FLUID MECHANICS.

(1) Fluid Mechanics: that banch of Engineering-Science which deal behaviour of fluid under conditions of rest and motion.

properties of fluids – General aspects:

The matter can be classified ~~as~~ on the basis of the spacing between molecules of the matter as follows:

1. Solid state, and
2. fluid state
   1. Liquid state, and (ii) Gaseous state

In solids, the molecules are closely spaced whereas in liquids the spacing between molecules is relatively large and in gases the spacing between the molecules is still large. It means that inter-molecular ~~distance~~ cohesive forces are large in solids, smaller in liquids and extremely small in gases, and on accaunt of this fact, solids possess compact and rigid form, liquid molecules can move freely within the liquid mass and the molecules of gases have greater freedom of movment so that the gases fill the container completely in which they are placed.

A solid can resist tensile, compressive and shear stresses upto a certain limit whereas a fluid has no tensile strength or very little of it and it can resist the compressive forces only when it is kept in a container. When a fluid is subjected to ~~shear stress in a fluid depends on the magnitude of the rate of deformation~~ shearing force it deform's continiously as long as the force is applied.

Liquids and gases exhibits different characteristics- The liquids under ordinary conditions are quite difficult to compress (and therefore they may be regarded as incompressible) whereas gases can be compressed much readily under the action of external pressure (and when the external pressure is removed the gases tends to expand indefinitely)

② FLUID.

A fluid may be defined as follows:

"A fluid is a substance which is capable of flowing"

OR

A fluid is a substance which deforms continiously when subjected to external sheering force.

A fluid has following characteristics:

1. It has no definite shape of its own, but conforms the shape of the containing vessel.
2. Even a small amount of shear force exerted on liquid/fluid will cause it to undergo a deformation which continue as the the force continued to ~~apply~~ applied.

A fluid may be classified as follows:

A. (i) Liquid (ii) Gas, and (iii) Vapour

B. (i) Ideal fluids and (ii) Real fluids.

A (i) Liquid: A liquid is a fluid, which possesses a definite volume, which varies only slightly with temperature and pressure. Since under ordinary conditions liquids are difficult to compress, so liquid is assumed to be ~~incom~~ incompressible.

(ii) Gas: A gas is a fluid, which is compressible and possess no definite volume but it always expands until its volume is equal to that of container. Even a slight change in the temperature of a gas has a significant effects ~~on~~ on its volume and pressure.

(iii) Vapour: A vapour is a gas whose temperature and pressure are such it is very near the liquid state. Thus steam may be considered a vapour because its state is normally not far from that water.

B.(i) Ideal fluids : are those fluids which has no viscocity and surface tension and they are incompressible. In true sense no such fluid exists in nature. However fluids which have low ~~viscocity~~ viscocities such as water and air can be treated as ideal fluids under certain conditions. The assumptions of ideal fluids helps in simplyfing the mathematical analysis.

(ii.) Practical or Real fluids : Real-fluids ~~are those~~ fluids which are actually available in nature. These fluids posses the properties such as viscocity, surface tension and ~~compressibility~~ compressibility and therefore a certain amount of resistance is always offered by these fluids when they are set in motion.

VISCOCITY :

Viscocity is defined as the property of a fluid which offer's resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say u and u+du as shown in figure.1 the relative velocity causes a shear stress acting between fluid layers.

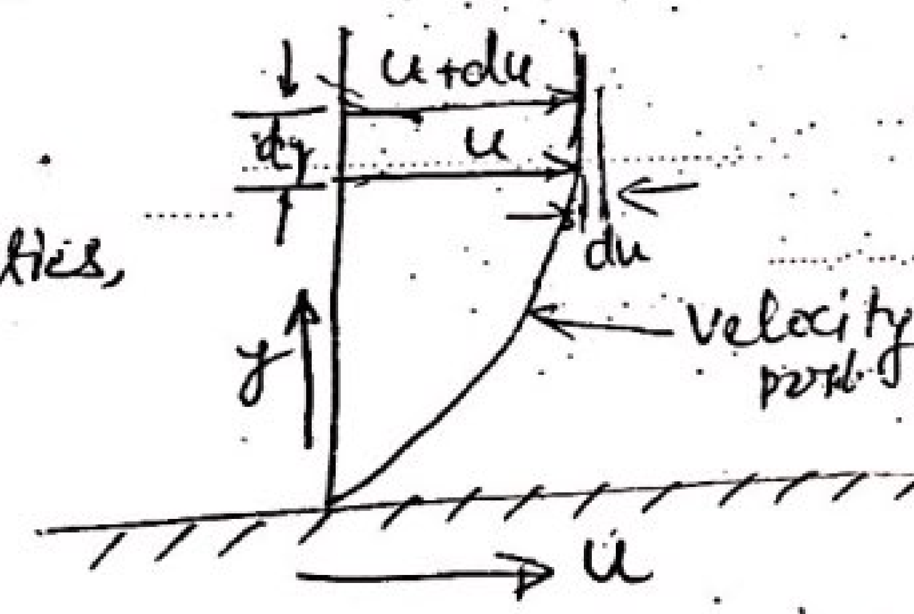


Fig.1. Velocity gradient near a Solid boundry

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by symbol τ called Tau.

So $\tau \propto \frac{du}{dy}$

$$\tau = \mu \frac{du}{dy} \quad \text{—— (1)}$$

μ = constant and is known as the Co-efficient of dynamic viscocity or only viscocity.

Unit of Viscosity.

$$\mu = \frac{\text{Shear Stress}}{\frac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/area}}{\left(\frac{\text{Length}}{\text{Time}}\right) \times \frac{1}{\text{length}}}$$

$$= \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

In MKS unit of viscosity $= \frac{kgf - sec}{m^2}$

C.G.S $= \frac{dyne - sec}{cm^2} = 1$ poise

SI $= \frac{\cancel{dyne}\ N - sec}{m^2} = \frac{N-S}{m^2}$

$$\frac{Kgf - sec}{m^2} = \frac{9.81\ N-Sec}{m^2} \qquad (1\ kgf = 9.81\ N)$$

one Newton $=$ one kg (mass) $\times$ one $\left(\frac{m}{sec^2}\right)$ acceleration

$$= \frac{1000\ gm \times 100\ cm}{sec^2} = 1000 \times 100 \frac{gm-cm}{sec^2}$$

$$= 10^5\ dyne \qquad \left[\because 1\ dyne = gm \times \frac{cm}{sec^2}\right]$$

$$1\ \frac{kgf - sec}{m^2} = 9.81 \times 10^5\ \frac{dyne-sec}{m^2}$$

$$= 9.81 \times 10^5\ \frac{dyne-sec}{100 \times 100\ cm^2}$$

$$= 98.1\ \frac{dyne-sec}{cm^2} = 98.1 \text{ poise}$$

$$\text{one } \frac{N\text{-}S}{m^2} = \frac{98.1}{9.81} \text{ poise} = 10 \text{ poise}$$

$$\text{one poise} = \frac{1}{10}\ \frac{N\text{-}S}{m^2}$$

Kinematic viscosity :- It is ratio of the dynamic viscosity and density of fluid. It is denoted by (ν)

$$\boxed{\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}}$$

Unit of Kinematic viscosity

$$\nu = \frac{\text{unit of } \mu}{\text{unit of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{length})^2 \times \frac{\text{Mass}}{(\text{length})^3}} = \frac{\text{Mass} \times \frac{\text{length}}{(\text{Time})^2}}{\left(\frac{\text{Mass}}{\text{length}}\right)}$$

$$= \frac{(\text{length})^2}{\text{Time}}$$

S.I unit of $\nu = \frac{(\text{metre})^2}{\text{sec}} = m^2/s$

C.G.S unit of $\nu = \frac{cm^2}{sec} = 1$ stoke

$1 \text{ stoke} = \frac{cm^2}{s} = \left(\frac{1}{10}\right)^2 m^2/s = 10^{-4} m^2/s$

1. centistoke $= \frac{1}{100}$ stoke.

Newton Law of Viscosity : The shear stress in fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

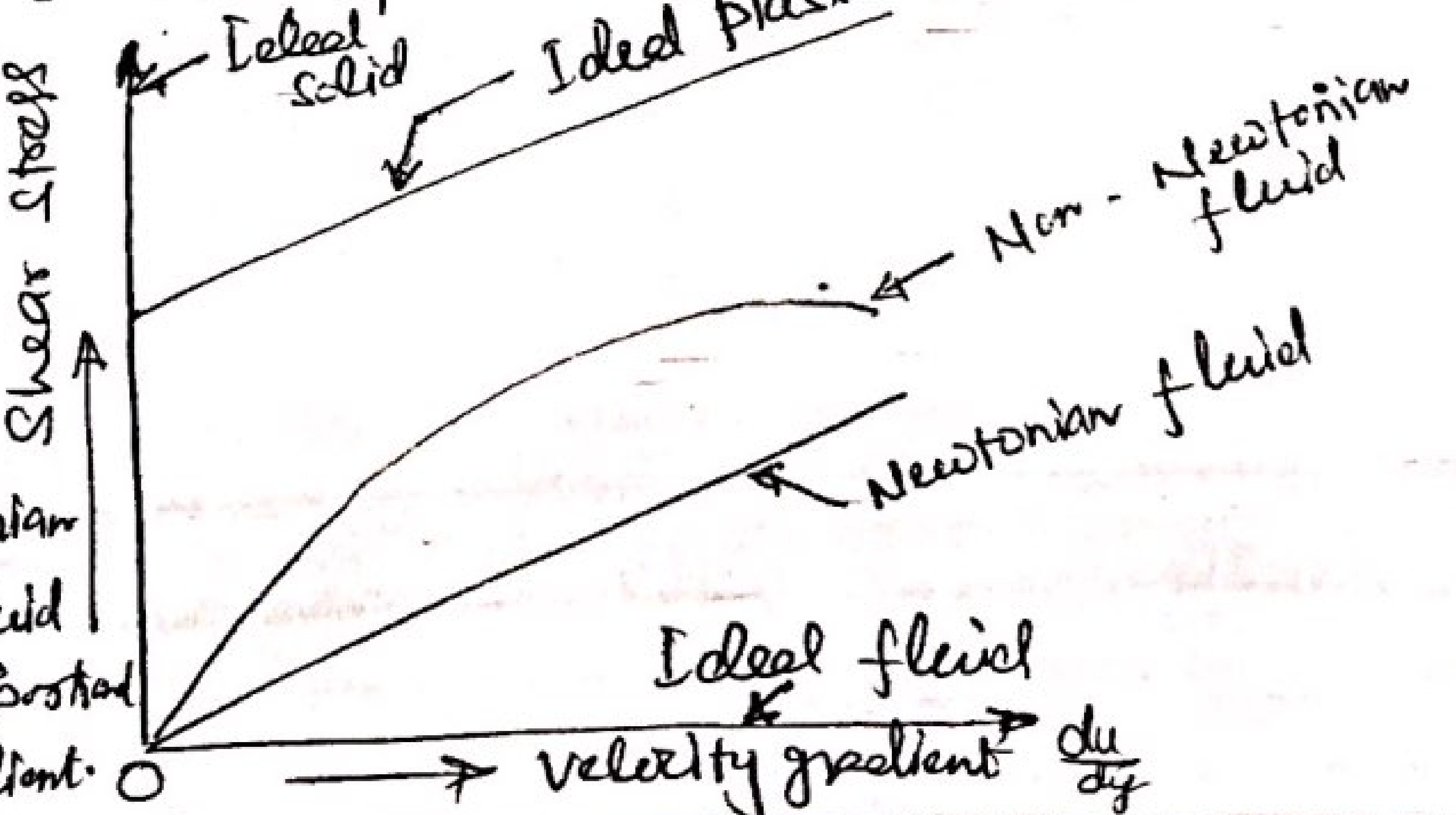
$$\boxed{\tau = \mu \frac{du}{dy}}$$

Fluid which obey above relation are called as Newtonian fluids and the fluids which donot obey above relation are called Non-newtonian fluids.

Ideal plastic fluid : A fluid in which the shear stress is not proportional to the rate of shear strain (or velocity gradient) is known as ideal plastic fluid.

Types of fluids.

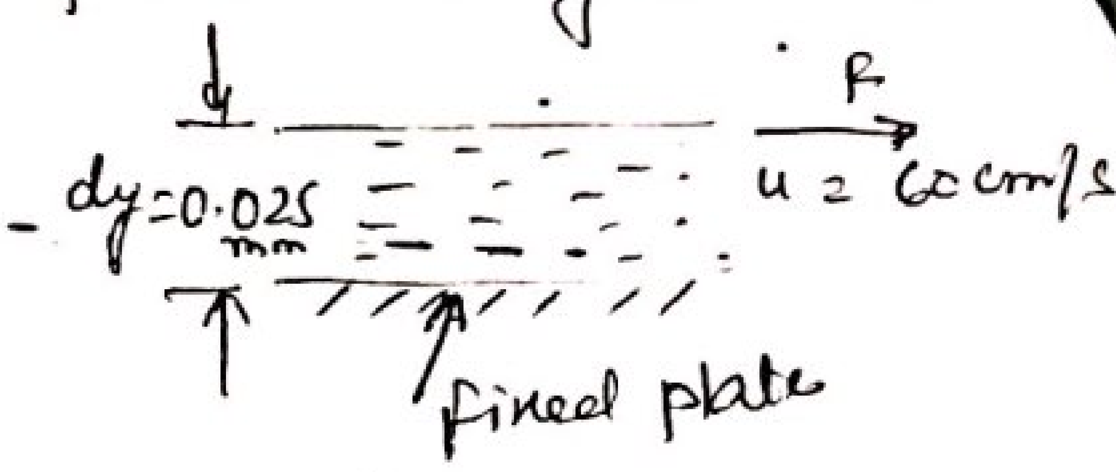
1. Ideal fluid 2. Real fluid 3. Newtonian fluid
4. Non-Newtonian fluid 5. Ideal plastic fluid



Newtonian fluid : A real fluid in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

Non-Newtonian fluid : A real fluid in which the shear stress is not proportional to the rate of shear strain or velocity gradient.

1) Q. A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area 1e 2N/m² to maintain this speed. Determine the fluid viscosity between the plates.



Given $dy = 0.025\ \text{mm}$
$= 0.025 \times 10^{-3}\ \text{m}$

velocity of upper plate $u = 60\ \text{cm/s}$
$= 0.6\ \text{m/s}$

$\tau = \frac{F}{A} = \frac{2}{1}\ \text{N/m}^2$

$du = u - 0 = 0.6\ \text{m/s}$
$dy = 0.025 \times 10^{-3}\ \text{m}$

$\tau = 2 = \mu \frac{0.6}{0.025 \times 10^{-3}}$

$\mu = 8.33 \times 10^{-5}\ \frac{Ns}{m^2}$
$= 8.33 \times 10^{-5} \times 10$ poise
$= 8.33 \times 10^{-4}$ poise. Ans

2(Q) Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in fig. 2. The weight of ~~incline~~ the square block is 300N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil ~~film~~ is 1.5 mm.

Solution: Area of plate $A = 0.8 \times 0.8 = 0.64\ \text{m}^2$

$\theta = 30°$
$W = 300\ \text{N}$
$u = 0.3\ \text{m/s}$

$dy = t = 1.5\ \text{mm} = 1.5 \times 10^{-3}$

component of weight along plane
$= W \sin 30° = 150\ \text{N}$

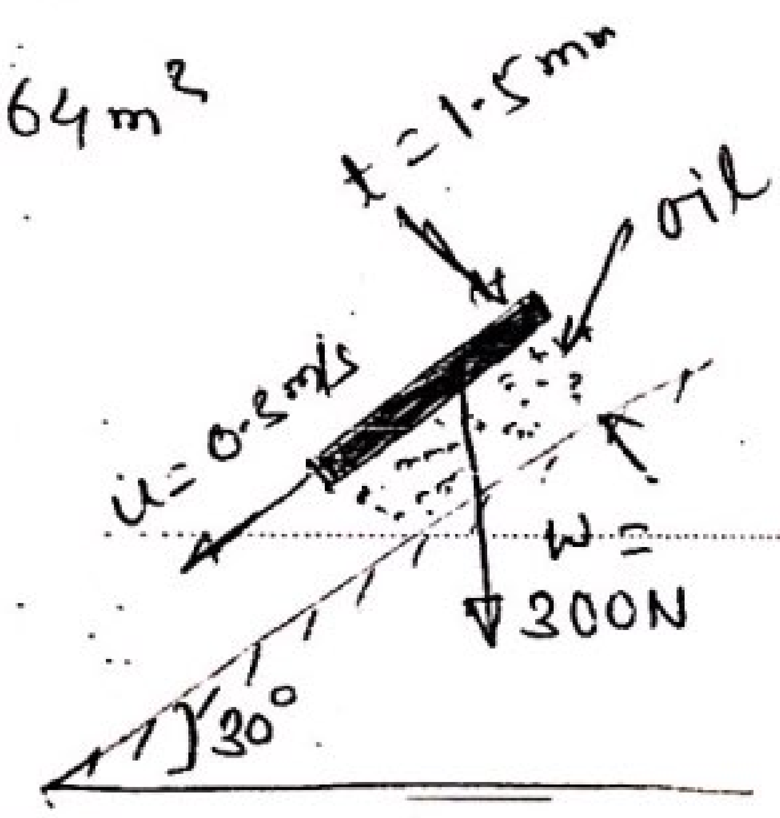


fig.2

$\tau = \frac{150}{0.64}\ \text{N/m}^2$

$\tau = \mu \frac{du}{dy}$ $\quad du = 0.3 - 0 = 0.3\ \text{m/s}$

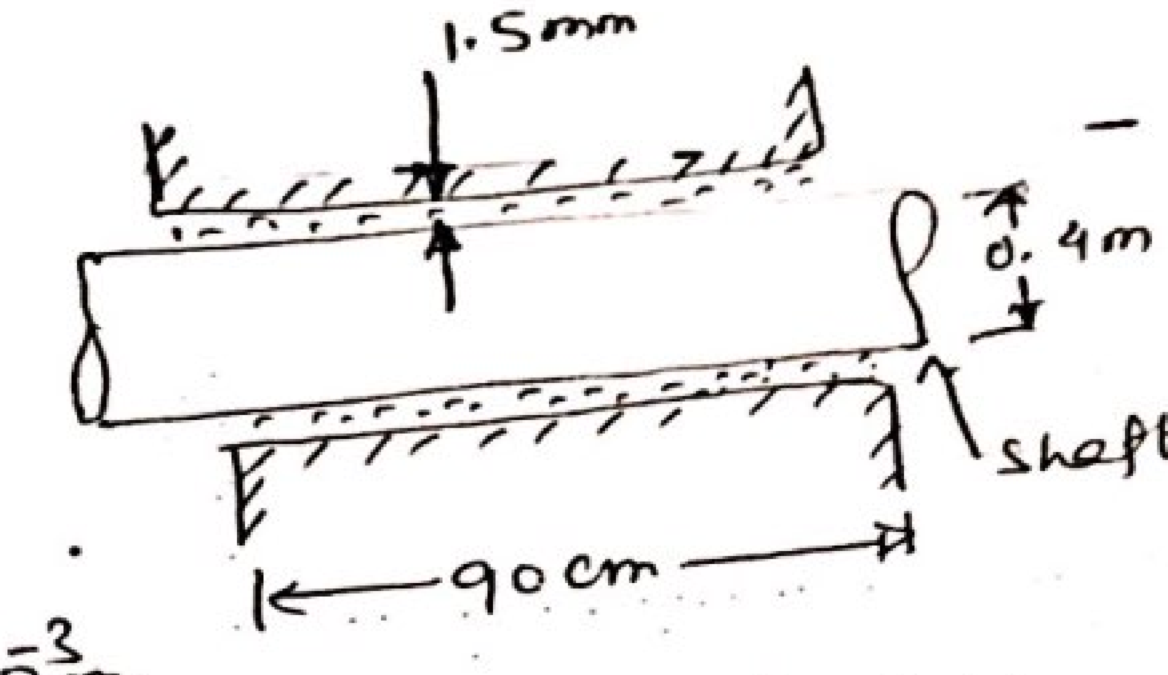
So $\frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$

$\Rightarrow \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17\ \text{Ns/m}^2 = 1.17 \times 10$ poise
$= 11.7$ poise.

3(a) The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of oil film is 1.5 mm.

Solⁿ : Given $\mu = 6$ poise

$= \frac{6}{10}\frac{NS}{m^2} = 0.6 \frac{NS}{m^2}$



Dia of shaft $D = 0.4m$

speed of shaft $N = 190$ rpm

sleeve length $L = 90$ mm $= 90\times10^{-3}$ m

Thickness of oil film $t = 1.5$ mm $= 1.5\times10^{-3}$ m

Tangential velocity of shaft $u = \frac{\pi D N}{60} = \frac{\pi\times0.4\times190}{60} = 3.98$ m/s

using the relation $\tau = \mu\frac{du}{dy} = 0.6\times\frac{3.98}{1.5\times10^{-3}} = 1592$ N/m²

Shear force on the shaft, F = Shear stress × area

$= 1592\times\pi D L = 1592\times\pi\times0.4\times90\times10^{-3}$

$= 180.05$ N

Torque on the shaft T = force × $\frac{D}{2}$ $= 180.05\times\frac{0.4}{2} = 36.01$ Nm

power lost $= \frac{2\pi N T}{60} = \frac{2\pi\times190\times36.01}{60} = 716.48$ watt

KINEMATICS OF FLOW

1. Kinematics ~~of~~ is defined as that branch of science which deals with motion of particles without considering the forces ~~acting~~ causing the motion. In this chapter, methods of determining velocity and acceleration are discussed.

2. TYPES OF FLUID FLOW

The fluid flow is classified as

i. steady and unsteady flows;
ii. Uniform and Non-uniform flows;
iii. Laminar and turbulent flows;
iv. Compressible and incompressible flows;
v. Rotational and irrotational flows; and
vi. One, two and three dimensional flows

1. Steady and Unsteady flows: steady flow is defined as that type of flow in which fluid characteristics like velocity, pressure, density etc at a point do not change with time.

For steady flow $\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0$, $\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0$, $\left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which velocity, pressure ~~and~~, density etc at a point changes with respect to time

For unsteady flow $\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$, $\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$ etc

ii. Uniform and Non-uniform flows: Uniform flow is defined as that type of flow in which velocity at any given time does not change with respect to space

For uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_t = \text{constant} = 0$$

where ∂V = change of velocity
∂s = length of flow in the direction S.

Non uniform flow is that type of flow in which the velocity at any given time changes with respect to space.

For non-uniform flow $\left(\frac{\partial V}{\partial s}\right)_t = \text{constant} \neq 0$

iii) Laminar and Turbulent flows: Laminar flow is defined that type of flow in which the fluid particles move along well defined paths or stream line and all the stream-lines are straight and parallel. This type of flow is also called stream-line or viscous flow.

Turbulent flow is ~~that~~ type of flow in which the fluid particles move in a zig-zag way.

For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{\rho V D}{\mu}$ called the Reynold number.

where D = Diameter of pipe
V = velocity of flow in pipe
ρ = density of fluid
μ = viscosity of fluid

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 to 4000, the flow ~~is called~~ may be laminar or turbulent.

iv) Compressive and Incompressible flows: Compressible flow is that type of flow in which the density of the fluid changes from point to point i.e density (ρ) is not constant for the fluid. For compressible flow

$$\rho \neq \text{constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow.

For incompressible flow

$$\rho = \text{constant}$$

v) Rotational and Irrotational flows: Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotates about their own axis. And if the fluid particles while flowing along stream-lines, do not rotates about their own axis that type of flow is called irrotational flow.

vi) One, Two and Three-Dimensional flows:

One-dimensional flow is that flow in which the flow parameter such ~~that~~ as velocity is a function of time and one space co-ordinate, say x.

For a steady one dimensional flow

$$u = f(x), \ v = 0, \ w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two - dimensional flow: For a steady two dimensional flow

$u = f_1(x, y)$, $v = f_2(x, y)$ and $w = 0$

Three - dimensional flow:

For a steady three dimensional flow

$u = f_1(x, y, z)$, $v = f_2(x, y, z)$, $w = f_3(x, y, z)$

RATE OF FLOW OR DISCHARGE (Q)

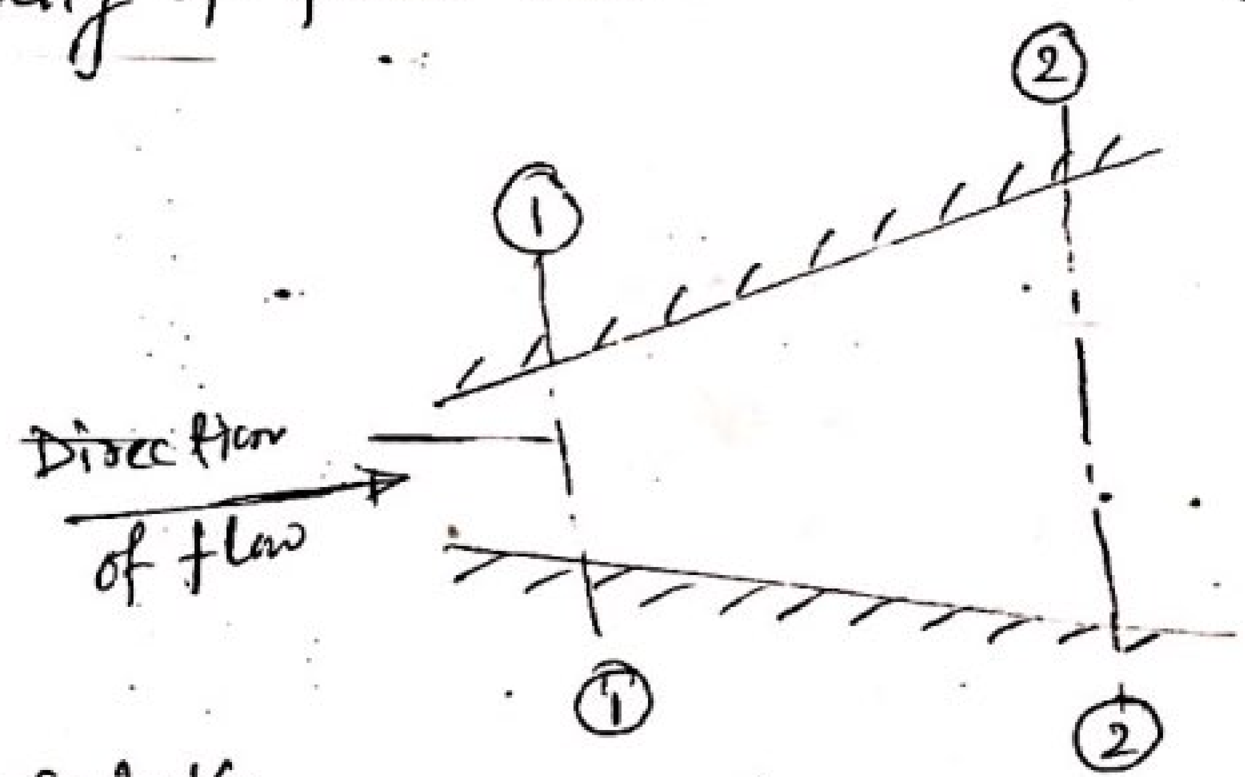
It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.

Discharge $Q = A \times V$

A = Cross-sectional area of pipe

V = Average velocity of fluid across the section.

CONTINUITY EQUATION



The equation based on the principle of the conservation of mass is called continuity equation.

Rate of flow at section 1-1

$= \rho_1 A_1 V_1$

Rate of flow at section 2-2 $= \rho_2 A_2 V_2$

According to law of conservation of mass

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

so $\boxed{\rho A V = \text{constant}}$ ——— (1)

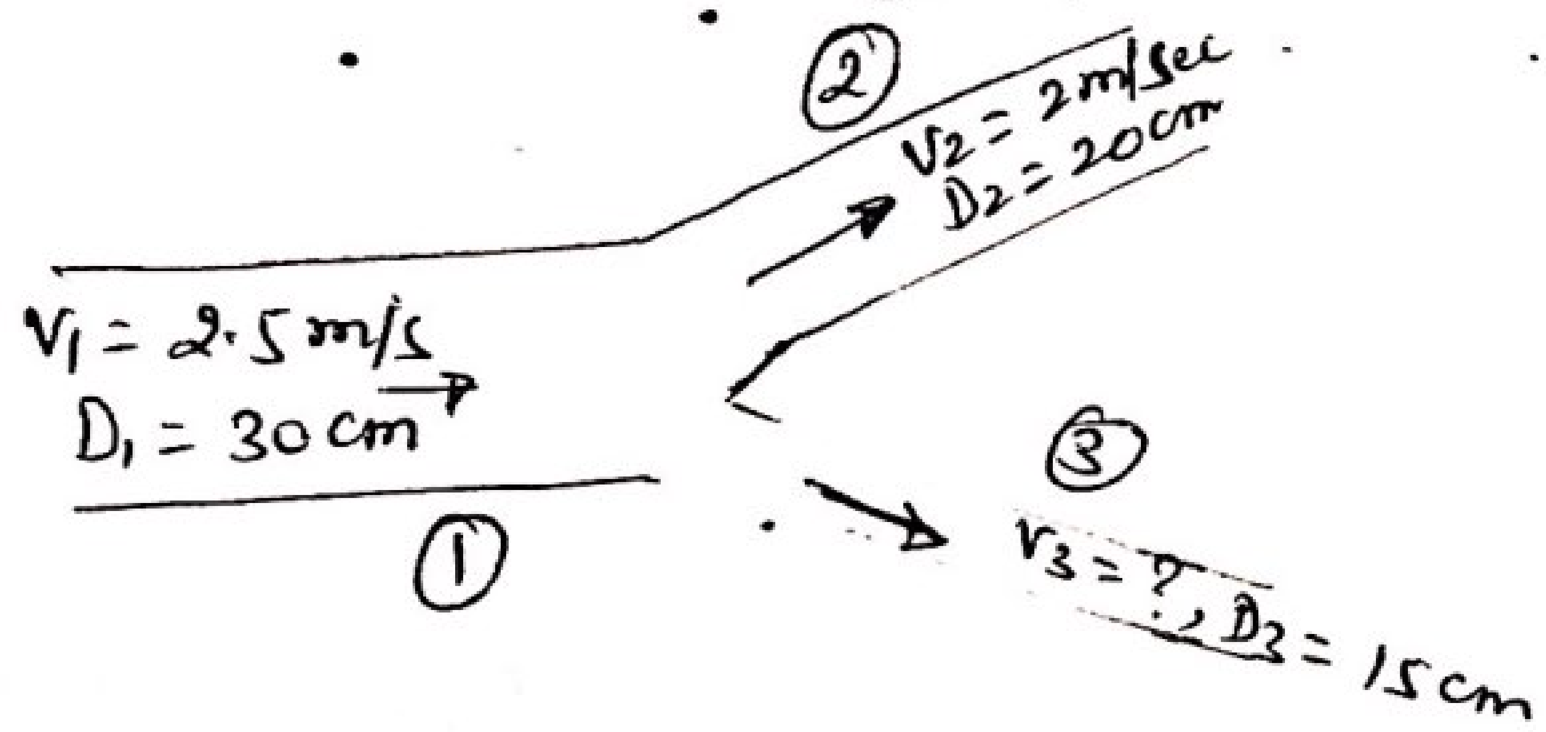
Equation (1) is ~~call~~ applicable to the compressible as well as incompressible fluids and called continuity equation. If the fluid is incompressible, then $\rho_1 = \rho_2$ then ~~eq~~ from above equation

$A_1 V_1 = A_2 V_2$

so AV = constant

Q. A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

Solution. Given



$D_1 = 30\text{ cm} = 0.3\text{ m}$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.3)^2$
$= 0.07068\text{ m}^2$

$V_1 = 2.5\text{ m/s}$

$D_2 = 20\text{ cm} = 0.2\text{ m}$

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314\text{ m}^2$

$V_2 = 2\text{ m/s}$

$D_3 = 15\text{ cm} = 0.15\text{ m}$

$A_3 = \frac{\pi}{4} (0.15)^2 = 0.01767\text{ m}^2$

According to continuity equation

$$Q_1 = Q_2 + Q_3$$

$Q_1 = A_1 V_1 = 0.07068 \times 2.5 = 0.1767\text{ m}^3/\text{s}$ Ans

$Q_2 = A_2 V_2 = 0.0314 \times 2.0 = 0.0628\text{ m}^3/\text{s}$

$Q_3 = Q_1 - Q_2 = 0.1767 - 0.0628 = 0.1139\text{ m}^3/\text{s}$

$0.1139 = A_3 V_3 = 0.01767 \times V_3$

$V_3 = \frac{0.1139}{0.01767} = 6.44\text{ m/s}$ Ans.

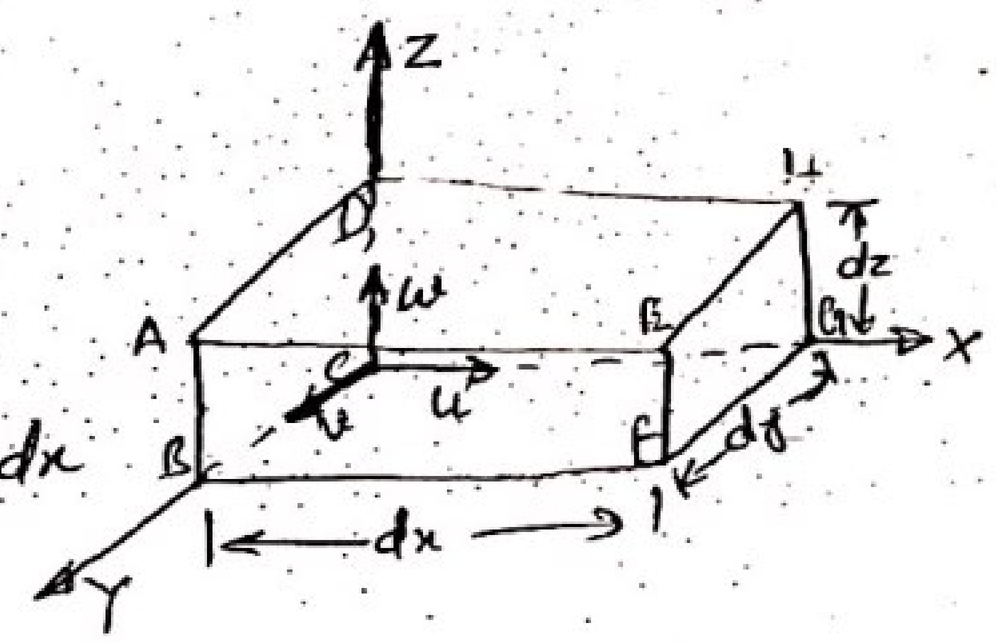
CONTINUITY EQUATION IN THREE-DIMENSIONS

Consider a fluid element of length dx, dy and dz in the direction of x, y and z. Let u, v and w are the inlet velocity component in x, y and z directions respectively. Mass of fluid entering the face ABCD per second

$$= \rho \times \text{velocity in x-direction} \times \text{area of ABCD}$$

$$= \rho \times u \times (dy \times dz)$$

Then mass of fluid leaving the face EFGH per second $= \rho u\, dy dz + \frac{\partial}{\partial x}(\rho u\, dy dz)\, dx$



Gain of mass in x-directions

$$= \rho u\, dy dz - \rho u\, dy dz - \frac{\partial}{\partial x}(\rho u\, dy dz)\, dx$$

$$= -\frac{\partial}{\partial x} \rho u\, dx dy dz$$

$$= -\frac{\partial}{\partial x}(\rho u)\, dx dy dz$$

Similarly net gain of mass in y ~~the~~ directions $= -\frac{\partial}{\partial y}(\rho v)\, dx dy dz$

" " " " in z direction $= -\frac{\partial}{\partial z}(\rho w)\, dx dy dz$

Net gain of masses $= -\left[\frac{\partial (\rho u)}{\partial x} + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx dy dz$

Since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in fluid element must be equal to the rate of increase of mass of fluid in the element. But mass of fluid in element is $\rho\, dx \cdot dy \cdot dz$ and its rate of increase with time is $\frac{\partial(\rho\, dx dy dz)}{\partial t}$ or $\frac{\partial \rho}{\partial t} dx dy dz$

Equating two equations

$$-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z}\right] dx dy dz = \frac{\partial \rho}{\partial t} dx dy dz$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \quad (1)$$

Eq^n (1) is continuity eq^n in Cartesian - Co-ordinates

For steady flow $\frac{\partial \rho}{\partial t} = 0$ hence above equation becomes

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial}{\partial y}(\rho v) + \frac{\partial (\rho w)}{\partial z} = 0$$

If the fluid is incompressible, then ρ is constant and the above equation becomes

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$ continuity equation in three-dimensions.

for two dimensions flow the component $w = 0$ so above equation becomes

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

VELOCITY AND ACCELERATION

Let V = resultant velocity at any point in a fluid flow. Let u, v and w are its component in x, y and z directions. The velocity components are functions of space-coordinates and time.

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

~~Resultant~~ velocity $V = ui + vj + wk$

$$|V| = \sqrt{u^2 + v^2 + w^2}$$

Let a_x, a_y and a_z are the total acceleration in x, y and z directions respectively. Then by the chain rule of differenciation

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\left[\ u = \frac{dx}{dt},\quad v = \frac{dy}{dt},\quad w = \frac{dz}{dt}\right.$$

$$a_y = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_z = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For steady flow $\frac{\partial V}{\partial t} = 0$, where V = resultant velocity.

So $\frac{\partial u}{\partial t} = 0$, $\frac{\partial v}{\partial t} = 0$, $\frac{\partial w}{\partial t} = 0$

Hence acceleration in x, y & z directions becomes

$$a_x = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Accelaration vector $A = a_x i + a_y j + a_z k$

$$|A| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Q. A fluid flow field is given by

$$V = x^2 y i + y^2 z j - (2xyz + yz^2)k.$$

prove that it is a case of possible steady incompressible fluid flow. Calculate velocity and acceleration at the point (2, 1, 3)

Solⁿ: For given fluid flow field $u = x^2 y$ $\frac{\partial u}{\partial x} = 2xy$

$v = y^2 z$ $\frac{\partial v}{\partial y} = 2yz$

$w = -2xyz - yz^2$ $\frac{\partial w}{\partial z} = -2xy - 2yz$

For a case of possible steady incompressible fluid-flow continuity eqⁿ should be satisfied

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

So $2xy + 2yz - 2xy - 2yz = 0$

So velocity field $V = x^2 y i + y^2 z j - (2xyz + yz^2)k$ is a possible case of fluid flow.

Velocity at (2, 1, 3)

$$V = 2^2 \times 1\, i + 1^2 \times 3 j - (2\times2\times1\times3 + 1\times3^2)k$$

$$= 4i + 3j - 21k \text{ Ans}$$

Resultant velocity $V = \sqrt{4^2 + 3^2 + (-21)^2} = \sqrt{466} = 21.587$ units Ans.

Acceleration at (2, 1, 3)

The

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$u = x^2y$, $\frac{\partial u}{\partial x} = 2xy$ $\frac{\partial u}{\partial y} = x^2$ and $\frac{\partial u}{\partial z} = 0$

$v = y^2z$, $\frac{\partial v}{\partial x} = 0$, $\frac{\partial v}{\partial y} = 2yz$, $\frac{\partial v}{\partial z} = y^2$

$w = -2xyz - yz^2$, $\frac{\partial w}{\partial x} = -2yz$, $\frac{\partial w}{\partial y} = -2xz - z^2$, $\frac{\partial w}{\partial z} = -2xy - 2yz$

At (2, 1, 3)

$a_x = x^2y(2xy) + y^2z(x^2) - (2xyz + yz^2) \times 0$

$= 2 \times 2^3 \times 1^2 + 2^2 \times 1^2 \times 3 = 28$ units

$a_y = x^2y(0) + y^2z(2yz) - (2xyz + yz^2)y^2$

$= 2 \times 1^3 \times 3^2 - 2 \times 2 \times 1^3 \times 3 - 1^3 \times 3^2 = 18 - 12 - 9 = -3$ units

$a_z = x^2y(-2xyz) + y^2z(-2xz - z^2) - (2xyz + yz^2)(-2xy - 2yz)$

$= 105$ units

$a = a_x i + a_y j + a_z k = 28i - 3j + 105k$ Ans.

STREAM FUNCTION

The stream function (ψ) is defined as scalar function of space and time, such that the partial derivative with respect to any direction gives the velocity component at right angles (in the ~~con~~ counter clockwise direction) to this direction

Stream function may be defined as $\psi(x, y, t)$ for unsteady flow & ~~dot~~ $\psi(x,y)$ for steady flow.

$$\left.\begin{aligned} \frac{\partial \psi}{\partial x} &= v \\ \frac{\partial \psi}{\partial y} &= -u \end{aligned}\right\} \quad \text{——— (1)}$$

VELOCITY POTENTIAL

The velocity potential ϕ is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction.

velocity potential is defined as $\phi(x,y,z,t)$ for unsteady flow ~~and~~ and $\phi(x,y,z)$ for steady flow.

$$\left.\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}\right\} \quad \text{——— (1)}$$

STREAM LINES : A property of stream function is that the difference of its value at two points represents the flow across any line joining the points. Therefore, when two points line on the same streamline, then since there is no flow across a streamline, the difference between stream functions ψ_1 and ψ_2 at these points is equal to zero i.e $(\psi_1 - \psi_2) = 0$

It means streamlines is given by $\psi = \text{constant}$