Nuclear Physics

On the basis of α-ray Scattering experiment, E. Rutherford described that inside the atom there is a very small nucleus in which all the positive charge and most of the mass is localized.

Structure of Nuclei :-

Two types of particles inside the nucleus called Nucleons

Nucleons → Protons (+ve charge) (Z or P)

Nucleons → Neutron → neutral (N)

Atomic Number → No. of protons inside the nucleus. ('Z')

Mass Number
↳ Total no. of nucleons (protons + Neutrons)

$$A = (Z + N)$$

Symbolic Representation of Nucleus :-

$^{A}_{Z}X$ or $_{Z}X^{A}$ or $^{A}X_{Z}$

A → Mass no.

Atomic no. ← Z

Date

1) **Isotopes** : Nuclei with same atomic no. 'Z' but diff. mass no. 'A'

eg :- $_1H^1$, $_1H^2$, $_1H^3$ Isotopes of Hydrogen.

2) **Isobars** : Nuclei with same mass no. A but diff. atomic no. 'Z'.

eg :- $_8O^{16}$ & $_7N^{16}$

3) **Isotones** : Nuclei with same no. of neutrons 'N'. eg :-

$_7N^{14}$, $_6C^{13}$ → Isotones.

4) **Mirror Nuclei** : A pair of Isobaric nuclei are known as mirror nuclei. where in the nucleus the proton no. Z & neutron no. N are interchanged & differ by one unit.

eg :- $_6C^{11}$ & $_5B^{11}$ → Mirror Nuclei

Binding Energy of Nucleus :-

ΔM → amount of mass disappeared

$$\Delta M = ZM_p + NM_N + ZM_e - M(A,Z)$$

$M(A,Z) \rightarrow$ mass of atom of mass no. A & atomic number Z.

$$E_B = [ZM_p + NM_n + ZM_e - M(A,Z)]c^2$$

$$= [ZM_p + NM_N + Z\cancel{M_e} - M_{nuc} - Z\cancel{M_e}]c^2$$

$$= [ZM_p + NM_N - M_{nuc}]c^2$$

$$\boxed{E_B = [ZM_p + NM_N - M_{nuc}]c^2}$$

$M_{nuc} \rightarrow$ mass of nucleus

Binding fraction Curve

Binding fraction 'f_B' is ratio of B·E to mass no.

$$f_B = \frac{E_B}{A} = \frac{(ZM_p + NM_N - M_{nuc})c^2}{A}$$

$$\boxed{f_B = \frac{[ZM_p + NM_N - M_{nuc}]c^2}{A}}$$

Graph

f_B is also called as Binding energy per Nucleon.

Date

Mass Defect : $\boxed{\Delta M' = M(A,z) - A}$

Difference b/w measured atomic mass $(M(A,z))$ and mass no. 'A' of nucleus.

for ${}_8O^{16}$, $\Delta M' = 15.994915 - 16$
$= -0.005085$ a.m.u.

Packing fraction :-

$$f = \frac{\Delta M'}{A} = \frac{M(A,Z) - A}{A}$$

$$\boxed{M(A,Z) = A(1+f)}$$

'Mass defect per nucleon in nucleus.

Nuclear size :-
From experimental results we can say that mass density of the nucleus almost remains constt :-

$$\rho_m = \frac{A}{V} = \text{constant}$$

$$V \propto A$$

$$\frac{4}{3}\pi R^3 \propto A \Rightarrow R \propto A^{1/3}$$

$$\boxed{R = R_0 A^{1/3}}$$ R_0 is proportionality constt.

$(R_0 = 1.2 \times 10^{-15}\ m)$

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Chapter 2

Nuclear Models

Introduction:

In previous chapter, we have studied about nuclear properties like variation of radius with mass number, charge density, binding energy, etc. But to understand the above behaviour of atomic nucleus, it is essential to know the exact nature of the forces between the nucleons. However, due to lack of knowledge of the exact internucleon force, it has not been possible to develop a satisfactory theory of nuclear structure. In absence of a comprehensive theory, attempts were made to develop nuclear models to explain the properties of nuclei. Several different models have been suggested, each with its successes and limitations. The important models are

(a) The liquid drop model
(b) Nuclear shell model
(c) Fermi gas model
(d) Alpha model
(d) Collective model
(e) Optical model.

A. **Liquid drop model:**

The liquid drop model of the nucleus was first proposed by Niels Bohr and F. Kalcar in the year 1937. They observed that there exists many similarities between the drop of a liquid and a nucleus. For instance,

(i) both the liquid drop and the nucleus possess constant density,
(ii) The constant binding energy per nucleon of a nucleus is similar to the latent heat of vaporization of a liquid,
(iii) The evaporation of a drop corresponds to the radioactive properties of the nucleus, and
(iv) The condensation of drops bears resemblance with the formation of compound nucleus, etc.

- According to this model, the nucleus is supposed to be spherical in shape in the stable state with radius $R = R_0 A^{1/3}$, just as a liquid drop is spherical due to symmetrical surface tension forces. The surface tension effects are analogous to the potential barrier effects on the surface of the nucleus.
- The density of a liquid drop is independent of the volume, as is the case with the nucleus. But whereas the nuclear density is independent of the type of nucleus, the density of a liquid does depend on its nature.
- Like the nucleons inside the nucleus, the molecules in the liquid drop interact only with their immediate neighbours.
- The non-independence of the binding energy per nucleon on the number of nucleons in the nucleus is analogous to the non-independence of the heat of vaporization of a liquid drop on the size of the drop.
- Molecules in a liquid drop evaporate from the liquid surface in raising the temperature of the liquid due to their increased energy of thermal agitation. Analogously, if high energy nuclear projectiles (or photons) bombard the nucleus, a compound nucleus is formed in which the nucleons quickly share the incident energy and the emission of nucleons (or photon) occurs.

- The phenomenon of nuclear fission is easily explained as the splitting of the liquid drop if set into vibration with sufficient energy.

Semi-empirical binding energy or mass formula:

C.V. Weizsacker, a German physicist, proposed the following semi-empirical formula for the nuclear binding energy B.E. (in MeV) for the nucleus (Z, A)

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} (\pm, 0) \frac{\delta}{A^{3/4}}$$

with the constants having the value,

$a_v = 15.75$, $a_s = 17.80$, $a_c = 0.71$, $a_a = 22.7$ and $\delta = 33.6$ all are in MeV.

These values are not absolute other sets of values are also possible.

(i) Volume energy:

The first term, $E_v = a_v A$, is the volume effect representing the volume energy of all nucleous. The more the total number of nucleons A, more difficult it becomes to remove an individual nucleon from the nucleus. Since the nuclear density is nearly constant, the nuclear mass is proportional to the nuclear volume, which is again proportional to R^3. But $R \propto A^{1/3} \Rightarrow R^3 \propto A$. So, the volume energy $E_v \propto A$.

$\Rightarrow \quad E_v = a_v A$

This energy corresponds to the amount of heat energy (the heat of vaporisation) required to transform a liquid to its vapour state being proportional to the mass of the liquid.

(ii) Surface energy:

The second term, $E_s = a_s A^{2/3}$, is the surface effect being similar to the surface tension in liquids, like the molecules on the surface of a liquid, the nucleons at the surface of the nucleus are not completely surrounded by other nucleons. It results in reducing the total binding energy due to nucleons on the surface. This correction due to surface energy E_s, which is proportional to the surface area of the nucleus i.e. to $4\pi R^2$ i.e. $E_s \propto R^2 \quad \Rightarrow \quad E_s \propto A^{2/3} \quad \Rightarrow E_s = a_s A^{2/3}$

(iii) Coulomb energy:

The third term, E_c, is the Coulomb electrostatic repulsion between the charged particles in the nucleus. Since each charged particle repulses all the other charged particles, this term would be directly proportional to the possible number of combinations for a given proton number Z, which is $Z(Z-1)/2$. The energy of interaction between protons is again inversely proportional to the distance of separation R, so the energy associated with Coulomb repulsion is:

$$E_c = k\frac{Z(Z-1)}{R} = a_c \frac{Z(Z-1)}{A^{1/3}}$$

where, $a_c = \frac{3}{5} \times \frac{e^2}{4\pi\varepsilon_0 R_0}$, binding energy decreases due to Coulomb terms.

(iv) Asymmetry energy:

The fourth term E_a originates from the asymmetry between the number of protons and the number of neutrons in the nucleus. Nuclear data for stable nuclei indicate that for lighter nuclei, the number of protons is almost equal to that of neutrons: $N = Z$. As A increases, the symmetry of proton and neutron number is lost and the number of neutrons exceeds that of protons to maintain the nuclear stability. This excess of neutrons over protons, i.e. $N-Z$, is the measure of the asymmetry and it decreases the stability or the B.E. of medium or heavy nuclei.

So, $\quad E_a \propto (N-Z)$, and $E_a \propto (N-Z)/A \quad \Rightarrow \quad E_a = a_a \frac{(N-Z)^2}{A} = a_a \frac{(A-2Z)^2}{A}$

(v) Pairing energy:

The last term, a pure corrective term, is called pairing energy E_p.

$$E_p = (\pm, 0)\frac{\delta}{A^{3/4}}$$

Z	N	A	δ	E_p
even	even	even	34	$+\delta / A^{3/4}$
even	odd	odd	0	0
odd	even	odd	0	0
odd	odd	even	35	$-\delta / A^{3/4}$

- Binding fraction (Or binding energy per nucleon),

$$f_B = \frac{B.E.}{A} = a_v - \frac{a_s}{A^{1/3}} - \frac{a_c Z(Z-1)}{A^{4/3}} - a_a\frac{(A-2Z)^2}{A^2}(\pm,0)\frac{\delta}{A^{7/4}}$$

Mass parabola and stability of nuclei against β-decay:

- Mass of nucleus, $M(A,Z) = Z M_p + (A-Z)M_n - B.E./c^2$

$$= ZM_p + (A-Z)M_n - \frac{1}{c^2}\left[a_v A - a_s A^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_a\frac{(A-2Z)^2}{A}(\pm,0)\frac{\delta}{A^{3/4}}\right]$$

The above formula is known as the semi-empirical mass formula.

Assuming, $F_A = A(M_n - a_v + a_a) + a_s A^{2/3}$

$$p = -4a_a - (M_n - M_p);\ q = \frac{1}{A}\left(a_c A^{2/3} + 4a_a\right)$$

$\Rightarrow$ $\boxed{M(A,Z) = F_A + pZ + qZ^2}$, it is equation of parabola for given A is called mass parab

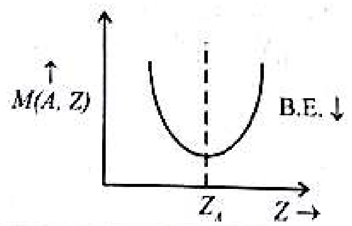
$\Rightarrow$ $\left(\frac{\partial M}{\partial Z}\right)_A = p + 2qZ = 0$ at $Z = Z_A$

$\Rightarrow$ $Z_A = -\frac{p}{2q} = \frac{(M_n - M_p + 4a_a)A}{2\left(a_c A^{2/3} + 4a_a\right)}$ $\Rightarrow$ $Z_A = A/\left(1.98 + 0.015A^{2/3}\right)$

In most cases, the value of Z nearest to Z_A gives the actual stablest nucleus for a given A.

All isobars having mass greater than that of stable isobar. They will decay by emission of β^-, β^+ ar capture.

1. The isobar to the left of the stable one decay by β^- emission.
2. The isobar to the right of the stable one decay by β^+ emission or electron-capture or by both.



Magic numbers : If the number of proton or neutron or both are 2, 8, 20, 28, 50, 82, 126 of a nucleous then it is very stable. These numbers are called as Magic numbers.

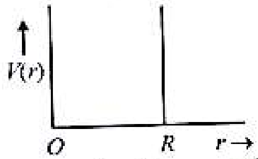
B. Shell model:

Liquid drop model can not explore why some nucleus having proton or neturon or both are 2, 8, 20, 28, 50, 82, 126 they have high binding energy than others. The scientists thought that there was some arrangement like electron distribution in the atom.

Since we do not know the exact force nature inside the nucleus so we need to take a guess. In shell model we can take any of the following three potentials.

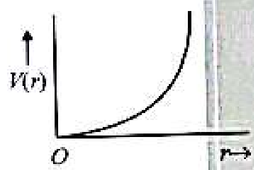
1. Three dimensional infinite spherical well potential

$$V(r)=\begin{cases}0 & \text{for } r\le R\\ \infty, & \text{for } r>R\end{cases}$$



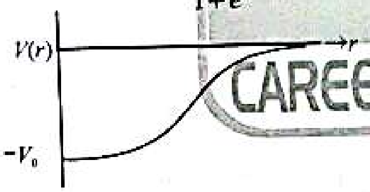
2. Three dimensional harmonic oscillator potential

$$V(r)=\frac{1}{2}kr^2,\ \text{where } k>0$$



3. Saxon-Woods model potential

$$V(r)=-\frac{V_0}{1+e^{(r-R)/a}}$$



where, $V_0\simeq 50\ \text{MeV}, R=r_0A^{1/3},\ a=0.52\ \text{fm}$

In all the above three types of potentials, we get the energy levels E_{nl}. For Saxon-Woods potential energy E_{nl} is given as

$$=\frac{\hbar^2\ell(\ell+1)}{2\mu R_0^2}\left(1+\frac{12a^2}{R_0^2}\right)-\frac{\hbar^2}{2\mu a^2}\left\{\frac{\left[\sqrt{1+192\ell(\ell+1)a^4}-2n-1\right]^2}{16}+\frac{4\left[\frac{\mu a^2V_0}{\hbar^2}-\frac{4\ell(\ell+1)a^3}{R_0^3}\right]^2}{\left[\frac{\sqrt{1+192\ell(\ell+1)a^4}}{R_0^4}-2n-1\right]^2}+\frac{\mu V_0a^2}{\hbar^2}\right\}$$

The lowest level is $1s$(i.e. n = 1, l = 0) which can contain up to 2 protons or neutrons. Then comes $1p$ which can contain up to a further 6 protons. This explains the first two magic numbers (2 and 8). Then there is a level $1d$ but this is quite close in energy to $2s$ so that they form the same shell. This allows a further 2 + 10 protons or neutrons giving us the next magic number of 20.

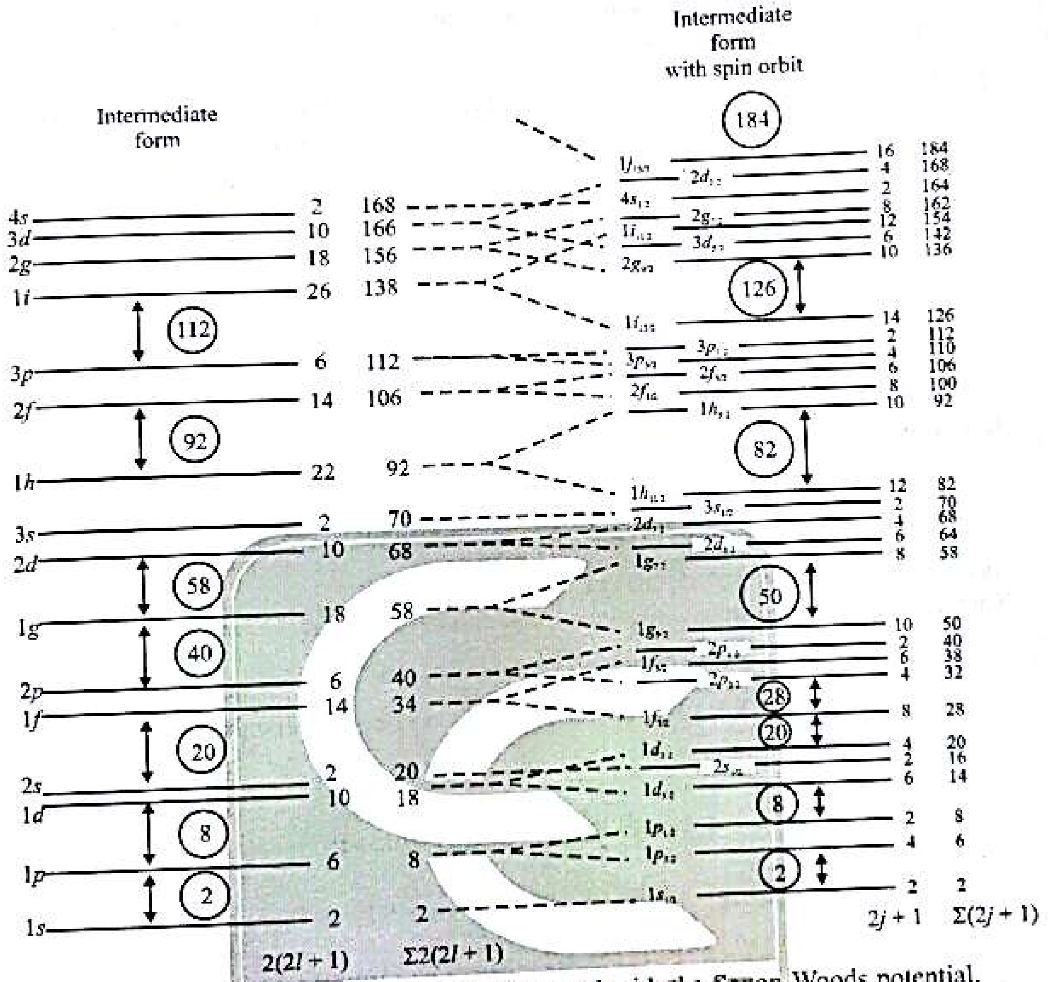


Figure: At the left are the energy levels calculated with the Saxon-Woods potential. To the right of each level are shown its capacity and the cumulative number of nucleons upto that level. The right side of the figure shows the effect of the spin-orbit interaction, which splits the levels with $l > 0$ into two new levels. The shell effect is quite apparent, and the magic numbers are exactly reproduced.

Success of shell model:

1. It very well explains the existence of magic numbers and the stability and high binding energy on the basis of closed shells.
2. The shell model provides explanation for the ground state spins and magnetic dipole moment of the nuclei. The neutrons and protons with opposite spins pair off so that the mechanical and magnetic dipole moment cancel and the odd or left out proton or neutron contributes to the spin and magnetic moment of the nuclei as a whole.
3. Nuclear isomerism, i.e., existence of isobaric, isotopic nuclei in different energy states of odd-A nuclei between 39-49, 69-81, 111 to 125 has been explained by shell model by the large difference in nuclear spins of isomeric states are their A-values are close to magic numbers.

Chapter 3

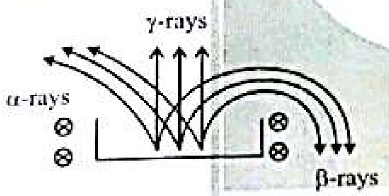
Radioactivity

Introduction:

In 1903, the M. Curie and P. Curie were awarded jointly with Becquerel, the Nobel Prize in physics for his discovery of radioactivity phenomenon. In this chapter, we shall discuss radioactive decay laws of α, β and γ-rays.

Radioactivity:

Radioactivity is the phenomenon of spontaneous distintegration, attended with emission of corpuscular or electromagnetic radiations, of heavy atomic nuclei $\left(\text{for } \frac{N}{Z} > 1.5\right)$ like Uranium, Radium etc. at a constant rate unaffected by any physical or chemical changes or influences such as temperature, pressure etc. to which the atom may be subjected. It is a nuclear property of the active element and in all radioactive processes a transmutation of the element occurs and altogether new nucleus is formed.



Radioactive Decay law:

1. On emission of α or β rays, which is usually but not invariably accompanied by γ-ray emission, the emitting parent nuclei transforms into a new daughter element. The daughter element again is radioactive so that the process of successive disintegration continuous till the original active parent nuclei gets transformed into stable one.

2. The rate of radioactive disintegration, that is, the number of atoms that break up at any instant of time t is directly proportional to the number N_t of active nuclei present in the sample under study at that instant.

Decay equation:

Let N_t be the number of active nucleides present in the sample at any time t then we have experimentally,

$$-\frac{dN_t}{dt} \propto N_t$$

$$\Rightarrow \quad \frac{dN_t}{dt} = -\lambda N_t$$

Where, λ, the constant of proportionality, is known as the decay constant – a characteristic constant of the element (nuclide). The negative sign hints at the fact that N_t decreases with t.

$$\frac{dN_t}{N_t} = -\lambda dt$$

$$\ell n\, N_t = -\lambda t + A$$

where, A is the constant of integration.

At $t = 0$, $N_t = N_0$ (the initial number of nuclides) and $A = \ell n\, N_0$

Therefore, $\ell n\left(\frac{N_t}{N_0}\right) = -\lambda t$

Or, $\boxed{N_t = N_0 e^{-\lambda t}}$

Half-life:

The half-life of a radioactive nuclide is defined as the time $T_{½}$ in which the original amount of of radioactive atoms is reduced by way of distintegration to half its value.

At $t = T_{½}$, $N_t = N_0/2$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{½}}$$

$$\therefore \quad T_{½} = \frac{\ell n 2}{\lambda} = \frac{2.303 \times \log 2}{\lambda} = \frac{0.693}{\lambda}$$

Or, $\boxed{\lambda T_{½} = 0.693 = \text{constant}}$

Average life:

The average or mean life time $\overline{T}$ of a radioelement is the average life time of all the atoms in the given sample and is defined as the ratio of the total life time of all the atoms to be total number of atoms.

$$\therefore \quad \overline{T} = \frac{t_1 dN_1 + t_2 dN_2 + \ldots}{dN_1 + dN_2 + \ldots} = \frac{\Sigma t dN}{\Sigma dN} = \frac{\int_{t=0}^{\infty} t dN}{\int_{N_0}^{0} dN} = \frac{\int_{t=0}^{\infty} t dN}{-N_0}$$

But, we have,

$$dN = d\left(N_0 e^{-\lambda t}\right) = -\lambda N_0 e^{-\lambda t} dt$$

$$\therefore \quad \overline{T} = \lambda \int_0^{\infty} t e^{-\lambda t} dt = \lambda \left[-\frac{t}{\lambda} e^{-\lambda t} + \frac{1}{\lambda} \int_0^{\infty} e^{-\lambda t} dt\right]$$

$$= \lambda \left[-\frac{t}{\lambda} e^{-\lambda t} - \frac{1}{\lambda^2} e^{-\lambda t}\right]_0^{\infty} = \frac{1}{\lambda}$$

Therefore, $\boxed{\overline{T} = \frac{1}{\lambda}}$

We can write also, $\boxed{T_{½} = \frac{0.693}{\lambda} = 0.693\overline{T}}$

Activity:

The activity or strength A_t of a radioactive sample at any instant t is defined as the number of disintegration occurring in the sample in unit time at t, that is,

$$\text{Activity,}\quad \boxed{A_t = \left|\frac{dN_t}{dt}\right| = \lambda N_t = \frac{0.693}{T} N_t}$$

Units of activity:

The customer unit of radioactivity is called the curie (Ci). It is defined as the activity of any radioactive substance that disintegrates at the rate of 3.7×10^{10} disintegration per second.

$$1\,\text{Ci} = 1\text{ curie} = 3.7\times10^{10}\text{ disint/sec}$$

$$1\,\text{mCi} = 10^{-3}\text{ curie}$$

$$1\,\mu\text{Ci} = 10^{-6}\text{ curie}$$

Radioactive displacement law:

1. The α particles have been identifies as Helium nuclei. Suppose we have parent nucleus having mass number A and atomic number Z, after α-decay the parent nucleus become a new daughter element whose atomic number will be $Z-2$ and mass number will be $A-4$. Its position in periodic table will be shifted towards left by two.

$$_Z X^A \xrightarrow{\alpha} {}_{Z-2}Y^{A-4}$$

2. The β-particle defined as $_{-1}e^0$ due to β-emission the parent elements $\left({}_Z X^A\right)$ become a daughter element $\left({}_{Z+1}X^A\right)$. So, it position in periodic table will be shifted towards right by one.

$$_Z X^A \xrightarrow{\beta} {}_{Z+1}Y^A$$

Natural radioactive decay series:

There are three naturally occuring radioactive series. These three series are

(i) Uranium series is called $(4n+2)$

(ii) Actinium series is called $(4n+3)$

(iii) Thorium series is called $(4n)$

Also there have another series is called Neptunium series

(iv) Neptunium series is called $(4n+1)$

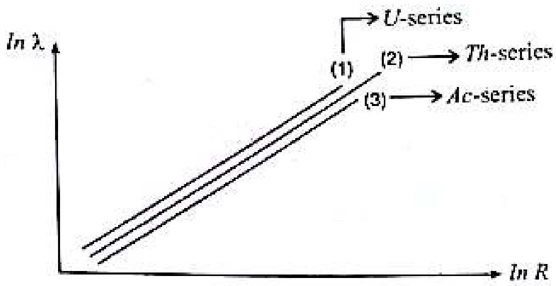


Figure: Variation of $ln\ \lambda$ with $ln\ R$: Geiger-Nuttall law

Since $R \propto E^{3/2} \quad \Rightarrow \ell n\, \lambda = C + D\, \ell n\, E$
where C, D are two constants.

- **α-disintegration energy:**

$$_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}$$

The Q-value of the decay process is known as the α-disintegration energy which is the total energy released in the disintegration process and is given by $Q_\alpha = (M_X - M_\alpha - M_Y)c^2$
where M's are the masses of the particles and c the velocity of light in vacuum.

For heavy nuclei, Q_α is positive, so the decay can occur spontaneously as it does. According to laws of conservation of momentum and energy.

$$0 = M_\alpha v_\alpha - M_Y v_Y$$

and $$Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y v_Y^2$$

$$\Rightarrow \quad Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2\left(1+\frac{M_\alpha}{M_Y}\right) = T_\alpha\left(1+\frac{M_\alpha}{M_Y}\right)$$

Kinetic energy of ejected α-particles, $T_\alpha = \dfrac{Q_\alpha}{1+M_\alpha/M_Y}$

$$\Rightarrow \quad Q_\alpha = T_\alpha \frac{[My + M_\alpha]}{M_y} = T_\alpha\left[\frac{A}{A-4}\right]$$

$$\Rightarrow \boxed{T_\alpha = Q_\alpha\left[1-\frac{4}{A}\right]} \qquad \& \ T_d = Q_\alpha - T_\alpha = \frac{4Q}{A}$$

Where K.E. of α-particle $= T_\alpha$
As $A \sim 200 \quad \Rightarrow \quad T_\alpha \sim Q_\alpha$

α - decay through tunnelling:

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + 2\text{He}^4$$

$$R = R_0 A^{1/3}$$

$$V(R)=\frac{2(Z-2)|e|^2}{4\pi\epsilon_0 R},\quad v=\sqrt{\frac{2E}{M}}$$

$$T=e^{-G},\ G=A(Z-2)E^{-1/2}-B(Z-2)^{1/2}R^{1/2}$$

$$A=1.587\times10^{-6}\ \mathrm{J}^{1/2}$$

$$B=94\times10^{6}\ \mathrm{m}^{-1/2}$$

$$\nu=\frac{v}{2R};\ \lambda=\nu T;\ T_{1/2}=\frac{\ell n\,2}{\lambda}$$

Example: ${}_{92}U^{238}\longrightarrow{}_{90}Th^{234}+{}_{2}He^{4}$ and the energy of the emitted α-particle is $E=6.72\times10^{-13}$ joule. Determine the half-life time of ${}_{92}\mathrm{U}^{238}$.

Soln. Nuclear radius of parent

$$R=R_0A^{1/3}=(1.4\ \mathrm{fm})(238)^{1/3}=8.7\mathrm{fm}=8.7\times10^{-15}\,\mathrm{m}$$

Atomic number of parent nucleus ${}_{92}U^{238}$, $Z=92$

Tunnelling probability,

$$T=e^{-G}=e^{-\left(1.587\times10^{-6}\,\mathrm{J}^{1/2}\right)(92-2)\left(6.72\times10^{-13}\,\mathrm{J}\right)+\left(94\times10^{6}\,\mathrm{m}^{-1/2}\right)(92-2)^{1/2}\left(8.7\times10^{-15}\,\mathrm{m}\right)^{1/2}}=\frac{6}{10^{39}}$$

Out of 10^{39} strikes α-emerges out six times only.

Velocity, $$v=\sqrt{\frac{2E}{m}}=\sqrt{\frac{2(6.72\times10^{-13}\,\mathrm{J})}{4\times1.67\times10^{-27}\,\mathrm{kg}}}=1.42\times10^{7}\ \mathrm{ms}^{-1}$$

Collision frequency, $$\nu=\frac{v}{2R}=\frac{1.42\times10^{7}\ \mathrm{ms}^{-1}}{2(8.7\times10^{-15}\,\mathrm{m})}=8.2\times10^{20}\,\mathrm{s}^{-1}$$

Decay constant, $$\lambda=\nu T=\left(8.2\times10^{20}\,\mathrm{s}^{-1}\right)\left(6\times10^{-39}\right)=4.92\times10^{-18}\,\mathrm{s}^{-1}$$

Half life $$T_{1/2}=\frac{\ell n\,2}{\lambda}=\frac{\ell n\,2}{4.92\times10^{-18}\,\mathrm{s}^{-1}}=1.4\times10^{17}\,\mathrm{s}$$

$$=\frac{1.4\times10^{17}}{365\times24\times60\times60}\ \text{years}=4.4\times10^{9}\ \text{years}$$

The experimental value of half life is 4.5×10^{9} years. So, we have obtained a rough estimate of half life ${}_{92}U^{238}$.

β-decay :

Let v be the velocity of a given β-particle, B the magnetic flux density, m the relativistic mass of the β-particle and r the radius of the circular track.

Then $\quad \frac{mv^2}{r} = Bev \quad \Rightarrow r = \frac{mv}{Be} = \frac{m_0}{\sqrt{1-v^2/c^2}} \cdot \frac{v}{Be}$

- β-decay, we write:

$${}_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0$$

The disintegration energy in β-decay is:

$$Q_{\beta^-} = \left[M_n(A,Z) - M_n(A,Z+1) - m_e\right]c^2$$
$$= \left[M(A,Z) - Zm_e - M(A,Z+1) + (Z+1)m_e - m_e\right]c^2 \text{ (in terms of atomic mass)}$$
$$= \left[M(A,Z) - M(A,Z+1)\right]c^2$$

where M_n is the nuclear mass, M the atomic mass and m_e the mass of electron.

$Q_{\beta^-} > 0$, if $M(A, Z) > M(A, Z+1)$

implying that β-decay occurs only if the mass of the parent atom is greater than that of the daughter atom.

- **β^+- decay:**

$${}_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0$$

$$\Rightarrow \quad Q_{\beta^+} = \left[M_n(A,Z) - M_n(A,Z-1) - m_e\right]c^2$$
$$= \left[M(A,Z) - Zm_e - M(A,Z-1) + (Z-1)m_e - m_e\right]c^2$$
$$= \left[M(A,Z) - M(A,Z-1) - 2m_e\right]c^2$$

For β^+- decay to occur : $Q_{\beta^+} > 0$, if $M(A,Z) > M(A,Z-1) + 2m_e$

i.e. the mass of the parent atom is greater than the daughter atom at least twice the electronic mass, i.e. 1.02 MeV.

- **Electron capture:**

$${}_ZY^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A$$

Therefore, disintegration energy, $Q_e = \left[M_n(A,Z) + m_e - M_n(A,Z-1)\right]c^2 - B_e$ where B_e is the binding energy of the electron to the orbit.

$$\Rightarrow \quad Q_e = \left[M(A,Z) - Zm_e + m_e - M(A,Z-1) + (Z-1)m_e\right]c^2 - B_e$$
$$= \left[M(A,Z) - M(A,Z-1)\right]c^2 - B_e$$

For electron capture to occur: $Q_e > 0$, if $M(A,Z) > M(A,Z-1) + B_e$

i.e. the mass of the parent atom is greater than that of the daughter atom by at least the binding energy of the electron.

$$\therefore \quad K_Y = \frac{m_x}{M_Y}K_x + \frac{m_y}{M_Y}K_y - \frac{2}{M_Y}\sqrt{m_x m_y K_x K_y}\cos\theta$$

But, $Q = K_Y + K_y - K_x$

$$\therefore \quad Q = K_y\left(1+\frac{m_y}{M_Y}\right) - K_x\left(1-\frac{m_x}{M_Y}\right) - \frac{2}{M_Y}\sqrt{m_x m_y K_x K_y}\cos\theta$$

This gives the Q-value of the reaction in terms of K_y, K_x and θ without involving the kinetic energy of the recoil nucleus, K_Y and the mass M_x of target nucleus and is called the standard form of Q-equation.

If $\theta = 90°$, the Q-expression simplifies to : $Q = K_y\left(1+\frac{m_y}{M_Y}\right) - K_x\left(1-\frac{m_x}{M_Y}\right)$

If Q is negative, it gives the minimum value of threshold kinetic energy to be given to the incident particle for an endoergic or endothermic reaction to proceed in the forward direction.

Q-value and threshold energy of nuclear reaction:

The law of conservation of energy and momentum imposes certain restrictions on the reactions. These restrictions are called as the kinematic restrictions and this mathematical method is known as kinematics. Consider the nuclear reaction

$$x + X \longrightarrow y + Y$$

Where x, X, y and Y are the bombarding particle, target nucleus, outgoing particle and product nucleus respectively. It is assumed that the target nucleus is in rest. Since total energy is conserved in the nuclear reaction, therefore we get,

$$\left(m_x c^2 + E_x\right) + M_X c^2 = \left(E_y + m_y c^2\right) + \left(E_Y + M_Y c^2\right)$$

E_x, E_y and E_Y are the kinetic energies of respective particles.

Now the quantity $Q = E_y + E_Y - E_x \Rightarrow Q = \left(m_x + M_X - m_y - M_Y\right)c^2$

Where Q is called the Q-value of nuclear reaction.

(i) If Q is positive, the reaction is said to be exoergic (exothermic) and

(ii) If Q is negative, the reaction is called endoergic (endothermic).

The minimum K.E. required for incident particle (x) to start the nuclear reaction is called the threshold energy $(E_x)^{th}$. The relation between Q-values and threshold energy is:

$$\boxed{E_x^{th} = -Q\frac{\left(m_y + M_Y\right)}{\left(M_Y + m_y - m_x\right)}}$$

If $E_x^{th} = 0$ for exoergic or exothermic reactions i.e. these reaction are spontaneous process.

Mechanism of nuclear reactions:

(i) ${}_3Li^7 + {}_1H^2 \rightarrow \left({}_4Be^9\right)^* \rightarrow {}_4Be^8 + {}_0n^1$

(ii) ${}_4Be^9 + {}_1H^2 \rightarrow \left({}_5B^{11}\right)^* \rightarrow {}_5B^{10} + {}_0n^1$

(iii) ${}_6C^{12} + {}_1H^2 \rightarrow \left({}_7N^{14}\right)^* \rightarrow {}_7N^{13} + {}_0n^1$

When two Deutrons Interact both the (d, n) and (d, p) reactions have been obtained.

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 \rightarrow {}_1H^3 + {}_1H^1 + 4.02\text{ MeV} \rightarrow {}_2He^3 + {}_0n^1 + 3.25\text{ MeV}$$

Nuclear fission and fusion :

Fission and fusion are two processes that alter the nucleus of an atom. Nuclear fission provides the energy in nuclear power plants and fusion is the source of the sun's energy. The use of fission in power plant can help conserve fossil fuels. Without the energy produced by the fusion of hydrogen in the sun, the earth would quickly change into a cold planet that could not support life as we know it.

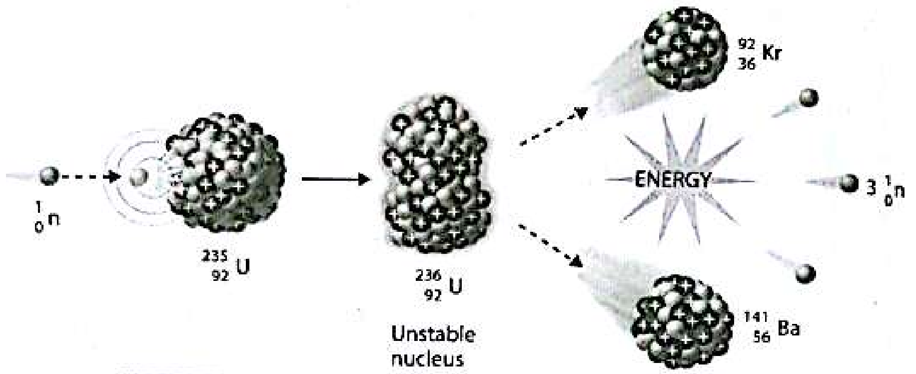
Nuclear transformation always obey two fundamental conservation laws

(i) Mass number is conserved
(ii) Electric charge is conserved.
Energy and mass are not conserved but can be interconverted according to Einstein's equation.

$$\boxed{E = mc^2}$$

Nuclear fission:

The process of fission occurs when a nucleus spits into smaller pieces. Fission can be induced by a nucleus capturing slow moving neutrons, which result in the nucleus becoming very unstable.



The following equation represent fission reactions

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$$

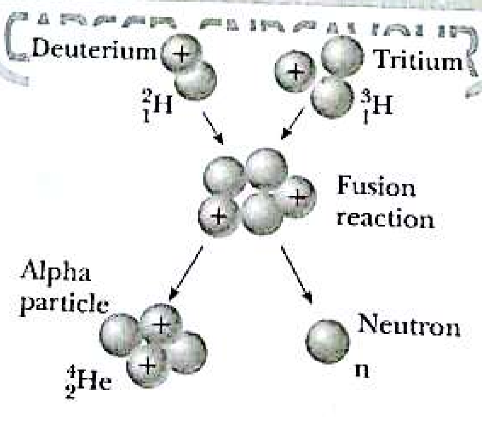
$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{137}_{52}Te + ^{97}_{40}Zr + 2^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{152}_{66}Nd + ^{81}_{32}Ge + 3^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{131}_{50}Sn + ^{103}_{42}Mo + 2^{1}_{0}n_0$$

Fusion :

Fusion occurs when two nuclei joint together to form a large nucleus. Fusion is brought about by bringing together two or more small nuclei under conditions of tremendous pressure and heat.



The following equation represent fusion reactions

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{1}H + ^{1}_{1}H$$

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n$$

$$^{3}_{2}He + ^{3}_{2}He \longrightarrow ^{4}_{2}He + 2^{1}_{1}H$$

3. The spin and parity of $_4Be^9$ nucleus, as predicted by the
(a) 3/2 and odd (b) 1/2 and odd (c) 3/2

4. The expression of a_c is:

(a) $\frac{3e^2}{4\pi \epsilon_0 R_0}$ (b) $\frac{3e^2}{}$ (c) $\frac{}{20}$

SOLVED EXAMPLES

Example 1. *The half life of radon is 3.8 day. After how many day will only 1/20of a radon sample be left over.*

Solution. Half-life of radon $T = 3.8$ day

Let N_0 be the initial amount of radon; after t day the remaining sample is

$$N = N_0/20$$

If λ is the disintegration constant of radon, then

$$N = N_0 e^{-\lambda t}$$

and $$\lambda = \frac{0.693}{T} = \frac{0.693}{3.8} \text{day}^{-1}$$

Therefore, $$\frac{N_0}{20} = N_0 e^{\frac{0.693}{3.8}t}$$

or $$20 = e^{\frac{0.693}{3.8}t}$$

or $$2.303 \log_{10} 20 = \frac{0.693}{3.8} t$$

$\therefore$ $$t = \frac{2.303 \times 1.3010 \times 3.8}{0.693} \text{ day} = 16.43 \text{ day}$$

Example 2. *If the activity of a radioactive sample drops to 1/16 of its initial value in 1 hr and 20 min what is its half-life?*

Solution. Let initial activity correspond to N_0 radioactive atoms, then after 80 minute the final activity corresponds to $\frac{N_0}{16}$ radioactive atoms. Let λ represent the disintegration constant, then

$\therefore$ $$\frac{N_0}{16} = N_0 e^{-80\lambda}$$

or $$16 = e^{80\lambda}$$

$$2.303 \log_{10} 16 = 80\lambda$$

or $$\lambda = \frac{2.303 \times 1.2041}{80} \text{ min}^{-1} = 0.03464 \text{ min}^{-1}$$

If T is the half-life of the radioactive sample, then

$$T = \frac{0.693}{\lambda} = \frac{0.693}{0.03463} \text{ min} = 20 \text{ minute}$$

Example 3. *The half of radium is 1500 year. In how many year will I g of pure radium (a) loose 1 mg, (b) be reduced to one centigram.*

Solution. Half-life of radium = T = 1500 year

λ = The disintegration constant of radium

$\therefore$ $$= \frac{0.693}{T} = \frac{0.693}{1500} \text{ year}^{-1}$$

Initial amount of radium = N_0 = 1 g = 1000 mg

(a) Let in t years the amount of radium lost be 1 mg. The remaining amount = 999 mg

Therefore,

$$999 = 1000\, e^{-\frac{0.693}{1500}t}$$

or

$$\frac{1000}{999} = e^{\frac{0.693}{1500}t}$$

$\therefore$

$$= \frac{2.303 \times 0.0004 \times 1{,}500}{0.693} \text{ year} = 1.995 \text{ year}$$

(b) Let in t_1 years the amount of radium left be 1 centigram (= 10 mg).

Then

$$10 = 1000\, e^{\frac{-0.693}{1500}t_1}$$

or

$$100 = e^{\frac{0.693}{1500}t_2}$$

$\therefore$

$$t_1 = \frac{2.303 \times 2.0000 \times 1{,}500}{0.693} \text{ year} = 9.972 \times 10^2 \text{ year}$$

Example 4. *10 milligrams of a radioactive material of half-life period two year are kept in store for four years. How much of the material remains unchanged ?*

Solution. N_0 = Initial amount of radioactive material = 10 mg

$$T = \text{half-life} = 2 \text{ year}$$

$\therefore$ The disintegration constant $\lambda = \frac{0.693}{2}$ year^{-1}

Let N be the amount of radioactive material left over after a time interval of 4 year. Then

$$N = 10\, e^{-\frac{0.693}{2} \times 4} = 10_e^{-1.386}$$

$$= \frac{10}{3.998} = 2.501 \text{ mg}$$

Example 5. *For a given sample, the counting rate is 47.5 α-particles/min. After 5 minute the count is reduced to 27 α-particle per minute. Find the decay constant and half-life of the sample.*

Solution. Initial counting rate = 47.5 α-particles/min

Final counting rate after 5 minutes = 27 α particles/min

Let λ be the disintegration constant of the sample

Then,

$$27 = 47.5e^{-5\lambda}$$

or

$$\frac{47.5}{27} = e^{5\lambda}$$

$\therefore$

$$\lambda = \frac{2.303 \times 0.2455}{5} \text{ min}^{-1} = 0.1157 \text{ min}^{-1}$$

$$T = \text{Half-life of the sample} = \frac{0.693}{\lambda}$$

$$= \frac{0.693}{0.1157} \text{ min} = 5.989 \text{ min}$$

The spin and parity of $_4Be^9$ nucleus, as predicted by the ...
(a) 3/2 and odd (b) 1/2 and odd (c) 3/2 ar...

4. The expression of a_s is:

(a) $\frac{3e^2}{4\pi\epsilon_0 R}$ ... (c) $\frac{3...}{20\pi...}$

Example 6. *Calculate the weight in gram of one curie of RaB (Pb^{214}) from its half-life of 26.8 minute.* (Punjab)

Solution. Activity of RaB $= \frac{dN}{dt} = 1$ curie $= 3.7 \times 10^{10}$ disintegrations/s

Half-life of RaB $= T = 26.8$ minute

Let λ be the disintegration constant of RaB

Then
$$\lambda = \frac{0.693}{T} = \frac{0.693}{26.8 \times 60} s^{-1}$$

Let N be the number of atoms of RaB having an activity of 1 curie. We know

$$\left|\frac{dN}{dt}\right| = \lambda N$$

$$N = \frac{3.7 \times 10^{10} \times 26.8 \times 60}{0.693}$$

We know that 6.02×10^{23} atoms = 1 gram atom = 214 g

∴ Weight of RaB having an activity of 1 curie

$$= \frac{214 \times 3.7 \times 10^{10} \times 26.8 \times 60}{6.02 \times 10^{23} \times 0.693} g$$

$$= 0.3052 \times 10^{-3} g = 30.52 \times 10^{-7} \mu g$$

Example 7. *One man of radium has an activity of one curie. What activity of radon will accumulate from one milligram of pure radon in 3.825 day which is the half-life of radon?*

Solution. No. of atoms in 1 mg of pure radon $= \frac{6.02 \times 10^{23}}{222} \times 10^{-3}$

Half-life of radon $= T = 3.825$ day

∴
$$\lambda = \frac{0.693}{3.825 \times 24 \times 60 \times 60} s^{-1}$$

∴ Activity of 1 mg of radon

$$= \frac{dN}{dt} = \lambda N$$

$$= \frac{0.693 \times 6.02 \times 10^{23}}{3.825 \times 24 \times 60 \times 60 \times 222} \text{ disintegration/s}$$

$$= \frac{0.693 \times 6.02 \times 10^{23}}{3.825 \times 24 \times 60 \times 60 \times 222 \times 3.7 \times 10^{10}} \text{ curie}$$

$$= 153.7 \text{ curie.}$$

Example 8. *One milligram of thorium emits 22 α-particles per unit solid angle per minute. Calculate the half-life of thorium.*

(At wt. of thorium = 232 and Avogadro's number 6.02×10^{23})

Solution. Number of α-particles per sec in all directions

$$\frac{22 \times 4\pi}{60} = \frac{22\pi}{15}$$

No. of atoms in 1 milligram of thorium $= \frac{0.001 \times 6.02 \times 10^{23}}{232}$

Decay rate per second $= \dfrac{22\pi}{15}$

$$-\frac{22\pi}{15} = -\frac{\lambda \times 0.001 \times 6.02 \times 10^{24}}{232}$$

$$\lambda = \frac{232 \times 22 \times \pi}{15 \times 0.001 \times 6.02 \times 10^{23}} = 1.775 \times 10s^{-13}$$

$$\text{Half-life} = \frac{0.693}{\lambda}$$

$$= \frac{0.693}{1.775 \times 10^{-16}} S = 1.238 \times 10^{10} \text{ year}$$

Example 9. *Th B decays into Th C with half-life 10.6 hour and Th C into Th D with half-life 60.5 minute. Calculate the time after which a freshly prepared sample of Th B would attain maximum Th C activity.*

Solution. Let T_1 be the half-life of Th B decays into Th C and T_2 that of Th C into Th D. If λ_1 and λ_2 represent the corresponding disintegration constants, we have

$$\lambda_1 = \frac{0.693}{T_1} = \frac{0.693}{10.6 \times 60} \text{ min}^{-1} = 0.00109 \text{ min}^{-1}$$

$$\lambda_1 = \frac{0.693}{T_2} = \frac{0.693}{60.5} \text{ min}^{-1} = 0.01145 \text{ min}^{-1}$$

Here λ_1 and λ_2 are of the same order of magnitude. If t_m is the time after which a freshly prepared sample of Th B would attain maximum Th C activity, then

$$t_m = \frac{1}{(\lambda_2 - \lambda_1)} l_n\left(\frac{\lambda_2}{\lambda_1}\right)$$

$$= \frac{1}{(0.01145 - 0.00109)} 2.303 \log_{10}\left(\frac{0.01145}{0.00109}\right) \text{min}$$

$$= \frac{1 \times 2.303 \times 1.0215}{1.036 \times 10^{-2}} \text{ min} = 227 \text{ minutes}$$

Example 10. *The mean half of radium (226) is 1600 year and that for radon (222) 3.8 day. Calculate the volume of radon gas that would be in equilibrium with 1 g of radium.*

Solution. Mean half-life of radium $= T_1 = 1600$ year

and Mean half-life of radon $= T_2 = 3.8$ day

Let N_1 and N_2 be the amount of radium and radon in equilibrium. Then

$$\frac{N_1}{N_2} = \frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2}$$

Given $N_1 = 1$ g

$\therefore$ $$N_2 = N_1 \frac{T_2}{T_1} = 1 \times \frac{3.8}{1600 \times 365} \text{g}$$

We know, at N.T.P., 222 g of radon occupies 22.4 litre. Therefore,

Volume of radon in equilibrium with 1 gm of radium

$$\frac{22.4 \times 10^3 \times 3.8}{222 \times 1600 \times 365} \text{ ml} = 0.656 \times 10^{-3} \text{ ml}$$

4. The expression of a_s is:

(a) $\frac{3e^2}{4\pi \epsilon_0 R_0}$ (b) $\frac{3e^2}{\quad}$ (c)

Example 11. *Given that the period of radon is 3.82 day and that the volume at normal temperature and pressure of the radon in equilibrium with 1 g of radium is 0.63 mm³; deduce the half-life period of radium.*

(Gram molecular volume = 22.4 litre and a atomic wt. of radium = 226)

Solution. Let the half-life period of radium = T_1

The half-life period of radon = T_2 = 3.8 day

Volume of radon at N.T.P. in equilibrium with 1 g of radium

$$= 0.63 \text{ mm}^3 = 0.63 \times 10^{-3} \text{ cm}^3$$

Let N_2 be the amount of radon in equilibrium with N_1 = 1 g or radium. Then

?

$$N_2 = \frac{222 \times 0.63 \times 10^{-3}}{22.4 \times 10^{-3}} \text{g}$$

224. — 0.63

Also, we know in secular equilibrium

$$\frac{N_1}{N_2} = \frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2}$$

$$T_1 = \frac{1 \times 3.8 \times 22.4 \times 10^2}{222 \times 0.63 \times 10^{-3}} \text{ day} = \frac{3.8 \times 22.4 \times 10^6}{222 \times 0.63 \times 365} \text{ year}$$

$$= 1676 \text{ year}$$

Example 12. *An accident occurs in a laboratory in which a large amount of radio active material with a known half-life of 20 day becomes embedded on floor, walls etc. Tests show that the level of radiation is 32 times the permissible level of normal occupancy of the room. Assuming that the last statement is correct, after how many days the laboratory can be safely occupied.*

Solution. Since the initial value of radiation is 32 times the permissible level, we have to find time t in which the activity drops to $\frac{1}{32}$ of its initial value, i.e.,

$$\frac{N}{N_0} = \frac{1}{32}$$

∴ Half life of the sample = 20 day

∴ Disintegration constant $\lambda = \frac{0.693}{20} \text{ day}^{-1}$

Let the laboratory become safe for use after t days :

Then

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore \quad \frac{1}{32} = \exp.\left[-\frac{0.693}{20} \times t\right]$$

$$\therefore \quad \frac{0.693}{20} t = 2.303 \times \log 32 = 2.303 \times 1.5051$$

$$t = \frac{2.303 \times 1.5051 \times 20}{0.693} = 100 \text{ day}$$

Chapter 11.3

Models of the Nucleus

Introduction

It is easy to realize that once a fairly detailed information about the nucleus became available, it was but natural to try and think of a plausible model of the nucleus. The model should help us to understand nuclear properties and the details of nucleon interactions, in much the same way as a model of the atom helps us to understand and explain the very many properties of the atoms.

Unlike the case of the atom, physicists however, still do not have a very clear understanding of the details of nuclear forces or the way the nucleons interact with each other. This lack of understanding has made them propose very many different models each 'designed' to explain a particular category of nuclear phenomenon or nuclear properties. At times, these models even tend to contradict each other. This has led to considerable confusion and attempts are constantly 'on' to clear this confusion and get a model that incorporates the essential details of these very many models.

Two Broad Categories of Nuclear Models

The nuclear models, proposed so far, can be categorized into two broad types. We refer to them as (*a*) the *strong interaction models* and (*b*) *the independent particle models*.

(*a*) **The strong Interaction Models:** These models are based on the assumption that *the nucleons, in a nucleus, are strongly coupled together.* The best known model of this category is the so called **liquid drop model**. This model has been quite successful in explaining the phenomenon of *nuclear fission*. It has also been used to arrive at the '*semi-empirical mass formula*' which enables us to calculate the nuclear masses in a fairly satisfactory way.

(*b*) **The Independent Particle Models:** These models are based on the assumption that there exists a common nuclear potential within the nucleus and all the nucleons move nearly independently within this common nuclear potential. The most successful model of this category, is the so called **shell model.** This model helps us to understand the periodicity observed in many properties of the nuclei. These periodicities are reminiscent of the periodicity shown in their chemical properties by the atoms of different elements. The (nuclear) *shell model* is, therefore, similar to the shell model of atom in very many ways.

We now talk about the *liquid drop model* and the *shell model* in some detail.

The Liquid Drop Model

The liquid drop model of the nucleus was first suggested by *Bohr*. He, along with *Wheelar*, used it to explain the essential details of the phenomenon of *Nuclear Fission*. This explanation has been the most significant achievement of this model but since it has not met with much success in understanding other nuclear properties, it is now a more or less obsolete model.

The liquid drop model was proposed and developed by highlighting some striking similarities between a liquid drop and a nucleus. We list below some of these similarities:

1. Small drops of a liquid are known to acquire a spherical shape. We know that this is because of the symmetrical surface tension forces acting towards the centre of the drop. The gravitational forces, acting vertically downwards, tend to flatten out the drop. Ultimately, it is the 'balance' between these two forces which decides the shape of the drop.

A nucleus also has two kinds of opposing forces acting within it. On the one hand there are the (short ranged and very strong) attractive nuclear forces that try to keep the nucleus 'intact'. On the other hand, there are the repulsive electrostatic forces between the protons contained in the nuclei. It is again a 'balance' between these two opposing forces which decides the stability level of a nucleus. For light stable nuclei, the attractive nuclear forces dominate the other repulsive (disruptive) forces. Such nuclei, therefore, tend to be spherical in shape.

2. For a spherical drop, the density is known to be independent of its volume. This also implies that the radius must vary as the cube root of its mass. It is known that the density of nuclear matter is (nearly) constant for all nuclei and nuclear radii vary as the cube root of their mass numbers. We are, therefore, apparently justified in regarding a nucleus as similar to a spherical drop of a liquid. Of course, liquid drops show a somewhat different behaviour here. We know that, unlike different nuclei, the densities of different liquids are different. However, there is still sufficient reason to think of a nucleus as if it were akin to a spherical drop.

3. The molecules of a liquid are known to be in continuous random thermal motion. We use this to explain the phenomenon of continuous 'evaporation' of a liquid and the increase in the rate of this evaporation with an increase in the temperature of the liquid. If we were to think of a similar random motion for the nucleons inside a nucleus, we could think of phenomenon like spontaneous emission of α-particles as essentially similar to the evaporation of molecules from the surface of a liquid.

There are thus quite a few apparent similarities between a liquid drop and a nucleus. The liquid drop model, besides helping us to understand the phenomenon of nuclear fission, also helps us to understand other nuclear reactions. It was also used to develop the fairly successful *'semi-empirical mass formula'* for different nuclei. We now look at the explanation of nuclear fission in terms of this model.

The Liquid Drop Model and the Phenomenon of Nuclear Fission

We know that if a drop of a liquid were to be suitably excited, it can oscillate in a variety of ways. The figure 11.3.1 shows a possible mode of this variation. Here a liquid drop is shown to become successively a prolate spheroid, a sphere, an oblate spheroid, a sphere and again a prolate spheroid. The cycle may then be repeated again and again.

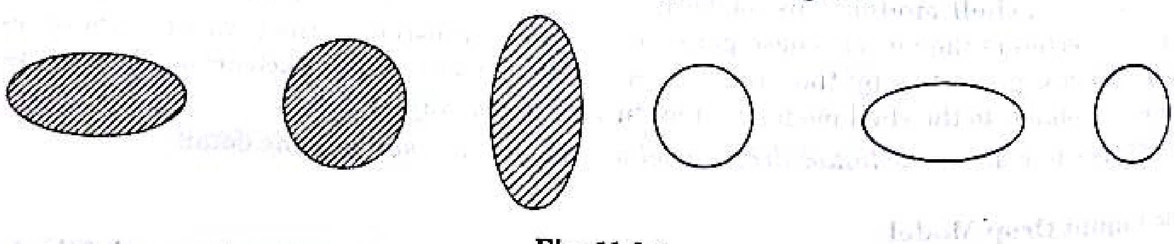


Fig. 11.3.1

These changes come primarily through the interplay of the forces of surface tension between ie liquid molecules and the natural *'inertia'* associated with moving liquid molecules. If a quid drop were to be slightly distorted, the forces of surface tension would tend to bring it back to its spherical shape. However, once this transition from a prolate spheroid to a sphere is taking place, the inertia, associated with the now moving liquid molecules, makes them go beyond the equilibrium spherical shape and makes the drop into an oblate spheroid. The forces of surface tension again set in and try to bring the drop back to its spherical shape. The inertia, now acquired takes the drop back to the prolate spheroidal shape and the process keeps on repeating itself.

One can think of a similar thing happening to a 'drop' of a nucleus once it is excited. Here the attractive (short ranged and strong) nuclear forces play the role of the surface tension forces. However, unlike the case of a liquid drop, the proton constituents of the nucleus also exert strong (long range) repulsive forces on each other. Hence when a nuclear drop is excited, say by the impact of an external neutron, its distortion effects are not only due to the inertia of its moving nucleons but also due to these long range repulsive electrical forces. The (short ranged) attractive nuclear forces (similar to the surface tension forces in a liquid drop) are able to cope up with the combined effects of these distortive forces only for small distortions. When the distortions are large, the nucleons may get separated from each other by distances that are larger than the range of the attractive nuclear forces. These forces are then no longer able to bring together the now widely separated groups of nucleons and the nucleus may, therefore, get split into parts. This is illustrated in the figure here.

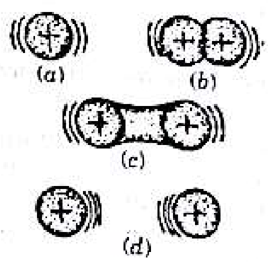


Fig. 11.3.2. "The Fission process", A separation of the nucleus into two parts occurs following a distortion of the spherical shape of the nucleus.

We believe this to be happening when thermal neutrons are made to hit the relatively unstable nuclei of the isotope U-235 of uranium. We thus have a reasonably plausible and pictorial explanation of the phenomenon of nuclear fission.

The Semi-Empirical Mass Formula

This formula, helps us to calculate the mass of a given nucleus from a knowledge of its mass number (A) and its atomic number (Z). It was arrived at by Von Weizsacker in 1935. His arguments were based on the assumption of a liquid drop type model for a nucleus. We now proceed to look at the details of the way this formula was arrived at.

By simple logic, we expect the mass, M, of a nucleus of atomic number Z and mass number A to be

$$M = Z\, m_p + (A - Z)\, m_n \qquad \text{...(11.3.1)}$$

Here m_p and m_n stand for the masses of the proton and the neutron respectively. The nucleus, of mass number A and atomic number Z, is known to be 'made up' of Z protons and (A—Z) neutrons.

Precise measurements of nuclear masses, however, indicated that the observed nuclear masses are *always less* than their 'ideally expected' mass given by the above formula. We now believe that this *'lost mass'* gets converted into energy as per the Einstein mass energy relation ($E = mc^2$). The energy equivalent of this 'lost mass' (called the *'mass defect'*) when divided by the number (A) of nucleons, is referred to as the *binding energy per nucleon* for the given nucleus. We call it so because unless each of the nucleons, within this nucleus, *were to get back this energy*, this nucleus cannot be 'broken back' or decomposed into Z (free) protons and (A—Z) free neutrons. The binding energy per nucleon varies from nucleus to nucleus but it is always there for all the nuclei. If, therefore, a given nucleus has a total binding energy B, its mass ${}^{A}_{Z}M$ can be correctly expressed through the relation

$$ {}^{A}_{Z}M = Z\,m_p + (A-Z)\,m_n - B/C^2 \qquad ...(11.3.2)$$

Von Weizsacker used the essential ideas of the liquid drop model to calculate this binding energy B in terms of a number of terms. These terms were based on the properties of liquid drops, the effect of the repulsive electrical forces on the stability of the nucleus and the experimentally observed features of the stable and not so stable nuclei. He expressed the disruptive effect of each of these terms in terms of a number of '*constants*' (whose values were 'estimated' from the relevant data about different nuclei) and the atomic number Z and the mass number A. The 'binding energy' of a given nuclear isotope could, therefore, be estimated from a knowledge of its Z (atomic number) and A (mass number) values. A knowledge of 'B', in turn, then helped us to calculate the mass ${}^{A}_{Z}M$ of the nucleus.

We now look at the various *terms*, that were used by Von Weizsacker, to calculate the binding energy 'B' of a given nucleus.

1. *Volume Energy.* We now know that the binding energy per nucleon has an approximately constant value over a wide range of mass numbers. Hence, as a first approximation, we can regard the total binding energy of a nucleus to be proportional to the total number of nucleons in it. Since the mass number A of a nucleus represents the total number of nucleons in it, the binding energy, can be expressed as

$$B_0 = a_v \times A$$

This relation tells us that the contribution to the binding energy comes from the entire volume of the nucleus.

Thus we have

$$B_0 = a_v A \qquad ...(11.3.3)$$

where a_v is an undetermined constant.

2. *Surface Energy.* In writing the above expression for volume energy, we have implicitly assumed that all the nucleons are attracted by all the other nucleons. However, this is not quite true. The surface nucleons have fewer nearer neighbours than the nucleons which are deep within the nuclear volume. This factor gives rise to a slight decrease in the total binding energy of the nucleus. This decrease is proportional to the number of nucleons on the surface. Hence it is proportional to the surface area of the nucleus. If r is the radius of the nucleus then we know that

$$r = r_0 A^{1/3} \qquad ...(11.3.4)$$

where r_0 is a constant. Since the surface area of a nucleus is proportional to r^2 it must be proportional to $A^{2/3}$. The contribution of the surface effect to the binding energy of the nucleus can, therefore, be written as

$$B_1 = -a_S A^{2/3} \qquad ...(11.3.5)$$

where a_S is an empirical constant.

The negative sign put here is to emphasize the fact that the existence of surface nucleons leads to a decrease in binding energy. This effect is similar to that in a liquid drop where the surface nucleons are less tightly bound and, therefore, 'evaporate out' more easily from the liquid.

3. *Coulomb Energy.* The Coulomb energy of a nucleus is equal to the potential energy of Z protons packed together in a symmetric assembly of radius r. The protons in a nucleus are known to exert repulsive forces on each other. These repulsive forces, therefore tend to make

the nucleus less stable and cause a decrease in its binding energy. The loss of binding energy due to the disruptive Coulomb forces is obtained by considering the potential energy of a uniformly charged sphere. This is given by

$$\frac{3}{5}\frac{Z^2e^2}{r}$$

where Ze is the total charge on the nucleus and r is its radius. The decrease in binding energy of the nucleus due to Coulomb repulsion is proportional to this potential energy and can be expressed as

$$B_2 = -a_{c'}\frac{3}{5}\frac{Z^2e^2}{r} = -a_c\frac{Z^2}{r}$$

where a'_c is a constant. Further size $r = r_0A^{1/3}$ we have

$$B_2 = -a_{c'}\frac{Z^2}{A^{1/3}}$$

where we have put

$$a_c = \frac{3}{5}a_{c'}\frac{e^2}{r} \qquad \text{...(11.3.6)}$$

More, precise calculations show that we must put

$$B_2 = -a_c\frac{Z(Z-1)}{A^{1/3}} \qquad \text{...(11.3.7)}$$

Asymmetry Energy

A detailed study of the known stable nuclei as well as the known unstable nuclei revealed some interesting features. It was found that when a plot was made for all the known nuclides using their neutron number N (= (A—Z)) as the ordinate and their proton number (= Z) as the abscissa, the 'smooth line', drawn through the stable nuclides, was almost coinciding with the line N = Z for A ≤ 40. For the other stable nuclides also, this '*smooth line*' was not too far away from the line of symmetry (the N = Z line) but was always above this line. It thus appeared that

(*i*) nature preferred symmetry ($Z \simeq N$) for $A \leq 40$

(*ii*) there was a decided preference for $N > Z$ for $A > 40$. We can understand this in terms of the simple fact that an increase in the number of protons increases the inherently disruptive electrical forces within the nucleus. However, an increase in the number of neutrons does not do anything of that sort.

Weizsacker used these experimentally observed features to conclude that among the different isotopes of an element, the ones having equal number of protons and neutrons will be most stable and will therefore, possess maximum binding energy. Thus we find that an excess of neutrons over protons leads to a decrease in binding energy and causes instability. Considering a nucleus of mass number A and atomic number Z, the excess of neutrons over protons is (A – 2Z). This (A – 2Z) excess of neutrons produce a decrease in binding energy. The fraction of nuclear volume so affected is $\frac{A-2Z}{A}$. The total decrease can, therefore, be taken to be proportional to

$$(A-2Z)\,\frac{(A-2Z)}{A}$$

We may hence write the contribution of the 'asymmetry effect' on the binding energy of the nucleus as

$$B_3 = -a_a \frac{(A-2Z)^2}{A} \qquad ...(11.3.8)$$

Odd Even Effect

A detailed look, at the composition of the stable nuclides, reveals another very interesting feature. We find that

(*i*) all nuclides having an even number of protons and an even number of neutrons (*i.e. even Z and even A*) are generally the *most stable* of all.

(*ii*) all nuclides having an odd number of protons and an odd number of neutrons (*i.e. odd Z and odd N*) are the *least stable* of all.

(*iii*) nuclides belonging to the even-odd (even Z, odd N) or odd-even category (odd Z, even N) have an intermediate level of stability. Of the two categories of these even-odd nuclides, the nuclides with even Z and odd N are relatively more stable than nuclides with odd Z and even N.

We now realize that this variation in the stability of nuclides, with variation in their proton and neutron numbers, can be linked to the 'pairing effect' associated with the Pauli exclusion principle. For even-even nuclides, al the protons as well as all the neutron are 'paired off' and this makes them the most stable of all. For odd-odd nuclides, there is at least one unpaired proton and one unpaired neutron and this makes them the least stable. For the even-odd (or odd-even) case, we have either one unpaired proton or one unpaired neutron and this gives them an intermediate level of stability.

We may, therefore, say that the odd-even effect causes an increase in binding energy when both the number of protons and neutrons are even. However when, in the nucleus, the number of neutrons and protons is odd, this effect would cause a decrease in binding energy. The following table may be used to indicate the change in binding energy, E_δ, due to the odd-even effect.

Table 11.3.1

Mass Number A	*No. of protons* Z	*No. of neutrons* N	E_δ
even	even	even	$+\frac{\delta}{A^{\frac{3}{4}}}$
odd	even	odd	0
odd	odd	even	0
even	odd	odd	$-\frac{\delta}{A^{\frac{3}{4}}}$

Combining the various terms as discussed above, the semi-empirical mass formula for a given nucleus can be represented as

$$M = Z\,M_p + (A-Z)\,M_n - \frac{1}{c^2}\left[a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + E_\delta\right]$$

Experimental result give the following values of the constants occurring in the semi-empirical mass formula

$$a_v = 15.753 \text{ MeV}$$
$$a_s = 17.80 \text{ MeV}$$
$$a_c = 0.7103 \text{ MeV}$$
$$a_\alpha = 23.6925 \text{ MeV}$$
$$E_\delta = 33.6 \text{ MeV}. A^{-3/4}$$

However, it must be kept in mind that no single unique set of values satisfies the equations for all known nuclides. Other sets of values of the constants may be found.

The semi-empirical formula can account for the stability of nuclei against α and β decay. It has been fairly successful in estimating the masses of a wide range of nuclides.

The Shell Model

The *shell model* is the simplest and the most successful of the so called *independent particle models* of the nucleus. In this model, *it is assumed that every nucleon, within a given nucleus, moves independently of the others, in an average force field produced by all the other nucleons.* A representative diagram of the nuclear potential, experienced by the proton and the neutron, is shown here. There is a slight different in the shapes and depths of this nuclear potentials experienced by the proton and the neutron. This is because of the presence of the additional *Coulomb interaction* between the protons and this interaction, as we know, is not there for the neutrons. The Coulomb interaction causes the depth of this nuclear potential to be somewhat less for the proton than that for the neutron. Typical depths of this potential for the neutron and the proton are of the order of 43 MeV and 37 MeV respectively.

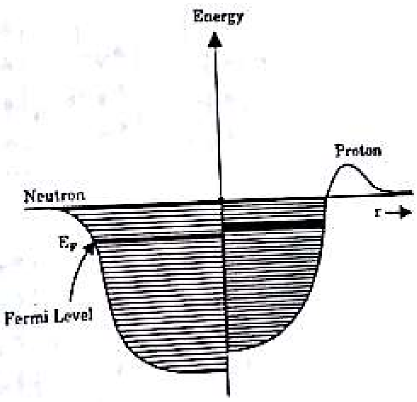


Fig. 11.3.3

Having thought of this nuclear potential for a nucleon, we can write and solve the Schrodinger equation for the motion of this nucleon. The results obtained are similar to those for the extra nuclear electrons orbiting around the force field of the nucleus. We again find that there exist a set of discrete allowed energy levels where each of these levels has a specific value of the *principal quantum number*, n, associated with it. We also find that we must also associate an 'orbital' *quantum number*, *l*, with a given nucleon. This quantum number tells us about the discrete set of values of the 'orbital' angular momentum of the nucleon.

The energy of a given nucleon is controlled by both its principal quantum number, n, and its orbital quantum number. Further, like in the case of the electrons, we have to think of the nucleons (both protons and neutrons) as having a value 1/2 for their *spin quantum number*. The association of this value 1/2 with the spin quantum number implies that both the proton and the neutron are *fermions* and hence obey the *Pauli exclusion principle*.

The similarity between these 'pictures' of the electrons and the nucleons implies that

(*i*) the nucleons too have a fermi energy level which is the highest energy level filled in the nucleus.

(*ii*) each state, with a given projection of the orbital angular momentum on the Z-axis, gets filled when it contains two nucleons with opposite spin directions.

(*iii*) an energy state, with a given value of *l*, can accommodate a total of $2(2l + 1)$ nucleons.

(*iv*) we can designate the nuclear energy states as 1s, 2p, 3d, ... etc. in much the same way as we do for the electrons in the atoms.

(*v*) we can work out the order of increasing energy for the different permitted energy states.

Energy Level Diagrams for Nuclides

We can use the ideas outlined above to draw the energy level diagrams for different nuclides. An important points, to be kept in mind here, is that we take the *zero* of the energy scale at the bottom of the nuclear potential. (This enables us to deal with positive energy values). Another significant point is that we must make a difference between the neutron energy levels and the proton energy levels. This, as we know, is because of the existence of the additional Coulomb repulsion for the protons. This Coulomb interaction causes a given neutron energy level to be slightly lower than the corresponding level for the proton.

An energy level, as we have noted above, gets *filled up* when it contains two nucleons with oppositely directed spins. It is now easy to realize the reason for the exceptional stability of '*even-even*' nuclides. For all such nuclides, the proton and the neutron energy levels each contain 'paired off' nucleons. This results in increase stability in much the same way as the existence of 'filled orbits' does for the *inert gases* in the case of the atoms. For exactly the same reason, we can understand why 'odd-odd' nuclides are the least stable and 'even-odd' or 'odd-even' nuclides have intermediate stabilities.

We now show below the schematic diagrams of the proton and neutron energy levels of some typical nuclides like the $^{12}_{6}C, ^{13}_{6}C, ^{14}_{7}N, ^{15}_{7}N, ^{15}_{8}O$ and $^{16}_{8}O$

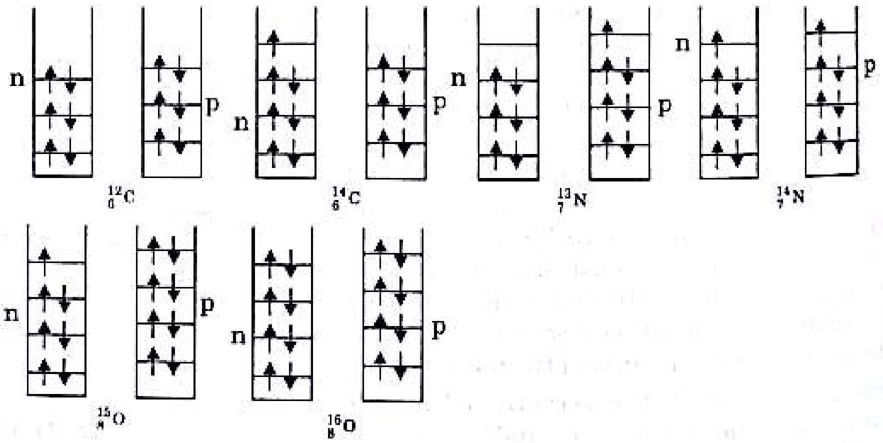


Fig. 11.3.4

Magic Numbers

We know that, in case of atoms, the *inert gases* are exceptionally stable. This high stability is associated with their 'completely filled' outermost shells. The inert gases have atoms with atomic numbers 2 (helium), 10 (neon), argon, krypton, xenon and radon all of which have completely filled up outermost shells.

In a similar way, we find that nuclides with number of protons or neutrons equal to **2, 8, 20, 50, 82 and 126** are exceptionally stable. We call these numbers as **magic numbers.** A nuclear *isotope* or *isotone*, with a Z (number of protons) or N (number of neutrons) value equal to one of these magic numbers, is found to be much more stable than the one in its neighbourhood with a slightly different value of Z or N.

In terms of the shell model, we can think of the *magic numbers* as those values of Z or N for which there can exist *closed* (or filled up) shells of protons or neutrons. Remembering that a nuclear energy level with a given value of the orbital angular momentum quantum number, l, can accommodate only up to $2(2l + 1)$ nucleons, we find that the nuclear closed shells can contain either 2 or 2 + 6 (= 8), or 2 + 6 + 10 (= 18), or 2 + 6 + 10 + 2 (= 20), or 2 + 6 + 10 + 2 + 14 (= 34)... and so on nucleons. The *magic numbers* **2, 8, 20** are thus understood easily. The other magic numbers (50, 82, and 126), however, cannot be explained on this basis only. We have to also introduce the concept of a *strong interaction between the spin angular momentum*, S, *and the orbital angular momentum*, l, *of a nucleon*. This causes a modification in the energy levels of the nucleons and the number of nucleons in the closed shells. The modified picture then helps us to understand the other magic numbers (50, 82 and 126) also.

The 'Successes' of the Shell Model

The '*shell model*' has proved useful and successful in explaining and accounting for the observed

(*i*) angular momentum

(*ii*) magnetic moment

(*iii*) electric quadrupole moment

of different nuclei.

Besides these, the shell model also helps us to understand the observed distribution of what are known as the *nuclear isomers*. Nuclear isomers are nuclei that can exist in an excited state for relatively much longer times (half life $\gtrsim$ 1s).

We thus find that this representative simplest nuclear model of the '*independent particle category*', is quite successful in understanding and explaining a wide range of nuclear properties.

Collective Model

Collective model is an attempt towards developing a general theory and view point of nuclear model. It is aimed to be a general model which can include the liquid drop model and the shell model as two special cases of this general view point. But again this model is not answering the fundamental question of nature of nuclear forces. This model has been developed by A. Bohr and Mottelson. It assumes that nucleus is made up of more or less stable core of nucleon's formed into closed shell and outside this closed shell extra nucleons move in potential of the core. A more detailed discussion of this model is beyond the scope of present work.

The other models which have been developed are the unified model and the optical model.

QUESTIONS

1. Why do we say that a nucleus behaves like a drop of liquid. What are the essential features which are common to a drop of liquid and a nucleus.
2. Discuss the significances of the charge effect and symmetry effect on nuclei stability.
3. Explain the postulates of the liquid drop model. Give a simple derivation of Weizsacker' semiempirical mass formula.
4. Discuss the various factors which contribute to the binding energy of nuclei and derive a formula for the atomic mass M (Z, A) of a nucleus based on these considerations.
5. What is the magic about the magic numbers? Explain how the shell model of the nucleus accounts for the existence of magic numbers.
6. Give salient features of nuclear shell model and point out its success and failures.