→ Multiple Integral ←

Maths Assignment -4

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A1 :-

$$= \int_0^1 \int_0^{2x} (x + xy)\, dy\, dx$$

$$= \int_0^1 \int_0^{2x} (x\,dy + xy\,dy)\, dx$$

$$= \int_0^1 \left[xy + \frac{xy^2}{2} \right]_0^{2x} dx$$

$$= \int_0^1 (2x^2 + 2x^3)\, dx$$

$$= \left[\frac{2x^3}{3} + \frac{x^4}{2} \right]_0^1 = \frac{2}{3} + \frac{1}{2}$$

$$= 7/6.$$

A2 :→

$$= \int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dy\, dx$$

$$= \int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{(\sqrt{1+x^2})^2 + y^2}\, dy\, dx$$

$$= \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} \tan^{-1}\left(\frac{y}{\sqrt{1+x^2}} \right) \right]_0^{\sqrt{1+x^2}} dx$$

$$= \int_0^1 \frac{1}{\sqrt{1+x^2}}\, dx \cdot \tan^{-1}(1)$$

Q4

$$= \frac{\pi}{4}\left[\log\left(x+\sqrt{1+x^2}\right)\right]_0^1$$

$$= \frac{\pi}{4}\left[\log(1+\sqrt{2}) - 0\right]$$

$$= \frac{\pi}{4}\log(1+\sqrt{2}).$$

Q3 let

$$\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx = I_1$$

$$\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy = I_2$$

Solving I_2

Put $x = t$ $\quad y = u$

$dx = dt$ $\quad dy = du$

$$I_2 = \int_0^1\int_0^1 \frac{t-u}{(t+u)^3}\,dt\,du$$

Put $t = y$ $\quad dt = dy$

$u = x$ $\quad du = dx$

$$I_2 = \int_0^1\int_0^1 \frac{y-x}{(x+y)^3}\,dy\,dx$$

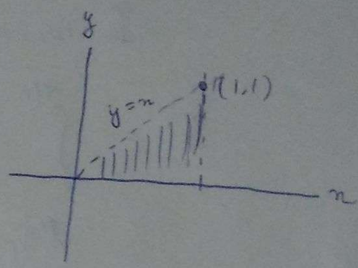
$$I_2 = (-1)\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx$$

$$I_2 = -I_1$$

Hence $I_2 \neq I_1$

Q4

$$\iint_R \sqrt{4x^2-y^2}\; dy\, dx$$



$$\therefore\; I = \int_0^1\int_0^x \sqrt{4x^2-y^2}\; dy\, dx.$$

$$= \int_0^1 \left[\frac{y}{2}\sqrt{4x^2-y^2} + \frac{4x^2}{2}\sin^{-1}\left(\frac{y}{2x}\right)\right]_0^x dx$$

$$= \int_0^1 \left[\frac{\sqrt{3}\,x^2}{2} + \frac{4x^2}{2}\cdot\frac{\pi}{6}\right] dx$$

$$= \int_0^1 x^2\left[\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right] dx.$$

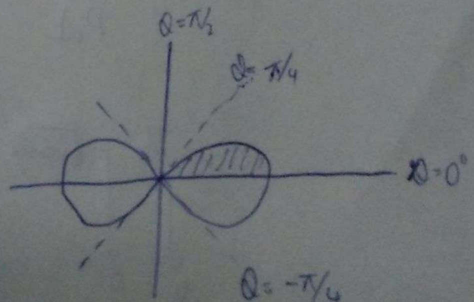
$$= \left[\frac{x^3}{3}\left[\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right]\right]_0^1$$

$$= \frac{1}{3}\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)$$

Hence, proved

Q5

$$\iint \frac{r\,dr\,d\theta}{\sqrt{a^2+r^2}}\;,\text{ over the loop },\; r^2 = a^2\cos 2\theta$$



$$I = 2\int_0^{\pi/4}\int_0^{a\sqrt{\cos 2\theta}} \frac{r\,dr\,d\theta}{\sqrt{a^2+r^2}}$$

Put $r^2 = t$

$2r\,dr = dt$

$r \Rightarrow 0 \rightarrow t = 0$

$r = a\sqrt{\cos 2\theta} \rightarrow t = a^2\cos 2\theta$

I becomes

$$I = \int_0^{\pi/4} \int_0^{a^2\cos 2\theta} \frac{1}{\sqrt{t+a^2}} \, dt \, d\theta$$

$$= \int_0^{\pi/4} \left[2\sqrt{t+a^2} \right]_0^{a^2\cos 2\theta} d\theta$$

$$= 2\int_0^{\pi/4} a \left[\sqrt{1+\cos 2\theta} - 1 \right] d\theta$$

$$= 2\int_0^{\pi/4} a \left[\sqrt{2}\cos\theta - 1 \right] d\theta$$

$$= 2a \left[\sqrt{2}\sin\theta - \theta \right]_0^{\pi/4}$$

$$= 2a \left[\sqrt{2} \times \frac{1}{\sqrt{2}} - \pi/4 - 0 \right]$$

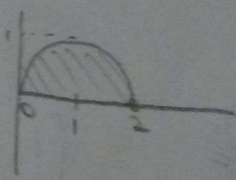
$$= 2a \left[1 - \pi/4 \right].$$

$x^2+y^2=4$

$x+z=2$

Q6

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}} \, dy \, dx.$$



Put $x = r\cos\theta$

$y = r\sin\theta$

$y = \sqrt{2x - x^2}$

$r\sin\theta = \sqrt{2r\cos\theta - r^2\cos^2\theta}$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

$$\therefore \int_0^{\pi/2} \int_0^{2\cos\theta} \frac{r\cos\theta}{r} \, r \, dr \, d\theta$$

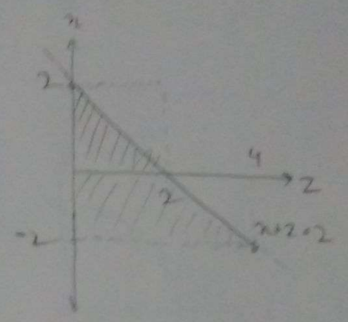
$$\int_0^{\pi/2} \left[\frac{r^2}{2}\cos\theta \right]_0^{2\cos\theta} d\theta$$

$$= \int_0^{\pi/2} 2\cos^3\theta \, d\theta$$

$$= 2 \cdot \frac{2}{3} = \frac{4}{3}$$

Q7

$$V = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{2-x} dz\, dy\, dx$$



Eqn of cylinder $x^2+y^2=4$

Eqn of plane $x+z=2$

$z>0$

$y = \pm\sqrt{4-x^2}$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left[z \right]_0^{2-x} dy\, dx$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2-x)\, dy\, dx$$

$$= \int_{-2}^{2} (2-x) \left[y \right]_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx$$

$$= \int_{-2}^{2} (2-x)\, 2\sqrt{4-x^2}\, dx$$

$$= 2\int_{-2}^{2} (2-x)\sqrt{(2-x)(2+x)}\, dx$$

Put $x = 2\cos\theta$

$\therefore dx = 2 \cdot -\sin\theta\, d\theta$

$$\therefore = 2\int_0^{\pi} 2\sin\theta \cdot 2(1-\cos\theta) \cdot 2 \cdot \sin\theta\, d\theta$$

$$= 16\int_0^{\pi} \sin^2\theta \cdot (1-\cos\theta)\, d\theta$$

$$= 16\left[2 \cdot \frac{1}{2} \cdot \frac{\pi}{2} \right] = \underline{\underline{8\pi}} \text{ cubic units}$$

Q5

$$\text{Volume} = \int_0^a \int_0^{b(1-\frac{x}{a})} \int_0^{c(1-\frac{x}{a}-\frac{y}{b})} dz\, dy\, dx$$

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

$$V = \int_0^a \int_0^{b(1-\frac{x}{a})} c\left(1-\frac{x}{a}-\frac{y}{b}\right) dy\, dx$$

$$= \int_0^a \left[c\left(y - \frac{x}{a}[y] - \frac{y^2}{2b}\right)\right]_0^{b(1-\frac{x}{a})}$$

$$= \int_0^a \left[c\left(b\left(1-\frac{x}{a}\right) - \frac{bx}{a}\left[1-\frac{x}{a}\right] - \frac{b}{2}\left[1-\frac{x}{a}\right]^2\right)\right] dx$$

$$= c\int_0^a \left[b - \frac{bx}{a} - \frac{bx}{a} + \frac{bx^2}{a^2} - \frac{b}{2} - \frac{bx^2}{2a^2} + \frac{bx}{a}\right] dx$$

$$= c\int_0^a \left[\frac{b}{2} - \frac{bx}{a} + \frac{x^2}{a^2}\left[b-\frac{b}{2}\right]\right] dx$$

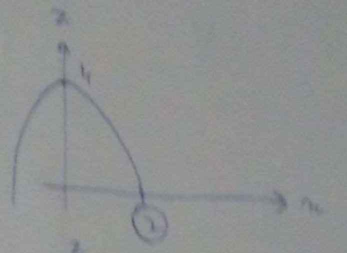
$$= c\left[\frac{bx}{2} - \frac{bx^2}{2a} + \frac{x^3}{3a^2}\left[\frac{b}{2}\right]\right]_0^a$$

$$= c\left[\frac{ab}{2} - \frac{ab}{2} + \frac{a}{3}\left[\frac{b}{2}\right]\right]$$

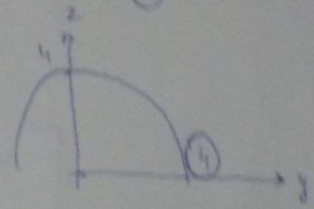
$$= \frac{abc}{6} \text{ units}^3$$

Q9 Volume bounded by $4z = 16 - 4x^2 - y^2$ and $z = 0$

$$V = \int_{-2}^{2} \int_{-\sqrt{16-4x^2}}^{\sqrt{16-4x^2}} \int_{0}^{\frac{16-4x^2-y^2}{4}} dz\, dy\, dx$$



$$= \int_{-2}^{2} \int_{-\sqrt{16-4x^2}}^{\sqrt{16-4x^2}} \frac{16-4x^2-y^2}{4}\, dy\, dx$$



Put $2x = X$ $\qquad$ $y = Y$

$2dx = dX$ $\qquad$ $dy = dY$

$$= \frac{1}{8} \int_{-2}^{2} \int_{-\sqrt{16-X^2}}^{\sqrt{16-X^2}} 16 - X^2 - Y^2 \; dY\, dX$$

Put $X = r\cos\theta$

$Y = r\sin\theta$

$$= \frac{1}{8} \int_{0}^{2\pi} \int_{0}^{4} (16 - r^2)\, r \; dr\, d\theta$$

$$= \frac{1}{8} \int \left[\frac{16r^2}{2} - \frac{r^4}{4} \right]_0^4 dr\, d\theta$$

$$= \frac{1}{8} \int_{0}^{2\pi} \left[\frac{16 \times 16}{2} - \frac{16 \times 16}{4} \right] d\theta$$

$$= \frac{1}{8} \times 2\pi \times \frac{16 \times 16}{4} = 16\pi$$

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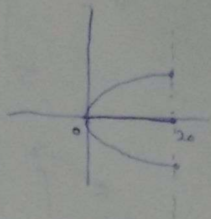
Q10 $x^2+y^2=2ax$ $\qquad$ $z^2=2ax$.



$$V = \int_0^{2a}\int_{-\sqrt{2ax-x^2}}^{\sqrt{2ax-x^2}}\int_{-\sqrt{2ax}}^{\sqrt{2ax}} dz\,dy\,dx$$

$$= \int_0^{2a}\int_{-\sqrt{2ax-x^2}}^{\sqrt{2ax-x^2}} 2\sqrt{2ax}\; dy\,dy$$

$$= \int_0^{2a} 4\sqrt{2ax-x^2}\,\sqrt{2ax}\; dx$$

$$= 4\sqrt{2a}\int_0^{2a} x\sqrt{2a-x}\; dx$$

Put $2a-x = t$

$\therefore -dx = dt$

$$= 4\sqrt{2a}\int_0^{2a}(2a-t)\sqrt{t}\; dt$$

$\frac{2a-12}{15}$

$$= 4\sqrt{2a}\left[\frac{t^{3/2}}{3/2}\cdot 2a - \frac{t^{5/2}}{5/2}\right]_0^{2a}$$

$$= 4\sqrt{2}\,a^3\left[2^{3/2}\cdot\frac{4}{3} - \frac{2}{5}2^{5/2}\right]$$

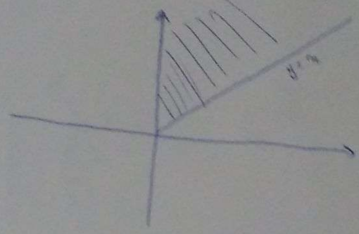
$$= \frac{4\sqrt{2}\cdot\sqrt{2}}{15}\left[\frac{8}{3}-\frac{8}{5}\right]a^3$$

$$= \frac{64}{15}[2]\,a^3$$

$$= \frac{128}{15}a^3 \text{ cubic units.}$$

Q11)

$$= \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$$



By changing order of integration

$$= \int_0^\infty \int_0^y \frac{e^{-y}}{y}\, dx\, dy$$

$$= \int_0^\infty \frac{e^{-y}}{\cancel{y}} \cdot \cancel{y} \cdot dy$$

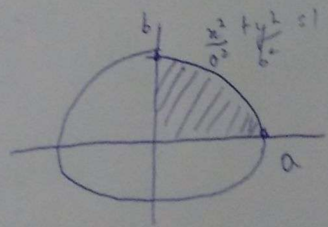
$$= (-1)\int_0^\infty e^{-y}$$

$$= (-1)\left[0 - 1\right]$$

$$= 1.$$

Q12)

$$A = \int_0^a \int_0^{b\left(\sqrt{1-x^2/a^2}\right)} dy\, dx$$



$$= \int_0^a b\sqrt{1-x^2/a^2}\, dx$$

$$= \int_0^a \frac{b}{a}\sqrt{a^2-x^2}$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(x/a\right).\right]_0^a$$

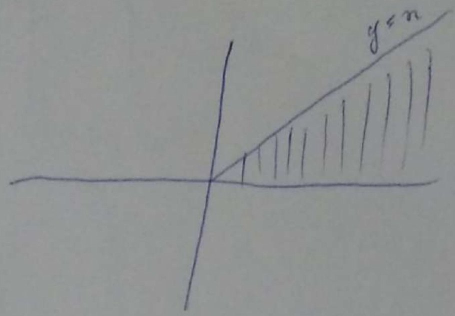
$$= \frac{b}{a}\left[0 + \frac{a^2}{2}\frac{\pi}{2} - 0 - 0\right] = \frac{\pi ab}{4} \text{ square}$$

Hence Area of ellipse $= 4 \times \frac{\pi ab}{4}$

$= \pi ab$ square units

A13

$$= \int_0^{\infty} \int_0^{x} x e^{-x^2/y} \, dy \, dx$$



By changing order of integration

$$= \int_0^{\infty} \int_y^{\infty} x e^{-x^2/y} \, dx \, dy$$

Put $x^2 = t$

$2x\,dx = dt$

$$= \frac{1}{2} \int_0^{\infty} \int_{y^2}^{\infty} e^{-t/y} \, dt \, dy$$

$$= \frac{-1}{2} \int_0^{\infty} \left[e^{-\infty} - e^{-y^2/y} \right] dy$$

$$= +\frac{1}{2} \int_0^{\infty} e^{-y^2} \, dy$$

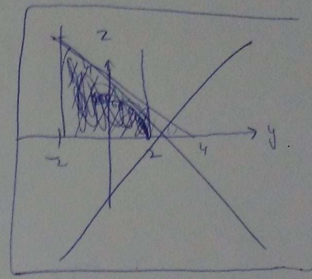
$\cancel{y^2 = t} \quad \cancel{2y\,dy = dt} \quad \cancel{dy = \frac{dt}{2\sqrt{t}}}$ $\quad e^{x}] \, d$

$$\cancel{= \frac{1}{4} \int_0^{\infty} t^{-1/2} e^{-t} \, dt}$$

$$\cancel{= \frac{1}{4} \sqrt{\frac{1}{2}}} = \frac{1}{2} [-1][-1] = \boxed{\frac{1}{2}}$$

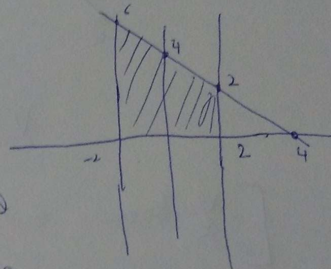
Q14 $V = \int_{-2}^{2} \int_{\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{0}^{4-2y} dz\, dx\, dy$

Volume = V



$= \int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} 4-2y \; dx\, dy$

$= \int_{-2}^{2} [4-2y][2]\sqrt{4-y^2}$



$= 2\int_{-2}^{2} [4-y]\left[\sqrt{4-y^2}\right] dy$

Put $y = 2\cos\theta$ $\quad dy = 2\text{-}\sin\theta\, d\theta$

$= 2\times 2\int_{0}^{\pi} (4-2\cos\theta)(2)(\sin^2\theta)\, d\theta$

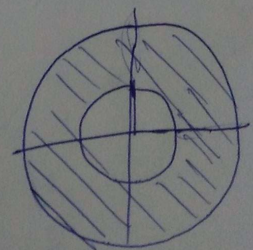
$= 16 \int_{0}^{\pi} \left[2\sin^2\theta - \cos\theta\sin^2\theta\right] d\theta$

$= 16 \left[\cancel{2}\times\cancel{2} \cdot \frac{1}{\cancel{2}} \cdot \frac{\pi}{\cancel{2}}\right.$

$= 16\pi$ cubic units

Q15 $V = 2\iint_R z \; dy\, dx$

$z = \sqrt{x^2+y^2-1}$



Converting into polar.

$\frac{V}{2} = 4\int_{0}^{\pi/2}\int_{1}^{2} \left(\sqrt{r^2-1}\right) r\, dr\, d\theta$

$$= \frac{4}{2}\int_0^{\pi/2}\left[\frac{(r^2-1)^{3/2}}{3} \times 2\right]_1^2$$

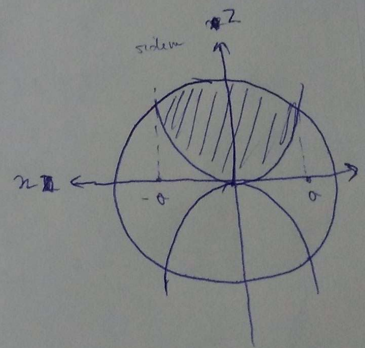
$$= 4 \int_0^{\pi/2}\left(\frac{3^{3/2}-0}{3}\right)d\theta \qquad \pi a^3$$

$$= 4 \quad \sqrt{3} \quad \frac{\pi}{2}$$

$$\frac{V}{2} = 2\sqrt{3}\,\pi$$

$V = 4\sqrt{3}\,\pi$ cubic units

[$\frac{V}{2}$ because one region above xy plane and other below].



Q 16

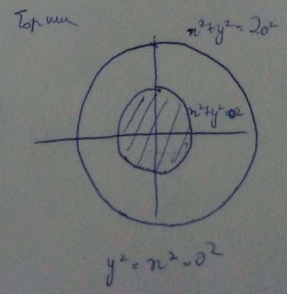
$$2\int_0^{a}\int_0^{\sqrt{a^2-x^2}}\int_{\frac{x^2+y^2}{a}}^{\sqrt{2a^2-x^2-y^2}} dz\, dy\, dx$$

$$= 2\int_0^{a}\int_0^{\sqrt{a^2-x^2}}\left(\sqrt{2a^2-x^2-y^2} - \frac{x^2+y^2}{a^2}\right)dy\, dx$$

not defined for $z < 0$

Put $x = r\cos\theta$
$y = r\sin\theta$

$$2\int_0^{\pi/2}\int_0^{a}\left(\sqrt{2a^2-r^2} - r^2/a^2\right) r\,dr\,d\theta$$



$$2\int_0^{\pi}\left[\frac{-1}{3}(2a^2-r^2)^{3/2} - \frac{r^4}{a}\right]_0^{a} d\theta$$

$$2\pi\left[\frac{2\sqrt{2}\,a^3 - a^3}{3} - \frac{a^3}{4}\right]$$

$$= 2\pi\left[\frac{2\sqrt{2}\,a^3}{3} - \frac{7a^3}{12}\right] = \left(\frac{4\sqrt{2}}{3} - \frac{7}{6}\right)\pi a^3$$

$$= \left(\frac{4\sqrt{2}}{3} - \frac{7}{6}\right)\pi a^3 \text{ cubic units.}$$

Q18.

$$\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$$

$$= \int_0^{\log 2}\int_0^{x} e^{x+y+x+\log y} - e^{x+y}\,dy\,dx$$

$$= \int_0^{\log 2}\int_0^{x}\left(y\,e^{2x+y} - e^{x+y}\right)dy\,dx$$

$$= \int_0^{\log 2} e^{2x}\left[e^{y}.y - e^{y}\right]_0^{x} - e^{x}\left[e^{y}\right]_0^{x}\,dx$$

$$= \int_0^{\log 2}\left[e^{2x}\left[e^{x}\cdot x - e^{x}\right] - \cancel{e^{2x}}\right] + \cancel{e^{2x}}[+1] + e^{x}\Big]\,dx$$

$$= \int_0^{\log 2}\left[e^{3x}(x-1) + e^{x}\right]dx$$

$$= \left[(x-1)\frac{e^{3x}}{3} - \frac{e^{3x}}{3\cdot 3} + e^{x}\right]_0^{\log 2}.$$

2

$\phi\, d\phi$

$\frac{\pi\ \ldots}{32}.$

Q20

$dx\,dy\,dz.$

$$= (\log 2 - 1)\frac{8}{3} - \left(\frac{-1}{3}\right) - \left(\frac{8-1}{9}\right) + (2-1)$$

$$= \frac{8}{3}\log 2 - \frac{7}{3} - \frac{7}{9} + 1$$

$$= \frac{8}{3}\log 2 - \frac{19}{9}$$

Q19

dz

$$= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz\,dy\,dx}{(x+y+z+1)^3}$$

$$= +\frac{1}{2} \int_0^1 \int_0^{1-x} \left[\frac{1}{(x+y+1)^2} - \frac{1}{2^2} \right] dy\,dx$$

$$= +\frac{1}{2} \int_0^1 \left[\frac{1}{(x+y+1)} + \frac{y}{4} \right]_{1-x}^{0} dx$$

$\phi\,d\phi.$

$$= -\frac{1}{2} \int_0^1 \left(\frac{1}{2} - \frac{1}{(1+x)} + \frac{1-x}{4} \right) dx$$

$$= -\frac{1}{2} \int \left[\frac{3}{4} - \frac{x}{4} - \frac{1}{(1+x)} \right]_0^1 dx$$

$$= -\frac{1}{2} \left[\frac{3}{4} - \frac{1}{8} - \ln(2) \right]$$

$$= +\frac{1}{2}\ln 2 - \frac{5}{16}.$$

Q22/5

Q20/5

$$g = \iiint_R \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}\right)^{1/2} dx\,dy\,dz$$

Put $\frac{x}{a} = X,\ \frac{y}{b} = Y,\ \frac{z}{c} = Z$

$\therefore\ dx = a\,dX$

$dy = b\,aY$

$dz = c\,dZ$

$$\therefore \frac{g}{abc} = \iiint_R (1 - X^2 - X^2 - Z^2)^{1/2}\, dX\,dY\,dZ$$

Converting into polar

$X = r\cos\theta\cos\phi$

$Y = r\cos\theta\sin\phi$

$Z = r\sin\theta$

$$\therefore \frac{g}{abc} = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 (1 - r^2)^{1/2}\, r^2\, dr\, d\theta\, \sin\phi\, d\phi.$$

Put $r = \sin\alpha$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{\pi/2} \cos^2\alpha\,\sin^2\alpha\,\sin\phi\; d\alpha\, d\theta\, d\phi.$$

[illegible]

$$= \frac{1}{2}\,\frac{\Gamma\frac{3}{2}\,\Gamma\frac{3}{2}}{\Gamma 3}\cdot\frac{\pi}{2}\cdot 1 = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\pi\cdot\frac{\pi}{2}$$

$$= \frac{\pi^2 abc}{32}.$$ cubic units

Q 21/5

$$V = \iint_R \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} dz\, dx\, dy$$

$$V = 2 \iint_R \sqrt{a^2-y^2}\, dx\, dy$$

Put $\frac{y}{a} = Y$, $\frac{x}{a} = X$.

$$\therefore V = 2a^3 \iint_R \sqrt{1-Y^2}\, dX\, dY$$

$$= 2a^3 \int_{-1}^{1} \int_{-\sqrt{1-Y^2}}^{+\sqrt{1-Y^2}} \sqrt{1-Y^2}\, dX\, dY$$

$$= 8a^3 \int_0^1 \int_0^{\sqrt{1-Y^2}} \sqrt{1-Y^2}\, dX\, dY$$

$$= 8a^3 \int_0^1 (1-Y^2)\, dY$$

$$= 8a^3 \left[Y - \frac{Y^3}{3} \right]_0^1$$

$$= \frac{16a^3}{3} \text{ cubic cm}$$

Q22

$2(x^2+y^2) = a^2$

$\boxed{r = a/\sqrt{2}}$

$$V = 2\iint \sqrt{a^2-(x^2+y^2)} - \sqrt{x^2+y^2}$$

$$V = 2a^3 \iint \sqrt{1-(X+Y)^2} - \sqrt{X^2+Y^2}\, dX\, dY$$

$$= 2a^3 \int_0^{2\pi}\int_0^{1/\sqrt{2}} \sqrt{1-r^2} - r)\, r\, dr\, d\theta$$

$$= 2a^3 \int_0^{2\pi}\int_0^{1/2} \left(-\frac{1}{2}\right)(-2r)(1-r^2)^{1/2} - r^2 \,.\, d\theta$$

$$= \int_0^{2\pi} 2a^3 \left[-\frac{1}{3}(1-r^2)^{3/2} - \frac{r^3}{3}\right]_0^{1/\sqrt{2}} d\theta$$

$$= 2a^3 \int_0^{2\pi} \left(\frac{-1}{3}\right)\left(\frac{1}{\sqrt{2}}-1\right)$$

$$= 2\pi a^3 \left(\frac{1}{3}\right)\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\frac{\sqrt{2}}{\cancel{}}$$

$$= \frac{2\pi a^3}{3}(2-\sqrt{2}) \text{ unit}^3$$

Q23

Put $x = e^{-t}$

$$I = \int_{\infty}^{0} (-1)^n e^{-mt}\, t^n\, e^{-t}\, dt(-1)$$

$$I = (-1)^n \int_{\infty}^{0} t^n e^{-(m+1)t}\, dt$$

$$= \frac{(-1)^n}{(m+1)^{n+1}} \int_{\infty}^{0} [m+1\,t]^n\, -e^{(m+1)t}\, (m+1)\, dt$$

$$= \frac{-1}{(m+1)^{n+1}} \int_{0}^{a} k^n e^{-k}\, dk\, (-1).$$

$$= \frac{(-1)^n}{(m+1)^{n+1}} \int_{0}^{\infty} k^n e^{-k}\, dk.$$

$$= \frac{(-1)^n}{(m+1)^{n+1}}\, n!.$$ HP

Q24

$$X = \frac{1}{1+y}.$$

$$B(m+n) = \int_{0}^{1} x^m (1-x)^n\, dx.$$

$\therefore$ Put $t = \left(\frac{1-x}{x}\right).$

$$B(m,n) = \int_{0}^{1} x^{m+n-2} \left(\frac{1-x}{x^{n-1}}\right)^{n-1} dx$$

Q23

Put $x = e^{-t}$

$I = \int^{0} \ldots^{n} \ldots^{-mt} \ldots^{n} \ldots t \ldots$

$$= \int_{\infty}^{0} \frac{t^{n-1}}{(1+t)^{m+n-2}} \frac{-1}{(1+t)^{2}}\, dt$$

$$= \int_{0}^{\infty} \frac{t^{n-1}}{(1+t)^{m+n}}\, dt$$

Put $t = y$

HP