UNIT ~~IV~~ III

Three Phase Circuits

INTRODUCTION

- In polyphase circuits
  These days balanced three-phase voltage system is widely used
- Electrical power is generated and transmitted in the form 3-phase a.c. system
- Single-phase ac voltage supply is used in homes, officies and industries.

Advantages of polyphase system over single-phase System

Transmission + Distribution

(i) The conductor material used in polyphase system is much less as compared to that used in 1-phase system.

(ii) There is only one/no return conductor used.

(iii) The return conductor if used is of comparatively smaller size, because phasor sum of current ~~(IV)~~ in all phases is zero.

(iv) For a given frame size, a polyphase ~~machine~~ machine gives a higher output than a 1-phase machine.

(v) - The power in a single line phase circuit is pulsating at twice the frequency.

- Sum of powers of all the phases in a polyphase system is constant

- Therefore, polyphase motors develop uniform torque whereas single-phase motors to develop pulsating torque.

(VI) – 1-phase motors are not self starting (Auxiliary arrangements are to be provided e.g. split phase etc)

- Polyphase motors are self starting.

(VIII) p.f. of polyphase motors is ↑

(IX) η " " " "

(X) Polyphase system is more reliable

(XI) Parallel operation of polyphase alternators is simpler as compared to single phase alternators

because

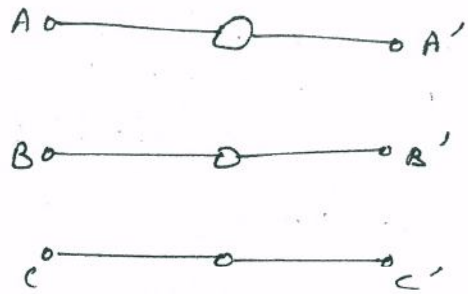
pulsating reaction in single phase alternator.

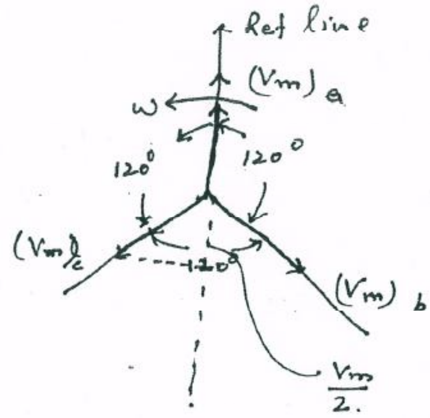
Commonly used Polyphase Systems

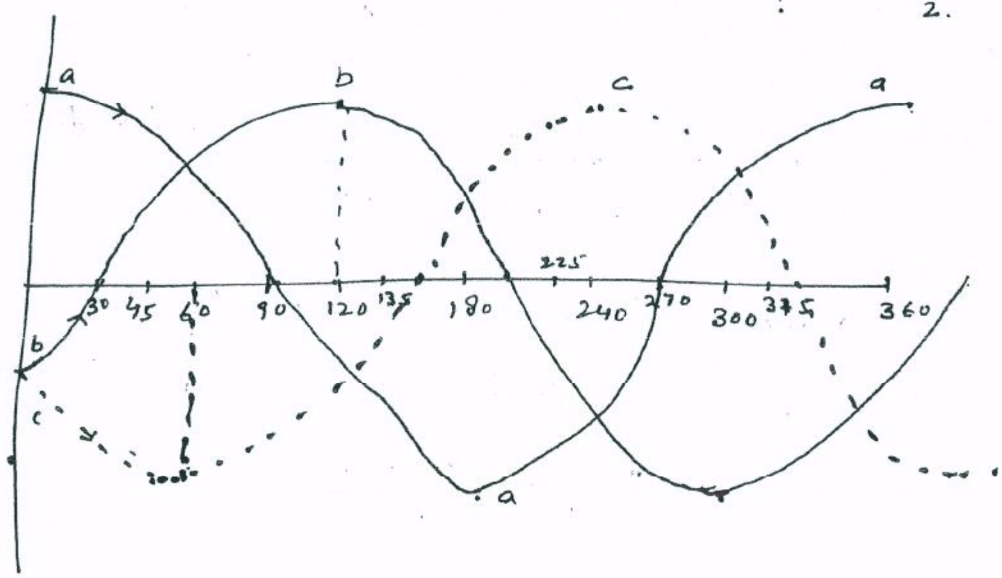
1. Various systems are 2, 3, and 6-phase system
2. 3-phase system is invariably adopted
3. 2-phase & 6-phase are obtained from 3-phase supply whenever required
4. Basic ckt network analysis techniques are directly applicable to 3-phase systems analysis.

Generation of 3-Phase EMFs

Three-Phase Voltage System







Phase order = Phase sequence = phase rotation = a b c

Anticlock wise rotation = +ve direction of rotation.

- Therefore, polyphase motors develop uniform torque whereas single-phase motors have pulsating torque.

(VI) - 1-phase motors not self starting. (Auxil. arrangements are to be provided e.g. split phase)
- Polyphase motors are self starting.

(VII) AC generators have no commutator
- large units are possible
- high speed is possible
- high voltage is possible

So,
- construction cost/kW is low ~~per kW~~
- operating cost/kW is low

*(VIII) Transformation of voltage is possible

(IX) The switchgear for AC are simpler than those for DC.

PTO

Commonly Used Poly phase Circuits

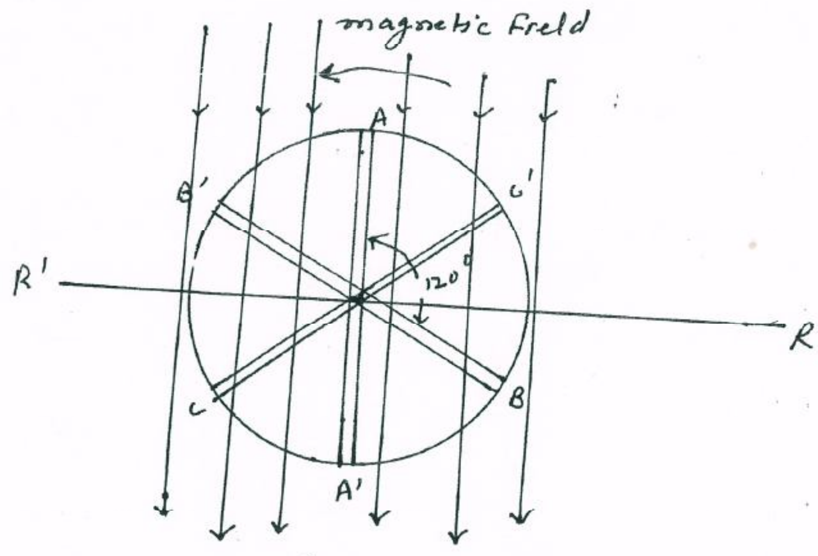
These are

2 - phase
3 - phase
6 - phase

But 3-phase is most common.
Generation is for 3-phase
others are obtained from 3-phase System.

Generation of 3-Phase EMF

Repeat from Unit III ~~pt~~ pages 6, 7 & 8. and modify the fig. as follows.



- We have 3 coils, AA', BB', CC' space displaced by 120° from one another.
- These coils wound on a rotor are placed in a magnetic field (uniform)
- These coils are rotated at a uniform angular speed ω in anticlock wise direction
- BB' follows AA' and CC' follows BB'.

- At the instant shown in fig. the coil AA' is along the direction of ~~sb~~ magnetic field, the induced emf in ~~k~~ it has +ve maximum value
- After 120^0 rotation BB' occupies this position and has +ve maximum value.
- After another 120^0 rotation cc' occupies this position and has +ve maximum value.
- If three coils are identical, the voltages induced will have same
  - shape
  - frequency
  - same maximum

  but, different

  phases.

We can write. with RR' taken as reference

$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin\left(\omega t - \frac{2\pi}{3}\right) = E_m \sin(\omega t - 120^0)$$

$$e_c = E_m \sin\left(\omega t - \frac{4\pi}{3}\right) = E_m \sin(\omega t - 240^0)$$

$$= E_m \sin(\omega t + 120^0)$$

There is another way

of generating 3-phase voltages. i.e.

- keep the ~~ma~~ coils stationary and rotate the magnetic field (in clockwise direction)

why? explain

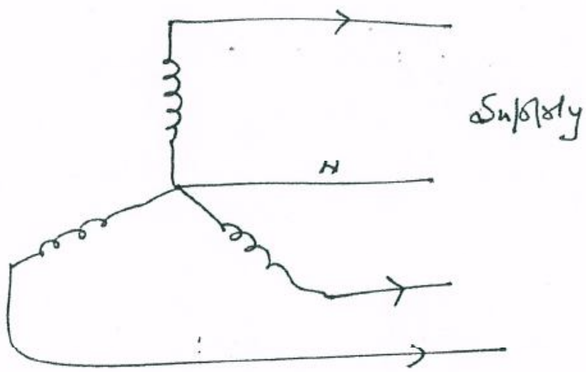
- The second method is preferable because
  - It is simpler to make external connection to the stationary coils than to the rotating coils.

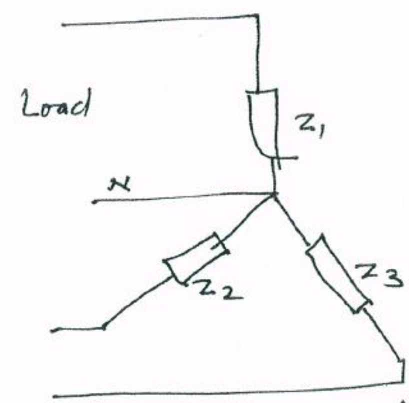
Balanced Circuits

λ / Δ Supply: Balanced (only to be considered); Unbalanced

λ / Δ Load: Balanced (only to be considered); Unbalanced (considered only for concepts)

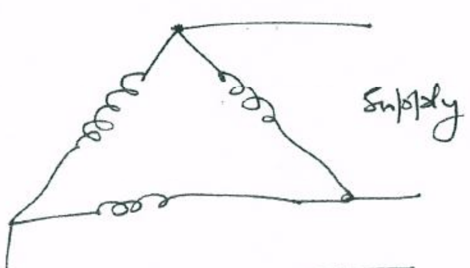
λ- Connections

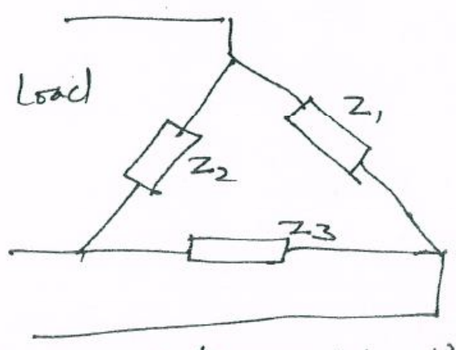




$Z_1 = Z_2 = Z_3$ for Balanced load

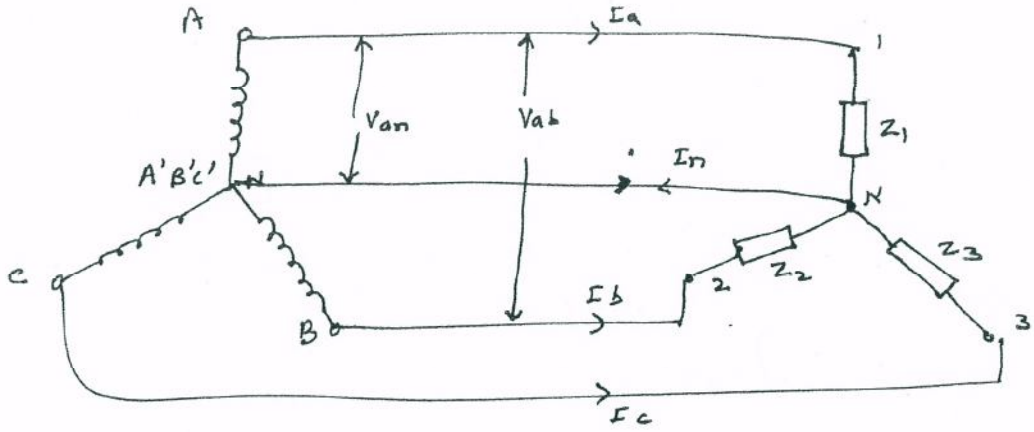
Δ- Connection



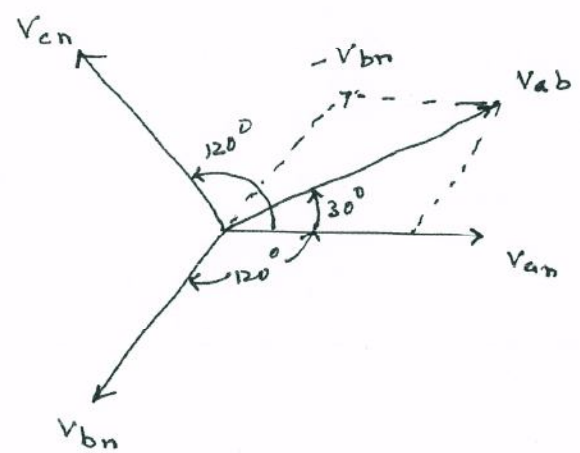


$Z_1 = Z_2 = Z_3$ (Banced load)

Current and Voltage Relationship for Y connection



$$\bar{I}_n = \bar{I}_a + \bar{I}_b + \bar{I}_c = 0 \text{ for a balanced system}$$



Line voltage $\vec{V}_{ab} = \bar{V}_{an} + \bar{V}_{nb}$

$$= \bar{V}_{an} - \bar{V}_{bn}$$

$$|\bar{V}_{an}| = |\bar{V}_{bn}| = |\bar{V}_{cn}| = V_{ph}.$$

$$V_{ab} = V_{ph}^2 + V_p^2 + 2 V_p^2 \cos 60$$

$$= V_p^2 + V_p^2 + 2 \cdot V_p^2 \cdot \frac{1}{2}$$

$$= 3 V_p^2$$

$\Rightarrow$ $\cancel{V_{ab} = \sqrt{3}\ V_{ab}}$ $\boxed{V_{ab} = \sqrt{3}\ V_p.}$

$\therefore$ ~~Phase voltage = $\sqrt{3}$~~

Alternatively,

$$\bar{V}_{an} = V_{ph} \angle 0$$

$$\bar{V}_{bn} = V_{ph} \angle -120^0$$

$$V_{cn} = V_p \angle -240^0 = V_p \angle 120^0$$

$$\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn}$$

$$\bar{V}_{ab} = V_p - V_p \angle -120 = V_p - V_p(\cos 120 - j \sin 120)$$

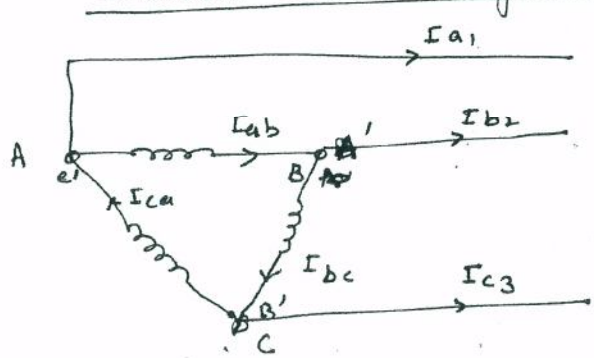
$$= V_p - V_p\left(-\frac{1}{2} - j\,\sqrt{3}/_2\right) = V_p\left[\frac{3}{2} + j\frac{\sqrt{3}}{2}\right]$$

$$\boxed{V_{ab} = V_p\sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}\,V_p}$$

$$\boxed{V_L = \sqrt{3}\,V_p}$$

Also $\boxed{\bar{I}_L = \bar{I}_p}$

Current and Voltage Relationship for Δ - Connection



$\boxed{V_L = V_{ph.}}$ obviously

$\bar{I}_{a1}$, $\bar{I}_{b2}$, $\bar{I}_{c3}$ are line current

$\bar{I}_{ab}$, $\bar{I}_{bc}$, $\bar{I}_{ca}$ are phase currents

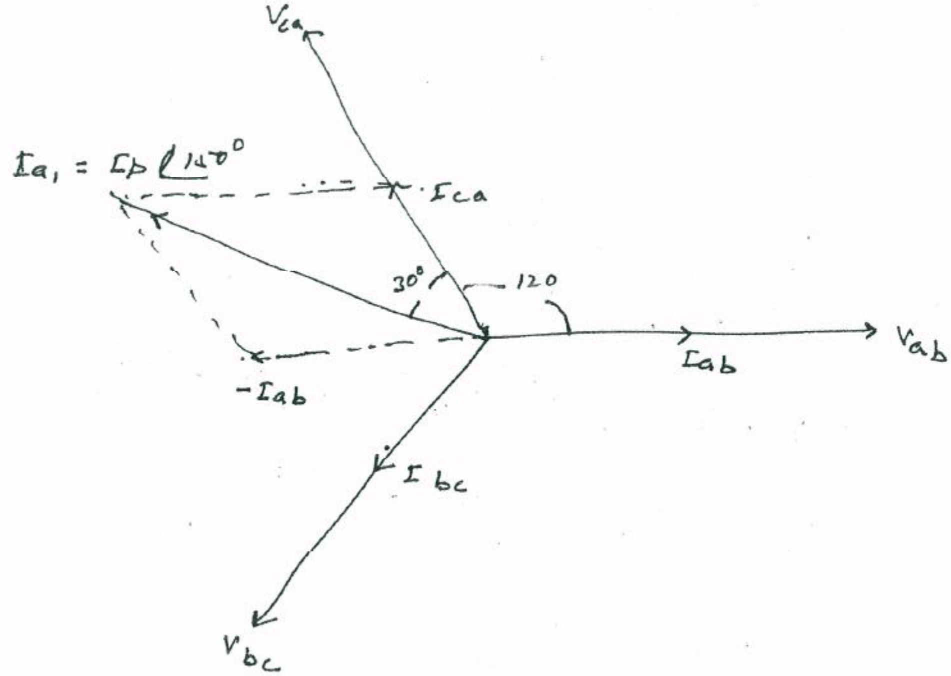
At node A

$$\bar{I}_a = \bar{I}_{ca} - \bar{I}_{ab}$$

Let

$$\bar{I}_{ab} = I_p \angle 0^\circ \qquad \bar{I}_{bc} = I_p \angle -120^\circ$$

$$\bar{I}_{ca} = I_p \angle 120^\circ$$



$$\bar{I}_{a_1} = \bar{I}_{ca} - \bar{I}_{ab} = I_p \angle 120^\circ - I_p \angle 0$$

$$= I_p\left(\cos 120^\circ + j \sin 120^\circ\right) - I_p$$

$$= I_p\left[-\tfrac{1}{2} + j\,\tfrac{\sqrt{3}}{2}\right] - I_p$$

$$= I_p\left[-\tfrac{3}{2} + j\,\tfrac{\sqrt{3}}{2}\right]$$

$$= \sqrt{3}\, I_p \angle 150^\circ$$

and similarly,

$$\bar{I}_{b_2} = \bar{I}_{ab} - \bar{I}_{bc} = \sqrt{3}\, I_p \angle 30^\circ$$

$$I_{c_3} = \bar{I}_{bc} - \bar{I}_{ca} = \sqrt{3}\, I_p \angle -90^\circ$$

$$\therefore \boxed{I_L = \sqrt{3}\, I_p}$$

Important Points

1. Unless otherwise mentioned, the voltages and currents specified are line values.

Example 1. A 400-V, 3-phase supply is connected across a balanced network of three impedances each consisting of a 32-Ω resistor and 24-Ω inductive reactance. Determine the current drawn from the supply mains, if impedances are connected in

(a) Y (b) Δ.

Soln:

$$\bar{Z} = R + jX = 32 + j24$$

$$|\bar{Z}| = \sqrt{32^2 + 24^2} = 40\,\Omega$$

(a) For Y-connection

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} =$$

$$I_p = \frac{V_p}{|Z|} = \frac{400/\sqrt{3}}{40} = \frac{10}{\sqrt{3}}\ A$$

$$I_L = I_p = \frac{10}{\sqrt{3}} = \underline{5.78\,A}$$ Ans.

For Δ-Connection

$$V_p = V_L = 400\ V$$

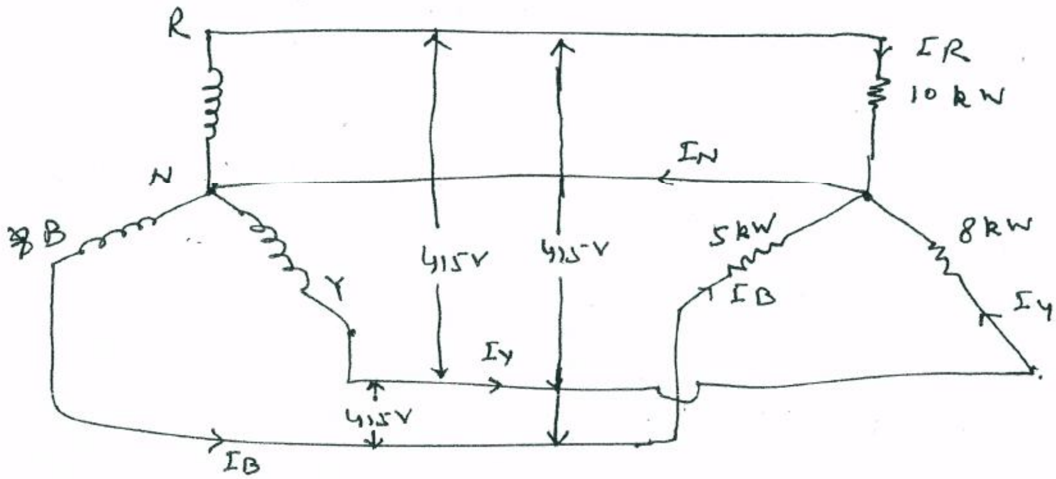
$$I_p = \frac{V_{ph}}{|Z|} = \frac{400}{40} = 10\ A$$

$$I_L = \sqrt{3}\ I_p = \sqrt{3} \times 10 = \underline{17.32\ A}$$ Ans.

Example 2: In a 3-phase, 4-wire system, the line voltage is 415 V and non-inductive loads of ~~80~~ 10 kW, 8 kW and 5 kW are connected between the three line conductor and the neutral as is shown in the figure. Calculate

(a) the current in each line

(b) the current in neutral conductor.



Soln:

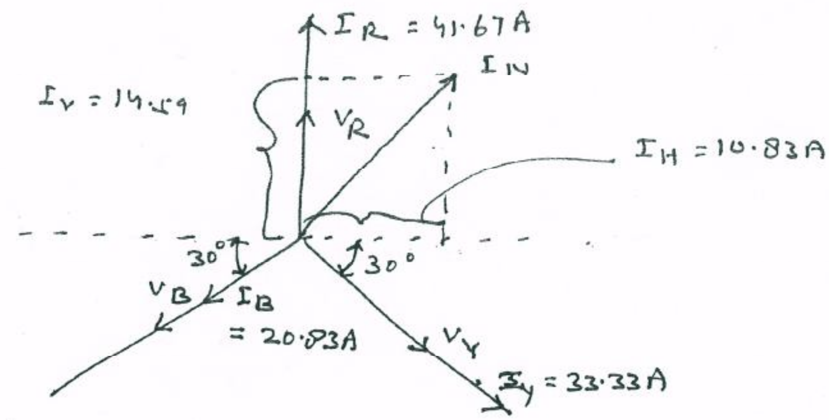
$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240\text{ V}$$

$$I_R = \frac{10 \times 1000}{240} = \underline{41.67\text{ A}}$$

$$I_Y = \frac{8 \times 1000}{240} = \underline{33.33\text{ A}}$$

$$I_B = \frac{5 \times 1000}{240} = \underline{20.83\text{ A}}$$

The phasor diagram is given below:



$\bar{I}_N = \bar{I}_R + \bar{I}_y + \bar{I}_B$

Horizontal Component of $\bar{I}_N$ (I_H) $= I_R \cdot \cos 90 + I_y \cos 30° - I_y \cos 30°$

$= 0 + 0.866\,(33.33 - 20.83) = 10.83A$

Vertical component of $\bar{I}_N$ (H_v) $= I_R \cdot \cancel{\sin 90°} \cos 0° - I_y \cos 60° - I_B \cos 60°$

$= 41.67 - 0.5(33.33 + 20.83) = 14.59A$

$\therefore \quad |\bar{I}_N| = \sqrt{(10.83)^2 + (14.59)^2} = \underline{18.2\ A}$

$\tan\theta = \dfrac{14.59}{10.83} = 1.347$

$\theta = \tan^{-1} 1.347 = 53.41°$

$\therefore \bar{I}_N = \underline{18.2A \angle 53.41°}$ Ans.

Alternatively,

$\cancel{\bar{I}_N = \bar{I}_R + \bar{I}_y + \bar{I}_B}$

$\bar{I}_R = \dfrac{\cancel{VA}\ 10 \times 1000}{240 \angle 90} = 41.67 \angle 90 \ A$

$\bar{I}_y = \dfrac{8000 \angle -30}{240 \angle \cancel{30}} = \quad 33.33 \angle -30° \ A$

$$\bar{I}_N = \bar{I}_R + \bar{I}_Y + \bar{I}_B$$

$$= 41.67\angle 90^\circ + 33.33\angle -30^\circ + 20.83\angle -150^\circ$$

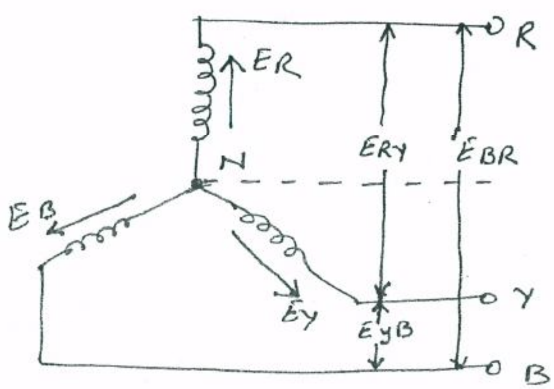
$$= (0 + j\,41.67) + (28.86 - j\,16.66) + (-18.04 - j\,10.42)$$

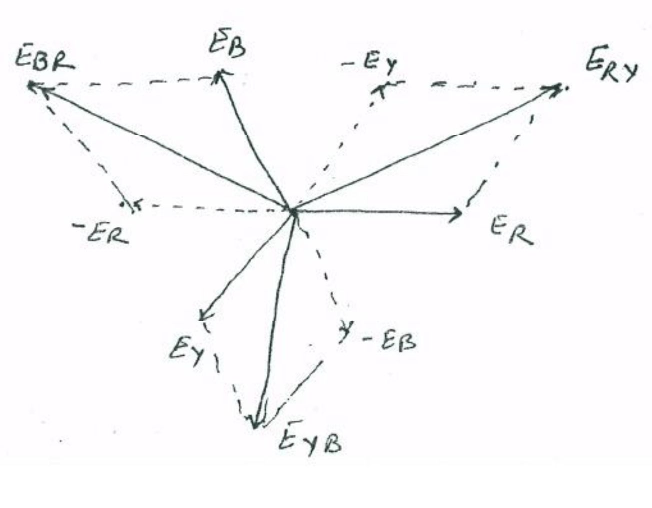
$$= 10.82 + j\,14.59 = 18.16\angle 53.43^\circ \quad \underline{\text{Ans.}}$$

Example 3 / JBG p 114 : If the phase voltages of a 3-phase star connected system is 200V, what will be the line voltage
(a) when the phases are correctly connected
(b) when ~~connected to~~ connections to one of the phases are reversed.

Soln: (a) Phases are correctly connected:
- i.e. similar ends of 3-coils are connected together as shown below
- All the line voltages would be equal in magnitude.

$$E_L = \sqrt{3}\,E_p = \sqrt{3} \times 200 = \underline{346.41\text{ V}} \quad \text{Ans}$$

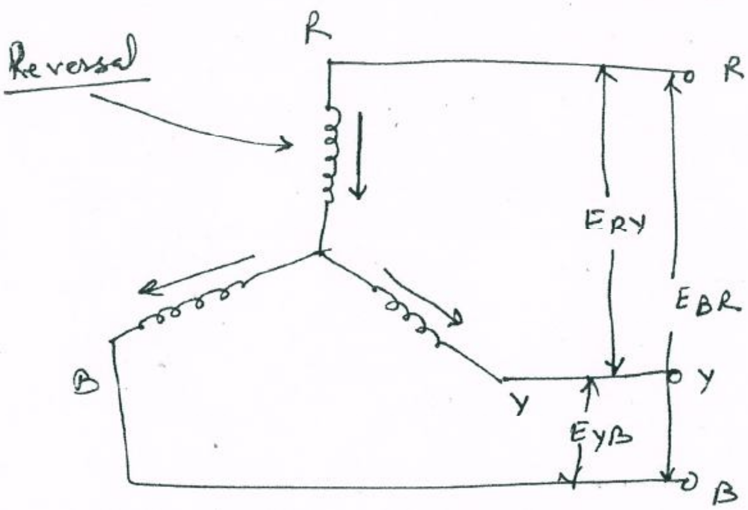


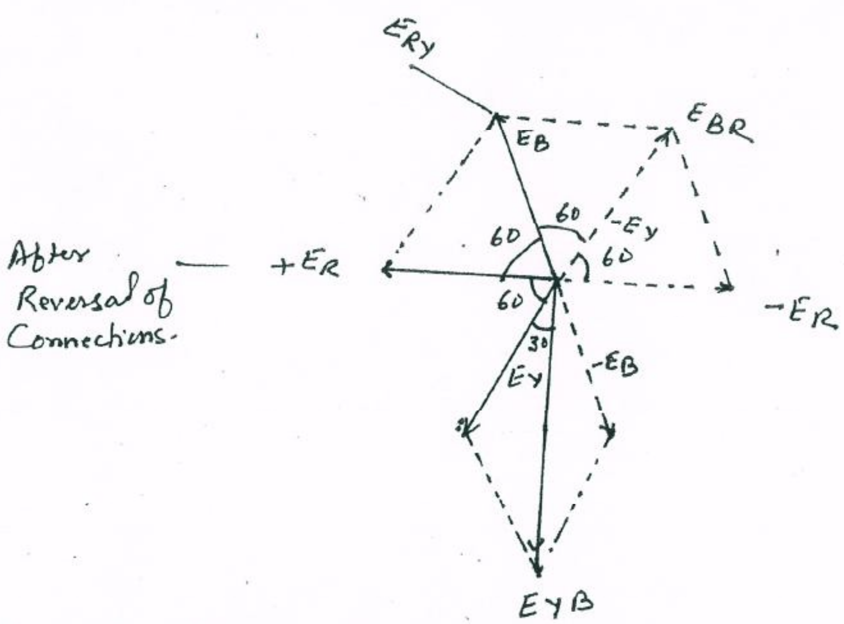


(b) When connections to one of phases are reversed.

- Here we consider the connections of R phase reversed.

- The corresponding phasor diagram is given bel[ow]





From the phasor diagram.

$$E_{RY} = E_R - E_Y = 2E_p \cos 60^\circ = E_p = \underline{200V} \quad \text{Ans}$$

$$E_{YB} = E_Y - E_B = 2E_p \cos 30^\circ = 2 \cdot 200 \cdot \frac{\sqrt{3}}{2} = \underline{346.41V} \quad \text{Ans}$$

$$E_{BR} = E_B - E_R = 2E_p \cos 60^\circ = \underline{200V} \quad \text{Ans}$$

Example 4 A delta connected load is arranged as in fig. below. Calculate

(a) phase currents

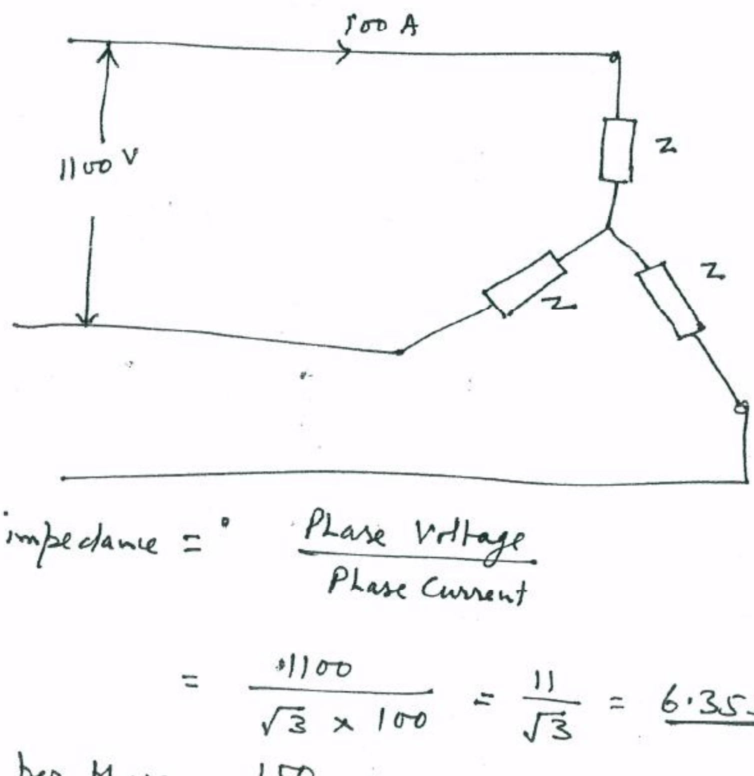
(b) line currents

Supply voltage is 415 V, 50 Hz.

2014/ Example 4 : A balanced 3-phase star connected load
PS 7.3/217

of 150 kW takes a leading current of 100 A with a line voltage of 1100 V, 50Hz. Find the circuit constants of the load.

Soln:



$$\text{Phase impedance} = \frac{\text{Phase Voltage}}{\text{Phase Current}}$$

$$= \frac{1100}{\sqrt{3} \times 100} = \frac{11}{\sqrt{3}} = \underline{6.35\,\Omega}$$

$$\text{Power per phase} = \frac{150}{3} = 50\text{ kW}$$

$$= I^2R \qquad \text{where } I = 100\text{ A}.$$

$$I^2R = 50 \times 10^3$$

$$\Rightarrow R = \frac{50 \times 10^3}{I^2} = \frac{50 \times 10^3}{10^4} = \underline{5\,\Omega} \quad \text{Ans.}$$

$$X = \sqrt{Z^2 - R^2} = \sqrt{6.35^2 - 5^2} = \underline{3.9\,\Omega}$$

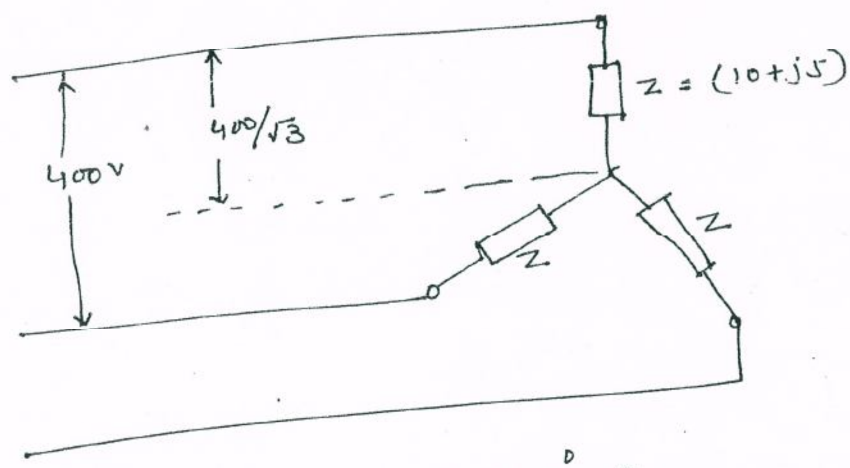
Since the current is leading X must be capacitive

$$\frac{1}{\omega C} = 3.9$$

$$C = \frac{1}{\omega \times 3.9} = \frac{1}{2\pi \times 50 \times 3.9} = \underline{810\,\mu F} \quad \text{Ans}$$

Example 5: A balanced star-connected load of $(10+j5)\,\Omega$ per phase is connected to a balanced 3-phase 400-V supply. Find
(i) line current (ii) power factor.

Soln:



$$\bar{Z}_p = (10+j5)\,\Omega = 11.18 \angle 26.57^\circ\,\Omega$$

$$\bar{V}_p = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \angle 0^\circ \text{ V}$$

(i) Phase current $\bar{I}_{ph} = \frac{\bar{V}_{ph}}{\bar{Z}_{ph}} = \frac{231 \angle 0^\circ}{11.18 \angle 26.57}$

$$= \underline{20.66 \angle -26.57^\circ \text{ A}}$$

(ii) p.f. $\cos\phi = \cos(-26.57^\circ) = \underline{0.894}$ (lagging)

Power in Three-Phase Circuits

– Let us consider one phase only

Average power in one phase =

$$P_1 = V_{ph} I_{ph} \cos\phi$$

$\cos\phi$ is the p.f. of the circuit.

For a balanced system, total power in all the three phases:

$$P = 3 V_{ph} I_{ph} \cos\phi$$

Converting phase quantities into line quantities

for $\curlywedge$ - connection

$$P = 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi = \underline{\sqrt{3} V_L I_L \cos\phi}$$

For Δ- connection

$$P = 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi = \underline{\sqrt{3} V_L I_L \cos\phi}$$

$\therefore$ In general the power in 3-phase system (balanced) is given by

$$P = \sqrt{3} V_L I_L \cos\phi = \text{Real Power}$$

Note: ϕ is angle of load impedance and not angle between V_L and I_L in general.

Apparent Power = $\sqrt{3} V_L I_L$

Reactive Power = $\sqrt{3} V_L I_L$ ~~cos~~ $\sin\phi$.

Total Instantaneous power in a balanced three phase system.

Let the phase voltages and phase currents in a balanced 3-phase system (with unity p.f. load) are:

$$v_R = V_m \sin \omega t \qquad i_R = I_m \sin \omega t$$

$$v_y = V_m \sin(\omega t - 120^\circ) \qquad i_y = I_m \sin(\omega t - 120^\circ)$$

$$v_B = V_m \sin(\omega t + 120) \qquad i_B = I_m \sin(\omega t + 120^\circ)$$

Total Instantaneous power of 3-phase system

$$p = V_m I_m \sin^2 \omega t + V_m I_m \sin^2(\omega t - 120) + V_m I_m \sin^2(\omega t + 120)$$

$$= V_m I_m \left[\sin^2 \omega t + \sin^2(\omega t - 120) + \sin^2(\omega t + 120) \right]$$

$$= V_m I_m \left[\sin^2 \omega t + \{ \sin(\omega t - 120) + \sin(\omega t + 120) \}^2 - 2 \sin(\omega t - 120) \sin(\omega t + 120) \right]$$

$$= V_m I_m \left[\sin^2 \omega t + \{ 2 \sin \omega t \cos(\omega t - 120) \}^2 - 2 \cdot \{ (\sin \omega t \cos 120 - \cos \omega t \sin 120) \times (\sin \omega t \cos 120 + \cos \omega t \sin 120) \} \right]$$

$$= V_m I_m \left[\sin^2 \omega t + 2 \sin^2 \omega t \{ \cos \omega t \cos 120 + \sin \omega t \sin 120 \}^2 - 2 \{ 2 \sin \omega t \cos 120 \} \right]$$

$$= V_m I_m \left[\sin^2 \omega t + \{ \sin \omega t \cos 120 - \cos \omega t \sin 120 \}^2 + \{ \sin \omega t \cos 120 + \cos \omega t \sin 120 \}^2 \right]$$

$$= \cancel{V_m I_m \left[\sin^2\omega t + \left\{ -\frac{\sin\omega t}{2} - \frac{\sqrt{3}}{2}\cos\omega t \right\}^2 + \left\{ -\frac{\sin\omega t}{2} \right. \right.}$$

$$(A-B)^2 + (A+B)^2 = 2A^2 + 2B^2$$

$$\therefore \; p = V_m I_m \left[\sin^2\omega t + 2\left\{ \sin^2\omega t \cos^2 120 + \cos^2\omega t \sin^2 120 \right\} \right]$$

$$= V_m I_m \left[\sin^2\omega t + 2 \cdot \left\{ \frac{\sin^2\omega t}{4} + \cos^2\omega t \left(\frac{\sqrt{3}}{2}\right)^2 \right\} \right]$$

$$= V_m . I_m \left[\sin^2\omega t + 2\left[\frac{\sin^2\omega t}{4} + \frac{3}{4}\cos^2\omega t \right] \right]$$

$$= V_m I_m \left[\sin^2\omega t + \frac{1}{2}\sin^2\omega t + \frac{3}{2}\cos^2\omega t \right]$$

$$= V_m I_m \left[\frac{3}{2}\sin^2\omega t + \frac{3}{2}\cos^2\omega t \right]$$

$$= V_m I_m . \frac{3}{2}\left[\sin^2\omega t + \cos^2\omega t \right]$$

$$= \frac{3}{2} V_m I_m = \frac{3}{2} . \sqrt{2}V . \sqrt{2}I = \underline{\underline{3VI}}$$

= 3 × (Average power per phase)

= Constant

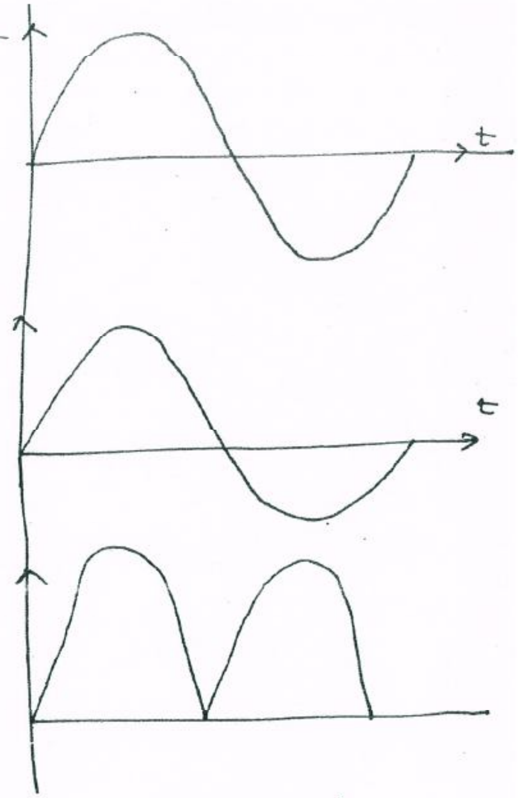
Exercise: (i) Consider p.f. of load as $\cos\phi$ and work out p

(ii) consider angle between V and I as θ and work out p.

For Single-phase circuit

p is pulsating at $2f$

- In 1-phase motors the torque is also pulsating with time
- In 3-phase motors the torque is constant with respect to time
- That is why for large size motors, 3-phase is used.



Example 1 : A 400-V, 3-phase supply is connected across a balanced network of three impedances each of a consisting of a $32\,\Omega$ resistance and $24\,\Omega$ inductive reactance. Determine the power consumed when these are impedances are (a) Y-connected (b) Δ-connected.

Soln.: p.f. of load impedance $(32+j24)$ is $\cancel{\tan^{-1}\frac{24}{32}}$

$$\cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right] = 0.8$$

$\cancel{P = \sqrt{3}\, V_L I_L \cos\phi}$ $\qquad \bar{Z} = 32 + j24 =$

$$|Z| = \sqrt{32^2 + 24^2} = 40$$

$\cancel{= \sqrt{3}\,.\,400}$

(a) Y-connection: $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} =$

$$I_{ph} = \frac{V_{ph}}{|Z|} = \frac{400/\sqrt{3}}{40} = \frac{10}{\sqrt{3}}\,A$$

$$P = 3\, V_{ph}\, I_{ph} \cos\phi$$

$$= 3 \cdot \frac{400}{\sqrt{3}} \cdot 5.78 \times 0.8 = \underline{3.2\ kW}$$

(b) <u>Δ – Connection</u>

$$V_L = V_{ph} = 400\ V$$

$$I_{ph} = \frac{V_{ph}}{|Z|} = \frac{400}{40} = 10\ A$$

$$\text{Power consumed} = 3\, V_{ph}\, I_{ph} \cos\phi$$

$$= 3 \cdot 400 \cdot 10 \times 0.8 = \underline{9.6\ kW}$$

<u>Example 2.</u> Three equal star-connected inductors take 9 kW at a power factor of 0.8 when connected to ~~a~~ 960 V, 3-ϕ, 50 Hz, 3 wire supply. Find per phase load resistance and inductance.

<u>Soln:</u>

$$\text{Phase voltage } V_p = \frac{V_L}{\sqrt{3}} = \frac{960}{\sqrt{3}} = 554.3\ V$$

Total power drawn = 9 kW = 9000 W

Power factor $\cos\phi = 0.8$ (lag)

$$P = 3\, V_p\, I_p \cos\phi$$

$$9000 = 3 \times 554.3 \times I_p \times 0.8$$

$$I_p = \frac{9000}{3 \times 554.3 \times 0.8} = 6.766\ A = I_L\ (\lambda\text{-conn})$$

$$\text{Load impedance/phase} = Z_L = \frac{V_p}{I_p} = \frac{554.3}{6.766} = 81.92\ \Omega$$

Load Resistance $R_L = Z\cos\phi = 81.92 \times 0.8 = \underline{65.54\ \Omega}$ Ans

Load reactance $X_L = Z\sin\phi = 81.92 \times 0.6 = 49.15\ \Omega$

Example 3 : A 3-phase, 400V supply is connected to a 3-phase star connected balanced load. The line current is 20 A and the power consumed by the load is 12 kW. Calculate the impedance of the load, phase current and power factor.

Soln:

Power consumed = 12 kW = 12000 W

Power factor, $\cos\phi = \frac{P}{\sqrt{3}\, V_L I_L} = \frac{12000}{\sqrt{3} \cdot 400 \cdot 20} = \underline{0.866}$ Ans

Phase current, $I_p = I_L = \underline{20\ A}$ Ans.

Phase voltage, $V_p = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\ V$

Load impedance, $Z_p = \frac{V_p}{I_p} = \frac{231}{20} = \underline{11.55\ \Omega}$ Ans.

Example 4: The load connected to a 3-phase supply comprises of three similar coils connected in star. The line current are 25 A and kVA and kW inputs are 20 and 11 respectively. Find the line and phase voltages and kVAR input.

Soln:

Line voltage $V_L = \frac{\text{input kVA} \times 1000}{\sqrt{3}\, I_L} = \frac{20 \times 1000}{\sqrt{3} \times 25} = \underline{461.88V}$ Ans

Phase voltage = $V_p = \frac{V_L}{\sqrt{3}} = \frac{461.88}{\sqrt{3}} = \underline{266.67V}$ Ans.

kVAR input, $Q = \sqrt{S^2 - P^2} = \sqrt{20^2 - 11^2} = \underline{16.70}$ Ans.

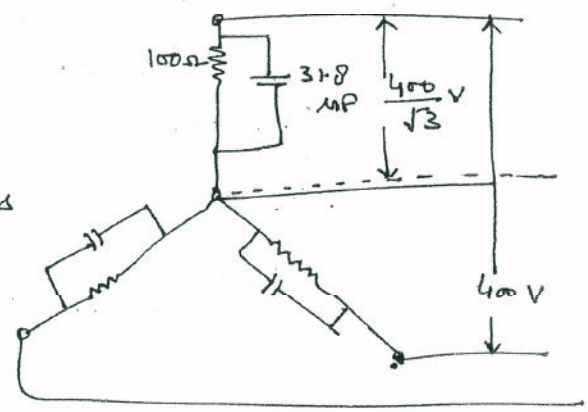
Example 5 : Each phase of a star-connected load consists of a non-reactive resistance of 100Ω in parallel with capacitance of 31.8 μF. Calculate the line current, the power absorbed, the total kVA and the power factor when it is connected to a 400V, 3-phase, 50 Hz supply.

Soln: Phase voltage, $V_p = \frac{V_L}{\sqrt{3}}$

$= \frac{400}{\sqrt{3}} = 231\,V$

Let phase voltage be taken as reference phasor, then

$\bar{V}_p = 231 \angle 0^\circ \, V$



Admittance per phase $Y_p = \frac{1}{R} + j\omega C$ [Why Y_p & not Z_p? $Z = R + jX$; what is conductance]

$= \frac{1}{100} + j \cdot 2\pi \cdot 50 \cdot 31.8 \times 10^{-6}$

$= 0.01 + j\,0.01 = 0.01414 \angle 45^\circ \, S$

line Current $I_L = I_p = V_p \times Y_p$

$= 231 \angle 0^\circ \times 0.01414 \angle 45^\circ = \underline{3.266 \angle 45^\circ \, A}$ Ans.

Power factor $\cos\phi = \cos 45^\circ = \underline{0.707}$ (leading) Ans.

Power absorbed: $P = \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3} \cdot 400 \cdot 3.266 \times 0.707 = \underline{1600\,W}$ Ans.

Total $kVA = \frac{\sqrt{3} V_L I_L}{1000} = \frac{\sqrt{3} \cdot 400 \cdot 3.266}{1000} = \underline{2.263}$ Ans.

Example 6: Calculate the active and reactive currents components in each phase of a star-connected 10,000 V, 3-phase alternator supplying 5000 kW at a p.f. of 0.8. If the total currents remains the same when the load power factor is raised to 0.9, find the new output.

Soln: Power supplied = 5000 kW

$$\text{Line Current} = \frac{P \text{ in kW} \times 1000}{\sqrt{3}\ V_L \cdot \cos\phi} = \frac{5000 \times 1000}{\sqrt{3} \cdot 10\,000 \cdot 0.8}$$

$$= 360.8A = \text{Phase Current} = I_p.$$

Active component of current in each phase $= I_p \cos\phi$

$$= 360.8 \cos\phi = 360.8 \times 0.8 = \underline{288.7A} \quad \text{Ans}$$

Reactive component of current in each phase $= I_p \sin\phi$

$$= 360.8 \sin\phi = 360.8 \times 0.6 = \underline{216.5A} \quad \text{Ans.}$$

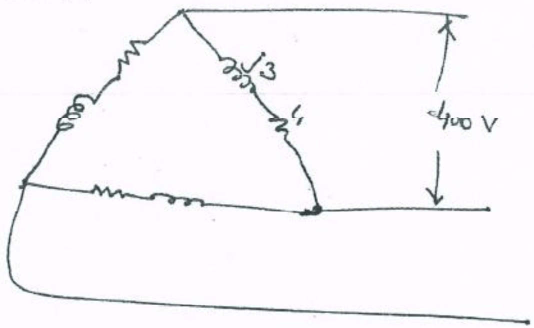
New output when power factor is raised to 0.9

$$P' = \frac{\sqrt{3}\, V_L I_L \cos\phi}{1000} = \frac{\sqrt{3} \cdot 10\,000 \cdot 360.8 \cdot 0.9}{1000}$$

$$= \underline{5625 \text{ kW}} \quad \text{Ans.}$$

Example 7: Three coils each of resistance 4 Ω and inductive reactance 3 Ω are connected in delta across 400V, 50 Hz supply. Find the current in each coil, line current, active, reactive and apparent power.

Soln.



Impedance in each phase $\bar{Z}_p = 4 + j3 =$

$$|\bar{Z}_p| = \sqrt{4^2 + 3^2} = 5\,\Omega$$

Phase voltage $V_p = V_L = 400\,V$

Current in each coil $= I_p = \frac{V_p}{Z_p} = \frac{400}{5} = \underline{80A}$ Ans.

Line Current, $I_L = \sqrt{3}\, I_p = \sqrt{3} \cdot 80 = \underline{138.6A}$ Ans

Active Power, $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \cdot 400 \cdot 138.6 \cdot (0.8)$

$= 76800W = \underline{76.8\,kW}$ Ans.

Reactive power $= \sqrt{3} V_L I_L \sin\phi = \sqrt{3} \cdot 400 \cdot 138.6 \cdot (0.6)$

$= \cancel{57.6}$ 57600 VAR $= \underline{57.6\ kVAR}$ Ans

Apparant Power $S = \sqrt{3} V_L I_L = \sqrt{3} \cdot 400 \cdot 138.6 = 96000$ VA

$= \underline{96\ kVA}$ Ans

Power factor: $\frac{R_p}{Z_p} = \frac{4}{5} = 0.8 = \cos\phi$

$\sin\phi = 0.6$.

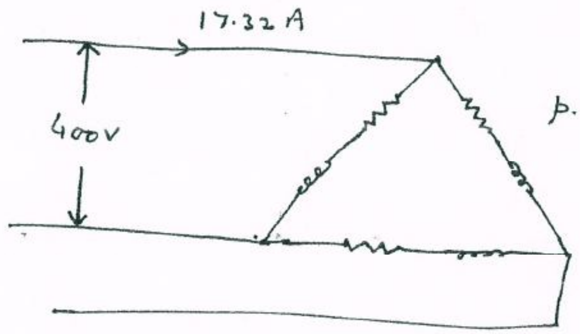
Example 8: Three identical coils connected in delta across 400V, 50 Hz, 3-phase a c supply ~~calculate~~ take a line current of 17.32 A at a power factor of 0.8 lagging. Calculate

(i) phase current

(ii) the resistance and inductance of each coil

(iii) power drawn by each coil.

Soln.



$V_p = V_L = 400$

(i) $I_p = \frac{I_L}{\sqrt{3}} = \frac{17.32}{\sqrt{3}} = \underline{10\ A}$ Ans

(ii) $Z_p = \frac{V_p}{I_p} = \frac{400}{10} = 40\,\Omega$

$R_p = Z_p \cos\theta = 40 \times 0.8 = \underline{32\,\Omega}$ Ans

$X_p = Z_p \sin\theta = 40 \times 0.6 = 24\,\Omega$

$XL_p = \frac{X_p}{2\pi f} = \frac{24}{2\pi \times 50} = \underline{0.0764\ H}$ Ans

(iii) Power by each coil = Power per phase $= I_p^2 R_p$

$= 10^2 \times 32 = \underline{3200\ W}$ Ans

Also

$P_{ph} = \frac{V_L I_L \cos\theta}{\sqrt{3}} = \frac{400 \times 17.32 \cdot 0.8}{\sqrt{3}}$

$= 400 \times \cancel{1732}^{10} \times 0.8 = \underline{3200 \times \cancel{1732}}\ W$

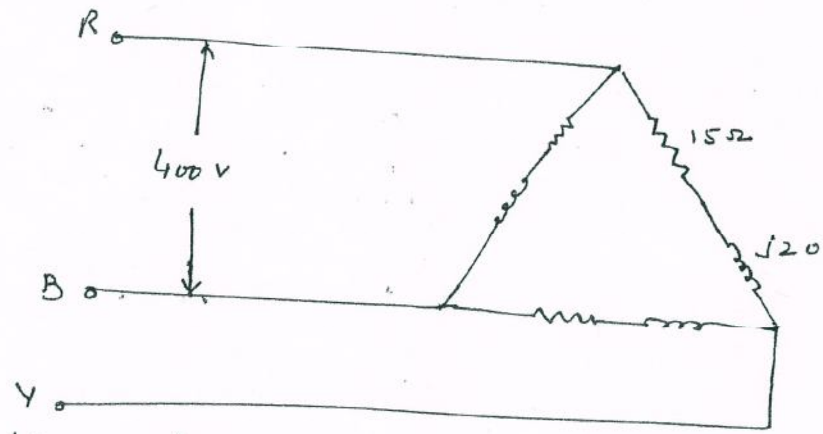
Example 9: A 400 V, 3-phase voltage is applied to a balanced 3-phase load of phase impedance $(15+j20)\,\Omega$. Find

(i) the phasor current in each line

(ii) power consumed in each phase

(iii) phasor sum of three line currents

Soln.



Let the phase sequence be RYB & $V_{RY} = 400\angle 0$ V

Then

$V_{YB} = 400\angle -120$ V and $V_{BR} = 400\angle 120$ V

$Z_p = (15+j20) = 25\angle 53.13\ \Omega$

phase Current, $I_{RY} = \dfrac{V_{RY}}{Z_p} = \dfrac{400\angle 0}{25\angle 53.13} =$

$= 16\angle -53.13 = (9.6 - j12.8)$ A

Phase Current, $I_{YB} = \dfrac{V_{YB}}{Z_p} = \dfrac{400\angle -120}{25\angle 53.13} = 16\angle -173.13$

$= (-15.88 - j1.91)$ A

Phase current $I_{BR} = \dfrac{V_{BR}}{Z_p} = \dfrac{400\angle 120}{25\angle 53.13} = (6.285 + j14.71)$ A

Line Currents

$$I_R = I_{RY} - I_{BR} = (9.6 - j12.8) - (6.285 + j14.71)$$

$$= (3.315 - j27.51) = \underline{27.7\angle -83.14\ A}$$

Ans.

$$I_Y = I_{YB} - I_{RY} = (-15.88 - j1.91) - (9.6 - j12.8)$$

$$= (-25.48 + j10.89) = \underline{27.7\angle 156.86\ A}$$

Ans

$$I_B = I_{BR} - I_{YB} = (6.285 + j14.71) - (-15.88 - j1.91)$$

$$= (22.165 + j16.62) = \underline{27.7\angle 36.86^\circ A}$$

Ans.

(ii) For R-Y phase, $s = V_{RY} \times I_{RY}$

$$= (400 - j0)(9.6 - j12.8) = (3840 - j5{,}120)\ VA$$

True Power = $\underline{3840\ W}$ Ans

(iii) Phasor Sum $= I_R + I_Y + I_B = 0$

Reason: It is a balanced load.

Comparison Between Y and Δ Systems

1. Y : similar ends are connected.

   Δ : Dissimilar ends are connected.

2. Y : $V_p = V_L/\sqrt{3}$

   Δ : $V_p = V_L$

   ∴ Y-connected alternators needs l
   1. no. of turns/phase
   2. Insulation

   for the same line voltage

   So 3-Φ alternators are so Y-connected.

3. Y : $I_p = I_L$

   Δ = $I_p = I_L/\sqrt{3}$

4. Y : neutral is available, so we can have 3-phase 4 wire system.

   So is used for power distribution.

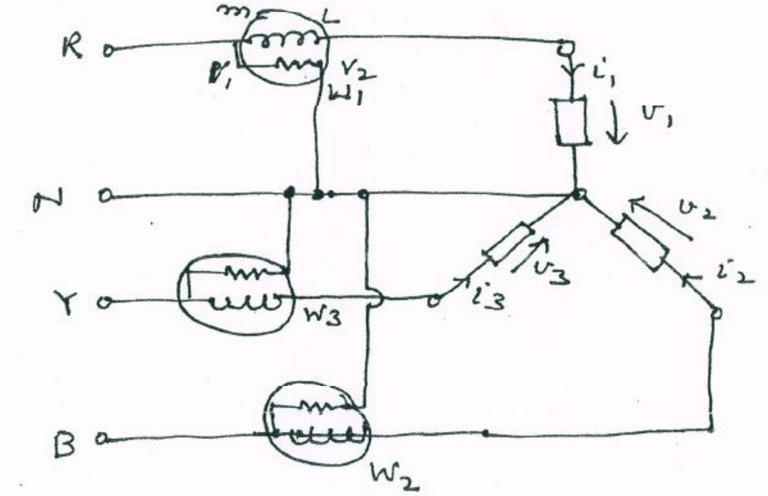
5. Y : neutral can be grounded & provide protection against ground faults.

6. Δ : Used for
   1. ~~Φ~~ Small Transformers for low voltage (440v)
   2. 3-phase motors
   3. Rotary Converters

Measurement of Power in Three Phase Circuits

Three Wattmeter Method

- This method can be used for 3-phase, 4-wire circuits.
- The connections are shown below.

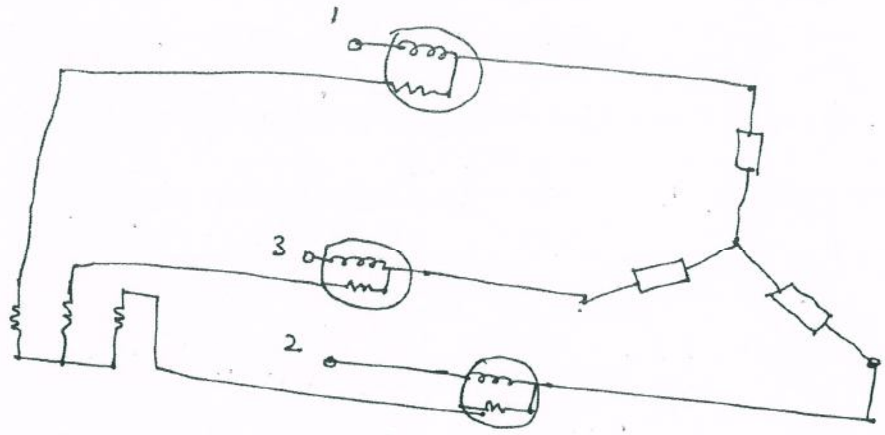


- Each wattmeter gives reading of power in ~~a~~ the respective phase.
- Total power $= W_1 + W_2 + W_3$
- In case of 3-phase 3-wire system ~~is not~~ ~~workab~~ This method can not be ~~to~~ used.

However, we have the following options.

- We can create an artificial star by using three high resistances as shown below.
- We can use 3-wattmeter method for measurement of power for Δ connected load too, but then the phase coils are required to be broken for inserting wattmeters.
- This process becomes too complicated and is not worth using.

PTO



One Wattmeter Method

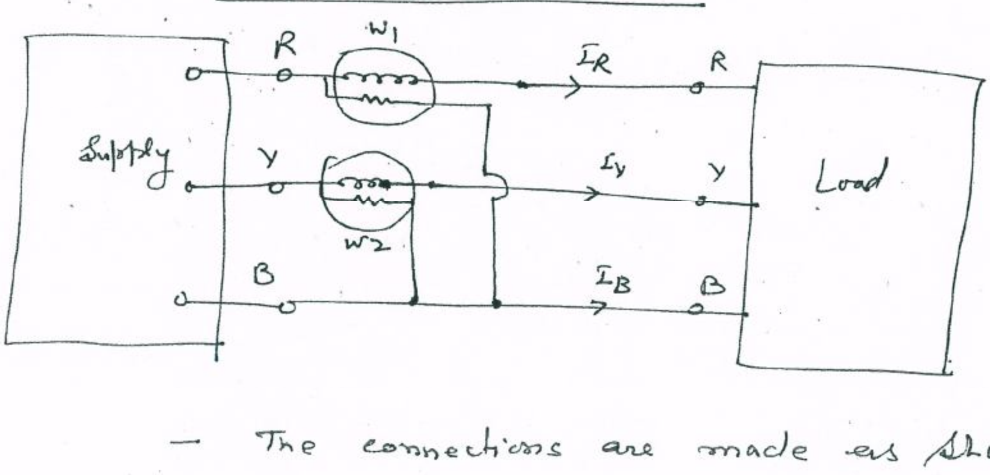
– This method is same as 3-wattmeter meth

– Here, we will connect a wattmeter in one phase Only and multiply the reading by through with an assumption that the load is balanced.

– This is valid for 3-phase 4-wire balanced system.

Limitation: – A little unbalance in load will produce large error in measurements.

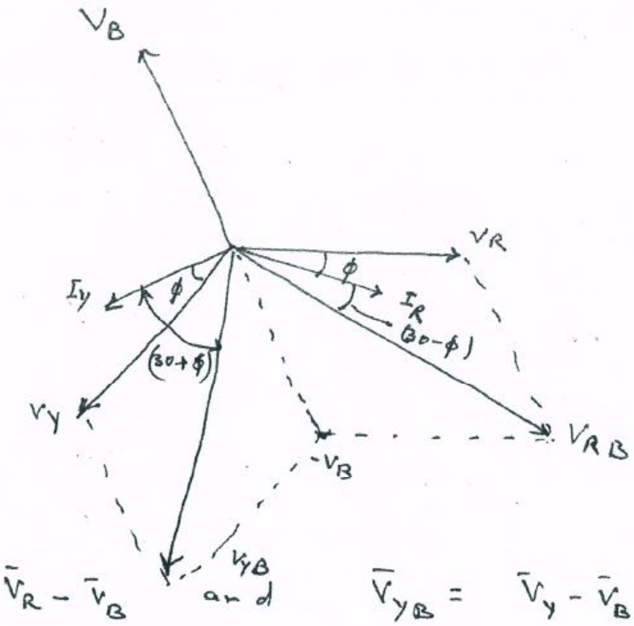
Two Wattmeter Method



– The connections are made as shown in fig. above.

– Consider the load as balanced (λ / Δ). But λ in this case

Then $I_R = I_Y = I_B$



$\bar{V}_{RB} = \bar{V}_R - \bar{V}_B$ and $\bar{V}_{yB} = \bar{V}_y - \bar{V}_B$

$\angle$ between $\bar{V}_{RB}$ and $\bar{I}_R = (30-\phi)$

$\angle$,, V_{yB} & $\bar{I}_y = (30+\phi)$

$$W_1 = V_{RB}\, I_R \cos(30-\phi) = \sqrt{3}\; V_p I_p \cos(30-\phi)$$

$$W_2 = V_{yB}\, I_y \cos(30+\phi) = \sqrt{3}\; V_p I_p \cos(30+\phi)$$

$$W_1 + W_2 = \sqrt{3}\; V_p I_p \left[\cos(30-\phi) + \cos(30+\phi)\right]$$

$$= \sqrt{3}\; V_p I_p \left[2 \cos 30 \cos\phi\right] = \sqrt{3}\; V_p I_p \left[2 \cdot \frac{\sqrt{3}}{2} \cos\phi\right]$$

$$= 3\, V_p I_p \cos\phi .$$

$$= \sqrt{3}\, V_L I_L \cos\phi .$$

Also

$$\frac{W_1}{W_2} = \frac{\cos(30-\phi)}{\cos(30+\phi)}$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\cos(30-\phi) - \cos(30+\phi)}{\cos(30-\phi) + \cos(30+\phi)}$$

$$= \frac{2 \sin 30 \sin\phi}{2 \cos 30 \cos\phi} = \tan 30^{\circ} \tan\phi$$

Comments:

1. If any of the wattmeter gives -ve readings, reverse the connections either for current coil or pressure coil and take the readings with -ve sign

2. The expression for total power has been derived for balanced load but it is true for unbalanced too

3. The reactive power is given by

$$Q = \sqrt{3}\,(W_1 - W_2)$$ [exercise: Derive]

4. If load is purely resistive, what would be the readings of wattmeters.

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

for $\cos\phi = 1$ $\sin\phi = 0$ & $\tan\phi = 0$

$$\therefore \boxed{W_1 = W_2}$$

5. If load is purely reactive, what would be the readings of wattmeters.

$\cos\phi = 0$ i.e. $W_1 + W_2 = 0 \Rightarrow W_1 = -W_2$

6. If one wattmeter reads, what is p-f?

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

put $W_2 = 0$

$$\tan\phi = \sqrt{3}\,\frac{W_1}{W_1} = \sqrt{3}$$

$$\theta = 60^\circ \quad \boxed{\cos\phi = 0.5}$$

Example: In a wattmeter method to measure power in a three-phase circuit, it was found that the two wattmeter read 3 kW and 1.5 kW respectively. Determine the total power consumed and the p.f. of the balanced three-phase circuit.

Soln. $W_1 = 3\ kW \qquad W_2 = 1.5\ kW$

$$P = W_1 + W_2 = 3 + 1.5 = \underline{4.5\ kW} \quad \text{Ans.}$$

p.f.

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\left[\frac{3-1.5}{3+1.5}\right] = \sqrt{3}\,\frac{1.5}{4.5}$$

$$= \frac{1}{\sqrt{3}}$$

$$\therefore \quad \phi = \tan^{-1}\frac{1}{\sqrt{3}} = 30^\circ$$

$$pf = \cos\phi = \cos 30 = \frac{\sqrt{3}}{2} = \underline{0.866} \quad \text{Ans.}$$

Example 2: The input power to a 3-phase motor was measured using two wattmeters. The readings were 5.2 kW and −1.7 kW, and the line voltage was 415 V. calculate:

(a) Total active power

(b) p.f.

(c) Line Current

Soln: (a) Total power $= W_1 + W_2 = 5.2 - 1.7 = \underline{3.5\ kW}$ Ans.

(b) p.f. : $\tan\phi = \sqrt{3}\,\dfrac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\dfrac{5.2+1.7}{5.2-1.7} = 3.415$

$\phi = 73.68^\circ \qquad \cos\phi = \underline{0.28}$ Ans

(c) $P = \sqrt{3}\, V_L I_L \cos\phi \;;\; I_L = \dfrac{P}{\sqrt{3}\, V_L \cos\phi}$

$$= \frac{3.5 \times 1000}{\sqrt{3}\ 415\ 0.28} = \underline{17.39A} \quad \text{Ans}$$