Therefore,

MA - 102

Assignment - II

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Q1 → Find the general solution for each of the following differential equation.

i) $y^{(3)} - 5y'' + 7y' - 3y = 0$.

The characteristic eqn for above eqn will be -

$$m^3 - 5m^2 + 7m - 3 = 0 \qquad - (1)$$

Solving for 'm',

$$(m-1)(m^2 - 4m + 3) = 0$$
$$(m-1)(m-1)(m-3) = 0$$
$$m = 1, 1, 3.$$

Hence, solution functions are e^x, e^{3x}

Since, '1' is repeated two times,

∴ coeffecient of e^x will be a polynomial of degree $(2-1) = 1$

∴ Complementary function $y_c = (C_1 x + C_2)e^x + C_3 e^{3x}$

Its the differential eqn is homogeneous, therefore

Complete solution → $y = y_c = (C_1 x + C_2)e^x + C_3 e^{3x}$.

ii) $y^{(5)} - 2y^{(4)} + y^{(3)} = 0$

$$(D^5 - 2D^4 + D^3)y = 0 \qquad - (1)$$

The characteristic eqn for eqn in (1) is,

$$m^5 - 2m^4 + m^3 = 0 \qquad - (2)$$

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Solving for m,

$$m^3(m^2 - 2m + 1) = 0$$
$$m^3(m-1)^2 = 0$$
$$m = 0, 0, 0, 1, 1.$$

$\therefore$ Solution func will be e^{0x}, e^{1x}.

$\therefore$ Since 0 is repeated three times, ~~the~~ coeffecient of e^{0x} will be polynomial of degree $(3-1) = 2$

And 1 is repeated two times, coefficient of e^{x} will be polynomial of degree $(2-1) = 1$.

$\therefore$ Complementary func $y_c = (c_1x^2 + c_2x + c_3)\,e^{0x} + (c_4x + c_5)\,e^{x}$.

Since, differential eqn in (1) is homogeneous.

There,

general soln $y = y_c = (c_1x^2 + c_2x + c_3) + (c_4x + c_5)\,e^{x}$.

Q2 → Find the general solution using operator method to solve non-homogeneous differential eq^n.

i) $y'' - 3y' + 2y = 2x^2 + e^{x} + 2xe^{x} + 4e^{3x}$.

Homogeneous eqn for above eqn is $\quad D \equiv \frac{d}{dx}$

$$(D^2 - 3D + 2)\,y = 0 \qquad -(1)$$

The characteristic eqn corresponding to (1).

$$m^2 - 3m + 2 = 0$$

Solving for m,

$$(m-1)(m-2) = 0$$
$$m = 1, 2.$$

Solution functions are e^{x}, e^{2x}.

Therefore,

complementary function $y_c = c_1 e^{x} + c_2 e^{2x}$ — (2)

Finding Particular integral,

$$y_p(x) = \frac{1}{D^2-3D+2}\left[2x^2 + e^x + 4e^{3x} + 2xe^x\right].$$

$$y_p(x) = \underbrace{\frac{1}{D^2-3D+2}[2x^2]}_{I} + \underbrace{\frac{1}{D^2-3D+2}\left[e^x + 4e^{3x}\right]}_{II} + \underbrace{\frac{1}{D^2-3D+2}\left[2xe^x\right]}_{III}. \quad -(3)$$

Solving I,

$$y_I = 2\left[\frac{1}{2+(D^2-3D)}\right]x^2$$

$$= \frac{2}{2}\,\frac{1}{1+\left(\frac{D^2-3D}{2}\right)}x^2$$

$$= \left[1 + \left(\frac{D^2-3D}{2}\right)\right]^{-1}x^2$$

$$= \left[1 - \left(\frac{D^2-3D}{2}\right) + \left(\frac{D^2-3D}{2}\right)^2 \cdots\right]x^2$$

$$= \left[1 + \frac{3D}{2} + \frac{7D^2}{4}\right]x^2$$

$$= \left(x^2 + 3x + 7/2\right). \quad -(4)$$

Solving II,

$$y_{II} = \frac{1}{D^2-30+2}\left[e^x + 4e^{3x}\right].$$

We know that

$\hookrightarrow$ $$\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax} \quad \text{if } f(a) \neq 0.$$

$$= \frac{1}{(D-a)^r}\,\frac{1}{f'(D)}e^{ax} = \frac{1}{f'(a)}\cdot\frac{x^r}{\lfloor r}e^{ax}. \quad \text{if } f(a) = 0$$

$$= \frac{1}{(D-1)(D-2)} e^{x} + 4 \frac{1}{(D-1)(D-2)} e^{3x}$$

$$= (-1) \frac{1}{(D-1)} e^{x} + 4 \frac{e^{3x}}{(3-1)(3-2)}$$

$$= (-1) \frac{x^{1} e^{x}}{1!} + 2e^{3x}$$

$$y_{II} = -xe^{x} + 2e^{3x} \qquad -(5)$$

Solving III,

$$y_{III} = 2 \frac{1}{(D-1)(D-2)} e^{x} x$$

We know that

$$\frac{1}{f(D)} e^{ax} g(x) = e^{ax} \frac{1}{f(D+a)} g(x)$$

$$\therefore y_{III} = 2e^{x} \frac{1}{((D+1)-1)((D+1)-2)} x$$

$$= 2e^{x} \frac{1}{D(D-1)} x$$

$$= -2e^{x} \frac{1}{D} (1-D)^{-1} x$$

$$= -2e^{x} \frac{1}{D} [1 + D - D^{2} \cdots] x$$

$$= -2e^{x} \frac{1}{D} [x+1]$$

$$= -2e^{x} \left[\frac{x^{2}}{2} + x\right]$$

$$y_{III} = -e^{x}[x^{2} + 2x] \qquad -(6)$$

$$\therefore y_{P} = y_{I} + y_{II} + y_{III}$$

[Using 4, 5 and 6]

$$= x^{2} + 3x + \frac{7}{2} - xe^{x} + 2e^{3x} - e^{x}x^{2} - 2xe^{x}$$

$$y_{P} = x^{2} + 3x + \frac{7}{2} - e^{x}x^{2} - 3xe^{x} + 2e^{3x} \qquad -(7)$$

Complete solution is $y_c + y_p$

$$\therefore \quad y = y_c + y_p$$
$$= c_1 e^{x} + c_2 e^{2x} + x^2 + 3x + \tfrac{7}{2} - x^2 e^{x} - 3xe^{x} + 2e^{3x}.$$

ii) $$y^{(4)} + y'' = 3x^2 + 4\sin x - 2\cos x.$$

The homogeneous eqn for above eqn is

$$(D^4 + D^2)y = 0 \qquad -(1) \qquad D \equiv \frac{d}{dx}$$

The characteristic eqn corresponding to (1)

$$m^4 + m^2 = 0.$$

Solving form, $$m^2(m^2+1) = 0.$$

$$m = 0, 0, i, -i.$$

Since 0 is repeated two times, the coefficient e^{0x} will be a polymonial of degree $(2-1) = 1$

$\therefore$ Complementory func $$y_c = (c_1 x + c_2)(1) + c_3' e^{ix} + c_4' e^{-ix}.$$

$$= c_1 x + c_2 + \cos x \underbrace{[c_3' + c_4']}_{c_3} + \sin x \underbrace{[ic_3' - ic_4']}_{c_4}$$

$$\therefore \quad y_c = c_1 x + c_2 + c_3 \cos x + c_4 \sin x. \qquad -(2)$$

Finding particular integral.

$$y_p(x) = \frac{1}{D^4 + D^2}\left[3x^2 + 4\sin x - 2\cos x\right] \qquad -(3)$$

$$= \underbrace{\frac{1}{D^2(D^2+1)}[3x^2]}_{\mathrm{I}} + \underbrace{\frac{1}{(1+D^2)(D^2)}\left[4\sin x - 2\cos x\right]}_{\mathrm{II}}.$$

Solving I

$$y_I = 3\frac{1}{D^2(1+D^2)}[x^2]$$

$$= 3\frac{1}{D^2}[1+D^2]^{-1}(x^2)$$

$$= 3\frac{1}{D^2}[1-D^2+D^4\dots](x^2)$$

$$= 3\frac{1}{D^2}[x^2-2]$$

$$= 3\left[\frac{x^4}{4\cdot 3} - \frac{\cancel{2}x^2}{\cancel{2}\cdot 1}\right]$$

$$y_I = \frac{x^4}{4} - 3x^2 \qquad -(4).$$

Solving II

$$y_{II} = \frac{1}{(D^2+1)(D^2)}[4\sin x - 2\cos x].$$

We know that

$$\frac{1}{f(D^2)}\cos x = \frac{1}{f(-a^2)}\cos x \quad \& \quad \frac{1}{f(D^2)}\sin x = \frac{1}{f(-a^2)}\sin x \qquad \text{if } f(-a^2) \neq 0$$

$$\frac{1}{D^2+a^2}\sin ax = -\frac{x}{2a}\cos ax \qquad \text{if } f(-a^2)=0$$

$$\frac{1}{D^2+a^2}\cos ax = \frac{x}{2a}\sin ax.$$

$$\therefore\ y_{II} = \frac{1}{(D^2+1)}\left[4\frac{1}{D^2}\sin x - 2\frac{1}{D^2}\cos x\right].$$

$$= (-1)\frac{1}{D^2+1}\left[4\sin x - 2\cos x\right].$$

$$= (-1)\left[4\frac{1}{(D^2+1)}\sin x - 2\frac{1}{D^2+1}\cos x\right].$$

$$= (+1)\left[4\cdot\frac{+x}{2}\cos x + 2\cdot\frac{x}{2}\cdot\sin x\right]$$

$$= x\cdot[2\cos x + \sin x] \qquad -(5).$$

$\therefore \quad y_P = y_I + y_{II}$

using (4), (5)

$$y_P = \left(\frac{x^4}{4} - 3x^2\right) + x\left[2\cos x + \sin x\right].$$

$\therefore$ Complete soln $\quad y = y_c + y_P$

$$= (xc_1 + c_2) + c_3\cos x + c_4 \sin x + \frac{x^4}{4} - 3x^2$$
$$+ x\left[2\cos x + \sin x\right].$$

Q3 Solve using variation of parameter.

i) $y'' + y = \tan(x)$.

The homogeneous eqn for above eqn is.

$$(D^2 + 1)y = 0$$

$\therefore$ Characteristic eqn $\Rightarrow \quad m^2 + 1 = 0$

$$m^2 = -1$$
$$m = \pm i.$$

$$\therefore \quad y_c = c_1' e^{ix} + c_2' e^{-ix}.$$
$$= (c_1' + c_2')\cos x + (ic_1' - ic_2')\sin x$$
$$= c_1 \cos x + c_2 \sin x$$

where $c_1 = c_1' + c_2'$
$c_2 = i(c_1' - c_2')$

Also

$$y_c = c_1 y_1(x) + c_2 y_2(x)$$

$\therefore \quad y_1(x) = \cos x \quad - (1)$

$y_2(x) = \sin x. \quad - (2)$

$a_0(x) = 1$ and $f(x) = \tan(x)$.

$$g(x) = \frac{f(x)}{a_0(x)} = \tan x \quad (3)$$

Using the method of variation of parameters.

PI $\Rightarrow y_p = u(x)\, y_1(x) + v(x)\, y_2(x)$

where $u_x = \int \frac{W_1}{W}\, dx$

$v_x = \int \frac{W_2}{W}\, dx$, where W_1, W_2, W are Wronskians

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x = 1$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ g(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & \sin x \\ \tan(x) & \cos x \end{vmatrix}$$

$$= -\sin x \cdot \tan x.$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & g(x) \end{vmatrix} = \begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix} = \cos x \tan x$$

$$= \sin x.$$

$$\therefore u_x = +\int \frac{-\sin x \cdot \tan x}{1}\, dx.$$

$$= \tan x \cdot \cos x - \int \sec^2 x \cdot \cos x \, dx$$

$$= \tan x \cdot \cos x - \ln|\sec x + \tan x|$$

$$= \sin x - \ln|\sec x + \tan x| \quad -(4)$$

$$u_v = \int \frac{\sin x}{1}\, dx = -\cos x \quad -(5)$$

$$\therefore y_p = \sin x \cos x - \cos x \ln|\sec x + \tan x| - \cancel{\cos x \sin x}$$

$$= + \cos x \ln|\sec x - \tan x|.$$

Complete solution

$$y = y_c + y_p = c_1 \cos x + c_2 \sin x + \cos x \ln|\sec x - \tan x|.$$

ii) $y'' + 4y' + 5y = e^{-2x} \sec(x)$.

The homogeneous eqn for the following is.

$$D^2 + 4D + 5 = 0.$$

The characteristic eqn is

$$m^2 + 4m + 5 = 0$$

Solving for m,

$$m = \frac{-4 \pm \sqrt{-4}}{2}$$

$$= -2 \pm i$$

$$\therefore y_c = c_1' e^{(-2+i)x} + c_2' e^{(-2-i)x}$$

$$= e^{-2x}\left[c_1' e^{ix} + c_2' e^{-ix}\right].$$

$$= e^{-2x}\left[c_1 \cos x + c_2 \sin x\right].$$

$$\therefore y_1 = e^{-2x} \cos x \quad -① $$

$$y_2 = e^{-2x} \sin x. \quad -②$$

Also, $a_0(x) = 1$ $\quad f(x) = e^{-2x} . \sec(x)$.

$$\therefore g(x) = \frac{f(x)}{a_0(x)} = e^{-2x} . \sec(x) \quad -③.$$

Using the method of variation of parameter for finding PI.

PI $\Rightarrow$ $y_p = u(x)\, y_1(x) + v(x)\, y_2(x)$.

where $u_x = \int \frac{W_1}{W}\, dx$ & $V_x = \int \frac{W_2}{W}\, dx$.

where W, W_1, W_2 are wronskians.

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Using the ...10 :

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-2x}\cos x & e^{-2x}\sin x \\ -e^{-2x}\sin x - 2e^{-2x}\cos x & e^{-2x}\cos x - 2e^{-2x}\sin x \end{vmatrix}$$

$$= \left(e^{-2x}\right)^2\left[\cos^2 x - \cos x\sin x + \sin^2 x + \sin x\cos x\right]$$

$$= \left(e^{-2x}\right)^2 = e^{-4x}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ g(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & e^{-2x}\sin x \\ e^{-2x}\sec x & e^{-2x}\cos x - 2e^{-2x}\sin x \end{vmatrix}$$

$$= -e^{-4x}\tan x.$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & g(x) \end{vmatrix} = \begin{vmatrix} e^{-2x}\cos x & 0 \\ -e^{-2x}\sin x - 2e^{-2x}\cos x & e^{-2x}\sec x \end{vmatrix}$$

$$= e^{-4x}.$$

$$u(x) = \int \frac{W_1}{W}\,dx = -\int \tan x\,dx$$

$$= \log|\cos x|$$

$$v(x) = \int \frac{W_2}{W}\,dx = \int \frac{e^{-4x}}{e^{-4x}}\,dx = x.$$

$$\therefore\ y_p = u(x)\,y_1 + y_2\,v(x)$$

$$= \log|\cos x| \cdot e^{-2x}\cos x + x e^{-2x}\sin x$$

$\therefore$ Complete soln $y = y_c + y_p$

$$= e^{-2x}c_1\cos x + e^{-2x}c_2\sin x + e^{-2x}\cos x\log|\cos x| + e^{-2x}x\sin x$$

$$= e^{-2x}\left[c_1\cos x + c_2\sin x + \cos x\log_e|\cos x| + x\sin x\right].$$

Q4 Find the general solution.

i) $x^2 y'' - 2xy' + 2y = x^3$.

$D \equiv \frac{d}{dx}$.

$\therefore (x^2 D^2 - 2xD + 2)\, y = x^3$. — (1)

Let $x = e^z$ — (2)

$\therefore x\,Dy = \frac{d}{dz} y$.

$= \theta y$ where $\theta \equiv \frac{d}{dz}$.

$x^2 D^2 y = \theta(\theta - 1)\, y$.

Putting back value.

$[\theta(\theta-1) - 2\theta + 2]\, y = e^{3z}$

$\theta^2 - 3\theta + 2 = e^{3z}$ — (3)

The homogeneous eqn for (3) will be

$[\theta^2 - 3\theta + 2]y = 0$

$\therefore$ Characteristic eqn

$m^2 - 3m + 2 = 0$

Solving for m,

$(m-1)(m-2) = 0$

$m = 1, 2$.

Solution functions e^z, e^{2z}.

$\therefore$ Complementary func $y_c = c_1 e^z + c_2 e^{2z}$

$y_c = c_1(x) + c_2(x^2)$. — (4).

$y_c = x^2 c_2 + x c_1$

Using the [illegible]

Solving 3 for particular integral.

$$y_p = \frac{1}{\theta^2 - 3\theta + 2} e^{3z}$$

$$= \frac{1}{(\theta-2)(\theta-1)} e^{3z}$$

We know that

$$\frac{1}{f(\theta)} e^{az} = \frac{1}{f(a)} e^{az} \quad , \text{ if } f(a) \neq 0 .$$

$$\therefore \quad y_p = \frac{1}{(3-1)(3-2)} e^{3z}$$

$$y_p = \frac{e^{3z}}{2}$$

Subsituting back $e^z = x$.

$$\therefore \quad y_p = \frac{x^3}{2} \quad - 5$$

$\therefore$ Complete solution

$$y = y_p + y_c$$

$$y = c_1 x + c_2 x^2 + \frac{x^3}{2} \qquad [\text{from } 4, 5].$$

ii) $x^2 y'' + 4xy' + 2y = 4\ln(x)$.

$$(x^2 D^2 + 4xD + 2)\, y = 4\ln(x) \qquad \text{where } D \equiv \frac{d}{dx}.$$

Let $x = e^z$

$$\therefore \quad xDy = \frac{d}{dz} y$$

$$= \theta y \qquad \text{where } \theta \equiv \frac{d}{dz}.$$

Also $\leftrightarrow$ $x^2 D^2 y = \theta(\theta - 1)\, y$

$= -\frac{1}{}[1 + \frac{D^2}{} - \frac{D^4}{} \dots](1-t) - \frac{e^{3t}}{}$

Putting back these value

$$[\theta(\theta-1) + 4\theta + 2]y = 4z$$

$$[\theta^2 + 3\theta + 2]y = 4z \quad - (2)$$

The homogeneous eqn for (2) is

$$[\theta^2 + 3\theta + 2]y = 0$$

Characteristic eqn,

$$m^2 + 3m + 2 = 0$$

Solving for m,

$$(m+1)(m+2) = 0$$

$$m = -1, -2.$$

Solution func $\rightarrow e^{-z}, e^{-2z}$

$\therefore$ complementary function

$$y_c = c_1 e^{-z} + c_2 e^{-2z}$$

Since, $e^z = x$

$$\therefore y_c = c_1 x^{-1} + c_2 x^{-2}$$

$$y_c = \frac{c_1}{x} + \frac{c_2}{x^2} \quad - (3)$$

Solving (2) for particular integral :

$$y_p = \frac{1}{\theta^2 + 3\theta + 2} 4z$$

$$= \frac{1}{\cancel{2}} \cancel{4}^{2} \left[\frac{1}{1 + \left(\frac{\theta^2 + 3\theta}{2}\right)}\right] z.$$

$$= 2\left[1 + \left(\frac{\theta^2 + 3\theta}{2}\right)\right]^{-1} z$$

$$= 2\left[1 - \left(\frac{\theta^2 + 3\theta}{2}\right) + \dots\right] z$$

$$= 2\left[z - \frac{3}{2}\right] = 2z - 3$$

$$y_p = 2z - 3.$$

In terms of x

$y_p = 2\ln(x) - 3$. — (5)

$\therefore$ Complete solution.

$y = y_c + y_p$

[from (4), (3)].

$y = \frac{c_1}{x} + \frac{c_2}{x^2} + 2\ln x - 3$.

Q5 Solve the following system, where x and y are dependent variables and t is independent variable.

1) $2x' - 2y' - 3x = t$

and

$2x' + 2y' + 3x + 8y = 2$.

$D \equiv \frac{d}{dt}$.

$\therefore$ [$(2D-3)x - 2Dy = t$.] $\times$ $2D+8$ — (1)

[$(2D+3)x + (2D+8)y = 2$.] $\times$ $2D$ — (2)

Adding $(2D+8)$(1) + $2D \times$ (2)

$\therefore$

$(2D+8)(2D-3)x + (2D+3)(2D)x = (2D+8)t + 0$.

$(8D^2 + 16D - 24)x = 2(D+4)t$.

$4(D^2 - 2D - 3)x = 4t + 1$

Characteristic eqn $\rightarrow$

$4(m^2 - 2m - 3) = 0$

$4(m+3)(m-1) = 0$

$\boxed{m = -3, 1}$.

Complementary function for 'x'. $\rightarrow$ $c_1e^{-3x} + c_2e^{x}$ — (3)

Finding particular integral for 'x'

$$x_{PI} = \frac{1}{4(D+3)(D-1)}\left[4t+1\right]$$

$$= \frac{-1}{12}\left[\frac{1}{1-\frac{(D^2+2D)}{3}}\right](4t+1).$$

$$= \frac{-1}{12}\left[1-\frac{(2D+D^2)}{3}\right]^{-1}(4t+1)$$

$$= \frac{-1}{12}\left[1+\frac{2D+D^2}{3}-\dots\dots\right]4t+1.$$

$$= \frac{-1}{12}\left[4t+1+\frac{2}{3}\cdot 4\right].$$

$$x_{PI} = -\frac{t}{3}-\frac{11}{36}. \quad -(4)$$

$$\therefore \quad x = x_c + x_{PI}$$

$$x = c_1e^{-3x} + c_2e^{x} - \frac{t}{3} - \frac{11}{36}$$ [Using (3) and (4)]

↳ (5).

Adding (1) + (2)

$$\therefore \quad 4Dx + 8y = t+2$$

$$\therefore \quad 8y = t+2 - 4Dx$$

$$= t+2-4\left[-3c_1e^{-3x}+c_2e^{x}-\frac{1}{3}\right]$$

$$= 12e^{-3x} - 4c_2e^{x} + t + \frac{10}{3}$$

Q5

$$x' + y' - x - 6y = e^{3t}$$
$$x' + 2y' - 2x - 6y = t$$

$D \equiv \frac{d}{dt}$

$$(D-1)x + (D-6)y = e^{3t} \quad -①$$
$$(D-2)x + (2D-6)y = t \quad -②$$

Subtracting, $(D-2)$ ① $-$ $(D-1)$ ②

we got

$$(D-1)(D-2)x - (D-2)(D-1)x + (D-6)(D-2)y - (2D-6)(D-1)y$$
$$= 3e^{3t} - 2e^{3t} - 1 + t$$

$$[D^2 - 8D + 12 - 2D^2 + 8D - 6]y = e^{3t} - 1 + t.$$

$$[-D^2 + 6]y = e^{3t} - 1 + t$$

$$(D^2 - 6)y = (1-t) - e^{3t} \quad -③$$

Auxillary eqn will be :→

$$m^2 - 6 = 0$$
$$m^2 = 6$$
$$m = \pm\sqrt{6}.$$

Complementary function

$$y_c = c_1 e^{\sqrt{6}x} + c_2 e^{-\sqrt{6}x} \quad -④$$

Finding particular integral for y.

$$y_{PI} = \frac{1}{D^2-6}\left[(1-t) - e^{3t}\right]$$

$$= \frac{1}{-6\left[1-\frac{D^2}{6}\right]}(1-t) - \frac{1}{(D^2-6)}e^{3t}$$

$$= -\frac{1}{6}\left[\left(1-\frac{D^2}{6}\right)^{-1}(1-t)\right] - \frac{1}{9-6}e^{3t}$$

$$= -\frac{1}{6}\left[1 + \frac{D^2}{6} - \frac{D^4}{36} \cdots \right](1-t) - \frac{e^{3t}}{3}$$

$$= -\frac{1}{6}[1-t] - \frac{e^{3t}}{3}$$

$$= -\frac{1}{3}\left[\frac{1}{2} - \frac{t}{2} + e^{3t}\right] \quad - (5)$$

$\therefore$ $y = y_c + y_{PI}$

$$y = c_1 e^{\sqrt{6}t} + c_2 e^{-\sqrt{6}t} - \frac{1}{3}\left[\frac{1}{2} - \frac{t}{2} + e^{3t}\right] \quad - (6)$$

Subtracting (2) from (1), we get

$$x - Dy = e^{3t} - t.$$

$$x = Dy + e^{3t} - t$$

$$= \sqrt{6}c_1 e^{\sqrt{6}t} - \sqrt{6}c_2 e^{-\sqrt{6}t} - \frac{1}{3}\left[-\frac{1}{2} + 3e^{3t}\right] + e^{3t} - t.$$

$$x = \sqrt{6}c_1 e^{\sqrt{6}t} - \sqrt{6}c_2 e^{-\sqrt{6}t} + \frac{1}{6} - t. \quad - (7)$$

Q_6
5

Check whether the following set of func are linearly independent or not :-

i) $\sin 2x$ and $\cos 2x$ in interval $(0,1)$

ii) e^{3x} and e^{-3x} on any interval I

iii) x, x^2 and x^3 on any interval I

i) $\sin 2x$ & $\cos 2x$.

To check for linear independence, consider the Wronskian with $y_1 = \sin 2x$

$y_2 = \cos 2x$.

$$\therefore \quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \sin 2x & \cos 2x \\ 2\cos 2x & -2\sin 2x \end{vmatrix}$$

$$= -2\left[\sin^2 2x + \cos^2 2x\right]$$

$$= -2.$$

→ Since $W \neq 0$, for each $x \in (0,1)$.

Hence, the set of function is linearly independent in interval $(0,1)$

ii) e^{3x} and e^{-2x}.

To check for linear independence, consider the Wronskian with $y_1 = e^{3x}$

$y_2 = e^{-2x}$.

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{3x} & e^{-2x} \\ 3e^{3x} & -2e^{-2x} \end{vmatrix}$$

$$= -5e^{x}.$$

Since $W \neq 0$, for any interval I,

Therefore, the set is linearly independent on any interval I

iii) x, x^2 and x^3

$y_1 = x$, $y_2 = x^2$, $y_3 = x^3$

Considering Wronskian.

$$W = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$$

$$W = x(12x^2 - 6x^2) - 1[6x^3 - 2x^2]$$

$$= 2x^3$$