$$y = x^n$$

$$\frac{d^n y}{dx^n} = \lfloor n$$

$$y = \sin(ax+b)$$

$$\frac{d^n y}{dx^n} = a^n \sin\left(ax+b+\frac{n\pi}{2}\right)$$

$$y = \frac{1}{x}$$

$$\frac{d^n y}{dx^n} = \frac{(-1)^n \lfloor n}{x^{n+1}}$$

$$y = \log_e x$$

$$\frac{d^n y}{dx^n} = \frac{(-1)^{n-1} \lfloor n-1}{x^n}$$

$$\frac{d^n}{dx^n} = D^n$$

$y = u \cdot v$ $\qquad u = u(x)$

$\qquad\qquad\qquad v = v(x)$

Lebinitz Rule

$$D^n y = D^n u \cdot v = v \cdot D^n u + {}^nC_1 D^{n-1} u \cdot Dv + {}^nC_2 D^{n-2} u\, D^2 v - \cdots + u \cdot D^n v$$

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$\Rightarrow \quad y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

To prove $(1-x^2)\dfrac{dy}{dx} = xy + 1$

To prove $(1-x^2)y_{n+1} - (2n+1)xy_n - n^2y_{n-1} = 0$

(·) diff. n times w.r.t x

$$(1-x^2)y_{n+1} + n(-2x)y_n + \frac{n(n-1)}{\underline{2}}(-2)y_{n-1}$$

$$- xy_n - n\cdot 1\cdot y_{n-1} = 0$$

$$(1-x^2)y_{n+1} - (2n+1)xy_n - n^2y_{n-1} = 0$$

at $x=0$

$y_{n+1}(0) = n^2y_{n-1}(0)$

$n \Rightarrow n-2$

$y_{n-1} = (n-2)^2 y_{n-3}$

$y_{n+1}(0) = n^2(n-2)^2y_{n-3}(0)$

$= n^2(n-2)^2(n-4)^2y_{n-5}(0)$

$= n^2(n-2)^2(n-4)^2 + \ldots$ $y_1(0)$ if n is even
$y_0(0)$ if n is odd

$y_5(0)$

$n = 4$

$y_5(0) = 4^2 \cdot \cancel{} \cdot 2^2 \, y_1(0)$

$y_{71}(0)$

$n = 70$

$y_{71}(0) = 70^2 \cdot 68^2 \cdot 66^2 \cdots \cdots y_1(0)$

→ $y = (\sin^{-1}x)^2$

Show that
$(1-x^2)\,y_{n+2} - (2n+1)\,x\,y_{n+1} - n^2 y_n = 0$

$\frac{dy}{dx} = \frac{2(\sin^{-1}x)}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\; y_1 = 2(\sin^{-1}x)$

$(1-x^2)\, y_1^2 = 4(\sin^{-1}x)^2$

MACLAURIN EXP.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots \infty$$

$a_i \Rightarrow i = 0, 1, 2, \ldots$ are const

$$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \cdots$$

$$f''(x) = 2! \cdot a_2 + 3 \cdot 2 a_3 x + \cdots$$

$$f^n(x) = n!\, a_n + \cdots$$

at $x = 0$

$$a_0 = f(0)$$

$$a_1 = f'(0)$$

$$a_2 = \frac{f''(0)}{2!}$$

$$a_n = \frac{f^n(0)}{n!}$$

$$\boxed{f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \cdots \frac{x^n}{n!} f^n(0) + \cdots}$$

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$f(x) = \sin x$ $\qquad f(0) = 0$

$f'(x) = \cos x$ $\qquad f'(0) = 1$

$f''(x) = -\sin x$ $\qquad f''(0) = 0$

$f'''(x) = -\cos x$ $\qquad f'''(0) = -1$

$$\underset{(\sin x)}{f(x)} = 0 + x + 0 - \frac{x^3}{3!} + - - - -$$

→ $\sin^{-1}(x) \Rightarrow$ MACLAURINS

$f(x) = \sin^{-1}(x)$ $\qquad f(0) = 0$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad f'(0) = 1$

$f''(x) = \frac{+\cancel{2}x}{\cancel{2}(1-x^2)^{3/2}}$ $\qquad f''(0) = 0$

$f'''(x) =$

$$\sin^{-1}(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + - - -$$

"at any real no other than '0' Maclaurins expansion fails"

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$\Rightarrow$ $y = f(x)$ $\quad x = a$

Taylor's Expansion

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \cdots$$

$$f(a+h) = a_0 + a_1 h + \frac{a_2 h^2}{2} + \cdots$$

$$f'(a+h) = a_1 + 2a_2 h + 3a_3 h^2 + \cdots$$

$$f''(a+h) = 2a_2 + 6a_3 h + \cdots$$

$$f^n(a+h) = n!\, a_n + \text{terms with } h$$

at $h=0$

$$f(a) = a_0$$

$$f'(a) = a_1$$

$$a_2 = \frac{f''(a)}{2!}$$

$$a_n = \frac{f^n(a)}{n!}$$

$\rightarrow$ $\rightarrow$ Expand the function about point 2 using Taylor exp

$\rightarrow$ Expand the function about power $(x-2)$ using Taylors m

$f(x) = x^3$ at $x = 2$

$f(x) = f(x-2+2)$ $\qquad x^3 = (x-2+2)^3$

$(x-2)^3 + 6(x-2)^2 + 12(x-2) + 8$

$\frac{dy}{dx}$

y_1

y_2

$f(x) = x^3 \rightarrow f(2) = 8$

$f'(x) = 3x^2 \rightarrow f'(2) = 12$

$f''(x) = 6x \rightarrow f''(2) = 12$

$f'''(x) = 6 \rightarrow f'''(2) = 6$

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$$x^3 = 8 + 12(x-2) + \frac{(x-2)^2}{2!} \cdot 12 + \frac{(x-2)^3}{3!} 6$$

$x(\cancel{2}) =$

$$f(x) = \log\sin(x+h)$$

let $f(y) = \log\sin y$

$f'(y) = \cot y$ $\qquad$ $f'(x) = \cot x$

$f''(x) = -\csc^2 y$ $\qquad$ $f''(x) = -\csc^2 x$

$$\log\sin(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \cdots$$

$$= \log\sin x + h\cot x + \frac{h^2}{2!}(-\csc^2 x) + \cdots$$

Expand $\sin x$ in power of $(x - \frac{\pi}{2})$

$$f(x) = \sin\left(\overbrace{x - \frac{\pi}{2}}^{h} + \overbrace{\frac{\pi}{2}}^{a}\right)$$

$$f'(x) = \cos\left(x - \frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$f''(x) = -\sin\left(x - \pi/2 + \pi/2\right)$$

$$= 1 - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!} + \cdots$$

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Expand $\log x$ in the power of $(x-1)$ & find value of $\log 1$

$$f(x) = \log(\overset{h}{x-1} + \overset{a}{1})$$

$$f'(x) = \frac{1}{(x-1+1)}$$

$$f''(x) = -\frac{1}{(x-1+1)^2}$$

$$\log x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \cdots$$

$$\log(1.1) = \log(1+0.1)$$

Compute the value of $\sqrt[3]{122}$ using taylor's exp.

$$(122)^{1/3} \, f(x) = (\underset{a}{125} - \underset{n}{3})^{1/3} \qquad a = 125,\; n = -3$$

$$f(x) = (x)^{1/3} \qquad f(125) = 5$$

$$f'(x) = \frac{1}{3(x)^{2/3}} \qquad f'(125) = \frac{1}{75}$$

$$f''(x) = \frac{-2}{9} \times \frac{1}{x^{5/3}} \qquad f''(125) = \frac{-2}{9} \times \frac{1}{5^5}$$

$$(122)^{1/3} = 5 + (-3)\frac{1}{75} + \frac{(-3)^2}{2!}\left(\frac{-2}{9}\right) \times \frac{1}{5^5} + \cdots$$

Q. Compute the value $\sqrt{2}$

$$(2)^{1/2} = (1+1)^{1/2}$$

$f(x) = x^{1/2}$ $\quad$ $f(1) = 1$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad$ $f'(1) = \frac{1}{2}$

$f''(x) = \cdot$

ERROR

$\delta x \rightarrow$ absolute error

$\frac{\delta x}{x} \rightarrow$ relative error

$\frac{\delta x}{x} \times 100 \rightarrow$ percentage error

$$y = f(x)$$
$$y + \delta y = f(x + \delta x)$$

$$\delta y = y + \delta y - y = f(x+\delta x) - f(x)$$

$$= f(x) + \delta x \cdot f'(x) + \frac{(\delta x)^2}{2} \cdot f''(x) + \ldots \infty - f(x)$$

$$\delta y = \delta x \cdot f'(x)$$

$$\boxed{\delta y = \frac{dy}{dx} \cdot \delta x}$$

Qn 1 Pressure P & volume V of gas are connected by relation PV^{γ} = constant value of $\gamma = 1.4$, find the % change in the pressure corresponding to a diminishin of $\frac{1}{2}$% of volume ?

Qn 2 Show that if $t = 2\pi\sqrt{l/g}$ an error of 2% is measuring l and g will cause an error of 1% in the time while error of 2% in l & g both may cause either no error or 2% in value of t ?

Ans 1.

$$\frac{\delta V}{V} \times 100 = \frac{1}{2}$$

$$\delta p = \frac{dt}{dV} \cdot \delta V$$

$$= \frac{dt}{dV}(pV^{1.4})$$

$$= -1.4\frac{p}{V}\delta V$$

$$\frac{\delta p}{p} \times 100 = +1.4 \times \frac{\delta V}{V} \times 100$$

$$= +1.4 \times \frac{1}{2}$$

$$= +0.7$$

∴ pressure will increase with 0.7%.

An 2

$t = 2\pi\sqrt{l/g}$

$\log t = \frac{1}{2}\log l - \frac{1}{2}\log g + \log 2\pi$

St. $\frac{d}{dt}\log t = \frac{1}{2}\delta l \frac{d}{dt}\log l - \frac{1}{2}\delta g \frac{d}{dt}\log g$

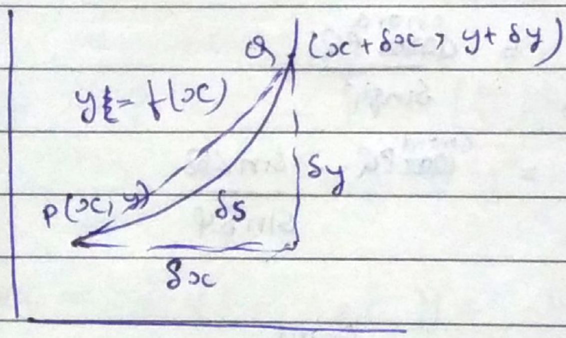
$\frac{\delta t}{t} = \frac{1}{2}\left(\frac{\delta l}{l} - \frac{\delta g}{g}\right)$

$\frac{\delta t}{t}\times 100 = \frac{1}{2}\left(\frac{\delta l}{l}\times 100 - \frac{\delta g}{g}\times 100\right)$

RADIUS OF CURVATURE - "ρ"

Curvature $= \frac{1}{\text{radius of curvature}}$

[radius ↓ses bending ↑ses]



δ ⇒ arc length

chord. $PQ = \sqrt{\delta x^2 + \delta y^2}$

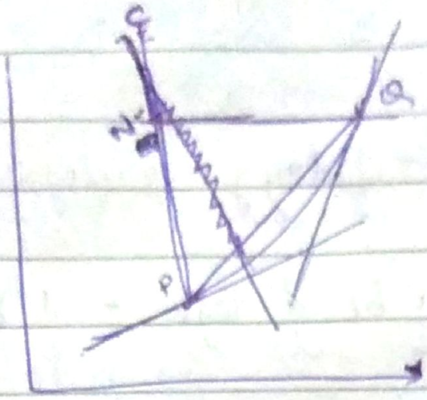
arc. $PQ = \delta s = \sqrt{\delta x^2 + \delta y^2}$

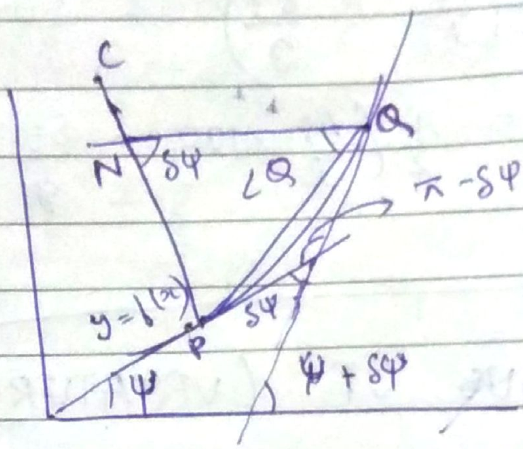
$\frac{\delta s}{\delta x} = \sqrt{1 + \left(\frac{\delta y}{\delta x}\right)^2}$

[In limiting situation arc will be taken as a line]

as $\delta x \to 0$

$$\boxed{\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}}$$





$$CP = \rho = \lim_{Q \to P} NP$$

ΔNPQ

$$\frac{NP}{\sin\angle Q} = \frac{\text{chord } PQ}{\sin\delta\psi}$$

$$NP = \text{chord } PQ \cdot \frac{\sin\angle Q}{\sin\delta\psi}$$

$$\rho = \lim_{Q \to P} \text{chord } PQ \cdot \frac{\sin\angle Q}{\sin\delta\psi} \qquad \text{arc } PQ = \delta s$$

$$= \lim_{\substack{Q \to P \\ \delta x \to 0}} \frac{\text{chord } PQ}{\text{arc } PQ} \cdot \frac{\delta s}{\delta\psi} \cdot \frac{\delta\psi}{\sin\delta\psi} \sin\angle Q$$

$$\boxed{\rho = \frac{ds}{d\psi}}$$

$\sin Q = \sin 90°$
when
$Q \to P$

$$\boxed{f(x, y) = g(s, \psi) \rightarrow \text{Intrinsic equation}}$$

$$\frac{dy}{dx} = \tan\psi$$

$$\frac{d^2y}{dx^2} \times \frac{dx}{dy} = \sec^2\psi \left(\frac{d\psi}{ds}\right) \rightarrow 1/\rho$$

$$\rho = \frac{\sec^2\psi}{\frac{d^2y}{dx^2} \cdot \frac{dx}{dy}}$$

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\frac{dy}{ds} = \sin\psi$$

$$= \frac{\sec^3\psi}{\frac{d^2y}{dx^2}}$$

$$\frac{dx}{ds} = \cos\psi$$

$$\boxed{\rho = \frac{\left(1+(y')^2\right)^{3/2}}{\frac{d^2y}{dx^2}}}$$

$$\rho = \frac{\left[1+(y')^2\right]^{3/2}}{y''}$$

Parametric form

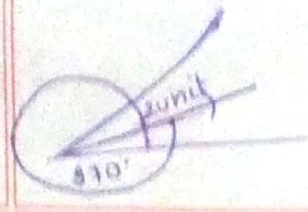
$$x = \phi(t), \quad y = \psi(t)$$

$$y' = \frac{dy/dt}{dx/dt}$$

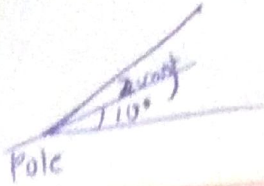
$$y'' = \frac{d}{dt}(\psi')\frac{dt}{dx} = \frac{d}{dt}\left(\frac{\psi'}{\phi'}\right)$$

$$\rho = \frac{\left[1+\left(\frac{dy/dt}{dx/dt}\right)^2\right]^{3/2}}{\frac{\phi'\psi'' - \phi''\psi'}{(\phi')^3}} = \frac{\frac{1}{(\phi' t)^3}\left[(\phi' t)^2+(\psi' t)^2\right]^{3/2}}{\frac{\phi'\psi'' - \phi''\psi'}{(\phi')^3}}$$

$$\rho = \boxed{\frac{\left[\phi'^2 + \psi'^2\right]^{3/2}}{\phi'\psi'' - \phi''\psi'}}$$



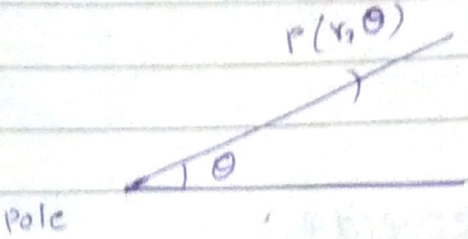
$P(2, 370^\circ)$

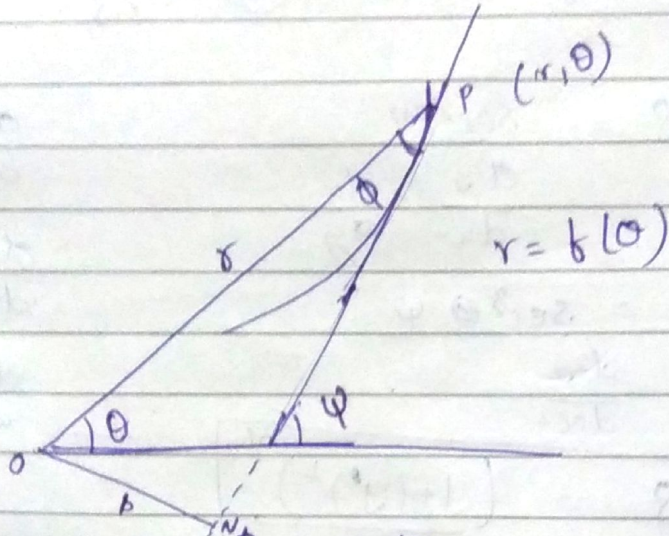


$P(2, 10)$

POLAR CURVE

$$\left[\begin{array}{l} r = f(\theta) \rightarrow \text{implicit} \\ g(r, \theta) = c \rightarrow \text{explicit} \end{array}\right.$$





ϕ = angle b/w radius vector & tangent at P

$$\tan\phi = r\frac{d\theta}{dr}$$

$$\frac{r\,d\theta}{ds} = \sin\phi$$

$$\frac{dr}{ds} = \cos\phi$$

$$\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

$$p = r\sin\phi$$

$$\frac{1}{p^2} = \frac{1}{r^2}\operatorname{cosec}^2\phi$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2 \qquad \left[\text{by } \tan\phi = r\frac{d\theta}{dr}\right]$$

$$\psi = \theta + \phi$$

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** $\left[f(r,\theta) = \underline{g(r,p)} = \text{constant} \Rightarrow \text{Pedal Equat.} \right]$

$$\frac{d\psi}{ds} = \frac{d\theta}{ds} + \frac{d\phi}{dr} \cdot \frac{dr}{ds}$$

$$\frac{1}{\rho} = \frac{1}{r}\sin\phi + \cos\phi \frac{d\phi}{dr}$$

$$= \frac{1}{r}\left(\sin\phi + r\cos\phi \frac{d\phi}{dr}\right)$$

$$\frac{1}{\rho} = \frac{1}{r}\frac{d}{dr}(r\sin\phi) \qquad \therefore \; p = r\sin\phi$$

$$\boxed{\frac{1}{\rho} = \frac{1}{r} \cdot \frac{dp}{dr}}$$

$$\boxed{\rho = \frac{r\,dr}{dp}} \Rightarrow \text{[Pedal Equation]}$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2 \quad \text{—— (1)}$$

differentiatin this equation

$$-\frac{2}{p^3}\frac{dp}{dr} = \frac{-2}{r^3} - \frac{4}{r^5}\left(\frac{dr}{d\theta}\right)^2 + \frac{2}{r^4}\cancel{\frac{dr}{d\theta}} \cdot \frac{d^2r}{d\theta^2} \cdot \cancel{\frac{d\theta}{dr}}$$

$$r' = \frac{dr}{d\theta} \qquad r'' = \frac{d^2r}{d\theta^2}$$

$$\frac{r^5}{p^3}\frac{dp}{dr} = r^2 + 2(r')^2 - rr''$$

$$\frac{r \times r^5}{p^3} \times \frac{1}{r}\frac{dp}{dr} = r^2 + 2(r')^2 - rr''$$

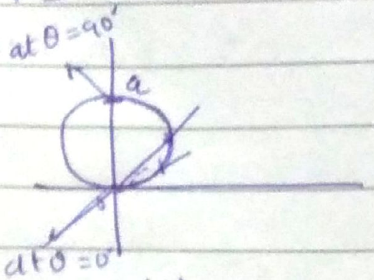
$$\frac{r^6/p^3}{\rho} = r^2 + 2(r')^2 - rr''$$

$$\boxed{\rho = \frac{r^6/p^3}{r^2 + 2(r')^2 - rr''}}$$

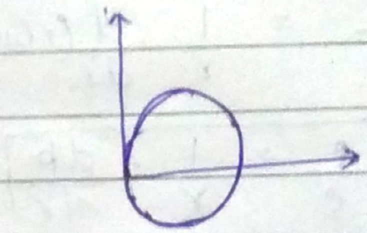
r^6/p^3 become $[r^2+(r')^2]^{3/2}$ from eq (1)

$$\rho = \frac{[r^2 + (r')^2]^{3/2}}{r^2 + 2(r')^2 - rr''}$$

$r = a\sin\theta$



$r = a\cos\theta$

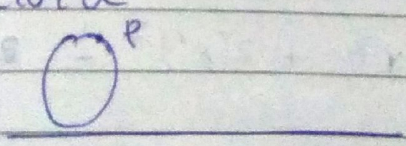


When we increase the θ by after 90° when sin become -ve we will back track the path of +ve angle of sin.

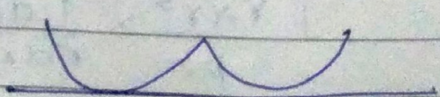
$s = c\tan\psi$ $\qquad$ c = constant

$\rho = \frac{ds}{d\psi} = c\sec^2\psi$

Cycloid



the path traced



$s =$

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Q. Show that the curvature at point $P\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the ~~follow~~ folium

$y = x^3 + y^3 = 3axy$ is $-\frac{8\sqrt{2}}{3a}$

~~Sol~~ $3x^2 + 3y^2y' = 3ay + 3axy'$

$$y' = \frac{3x^2 - 3ay}{3ax - 3y^2}$$

$$y' = \frac{x^2 - ay}{ax - y^2}$$

$$y'_P = \frac{9a^2/4 - 3a^2/2}{3a^2/2 - \frac{9a^2}{4}} \Rightarrow -1$$

$$6x + 6y(y')^2 + 3y^2y'' = 3ay' + 3ay' + 3axy''$$

$$6x + 6y(y')^2 - 6ay' = y''(3ax - 3y^2)$$

$$2x + 2y(y')^2 - 2ay' = y''(ax - y^2)$$

at P $y' = -1$

$$2x + 2y + 2a = y''(ax - y^2)$$

$$3a + 3a + 2a = y''\left(\frac{3a^2}{2} - \frac{9a^2}{4}\right)$$

$$y'' = \frac{-8a \times 4}{3a^2}$$

$$y'' = -\frac{32}{3a}$$

$$\rho = \frac{(1 + (y')^2)^{3/2}}{y''} = \frac{2^{3/2}}{-32/3a}$$

$$= -\frac{3\sqrt{2}a}{16}$$

curvature $= \frac{1}{\rho} = -\frac{16}{3\sqrt{2}a} = -\frac{8\sqrt{2}}{3a}$

$x = a(t - \sin t)$

$y = a(1 - \cos t)$

$$\rho = \frac{(x'^2 + y'^2)^3}{y'x'' - y''x'} = \frac{(1+y'^2)^{3/2}}{y''}$$

$$x' = \frac{dx}{dt} = a(1-\cos t) \quad | \quad x'' = a\sin t$$

$$y' = a\sin t \quad | \quad y'' = a\cos t$$

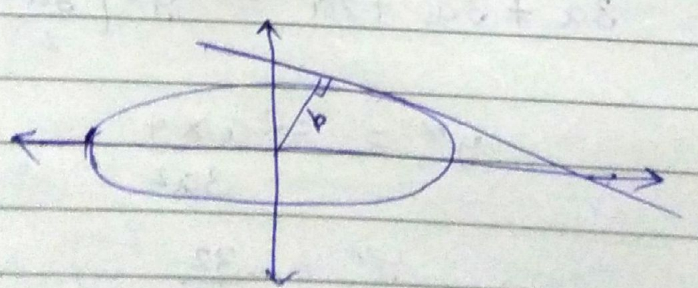
$$\frac{dy}{dx} = y'/x' = \frac{a\sin t}{a(1-\cos t)} = \frac{2\sin t/2 \cos t/2}{2\sin^2 t/2}$$

$$= \cot t/2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\cot t/2 \cdot \frac{dt}{dx} = -\frac{1}{2}\operatorname{cosec}^2 t/2 \times \frac{1}{a(1-\cos t)}$$

4 Prove that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

$\rho = \frac{a^2b^2}{p^3}$; $\cancel{p}$



$$\frac{xx'}{a^2} + \frac{yy'}{b^2} = 1$$

$$p = ON = \frac{1}{\sqrt{\frac{x'^2}{a^2} + \frac{y'^2}{b^2}}}$$

$$\frac{1}{p^2} = \left(\sqrt{\frac{x'^2}{a^2} + \frac{y'^2}{b^2}}\right)^2$$

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Supplementary problem

Show that the pedal eq for this ellipse is

$$a^2 + b^2 - r^2 = \frac{a^2b^2}{p^2} \rightarrow \cancel{2}r\frac{dr}{dp} = \cancel{2}\frac{a^2b^2}{p^3}$$

$$\boxed{r\frac{dr}{dp} = \rho = \frac{a^2b^2}{p^3}}$$

$$y' = \frac{-xb^2}{ya^2}$$

$$y'' = \frac{-b^2}{a^2}\left[\frac{y - xy'}{y^2}\right]$$

$$y'' = -\frac{b^2}{a^2}\left[\frac{y + \frac{x^2b^2}{ya^2}}{y^2}\right]$$

$$y'' = \frac{-b^2}{a^4y^3}\left[a^2y^2 + x^2b^2\right]$$

$$\rho = \frac{(1+y'^2)^{3/2}}{y''} = \frac{\left[1 + \frac{x^2}{y^2}\frac{b^4}{a^4}\right]^{3/2}}{-\frac{b^2}{a^4y^3}\left[a^2y^2 + x^2b^2\right]}$$

Qus Show that at any point on equiangular spiral

$r = ae^{\theta\cot\alpha}$, are

$\rho = r\operatorname{cosec}\alpha$

and that it subtends the right angle at pole.

Ans

$$\rho = \frac{(r^2 + r'^2)^{3/2}}{r^2 + 2r'^2 - rr''}$$

$$r' = \frac{dr}{d\theta} = a\cot\alpha\, e^{\theta\cot\alpha}$$

$$= r\cot\alpha$$

$$r'' = \frac{d^2r}{d\theta^2} = r'\cot\alpha \rightarrow r\cot^2\alpha$$

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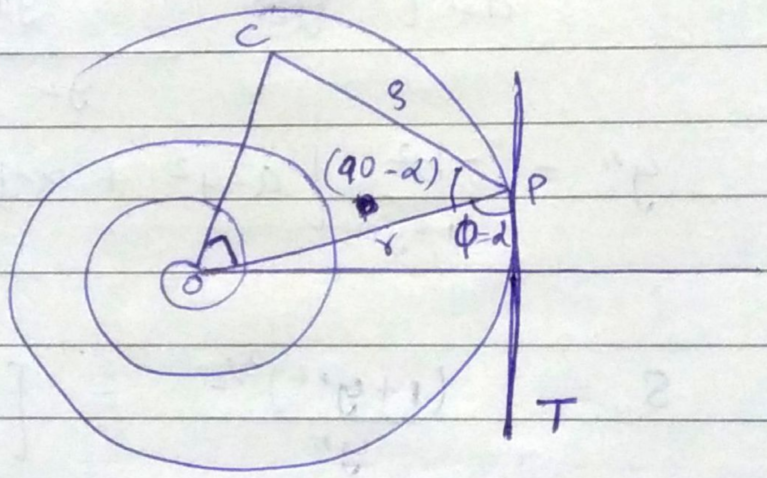
$$\rho = \frac{(r^2 + r^2\cot^2\alpha)^{3/2}}{r^2 + 2r^2\cot^2\alpha - r^2\cot^2\alpha}$$

$$= \frac{r^3(1+\cot^2\alpha)^{3/2}}{r^2(1+2\cot^2\alpha - \cot^2\alpha)}$$

$$= \frac{r(\operatorname{cosec}^2\alpha)^{3/2}}{\operatorname{cosec}^2\alpha}$$

$\rho = r\operatorname{cosec}\alpha$ Hence proved.



$$\tan\phi = \frac{r\,d\theta}{dr}$$

$$= \frac{r}{r\cot\alpha} = \tan\alpha$$

$$\phi = \alpha$$

In ΔCOP
if $\angle O$ is $\pi/2$

$$\cos(\pi/2 - \alpha) = \frac{r}{\rho}$$

$$\rho = r\operatorname{cosec}\alpha$$

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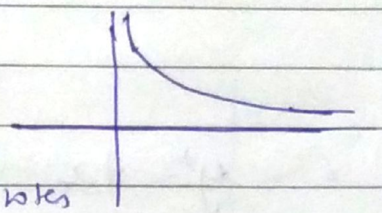
ASYMPTOTES is line which tends to tangent at ∞ to the curve remaining at a finite distance.

Equate the coff of highest power of x equal to zero provided it is not a constant is will give the asmptotes // to x-axis and asymptotes // to y-axis will be obtained by the equating highest power of y equals to zero.

Rectangular hyperbola $xy = 1$

$y = 0$ } asymptotes
$x = 0$

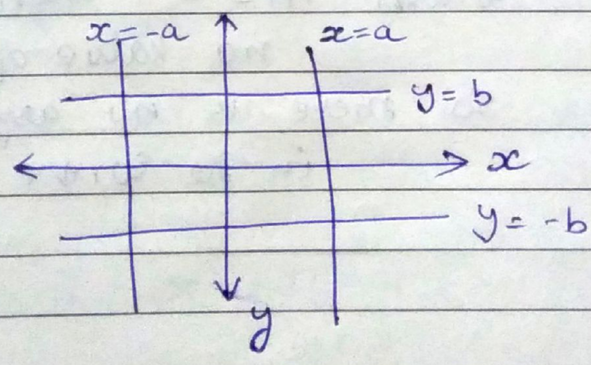


$$(x^2 - a^2)(y^2 - b^2) = a^2b^2$$

$y^2 - b^2 = 0 \Rightarrow y = \pm b$

coeff of x

coeff of $y = 0 \Rightarrow x^2 - a^2 = 0 \Rightarrow x = \pm a$



$\Rightarrow$ $f(x, y) = 0$

let $y = mx + c$ be the asymptotes
put $y = mx + c$ in $f(x, y)$ and arrange all power of x in descending order.

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$f(x,y)=0 \quad = a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \cdots a_ny^n$

$x^n(\quad\downarrow\quad) + x^{n-1}(\quad\downarrow\quad) + \cdots$

$\phi(m) \qquad\qquad \phi_2(m,c)$

$\phi(m)=0 \Rightarrow m_1, m_2 \cdots m_n$ are roots.

$\phi_1(m,c)=0$

$\phi_2(m,c)=0$

le

$y^2=4x$

$(mx+c)^2=4x$

$m^2x^2+c^2+2mxc=4x$

$m^2x^2+x(2mc-4)+c^2=0$

$m^2=0 \Rightarrow m=0,0$

$2mc-4=0$

$mc-2=0$

$mc=2$

when $m=0$ then there is

$\therefore$ no value of c

so there is no asymptotes line

in the curve $y^2=4x$

$x \cdot 0 = 0 \rightarrow$ indeterminate, infinite solution

$x \cdot 1 = 0 \Rightarrow$ unique solution

$x \cdot 0 = 1 \rightarrow$ no solution

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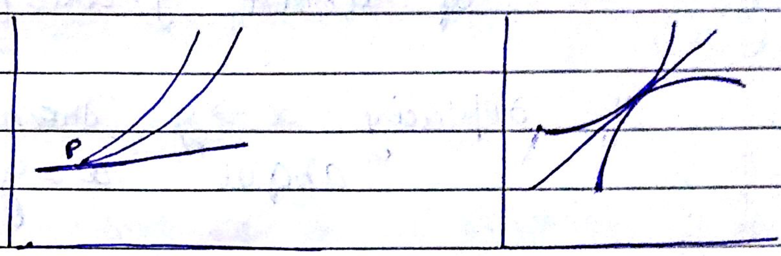
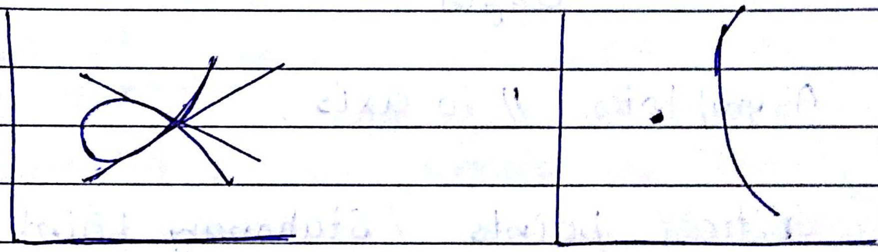
$a_n y^n$

MULTIPLE POINTS

A point where more than one brances of curve passes.



cusp



Node

Conjugate point

(*) The equation of tangent at the origin for the curve can be obtained by equating the lowest degree of curve equal to zero.

for eg:- $x^2 + 3y^2 + x^3 - 3y^2x + 2x + 3y = 0$

$2x + 3y = 0$ so this is the equation of tangent at origin.

if $x^2 + 3y^2 + x^3 - 3y^2x$

$x^2 + 3y^2 = 0$

$y = \pm\sqrt{-\frac{x^2}{3}}$

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CURVE TRACING

(*) Symmetry - If all terms of $x(y)$ are of even degree than curve has symmetry about y-axis (x-axis).

(*) If replacing $x \to y$ then curve has symmetry about $x = y$.

(*) Origin - check whether curve passing through origin

(*) Asymptotes ∥ to axis.

(*) Critical points / Stationary point → there is maxima or minima exsist

(*) solve for x or y as per convenience.

For polar curve
$r = f(\theta)$; $f(r, \theta) = 0$

Symmetry $\theta \to -\theta$

	θ_1	θ_2	--	θ_n
r	r_1	r_2	--	r_n

for tracing.

$x^3 + 3xy^2 + y^2 + 2x + y = 0$

as coeff of x^3 is constant so there is no asymptote $\parallel$ to x-axis

$3x + 1 = 0$

$x = -1/3$ asymptote $\parallel$ to y-axis.

$y = \frac{x^2}{1-x^2}$

Symmetry → about y-axis as term of all x are even degree

Origin → ~~passes through~~ (0, 0) satisfy the equation

$y(1-x^2) = x^2$

$y - yx^2 = x^2$

$x^2 + yx^2 - y = 0$

$\boxed{y = 0}$ eq of tangent at origin

∴ curve is passing through origin.

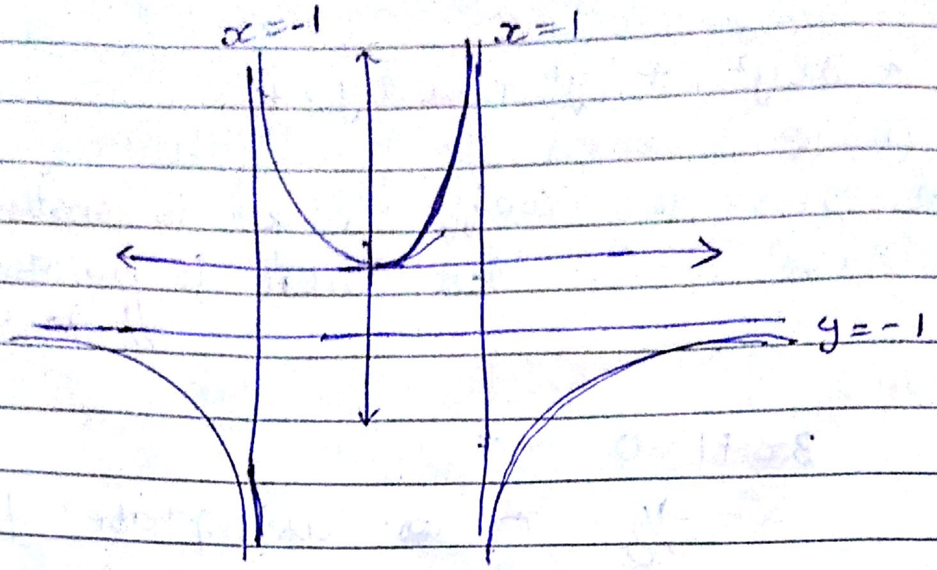
asymptotes

$x = \pm 1$; $y = -1$

↓ ↓

$x^2 - 1 = 0$ $y = -1$

$x = \pm 1$



$0 < x < 1$

$y = +ve$

$x \to 1$

$y \to \infty$

$1 < x$

$y = -ve$

$x \to \infty$

$y \to -1$

Qn $\quad x = (y-1)(y-2)(y-3)$

Symmetry = No symmetry
No asymptotes
no of satisfying origin.

$x = 0 \Rightarrow y = 1, 2, 3$

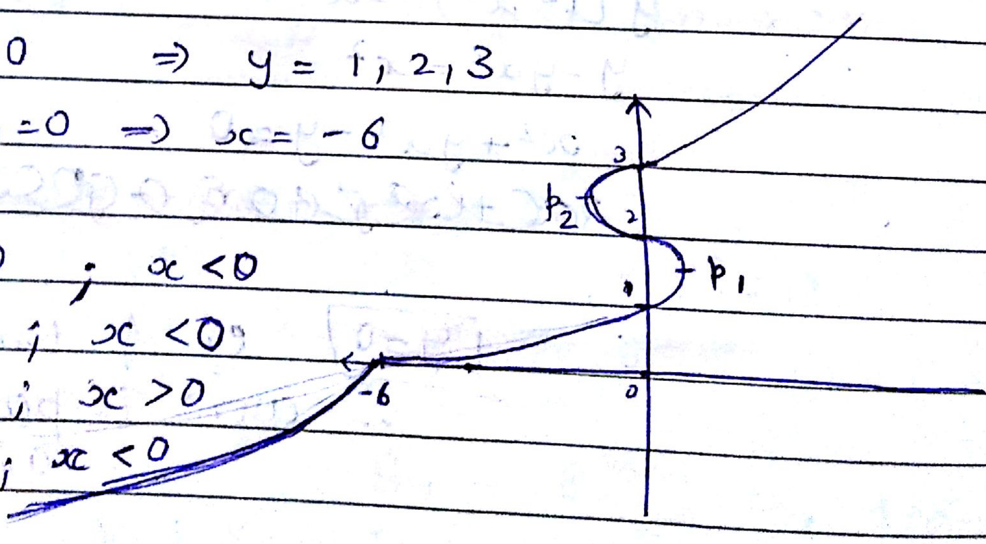
$y = 0 \Rightarrow x = -6$

if $y < 0$; $x < 0$

if $1 > y > 0$; $x < 0$

if $1 < y < 2$; $x > 0$

if $2 < y < 3$; $x < 0$



$\frac{dx}{dy} = 0$ we'll get p_1 & p_2

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$$y^2(x^2+y^2) - 4x(x^2+2y^2) + 16x^2 = 0$$

Symmetry along x-axis as y-degree even.
$(0,0)$ satisfy the equation

$16x^2 = 0 \Rightarrow x=0, \; x=0$ [two coincident tangent]
$\therefore$ there will be cusp

$$y^4 + y^2(x^2-8x) + 16x^2 - 4x^3 = 0$$

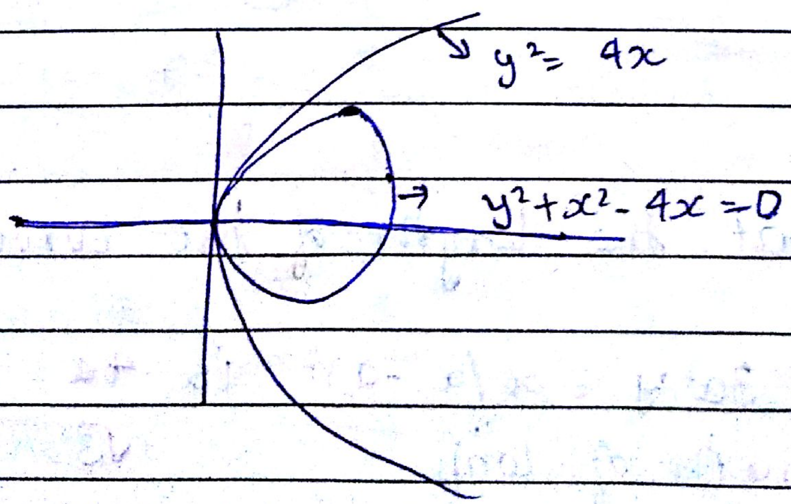
$$y^2 = \frac{8x - x^2 \pm \sqrt{(8x-x^2)^2 - 4(16x^2-4x^3)}}{2}$$

$$y^2 = \frac{8x-x^2 \pm \sqrt{\cancel{64x^2}+x^4-\cancel{16x^3}-\cancel{64x^2}+\cancel{16x^3}}}{2}$$

$$y^2 = \frac{8x-x^2 \pm \sqrt{x^4}}{2}$$

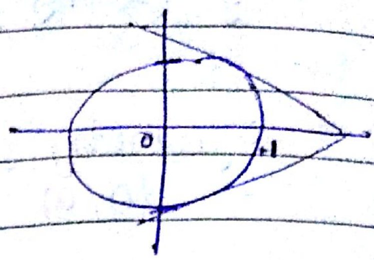
$$y^2 = \frac{8x-x^2 \pm x^2}{2}$$

$y^2 = 4x$; $y^2 = 4x - x^2$
$y^2 + x^2 - 4x = 0$



Cusp

$$ds = \sqrt{1+y'^2}\,dx$$
$$ds = \sqrt{r^2 + r'^2}\,d\theta$$



Qn [illegible]

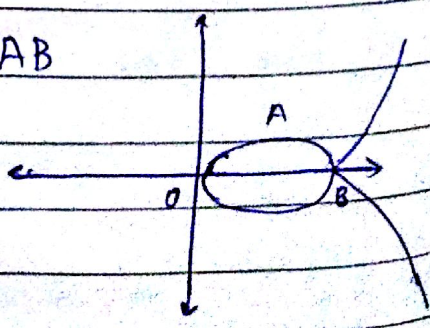
$$x^2 + y^2 = 1$$
$$y' = \frac{-x}{y}$$
$$\sqrt{1+y'^2} = \sqrt{1 + \frac{x^2}{y^2}} = \frac{1}{y}$$
$$\int_0^s ds = \int_0^1 \frac{1}{y}\,dx$$
$$s = \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$$
$$x = \sin\theta$$
$$dx = \cos\theta\, d\theta$$
$$s = \int_0^{\pi/2} \frac{\cos\theta\, d\theta}{\cos\theta}$$
$$s = \pi/2$$

Qn Prove that the length of the curve

$3a^2 y = x(x-a)^2$ is $\frac{4a}{\sqrt{3}}$

length of loop

$= 2$ length of arc OAB



$$S_{OAB} = \int_{x_0}^{x_B} \sqrt{1+y'^2}\,dx$$

$$6ayy' = (x-a)^2 + 2x(x-a)$$

$$6ayy' = (x-a)[3x-a]$$

$$(y')^2 = \frac{(x-a)^2(3x-a)^2}{36a^2y^2}$$

$\because 3ay^2 = x(x-a)^2$

$$1+(y')^2 = \frac{\cancel{(x-a)^2}(3x-a)^2}{12ax\cancel{(x-a)^2}} + 1$$

$$\sqrt{1+(y')^2} = \sqrt{\frac{(3x+a)^2}{12ax}}$$

$$= \frac{(3x+a)}{\sqrt{12ax}}$$

$$S_{OAB} = \frac{1}{\sqrt{12a}} \int_0^{a} 3\sqrt{x} + \frac{a}{\sqrt{x}}\, dx$$

$$S_{OAB} = \frac{2a}{\sqrt{3}}$$

(*) $\boxed{\sinh x = \frac{e^x - e^{-x}}{2}}$

(*) $\boxed{\cosh x = \frac{e^x + e^{-x}}{2}}$

(*) $\boxed{\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}}$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

let $\theta = ix$

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$\cos ix = \frac{e^{-x}+e^{x}}{2}$

$\boxed{\cos ix = \cosh x}$

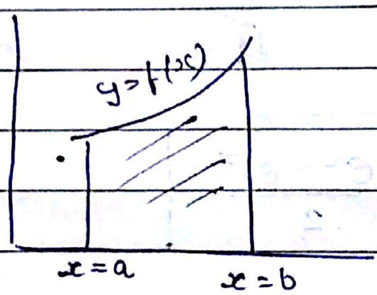
$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2}$

$\sin ix = \frac{e^{-x}-e^{x}}{2i}$

$= i\, \frac{e^{x}-e^{-x}}{2}$

$\boxed{\sin ix = i\sinh x}$

$y = f(x)$; $x=a$, $x=b$



$A = \int_a^b f(x)\,dx$

$x^2+y^2=1$

$\int_{-1}^{1} y\,dx = \int_{-1}^{1}\sqrt{1-x^2}\,dx$

$= 2\int_0^1 \sqrt{1-x^2}\,dx$

$= 2\int_0^1 \cos\theta\cos\theta\,d\theta = 2\int \cos^2\theta\,d\theta$

$= 2\times \pi/4 = \pi/2$

$$\left[\int_0^{\pi/2} \sin^m\theta \cos^n\theta = \frac{\Gamma\frac{m+1}{2}\,\Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+2}{2}}\right]$$ Gamma function

$\Gamma 1 = 1$

$\Gamma(n+1) = n\Gamma n$

$= n\Gamma(n-1+1) \quad 0<n<1$

$= n(n-1)\Gamma(n-1)$

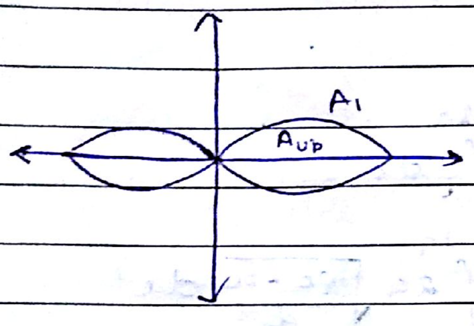
$\Gamma n = \int_0^{\infty} e^{-t} t^{n-1} dt$

Qu Trace the curve and find the area

(1) $a^2y^2 = a^2x^2 - x^4$

$a^2y^2 = x^2(a^2 - x^2)$

$y = \pm \frac{x}{a}\sqrt{a^2-x^2}$



Total area $A = 4 \times A_1$

$A_1 = \int_0^a y\,dx$

$= \int_0^a \frac{x}{a}\sqrt{a^2-x^2}\,dx$

$= \frac{a^2}{3}$

Total Area $= \frac{4a^2}{3}$

$\boxed{n \in I^+ \quad \Gamma(n+1) = n!}$

$\Gamma\frac{11}{3} = \frac{8}{3}\Gamma\frac{8}{3}$

$= \frac{8}{3} \times \frac{5}{3}\Gamma\frac{5}{3}$

$= \frac{8}{3} \times \frac{5}{3} \times \frac{2}{3}\Gamma\frac{2}{3}$

$\boxed{\Gamma\tfrac{1}{2} = \sqrt{\pi}}$

Qu Find the area included b/w the curve

$xy^2 = 4a^2(2a - x)$ and its asymptotes.

An $y = \pm 2a\sqrt{\frac{2a-x}{x}}$

asymptotes, $x = 0$

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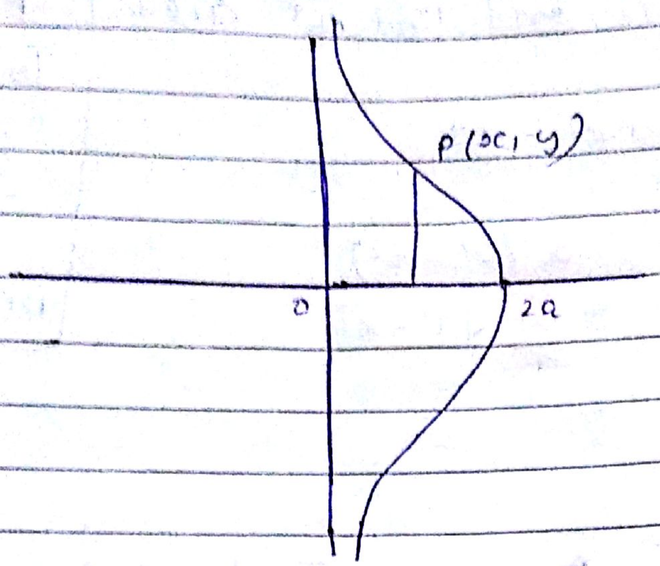
Total $A = 2A_{up}$



$$A_{up} = \int_0^{2a} y\,dx$$

$$= \int_0^{2a} 2a\sqrt{\frac{2a-x}{x}}\,dx$$

$x = 2a\sin^2\theta$

$dx = 4a\sin\theta\cos\theta\,d\theta$ $\quad = \int_0 2a \times \sqrt{\frac{\cos^2\theta}{\sin^2\theta}}\; 4a\sin\theta\cos\theta\,d\theta$

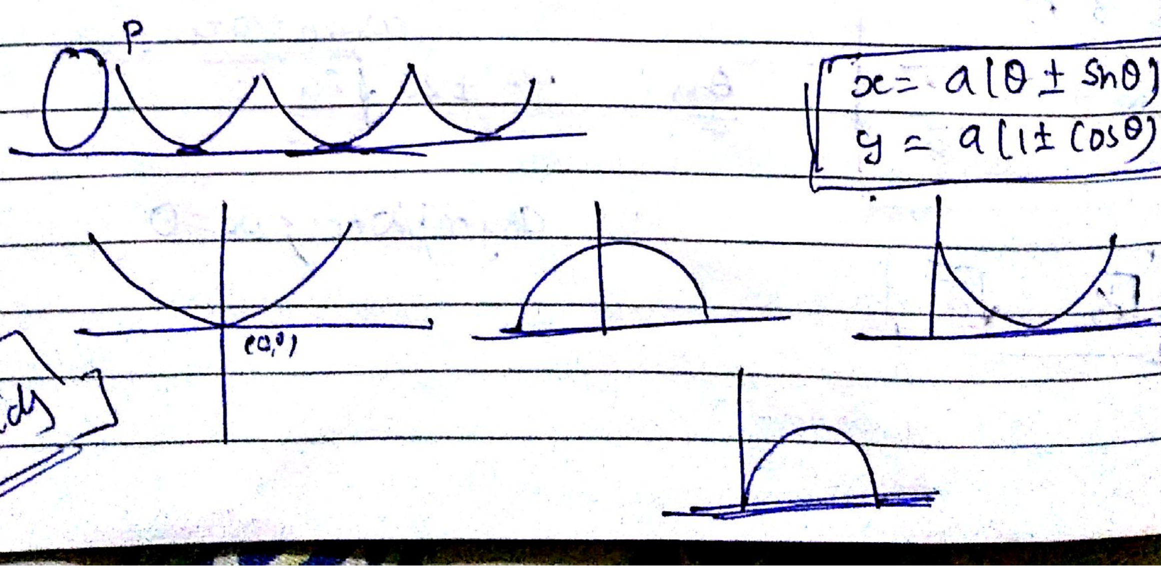
$$= 8a^2 \int_0^{2a} \frac{\cos\theta}{\sin\theta} \times \sin\theta \times \cos\theta\,d\theta$$

$$= 8a^2 \int_0^{\pi/2} \cos^2\theta\,d\theta$$

$$= \frac{8\pi a^2}{4} = 2\pi a^2$$

$$\text{Total } A = 2 \times 2\pi a^2$$
$$= 4\pi a^2$$

(*)



$$\boxed{\begin{aligned} x &= a(\theta \pm \sin\theta) \\ y &= a(1 \pm \cos\theta) \end{aligned}}$$

$\boxed{\text{Four cycloids}}$

find the area included b/w the cycloid $x = a(\theta - \sin\theta)$ $y = a(1-\cos\theta)$ and its base

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{a\sin\theta}{a(1-\cos\theta)}$$

$$= \frac{2\sin\theta/2 \cos\theta/2}{2\sin^2\theta/2}$$

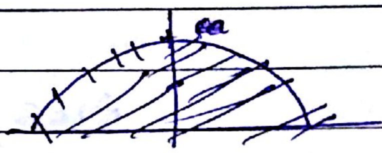
$$= \cot\theta/2$$

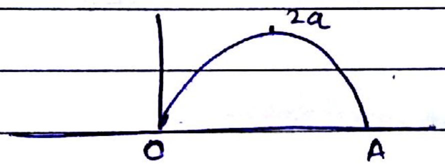
$$\frac{dy}{dx} = 0 \quad = \cot\theta/2 \Rightarrow \theta = \pi$$

Vertex $= \theta = \pi$

$x = a\pi$

$y = 2a$





$$A = \int_{x_0}^{x_A} y\, dx$$

$$= \int_0^{2\pi} a(1-\cos\theta) \cdot a(1-\cos\theta)\, d\theta$$

$$= \int_0^{2\pi} 4a^2 \sin^2\theta/2 \; \sin^2\theta/2 \, d\theta$$

$$= 8a^2 \int_0^{\pi} \sin^4\theta/2 \, d\theta$$

let $\theta/2 = \phi \Rightarrow d\theta = 2d\phi$

$$= 16a^2 \int_0^{\pi/2} \sin^4\phi \, d\phi$$

$$= 16a^2 \frac{\Gamma\frac{5}{2}\,\Gamma\frac{1}{2}}{2\cdot\Gamma 3}$$

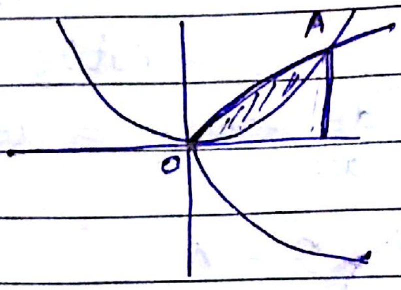
$$= \frac{16a^2}{4}\sqrt{\pi}\left\{\frac{3}{2}\times\frac{1}{2}\sqrt{\pi}\right.$$

$$= 3\pi a^2$$

Qn Find the area of the curve

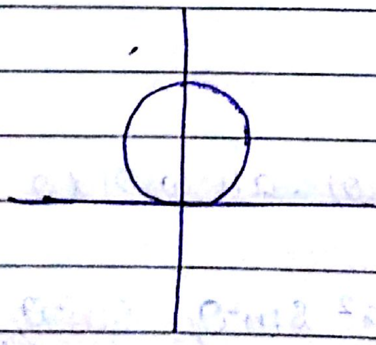
$y^2 = 4ax$

$x^2 = 4ay$



$$\text{Area} = \int_{x_0}^{x_A} \sqrt{4ax}\,dx - \int_{x_0}^{x_A} \frac{x^2}{4a}\,dx$$

Qn $r = 2a\sin\theta$



$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\,d\theta$$

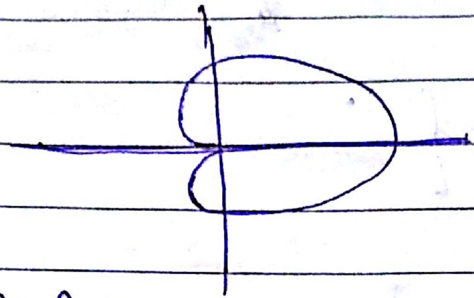
$$= \frac{1}{2}\int_{0}^{\pi} 4a^2\sin^2\theta\,d\theta$$

$$= 4a^2\int_{0}^{\pi/2} \sin^2\theta\,d\theta$$

$$= 4a^2 \times \frac{\pi}{4}$$

$$= \pi a^2$$

Q) $r = a(1+\cos\theta)$



$A = 2 \times A_{up}$

$$A_{up} = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$$

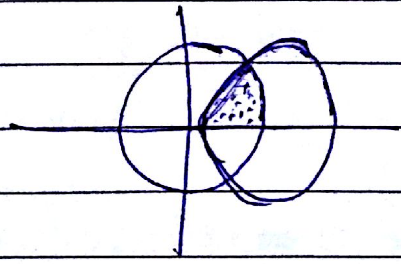
$$= \frac{1}{2}\int_{0}^{\pi} a^2(1+\cos\theta)^2 d\theta$$

$$= \frac{a^2}{2}\int_{0}^{\pi} 4\cos^4\theta/_2 \, d\theta \qquad \theta/_2 = \phi, \quad d\theta = 2d\phi$$

$$= 2a^2 \times 2\int_{0}^{\pi/2} \cos^4\phi \, d\phi$$

$$= 4a^2\int_{0}^{\pi/2} \cos^4\phi \, d\phi$$

Ques Find the area common to the circle $r = a\sqrt{2}$ and $r = 2a\cos\theta$



$a\sqrt{2} = 2a\cos\theta$

$\cos\theta = \frac{1}{\sqrt{2}}$

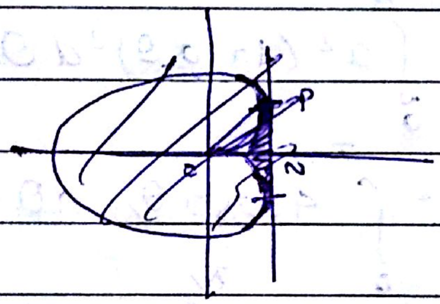
$\theta = \pi/4$

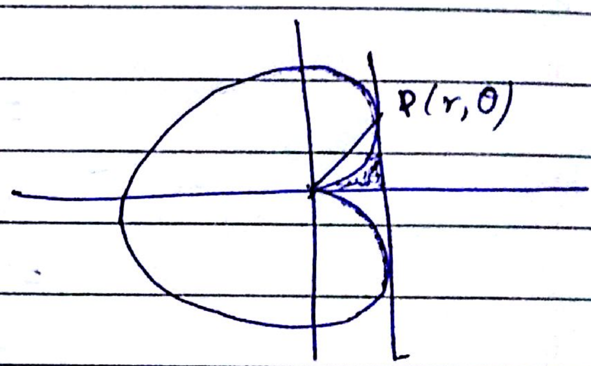
$$A_{up} = \frac{1}{2}\left[\int_{0}^{\pi/4} (a\sqrt{2})^2 d\theta + \int_{\pi/4}^{\pi/2} (2a\cos\theta)^2 d\theta\right]$$

$$\frac{1}{2}\left[\int_0^{\pi/4} 2a^2\,d\theta + \int_{\pi/4}^{\pi/2} 4a^2\cos^2\theta\,d\theta\right]$$

$$= \frac{a^2}{2}\times 2\left[\right.$$

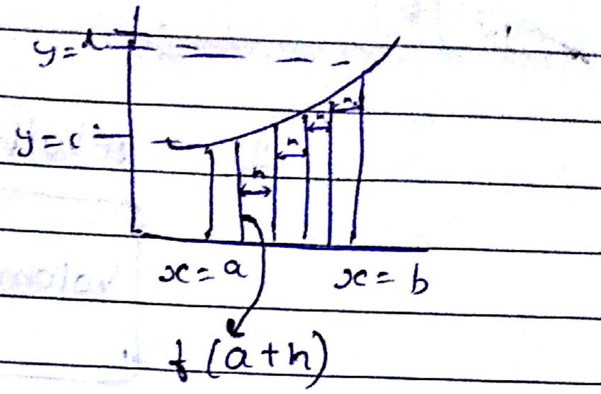
Find the area lying b/w cardiac $r = a(1-\cos\theta)$ and its double tangent.





VOLUME

$$y = f(x)$$

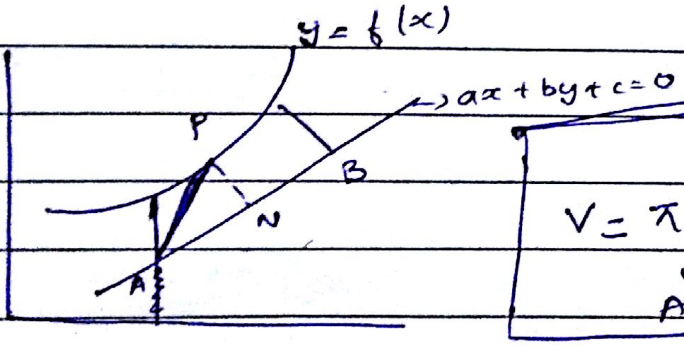


$$\pi f^2(a)\, h < \delta V_1 < \pi f^2(a+h)\, h$$

$$\pi \int y^2 dx \le V \le \pi \int y^2 dx$$

$$\boxed{V = \pi \int_a^b y^2 dx}$$

$$\boxed{V = \pi \int_c^d x^2 dy}$$



$$\boxed{V = \pi \int_A^B (PN)^2\, d(AN)}$$

(*) Surface area = $\boxed{2\pi \int_a^b f(x)\, ds}$ $\quad ds = \sqrt{1+y'^2}\, dx$

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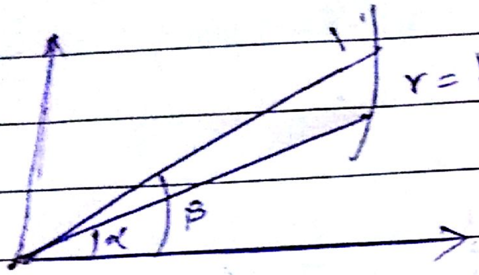

$r = f(\theta)$

$$\boxed{\text{Volume} = \int_{\alpha}^{\beta} \pi \, \frac{2}{3} r^3 \sin\theta \, d\theta}$$

if revolving around initial line

If revolving around vertical line

$$\boxed{\text{Volume} = \pi \int_{\alpha}^{\beta} \frac{2}{3} r^3 \cos\theta \, d\theta}$$