Assignment - 4

of

Applied Mathematics

Submitted By:-

Ishita Sharma

DTU/2K16/B1/134

Q1. Using the defination find the laplace transform of the following functions.

a) at^2+bt+c

Ans:- By defination,

$$L\{at^2+bt+c\} = \int_0^\infty (at^2+bt+c)\, e^{-st}\, dt$$

$$\Rightarrow \int_0^\infty e^{-st} at^2 dt + \int_0^\infty e^{-st} bt\, dt + \int_0^\infty e^{-st} c\, dt$$

$$\Rightarrow at^2 \frac{e^{-st}}{-s}\Big|_0^\infty + \frac{2}{s}\int_0^\infty at\, e^{-st} dt + bt\frac{e^{-st}}{-s}\Big|_0^\infty + \frac{b}{s}\int_0^\infty e^{-st} dt + c\frac{e^{-st}}{-s}\Big|_0^\infty$$

$$\Rightarrow 0 + \frac{2}{s}\left[at\frac{e^{-st}}{-s}\Big|_0^\infty + \frac{a}{s}\int_0^\infty e^{-st} dt\right] + 0 + \frac{b}{s}\left[\frac{e^{-st}}{-s}\right]_0^\infty + \frac{c}{s}$$

$$\Rightarrow \frac{2}{s}\left[\frac{a}{s}\frac{e^{-st}}{-s}\right]_0^\infty + \frac{b}{s^2} + \frac{c}{s}$$

$$\Rightarrow \frac{2a}{s^3} + \frac{b}{s^2} + \frac{c}{s}$$

$$\Rightarrow \frac{2a+bs+cs^2}{s^3}$$

b) at^2+bt+c

Ans: Same as above

c) $\cos(at+b)$

Ans : By defination of Laplace transformation,

$$L\{\cos(at+b)\} = \int_0^\infty e^{-st}\cos(at+b)\,dt$$

$$= Re\left[\int_0^\infty e^{-st} e^{i(at+b)}\,dt\right]$$

$$= Re\left[e^{ib}\int_0^\infty e^{-(s-ia)t}\,dt\right]$$

$$= Re\left[e^{ib}\frac{e^{-(s-ia)t}}{-(s-ia)}\right]_0^\infty$$

$$= Re\left[\frac{e^{ib}}{s-ia}\right]$$

$$= Re\frac{(\cos b + i\sin b)(s+ia)}{(s-ia)(s+ia)}$$

$$= Re\frac{(\cos b + i\sin b)(s+ia)}{s^2+a^2}$$

$$= \frac{s\cos b - a\sin b}{s^2+a^2}$$

d) te^t

By defination of Laplace transformation,

$$L\{te^t\} = \int_0^\infty e^{-st}\,te^t\,dt$$

$$= \int_0^\infty te^{-(s-1)t}\,dt$$

$$= t\frac{e^{-(s-1)t}}{-(s-1)}\Big|_0^\infty + \frac{1}{s-1}\int_0^\infty e^{-(s-1)t}\,dt$$

$$= 0 + \frac{1}{s-1}\frac{e^{-(s-1)t}}{-(s-1)}\Big|_0^\infty$$

$$= \frac{1}{(s-1)^2}$$

e) $f(t) = \begin{cases} 0 & 0 \leq t < \pi \\ \sin t & t \geq \pi \end{cases}$

Ans :- By defination of Laplace Transformation :-

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$\Rightarrow \int_0^\pi e^{-st} f(t)\,dt + \int_\pi^\infty e^{-st} f(t)\,dt$$

$$\Rightarrow 0 + \int_\pi^\infty e^{-st} \sin t\,dt$$

$$\Rightarrow \text{Img} \int_\pi^\infty e^{-st} e^{-it}\,dt$$

$$\Rightarrow \text{Img} \int_\pi^\infty e^{-(s+i)t}\,dt$$

$$\Rightarrow \text{Img} \left[\frac{e^{-(s+i)t}}{-(s+i)} \Big|_\pi^\infty \right]$$

$$\Rightarrow \text{Img} \left[\frac{e^{-(s+i)\pi}}{s-i} \times \frac{s+i}{s-i} \right]$$

$$\Rightarrow \text{Img} \left[\frac{(s+i)(e^{-s\pi}(-1))}{s^2+1} \right]$$

$$= \text{Img} \left[\frac{-se^{-s\pi} - ie^{-s\pi}}{s^2+1} \right]$$

$$= \frac{-e^{-s\pi}}{s^2+1}$$

Q2. Find the Laplace transformation of following functions :-

a) $t \sin 4t$

Ans : $L\{t\sin 4t\} = \text{img}\, L\{t\, e^{+4it}\}$

We know,

$$L\{t\} = \frac{1}{s^2}$$

Scanned by CamScanner

By first shifting theorm,

we know, if $L\{f(t)\} = \bar{f}(s)$

then $L\{e^{at} \cdot f(t)\} = \bar{f}(s-a)$

$\therefore$ We have,

$$L\{t\} = \frac{1}{s^2}$$

$$\therefore L\{t e^{+4it}\} = \frac{1}{(s-4i)^2}$$

$$\therefore L\{t \sin 4t\} = \text{img } L\{t e^{4it}\}$$

$$= \text{img } \frac{1}{(s-4i)^2}$$

$$= \text{img } \frac{(s+4i)^2}{(s^2+16)^2}$$

$$= \text{img } \frac{s^2 - 16 + 8is}{(s^2+16)^2}$$

$$= \frac{8s}{(s^2+16)^2}$$

ii) $t^2 \cos 3t$

It can be expressed as $\text{Re} L\{t^2 e^{3it}\}$

We know, $L\{t^2\} = \frac{\sqrt{3}}{s^3} = \frac{2}{s^3}$

$\therefore$ By first shifting theorm,

$$L\{t^2 e^{3it}\} = \frac{2}{(s-3i)^3}$$

$$\text{Re } L\{t^2 e^{3it}\} = \text{Re}\left\{\frac{2}{(s-3i)^3} \cdot \frac{(s+3i)^3}{(s+3i)^3}\right\}$$

$$= \text{Re}\left\{\frac{2(s^3 - 27i + 9.0is - 27s)}{(s^2+9)^3}\right\}$$

Scanned by CamScanner

$$L\{t^2\cos 3t\} = \frac{2(s^3-27s)}{(s^2+9)^3}$$

$$= \frac{2s(s^2-27)}{(s^2+9)^3}$$

c) $t^2 e^{-2t}$

We know, $L\{t^2\} = \frac{\lceil 3}{s^3} = \frac{2}{s^3}$

$\therefore$ By first shifting theorm,

$$L\{t^2 e^{-2t}\} = \frac{2}{(s+2)^3}$$

Q3. In the following problems, the laplace transform is given, i.e, $F(s) = L\{f(t)\}$. find the inverse Laplace transform

Ans :- i) Given $\bar{f}(s) = \frac{3}{s-5}$

Let $L\{f(t)\} = \frac{3}{s-5}$

We have, $L\{1\} = \frac{1}{s}$

$L\{3\} = \frac{3}{s}$

By first shifting theorm

$$L\{3e^{5t}\} = \frac{3}{s-5}$$

$$3e^{5t} = L^{-1}\left\{\frac{3}{s-5}\right\}$$

$$\therefore f(t) = 3e^{5t}$$

ii) $\frac{\pi}{s^2+\pi^2}$

Given, $\bar{f}(s) = L\{f(t)\} = \frac{\pi}{s^2+\pi^2}$

We know, $L\{\sin \pi t\} = \frac{\pi}{s^2+\pi^2}$

$\therefore \sin \pi t = L^{-1}\left\{\frac{\pi}{s^2+\pi^2}\right\}$

$\therefore f(t) = \sin \pi t$

c) $\frac{(s+3)}{(s-1)(s+2)}$

Given, $\bar{f}(s) = L\{f(t)\} = \frac{s+3}{(s-1)(s+2)}$

Let $\frac{s+3}{(s-1)(s+2)} = \frac{A}{s-1} + \frac{B}{s+2}$

$\therefore A = \frac{4}{3}, \quad B = \frac{-1}{3}$

$\therefore$ We have,

$L\{f(t)\} = \frac{4}{3(s-1)} + \frac{(-1)}{3(s+2)}$

$f(t) = L^{-1}\left\{\frac{4}{3(s-1)}\right\} - L^{-1}\left\{\frac{1}{3(s+2)}\right\}$

By appeying first shifting theorm:-

$f(t) = \frac{4}{3}e^{t} - \frac{1}{3}e^{-2t}$

$= \frac{1}{3}(4e^{t} - e^{-2t})$

Q4. Find the Laplace Transform of the function :-

$$f(t) = \begin{cases} k, & 0 \le t < 2 \\ 0, & 2 \le t < 4 \\ k, & t \ge 4 \end{cases}$$

Ans :- By defination :-

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$\Rightarrow \int_0^2 e^{-st} f(t)\,dt + \int_2^4 e^{-st} f(t)\,dt + \int_4^\infty e^{-st} f(t)\,dt$$

$$\Rightarrow k\int_0^2 e^{-st}\,dt + 0 + k\int_4^\infty e^{-st}\,dt$$

$$\Rightarrow k\left[\frac{e^{-st}}{-s}\right]_0^2 + k\left[\frac{e^{-st}}{-s}\right]_4^\infty$$

$$\Rightarrow k\left[\frac{e^{-2s}}{-s} + \frac{1}{s}\right] + k\left[0 + \frac{e^{-4s}}{s}\right]$$

$$\Rightarrow \frac{k\left(1 - e^{-2s} + e^{-4s}\right)}{s}$$

Q5. Find the Laplace transform of the function :-

$$f(t) = \begin{cases} 0, & 0 \le t < 3 \\ (t-3)^2, & t \ge 3 \end{cases}$$

Ans :- By defination,

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$\Rightarrow \int_0^3 f(t)\, e^{-st}\,dt + \int_3^\infty f(t)\, e^{-st}\,dt$$

$$L\{f(t)\} = \int_3^\infty (t-3)^2 e^{-st}\,dt$$

$$= (t-3)^2 \frac{e^{-st}}{-s}\Big|_3^\infty + \frac{2}{s}\int_3^\infty (t-3)\, e^{-st}\,dt$$

$$= (t-3)^2 \frac{e^{-st}}{-s}\Big|_3^\infty + \frac{2}{s}\left[\frac{(t-3)e^{-st}}{-s}\Big|_3^\infty + \frac{1}{s}\int_3^\infty e^{-st}dt \right]$$

$$= 0 + \frac{2}{s}\left[0 + \frac{e^{-st}}{-s^2}\Big|_3^\infty \right]$$

$$= \frac{2}{s}\,\frac{e^{-3s}}{s^2}$$

$$= \frac{2e^{-3s}}{s^3}$$

Q6. Solve the initial value problem using Laplace transformation.

a) $y'' + 2y' - 3y = 3 \quad , \; y(0) = 4 \, , \; y'(0) = -7$

By obtaining Laplace transform of each term,

$$L\{D^2y\} + 2L\{Dy\} - 3L\{y\} = L\{3\}$$

Let $L\{y\} = \bar{y}$

$$(s^2 + 2s - 3)\,\bar{y} = \frac{3}{s} + (4)(s+2) + (-7)(1)$$

$$\bar{y} = \frac{3 + 4s(s+2) - 7s}{s(s^2+2s-3)}$$

$$\bar{y} = \frac{3 + 4s^2 + 8s - 7s}{s(s^2+2s-3)}$$

$$\bar{y} = \frac{4s^2 + s + 3}{s(s^2+2s-3)}$$

$$\bar{y} = \frac{4s^2+s+3}{s(s^2+2s-3)}$$

$$\bar{y} = \frac{4s^2+4s-3s+3}{s(s-1)(s+3)}$$

let $$\frac{4s^2+s+3}{s(s-1)(s+3)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+3}$$

By comparing coefficients,

$A=-1$, $B=2$, $C=3$

$$\therefore \quad \bar{y} = \frac{-1}{s} + \frac{2}{s-1} + \frac{3}{s+3}$$

$$y = L^{-1}\left\{\frac{-1}{s}\right\} + L^{-1}\left\{\frac{2}{s-1}\right\} + L^{-1}\left\{\frac{3}{s+3}\right\}$$

$$y = -1 + 2e^{t} + 3e^{-3t}$$

b) $y'' - 5y' + 4y = e^{2t}$, $y(0) = \frac{19}{2}$, $y'(0) = \frac{8}{3}$

By obtaining Laplace of each term,

$$L\{D^2y\} - L\{5Dy\} + L\{4y\} = L\{e^{2t}\}$$

let $L\{y\} = \bar{y}$

$$(s^2-5s+4)\bar{y} = \frac{1}{s-2} + \frac{19}{2}(s-5) + \frac{8}{3}(1)$$

$$(s^2-5s+4)\bar{y} = \frac{6+19\times3(s-2)(s-5)+16(s-2)}{6(s-2)}$$

$$\bar{y} = \frac{6+57(s^2-7s+10)+16s-32}{6(s-2)(s^2-5s+4)}$$

$$\bar{y} = \frac{57s^2-383s+544}{6(s-2)(s^2-5s+4)}$$

Scanned by CamScanner

$$\therefore \quad \bar{y} = \frac{57s^2 - 383s + 544}{6(s-2)(s-4)(s-1)}$$

Let $$\frac{57s^2 - 383s + 544}{6(s-2)(s-4)(s-1)} = \frac{A}{s-2} + \frac{B}{s-4} + \frac{C}{s-1}$$

By compairing coefficients :-

$$A = 1 \; , \; B = \frac{38}{3} \; , \; C = \frac{73}{2}$$

$$\therefore \quad \bar{y} = \frac{1}{s-2} + \frac{38}{3(s-4)} + \frac{73}{2(s-1)}$$

$$y = L^{-1}\left\{\frac{1}{s-2}\right\} + L^{-1}\left\{\frac{38}{3(s-4)}\right\} + L^{-1}\left\{\frac{73}{2(s-1)}\right\}$$

$$y = e^{2t} + \frac{38}{3} e^{4t} + \frac{73}{2} e^{t}$$

c) $$y' + 3y + 2\int_0^t y(\tau)\,d\tau = t \qquad y(0) = 0$$

By obtaining Laplace transform of each term,

$$L\{y'\} + 3L\{y\} + 2L\left\{\int_0^t y(u)\,du\right\} = L\{t\}$$

Let $L\{y\} = \bar{y}$

$$s\bar{y} - 3\bar{y} + 2\frac{\bar{y}}{s} = \frac{1}{s^2}$$

$$s^2\bar{y} - 3s\bar{y} + 2\bar{y} = \frac{1}{s}$$

$$\bar{y}(s^2 - 3s + 2) = \frac{1}{s}$$

$$\bar{y} = \frac{1}{s(s-1)(s-2)}$$

Let $$\frac{1}{s(s-1)(s-2)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-2}$$

By comparinig coefficients

$$A = \frac{1}{2} \; , \; B = -1 \; , \; C = 2$$

$$y = L^{-1}\left\{\frac{1}{2s} - \frac{1}{s-1} + \frac{2}{s-2}\right\}$$

By applying first shifting theorm,

We get →

$$y = L^{-1}\left\{\frac{1}{2s}\right\} - L^{-1}\left\{\frac{1}{s-1}\right\} + L^{-1}\left\{\frac{2}{s-2}\right\}$$

$$y = \frac{1}{2} L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{1}{s-1}\right\} + 2L^{-1}\left\{\frac{1}{s-2}\right\}$$

$$y = \frac{1}{2}\times 1 - 1\times e^{t} + 2\times e^{2t}$$

$$y = \frac{1}{2} - e^{t} + 2e^{2t}$$

Q7. Find the solution of the initial value problem, using Laplace Transformation :-

$$ty'' + 2ty' + 2y = 2 \quad , y(0) = 1 \ , y'(0) \text{ is arbitrary}$$

Let $y'(0) = a$

Ans :- Let $L\{y\} = \bar{y}$

$$L\{ty''\} + 2L\{ty'\} + 2L\{y\} = L\{2\}$$

$$-\frac{d}{ds}(s^2\bar{y} - s\cdot 1 - a) + 2\left(-\frac{d}{ds}(s\bar{y} - 1)\right) + 2\bar{y} = \frac{2}{s}$$

$$-\bar{y}'(s^2+2s) - \bar{y}(2s+2-2) + 1 = \frac{2}{s}$$

$$\bar{y}'(s^2+2s) + \bar{y}(2s) - 1 = -\frac{2}{s}$$

$$\bar{y}' + \frac{2s}{s^2+2s}\bar{y} = \frac{s-2}{s(s^2+2s)}$$

$$\bar{y}' + \frac{2}{s+2}\bar{y} = \frac{s-2}{s^2(s+2)}$$

This is linear differential eq^n in $\bar{y}$

$\therefore$ $I.F. = e^{\int \frac{2}{s+2} ds}$

$= (s+2)^2$

$\therefore$ Solution is :-

$$\bar{y}(s+2)^2 = \int \frac{(s-2)(s+2)^2}{s^2(s+2)} ds + c$$

$$\bar{y}(s+2)^2 = \int \frac{s^2-4}{s^2} ds + c$$

$$\bar{y}(s+2)^2 = s + \frac{4}{s} + c$$

$$\bar{y} = \frac{s^2+4}{s(s+2)^2} + \frac{c}{(s+2)^2}$$

$$y = L^{-1}\left\{\frac{s^2+4}{s(s+2)^2} + \frac{c}{(s+2)^2}\right\}$$

Let $\frac{s^2+4}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

By comparing coefficients,

$A = 1, \quad B = 0, \quad C = -4$

$$\therefore \quad y = L^{-1}\left\{\frac{1}{s}\right\} + (-4)L^{-1}\left\{\frac{1}{(s+2)^2}\right\} + cL^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

By Applying first shifting theorm,

$$y = 1 + (c-4)te^{-2t}$$

$$y' = (c-4)\left[e^{-2t} - 2te^{-2t}\right]$$

At $t = 0$

$a = c - 4$

$\therefore c = a + 4$

$$y = 1 + ate^{-2t}$$

Q8. Using convolution, solve the initial value problem :-

$$y'' + 9y = \sin 3t \qquad y(0)=0\ ,\ y'(0)=0$$

Let $L\{y\} = \bar{y}$

$$\therefore\ s^2\bar{y} + 9\bar{y} = \frac{3}{s^2+9}$$

$$\bar{y} = \frac{3}{(s^2+9)^2}$$

$$y = L^{-1}\left\{\frac{3}{(s^2+9)^2}\right\}$$

$$y = 3L^{-1}\left\{\frac{1}{(s^2+9)^2}\right\}$$

First find $L^{-1}\left\{\frac{1}{s^2+9}\right\}$

$$\frac{1}{s^2+9} = \frac{3}{3(s^2+9)} = L\left\{\frac{\sin 3t}{3}\right\}$$

$$\therefore\ L^{-1}\left\{\frac{3}{3(s^2+9)}\right\} = \frac{\sin 3t}{3}$$

Let $\bar{f}_1(t) = \frac{1}{s^2+9} = \bar{f}_2(t)$

$$\therefore\ L^{-1}\{\bar{f}_1(t)\} = L^{-1}\{\bar{f}_2(t)\} = \frac{\sin 3t}{3} = f_1(s) = f_2(s)$$

$\therefore$ By convolution theorm,

$$L^{-1}\{\bar{f}_1(s)\,\bar{f}_2(s)\} = \int_0^t f_1(t-u)\,f_2(u)\,du$$

$$L^{-1}\left\{\frac{1}{(s^2+9)^2}\right\} = \frac{1}{9}\int_0^t \sin(3t-3u)\sin 3u\,du$$

$$y = \frac{3}{9\times 2}\int_0^t 2\sin(3t-3u)\sin 3u\,du$$

Scanned by CamScanner

$$y = \frac{1}{6}\left[\int_0^t (\cos(3t-6u) - \cos 3t)\, du\right]$$

$$y = \frac{1}{6}\int_0^t (\cos 3t \cos 6u + \sin 3t \sin 6u)\, du - \frac{1}{6}\int_0^t \cos 3t\, du$$

$$y = \frac{1}{6}\cos 3t \frac{\sin 6u}{6}\Big|_0^t + \frac{1}{6}\sin 3t \left(\frac{-\cos 6u}{6}\right)\Big|_0^t$$

$$- \frac{t\cos 3t}{6}$$

$$y = \frac{1}{36}\cos 3t \sin 6t + \frac{1}{36}\sin 3t - \frac{1}{36}\sin 3t \cos 6t - \frac{t\cos 3t}{6}$$

$$y = \frac{\cos 3t \sin 6t + \sin 3t - \sin 3t \cos 6t - 6t\cos 3t}{36}$$

$$y = \frac{\sin 3t + \sin 3t - 6t\cos 3t}{36}$$

$$y = \frac{\sin 3t - 3t\cos 3t}{18}$$

Q9. In the following problems use the convolution theorm to find the inverse Laplace Transformation :-

a) $\frac{1}{(s-a)(s-b)}$

Ans :- Let $\bar{f}_1(s) = \frac{1}{s-a}$ $\therefore f_1(s) = e^{at}$

$\bar{f}_2(s) = \frac{1}{s-b}$ $\therefore f_2(s) = e^{bt}$

$\therefore$ By convolution theorm,

$$L^{-1}\left\{\frac{1}{(s-a)} \times \frac{1}{(s-b)}\right\} = \left[\cancel{\int_0^t \frac{1}{(t-u-a)(u-b)}\, du}\right]$$

$$= \int_0^t e^{a(t-u)} e^{bu}\, du$$

4t

$$= \int_0^t e^{at-au+bu}\,du$$

$$= e^{at}\int_0^t e^{(b-a)u}\,du$$

$$= e^{at}\left.\frac{e^{(b-a)u}}{(b-a)}\right|_0^t$$

$$= e^{at}\frac{\left(e^{(b-a)t}-1\right)}{(b-a)}$$

b) $\dfrac{1}{s^2(s^2+16)}$

$\bar{f}_1(s) = \dfrac{1}{s^2}$ $\qquad$ $\bar{f}_2(s) = \dfrac{1\times 4}{(s^2+16)\times 4}$

$f_1(s) = t$ $\qquad$ $f_2(s) = \dfrac{\sin 4t}{4}$

By convolution theorm :-

$$L^{-1}\{\bar{f}_1(s)\,\bar{f}_2(s)\} = \int_0^t f_1(t-u)\,f_2(u)\,du$$

$$= \int_0^t \frac{(t-u)\sin 4u}{4}\,du$$

$$= \frac{1}{4}\int_0^t t\sin 4u\,du - \frac{1}{4}\int_0^t u\sin 4u\,du$$

$$= \frac{1}{4}t\left.\left(\frac{-\cos 4u}{4}\right)\right|_0^t - \frac{u}{4}\left.\left(\frac{-\cos 4u}{4}\right)\right|_0^t + \frac{1}{4}\int_0^t \frac{(-\cos 4u)}{4}\,du$$

$$= \frac{1}{4}t\frac{(1-\cos 4t)}{4} + \frac{t\cos 4t}{16} - 0 + \frac{1}{4\times 4} - \frac{\cos 4t}{4\times 4}$$

$$= \frac{t - t\cos 4t + t\cos 4t + 1 - \cos 4t}{16} = \frac{t+1-\cos 4t}{16}$$

c) $\dfrac{1}{(s^2+9)^2}$

Same as que 8.

d) $\dfrac{s}{(s^2+4)^2}$

$\bar{f}_1(s) = \dfrac{s}{s^2+4}$ $\quad\quad$ $\bar{f}_2(s) = \dfrac{1\times 2}{(s^2+4)\times 2}$

$f_1(s) = \cos 2t$ $\quad\quad$ $\bar{f}_2(s) = \dfrac{\sin 2t}{2}$

By convolution theorm,

$$L\{\bar{f}_1(s)\,\bar{f}_2(s)\} = \int_0^t f_1(-u+t)\,f_2(u)\,du$$

$$= \frac{1}{2\cdot 2}\int_0^t 2\cos 2(-u+t)\sin 2u\,du$$

$$= \frac{1}{4}\int_0^t (\sin(-2u+2t+2u) - \sin(-2u+2t-2u))\,du$$

$$= \frac{1}{4}\int_0^t (\sin(-4u+2t) + \sin 2t)\,du$$

$$= \frac{1}{4}\int_0^t (\sin 2t\cos 4u - \cos 2t\sin 4u)\,du + \frac{u\sin 2t}{4}\Big|_0^t$$

$$= \frac{1}{4}\left[\frac{\sin 2t\sin 4u}{4} + \frac{\cos 2t\cos 4u}{4}\right]_0^t + \frac{t\sin 2t}{4}$$

$$= \frac{1}{4}\left[\frac{\sin 2t\sin 4t}{4} + \frac{\cos 2t\cos 4t}{4} - \frac{\cos 2t}{4}\right] + \frac{t\sin 2t}{4}$$

$$= \frac{1}{4}\left[\frac{\cos 2t - \cos 2t}{4}\right] + \frac{t\sin 2t}{4}$$

$$= \frac{t\sin 2t}{4}$$

Q10. Using the Laplace transforms of the periodic function, verify that,

$$L\{\sin \omega t\} = \frac{\omega}{\omega^2+s^2}$$

Ans :- Let L.H.S. →

$L\{\sin \omega t\}$

We know,

fore periodic function

$$L\{f(t)\} = \frac{1}{1-e^{-as}} \int_0^a e^{-st} f(t)\, dt$$

where a is period of $f(t)$.

$$L\{\sin \omega t\} = \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} e^{-st} \sin \omega t\, dt$$

$$= \text{img} \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} e^{-st} e^{i\omega t}\, dt$$

$$= \text{img} \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} e^{-(s-i\omega)t}\, dt$$

$$= \text{img} \frac{1}{1-e^{-2\pi s}} \left. \frac{e^{-(s-i\omega)t}}{-(s-i\omega)} \right|_0^{2\pi}$$

$$= \text{img} \frac{1}{1-e^{-2\pi s}} \frac{e^{-(s-i\omega)2\pi} - 1}{-(s-i\omega)}$$

$$= \text{img} \frac{1}{1-e^{-2\pi s}} \frac{e^{-2\pi s} e^{i\omega 2\pi} - 1}{-(s-i\omega)}$$

$$= \text{img} \frac{1}{1-e^{-2\pi s}} \frac{e^{-2\pi s}(\cos 2\pi\omega + i\sin 2\pi\omega) - 1}{-(s-i\omega)}$$

$$= \text{img} \frac{1}{1-e^{-2\pi s}} \frac{e^{-2\pi s}(1+0) - 1}{-(s-i\omega)}$$

$$= \text{img} \frac{1}{s-i\omega}$$

$$\Rightarrow \quad Img \ \frac{1}{s-iw} \times \frac{s+iw}{s+iw}$$

$$\Rightarrow \quad img \ \frac{s+iw}{s^2+w^2}$$

$$\Rightarrow \quad \frac{w}{s^2+w^2} = R.H.S.$$

Hence, Proved