Self and Mutual Inductances

8.1 SELF INDUCTANCE

Inductance is the property of a circuit element in which a voltage is induced by changing the current in it. When the voltage is induced in the element by the current changing in its *own circuit* the property is called the *self inductance* or simply inductance. It is denoted by the symbol L.

The induced voltage is proportional to the rate of change of current.

Let e = self-induced voltage

i = instantaneous current

Rate of change of current

$$= \frac{di}{dt} \text{ A/s}$$

$$e \propto \frac{di}{dt}$$

$$e = L\frac{di}{dt} \text{ volts} \tag{8.1.1}$$

The proportionality constant L is called the self inductance. The unit of self inductance is henry (H).

If $\frac{di}{dt} = 1$ A/s, and $e = 1$ V

then $L = 1$ H

Thus, henry is defined as follows :

A circuit element has a self inductance of 1 H when a voltage of 1 V is induced in it by a current in the element changing at the rate of 1 A/s.

The direction of induced voltage is given by Lenz's law. Hence the induced voltage is given by

$$e = -L\frac{di}{dt} \tag{8.1.2}$$

The negative sign is used to satisfy Lenz's law, since when the rate of change of current is positive the induced voltage is negative. Thus, it acts in a negative direction around the circuit to oppose the change in current.

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8.2 VOLT-AMPERE RELATIONSHIP

The symbol for self inductance is L. It is shown in Fig. 8.1. If v is the applied voltage and e the induced voltage by KVL,

$$v + e = 0\ ;\ v = -e;\ v = L\frac{di}{dt} \qquad (8.2.1)$$

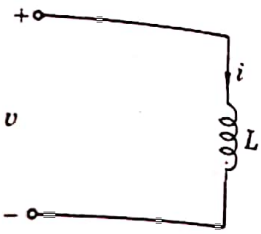

Fig. 8.1

8.3 COIL INDUCTANCE

Consider a coil of N turns carrying a current i which produces a flux Φ linking with the coil. The induced voltage in the coil is given by

$$e = -\frac{d}{dt}(\Phi N) \qquad (8.3.1)$$

If L be the self inductance of the coil, the induced voltage is also given by

$$e = -L\frac{di}{dt} \qquad (8.3.2)$$

Equating the two expressions for the induced voltage we get

$$L = \frac{d(\Phi N)}{di} \qquad (8.3.3)$$

$$L = \frac{\text{change in flux linkages}}{\text{change in current}} \qquad (8.3.4)$$

Equation (8.3.3) may be written as

$$L = N\frac{d\Phi}{di} \qquad (8.3.5)$$

Since $d\Phi/di$ is the slope of the magnetisation curve, Eq. (8.3.5) shows that the inductance varies with the slope of the magnetisation curve. Thus, the inductance of a coil with a magnetic core is not constant for all values of current.

For a coil with non-magnetic core, the flux is directly proportional to the current. Hence

$$\frac{d\Phi}{di} = \frac{\Phi}{i} = \text{a constant}$$

Therefore, the inductance is also constant. In such a case the inductance is given by

$$L = \frac{\Phi N}{i} \qquad (8.3.6)$$

This relation shows that the inductance of the coil is equal to the flux linkages per ampere. It can be taken as a defining equation of a *linear inductor*. For a linear inductor, the flux linkage is proportional to the current through the inductor for all values of the current.

Let I_1 = initial current in the coil ; I_2 = final current in the coil

Φ_1 = initial flux ; Φ_2 = final flux

N = number of turns in the coil

Initial flux linkages = $\Phi_1 N$ Final flux linkages = $\Phi_2 N$

Change in flux linkages = final flux linkages − initial flux linkages

$= \Phi_2 N - \Phi_1 N = (\Phi_2 - \Phi_1) N$

Change in current $= I_2 - I_1$

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$$L = \frac{\text{change in flux linkages}}{\text{change in current}}$$

$$L = \frac{(\Phi_2 - \Phi_1)N}{I_2 - I_1} \tag{8.3.7}$$

EXAMPLE 8.1

A coil of 400 turns has a flux of 0.5 mWb linking with it when carrying a current of 2 A. Determine the inductance of the coil.

SOLUTION

$$N = 400, \quad \Phi = 0.5 \text{ mWb} = 0.5 \times 10^{-3} \text{ Wb}$$

$$i = 2 \text{ A}$$

$$L = \frac{\Phi N}{i}$$

$$= \frac{0.5 \times 10^{-3} \times 400}{2} = 0.1 \text{ H}$$

EXAMPLE 8.2

A coil consists of 750 turns and a current of 10 A in the coil gives rise to a magnetic flux of 1.2 mWb. Calculate the inductance of the coil. If the current in the coil is reversed in 0.01 s, determine the average voltage induced in the coil.

SOLUTION

$$N = 750, \quad i = 10 \text{ A}$$

$$\Phi = 1.2 \text{ mWb} = 1.2 \times 10^{-3} \text{ Wb}$$

$$L = \frac{\Phi N}{i} = \frac{1.2 \times 10^{-3} \times 750}{10} = 0.09 \text{ H}$$

Initial current = 10 A Final current = – 10 A

Change in current = final current – initial current

$$di = -10 - 10 = -20 \text{ A}$$

Time taken for this change

$$dt = 0.01 \text{ s}$$

Induced voltage

$$e = -L\frac{di}{dt} = -0.09 \times \frac{(-20)}{0.01} = 180 \text{ V}$$

8.4 INDUCTANCE OF A COIL IN TERMS OF ITS DIMENSIONS

Consider a ring (Fig. 8.2) of constant permeability uniformly wound with N turns of a coil.

Let l = mean circumference of the ring(m)

a = cross-sectional area of the ring (m^2)

I = current in the coil (A)

S = reluctance of the magnetic circuit (A/Wb)

L = inductance of the coil

μ_r = relative permeability of the ring

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It is not necessary that the cross-section of the coil be circular.

$$L = \frac{\Phi N}{I} \tag{8.4.1}$$

$$\Phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{IN}{S}$$

$$L = \frac{IN}{S} \cdot \frac{N}{I}; \; L = \frac{N^2}{S} \tag{8.4.2}$$

But $$S = \frac{l}{\mu_0 \mu_r a} \tag{8.4.3}$$

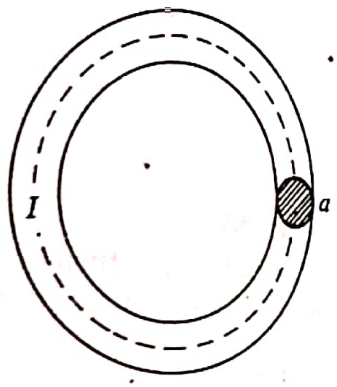

Fig. 8.2

Substituting the value of S from Eq. (8.4.3) in Eq. (8.4.2) gives

$$L = \mu_0 \mu_r N^2 \frac{a}{l} \tag{8.4.4}$$

Equation (8.4.4) gives the inductance of a coil in terms of its dimensions. Equation (8.4.4) shows that the inductance of a coil depends upon its length, area of cross-section, the square of number of turns and the core material on which the coil is wound. A coil with a ferromagnetic core has a very much higher inductance as compared with a coil of the same dimensions with an air, or non-magnetic, core. A choke is made of a ferromagnetic material with a small airgap. The provision of the airgap reduces the inductance slightly, but it makes the $B-H$ curve of the choke approximately linear, with the result that the inductance becomes nearly constant over a range of currents.

EXAMPLE 8.3

Calculate the inductance of a ring-shaped coil having a mean diameter of 200 mm wound on a wooden core of diameter 20 mm. The winding is evenly wound and contains 500 turns.

If the wooden core is replaced by an iron core which has a relative permeability of 600 when the current is 5 A, calculate the new value of inductance.

SOLUTION

Mean diameter of the ring $d = 200 \text{ mm} = 0.2 \text{ m}$

Mean length of the magnetic path $l = \pi d = 0.2\,\pi$

Diameter of the core $d_e = 20 \text{ mm} = 0.02 \text{ m}$

Cross-sectional area of the core $a = \frac{\pi}{4} d_c^2 = \frac{\pi}{4}(0.02)^2 = \pi \times 10^{-4} \text{ m}^2$

Number of turns in the coil

$$N = 500$$

$$\mu_0 = 4\pi \times 10^{-7}$$

For *wooden core*, which is non-magnetic, $\mu_r = 1$

Inductance of the coil,

$$L = \mu_0 \mu_r N^2 \frac{a}{l}$$

$$= 4\pi \times 10^{-7} \times 1 \times (500)^2 \times \frac{\pi \times 10^{-4}}{\pi \times 0.2}$$

$$= 157 \times 10^{-6} \text{ H} = 157 \; \mu\text{H}$$

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For *iron core*, $\mu_r = 600$

$$L = 4\pi \times 10^{-7} \times 600 \times (500)^2 \times \frac{\pi \times 10^{-4}}{\pi \times 0.2}$$

$$= 94.2 \times 10^{-3}\ \text{H} = 94.2\ \text{mH}$$

8.5 EFFECT OF SELF INDUCTANCE

When current in a circuit changes, a voltage is induced in the circuit due to its inductance. According to Lenz's law the induced voltage opposes the changes in current. The magnitude of the induced voltage and hence the opposition to the changing current depends upon the inductance of the circuit. The larger is the inductance of the circuit, the greater is the induced voltage and, therefore, the greater is the opposition to the changing current. Thus, inductance is the property of a circuit due to which it opposes the change in circuit current. The following effects of circuit inductance are to be noted :

(*a*) The property of inductance comes into play only when current in the circuit changes.

(*b*) When circuit current changes, inductance opposes this change.

(*c*) When circuit current is constant, $di/dt = 0$ and there is no induced voltage.

8.6 ENERGY STORED IN AN INDUCTOR

Consider an inductor of inductance L henry in which the current rises *uniformly* from zero to I amperes in time t seconds.

Rate of change of current $= \frac{I}{t}$ A/s

Induced voltage, $e = -L\frac{I}{t}$

Supply voltage, $v = -e = L\frac{I}{t}$

Average current $= \frac{0+I}{2} = \frac{I}{2}$

Energy stored in the magnetic field of the inductor

= energy supplied

= voltage × average current × time

$$= \frac{LI}{t} \cdot \frac{I}{2} \cdot t = \frac{1}{2}LI^2 \text{ joules}$$

In the general case when the current increases at a uniform or a non-uniform rate : let the inductance be constant. Suppose that the current increases by di amperes in dt seconds.

Rate of change of current $= \frac{di}{dt}$ A/s

Induced voltage, $e = -L\frac{di}{dt}$

Supply voltage, $v = -e = L\frac{di}{dt}$

Energy stored in the magnetic field of the inductor during time dt seconds

= energy supplied in dt seconds

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$$= \text{voltage} \times \text{current} \times \text{time}$$

$$= L\frac{di}{dt} \cdot i \cdot dt = Li\, di \text{ joules}$$

Hence the total energy stored in the magnetic field of the inductor when the current increases from zero to I amperes is

$$W_f = \int_0^I Li \cdot di$$

$$= L\left[\frac{i^2}{2}\right]_0^I = \frac{1}{2}LI^2 \text{ joules}$$

8.7 MUTUAL INDUCTANCE

When two circuits are so placed that a portion of the magnetic flux produced by one links with the turns of both the circuits, they are said to be *mutually coupled*. Mutual inductance is the property of two circuits when a voltage is induced in one circuit by a change of current in the other circuit.

Consider two coils 1 and 2 with N_1 and N_2 turns respectively as shown in Fig. 8.3. The coils are located physically close to one another so that a part of the flux produced by current in one coil links the second coil also. Let the current in coil 1 be i_1. It produces a magnetic flux Φ_1. This flux has two components Φ_{11} and Φ_{12}, that is.

$$\Phi_1 = \Phi_{11} + \Phi_{12} \tag{8.7.1}$$

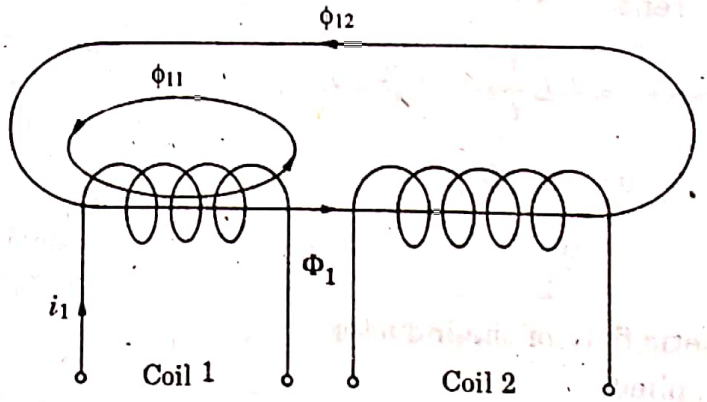

Fig. 8.3. Mutually coupled coils.

The component Φ_{11} links with turns of coil 1 only and does not link with turns of coil 2. The flux Φ_{11} is called the *self flux* of coil 1. The other component Φ_{12} links with turns of *both* the coils. It is called the *mutual flux*.

If the current i changes with time the flux produced by it will also vary with time. Thus, the change of current in coil 1 changes the flux linkages in both the coils. Hence voltages are induced in both the coils. The voltage induced in coil 1 is called the *self-induced voltage*, and the voltage induced in coil 2 is called the *mutually-induced voltage*.

The voltage induced in coil 2 due to change of current in coil 1 is proportional to the rate of change of i_1. Mathematically it can be expressed as

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$$e_2 \propto \frac{di_1}{dt}$$

$$e_2 = -M\frac{di_1}{dt} \tag{8.7.2}$$

where M is called the coefficient of mutual inductance or simply *mutual inductance*. The negative sign is used to satisfy Lenz's law.

Unit of Mutual Inductance

The unit of mutual inductance is henry (H).

In Eq. (8.7.2), if $di_1/dt = 1$ A/s and $e_2 = 1$ V then $M = 1$ H. Thus, henry is defined as follows :

Two coils have a mutual inductance of 1 H when a current changing at the rate of 1 A/s in one coil induces a voltage of 1 V in the other.

8.8 MUTUAL INDUCTANCE IN TERMS OF FLUX-LINKAGES PER AMPERE

If a flux $d\,\Phi_{12}$ links with N_2 turns of coil 2 due to a change di_1 in current in coil 1 in time dt then the voltage induced in coils 2 is

$$e_2 = -\frac{d}{dt}(\Phi_{12} N_2) \tag{8.8.1}$$

Equating the expression for e_2 obtained in Eqs. (8.7.2) and (8.8.1)

$$-M\frac{di_1}{dt} = -\frac{d}{dt}(\Phi_{12}N_2)$$

Hence

$$M = \frac{d}{di_1}(\Phi_{12}N_2) \tag{8.8.2}$$

$$= \frac{\text{change of flux-linkages with coil 2}}{\text{change of current in coil 1}}$$

If the relative permeability of the magnetic circuit is constant, then the flux is directly proportional to the current. Hence

$$\frac{d\Phi_{12}}{di_1} = \frac{\Phi_{12}}{i_1} = \text{a constant} \tag{8.8.3}$$

From Eqs. (8.8.2) and (8.8.3)

$$M = \frac{\Phi_{12}N_2}{i_1} = \frac{\text{flux-linkages with coil 2}}{\text{current in coil 1}} \tag{8.8.4}$$

Thus, the mutual inductance may also be defined in terms of the flux-linkages in one coil per unit current in the other.

The mutual inductance between two coils is 1 H if 1 A flowing in one coil produces flux linkages of 1 weber turn in the other coil.

8.9 COUPLING COEFFICIENT

Consider two coils 1 and 2 having N_1 and N_2 turns respectively to be wound uniformly very close to each other on a ring of constant reluctance. Consider an ideal case when the whole of the flux produced in one coil links with all the turns of the other coil.

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Let Φ_1 = flux produced by I_1 in coil 1 ; Φ_2 = flux produced by I_2 in coil 2

L_1 = self inductance of coil 1 ; L_2 = self inductance of coil 2

M = mutual inductance between the coils

S = reluctance of the common flux path

$$L_1 = \frac{\Phi_1 N_1}{I_1} \tag{8.9.1}$$

$$L_2 = \frac{\Phi_2 N_2}{I_2} \tag{8.9.2}$$

$$M = \frac{\Phi_1 N_2}{I_1} \tag{8.9.3}$$

$$M = \frac{\Phi_2 N_1}{I_2} \tag{8.9.4}$$

$$S = \frac{\text{m.m.f.}}{\text{flux}} = \frac{I_1 N_1}{\Phi_1} = \frac{I_2 N_2}{\Phi_2}$$

From Eq. (8.9.3)

$$M = \frac{\Phi_1 N_2 N_1}{I_1 N_1} = \frac{N_1 N_2}{(I_1 N_1)/\Phi_1}$$

$$M = \frac{N_1 N_2}{S} \tag{8.9.5}$$

If l = length of the magnetic path

a = cross-sectional area of the magnetic path

μ_r = relative permeability

$$S = \frac{l}{\mu_0 \mu_r a} \tag{8.9.6}$$

From Eqs. (8.9.5) and (8.9.6)

$$M = \mu_0 \mu_r N_1 N_2 \frac{a}{l} \tag{8.9.7}$$

Multiplying Eqs. (8.9.1) and (8.9.2)

$$L_1 L_2 = \frac{\Phi_1 \Phi_2 N_1 N_2}{I_1 I_2} \tag{8.9.8}$$

Multiplying Eqs. (8.9.3) and (8.9.4)

$$M^2 = \frac{\Phi_1 \Phi_2 N_1 N_2}{I_1 I_2} \tag{8.9.9}$$

Comparison of Eqs. (8.9.8) and (8.9.9) gives

$$M^2 = L_1 L_2$$

$$M = \sqrt{L_1 L_2} \tag{8.9.10}$$

Equation (8.9.10) gives theoretical maximum possible mutual inductance. In practice, there is always some leakage flux produced by each current. This leakage flux links with its own turns but not with the turns of both coils. Thus, the actual mutual inductance between two coils is different

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from the theoretical mutual inductance between them. The ratio of the actual value of mutual inductance to the theoretical maximum value is called the *coupling coefficient*. It is denoted by k.

$$k = \frac{\text{actual value of mutual inductance}}{\text{theoretical value of mutual inductance}}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} \tag{8.9.11}$$

The value of k lies between 0 and 1. The maximum value of k is 1 when the entire self flux of each coil links with the other. If most of the flux produced in one coil links with both coils, the coils are said to be *tightly coupled*. If only a small portion of the flux produced by one coil links with the other, the coils are said to be *loosely coupled*.

EXAMPLE 8.4

Two coils having 150 and 200 turns respectively are wound side by side on a closed magnetic circuit of cross-section 1.5×10^{-2} m^2 and mean length 3 m. The relative permeability of the magnetic circuit is 2000. Calculate : (*a*) the mutual inductance between the coils; (*b*) the voltage induced in the second coil if the current changes from 0 to 10 A in the first coils in 20 ms.

SOLUTION

$$N_1 = 150, N_2 = 200$$

$$a = 1.5 \times 10^{-2}, l = 3 \text{ m}, \mu_r = 2000$$

(*a*) $$M = \mu_0 \mu_r N_1 N_2 \frac{a}{l} = 4\pi \times 10^{-7} \times 2000 \times 150 \times 200 \times \frac{1.5 \times 10^{-2}}{3}$$

$$M = 0.377 \text{ H}$$

(*b*) $$di_1 = 10 - 0 = 10 \text{ A}$$

$$dt = 20 \text{ ms} = 20 \times 10^{-3} \text{ s}$$

Voltage induced in the second coil

$$e_2 = M \frac{di_1}{dt} = 0.377 \times \frac{10}{20 \times 10^{-3}} = 188.5 \text{ V}$$

EXAMPLE 8.5

A coil of 50 turns having a mean diameter of 30 mm is placed coaxially at the centre of a solenoid 0.6 m long wound with 2500 turns and carrying a current of 2 A. Determine the mutual inductance of the arrangement.

SOLUTION

Flux density produced by the solenoid

$$B = \mu_0 \mu_r H = \mu_0 H \qquad (\because \mu_r = 1 \text{ for air})$$

$$H = \frac{NI}{l}, \quad B = \mu_0 \frac{NI}{l}$$

$$= 4\pi \times 10^{-7} \times 2500 \times \frac{2}{0.6} = 1.047 \times 10^{-2} \text{ T}$$

Diameter of inner coil, $d = 30 \text{ mm} = 30 \times 10^{-3}$ m. Area of cross-section of the inner coil

$$a_2 = \frac{\pi}{4} d^2 = \frac{\pi}{4} (30 \times 10^{-3})^2 = 7.068 \times 10^{-4} \text{ m}^2$$

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Flux linked with the inner coil

$$\Phi_2 = Ba_2 = 1.047 \times 10^{-2} \times 7.068 \times 10^{-4} = 7.4 \times 10^{-6} \text{ Wb}$$

Mutual inductance of the arrangement

$$M = \frac{\Phi_2 N_2}{I_1} = 7.4 \times 10^{-6} \times \frac{50}{2} \text{ H}$$

$$M = 185 \ \mu \text{ H}$$

EXAMPLE 8.6

Two long single-layer solenoids have the same length and the same number of turns but are placed coaxially one within the other. The diameter of the inner coil is 60 mm and that of the outer coil is 75 mm. Determine the coefficient of coupling between the coils.

SOLUTION

Suppose that suffixes 1 and 2 are used for outer and inner solenoids respectively. Let N be the number of turns of each solenoid and l the length of each solenoid.

$$L_1 = \mu_0 \mu_r N^2 \frac{a_1}{l} \ ; \ L_2 = \mu_0 \mu_r N^2 \frac{a_2}{l}$$

$$\sqrt{L_1 L_2} = \mu_0 \mu_r \frac{N^2}{l} \sqrt{a_1 a_2}$$

Flux density produced by the outer solenoid

$$B = \mu_0 \mu_r H = \mu_0 \mu_r \frac{NI_1}{l}$$

Flux linked with the inner solenoid is

$$\Phi_2 = Ba_2 = \mu_0 \mu_r N I_1 \frac{a_2}{l}$$

$$M = \frac{\Phi_2 N_2}{I_1} = \frac{\Phi_2 N}{I_1} = \mu_0 \mu_r N^2 \frac{a_2}{l}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{a_2}{\sqrt{a_1 a_2}} = \sqrt{\frac{a_2}{a_1}}$$

$$= \sqrt{\frac{\frac{\pi}{4} d_2^2}{\frac{\pi}{4} d_1^2}} = \frac{d_2}{d_1} = \frac{60}{75} = 0.8$$

8.10 INDUCTANCE OF COUPLED COILS

Consider a circuit containing two mutually coupled coils connected in series. There are two possible ways to connect the coils in series. In one connection (Fig. 8.4) the fluxes produced by the coils act

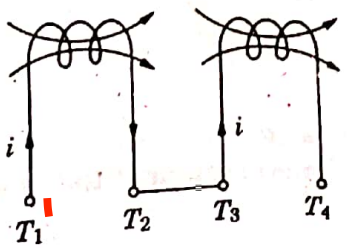

Fig. 8.4. Series-aiding connection of coupled coils.

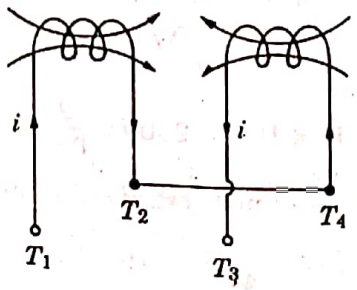

Fig. 8.5

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in the same direction. Then the coils are said to be connected in *series-aiding*. If the current is reversed in the other coil so that the fluxes produced by the coils oppose each other, the coils are said to be connected in *series-opposition*. Such a connection is shown in Fig. 8.5.

Let the self inductance of coil 1 be L_1 and that of coil 2 be L_2. The two coils are coupled by mutual inductance M.

(*a*) *Series-aiding connection*

Self-induced voltage in coil 1

$$e_{S1} = -L_1 \frac{di}{dt}$$

Mutual-induced voltage in coil 1

$$e_{M1} = -M \frac{di}{dt}$$

Self-induced voltage in coil 2

$$e_{S2} = -L_2 \frac{di}{dt}$$

Mutual-induced voltage in coil 2

$$e_{M2} = -M \frac{di}{dt}$$

Total induced voltage in the circuit

$$e = e_{S1} + e_{M1} + e_{S2} + e_{M2}$$

$$= -L_1 \frac{di}{dt} - M \frac{di}{dt} - L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$e = -(L_1 + L_2 + 2M) \frac{di}{dt} \qquad (8.10.1)$$

If L_{eq} is the equivalent inductance of the two coils in series-aiding connection, then the induced voltage in the circuit is given by

$$e = -L_{eq} \frac{di}{dt} \qquad (8.10.2)$$

Comparing Eqs. (8.10.1) and (8.10.2) we get

$$L_{eq} = L_1 + L_2 + 2M \qquad (8.10.3)$$

Equation (8.10.3) gives the equivalent inductance of two cumulatively coupled c ils.

(*b*) *Series-opposition connection*

Let the two coils be so connected that their fluxes oppose each other as shown in Fig. 8.5

Self-induced voltage in coil 1

$$e_{S1} = -L_1 \frac{di}{dt}$$

Mutual-induced voltage in coil 1

$$e_{M1} = +M \frac{di}{dt}$$

Self-induced voltage in coil 2

$$e_{S2} = -L_2 \frac{di}{dt}$$

Mutual-induced voltage in coil 2

$$e_{M2} = +M \frac{di}{dt}$$

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It is to be noted that the polarity of the mutually induced voltages in reversed.

Total induced voltage in the circuit

$$e = e_{S1} + e_{M1} + e_{S2} + e_{M2}$$

$$= -L_1 \frac{di}{dt} + M\frac{di}{dt} - L_2 \frac{di}{dt} + M\frac{di}{dt}$$

$$e = -(L_1 + L_2 - 2M)\frac{di}{dt} \tag{8.10.4}$$

If L'_{eq} is the equivalent inductance of the two coils in series opposition connection, then the induced voltage in the circuit is given by

$$e = -L'_{eq}\frac{di}{dt} \tag{8.10.5}$$

Comparing Eqs. (8.10.4) and (8.10.5) we get

$$L'_{eq} = L_1 + L_2 - 2M \tag{8.10.6}$$

Equation (8.10.6) gives the equivalent inductance of two coils connected in series opposition. It is observed that

$$L_{eq} - L'_{eq} = (L_1 + L_2 + 2M) - (L_1 + L_2 - 2M) = 4M$$

$$M = \frac{1}{4}(L_{eq} - L'_{eq}) \tag{8.10.7}$$

Equation (8.10.7) gives the mutual inductance of the coils. Equations (8.10.3), (8.10.6) and (8.10.7) may be used to measure the mutual inductance between the two coils. The coils are connected in series and the total inductance is measured. The current is reversed in coil 2 and the inductance is measured again. The smaller value is subtracted from the larger one and the difference is divided by 4 to get the mutual inductance between the coils.

EXAMPLE 8.7

When two identical coupled coils are connected in series, the inductance of the combination is found to be 80 mH. When the connections to one of the coils are reversed, a similar measurement indicates 20 mH. Find the coupling coefficient between the coils.

SOLUTION

$$L_{eq} = L_1 + L_2 + 2M$$

$$L'_{eq} = L_1 + L_2 - 2M$$

$$L_{eq} - L'_{eq} = 4M$$

$$M = \frac{1}{4}(L_{eq} - L'_{eq}) = \frac{1}{4}(80 - 20)\ M = 15 \text{ mH}$$

Since the coils are identical, $L_1 = L_2$

$$L_{eq} = 2L_1 + 2M$$

$$80 = 2L_1 + 2 \times 15$$

$$L_1 = \frac{1}{2}(80 - 30) = 25 \text{ mH}$$

$$\therefore \quad L_1 = L_2 = 25 \text{ mH}$$

Coupling coefficient,

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{15}{\sqrt{25 \times 25}} = \frac{15}{25} = 0.6$$

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EXAMPLE 8.8

Two coils have self inductance of 3 H and 2 H respectively and the mutual inductance is 2 H. They are connected in series and a current of 4 A is passed through them. Calculate the energy of the magnetic field when the self and mutual fluxes are (*a*) in the same direction, (*b*) in opposition. Find also the coupling coefficient.

SOLUTION

$$L_1 = 3\text{ H}, L_2 = 2\text{ H}, M = 2\text{ H}$$

$$L_{eq} = L_1 + L_2 + 2M = 3 + 2 + 2 \times 2 = 9\text{ H}$$

$$L'_{eq} = L_1 + L_2 - 2M = 3 + 2 - 2 \times 2 = 1\text{ H}$$

(*a*) Energy $= \frac{1}{2} L_{eq} I^2 = \frac{1}{2} \times 9 \times 4^2 = 72\text{ J}$

(*b*) Energy $= \frac{1}{2} L'_{eq} I^2 = \frac{1}{2} \times 1 \times 4^2 = 8\text{J}$

Coupling coefficient

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2}{\sqrt{2 \times 3}} = 0.816$$

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