DELTA Pg No.
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MATHS ASSIGNMENT

Ans (1) $y'' - xy \Rightarrow 0$ ...... (i)

→ Using power series solution method →

At $x=0$.

$$y \longrightarrow \sum_{n=0}^{\infty} a_n x^n \quad \text{.... (2)}$$

$$y' \longrightarrow \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' \longrightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

putting both in eq (i)

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=0}^{\infty} a_n x^{n+1} \Rightarrow 0$$

let $n = m+2$

$$\sum_{m=0}^{\infty} (m+2)(m+1) a_{m+2} x^{m} - \sum_{n=0}^{\infty} a_n x^{n+1} \Rightarrow 0$$

→ putting $m = n$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^{n} - \sum_{n=0}^{\infty} a_n x^{n+1} \Rightarrow 0$$

coefficient of x^0.

$2 \cdot 1 \cdot a_2 \Rightarrow 0$. , $a_2 \Rightarrow 0$.

coefficient of x^1.

$3 \cdot 2 a_3 - a_0 \Rightarrow 0$, $a_3 = \frac{a_0}{3 \cdot 2}$

similarly $\div\rightarrow$

coefficient of x^n $\div\rightarrow$

$$(n+2)(n+1)a_{n+2} - a_{n-1} \Rightarrow 0$$

$$\boxed{a_{n+2} = \frac{a_{n-1}}{(n+2)(n+1)}}$$

$$a_5 = \frac{a_0}{(n+2)(n+1)} = 0.$$

so, $a_8 = 0, \ldots$ and so on.

$$a_6 = \frac{a_3}{6.5} \Rightarrow \frac{a_0}{3.2.6.5} \Rightarrow \frac{a_0}{2.3.\cancel{6}.\cancel{2}.5}$$

General $a_4 \Rightarrow \dfrac{a_1}{4.3}$

$$a_7 \Rightarrow \frac{a_4}{7.6} = \frac{a_1}{4.3\ 7.6}$$

$$y \Rightarrow \left(a_0 + \frac{a_0 x^3}{\cancel{3}.3.2} + \frac{a_0 x^6}{6.5.3.2} + \ldots\right)$$

$$+a_1\left(x + \frac{x^4}{4.3} + \frac{x^7}{7.6.4.3} + \ldots\right)$$

$$y \Rightarrow a_0\left[1+\sum_{n=1}^{\infty}\frac{x^{3n}}{(3n)(3n-1)(3n-3)\ldots 3.2}\right]$$

$$+a_1\left[x+\sum_{n=1}^{\infty}\frac{x^{3n+1}}{(3n+1)(3n)(3n-2)\ldots 4.3}\right]$$

→ About $x=1$:-

$$y \to \sum_{n=0}^{\infty} a_n(x-1)^n$$

$$y' \Rightarrow \sum_{n=1}^{\infty} n(x-1)^{n-1} a_n.$$

$$y'' \Rightarrow \sum_{n=2}^{\infty} n(n-1)\, a_n\,(x-1)^{n-2}.$$

putting all these in equation (i).

$$\sum_{n=2}^{\infty} n(n-1)a_n(x-1)^{n-2} - x\sum_{n=0}^{\infty} a_n(x-1)^n \Rightarrow 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n(x-1)^{n-2} - (x-1+1)\sum_{n=0}^{\infty} a_n(x-1)^n \Rightarrow 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n(x-1)^{n-2} - \sum_{n=0}^{\infty} a_n(x-1)^{n+1} - \sum_{n=0}^{\infty} a_n(x-1)^n \Rightarrow 0.$$

put $n=m+2$

$$\sum_{m=0}^{\infty}(m+2)(m+1)a_{m+2}(x-1)^m - \sum_{n=0}^{\infty} a_n(x-1)^{n+1} - \sum_{n=0}^{\infty} a_n(x-1)^n \Rightarrow 0$$

put $m=n$.

$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}(x-1)^n - \sum_{n=0}^{\infty} a_n(x-1)^{n+1} - \sum_{n=0}^{\infty} a_n(x-1)^n \Rightarrow 0$$

coefficient of $(x-1)^0$

$$2\cdot1\,a_2 \Rightarrow a_0$$

$$a_2 = \frac{a_0}{1\cdot2}$$

coefficient of $(x-1)^n$

$$(n+2)(n+1)a_{n+2} - a_{n-1} - a_n \Rightarrow 0$$

$$a_{n+2} \Rightarrow \frac{a_{n-1}+a_n}{(n+2)(n+1)}$$

$$a_3 \Rightarrow \frac{a_0+a_1}{6}$$

$$a_4 \Longrightarrow \frac{a_1+a_2}{4\cdot3} \Rightarrow \frac{a_1}{4\cdot3} + \frac{a_2}{4\cdot3}$$

$$\boxed{a_4 \Rightarrow \frac{a_1}{4\cdot3} + \frac{a_0}{4\cdot3\cdot2\cdot1}}$$

$$a_5 \Rightarrow \frac{a_2+a_3}{5\cdot4}$$

$$\Rightarrow \frac{a_0}{5\cdot4\cdot1\cdot2} + \frac{a_0+a_1}{6\cdot5\cdot4}.$$

$$\Rightarrow \frac{a_0}{40} + \frac{a_0}{120} + \frac{a_1}{120}.$$

$$= a_0\left[\frac{1}{30}\right] + \frac{a_1}{120}.$$

$$y \rightarrow a_0\left[1 + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6} + \frac{(x-1)^4}{24} + \frac{(x-1)^5}{30} + \cdots\right]$$

$$+ a_1\left[(x-1) + \frac{(x-1)^3}{6} + \frac{(x-1)^4}{12} + \frac{(x-1)^5}{120} + \cdots\right]$$

2. classify the singular points of the following equations :→

ⓐ $(1-x^2)y'' + 2xy' + n(n+1)y \Rightarrow 0.$

→ $p(x) \Rightarrow 0.$

$1-x^2 \neq 0$

$\boxed{x \rightarrow \pm 1}$

$\boxed{x_0 = \pm 1}$

$x_0 \Rightarrow 1$:→

$$\Rightarrow \lim_{x\to 1} \frac{2x}{(1-x^2)} \times (x-1). \rightarrow \lim_{x\to 1} \frac{-2x}{x+1} \Rightarrow -1.$$

$$\Rightarrow \lim_{x\to 1} \frac{n(n+1)}{(1-x^2)} (x-1)^2 \Rightarrow \lim_{x\to 1} \frac{-n(n+1) \times (x-1)}{(x+1)} \Rightarrow 0$$

so, $x_0 = 1$ is singular point.
and $x_0 = 1$ is regular singular point.

$x_0 = -1$:→

$$\Rightarrow \lim_{x\to -1} \frac{2x}{(1-x^2)} (x+1) \Rightarrow \lim_{x\to -1} \frac{-2x}{x-1} \Rightarrow -1.$$

$$\lim_{x\to -1} \frac{n(n+1)}{(1-x^2)} (x+1)^2 \Rightarrow \lim_{x\to -1} \frac{-n(n+1)(x+1)}{x-1} \Rightarrow 0.$$

So, $x_0 = -1$ is also a regular singular point.

So, $x_0 = 1, -1$ are analytical point of given equations.

(b) $x^3(x-2)y'' + x^3y' + 6y \Rightarrow 0$.

$\Longrightarrow a_0(x)y'' + a_1(x)y' + a_2(x)y \Rightarrow 0$.

$a_0(x) = 0$

$x^3(x-2) = 0$

$x=0, \ x=2.$ are singular point.

For singular point: $\longrightarrow \ x \Rightarrow 0.$

$$\lim_{x\to 0} \frac{\cancel{x^3} \times x}{\cancel{x^3}(x-2)} \Longrightarrow 0$$

$$\lim_{x\to 0} \frac{6\cancel{x^2}}{x\cancel{x^3}(x-2)} \Longrightarrow \infty.$$

$x=0$ is irregular singular point.

For $x=2$.

$$\lim_{x\to 2} \frac{\cancel{x^3}\ \cancel{(x-2)}}{\cancel{x^3}\cancel{(x-2)}} \Longrightarrow 1.$$

$$\lim_{x\to 2} \frac{6\ \cancel{(x-2)}^2 (x-2)}{x^3\cancel{(x-2)}} \Longrightarrow 0.$$

So, $x=2$ is regular singular point.

(c) $\left(x-\frac{\pi}{2}\right)^2 y'' + \cos x y' + \sin x y \Rightarrow 0$

$\rightarrow$ $a_0(x) y'' + a_1(x) y' + a_2(x) y \Rightarrow 0.$

$a_0(x) = 0$

$\left(x - \frac{\pi}{2}\right)^2 = 0$

$x \Rightarrow \frac{\pi}{2}.$

Singular point at $x \Rightarrow \frac{\pi}{2}.$

$\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\left(x-\frac{\pi}{2}\right)^2}\left(x-\frac{\pi}{2}\right) \Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\left(x-\frac{\pi}{2}\right)} \Rightarrow \lim_{h\to 0} \frac{-\sin h}{h} \Rightarrow -1$

$\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{\left(x-\frac{\pi}{2}\right)^2}\left(x-\frac{\pi}{2}\right)^2 \Rightarrow \lim_{x\to\frac{\pi}{2}} \sin x \Rightarrow 0$

So, $x \Rightarrow \pi/2$ is regular singular point.

3(a) $y'' + (x-1)y' + y = 0$ about $x = 2$.

$\longrightarrow$ $y \Rightarrow \sum_{n=0}^{\infty} a_n (x-2)^n$

$y' \Longrightarrow \sum_{n=1}^{\infty} n a_n (x-2)^{n-1}.$

$y'' \Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n (x-2)^{n-2}$

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putting in equation :→

$$\sum_{n=2}^{\infty} n(n-1)a_n(x-2)^{n-2} + (x-1)\sum_{n=1}^{\infty} na_n(x-2)^{n-1}$$

putting $n=m+2$ & $m=n$

$$+\sum_{n=0}^{\infty} a_n(x-2)^n \Rightarrow 0$$

$$\Rightarrow \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}(x-2)^n + (x-2+1)\sum_{n=1}^{\infty} na_n(x-2)^{n-1}$$

$$+\sum_{n=0}^{\infty} a_n(x-2)^n \Rightarrow 0.$$

$$\Rightarrow \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}(x-2)^n + \sum_{n=1}^{\infty} na_n(x-2)^n + \sum_{n=1}^{\infty} na_n(x-2)^{n-1}$$

$$+\sum_{n=0}^{\infty} a_n(x-2)^n \Rightarrow 0.$$

$$\Rightarrow \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}(x-2)^n + \sum_{n=0}^{\infty}(n+1)a_{n+1}(x-2)^{n+1}$$

$$+\sum_{n=0}^{\infty}(n+1)a_{n+1}(x-2)^n + \sum_{n=0}^{\infty} a_n(x-2)^n \Rightarrow 0$$

coefficient of $(x-2)^0$:

$$2\cdot 1 a_2 + a_1 + a_0 \Rightarrow 0.$$

coefficient of $(x-2)^n$.

$$(n+2)(n+1)a_{n+2} + na_n + (n+1)a_{n+1} + a_n \Rightarrow 0$$

$$(n+2)(n+1)a_{n+2} + (n+1)[a_n + a_{n+1}] = 0.$$

$$\boxed{a_{n+2} = \frac{-[a_n + a_{n+1}]}{n+2}.}$$

$$a_3 \Rightarrow -\frac{[a_1 + a_2]}{3} = -\frac{a_1}{3} - \frac{a_2}{3}$$

$$a_4 \Rightarrow a_3 = \frac{-a_1}{3} + \frac{1}{3}\left[\frac{a_1 + a_0}{2}\right]$$

$$a_3 \Rightarrow -\frac{a_1}{3} + \frac{a_1}{6} + \frac{a_0}{6}$$

$$a_3 \Rightarrow \frac{-a_1}{6} + \frac{a_0}{6}$$

$$a_4 \Rightarrow \frac{-[a_2 + a_3]}{4}$$

$$a_4 \Rightarrow -\frac{a_2}{4} - \frac{a_3}{4}$$

$$\Rightarrow \frac{1}{4} \times \left(\frac{a_1 + a_0}{2}\right) + \frac{1}{4} \times \left(\frac{a_0 - a_1}{6}\right)$$

$$= \frac{a_1}{8} + \frac{a_0}{8} + \frac{a_0}{24} - \frac{a_1}{24}$$

$$a_4 = \frac{a_1}{6} + \frac{a_0}{12}$$

$$a_5 = \frac{-[a_3 + a_4]}{5}$$

$$a_5 \Rightarrow \frac{-a_3}{5} - \frac{a_4}{5} \Rightarrow \cancel{\frac{+a_1}{30}} - \frac{a_0}{30} - \frac{a_0}{60} \cancel{-\frac{a_1}{30}}$$

$$a_5 \Rightarrow \text{[illegible]} - \frac{a_0}{20}$$

$$y \rightarrow a_0 + a_1(x-2) - \frac{(a_0+a_1)}{2}(x-2)^2 + \frac{(a_0-a_1)}{6}(x-2)^3 + \left(\frac{a_1}{6} + \frac{a_0}{12}\right)(x-2)^4 + \left(\frac{a_1}{60} - \frac{a_0}{20}\right)(x-2)^5$$

$$y \Rightarrow a_0\left[1 - \frac{(x-2)^2}{2} + \frac{(x-2)^3}{6} + \frac{(x-2)^4}{12} - \frac{(x-2)^5}{20} + \cdots\right] + a_1\left[(x-2) - \frac{(x-2)^2}{2} - \frac{(x-2)^3}{6} + \frac{(x-2)^4}{6} + \frac{(x-2)^5}{60} + \cdots\right]$$

(b) $(1-x^2)y'' + 2xy' + y = 0$ about $x=0$.

$\rightarrow$ $$y \Rightarrow \sum_{n=0}^{\infty} a_n x^n$$

$$y' \Rightarrow \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' \Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

putting all these in equation,

$$\Rightarrow (1-x^2)\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + 2x\sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n \Rightarrow 0$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_n x^n + 2\sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n \Rightarrow 0$$

$n = m+2$ & $m = n$. $\quad$ $n = m+2$ & $m = n$ $\quad$ $n = m+1$ & $m = n$.

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n - \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^{n+2} + \sum_{n=0}^{\infty} 2(n+1)a_{n+1}x^{n+1} + \sum_{n=0}^{\infty} a_n x^n \Rightarrow 0$$

$\Rightarrow$ constant term coefficient $(x^0) \Rightarrow$

$2\cdot 1 a_2 + a_0 \Rightarrow 0$

$$\boxed{a_2 \Rightarrow \frac{-a_0}{2\cdot 1}}$$

coefficient of x^n :$\rightarrow$

$(n+2)(n+1) a_{n+2} - n(n-1)a_n + 2n a_n + a_n \Rightarrow 0$

$(n+2)(n+1) a_{n+2} = [n(n-1) - 2n - 1] a_n$

$$a_{n+2} \Rightarrow \frac{[n^2 - 3n - 1] a_n}{(n+2)(n+1)}$$

$$\boxed{a_3 \Rightarrow \frac{-a_1}{2}}$$

$$a_5 = \frac{-1}{5\cdot 4} \times \frac{-a_1}{2} \Rightarrow \frac{a_1}{(5\cdot 4)(2\cdot 1)} = \frac{a_1}{40}$$

$$a_4 = \frac{-a_2}{4} \Rightarrow \frac{a_0}{4\cdot 2\cdot 1}$$

$y \Rightarrow a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \ldots$

$\Rightarrow a_0 + a_1 x - \frac{a_0 x^2}{2} - \frac{a_1 x^3}{2} + \frac{a_0 x^4}{8} + \frac{a_1 x^5}{40} + \ldots$

$\Rightarrow a_0 \left[1 - \frac{x^2}{2} + \frac{1}{8} x^4 + \ldots\right]$

$+ a_1 \left[x - \frac{x^3}{2} + \frac{x^5}{40} + \ldots\right]$

(c) $y'' + (\cos x) y \Rightarrow 0$ about $x \Rightarrow 0$.

$\Rightarrow$ let $y \Rightarrow \sum_{n=0}^{\infty} a_n x^n$

$\Rightarrow y' \Rightarrow \sum_{n=1}^{\infty} n a_n x^{n-1} \Rightarrow \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$

$y'' \Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

$\Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \cos x \sum_{n=0}^{\infty} a_n x^n \Rightarrow 0$

$n = m+2$
$m = n$

$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{r=0}^{\infty} \frac{(-1)^r x^{2r}}{(2r)!} \times \sum_{n=0}^{\infty} a_n x^n \Rightarrow 0$

$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=0}^{\infty} \left(a_n - \frac{a_{n-2}}{2!} + \frac{a_{n-4}}{4!} + \cdots\right) x^n$

$\Rightarrow$ coefficient of $x^0 \rightarrow$

$(2 \cdot 1 a_2 + a_0) \Rightarrow 0$

$$\boxed{a_2 = \frac{-a_0}{2 \cdot 1}}$$

coefficient of $x^1 \rightarrow$

$(3 \cdot 2 a_3 + a_1) x \Rightarrow 0$

$$\boxed{a_3 = \frac{-a_1}{3 \cdot 2}}$$

coefficient of $x^2 \rightarrow$

$(4 \cdot 3) a_4 + \left(a_2 - \frac{a_0}{2}\right) \Rightarrow 0$

$12 a_4 \Rightarrow a_0$

$$\boxed{a_4 = \frac{a_0}{12}}$$

coefficient of x^3 :→

$$\left(a_3 - \frac{a_1}{2}\right) + 5\cdot4 a_5 \Rightarrow 0$$

$$(5\cdot4)a_5 \Rightarrow \frac{a_1}{2} - a_3$$

$$a_5 \Rightarrow \frac{a_1}{40} + \frac{a_1}{120} \Rightarrow \frac{a_1}{30}$$

coefficient of x^4 :→

$$(6\cdot5)a_6 + \left(a_4 - \frac{a_1}{2} + \frac{a_0}{4!}\right)$$

$$30a_6 \Rightarrow -\left[\frac{a_0}{12} + \frac{a_0}{2} + \frac{a_0}{24}\right]$$

$$30a_6 \Rightarrow \frac{-15a_0}{24}$$

$$\boxed{a_6 \Rightarrow \frac{-a_0}{48}}$$

$$y \Rightarrow a_0 + a_1 x - \frac{a_0}{2}x^2 - \frac{a_1}{6}x^3 + \frac{a_0}{12}x^4 + \frac{a_1}{30}x^5 - \frac{a_0}{48}x^6 + - - -$$

$$\Rightarrow a_0\left[1 - \frac{x^2}{2} + \frac{x^4}{12} - \frac{x^6}{48} + \cdots\right] + a_1\left[x - \frac{x^3}{6} + \frac{x^5}{30} + \cdots\right]$$

(4) Find a power series solution of
$(x^2-1)y''(x) + 3xy'(x) + xy(x) \Rightarrow 0$ subject to
(a) $y(0) = 4$, $y'(0) = 6$ and
(b) $y(2) = 4$, $y'(2) = 6$.

$\Rightarrow$ Let $y \Rightarrow \sum_{n=0}^{\infty} a_n x^n$

$y' \Rightarrow \sum_{n=1}^{\infty} n a_n x^{n-1}$

$y'' \Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

$\Rightarrow (x^2-1) \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + 3x \sum_{n=1}^{\infty} n a_n x^{n-1} + x \sum_{n=0}^{\infty} a_n x^n \Rightarrow 0.$

$\Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^n - \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + 3\sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^{n+1} \Rightarrow 0$

shifting at $n=0$.

$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^{n+2} - \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=0}^{\infty} 3(n+1) a_{n+1} x^{n+1} + \sum_{n=0}^{\infty} a_n x^{n+1} \Rightarrow 0$

constant term :→

$2 \cdot 1 a_2 \Rightarrow 0.$

$a_2 \Rightarrow 0.$

coefficient of x^n :→

$\Rightarrow n(n-1)a_n - (n+2)(n+1)a_{n+2} + 3na_n + a_{n-1} x^n \Rightarrow 0$

$\Rightarrow (n^2 - n + 3n)a_n - (n+2)(n+1)a_{n+2} + a_{n-1} \Rightarrow 0$

$\Rightarrow n(n+2)a_n + a_{n-1} = (n+2)(n+1)a_{n+2}$

$$\boxed{a_{n+2} \Rightarrow \frac{n(n+2)a_n + a_{n-1}}{(n+2)(n+1)}}$$

$$a_{n+2} \Rightarrow \frac{n}{n+1}a_n + \frac{a_{n-1}}{(n+2)(n+1)}$$

$$\boxed{a_3 \Rightarrow \frac{a_1}{2} + \frac{a_0}{3.2}}$$

$$a_4 \Rightarrow \frac{2}{3}a_2 + \frac{a_1}{4.3.}$$

$$\boxed{a_4 = \frac{a_1}{12}}$$

$$a_5 = \frac{3}{4}a_3 + \frac{a_2}{5.4} = \frac{3}{4}a_3.$$

so. $a_5 = \frac{3}{4} \times \frac{a_1}{2} + \frac{3}{4} \times \frac{a_0}{3.2}$

$$\boxed{a_5 = \frac{3a_1}{8} + \frac{a_0}{8}}$$

$$y \Rightarrow a_0 + a_1 x + \left(\frac{a_1}{2} + \frac{a_0}{3.2}\right)x^3$$

$$+ \frac{a_1}{12}x^4 + \left(\frac{3a_1}{8} + \frac{a_0}{8}\right)x^5 + \ldots$$

$$y = a_0\left[1 + \frac{x^3}{6} + \frac{x^5}{8} + \cdots\right] + a_1\left[x + \frac{x^3}{2} + \frac{x^4}{12} + \frac{3}{8}x^5 + \cdots\right]$$

$y(0) \Rightarrow a_0 \Rightarrow 4.$
$y'(0) \Rightarrow a_1 = 6.$

$$y = 4\left[1 + \frac{x^3}{6} + \frac{x^5}{8} + \cdots\right] + 6\left[x + \frac{x^3}{2} + \frac{x^4}{12} + \frac{3}{8}x^5 + \cdots\right]$$

$$\underline{\underline{y}} = 4 + 6x + \frac{11}{3}x^3 + \frac{x^4}{2} + \frac{11}{48}x^5 + \cdots$$

(b) $y(2) = 4, \quad y'(2) = 6.$

$$\Rightarrow \boxed{y = \sum_{n=0}^{\infty} a_n (x-2)^n}$$

$$\Rightarrow \quad y' \Rightarrow \sum_{n=1}^{\infty} n a_n (x-2)^{n-1}.$$

$$y'' \Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n (x-2)^{n-2}.$$

$$(x^2-1)\sum_{n=2}^{\infty} n(n-1) a_n (x-2)^{n-2} + 3x\sum_{n=1}^{\infty} n a_n (x-2)^{n-1} + x\sum_{n=0}^{\infty} a_n (x-2)^n \Rightarrow 0.$$

$$\Rightarrow \left[(x-2+2)^2-1\right]\sum_{n=2}^{\infty} n(n-1)a_n(x-2)^{n-2}$$
$$+3(x-2+2)\sum_{n=1}^{\infty} na_n(x-2)^{n-1} + (x-2+2)\sum_{n=0}^{\infty} a_n(x-2)^n = 0$$

$$\Rightarrow \left[(x-2)^2+4(x-2)+3\right]\sum_{n=2}^{\infty} n(n-1)a_n(x-2)^{n-2}$$
$$+3(x-2)\sum_{n=1}^{\infty} na_n(x-2)^{n-1} + 6\sum_{n=1}^{\infty} na_n(x-2)^{n-1}$$
$$+(x-2)\sum_{n=0}^{\infty} a_n(x-2)^n + 2\sum_{n=0}^{\infty} a_n(x-2)^n = 0$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_n(x-2)^n + 4\sum_{n=2}^{\infty} n(n-1)a_n(x-2)^{n-1} + 3\sum_{n=2}^{\infty} n(n-1)a_n(x-2)^{n-2}$$
$$+3\sum_{n=1}^{\infty} na_n(x-2)^n + 6\sum_{n=1}^{\infty} na_n(x-2)^{n-1}$$
$$+\sum_{n=0}^{\infty} a_n(x-2)^{n+1} + \sum_{n=0}^{\infty} 2a_n(x-2)^n = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}(x-2)^{n+2} + 4\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}(x-2)^{n+1}$$
$$+3\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}(x-2)^n + 3\sum_{n=0}^{\infty} (n+1)a_{n+1}(x-2)^{n+1}$$
$$+6\sum_{n=0}^{\infty} (n+1)a_{n+1}(x-2)^n + \sum_{n=0}^{\infty} a_n(x-2)^{n+1}$$
$$+\sum_{n=0}^{\infty} 2a_n(x-2)^n = 0$$

coefficient of $(x-2)^0$:

$$3\cdot 2\cdot a_2 + 6\cdot a_1 + 2a_0 = 0$$

$$\boxed{a_2 \Rightarrow -a_1 - \frac{a_0}{3}}$$

coefficient of $(x-2)^n$.

$\Rightarrow n(n-1)a_n + 4(n+1)n a_{n+1} + 3(n+2)(n+1)a_{n+2}$
$+ \cancel{3(n+1)a_{n+1}}\ 3n a_n + 6(n+1)a_{n+1} + a_{n-1}$
For $\underline{a_3 \div ?}$ $+ 2a_n \Rightarrow 0$
$\cancel{3\times 2 a_3 + 4\times 4\times 3 a_4}$

$\Rightarrow 3(n+2)(n+1)a_{n+2} + [n(n-1) + 3n + 2]a_n$
$+ [4(n+1)n + 6(n+1)]a_{n+1} + a_{n-1} \Rightarrow 0.$

$\Rightarrow 3(n+2)(n+1)a_{n+2} + (n^2+2n+2)a_n$
$+ (4n+6)(n+1)a_{n+1} + a_{n-1} \Rightarrow 0.$

$\Rightarrow a_{n+2}\times 3(n+2)(n+1)$
$\Rightarrow -(n^2+2n+2)a_n - 2(2n+3)(n+1)a_{n+1} - a_{n-1}$

$$\Rightarrow a_{n+2} \Rightarrow -\frac{[n^2+2n+2]a_n}{3[n^2+3n+2]} - \frac{2(2n+3)}{3(n+2)}a_{n+1} - \frac{a_{n-1}}{3(n+2)(n+1)}$$

$$\Rightarrow a_3 \Rightarrow -\frac{5}{18}a_1 - \frac{2}{3}\times\frac{5}{3}a_2 - \frac{a_0}{3\times 3\times 2}$$

$$\Rightarrow a_3 \Rightarrow -\frac{5}{18}a_1 - \frac{a_0}{18} + \frac{10}{9}\left(a_1 + \frac{a_0}{3}\right).$$

$$\Rightarrow \frac{-5a_1}{18} + \frac{10}{9}a_1 - \frac{a_0}{18} + \frac{10}{27}a_0$$

$$a_3 \Rightarrow \frac{15}{18}a_1 + \frac{17}{54}a_0,$$

$$a_3 = \frac{5}{6}a_1 + \frac{17}{54}a_0$$

$$a_4 \Rightarrow \frac{-10}{3\times 12}a_2 - \frac{7}{6}a_3 - \frac{a_1}{36}$$

$$a_4 = \frac{+5}{18}\left(a_1 + \frac{a_0}{3}\right) - \frac{7}{6}\left(\frac{5}{6}a_1 + \frac{17}{54}a_0\right) - \frac{a_1}{36}$$

$$a_4 = \left(\frac{5}{18}a_1 - \frac{35a_1}{36} - \frac{a_1}{36}\right) - \frac{89}{324}a_0$$

$$a_4 = \frac{-13}{18}a_1 - \frac{89}{324}a_0$$

$$y \Rightarrow a_0 + a_1(x-2) - \left(a_1 + \frac{a_0}{3}\right)(x-2)^2 + \left(\frac{5}{6}a_1 + \frac{17}{54}a_0\right)(x-2)^3 + \left(\frac{-13}{18}a_1 - \frac{89}{324}a_0\right)(x-2)^4 + \dots$$

$$y \Rightarrow a_0 - \frac{a_0}{3}(x-2)^2 + \frac{17a_0}{54}(x-2)^3 - \frac{89a_0}{324}(x-2)^4 + \dots$$

$$+ a_1\left[(x-2) - (x-2)^2 + \frac{5}{6}(x-2)^3 - \frac{13}{18}(x-2)^4 + \dots\right].$$

For $y(2) = 4$, $a_0 = 4$

$y'(2) = 6$, $a_1 = 6$.

$$y = a_0\left[1 - \frac{(x-2)^2}{3} + \frac{17}{54}(x-2)^3 - \frac{89}{324}(x-2)^4 + \dots\right]$$
$$+ a_1\left[(x-2) - (x-2)^2 + \frac{5}{6}(x-2)^3 - \frac{13}{18}(x-2)^4 + \dots\right]$$

$$\text{so } y = 4\left[1 - \frac{(x-2)^2}{3} + \frac{17}{54}(x-2)^3 - \frac{89}{324}(x-2)^4 + \dots\right]$$
$$+ 6\left[(x-2) - (x-2)^2 + \frac{5}{6}(x-2)^3 - \frac{13}{18}(x-2)^4 + \dots\right]$$

$$y = 4 + 6(x-2) - \frac{22}{3}(x-2)^2 + \frac{169}{27}(x-2)^3 - \frac{440}{81}(x-2)^4 + \dots$$

(5) use the method of frobenius to find solutions of the following differential eqn:→

(a) $2x^2y'' - xy' + (x-5)y \Rightarrow 0.$

→ $x=0$ is a singular point.

and $\lim_{x\to 0} \frac{-x}{2x^2} \times x \implies \frac{-1}{2}.$

$\lim_{x\to 0} \frac{(x-5)\,x^2}{2x^2} \Rightarrow -\frac{5}{2}.$

so, $x=0$ is a regular singular point.

$\rightarrow$ $$y \Rightarrow \sum_{n=0}^{\infty} a_n x^{n+r}$$

$$y' \Rightarrow \sum_{n=0}^{\infty} (n+r) a_n x^{n+r-1}$$

$$y'' \Rightarrow \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n x^{n+r-2}$$

$\rightarrow$ $$2\sum_{n=0}^{\infty} (n+r)(n+r-1) a_n x^{n+r} - \sum_{n=0}^{\infty} (n+r) a_n x^{n+r} + \sum_{n=0}^{\infty} a_n x^{n+r+1} - \sum_{n=0}^{\infty} 5 a_n x^{n+r} \Rightarrow 0$$

coefficient of $x^r :\rightarrow (n=0)$

$\rightarrow$ $$[2r(r-1) - r - 5] a_0 \Rightarrow 0$$

$$2r^2 - 3r - 5 \Rightarrow 0$$

$$2r^2 - 5r + 2r - 5 \Rightarrow 0$$

$$r(2r-5) + 1(2r-5) \Rightarrow 0$$

$$\boxed{r \Rightarrow -1, \frac{5}{2}}$$

coefficient of $x^{n+r} :\rightarrow$

$$2(n+r)(n+r-1) a_n - (n+r) a_n + a_{n-1} - 5a_n \Rightarrow 0$$

$$[2(n+r)(n+r-1) - (n+r) - 5] a_n + a_{n-1} \Rightarrow 0$$

$$a_n \Rightarrow \frac{-a_{n-1}}{\cancel{2}[(n+r)(2n+2r-3) - 5]}$$

For $r = -1 :\rightarrow$

$$a_n \Rightarrow \frac{-a_{n-1}}{[(n-1)(2n-5) - 5]}$$

$$\boxed{a_1 \Rightarrow \frac{+a_0}{\cdot 5}}$$

$$a_2 \Rightarrow \frac{-a_1}{-6} \Rightarrow \frac{a_1}{6} \Rightarrow \frac{a_0}{30}$$

$$a_3 \Rightarrow \frac{-a_2}{-3} \Rightarrow \frac{a_0}{90}$$

$$a_4 \Rightarrow \frac{-a_3}{4} \Rightarrow \frac{-a_0}{360}$$

For $r \Rightarrow \frac{5}{2}$;

$$a_n = \frac{-a_{n-1}}{\left[\left(n+\frac{5}{2}\right)(2n+2) - 5\right]}$$

$$a_n = \frac{-a_{n-1}}{(2n+5)(2n+2) - 5}$$

$$a_n = \frac{-a_{n-1}}{n(2n+7)}$$

$$a_1 \Rightarrow \frac{-a_0}{9}$$

$$a_2 = \frac{-a_1}{22} \Rightarrow \frac{a_0}{22\times 9} \Rightarrow \frac{a_0}{198}$$

$$a_3 \Rightarrow \frac{-a_2}{3\times 13} \Rightarrow \frac{-a_0}{198\times 39} \Rightarrow \frac{-a_0}{7722}$$

$$a_4 \Rightarrow \frac{-a_3}{4 \times 15} \Rightarrow \frac{a_0}{7722 \times 60} \Rightarrow \frac{a_0}{463320}$$

complete solution :→

$$y \to c_1 a_0 \left[1 + \frac{x}{5} + \frac{x^2}{30} + \frac{x^3}{90} - \frac{x^4}{360} + \cdots\right]$$

$$+ c_2 a_0 \left[1 - \frac{x}{9} + \frac{x^2}{198} - \frac{x^3}{7722} + \frac{x^4}{463320} + \cdots\right]$$

$$y = \frac{c_1}{x}\left[1 + \frac{x}{5} + \frac{x^2}{30} + \frac{x^3}{90} - \frac{x^4}{360} + \cdots\right]$$

$$+ c_2 x^{5/2}\left[1 - \frac{x}{9} + \frac{x^2}{198} - \frac{x^3}{7722} + \frac{x^4}{463320} + \cdots\right]$$

(b) $2x^2y'' + xy' + (x^2-3)y \Rightarrow 0$

$$\Rightarrow \quad y \Rightarrow \sum_{n=0}^{\infty} a_n x^{n+r}.$$

$$y' \Rightarrow \sum_{n=0}^{\infty} (n+r)\, a_n x^{n+r-1}$$

$$y'' \Rightarrow \sum_{n=0}^{\infty} (n+r)(n+r-1)\, a_n x^{n+r-2}.$$

putting in equation :→

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$$\Rightarrow 2\sum_{n=0}^{\infty}(n+r)(n+r-1)a_n x^{n+r} + \sum_{n=0}^{\infty}(n+r)a_n x^{n+r}$$

$$+ (x^2-3)\sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow 0$$

$$\Rightarrow \sum_{n=0}^{\infty} 2(n+r)(n+r-1)a_n x^{n+r} + \sum_{n=0}^{\infty}(n+r)a_n x^{n+r}$$

$$+ \sum_{n=0}^{\infty} a_n x^{n+r+2} - \sum_{n=0}^{\infty} 3a_n x^{n+r} \Rightarrow 0$$

coefficient of x^r :$\longrightarrow$

$$2r(r-1) + r - 3 \Rightarrow 0$$

$$2r^2 - r - 3 \Rightarrow 0.$$

$$2r^2 - 3r + 2r - 3 = 0$$

$$r(2r-3) + 1(2r-3) = 0$$

$$\boxed{r = -1, \tfrac{3}{2}}$$

$\Rightarrow$ coefficient of x^{n+r} :$\longrightarrow$

$$2((n+r)(n+r-1)a_n + (n+r)a_n + a_{n-2} - 3a_n$$

$$[(n+r)(2n+2r-1) - 3]a_n + a_{n-2} \Rightarrow 0.$$

$$a_n \Rightarrow \frac{-a_{n-2}}{[(n+r)(2n+2r-1)-3]}$$

<u>For $r=-1$:$\longrightarrow$</u>

$$a_2 \Rightarrow \frac{-a_0}{[(1)(1)-3]} \Rightarrow \frac{a_0}{2}$$

$$a_4 \Rightarrow \frac{-a_2}{12} \Rightarrow \frac{-a_0}{24}$$

for coefficient of x^{r+1} :→

$$[2(r+1)r + (r+1) - 3]\, a_1 \Rightarrow 0.$$

For $(r = -1)$:→

$$\boxed{a_1 \Rightarrow 0.}$$

so, $a_3 = \dfrac{-a_1}{2\times 3 - 3} \Rightarrow 0.$

and so on $a_5 \Rightarrow a_7 \Rightarrow 0$:-

For $r = \frac{3}{2}$;

$$a_n \Rightarrow \frac{-a_{n-2}}{\left[(2n+\frac{3}{2})(n+1) - 3\right]}$$

$$\boxed{a_n \Rightarrow \frac{-a_{n-2}}{n(2n+5)}}$$

coefficient of x^{r+1} :→

$$[2(r+1)r + (r+1) - 3]\, a_1 = 0$$

For $r = \frac{3}{2}$ $\boxed{a_1 \Rightarrow 0}$

So, $a_5 = a_3 = a_1 \Rightarrow 0$

$$a_2 = \frac{-a_0}{18}$$

$$a_4 \Rightarrow \frac{-a_2}{52} = \frac{a_0}{18\times 52} \Rightarrow \frac{a_0}{936}.$$

$$y \Rightarrow c_1 x^{-1}\left[a_0 + \frac{a_0 x^2}{2} - \frac{a_0 x^4}{24} + \cdots\right] + c_2 x^{3/2}\left[a_0 - \frac{a_0}{18}x^2 - \frac{a_0}{936}x^4 + \cdots\right]$$

$$y = c_1 a_0 x^{-1}\left[1 + \frac{x^2}{2} - \frac{x^4}{24} + \cdots\right] + c_2 a_0 x^{3/2}\left[1 - \frac{x^2}{18} - \frac{x^4}{936} + \cdots\right]$$

$$y = c_1 x^{-1}\left[1 + \frac{x^2}{2} - \frac{x^4}{24} + \cdots\right] + c_2 x^{3/2}\left[1 - \frac{x^2}{18} + \frac{x^4}{936} + \cdots\right].$$

(6) $x^2y'' - xy' - \left(x^2 + \frac{5}{4}\right)y = 0.$

→ Let the summation be $y = \sum_{n=0}^{\infty} a_n x^{n+r}$

$$y' \Rightarrow \sum_{n=0}^{\infty} (n+r) a_n x^{n+r-1}$$

$$y'' \Rightarrow \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n x^{n+r-2}.$$

putting in equation:$\rightarrow$

$$\Rightarrow \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n x^{n+r} - \sum_{n=0}^{\infty} (n+r) a_n x^{n+r} - \left(x^2 + \frac{5}{4}\right) \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n x^{n+r} - \sum_{n=0}^{\infty} (n+r) a_n x^{n+r} - \sum_{n=0}^{\infty} a_n x^{n+r+2} - \sum_{n=0}^{\infty} \frac{5}{4} a_n x^{n+r} \Rightarrow 0.$$

coefficient of x^r:$\rightarrow$

$$\rightarrow \left[r(r-1) - r - \frac{5}{4} \right] a_0 \Rightarrow 0$$

$$\left[r^2 - 2r - \frac{5}{4} \right] \Rightarrow 0 \qquad (a_0 \neq 0).$$

$$\left[4r^2 - 8r - 5 \right] = 0.$$

$$4r^2 - 10r + 2r - 5 = 0$$

$$2r(2r-5) + 1(2r-5)$$

$$r = \frac{5}{2}, \ -\frac{1}{2}.$$

Since, $\frac{5}{2} + \frac{1}{2} \Rightarrow 3$

coefficient of x^{n+r} $:\longrightarrow$

$$(n+r)(n+r-1)a_n - (n+r)a_n - a_{n-2} - \frac{5}{4}a_n \Rightarrow 0$$

$$a_{n-2} \Rightarrow \left[(n+r)(n+r-2) - \frac{5}{4}\right]a_n$$

$$a_n \Rightarrow \frac{4a_{n-2}}{4(n+r)(n+r-2)-5}$$

coefficient of x^{r+1} $:\longrightarrow$

$$\left[(r+1)r\,a_1 - (r+1)a_1 - \frac{5}{4}a_1\right] = 0$$

$$\left[r^2 + \cancel{r} - \cancel{r} - 1 - \frac{5}{4}\right]a_1 = 0.$$

$$\left[r^2 - \frac{9}{4}\right]a_1 = 0$$

$$\boxed{a_1 = 0}$$

$$a_3 \Rightarrow \frac{4a_1}{4(3+\frac{5}{2})(3+\frac{5}{2}-2)-5} \Rightarrow 0.$$

$$a_5 = a_3 = a_1 \Rightarrow 0$$

For $(r = 5/2)$ $:\longrightarrow$

$$a_2 \Rightarrow \frac{4a_0}{4(2+\frac{5}{2})(2+\frac{5}{2}-2)}$$

$$a_2 = \frac{4a_0}{40} = \frac{a_0}{10}$$

$$a_4 = \frac{4a_2}{4\times\left[4+\frac{5}{2}\right]\left[4+\frac{5}{2}-2\right]-5} = \frac{a_2}{28} = \frac{a_0}{280}$$

$$y_1 \Rightarrow a_0 x^{5/2}\left[1+\frac{x^2}{10}+\frac{x^4}{280}+\cdots\right]$$

For $r \Rightarrow -1/2$;

coefficient of $x^{r+1} \Rightarrow 0$

$$\left[r(r+1)a_1 - (r+1)a_1 - \frac{5}{4}a_1\right] = 0$$

$$\left(r^2 - \frac{9}{4}\right)a_1 = 0$$

$$a_1 = 0.$$

coefficient of x^{n+r} ⟶

$\boxed{\text{For } r = -\frac{1}{2}}$

$$a_n = \frac{4a_{n-2}}{4\left(n-\frac{1}{2}\right)\left(n-\frac{5}{2}\right)-5}$$

$$a_n \Rightarrow \frac{4a_{n-2}}{(2n-1)(2n-5)-5}$$

$$a_n \Rightarrow \frac{4a_{n-2}}{4n(n-3)}$$

$$a_n \times 4n \times (n-3) \Rightarrow 4a_{n-2}.$$

$4n\, a_n (n-3) - a_{n-2} \Rightarrow 0.$

For $\boxed{n=3.}$

$a_3 = \frac{0}{0}$ intederminate form :→

so, for $r = -\frac{1}{2}$, gives us solution containing two arbitrary constant a_3 & a_0.

$$\boxed{a_5 \Rightarrow \frac{a_3}{10}}$$

$$a_7 = \frac{a_5}{28} = \frac{a_3}{280}.$$

and $a_2 = \frac{4a_0}{-8} = \frac{-a_0}{2}$.

$$a_4 = \frac{4a_2}{16} = \frac{a_2}{4} = \frac{-a_0}{8}.$$

$$a_6 = \frac{a_4 \times 4}{72} = \frac{a_4}{18} = \frac{-a_0}{18 \times 8} = \frac{-a_0}{144}.$$

so solⁿ :→

$$y_1 \Rightarrow \cancel{a_3} x^{-1/2} \left[a_0 \cancel{+} - \frac{a_0 x^2}{2} + a_3 x^3 - \frac{a_0}{8} x^4 + \frac{a_3}{10} x^5 \cancel{+} - \frac{a_0 x^6}{144} + \frac{a_3 x^7}{280} + \ldots \right]$$

$$y = a_0 x^{-1/2} \left[1 - \frac{x^2}{2} \cancel{+} - \frac{x^4}{8} - \frac{x^6}{144} \ldots \right]$$

$$+ a_3 x^{-1/2} \left[x^3 + \frac{x^5}{10} + \frac{x^7}{280} + - \right]$$

Hence,
solution is also containing the solution of
$\gamma \Rightarrow \frac{5}{2}$.

$$y \Rightarrow a_0 x^{-1/2}\left[1 - \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^6}{144} + \cdots\right]$$

$$+ a_3 x^{-1/2}\left[x^3 + \frac{x^5}{10} + \frac{x^7}{280} + \cdots\right]$$

This is also part 1. (solution).

ⓓ $x^2 y'' + (x^2 - 3x)y' + y \Rightarrow 0$.

$\rightarrow$ let $y = \sum_{n=0}^{\infty} a_n x^{n+r}$

$$y' = \sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-2}$$

$$\rightarrow x^2 \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-2} + (x^2 - 3x)\sum_{n=0}^{\infty} a_n (n+r) x^{n+r-1} + \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow 0$$

$$\rightarrow \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r} + \sum_{n=0}^{\infty} a_n (n+r) x^{n+r+1}$$

$$-3\sum_{n=0}^{\infty} a_n (n+r) x^{n+r} + \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow 0$$

coefficient of x^r :→

$r(r-1) - 3r + 1 \Rightarrow 0$

$r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$

$r_1 - r_2 = 2\sqrt{3}$. (which is not integer).

since, both roots are distinct and do not differ by integer.

coefficient of x^{n+r} :→

$\Rightarrow a_n(n+r)(n+r-1) + a_{n-1}(n+r-1) - 3a_n(n+r) + a_n \Rightarrow 0$

$\Rightarrow a_n[(n+r)(n+r-1) - 3(n+r) + 1] \Rightarrow - a_{n-1}(n+r-1)$.

$a_n \Rightarrow \frac{-a_{n-1}(n+r-1)}{[(n+r)(n+r-4)+1]}$

$a_1 \Rightarrow \frac{-a_0(r)}{(r+1)(r-3)+1}$

For $r \Rightarrow 2+\sqrt{3}$

$a_1 \Rightarrow \frac{-a_0(2+\sqrt{3})}{(3+\sqrt{3})(\sqrt{3}-1)+1}$

$$\boxed{a_1 \Rightarrow \frac{-a_0(2+\sqrt{3})}{2\sqrt{3}+1}}$$

$$a_2 \Rightarrow \frac{-a_1(r+1)}{(r+2)(r-2)+1}$$

$$a_2 \Rightarrow \frac{-a_1(3+\sqrt{3})}{(4+\sqrt{3})(\sqrt{3})+1}$$

$$\Rightarrow \frac{-a_1(3+\sqrt{3})}{4(\sqrt{3}+1)} \Rightarrow \frac{-a_1 \times \cancel{\sqrt{3}}(\cancel{\sqrt{3}+1})}{4\cancel{\sqrt{3}}(\cancel{\sqrt{3}+1})}$$

$$= \frac{-a_1\sqrt{3}}{4}.$$

$$\Rightarrow \frac{+a_0(2+\sqrt{3})\sqrt{3}}{4(2\sqrt{3}+1)} \Rightarrow \frac{a_0}{4} \times \frac{(3+2\sqrt{3})}{(1+2\sqrt{3})}$$

For $r = 2-\sqrt{3}$

$$a_n = \frac{-a_{n-1}(n+r-1)}{[(n+r)(n+r-4)+1]}$$

$$a_1 = \frac{-a_0(\cancel{1}+2-\sqrt{3}\cancel{-1})}{[1+2-\sqrt{3})(-1-\sqrt{3})+1]} \Rightarrow \frac{-a_0(2-\sqrt{3})}{-\sqrt{3}(\sqrt{3}-1)(\sqrt{3}+1)+1}$$

$$\Rightarrow \frac{-a_0(2-\sqrt{3})}{(1-2\sqrt{3})} \Rightarrow \frac{a_0(2-\sqrt{3})}{(2\sqrt{3}-1)}$$

$$a_2 = \frac{-a_1(2+2-\sqrt{3}-1)}{[(2+2-\sqrt{3})(-\sqrt{3})+1]} = \frac{-a_1(3-\sqrt{3})}{4-4\sqrt{3}}$$

$$= \frac{-a_1\sqrt{3}(\sqrt{3}-1)}{4(1-\sqrt{3})} = \frac{a_1\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{4} \times \frac{a_0(2-\sqrt{3})}{(2\sqrt{3}-1)}$$

$$y = c_1 x^{2+\sqrt{3}}\left[a_0 - \frac{a_0(2+\sqrt{3})}{2\sqrt{3}+1}x + \frac{a_0}{4}\frac{(3+2\sqrt{3})x^2}{1+2\sqrt{3}} - \cdots\right]$$

$$+ c_2 x^{2-\sqrt{3}}\left[a_0 + \frac{a_0(2-\sqrt{3})x}{2\sqrt{3}-1} + \frac{a_0}{4}\frac{(2\sqrt{3}-3)}{(2\sqrt{3}-1)}x^2 + \cdots\right]$$

$$y = C_1 x^{2+\sqrt{3}}\left[1 - \left(\frac{2+\sqrt{3}}{2\sqrt{3}+1}\right)x + \frac{(3+2\sqrt{3})x^2}{(1+2\sqrt{3})} + \cdots\right]$$

$$+ C_2 x^{2-\sqrt{3}}\left[1 + \left(\frac{2-\sqrt{3}}{2\sqrt{3}-1}\right)x + \frac{(2\sqrt{3}-3)}{(2\sqrt{3}-1)}x^2 + \cdots\right]$$

(6) Show that for $n = 0, 1, 2, 3$ the following legendre polynomial is given by.

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n.$$

→ Let $u \Rightarrow (x^2-1)^n$.

$$u' \Rightarrow n(x^2-1)^{n-1} \times 2x$$

$$u' \Rightarrow \frac{n(x^2-1)^n \times 2x}{x^2-1}$$

$$\boxed{u'(x^2-1) \Rightarrow 2nux}.$$

differentiating $(n+1)$ w.r.t x.

$$u^{n+2}(x^2-1) + {}^{(n+1)}C_1 u^{n+1} \times 2x + \frac{(n+1)n}{2} u^n \times \cancel{2}$$

$$\Rightarrow 2n\left[u^{n+1}x + (\cancel{n})u^n\right]$$

$$u^{n+2}(x^2-1) + 2(n+1)u^{n+1}x + n(n+1)u^n$$
$$\Rightarrow 2n\,u^{n+1}x + 2n(n+1)u^n.$$

$$(1-x^2)u^{n+2} - 2x\,u^{n+1} + n(n+1)u^n = 0.$$

This is a legendarge's equation.

Hence,

$\boxed{P_n(x) = c_n U^n(x)}$ is the solution of eq^n.

$$P_n(x) \Rightarrow c_n \frac{d^n}{dx^n}\left[(x^2-1)^n\right].$$

setting $P_n(1) = 1$:→

$$1 \Rightarrow c_n \left[\frac{d^n}{dx^n}\left[(x+1)^n(x-1)^n\right]\right]$$

$$1 = c_n\left[n!(x-1)^n + \cdots + n!(x+1)^n\right]$$

For $x = 1$.

$$1 = c_n \times n!\,2^n.$$

$$c_n = \frac{1}{n!\,2^n}.$$

Hence,

$$\boxed{P_n(x) \Rightarrow \frac{1}{n!\,2^n} \times \frac{d^n}{dx^n}\left[(x^2-1)^n\right]}$$

Hence, This legendarge's equation is satisfied by $P_n(x)$.

(7) Find 2 linearly independent solutions of the bessel equation of order $\frac{3}{4}$ for all $x>0$.

$\Rightarrow$ differential eq^n :$\rightarrow$

$$x^2y'' + xy' + \left(x^2 - \frac{9}{16}\right)y \Rightarrow 0.$$

solution for order n :$\rightarrow$

$$J_v(x) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma n+v+1}\left(\frac{x}{2}\right)^{2n+v}$$

For order $\frac{3}{4}$. , $v = \frac{3}{4}$.

$$J_{\frac{3}{4}}(x) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\sqrt{n+\frac{7}{4}}}\left(\frac{x}{2}\right)^{2n+\frac{3}{4}}$$

$$J_{-3/4}(x) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\sqrt{n-\frac{3}{4}}}\left(\frac{x}{2}\right)^{2n-\frac{3}{4}}.$$

complete solution is :$\rightarrow$

$$y = c_1 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\sqrt{n+\frac{7}{4}}}\left(\frac{x}{2}\right)^{2n+\frac{3}{4}}$$

$$+ c_2 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\sqrt{n-\frac{3}{4}}}\left(\frac{x}{2}\right)^{2n-3/4}$$

(8) Define legendre polynomial $P_n(x)$. If m & n are non-negative integers, then show that $\int_{-1}^{1} P_n(x) P_m(x)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1} & \text{if } m = n. \end{cases}$

$\Rightarrow$ legendre's equation :- $\rightarrow$

$$(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y \Rightarrow 0$$

since, $P_n(x)$ & $P_m(x)$ are its solution for $\alpha = n$ and $\alpha = m$ respectively.

$$\Rightarrow (1-x^2)(P_n'') - 2xP_n' + n(n+1)P_n(x) \Rightarrow 0. \quad \ldots (i)$$

$$(1-x^2)P_m''(x) - 2xP_m'(x) + m(m+1)P_m(x) \Rightarrow 0 \quad \ldots (ii)$$

multiplying eq (i) by $P_m(x)$ and eq (ii) by $P_n(x)$ and subtracting.

$$\Rightarrow (1-x^2)(P_n''P_m - P_m''P_n) - 2x(P_n'P_m - P_m'P_n) + [n(n+1) - m(m+1)]P_mP_n \Rightarrow 0$$

$$\Rightarrow \frac{d}{dx}\left[(1-x^2)(P_n'P_m - P_m'P_n)\right] + n(n+1) - m(m+1)\, P_mP_n \Rightarrow 0$$

$$\Rightarrow \underbrace{\int_{-1}^{1} \frac{d}{dx}(1-x^2)(P_n'P_m - P_m'P_n)\,dx}_{0} + n(n+1) - m(m+1)\int_{-1}^{1} P_mP_n\, dx = 0$$

$$\int_{-1}^{1} P_m P_n \, dx \Rightarrow 0.$$

For $(n = m)$:-→
using Generating function.

$$(1-2xt+t^2)^{-1/2} \Rightarrow \sum_{n=0}^{\infty} P_n(x) t^n.$$

squaring both sides :-→

$$(1-2xt+t^2)^{-1} \Rightarrow \sum_{n=0}^{\infty} P_n^2(x) t^{2n}.$$

$$\frac{1}{1-2xt+t^2} \Rightarrow \sum_{n=0}^{\infty} P_n^2(x) t^{2n}.$$

integrating both sides :-→

$$\int_{-1}^{1} \frac{1}{1-2xt+t^2} = \sum_{n=0}^{\infty} \int_{-1}^{1} P_n^2 dx . t^{2n}.$$

coefficient of t^{2n} gives us $\int_{-1}^{1} P_m P_n dx$

$$\Rightarrow \left[\frac{\ln|1-2xt+t^2|}{-2t}\right]_{-1}^{1} \Rightarrow \sum_{n=0}^{\infty} \int_{-1}^{1} P_n^2(x) \, dx \, t^{2n}$$

$$\frac{\ln|1-t|^2 - \ln|1+t|^2}{-2t} \Rightarrow \sum_{n=0}^{\infty} \int_{-1}^{1} P_n^2(x) dx \, t^{2n}$$

$$\frac{\ln|1+t| - \ln|1-t|}{} \Rightarrow \sum_{n=0}^{\infty} \int_{1}^{1} P_n^2(x)\,dx\; t^{2n}$$

$$\frac{\left(t - \frac{t^2}{2} + \cdots \frac{t^n}{n}\right) + \left(t + \frac{t^2}{2} + \cdots\right)}{t} \Rightarrow \sum_{n=0}^{\infty} \int_{1}^{1} P_n^2(x)\,dx\; t^{2n}.$$

$$2 + \frac{2t^2}{3} + \cdots + \frac{2t^{2n}}{2n+1} \Rightarrow \sum_{n=0}^{\infty} \int_{-1}^{1} P_n^2(x)\,dx\; t^{2n}.$$

coefficient of t^{2n} is $\frac{2}{2n+1}$.

so $\boxed{\int_{-1}^{1} P_n^2(x)\,dx = \frac{2}{2n+1}}$

(9) at $x \Rightarrow 0$.

$P_0(x) = 0$.

so, $x=0$ is a ~~regular~~ singular point.

For regular $\rightarrow$

$$\lim_{x\to 0} \frac{3x-1}{\cancel{x}(x-1)} \cancel{x} = 1.$$

$$\lim_{x\to 0} \frac{x^2}{x(x-1)} = 0.$$

Hence, $x=0$ is regular singular point

$\rightarrow$ let $y \Rightarrow \sum_{n=0}^{\infty} a_n x^{n+r}$.

$$y' \Rightarrow \sum_{n=0}^{\infty} (n+r)\, a_n\, x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1)a_n x^{n+r-2}.$$

$$x^2y'' - xy'' + 3xy' - y' + y \Rightarrow 0$$

$$\sum_{n=0}^{\infty} (n+r)(n+r-1)a_n x^{n+r} - \sum_{n=0}^{\infty}(n+r)(n+r-1)a_n x^{n+r-1}$$

$$+ 3\sum_{n=0}^{\infty} (n+r)a_n x^{n+r} - \sum_{n=0}^{\infty} (n+r)a_n x^{n+r-1}$$

$$+ \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow 0.$$

coefficient of x^{r-1} $\xrightarrow{}$

$$[r(r-1) - r]a_0 = 0 \qquad a_0 \neq 0.$$

$$r(r-2) = 0$$

$$r = 0, \quad r = 2.$$

coefficient of x^r $\xrightarrow{}$

$$r(r-1)a_0 - (r+1)r\, a_1 + 3(r)a_0$$

$$-(r+1)a_1 + a_0 \Rightarrow 0.$$

$$[r(r-1) + 3r + 1]a_0 \Rightarrow (r+1)^2 a_1$$

$$\cancel{(r+1)^2} a_0 = \cancel{(r+1)^2} a_1.$$

$$\boxed{a_1 \Rightarrow a_0}$$

coefficient of x^{n+r} $\xrightarrow{}$

$$\Rightarrow (n+r)(n+r-1)a_n - (n+r+1)(n+r)a_{n+1}$$

$$+ 3(n+r)a_n - (n+r+1)a_{n+1} + a_n = 0.$$

$$\Rightarrow [(n+r)^2 - (n+r)]a_n + 3(n+r)a_n + a_n$$

$$\Longrightarrow (n+r+1)^2 a_{n+1}$$

$\cancel{(n+r+1)^2 a_n \Rightarrow (n+r+1)^2 a_{n+1}}$

$\boxed{a_n \Rightarrow a_{n+1}}$

For $r=2$:→

$y_1 \Rightarrow \cancel{x_0}\, x^2 [a_0 + a_0 x + \text{———} + a_0 x^n + \text{—}]$

For $r=0$

$$y_2 = \left(\frac{\partial y}{\partial r}\right)_{r=0} \Rightarrow \frac{\partial}{\partial r}\left[x^r [a_0 + a_0 x + \cdots + a_0 x^n +]\right]$$

$\Rightarrow x^r \log x\, a_0 [1 + x + \cdots x^n \cdots]$

$\Rightarrow a_0 \log x\, [1 + \cdots x^n]$

complete solution is :→

$$y = c_1 y_1 + c_2 y_2$$

$$= c_1 a_0 x^2 [1 + x + \cdots + x^n + \cdots] + c_2 a_0 \log x [1 + \cdots + x^n + \cdots]$$

$$= (c_1 x^2 + c2 \log x)[1 + \cdots + x^n + \cdots]$$

(10) show that chebyshev's equation $(1-x^2)y''(x) - xy'(x) + a^2 y(x) = 0$ with $a \in (0, \infty)$ has the following linear independent power series solutions.

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$$y_1(x) \Rightarrow 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left[\prod_{k=0}^{n-1} (4k^2 - a^2)\right] x^{2n} \quad \text{and}$$

$$y_2(x) = x + \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left[\prod_{k=0}^{n-1} (4k^2 + 4k + 1 - a^2)\right] x^{2n+1}.$$

$\Rightarrow$ $x = 0$ is an ordinary point.

$y \Rightarrow \sum_{n=0}^{\infty} a_n x^n$ is an infinite series.

$$y' \Rightarrow \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' \Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\Rightarrow y''(x) - x^2 y''(x) - xy'(x) + a^2 y(x) \Rightarrow 0$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1) a_n x^n - \sum_{n=1}^{\infty} n a_n x^n$$
$$+ a^2 \sum_{n=0}^{\infty} a_n x^n \Rightarrow 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^{n+2}$$
$$- \sum_{n=0}^{\infty} (n+1) a_{n+1} x^{n+1} + a^2 \sum_{n=0}^{\infty} a_n x^n \Rightarrow 0$$

coefficient of x^0 $\longrightarrow$

$$2 \cdot 1\, a_2 + a^2 a_0 \Rightarrow 0.$$

$$\boxed{a_2 \Rightarrow \frac{-a^2 a_0}{2 \cdot 1}}$$

coefficient of x^n :→

$(n+2)(n+1)a_{n+2} - n(n-1)a_n - na_n + a^2 a_n = 0$

$(n+2)(n+1)a_{n+2} = [n(n-1)+n-a^2]a_n$

$$a_{n+2} = \frac{(n^2-a^2)a_n}{(n+2)(n+1)}$$

$$a_2 = \left(\frac{0-a^2}{2\cdot 1}\right)a_0$$

$$a_4 = \frac{(2^2-a^2)a_2}{4\cdot 3}$$

⋮

$$a_{2n+2} \Rightarrow \frac{(4n^2-a^2)}{2n\cdot(2n-1)} \cdot a_{2n}$$

$$a_{2n+2} = \frac{(4n^2-a^2)\cdots\cdots(-a^2)}{(2n+2)}$$

so $$a_{2n} = \frac{[(2n-2)^2-a^2]\cdots\cdots(-a^2)}{(2n)!}$$

similarly :→

$$a_3 = \frac{(1-a^2)a_1}{3\cdot 2}$$

$$a_5 \Rightarrow \left(\frac{3^2-a^2}{5\cdot 4}\right)a_3 = \frac{(3^2-a^2)}{5\cdot 4}\times\left(\frac{1-a^2}{3\cdot 2}\right)a_1$$

⋮

$$a_{2n+1} \Rightarrow \frac{(2n-1)^2-a^2 \times [(2n-3)^2-a^2]\cdots(1-a^2)a_1}{(2n+1)!}$$

so :→

$$y = c_1\left[1+\sum_{n=1}^{\infty}\frac{1}{(2n)!}\left[\prod_{k=0}^{n-1}(4k^2-a^2)\right]x^{2n}\right] + c_2\left[x+\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}\left[\prod_{k=0}^{n-1}((2k+1)^2-a^2)\right]x^{2n+1}\right]$$

$$\Rightarrow y_1(x) = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left[\prod_{k=0}^{n-1} (4k^2 - a^2) \right] x^{2n}$$

and

$$y_2(x) = x + \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left[\prod_{k=0}^{n-1} (4k^2 + 4k + 1 - a^2) \right] x^{2n+1}.$$

11. Given $n \in N$ and $x \in (0, \infty)$. Then prove the following :→

(a) $x J_v'(x) \Rightarrow v J_v(x) - x J_{v+1}(x)$

(b) $x J_v'(x) \Rightarrow -v J_v(x) + x J_{v-1}(x)$

(c) $2 J_v'(x) \Rightarrow J_{v-1}(x) - J_{v+1}(x)$

(d) $2 v J_v(x) = x [J_{v-1}(x) + J_{v+1}(x)]$

(e) $\frac{d}{dx}[x^{-v} J_v] = -x^{-v} J_{v+1}(x)$

(f) $\frac{d}{dx}[x^v J_v] = x^v J_{v-1}(x)$

$$\rightarrow J_v(x) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+v}}{n! \, \sqrt{n+v+1} \; 2^{2n+v}}$$

$$x^v J_v \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2v}}{n! \, \sqrt{n+v+1} \; 2^{2n+v}}$$

$$\frac{d}{dx}[x^v J_v(x)] \Rightarrow \sum_{n=0}^{\infty} \frac{(2n+2v)(-1)^n x^{2n+2v-1}}{2^{2n+v} \, n! \, \sqrt{n+v+1}}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2v-1}}{2^{2n+v-1} \, n! \, \sqrt{n+v}}$$

$$\Rightarrow x^{v} \sum_{n=0}^{\infty} \frac{x^{2n+v-1}}{2^{2n+v-1}\, n!\, \sqrt{n+v}}$$

$$\Rightarrow x^{v} \sum_{n=0}^{\infty} \frac{x^{2(n)+v-1}}{2^{2n+v-1} \times n!\, \sqrt{n+1+v-1}}$$

$$\Rightarrow x^{v} J_{v-1}(x)$$ (Part f).

(Hence proved).

Part f is proved.

$$\frac{d}{dx}(x^{-v} J_v) \Rightarrow \frac{d}{dx}\left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+v-v}}{2^{2n+v}\, n!\, \sqrt{n+v+1}} \right)$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n \times 2n\, x^{2n-1}}{2^{2n+v}\, n!\, \sqrt{n+v+1}}$$

$$\Rightarrow x^{-v} \sum_{n=1}^{\infty} \frac{(-1)^n \times 2n \times x^{2n-1} \times x^{+v}}{2^{2n+v}\, n!\, \sqrt{n+v+1}}.$$

$$\Rightarrow x^{-v} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}\, 2(n+1)\, x^{2n+v+1}}{2^{2n+2+v}\, (n+1)!\, \sqrt{n+v+2}}$$

$$\Rightarrow -x^{-v} \sum_{n=0}^{\infty} \frac{(-1)^n (-1) \times 2(n+1)\, x^{2n+v+1}}{2^{2n+v+1}\, (n+1)!\, \sqrt{n+v+2}}$$

$$\Rightarrow -x^{-v} J_{v+1}(x).$$

Part e is proved.

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$$\frac{d}{dx}\left[x^{\nu} J_{\nu}(x)\right] = x^{\nu} J_{\nu-1}(x)$$

$$\boxed{\nu x^{\nu-1} J_{\nu}(x) + x^{\nu} J'_{\nu}(x) \Rightarrow x^{\nu} J_{\nu-1}(x).}$$

$$\nu J_{\nu}(x) + x J'_{\nu}(x) \Rightarrow x J_{\nu-1}(x)$$

$$\boxed{x J'_{\nu}(x) \Longrightarrow x J_{\nu-1}(x) - \nu J_{\nu}(x)} \quad \ldots \text{(iii)}$$

proof of part (b).

similarly

$$\frac{d}{dx}\left[x^{-\nu} J_{\nu}(x)\right] = -x^{-\nu} J_{\nu+1}(x)$$

$$-\nu x^{-\nu-1} J_{\nu}(x) + J'_{\nu}(x) x^{-\nu} \Longrightarrow -\left[x^{-\nu} J_{\nu+1}(x).\right]$$

$$-\nu x^{-\nu-1} J_{\nu}(x) = -x^{-\nu}\left(J_{\nu+1}(x) - J'_{\nu}(x)\right]$$

$$\nu J_{\nu}(x) \Rightarrow x\left(J_{\nu+1}(x) + J'_{\nu}(x)\right]$$

$$\boxed{x J'_{\nu}(x) \Longrightarrow \nu J_{\nu}(x) - x J_{\nu+1}(x)} \quad \ldots \text{(iv)}$$

proof of part A.

adding eq (iii) & (iv)

$$2x J'_{\nu}(x) = x\left(J_{\nu-1}(x) - J_{\nu+1}(x)\right]$$

$$\boxed{2 J'_{\nu}(x) = J_{\nu-1}(x) - J_{\nu+1}(x)}$$

proof of part C.

subtracting eq (iii) from (iv)

$$0 \Rightarrow x J_{\nu-1}(x) + x J_{\nu+1} - 2\nu J_{\nu}(x)$$

$$2\nu J_{\nu}(x) = x\left[J_{\nu-1}(x) + J_{\nu+1}(x)\right]$$

proof of part d