DELTA Pg No.
Date / /

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ROLL No → 66

BATCH → B-5

MATHS ASSIGNMENT

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1. Define rank of a matrix. Hence, find the rank of the following matrices.

(a) $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & 2 & 6 & -7 \end{bmatrix}$

$\Rightarrow$ A matrix is said to be of rank r when
(i) it has at-least one non-zero minor of order r
(ii) every minor of order higher than r vanishes.

$\Rightarrow$ (a) $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 4 & 5 & -3 \end{bmatrix}$ [By $R_3 + R_1$, and $R_2 - 2R_1$]

$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 4 & 5 & -3 \\ 0 & 0 & 5 & -3 \end{bmatrix}$ [By R_{23}]

By Echelon's form,
Number of non-zeroes rows $= 3$.
$\therefore \rho(A) = 3$.

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(b) $A = \begin{bmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 3 & -2 & 4 \\ 0 & -13/2 & 4 & -4 \\ 0 & +4 & 4 & 2 \\ 0 & 7 & -2 & 9 \end{bmatrix}$ $\left[\text{operate } R_4 + R_1,\ R_3 - R_2,\ R_2 - \frac{3}{2}R_1.\right]$

$\Rightarrow \begin{bmatrix} 2 & 3 & -2 & 4 \\ 0 & -13/2 & 4 & -4 \\ 0 & 0 & 58/13 & -6/13 \\ 0 & 0 & 30/13 & \frac{61}{13} \end{bmatrix}$ $\left[R_4 \leftrightarrow R_4 + \left(\frac{14}{13}\right)R_2 \text{ and } R_3 \rightarrow R_3 + \left(\frac{8}{13}\right)R_2\right.$

$\Rightarrow \begin{bmatrix} 2 & 3 & -2 & 4 \\ 0 & -13/2 & 4 & -4 \\ 0 & 0 & 58/13 & -6/13 \\ 0 & 0 & 0 & \frac{1859}{377} \end{bmatrix}$ $\left[R_4 \rightarrow R_4 - R_3 \times \frac{30}{58}\right]$

By echelon's form,
No. of non zero row is 4.
So, $\underline{\rho(A) \Rightarrow 4}$

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(2) Reduce the following matrices into (a) Row-echelon form (b) Normal form. Hence, find the rank.

(a) $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

USING ECHELON FORM $\rightarrow$

$\Rightarrow A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 5 & -3 \end{bmatrix}$ $\left[R_2 \rightarrow R_2 - 2R_1,\ R_3 \rightarrow R_3 - R_1,\ R_4 \rightarrow R_4 + R_1 \right]$

$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\left[R_4 = R_4 - R_2 \right]$

$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\left[R_3 \rightarrow \frac{R_3}{4} \right]$

$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 0 & 4/5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\left[R_3 \rightarrow R_3 - R_2 \times \frac{1}{5} \right]$

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$$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 0 & 4/5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \left[R_4 \to R_4 - R_3 \times \frac{5}{4} \right]$$

In Row echelon form,
$\underline{\rho(A) = 3}$

<u>Using NORMAL FORM</u> $\Rightarrow$

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix} \quad \left[\text{operating } R_2 = R_2 - 2R_1,\ R_3 \to R_3 - R_1,\ R_4 \to R_4 + R_1 \right]$$

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 5 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 5 & -3 \end{bmatrix} \quad \left[\text{operating } C_2 = C_2 - 2C_1,\ C_3 = C_3 + C_1,\ C_4 = C_4 - 4C_1. \right]$$

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 5 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & -3 & 5 & 0 \end{bmatrix} \quad \left[\text{operating } C_{24}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -3 & 5 & 0 \end{bmatrix} \quad \left[R_2 = R_2 - R_4 \text{ and } R_3 = \frac{R_3}{4}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \left[R_4 - 3R_2 \text{ and } R_4 - 5R_3\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \left[R_2 = \frac{R_2}{(-1)}\right]$$

$$= \left[\begin{array}{c|c} I_3 & 0 \\ \hline 0 & 0 \end{array}\right]$$

$\rho(A) \Rightarrow 3$.

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(b) $A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$

⇒ <u>Using Echelon form</u> :→

$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -2 & 4 \end{bmatrix}$ [using $R_3 - R_1$ and $R_2 - R_1$]

$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ [$R_3 = R_3 - 2R_2$]

$\boxed{\rho(A) \Rightarrow 2.}$

<u>Using Normal form</u> :→

$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -2 & 4 \end{bmatrix}$ [using $R_3 - R_1$ and $R_2 - R_1$]

$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -2 & 4 \end{bmatrix}$ [~~using~~ operating $C_2 = C_2 - 3C_1$, $C_3 = C_3 - 6C_1$, $C_4 = C_4 + C_1$]

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$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$ [operating $C_3 = C_3 + C_2$ and $C_4 = C_4 - 2C_2$]

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ [operating $R_3 \Rightarrow R_3 - 2R_2$]

$$A = \left[\begin{array}{c|c} I_2 & 0 \\ \hline 0 & 0 \end{array}\right]$$

$$\boxed{\rho(A) = 2}$$

Q3 :→ Find non-singular matrices P and Q such that PAQ is the normal form where

$$A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$$

$\Rightarrow$ $A = IAI$

$$\begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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We shall affect every elementary row (column) transformation of the product by subjecting the prefactor and postfactor of A to same.

$$\begin{bmatrix} 1 & -1 & 2 & -1 \\ 0 & 6 & -9 & 6 \\ 0 & 4 & -6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(operating $R_2 - 4R_1$, $R_3 - 2R_1$).

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & -9 & 6 \\ 0 & 4 & -6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(operating $C_2 + C_1$, $C_3 - 2C_1$, $C_3 + C_1$).

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 4 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1/2 & 0 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(operating $C_4 - C_2$, $C_3 + \frac{3}{2} C_2$)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ \frac{2}{3} & -\frac{2}{3} & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1/2 & 0 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(operating $R_3 - \frac{2}{3}R_2$)

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/6 & 0 \\ -1/3 & 1/3 & -1/2 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1/2 & 0 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[\text{operating } R_2 = \frac{R_2}{6},\ R_3 = \frac{R_3}{(-2)}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/6 & 0 \\ -1/3 & 1/3 & -1/2 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 & -1/2 \\ 0 & 0 & -1 & 3/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(C_{34})

Q4) Find the inverse of $\begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$ using elementary row transformation.

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix} X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(By R_{12})

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$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 1 & -1 & -3 \end{bmatrix} X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix}$$

(By $R_3 - 2R_1$, $R_4 - 2R_1$).

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & -3 & -5 \end{bmatrix} X = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ -1 & -2 & 0 & 1 \end{bmatrix}$$

(By $R_1 - R_2$, $R_4 - R_2$).

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix} X = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ -1 & 1 & -\frac{3}{2} & 1 \end{bmatrix}$$

[$R_2 \Rightarrow R_2 + R_3$ and $R_4 \Rightarrow R_4 - \frac{R_3 \times 3}{2}$]

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -1/2 & 0 \\ 2 & -2 & 3 & -2 \end{bmatrix}$$

(By $R_3 \Rightarrow \frac{R_3}{(-2)}$ & $R_4 = -2(R_4)$]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X \Rightarrow \begin{bmatrix} -3 & 3 & -3 & 2 \\ 3 & -4 & 4 & -2 \\ -3 & 4 & -5 & 3 \\ 2 & -2 & 3 & -2 \end{bmatrix}$$

(By $R_1 - R_4$, $R_2 + R_4$, $R_3 - \frac{3}{2}R_4$).

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(5) check the consistency of the following system of linear non homogeneous equations and find the solution, if exists:-

(a) $2a - 2b + 4c + 3d = 9$; $a - b + 2c + 2d = 6$;
$2a - 2b + c + 2d = 3$; $a - b + d = 2$.

$$\Rightarrow \begin{bmatrix} 2 & -2 & 4 & 3 \\ 1 & -1 & 2 & 2 \\ 2 & -2 & 1 & 2 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 0 & 0 & 1/2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & -2 & -1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ -6 \\ -5/2 \end{bmatrix}$$

(By $R_2 - \frac{R_1}{2}$, $R_3 - R_1$, $R_4 - R_1$)

$$\Rightarrow \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 0 & 0 & 1/2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & \frac{1}{6} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ -6 \\ +3/2 \end{bmatrix}$$

(By $R_4 - R_3 \times \frac{2}{3}$)

$$\Rightarrow \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 0 & 0 & 1/2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ -6 \\ 1 \end{bmatrix}$$

(By $R_4 - \frac{R_2}{3}$)

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$\therefore$ rank of coefficient matrix $\Rightarrow 3$,
rank of augmented matrix $= 4$.

$\therefore$ No solution.

$$\begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \\ 3/2 \\ 1 \end{bmatrix}$$

(b) $4a - b = 12$, $-a + 5b - 2c = 0$, $-2b + 4c = -8$.

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ -8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 0 & 60/19 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ \frac{-128}{19} \end{bmatrix}$$

rank of coefficient matrix = rank of augmented = 3.
= no. of variable.
so unique solution.

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$\therefore \frac{60}{19} c = \frac{-128}{19}$

$$\boxed{c = \frac{-64}{30} = \frac{-32}{15}} \quad \ldots (i)$$

$4a - b = 12. \quad \ldots (ii)$

$19b - 8c = 12 \quad \ldots (iii)$

$19b + \frac{32 \times 8}{15} = 12$ (From eq (i))

$19b = 12 - \frac{32 \times 8}{15}$

$19b = \frac{180 - 256}{15}$

$19b = \frac{-76}{15}$

$$\boxed{b = \frac{-4}{15}}$$

$4a + \frac{4}{15} = 12$

$$\boxed{a \Rightarrow 3 - \frac{1}{15} = \frac{44}{15}}$$

(6) For what values of λ and μ, the equations $a + b + c = 0$ and $a + 2b + 3c = 10$ and $a + 2b + \lambda c = \mu$ has

(i) no solution. (ii) unique solⁿ

→ (iii) infinite number of solution.

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$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

Δ of coefficient matrix

$\Rightarrow 1(2\lambda-6) - 1(\lambda-3) + 1(2-2)$

$\Rightarrow 2\lambda - 6 - \lambda + 3$

$\Rightarrow \lambda - 3 = 0$

$\boxed{\lambda = 3.}$

<u>augmented matrix</u>

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & 3 & \mu \end{bmatrix}$$

$\Rightarrow 1(3\mu - 30) - 1(2\mu - 20) + 6(6-6) = 0$

$3\mu - 30 - 2\mu + 20$

$\boxed{\mu = 10.}$

If $\lambda = 3$, rank of coefficient matrix is 2.
and $\mu = 10$, rank of augmented matrix is 2.
$\therefore$ $\lambda = 3$, $\mu = 10$ is consistent case.

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(i) If $\lambda = 3$, $\mu \neq 10$,
rank of coefficient and augmented matrix is different means no solution.

(ii) If $\lambda \neq 3$, μ any value (real number),
rank of coefficient and augmented matrix are same and also equal to no. of variable $= 3$.
$\therefore$ So, unique sol^n. when $\lambda \neq 3$ and μ any value.

(iii) If $\lambda = 3$, $\mu = 10$,
rank of coefficient and augmented are 2 which is less than no. of variables.
So, For infinite sol^n., $\lambda = 3$, $\mu = 10$

(7) Show that the vectors $\vec{a} = [3, 3, 1, -1]$, $\vec{b} = [2, 3, 1, -2]$ and $\vec{c} = [4, 6, 2, -3]$ are linearly independent.

$\Rightarrow$ $\lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c} \Rightarrow 0.$

$$\lambda_1 \begin{bmatrix} 3 \\ 3 \\ 1 \\ -1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix} + \lambda_3 \begin{bmatrix} 4 \\ 6 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} 2\lambda_1 + 2\lambda_2 + 4\lambda_3 \\ 3\lambda_1 + 3\lambda_2 + 6\lambda_3 \\ \lambda_1 + \lambda_2 + 2\lambda_3 \\ -\lambda_1 - 2\lambda_2 - 3\lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 4 \\ 3 & 3 & 6 \\ 1 & 1 & 2 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since, rank of coefficient matrix and augmented is 1.
and no. of variable is 3.
It ~~is~~ has infinite no. of solutions.

For only, $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0$ solutions vectors will be independent.

8. Examine the vectors $\vec{a} = [1, 0, 2, 1]$, $\vec{b} = [3, 1, 2, 1]$, $\vec{c} = [4, 6, 2, -4]$ and $\vec{d} = [-6, 0, -3, -4]$ for linear independence and dependence. Also find the relation between them if it exists.

$\Rightarrow$ Let the vectors be connected by numbers $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively.

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$$\Rightarrow \lambda_1 \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \lambda_3 \begin{bmatrix} 4 \\ 6 \\ 2 \\ -4 \end{bmatrix} + \lambda_4 \begin{bmatrix} -6 \\ 0 \\ -3 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \lambda_1 + 3\lambda_2 + 4\lambda_3 - 6\lambda_4 \\ \lambda_2 + 6\lambda_3 \\ 2\lambda_1 + \lambda_2 + 2\lambda_3 - 3\lambda_4 \\ \lambda_1 + \lambda_2 - 4\lambda_3 - 4\lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 4 & -6 \\ 0 & 1 & 6 & 0 \\ 2 & 1 & 2 & -3 \\ 1 & 1 & -4 & -4 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Δ of coefficient matrix $\neq 0$, so rank $= 4$.
rank of augmented matrix $= 4$.
no. of variable $= 4$.
$\therefore$ so, unique solution. (trivial solution).
$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$.
so, vectors are linearly independent of each other.

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9. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$. Also find the eigen values and eigen vectors of this matrix.

$\Rightarrow |A - \lambda I| \Rightarrow 0$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 0 & 2-\lambda & 1 \\ -1 & 2 & 2-\lambda \end{vmatrix} \Rightarrow 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 2 \\ 0 & 2-\lambda & 1 \\ -1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$= (1-\lambda)\left[(2-\lambda)^2 - 2\right] - 1(2 - 4 + 2\lambda) \Rightarrow 0$

$\Rightarrow (1-\lambda)\left[4 + \lambda^2 - 4\lambda - 2\right] - 1(2\lambda - 2) \Rightarrow 0$

$\Rightarrow (1-\lambda)\left[\lambda^2 - 4\lambda + 2\right] - 2\lambda + 2 \Rightarrow 0$

$\Rightarrow \lambda^2 - 4\lambda + 2 - \lambda^3 + 4\lambda^2 - 2\lambda - 2\lambda + 2 \Rightarrow 0$

$\Rightarrow +5\lambda^2 - \lambda^3 - 8\lambda + 4 \Rightarrow 0$

$\underline{\lambda^3 - 5\lambda^2 + 8\lambda - 4 \Rightarrow 0}$

$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$ is a characteristic equation of given matrix.

$(\lambda - 1)(\lambda - 2)(\lambda - 2) \Rightarrow 0$

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roots are $\lambda = 1, 2$.

For $\lambda = 1 \rightarrow$

$$\begin{bmatrix} 0 & 2 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2y + 2z = 0$$
$$y + z = 0$$
$$-x + 2y + z = 0$$

$$\frac{x}{1} = \frac{y}{1} = \frac{z}{-1} = k.$$

$$\begin{bmatrix} k \\ k \\ -k \end{bmatrix}, \quad \text{For } k = 1 \quad \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

For $\lambda_2 = \lambda_3 = 2 \rightarrow$

$$\begin{bmatrix} -1 & 2 & 2 \\ 0 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-x + 2y + 2z = 0$$
$$z = 0$$
$$-x + 2y = 0$$
$$\frac{x}{2} = \frac{y}{1} = \frac{z}{0} = k_2.$$

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$$\begin{bmatrix} 2k \\ k \\ 0 \end{bmatrix}, \quad \text{for } k=1, \quad \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

(10) which of the matrix is diagonalisable.

(a) $$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$\Rightarrow \quad |A - \lambda I| = 0.$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (6-\lambda)\left[(3-\lambda)^2-1\right] + 2\left(-2(3-\lambda)+2\right) + 2\left[2-2(3-\lambda)\right] = 0$

$\Rightarrow \quad 6(\lambda^2+9-6\lambda-1) - \lambda(9+\lambda^2-6\lambda-1) + 4(-3+\lambda+1)$
$\quad + 4(1-3+\lambda) = 0$

$\Rightarrow \quad 6(\lambda^2-6\lambda+8) - \lambda(\lambda^2-6\lambda+8) + 4(\lambda-2) + 4(\lambda-2)$

$\Rightarrow \quad (6-\lambda)(\lambda^2-6\lambda+8) + 8(\lambda-2)$

$\Rightarrow \quad (6-\lambda)(\lambda-2)(\lambda-4) + 8(\lambda-2)$

$\Rightarrow \quad (\lambda-2)\left[(6-\lambda)(\lambda-4)+8\right]$

$\Rightarrow \quad (\lambda-2)\left[6\lambda-24-\lambda^2+4\lambda+8\right]$

$\Rightarrow \quad (\lambda-2)\left[-\lambda^2+10\lambda-16\right]$

$\Rightarrow \quad (\lambda-2)\left[\lambda^2-10\lambda+16\right] = 0$

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$(\lambda-2)(\lambda-8)(\lambda-2) = 0$

$\lambda = 2, 2, 8$

$\lambda = 8$ gives unique eigen vector.
we have to check for $\lambda = 2$.

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 4 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

(using $R_3 + R_2$, $R_3 - \frac{R_1}{2}$)

$\underline{4x - 2y + 2z} \Rightarrow 0.$

↳ let ~~x_1, x_2~~

$y = a, \quad z \Rightarrow b.$

$x \Rightarrow \cancel{\frac{b-a}{2}} \quad \frac{a-b}{2}$

$$\text{eigen vector} = \begin{bmatrix} \frac{a-b}{2} \\ a \\ b. \end{bmatrix} \Rightarrow a \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow a \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + b \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

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For $\lambda = 2$,
we get two eigen vectors

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$\therefore$ Hence, 3 eigen vectors are possible which are different. So, matrix is diagonalisable.

(b) $$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

$\Rightarrow |A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 10 & 5 \\ -2 & -3-\lambda & -4 \\ 3 & 5 & 7-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)\left[-(\lambda+3)(7-\lambda)+20\right] - 10\left[-2(7-\lambda)+12\right] + 5\left[-10+3(3+\lambda)\right] = 0$$

$$\Rightarrow -\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0$$

$$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$$

$$(\lambda-2)(\lambda-2)(\lambda-3) = 0$$

$$\lambda = 2, 2, 3.$$

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eigen values are 2, 2, 3.

For $\lambda = 3 \rightarrow$

$$\begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow 0.$$

$$\begin{bmatrix} 0 & 10 & +5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow 0.$$

$$10y + 5z \Rightarrow 0.$$
$$-2x - 6y - 4z = 0.$$
$$3x + 5y + 4z = 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k_1 \\ k_1 \\ -2k_1 \end{bmatrix}, \text{ For } k = 1, \Rightarrow \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

For $\lambda = 2 \rightarrow$

$$\begin{bmatrix} 1 & 10 & 5 \\ -2 & -5 & -4 \\ 3 & 5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x + 10y + 5z \Rightarrow 0$$
$$-2x - 5y - 4z = 0$$
$$3x + 5y + 5z = 0$$

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$$\frac{x}{-5} = \frac{y}{-2} \Rightarrow \frac{k}{5}.$$

$$\begin{bmatrix} -5k \\ -2k \\ 5k \end{bmatrix}, \quad \text{For } k=1 \rightarrow \begin{bmatrix} -5 \\ -2 \\ 5 \end{bmatrix}$$

So, vector is same for both eigen value, so non-diagonalisable.

11. Find the characteristic equation of the matrix $\begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$. verify cayley hamilton theorem and hence find A^{-1}.

$$\Rightarrow |A-\lambda I| = \begin{vmatrix} 4-\lambda & 3 & 1 \\ 2 & 1-\lambda & -2 \\ 1 & 2 & 1-\lambda \end{vmatrix} \Rightarrow 0$$

$$\Rightarrow (4-\lambda)\left[(1-\lambda)^2+4\right] - 3\left[2-2\lambda+2\right] + 1(4-1+\lambda) \Rightarrow 0$$

$$\Rightarrow (4-\lambda)\left[(1-\lambda)^2+4\right] - 12 + 6\lambda + 3 + \lambda \Rightarrow 0$$

$$\Rightarrow -\lambda^3 + 6\lambda^2 - 6\lambda + 11$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 6\lambda - 11 \Rightarrow 0$$

Every square matrix should satisfy its characteristic equation as per Hamilton theorem.

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verification of Hamilton Theorem :→

$$A^2 = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 17 & -1 \\ 8 & 3 & -2 \\ 9 & 7 & -2 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 23 & 17 & -1 \\ 8 & 3 & -2 \\ 9 & 7 & -2 \end{bmatrix} \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 125 & 84 & -12 \\ 36 & 23 & 0 \\ 48 & 30 & -7 \end{bmatrix}$$

$$A^3 - 6A^2 + 6A - 11I$$

$$\Rightarrow \begin{bmatrix} 125 & 84 & -12 \\ 36 & 23 & 0 \\ 48 & 30 & -7 \end{bmatrix} + \begin{bmatrix} -138 & -102 & 6 \\ -48 & -18 & 12 \\ -54 & -42 & 12 \end{bmatrix} + \begin{bmatrix} 24 & 18 & 6 \\ 12 & 6 & -12 \\ 6 & 12 & 6 \end{bmatrix}$$

$$- \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} -13 & -18 & -6 \\ -12 & 5 & 12 \\ -6 & -12 & 5 \end{bmatrix} + \begin{bmatrix} 24 & 18 & 6 \\ 12 & 6 & 12 \\ -6 & -12 & 6 \end{bmatrix} + \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ (verified).

since, $A^3 - 6A^2 + 6A - 11 = 0$.

$A^{-1}(A^3 - 6A^2 + 6A - 11) = 0$

$$\boxed{\frac{A^2 - 6A + 6}{11} = A^{-1}}$$

$$A^{-1} = \frac{1}{11}\left(\begin{bmatrix} 23 & 17 & -1 \\ 8 & 3 & -2 \\ 9 & 7 & -2 \end{bmatrix} + \begin{bmatrix} -24 & -18 & -6 \\ -12 & -6 & 12 \\ -6 & -12 & -6 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}\right)$$

$$A^{-1} = \frac{1}{11}\begin{bmatrix} 5 & -1 & -7 \\ -4 & 3 & 10 \\ 3 & -5 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 5/11 & -1/11 & -7/11 \\ -4/11 & 3/11 & 10/11 \\ 3/11 & -5/11 & -2/11 \end{bmatrix}.$$

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12. Diagonalise the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$

and hence find A^{-1}.

$\Rightarrow |A| = 11(12-20) + 4(-42+50) - 7(-28+20)$

$= -88 + 32 + 56$

$\Rightarrow 0$;

clearly, one of its eigen value is zero.

and A^{-1}, not exist because the matrix is singular.

$$A - \lambda I = \begin{bmatrix} 11-\lambda & -4 & -7 \\ 7 & -2-\lambda & -5 \\ 10 & -4 & -6-\lambda \end{bmatrix} =$$

$|A - \lambda I| = 0$

$\Rightarrow (11-\lambda)\left[(\lambda+2)(\lambda+6) - 20\right] + 4\left[-42 - 7\lambda + 50\right]$

$- 7\left[-28 + 20 + 1 - \lambda\right]$

$\Rightarrow -\lambda^3 - 2\lambda + 3\lambda^2 = 0$

$= -\lambda(\lambda^2 - 3\lambda + 2) = 0$

$\lambda = 0, \quad \lambda = 2, \lambda = 1.$

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For $\lambda = 0$.

$$\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$11x - 4y - 7z = 0$$
$$7x - 2y - 5z = 0$$
$$10x - 4y - 6z = 0$$

$$\frac{x}{6} \Rightarrow \frac{-y}{-6} \Rightarrow \frac{z}{6}$$

$$\frac{x}{1} = \frac{y}{1} = \frac{z}{1}.$$

(eigen) Dir^n vector $\rightarrow$ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

For $\lambda = 2$.

$$\begin{bmatrix} 9 & -4 & -7 \\ 7 & -4 & -5 \\ 10 & -4 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$9x - 4y - 7z = 0$$
$$7x - 4y - 5z = 0$$
$$10x - 4y - 8z = 0$$

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$$\frac{x}{2} = \frac{y}{1} = \frac{z}{2}.$$

eigen vector :→ $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

For $\lambda = 1$.

$$\begin{bmatrix} 10 & -4 & -7 \\ 7 & -3 & -5 \\ 10 & -4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$10x - 4y - 7z = 0$
$7x - 3y - 5z = 0$
$10x - 4y - 7z = 0$

$$\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}.$$

eigen vector :→ $\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$.

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Modal matrix

$$P = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

and $P^{-1} \Rightarrow \begin{bmatrix} -4 & 2 & 3 \\ 3 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$

Diagonal matrix $\Rightarrow P^{-1}AP$

$$= \begin{bmatrix} -4 & 2 & 3 \\ 3 & -1 & 2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 6 & -2 & -4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

13. Let $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. find a matrix P such that $P^{-1}AP$ is diagonal.

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$\Rightarrow |A-\lambda I| = 0 \quad \begin{vmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix} = 0$

$= (4-\lambda)(3-\lambda) - 2 = 0$

$12 - 4\lambda - 3\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 7\lambda + 10 = 0$

$(\lambda-5)(\lambda-2) = 0, \quad \lambda = 2, 5.$

For $\lambda = 2$.

$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0.$

$2x + y = 0$

$\boxed{\frac{x}{-1} = \frac{y}{+2}} \Rightarrow \begin{bmatrix} -1 \\ +2 \end{bmatrix}$ (eigen vector)

For $\lambda = 5$.

$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-x + y = 0, \quad \frac{x}{1} = \frac{y}{1}. \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ eigen vector.

so, $P = \begin{bmatrix} -1 & 1 \\ +2 & 1 \end{bmatrix}.$

$P^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$D = P^{-1}AP = \begin{bmatrix} -1/3 & +1/3 \\ 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ +2 & 1 \end{bmatrix}$

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$$= \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$ verified..

So, $P = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$ is a modal matrix.

14 If $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$. Find the eigen value of A^{-2} & A^3.

$$\Rightarrow |A-\lambda I| = \begin{vmatrix} -1-\lambda & 2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0.$$

$$\Rightarrow (-1-\lambda)\left(-\lambda(2-\lambda)+1\right) - 2(-\lambda+1) - 2(-1+2-\lambda) = 0.$$

$$\Rightarrow -(\lambda+1)(\lambda(\lambda-2)+1) + 2(\lambda-1) + 2(\lambda-1) = 0$$

$$\Rightarrow -(\lambda+1)(\lambda-1)^2 + 4(\lambda-1) = 0$$

$$\Rightarrow (\lambda-1)\left[5-\lambda^2\right] = 0$$

$\underline{\lambda = 1}$, $\underline{\lambda = \sqrt{5}}$, $\underline{-\sqrt{5}}$.

$\Rightarrow$ $Ax = \lambda x$.

$A(Ax) \Rightarrow A(\lambda x) = \lambda(\lambda x) = \lambda^2(x)$ (multiplying by A).

similarly $\Rightarrow$

$$\boxed{A^m x = \lambda^m x.}$$

For A^m, eigen values are λ^m.

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For A^3,
eigen values are
$$|1|^3, |\sqrt{5}|^3, |-\sqrt{5}|^3.$$

For A^{-2}.
eigen values are
$$|1|^2, \left(\frac{1}{\sqrt{5}}\right)^2, \left(\frac{-1}{\sqrt{5}}\right)^2$$
$$= 1, \frac{1}{5}, +\frac{1}{5}$$
$$= 1, 0.2, +0.2.$$

15. State cayley - hamilton theorem use it to express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

→ cayley - hamilton theorem states that every square matrix satisfies its own characteristic eq^n.

$$\Rightarrow |A - \lambda I| = \begin{vmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(2-\lambda) + 1 = 0$$
$$6 - 3\lambda - 2\lambda + \lambda^2 + 1 = 0.$$
$$\lambda^2 - 5\lambda + 7 = 0.$$

since, it is characteristic eq^n of cayley - hamilton.

$$\therefore A^2 - 5A + 7 = 0. \quad \ldots (i)$$

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$\Rightarrow 2A^5 + 3A^4 + A^2 - 4$

$\Rightarrow 2A^3(A^2 - 5A + 7) + 7A^2(A^2 - 5A + 7)$
$+ 21(A^2 - 5A + 7) + 57(A^2 - 5A + 7)$
$+ 138A - 395I.$

$\Rightarrow 2A^3 \times 0 + 7A^2 \times 0 + 21 \times 0 + 57 \times 0$
$+ 138A - 395I$

$\Rightarrow 138A - 395.$

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