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B1

Q1

a.)

$$\begin{bmatrix} 1 & 5 & 6 & 3 \\ 2 & 1 & 4 & 1 \\ 3 & 5 & 5 & 4 \end{bmatrix}.$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & 5 & 6 & 3 \\ 0 & -9 & -8 & -5 \\ 0 & -10 & -13 & -5 \end{bmatrix}.$$

$R_3 \to R_3 - \frac{10}{9} R_2$

$$\sim \begin{bmatrix} 1 & 5 & 6 & 3 \\ 0 & -9 & -8 & 3 \\ 0 & 0 & -\frac{37}{9} & \frac{5}{9} \end{bmatrix}.$$

$R_2 \to \frac{-R_2}{9}$, $R_3 \to R_3 \cdot \frac{-9}{37}$.

$$\sim \begin{bmatrix} 1 & 5 & 6 & 3 \\ 0 & 1 & 8/9 & -1/3 \\ 0 & 0 & 1 & -5/37 \end{bmatrix}.$$

There are 3 non-zero rows, in echelon form, hence there are 3 linearly independent rows.
$\therefore$ Rank $= 3$.

b.)

$$\begin{bmatrix} 1 & 5 & 6 \\ 2 & 6 & 8 \\ 7 & 1 & 8 \end{bmatrix}.$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 7R_1$

$$\sim \begin{bmatrix} 1 & 5 & 6 \\ 0 & -4 & -4 \\ 0 & -34 & -34 \end{bmatrix}.$$

$R_3 \to R_3 + \left(-\frac{34}{4}\right) R_2$

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$$\sim \begin{bmatrix} 1 & 5 & 6 \\ 0 & -4 & -4 \\ 0 & 0 & 0 \end{bmatrix}.$$

$R_2 \to R_2\left(-\frac{1}{4}\right).$

$$\sim \begin{bmatrix} 1 & 5 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

There are 2 non-zero rows in echlon form, hence there are 2 linearly independent vectors.

$\therefore$ Rank $= 2$

c) $$\begin{bmatrix} 2 & 3 & 1 & 0 & 4 \\ 3 & 1 & 2 & -1 & 1 \\ 4 & -1 & 3 & -2 & -2 \\ 5 & 4 & 3 & -9 & 5 \end{bmatrix}.$$

$R_4 \to R_4 - R_3$

$$\sim \begin{bmatrix} 2 & 3 & 1 & 0 & 4 \\ 3 & 1 & 2 & -1 & 1 \\ 4 & -1 & 3 & -2 & -2 \\ 1 & 5 & 0 & -7 & 7 \end{bmatrix}$$

$R_1 \leftrightarrow R_4$

$$\sim \begin{bmatrix} 1 & 5 & 0 & -7 & 7 \\ 3 & 1 & 2 & -1 & 1 \\ 4 & -1 & 3 & -2 & -2 \\ 2 & 3 & 1 & 0 & 4 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 4R_1$

$R_4 \to R_2 - 2R_1$

$$\sim \begin{bmatrix} 1 & 5 & 0 & -7 & 7 \\ 0 & -14 & 2 & 20 & -20 \\ 0 & -21 & 3 & 26 & -30 \\ 0 & -7 & 1 & 4 & -10 \end{bmatrix}.$$

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$$\sim \begin{bmatrix} 1 & 5 & 0 & -7 & 7 \\ 0 & 0 & 0 & -8 & 0 \\ 0 & 0 & 0 & -16 & 0 \\ 0 & -7 & 1 & 14 & -10 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\sim \begin{bmatrix} 1 & 5 & 0 & -7 & 7 \\ 0 & -7 & 1 & 14 & -10 \\ 0 & 0 & 0 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \boxed{\text{Rank} = 3}$$

Q2

a)

$$\begin{pmatrix} 1 & 1 & 0 \\ 4 & 5 & 3 \\ 1 & 0 & 2 \end{pmatrix}$$

Applying gauss - jordan method

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 4 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right].$$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & -4 & 1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 1 \end{array}\right].$$

$R_3 \to R_3 + R_2$

$R_1 \to R_1 - R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 5 & -1 & 0 \\ 0 & 1 & 3 & -4 & 1 & 0 \\ 0 & 0 & 5 & -5 & 1 & 1 \end{array}\right]$$

$R_3 \to \frac{R_3}{5}$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 5 & -1 & 0 \\ 0 & 1 & 3 & -4 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1/5 & 1/5 \end{array}\right].$$

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$R_2 \to R_2 - 3R_3$

$R_1 \to R_1 + 3R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -2/5 & 3/5 \\ 0 & 1 & 0 & -1 & 2/5 & -3/5 \\ 0 & 0 & 1 & -1 & 1/5 & 1/5 \end{array}\right]$$

Since A is reduced to 'I'

Hence,

Inverse of matrix is $\begin{bmatrix} 2 & -2/5 & 3/5 \\ -1 & 2/5 & -3/5 \\ -1 & 1/5 & 1/5 \end{bmatrix}$.

b) $\begin{pmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$.

Applying gauss - jordon method.

$$[A|I] = \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{array}\right].$$

$R_2 \to R_2 + 3R_1$, $R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 7 & 3 & 1 & 0 \\ 0 & 2 & 2 & -1 & 0 & 1 \end{array}\right].$$

$R_1 \to -R_1$,

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 0 & 2 & 7 & 3 & 1 & 0 \\ 0 & 2 & 2 & -1 & 0 & 1 \end{array}\right].$$

$R_3 \to R_3 - R_2$.

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$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 0 & 2 & 7 & 3 & 1 & 0 \\ 0 & 0 & -5 & -4 & -1 & 1 \end{array}\right].$$

$R_2 \to \frac{R_2}{2}$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 0 & 1 & 7/2 & 3/2 & 1/2 & 0 \\ 0 & 0 & -5 & -4 & -1 & 1 \end{array}\right].$$

$R_1 \to R_1 + R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3/2 & 1/2 & 1/2 & 0 \\ 0 & 1 & 7/2 & 3/2 & 1/2 & 0 \\ 0 & 0 & -1 & -4/5 & -1/5 & 1/5 \end{array}\right]$$

(1) $R_1 \to R_1 + \frac{3}{2} R_3$ (2)

$R_2 \to R_2 + \frac{7}{2} R_2$; $R_3 \to -R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/10 & 2/10 & 3/10 \\ 0 & 1 & 0 & -13/10 & -2/10 & 7/10 \\ 0 & 0 & +1 & 4/5 & 1/5 & -1/5 \end{array}\right].$$

Since, A is reduced to I.

Hence,

inverse of matrix is $\begin{bmatrix} -7/10 & 2/10 & 3/10 \\ -13/10 & -2/10 & 7/10 \\ 4/5 & 1/5 & -1/5 \end{bmatrix}$.

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c) $\begin{pmatrix} -0.5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$[A|I] = \left[\begin{array}{ccc|ccc} -0.5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right].$$

$R_1 \to -2R_1$

$R_2 \to \frac{R_2}{4}$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right].$$

Since A is reduced to I,

Inverse is $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Q3

a) $2x + y - 2z = 10$;
$3x + 2y + 2z = 1$;
$5x + 4y + 3z = 4$.

$\Rightarrow$ $A = \begin{bmatrix} 2 & 1 & -2 \\ 3 & 2 & 2 \\ 5 & 4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 10 \\ 1 \\ 4 \end{bmatrix}$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

Consider the augmented matrix,

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 3 & 2 & 2 & 1 \\ 5 & 4 & 3 & 4 \end{array}\right].$$

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$R_2 \to R_2 - R_1$

$$\sim \left[\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 1 & 1 & 4 & -9 \\ 5 & 4 & 3 & 4 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 4 & -9 \\ 2 & 1 & -2 & 10 \\ 5 & 4 & 3 & 4 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 5R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 4 & -9 \\ 0 & -1 & -10 & 28 \\ 0 & -1 & -17 & 49 \end{array}\right].$$

$R_3 \to R_3 \to R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 4 & -9 \\ 0 & -1 & -10 & 28 \\ 0 & 0 & -7 & 21 \end{array}\right].$$

~~It~~ Since, it has 3 independent rows.

$\rho(A|B) = \rho(A) = 3$

$= 3$ ~~is~~ un-knowns in eqn.

$\therefore$ System is consistent & unique.

$\therefore$

$$x + y + 4z = -9$$
$$-y - 10z = 28$$
$$-7z = 21$$
$$\boxed{z = -3}$$

$\boxed{y = 2}$, $\boxed{x = 1}$

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Hence, soln is

$$= \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$x = 1$
$y = 2$
$z = -3$

b.) $2x + y - 3z = 1$
$5x + 2y - 6z = 5$
$3x - y - 4z = 7$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & -3 & 1 \\ 5 & 2 & -6 & 5 \\ 3 & -1 & -4 & 7 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 2 & 1 & -3 & 1 \\ 1 & 0 & 0 & 3 \\ 3 & -1 & -4 & 7 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$
$R_3 \to R_1 - 3R_2$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & -3 & -5 \\ 1 & 0 & 0 & 3 \\ 0 & -1 & -4 & -2 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -3 & -5 \\ 0 & -1 & -4 & -2 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & -7 & -7 \end{array}\right]$$

$\rho(A|B) = 3$
$\rho(A) = 3$

$\therefore$ System is consistent with unique soln

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$$-7z = -7$$

$$\boxed{z = 1}$$

$$y - 3z = -5$$

$$y = -5 + 3$$

$$\boxed{y = -2}$$

$$\boxed{x = 3}$$

Soln is

$$x = 3$$
$$y = -2$$
$$z = 1$$

c)

$$x - y + 5z - w = 0$$
$$x + y - 2z + 3w = 0$$
$$3x - y + 8z + w = 0$$
$$x + 3y - 9z + 7w = 0$$

$$[A|B] = \left[\begin{array}{cccc|c} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - R_1$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 5 & -1 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 4 & -14 & 8 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - 2R_2$

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$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 5 & -1 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

$$\rho(A|B) = \rho(A)$$

$$\rho(A) < 4 \quad [\text{Number of unknowns}]$$

System is consistent with ∞ soln.

Let $z = \beta$

$w = \alpha$

$\therefore \quad 2y - 7z + 4w = 0$

$$y = \frac{7\beta}{2} - 2\alpha$$

$$x = y - 5z + w$$

$$= \frac{7\beta}{2} + \beta - 2\alpha - 5\alpha$$

$$x = \frac{9\beta}{2} - 7\alpha$$

$$x, y, z, w = \frac{9\beta}{2} - 7\alpha, \ \frac{7\beta}{2} - 2\alpha, \ \beta, \ \alpha$$

where $(\alpha, \beta) \in R$

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d) $2x + 3y + z = 7$

$x + 2y - 2z = 8$

$3x + 4y + 4z = 20$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 3 & 1 & 7 \\ 1 & 2 & -2 & 8 \\ 3 & 4 & 4 & 20 \end{array}\right].$$

$R_1 \to R_1 - 2R_2$

$R_3 \to R_3 - 3R_2$

$$\sim \left[\begin{array}{ccc|c} 0 & -1 & 5 & -9 \\ 1 & 2 & -2 & 8 \\ 0 & -2 & 10 & -4 \end{array}\right].$$

$R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 0 & -1 & 5 & -9 \\ 1 & 2 & -2 & 8 \\ 0 & 0 & 0 & 14 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -2 & 8 \\ 0 & -1 & 5 & -9 \\ 0 & 0 & 0 & 14 \end{array}\right].$$

$\rho(A|B) \neq \rho(A)$

system. Inconsistent

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Q4 $\alpha x + \beta y + 2z = 1$

$\alpha x + (2\beta - 1) y + 3z = 1$

$\alpha x + \beta y + (\beta + 3) z = 2\beta - 1$

$$[A|B] = \left[\begin{array}{ccc|c} \alpha & \beta & 2 & 1 \\ \alpha & 2\beta-1 & 3 & 1 \\ \alpha & \beta & \beta+3 & 2\beta-1 \end{array}\right].$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} \alpha & \beta & 2 & 1 \\ 0 & \beta-1 & -1 & 0 \\ 0 & 0 & \beta+1 & 2\beta-2 \end{array}\right].$$

For soln to exist,

$\rho(A)$ should be equal to $\rho(A|B)$.

$\rho(A) = 3 = \rho(A|B)$

when

$\beta + 1 \neq 0$

$\boxed{\beta \neq -1}$

Hence for all $\boxed{\beta \neq -1}$, a unique soln will exist.

independent of the value of α, hence 'α' can have any value.

UK

Q.5

a) $x + 2y = 0$

$3x + 6y = 0$

$2x + ky = 0$.

$$A|B = \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 3 & 6 & 0 \\ 2 & k & 0 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & k-4 & 0 \end{array}\right].$$

$R_2 \longleftrightarrow R_3$

$$\sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & k-4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

When $k \neq +4$

$\rho(A/B) = \rho(A) = 2$

Trivial soln will exist

$x = 0$, $y = 0$

At $k = 4$

$\rho(A/B) = \rho(A) = 1$

which is less than no of unknowns

$\therefore$ Infinite solns will exist.

$x + 2y = 0$

Let $y = \alpha$

$\alpha \in R$

$x = -2\alpha$

$\therefore x = -2\alpha$, $y = \alpha$

OR $\boxed{x = -2, y = 1}$

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b) $x + y + z = 1$

$x + 2y + 4z = \lambda$

$x + 4y + 10z = \lambda^2$

$$\rho(A/B) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right].$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 3 & 9 & \lambda^2 - 1 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 0 & 0 & \lambda^2 - 1 - 3\lambda + 3 \end{array}\right]$$

For soln to exist

$\rho(A/B) = \rho(A)$

Since $\rho(A) = 2$

$\therefore \rho(A/B) = 2$

Hence $\lambda^2 - 1 - 3\lambda + 3 = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\boxed{\lambda = 2, 1}$

$\boxed{\lambda = 1} \to y + 3z = 0$ Let $z = t$ $t \in \mathbb{R}$.

$y = -3t$, $x = 2t + 1$.

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$$\sim \begin{bmatrix} 2 & 0 & 8_1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \to R_1/2$$

$$\sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

System becomes :→

$$x + 2z = 0$$

$$x = -2t$$

$$3y = 0$$

$$y = 0$$

Let $z = t$

$t \in R$

$$X = \begin{bmatrix} -2t \\ 0 \\ t \end{bmatrix}.$$

Linearly independent soln

$$X_1 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}.$$

When $\boxed{\lambda = 5}$

$$[A - 5I]$$

$$\sim \begin{bmatrix} -4 & 3 & 0 \\ 0 & -4 & 0 \\ 2 & 1 & 0 \end{bmatrix}.$$

$$R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 2 & 1 & 0 \\ 0 & -4 & 0 \\ -4 & 3 & 0 \end{bmatrix}.$$

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Soln is $x = 2t+1$, $y = -3t$, $z = t$

$\boxed{\lambda = 2}$

$y + 3z = 1$ Let $z = t$

$y = 1 - 3t$ $t \in R$

$x = 2t$

$x = 2t$, $y = 1-3t$, $z = t$.

Q6 a)

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 5 \end{pmatrix}$$

Characteristic eqn $= 0$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 3 & 0 \\ 0 & 1-\lambda & 0 \\ 2 & 1 & 5-\lambda \end{vmatrix} = 0$$

Expanding along C_3

$(5-\lambda)(1-\lambda)(1-\lambda) = 0$

$\boxed{\lambda = 1, 1, 5}$

When $\boxed{\lambda = 1}$ $[A - \lambda I] =$

$$= \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 0 \\ 2 & 1 & 4 \end{bmatrix}.$$

① $R_1 \leftrightarrow R_3$

② $R_3 \leftrightarrow R_2$.

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$R_3 \to R_3 + 2R_1$

$$\sim \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 5 & 0 \end{bmatrix}$$

$R_3 \to R_3 + \frac{5}{4} R_2$

$$\sim \begin{bmatrix} 2 & 1 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

System becomes

$2x + y = 0 \Rightarrow x = 0$

$-4y = 0 \Rightarrow y = 0$

Let $z = t \quad t \in R$

$$X = \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix}$$

linearly independent soln

$$X_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ Eigen value $= 1, 1$

Eigen vector $= \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

Eigen value $= 5$

Eigen vector $= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

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Characteristic eqn

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1-\lambda)(1-\lambda) = 0$$

$$\boxed{\lambda = 1, 1, 1}$$

$\boxed{\text{When } \lambda = 1}$

System becomes

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let $x = \alpha$
$y = \beta$ when $\alpha, \beta, \gamma \in R$
$z = \gamma$

linearly independent soln

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad X_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad X_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

when $\alpha = 1, \beta = 0, \gamma = 0$ when $\alpha = 0, \beta = 1, \gamma = 0$ when $\alpha = 0, \beta = 0, \gamma = 1$

c) $$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

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Characteristic eqn,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$0 = -(2+\lambda)\left[-\lambda + \lambda^2 - 12\right] + 2\left[+2\lambda + 6\right] + 3\left[+4 + (\lambda - 1)\right]$$

$$0 = -(2+\lambda)\left[\lambda^2 - \lambda - 12\right] + 7\lambda + 12 + 12 - 3$$

$$0 = -2\lambda^2 + 2\lambda + 24 - \lambda^3 + \lambda^2 + 12\lambda + 7\lambda + 21$$

$$0 = 21\lambda - \lambda^2 - \lambda^3 + 45$$

$$0 = \lambda^3 + \lambda^2 - 21\lambda - 45.$$

$$0 = (\lambda+3)(\lambda^2 - 2\lambda - 15)$$

$$0 = (\lambda+3)((\lambda-5)(\lambda+3))$$

$$\lambda = -3, -3, 5$$

$\boxed{\text{When } \lambda = -3}$

System becomes

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$R_2 \to R_2 - 2R_1$

$$\sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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$x + 2y - 3z = 0$

Let $z = \alpha$ when $X = \begin{bmatrix} 3\alpha - 2\beta \\ \beta \\ \alpha \end{bmatrix}$.

$y = \beta$ $\alpha, \beta \in R$

$\therefore x = 3\alpha - 2\beta$

$\therefore$ Linearly independent solu

$\alpha = 0, \beta = 1$ $\alpha = 1, \beta = 0$

$X_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ $X_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$.

When $\lambda = 5$,

System becomes

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} -1 & -2 & -5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix}.$$

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 - 7R_1$

$$\sim \begin{bmatrix} 1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\sim \begin{bmatrix} 1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix}.$$

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a. $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

Characteristic eqn,

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

Expanding along C_1

$(1-\lambda)(1-\lambda)(1-\lambda) = 0$

$\boxed{\lambda = 1, 1, 1}$

When $\lambda = 1$,

System becomes

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$y = 0$

$z = 0$

Let $x = \alpha$ where $\alpha \in R$

$$X_1 = \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$$

Linearly independent soln

$X_1 = (1, 0, 0)^T$

Hence eigen value $= 1, 1, 1$

eigen vector $= (1, 0, 0)^T$

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$\therefore \quad R_2 \to \frac{-R_2}{8}$

$$\sim \begin{bmatrix} 1 & -2 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$y + 2z = 0$

$x - 2y - 5z = 0$

$\therefore \quad y = -2\alpha$

$x = 2(-2\alpha) + 5(\alpha)$

$= \alpha$

Let $z = \alpha$ where $\alpha \in R$

$$\therefore \quad X = \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$$

linearly independent sol^n

$$X_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Hence,

Eigen value $= -3, -3$

Eigen vector $= \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$.

Eigen value $= 5$.

Eigen vector $= \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

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Q7 →

e) $\begin{pmatrix} 4 & -6 & -6 \\ 0 & -2 & 0 \\ 1 & -1 & -1 \end{pmatrix}$

Characteristic eqn

$|A - \lambda I| = 0$

$\begin{vmatrix} 4-\lambda & -6 & -6 \\ 0 & -2-\lambda & 0 \\ 1 & -1 & -1-\lambda \end{vmatrix} = 0$

$[4-\lambda]\left[(2+\lambda)(1+\lambda)\right] + 1\left[-6(2+\lambda)\right] = 0$

$(2+\lambda)\left[(\lambda-4)(\lambda+1) + 6\right] = 0$

$(2+\lambda)\left[\lambda^2 - 3\lambda + 2\right] = 0$

$(\lambda+2)(\lambda-1)(\lambda-2) = 0$

$\lambda = 1, 2, -2$

When $\boxed{\lambda = 1}$

$\begin{pmatrix} 3 & -6 & -6 \\ 0 & -3 & 0 \\ 1 & -1 & -2 \end{pmatrix}$

$R_1 \to R_1 - 3R_3 - R_2$

$\sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3 & 0 \\ 1 & -1 & -2 \end{pmatrix}$

$-3y = 0$

$y = 0$

$\boxed{x = 2z}$

Let $z = a$, where

$x = 2a$, $a \in R$.

$X = [2a, 0, a]^T$

Linearly independent soln for $a = 1$.

$X_1 = [2 \quad 0 \quad 1]^T$

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Q7 →

For $\lambda = -2$

Eu

$$\sim \begin{bmatrix} 6 & -6 & -6 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Prog

$R_1 \to R_1 - 6R_3$

$$\sim \begin{bmatrix} 0 & 0 & -12 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

OR

$\Rightarrow -12z = 0 \quad \Rightarrow z = 0$

$x - y + z = 0$

Let $y = a, \quad a \in R \quad \Rightarrow x = a$

$X = [a \ a \ 0]^T$

linearly independent soln

$a = 1$

$X_2 = [1 \ 1 \ 0]^T$

when

For $\lambda = 2$

$$\sim \begin{bmatrix} 2 & -6 & -6 \\ 0 & -4 & 0 \\ 1 & -1 & -3 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & -4 & 0 \\ 2 & -6 & -6 \end{bmatrix}$$

Thus

$R_3 \to R_3 - 2R_1 - R_2$

$$\sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x - y - 3z = 0$

$-4y = 0 \quad \Rightarrow y = 0$

Let $z = a \quad a \in R$

$x = 3a$

$X_3 = [3 \ 0 \ 1]^T$

So, for given matrix eigen values and corresponding eigen vectors are

$1, \ X_1 = [2 \ 0 \ 1]^T$

$-2, \ X_2 = [1 \ 1 \ 0]^T$

$2, \ X_3 = [3 \ 0 \ 1]^T$

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Q7 → Cayley - Hamilton thm →

Every Square matrix satisfies its characteric eqn.

Proof,

Let A be a square matrix of order $n \times n$, then its characteristic eqn is

$$|A - \lambda I| = 0$$

OR $(-1)^n \lambda^n + p_1 \lambda^{n-1} + p_2 \lambda^{n-2} + \cdots p_n = 0$

Let B be adjoint of matrix $A - \lambda I$, then elements of B will be polynomials of degree $(n-1)$: $n\lambda$. Thus B can be reduced as

$$B = B_1 \lambda^{n-1} + B_2 \lambda^{n-2} \cdots B_{n-1} \lambda + B_n$$

where B_i is a square matrix of order n, whose elements are a func of elements of A.

We know that.

$$[A - \lambda I] \operatorname{adj}(A - \lambda I) = |A - \lambda I| \, I$$

Thus

$$(A - \lambda I)\left[B_1 \lambda^{n-1} + B_2 \lambda^{n-2} \cdots + B_n\right]$$
$$= \left[(-1)^n \lambda^n + p_1 \lambda^{n-1} \cdots p_n\right] I.$$

Equating the coefficients of various powers of

$$-B_1 = (-1)^n I.$$

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Similarly, $AB_1 - B_2 = p_1 I$

$AB_2 - B_3 = p_2 I$

$\vdots$

$AB_{n-1} - B_n = p_{n-1} I$

$AB_n = p_n I$

By multiplying the eqn from top to bottom respectively and A^n, A^{n-1}, A, I on adding

$$A^n[-\cancel{B_1}] = (-1)^n I$$
$$A^{n-1}[\cancel{AB_1} - \cancel{B_2}] = p_1 I$$
$$\vdots$$
$$A^2[\cancel{AB_{n-2}} - \cancel{B_{n-1}}] = p_{n-2} I$$
$$+ \quad A[\cancel{AB_{n-1}} - \cancel{B_n}] = p_{n-1} I$$
$$\cancel{AB_n} = p_n I$$

Hence, $0 = (-1)^n A^n + p_1 A^{n-1} \ldots p_n$

Hence, it satisfies the characteristic eqn.

a) $\begin{pmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix}$

Characteristic eqn

$|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 5 & 3 \\ 0 & 4-\lambda & 6 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$(3-\lambda)(4-\lambda)(1-\lambda) = 0$

$(\lambda-3)(\lambda-4)(\lambda-1) = 0$

$(\lambda^2 - 7\lambda + 12)(\lambda - 1) = 0$

$\lambda^3 - 7\lambda^2 + 12\lambda + 7\lambda - \lambda^2 - 12 = 0$

$\lambda^3 - 8\lambda^2 + 19\lambda - 12 = 0$

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Replacing λ by 'A'

we get

$$A^3 - 8A^2 + 19A - 12I = 0 \qquad -(1)$$

$$A^2 = A \cdot A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 35 & 42 \\ 0 & 16 & 30 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$A^3 = A \cdot A^2 = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 & 35 & 42 \\ 0 & 16 & 30 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 27 & 185 & 279 \\ 0 & 64 & 126 \\ 0 & 0 & 1 \end{bmatrix}.$$

Hence, putting value of A^3, A^2, A, I in (1) ; we get.

LHS

$$= \begin{bmatrix} 27 & 185 & 279 \\ 0 & 64 & 126 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -72 & -280 & -336 \\ 0 & -128 & -240 \\ 0 & 0 & -8 \end{bmatrix} + \begin{bmatrix} 57 & 85 & 57 \\ 0 & 76 & 114 \\ 0 & 0 & 19 \end{bmatrix} + \begin{bmatrix} -12 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 27+57-72-12 & 185+85-280 & 279+57-336 \\ 0 & 64-128+76-12 & 126+114-240 \\ 0 & 0 & 20-20 \end{bmatrix}.$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{zero matrix}$$

$$RHS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{zero matrix}$$

LHS = RHS

Hence Cayley Hamilton thm verified.

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Since

$$A^3 - 8A^2 + 19A - 12I = 0$$

Premultiply by A^{-1}

$$A^2 - 8A + 19I - 12A^{-1} = 0$$

$$\therefore A^{-1} = \frac{1}{12}\left[A^2 - 8A + 19I\right]$$

$$= \frac{1}{12}\begin{bmatrix} 9-24+19 & 35-40 & 42-24 \\ 0-0 & 16+19-32 & 30-48 \\ 0-0 & 0-0 & 1+19-8 \end{bmatrix}$$

$$A^{-1} = \frac{1}{12}\begin{bmatrix} 4 & -5 & +18 \\ 0 & 3 & -18 \\ 0 & 0 & 12 \end{bmatrix}.$$

b) $\begin{pmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

Characteristic eqn

$$\Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} a-\lambda & 1 & 0 \\ 1 & a-\lambda & 1 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$$

Expanding along R_3

$$\Rightarrow +\lambda\left[(a-\lambda)^2 - 1\right] = 0$$

$$\Rightarrow \lambda\left[\lambda^2 + a^2 - 2\lambda a - 1\right] = 0$$

$$\Rightarrow \lambda^3 - 2\lambda^2 a + (a^2-1)\lambda = 0$$

Replacing λ with 'A'

we get

$$A^3 - 2A^2 a + (a^2-1)A = 0. \quad - (1)$$

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$$A^2 = A \cdot A = \begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+1 & 2a & 1 \\ 2a & a^2+1 & a \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 = A \cdot A^2 = \begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a^2+1 & 2a & 1 \\ 2a & a^2+1 & a \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a^3+3a & 3a^2+1 & 2a \\ 3a^2+1 & a^3+3a & a^2 \\ 0 & 0 & 0 \end{bmatrix}$$

Eqn (1) becomes

LHS

$$= \begin{bmatrix} a^3+3a & 3a^2+1 & 2a \\ 3a^2+1 & a^3+3a & a^2 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 2a^3+2a & 4a^2 & 2a \\ 4a^2 & 2a^3+2a & 2a^2 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} a^3-a & a^2-1 & 0 \\ a^2-1 & a^3-a & a^2-1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{zero matrix}$$

$$RHS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{zero matrix}$$

LHS = RHS

Hence cayley hamilton thm verified.

Inverse will not exist bcoz $|A| = 0$, since R_3 is zero.

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Q8

a) $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$

Characteristic eqn

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(\lambda-5)(\lambda-2) = 4$$

$$\lambda^2 - 7\lambda + 10 = 4$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$(\lambda-6)(\lambda-1) = 0$$

$$\boxed{\lambda = 1, 6}$$

When $\lambda = 1$

$$\begin{pmatrix} 4 & 4 \\ 1 & 1 \end{pmatrix}$$

$R_1 \to R_1 - 4R_2$

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

System is

$x + y = 0$

Let $y = \alpha$

$\alpha \in R$

$\therefore x = -\alpha$

$\therefore$ Linearly independent soln

$$X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

When $\lambda = 6$

$$\begin{pmatrix} -1 & 4 \\ 1 & -4 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{pmatrix} -1 & 4 \\ 0 & 0 \end{pmatrix}$$

System becomes

$-x + 4y = 0$

Let $y = \alpha$

$\alpha \in R$

$x = 4\alpha$

Linearly independent soln

$$X_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Hence,

$$P = \begin{bmatrix} -1 & 4 \\ 1 & 1 \end{bmatrix}.$$

Finding $P^{-1} \Rightarrow$ $[P|I] = \left[\begin{array}{cc|cc} -1 & 4 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right].$

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$R_2 \to R_2 + R_1$

$$\left[\begin{array}{cc|cc} -1 & 4 & 1 & 0 \\ 0 & 5 & 1 & 1 \end{array}\right]$$

$R_2 \to R_2/5$

$$\left[\begin{array}{cc|cc} -1 & 4 & 1 & 0 \\ 0 & 1 & 1/5 & 1/5 \end{array}\right]$$

$R_1 \to R_1 - 4R_2$

$$\left[\begin{array}{cc|cc} -1 & 0 & 1-4/5 & 0-4/5 \\ 0 & 1 & 1/5 & 1/5 \end{array}\right]$$

$R_1 \to -R_1$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1/5 & +4/5 \\ 0 & 1 & 1/5 & 1/5 \end{array}\right]$$

$$\therefore P^{-1} = \frac{1}{5}\begin{bmatrix} -1 & 4 \\ 1 & 1 \end{bmatrix}.$$

$$\therefore P^{-1}AP = \frac{1}{5}\begin{bmatrix} -1 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} -1 & 4 \\ 6 & 6 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 5 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}.$$

b) $\begin{pmatrix} 7 & 0 & 3 \\ 2 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix}.$

Characteristic eqn

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 7-\lambda & 0 & 3 \\ 2 & 1-\lambda & 1 \\ 2 & 0 & 2-\lambda \end{vmatrix} = 0$$

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$$(7-\lambda)(2-\lambda)(1-\lambda) - 3(2)(1-\lambda) = 0$$

$$[1-\lambda]\left[\lambda^2+14-9\lambda-6\right] = 0$$

$$[1-\lambda][\lambda^2-9\lambda+8] = 0$$

$$[1-\lambda][\lambda-8][\lambda-1] = 0$$

$$\lambda = 1, 1, 8$$

When $\lambda = 1$

System becomes

$$= \begin{pmatrix} 6 & 0 & 3 \\ 2 & 0 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad R_3 \to R_3 - R_2, \quad R_1 \to R_1 - 3R_2$$

$$\sim \begin{pmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R_1 \leftrightarrow R_2$$

$$2x + z = 0$$

$$x = -z/2$$

Let $z = 2\beta$, $\beta \in R$

$$x = -2\beta/2 = -\beta$$

Let $y = \alpha$, $\alpha \in R$

$$\therefore X = [-\beta, \alpha, 2\beta]^T$$

$$X_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad X_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

when $\alpha = 1$, $\beta = 0$

when $\beta = 1$, $\alpha = 0$.

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$= \cancel{(7-\lambda)(1-\lambda)(2-\lambda)} \cancel{-3(1-\lambda)(2)}$

$\cancel{= (1-\lambda)}$

When $\lambda = 8$

$$\begin{bmatrix} -1 & 0 & 3 \\ 2 & -7 & 1 \\ 2 & 0 & -6 \end{bmatrix}$$

$R_3 \to R_3 + 2R_1$

$$\sim \begin{bmatrix} -1 & 0 & 3 \\ 2 & -7 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$$\sim \begin{bmatrix} -1 & 0 & 3 \\ 0 & -7 & 7 \\ 0 & 0 & 0 \end{bmatrix}.$$

$\therefore \quad -x + 3z = 0$

$-7y + 7z = 0$

Let $z = \alpha$, $\alpha \in R$

$\therefore \boxed{x = 3\alpha}$

$y = z$

Linearly independent soln

$$X_3 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}.$$

$$\therefore P = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}.$$

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Diagonalisable matrix - D

$$D = P^{-1} A P$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}.$$

Scanned by CamScanner

c) $\begin{pmatrix} 10 & -3 & 5 \\ 0 & 1 & 0 \\ -15 & 9 & -10 \end{pmatrix}$

Characteristic eqn

$|A - \lambda I| = 0$

$$\begin{vmatrix} 10-\lambda & -3 & 5 \\ 0 & 1-\lambda & 0 \\ -15 & 9 & -10-\lambda \end{vmatrix} = 0$$

Expanding along R_2

$$0 = [1-\lambda]\left[+(10-\lambda)(10+\lambda) - 5\times 15.\right]$$

$$0 = [1-\lambda][100-\lambda^2 - 75]$$

$$0 = [1-\lambda][5-\lambda][5+\lambda]$$

$$\lambda = 1, 5, -5.$$

when $\lambda = 1$

System becomes

$$\begin{pmatrix} 9 & -3 & 5 \\ 0 & 0 & 0 \\ -15 & 9 & -11 \end{pmatrix}$$

$R_3 \to R_3 + 3R_1$

$$\sim \begin{pmatrix} 9 & -3 & 5 \\ 0 & 0 & 0 \\ 12 & 0 & 4 \end{pmatrix}$$

$R_3 \to R_3/4$

$$\sim \begin{pmatrix} 9 & -3 & 5 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

$R_1 \to R_1 - 3R_1$

$$\sim \begin{pmatrix} 0 & -3 & 3 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

Hence

$-3y + 3z = 0$

$\boxed{y = 2}$

$\boxed{3x = -2}$

Let $x = \alpha$, $\alpha \in R$

$\therefore$ linearly independent sol^n

$$X_1 = \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}.$$

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When $\lambda = 5$

$$\begin{pmatrix} 5 & -3 & 5 \\ 0 & -4 & 0 \\ -15 & 9 & -15 \end{pmatrix}$$

$R_2 \to \frac{R_2}{-4}$

$R_3 \to R_3 - 3R_1$

$$\begin{pmatrix} 5 & -3 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$5x - 3y + 5z = 0$

$y = 0$

let $z = \alpha$, $\alpha \in R$

$\therefore \boxed{x = -\alpha}$

linearly independent soln

$$X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

When $\lambda = -5$

$$\begin{pmatrix} 15 & -3 & 5 \\ 0 & 6 & 0 \\ -15 & 9 & -5 \end{pmatrix}$$

① $R_3 \to R_3 + R_1 - R_2$

② $R_2 \to R_2/6$

$$\begin{pmatrix} 15 & -3 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$15x - 3y + 5z = 0$

$y = 0$

let $x = \alpha$, $\alpha \in R$.

$\therefore x = \alpha,\ y = 0,\ z = -3\alpha.$

linearly idepent soln

$$X_3 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}.$$

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D is the diagonalisable matrix

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -5 \end{bmatrix}.$$

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Q4 (a) $x + y - z = 0$
$2x - 3y + z = 0$
$x - 4y + 2z = 0$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -3 & 1 & 0 \\ 1 & -4 & 2 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & 0 \\ 0 & -5 & 3 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$\rho(A/B) = \rho(B) = 2$, less than no of unknown

Hence, infinite soln (non-trivial soln) will exist

System becomes

$x + y - z = 0$

$-5y = 3z$

Let $z = k$, $k \in R$

$$\boxed{y = \frac{3}{5}k, \quad z = k \quad \text{and} \quad x = \frac{2}{5}k.}$$

~~Let $k = 5$~~

~~Hence linearly independent soln becomes~~

$$\boxed{\begin{array}{l} x = 2 \\ y = 3 \\ z = 5 \end{array}}$$

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b)

$$x+y-z=0$$
$$2x+4y-z=0$$
$$3x+2y+2z=0$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & 4 & -1 & 0 \\ 3 & 2 & 2 & 0 \end{array}\right].$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right]$$

$R_2 \to R_2 + 2R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 11 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 0 & 11 & 0 \end{array}\right].$$

$\rho(A) = \rho(A|B) = 3$, which equal number of unknowns.

Hence, only trivial sol'n will exist

which is $\boxed{x=0,\ y=0,\ z=0}$

Q10 a)

$$-x+y+2z=2$$
$$3x-y+z=6$$
$$-x+3y+4z=4$$

$$[A|B] = \left[\begin{array}{ccc|c} -1 & 1 & 2 & 2 \\ 3 & -1 & 1 & 6 \\ -1 & 3 & 4 & 4 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$R_2 \to R_2 + 3R_1$

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$$\sim \left(\begin{array}{ccc|c} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 8 \\ 0 & 2 & 2 & 2 \end{array}\right)$$

$R_3 \to R_3 - R_2$

$$\sim \left(\begin{array}{ccc|c} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 8 \\ 0 & 0 & -5 & -6 \end{array}\right)$$

$\rho(A) = \rho(A|B) = 3$, equal number of unknown

Hence, a unique soln exists.

b)

$5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5.$

$$(A|B) = \left(\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right)$$

$R_1 \to R_1 + R_2 - R_3$

$$\sim \left(\begin{array}{ccc|c} 1 & 27 & -1 & 8 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right)$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 7R_1$

$$\sim \left(\begin{array}{ccc|c} 1 & 27 & -1 & 8 \\ 0 & -52 & 5 & -15 \\ 0 & -187 & 17 & -51 \end{array}\right)$$

$\cancel{R_3 \to R_3 - 3R_2}$

$$\cancel{\sim \left(\begin{array}{ccc|c} 1 & 27 & -1 & 8 \\ 0 & -52 & 5 & -15 \\ 0 & 0 & 2 & -6 \end{array}\right)}$$

$R_3 \to R_3/17$

$R_2 \to R_2/5$

$$\left(\begin{array}{ccc|c} 1 & 27 & -1 & 8 \\ 0 & -11 & 1 & -3 \\ 0 & -11 & 1 & -3 \end{array}\right)$$

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$\rho(A) = \rho(A|B) = 2$, less than ~~equal~~ number of unknowns.

Hence, system has ~~unique~~ infinite soln.

c) $x + 3y - 2z + 5w = 4$

$2x + 8y - z + 9w = 9$

$3x + 5y - 12z + 17w = 7$

$$[A|B] = \begin{bmatrix} 1 & 3 & -2 & 5 & 4 \\ 2 & 8 & -1 & 9 & 9 \\ 3 & 5 & -12 & 17 & 7 \end{bmatrix}.$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & -2 & 5 & 4 \\ 0 & 2 & 3 & -1 & 1 \\ 0 & -4 & -6 & 2 & -5 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & -2 & 5 & 4 \\ 0 & 2 & 3 & -1 & 1 \\ 0 & 0 & 0 & 0 & -7 \end{array}\right]$$

$\rho(A/B) \neq \rho(A)$

System is inconsistent

No soln.

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