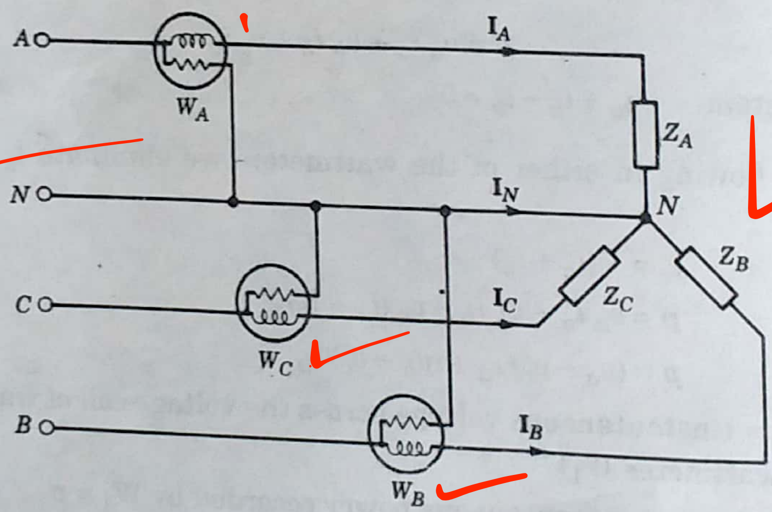

Fig. 29.4. Measurement of power in 3–phase, 4–wire system.

The total power consumed by the load

$$P_{3\phi} = P_A + P_B + P_C$$

where P_A, P_B and P_C are the readings of wattmeters W_a, W_B and W_C.

29.4 POWER FACTOR OF A BALANCED LOAD

If the load is inductive, the power factor $\cos\phi$ will be lagging. In case the load is capacitive the power factor $\cos\phi$ will be leading.

29.4.1 Lagging Power Factor Load

In case of lagging power factor the phase currents $\mathbf{I}_a$, $\mathbf{I}_b$ and $\mathbf{I}_c$ will be lagging behind their respective phase voltages by an angle ϕ. The phasor diagram under balanced load conditions for lagging power factor $\cos\phi$ is shown in Fig. 29.5.

The power indicated by a wattmeter in an ac circuit is equal to the product (Voltage across the voltage coil) × (current through the current coil) × (cosine of the angle between the voltage and current in the coils)

In the phasor diagram of Fig. 29.5,

$$|\mathbf{I}_a| = |\mathbf{I}_b| = \mathbf{I}_c| = I_L$$

$$|\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}| = V_L$$

Reading of wattmeter W_1 is given by

$$P_1 = (V_{ac}\,\mathbf{I}_a)\ (\text{cosine of the angle between } \mathbf{V}_{ac} \text{ and } \mathbf{I}_a)$$

$$P_1 = V_L\,I_L \cos(30° - \phi) \tag{29.4.1}$$

Reading of wattmeter W_2 is given by

$$P_2 = (V_{bc}\,\mathbf{I}_b)(\text{ cosine of angle between } \mathbf{V}_{bc} \text{ and } \mathbf{I}_b)$$

$$P_2 = V_L\,I_L \cos(30° + \phi) \tag{29.4.2}$$

$$P_1 + P_2 = V_L\,I_L \cos(30° - \phi) + V_L\,I_L \cos(30° + \phi)$$

$$= V_L\,I_L\,[(\cos 30° \cos\phi + \sin 30° \sin\phi) + (\cos 30° \cos\phi - \sin 30° \sin\phi)]$$

$$\boxed{P_1 + P_2 = \sqrt{3}\,V_L\,I_L \cos\phi} \tag{29.4.3}$$

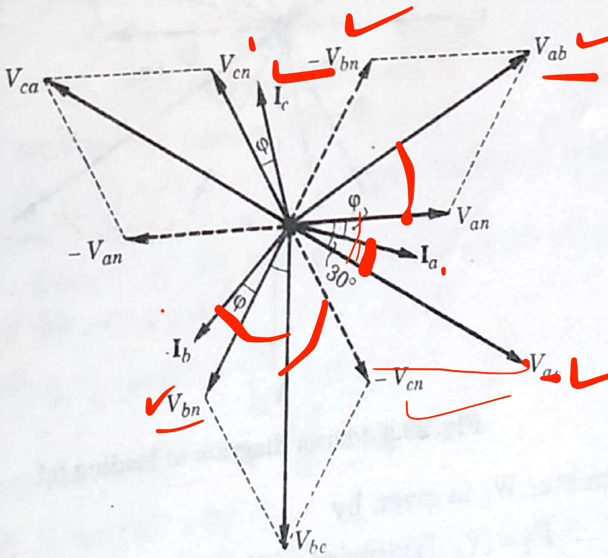

Fig. 29.5. Phasor diagram for lagging p.f. $\cos\phi$.

This relation also confirms that the sum of the wattmeter readings gives the total power.

Also,

$$P_1 - P_2 = V_L\,I_L \cos(30° - \phi) - V_L\,I_L \cos(30° + \phi)$$

$$P_1 - P_2 = V_L\,I_L \sin\phi \tag{29.4.4}$$

Dividing Eq. (29.4.4) by Eq. (29.4.3)

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{V_L\,V_L \sin\phi}{\sqrt{3}\,V_L\,I_L \cos\phi}$$

$\phi =$

$$\boxed{\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)} \tag{29.4.5}$$

or
$$\tan\phi = \sqrt{3}\,\frac{\text{difference of wattmeter readings}}{\text{sum of wattmeter readings}}$$

Eq. (29.4.5) gives the value of $\tan\phi$. Hence the value of power factor $\cos\phi$ can be calculated. In making calculations it is usual to take P_1 as the greater value of the two readings of the wattmeter.

29.4.2 Leading Power Factor Load

If the load in Fig. 29.3 is capacitive, the power factor $\cos\phi$ will be leading. The phase currents $\mathbf{I}_a$, $\mathbf{I}_b$ and $\mathbf{I}_c$ will be leading their respective phase voltages by an angle ϕ. The phasor diagram at leading power factor load is shown in Fig. 29.6.

Reading of wattmeter W_1 is given by

$$P_1 = (V_{ac}\, I_a)\text{ (cosine of the angle between } \mathbf{V}_{ac} \text{ and } \mathbf{I}_a)$$

or
$$P_1 = V_L\, I_L \cos(30° + \phi) \tag{29.4.6}$$

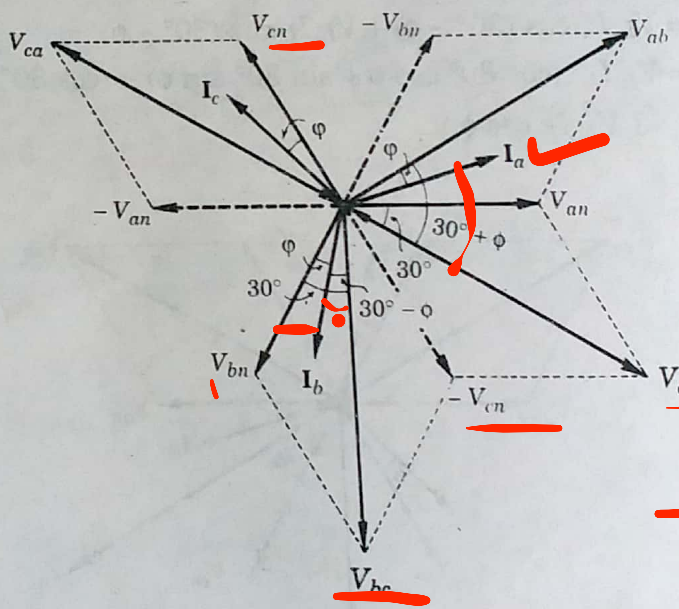

Fig. 29.6. Phasor diagram at leading p.f.

Reading of wattmeter W_2 is given by

$$P_2 = (V_{bc}\, I_b)\text{ (cosine of the angle between } \mathbf{V}_{bc} \text{ and } \mathbf{I}_b)$$

or
$$P_2 = V_L\, I_L \cos(30° - \phi)$$

Thus, for the leading power factor, the readings of the wattmeters are interchanged when compared to the wattmeter readings for the lagging power factor.

Since $\quad \cos(30° + \phi) < \cos(30° - \phi)$

$$P_1 < P_2$$

In case of lagging power factor, $P_1 > P_2$.

Also,
$$P_{3\phi} = P_1 + P_2$$
$$= V_L\, I_L \cos(30° + \phi) + V_L\, I_L \cos(30° - \phi)$$

$$P_1 + P_2 = \sqrt{3}\, V_L I_L \cos\phi \qquad (29.4.8)$$

$\therefore$

$$P_1 - P_2 = V_L I_L \cos(30^\circ + \phi) - V_L I_L \cos(30^\circ - \phi)$$

$$= V_L I_L\, [(\cos 30^\circ \cos\phi - \sin 30^\circ \sin\phi) - (\cos 30^\circ \cos\phi + \sin 30^\circ \sin\phi)]$$

$$= -2\, V_L I_L \sin 30^\circ \sin\phi$$

$$= -V_L I_L \sin\phi$$

$\therefore$

$$P_2 - P_1 = V_L I_L \sin\phi \qquad (29.4.9)$$

Dividing Eq. (29.4.9) by Eq. (29.4.8)

$$\frac{P_2 - P_1}{P_2 + P_1} = \frac{V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi}$$

or

$$\tan\phi = \sqrt{3}\left[\frac{P_2 - P_1}{P_2 + P_1}\right] \qquad (29.4.10)$$

Thus, it is seen that in both the cases of lagging and leading power factors, the smaller reading is to be subtracted from the greater reading in the numerator of Eq. (29.4.10). That is, for both lagging and leading power factor $\cos\phi$,

$$\boxed{\tan\phi = \sqrt{3} \times \frac{\text{greater wattmeter reading} - \text{smaller wattmeter reading}}{\text{sum of wattmeter readings}}} \qquad (29.4.11)$$

29.5 VARIATION OF WATTMETER READINGS WITH LOAD POWER FACTOR

For lagging power factor load (inductive load) the readings of the two wattmeters are given by

$$P_1 = V_L I_L \cos(30^\circ - \phi)$$

$$P_2 = V_L I_L \cos(30^\circ + \phi)$$

These readings depend on the load power factor angle ϕ.

(*a*) When $\phi = 0^\circ$, $\cos\phi = 1$, $P_1 = P_2$. Hence the two wattmeters indicate equal and positive values at unity power factor of the load.

(*b*) When ϕ increases from 0° to 60°, that is upto $\cos\phi = 0.5$, P_1 and P_2 are positive but $P_1 > P_2$. When $\phi = 60^\circ$, $\cos\phi = 0.5$

$$\text{Total power} = \sqrt{3}\, V_L I_L \cos 60^\circ = \frac{\sqrt{3}}{2} V_L I_L$$

$$P_1 = V_L I_L \cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2} V_L I_L$$

$$P_2 = V_L I_L \cos(30^\circ + 60^\circ) = 0$$

Hence one wattmeter will read zero and the other will read the total power.

(*c*) When $\cos\phi > 0.5$, $\phi < 60^\circ$. In this case both the wattmeters indicate positive readings.

(*d*) When $\cos\phi < 0.5$, $\phi > 60^\circ$. In this case wattmeter W_1 gives a positive reading while wattmeter W_2 gives a negative reading. The pointer of this wattmeter tries to go on the left side of the zero point of the scale. To take this reading on the wattmeter, it is necessary to reverse the connections of either current coil or voltage coil. The reading thus obtained must be given a negative sign. Then the total power in the circuit

$$P = P_1 + (-P_2) = P_1 - P_2$$

Wattmeter W_2 reads downscale for the phase angle ϕ between 60° and 90°.

(e) When $\phi = 90°$, $\cos\phi = 0$

$$P_1 = V_L I_L \cos(30° - 90°) = 0.5\, V_L I_L$$

$$P_2 = V_L I_L \cos(30° + 90°) = -0.5\, V_L I_L$$

Thus, at zero power factor (lag or lead), the two wattmeters read equal and opposite values.

The variation in the wattmeter readings with the capacitive load (leading power factor) follows the same sequence as in inductive load (lagging power factor), with a change in roles of the wattmeters.

29.6 MEASUREMENT OF REACTIVE VOLTAMPERES

Two-wattmeter method may also be used to measure the reactive voltamperes in all the three phases.

Total reactive voltamperes

$$Q = \sqrt{3}\, V_L I_L \sin\phi$$

From Eq. (29.4.4)

$$P_1 - P_2 = V_L I_L \sin\phi$$

Therefore,
$$Q = \sqrt{3}\,(P_1 - P_2) \tag{29.6.1}$$

EXAMPLE 29.1

Two wattmeters connected to measure the total power in a 3–phase load read 5 kW and 2 kW. Calculate the total power and the power factor

SOLUTION

Total power $P = P_1 + P_2$

$$= 5 + 2 = 7 \text{ kW}$$

$$\tan\phi = \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2} = \sqrt{3}\,\frac{5-2}{5+2} = 0.7423$$

$$\phi = 36.58°,\ \cos\phi = 0.803$$

EXAMPLE 29.2

The input power to a 3–phase motor was measured by the 2–wattmeter method. The readings were 5.2 kW and 1.7 kW, the latter reading was obtained after reversal of current-coil connections. The line voltage was 400 V. Calculate (a) the total power, (b) the power factor, and (c) the line current.

SOLUTION

$$P_1 = 5.2 \text{ kW},\ P_2 = -1.7 \text{ kW}$$

(a) Total power $P = P_1 + P_2$

$$= 5.2 + (-1.7) = 3.5 \text{ kW}$$

(b)
$$\tan\phi = \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2} = \sqrt{3} \times \frac{5.2 - (-1.7)}{5.2 + (-1.7)} = \frac{\sqrt{3} \times 6.9}{3.5} = 3.41$$

$$\phi = 73.67°,\ \cos\phi = 0.281$$

(c) $P = \sqrt{3}\, V_L I_L \cos\phi$

$$I_L = \frac{P}{\sqrt{3}\, V_L \cos\phi} = \frac{3.5 \times 10^3}{\sqrt{3} \times 400 \times 0.281} = 17.98 \text{ A}$$

Q.3 A balanced load of 20 kVA is connected to a three phase three-wire system. Two wattmeters are connected in the usual manner to measure power. Determine the readings of the two wattmeters if the power factor of the load is (*a*) unity, (*b*) 0.866 lagging, (*c*) 0.5 leading, and (*d*) zero lagging. What is the maximum possible reading of either wattmeter ?

SOLUTION

The total power consumed by the load

$$= \text{kVA} \cos\phi = 20 \cos\phi \text{ kW}$$

$$\therefore \quad P_1 + P_2 = 20 \cos\phi$$

Also,

$$\tan\phi = \frac{\sqrt{3}\,(P_1 - P_2)}{P_1 + P_2}$$

$$\therefore \quad \tan\phi = \frac{\sqrt{3}\,(P_1 - P_2)}{20 \cos\phi}$$

$$P_1 - P_2 = \frac{1}{\sqrt{3}}(20 \cos\phi \tan\phi) = \frac{20}{\sqrt{3}} \sin\phi$$

(*a*) $\cos\phi = 1, (\phi = 0°)$

$$P_1 + P_2 = 20 \cos\phi = 20 \times 1 = 20$$

$$P_1 - P_2 = \frac{20}{\sqrt{3}} \sin\phi = 20 \times 0 = 0$$

$$\therefore \quad P_1 = P_2 = 10 \text{ kW}$$

(*b*) $\cos\phi = 0.866$ (lagging), $\phi = 30°$ lagging

$$P_1 + P_2 = 20 \cos\phi = 20 \times 0.866 = 17.32$$

$$P_1 - P_2 = \frac{20}{\sqrt{3}} \sin\phi = \frac{20}{\sqrt{3}} \sin 30° = 5.773$$

$$\therefore \quad P_1 = 11.546 \text{ kW}, P_2 = 5.773 \text{ kW}$$

(c) $\cos\phi = 0.5$ leading, ($\phi = 60°$ leading) since ϕ is leading, we shall take $\phi = -60°$.

$$P_1 + P_2 = 20 \cos\phi = 20 \cos(-60°) = 10 \text{ kW}$$

$$P_1 - P_2 = \frac{20}{\sqrt{3}} \sin\phi = \frac{20}{\sqrt{3}} \sin(-60°) = -10 \text{ kW}$$

$$\therefore \quad P_1 = 0, \quad P_2 = 10 \text{ kW}$$

(*d*) $\cos\phi = 0. (\phi = 90°)$

$$P_1 + P_2 = 20 \cos\phi = 20 \cos 90° = 0$$

$$P_1 - P_2 = \frac{20}{\sqrt{3}} \sin\phi = \frac{20}{\sqrt{3}} 90° = 11.547$$

$$\therefore \quad P_1 = 5.7736 \text{ kW}, P_2 = -5.7736 \text{ kW}$$

The maximum reading on either wattmeter occurs when the current through the current coil of the wattmeter is in phase with the voltage across the voltage coil. This occurs for wattmeter W_1 at a phase angle of 30° lagging, and for W_2 at a phase angle of 30° lagging, In either case the maximum reading is 11.546 kW as seen in solution (*b*).