Assignment-2, Mathematics.
Topic → Calculus in single variable.

Q1 Expand $\log\sin x$ in power of $(x-3)$ by using Taylor series expansion.

$y = \log\sin x$

By Taylor Series expansion ⇒ $x = 3$, $h = x-3$

$$f(x) = f(3) + \frac{x-3}{1!}f'(3) + \frac{(x-3)^2}{2!}f''(3) + \frac{(x-3)^3}{3!}f'''(3) \ldots \quad (1)$$

$$y' = \frac{1}{\sin x}\cos x = \cot x \qquad y' = f'(x) = \cot x$$

$$\therefore f'(3) = \cot 3. \quad -(2)$$

$$y'' = -\operatorname{cosec}^2 x.$$

$$f''(3) = -\operatorname{cosec}^2(3). \quad -(3)$$

$$y''' = -[-2\operatorname{cosec} x \cot x . \operatorname{cosec} x]$$

$$= 2\operatorname{cosec}^2 x \cot x$$

$$f'''(3) = 2\operatorname{cosec}^2(3) . \cot(3). \quad -(4)$$

Putting values in (1) from 2, 3, 4

$$\therefore f(x) = \log\sin x = \log\sin 3 + (x-3)\cot 3 - \frac{(x-3)^2}{2}\operatorname{cosec}^2 3$$

$$+ \frac{(x-3)^3}{6} \times 2\operatorname{cosec}^2 3 \cot 3 \ldots$$

$$\log\sin x = \log\sin 3 + (x-3)\cot 3 - \frac{(x-3)^2}{2}\operatorname{cosec}^2 3 + \frac{(x-3)^3}{3}\operatorname{cosec}^2 3\cot 3 \ldots + \ldots$$

Q2 Use taylor's series expansion to compute the value of $\cos 32^\circ$ correct to four decimal places.

$$f(x) = \cos x = \cos(\underset{(x)}{30^\circ} + \underset{(h)}{2^\circ}) = \qquad h = 2 \times \frac{\pi}{180}$$

$$= f(30^\circ) + + h f'(30^\circ) + \frac{h^2}{2!}f''(30^\circ) + \frac{h^3}{3!}f'''(30^\circ) \ldots$$

$f(30) = \cos 30° = \sqrt{3}/2$

$f'(30) = -\sin 30° = -1/2$

$f''(30) = -\cos 30° = -\sqrt{3}/2$

$f'''(30) = \sin 30° = 1/2$

$\therefore \cos 32° = \frac{\sqrt{3}}{2} - \left(\frac{\pi}{90}\right)\frac{1}{2} - \left(\frac{\pi}{90}\right)^2 \frac{\sqrt{3}}{2} \times \frac{1}{2} + \left(\frac{\pi}{90}\right)^3 \times \frac{1}{2} \times \frac{1}{6} \cdots$

$= 0.86605 - 0.017444 - 0.00052 + 0.0000035$

$= 0.84809$

$\boxed{= 0.8481} \Rightarrow \cos 32°$.

Q3 Show that

i) $\sin^{-1}(x) = x + \frac{1}{2}\frac{x^2}{3} + \frac{1 \cdot 3}{2 \cdot 4}\frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\frac{x^7}{7} + \cdots$

$y = \sin^{-1}(x)$

$y' = \frac{1}{\sqrt{1-x^2}}$

$y'(\sqrt{1-x^2}) = 0$

Squaring both sides

$y'^2(1-x^2) = 0$

differentiating it $\rightarrow$

$\cancel{2}y' y''(1-x^2) + y'^2(-\cancel{2}x) = 0$

$y'[y''(1-x^2) + y'(2x)] = 0$

Either $y' = 0$ $\boxed{y''(1-x^2) + y'x = 0}$ $-(1)$

$\Downarrow$

diff (1) n-times.

$\left\{(1-x^2)\, y_{n+2}^{(x)} + {}^nC_1(-2x)\left(y_{n+1}^{(x)}\right) + {}^nC_2(-2)\, y_n^{(x)}\right\}$

$- \left\{y_{n+1}^{(x)} \cdot x + {}^nC_1(1)(y_n)\right\} = 0$

$$y_{n+2}(x)\left[1-x^2\right] + y_{n+1}(x)\left[-2nx - x\right] + y_n(x)\left[\frac{n(n-1)}{2}(-2) - 1\right] = 0$$

$\boxed{\text{Put } x = 0}$

$$y_{n+2}(0) = 0 + y_n(0)\left[n^2\right].$$

We know that

$$y(0) = 0$$

$$y_2(0) = 0^2(y(0)) = 0$$

$$y_4(0) = 2^2(y_2(0)) = 0$$

$$y_1(0) = 1$$

$$y_3(0) = 1^2(1)$$

$$y_5 = 3^2 \cdot 1^2$$

$$y_7 = 5^2 \cdot 3^2 \cdot 1^2 \ldots$$

$$y_n = \begin{cases} 0 & n \text{ is even} \\ 1^2 \cdot 3^2 \cdot 5^2 \cdots (n-2)^2 & n \text{ is odd.} \end{cases}$$

$$\sin^{-1}(x) = \sin^{-1}(0) + \frac{y_1(0)}{1!}x + \frac{y_2(0)}{2!}x^2 + \frac{y_3(0)}{3!}x^3 + \frac{y_4(0)}{4!}x^4 + \frac{y_5(0)}{5!}x^5 + \frac{y_6(0)}{6!}x^6 + \frac{y_7(0)}{7!}x^7 \ldots$$

$$= 0 + x + \frac{x^3}{3!}(1) + \frac{x^5}{5!}(1^2 \cdot 3^2) + \frac{x^7}{7!}(1^2 \cdot 3^2 \cdot 5^2)$$

$$= x + \frac{x^3}{3\times 2} + \frac{x^5}{5}\left(\frac{1\cdot 3}{2\cdot 4}\right) + \frac{x^7}{7}\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right) \ldots$$

$$\sin^{-1}\left(\frac{1}{2}\right) = (0.5) + \frac{(0.5)^3}{6} + \frac{(0.5)^5 3}{40} + \frac{5(0.5)^7}{112} + \frac{35(0.5)^9}{1152} \ldots$$

$$\frac{\pi}{6} = 0.5 + 0.02083 + 0.00234 + 0.000348 \ldots$$

$$\frac{\pi}{6} = 0.52352 \Rightarrow \boxed{\pi = 3.1411}$$

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ii) $\log(x + \sqrt{1+x^2}) = x - 1^2 \cdot \frac{x^3}{3!} + 1^2 \cdot 3^2 \cdot \frac{x^5}{5!} - 1^2 3^2 5^2 \frac{x^7}{7!} \ldots$

$$y = \log(x + \sqrt{1+x^2})$$

$$y' = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \frac{2x}{2\sqrt{1+x^2}}\right)$$

$$y' = \frac{1}{\sqrt{1+x^2}}$$

Squaring both sides

$$y'^2 (1+x^2) = 1.$$

differentiating it →

$$2y' y'' (1+x^2) + y'^2 (2x) = 0$$

$$y'[y''(1+x^2) + y'(x)] = 0$$

$$\therefore \quad y''[1+x^2] + y'[x] = 0 \qquad - (1)$$

differentiating it n - times,

$$[y_{n+2}(x)][1+x^2] + n[y_{n+1}(x)][2x] + \frac{n(n-1)}{2}[y_n(x)][2]$$

$$+ [y_{n+1}(x)][x] + [y_n][1][n] = 0$$

$$\boxed{\text{Put } x = 0}$$

$$y_{n+2}(x) = -n^2[y_n(x)]$$

$\therefore \; y_0 = 0, \; \therefore \; y_2, y_4, y_6 \ldots = 0.$

$y_1 = 1$

$$y_3(0) = -1^2 \, y_1(0) = -1$$

$$y_5(0) = 3^2$$

$$y_7(0) = -1^2 \cdot 3^2 \cdot 5^2 \ldots$$

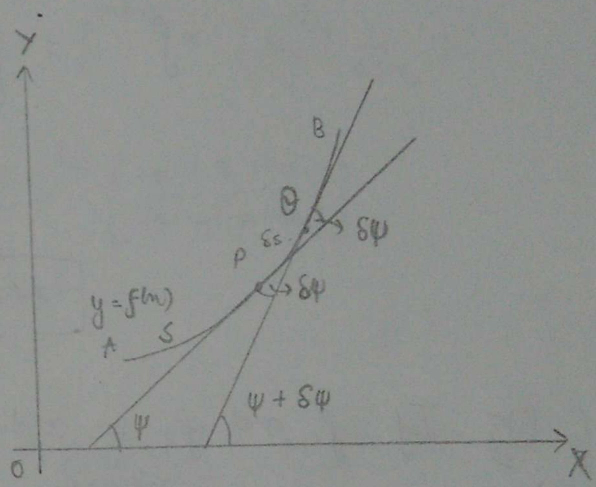
Arc surface Area

$$\rho = [1 + (dy)^2]^{3/2} \qquad = -[n \cdot , n]^{3/2}$$

$$\log(x + \sqrt{1+x^2}) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)x^2}{2!} + \frac{x^3}{3!}f'''(0) + f''''(0)\frac{x^4}{4!} \; ----$$

$$= x - 1^2 \cdot \frac{x^3}{3!} + 1^2 \cdot 3^2 \frac{x^5}{5!} - 1^2 \cdot 3^2 \cdot 5^2 \cdot \frac{x^7}{7!} \cdots -$$

Q4. Prove that radius of curvature (ρ) at any point (x, y) of the curve $y = f(x)$ is given by
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$



$\delta\psi$ is the angle through which the tangents turns as a point moves along the curve from $P \to Q$, through a dist δs.

$$\lim_{Q \to P} \frac{\delta\psi}{\delta s} = \lim_{\delta s \to 0} \frac{\delta\psi}{\delta s} = \frac{d\psi}{ds}.$$

Curvature at pt P $= \dfrac{d\psi}{ds}$.

Radius of curvature $= \dfrac{1}{\text{Curvature}}$

$$\boxed{\rho = \frac{ds}{d\psi}} \qquad - (1).$$

Now, we know that

$$\underbrace{\tan\psi}_{\text{Slope of tangent}} = \frac{dy}{dx}$$

Differentiating wrt to s

$$\sec^2\psi \cdot \frac{d\psi}{ds} = \frac{d^2y}{dx^2} \cdot \frac{dx}{ds}.$$

$$\frac{ds}{d\psi} = \frac{(1 + \tan^2\psi)\frac{ds}{dx}}{\frac{d^2y}{dx^2}}$$

$$\tan^2\psi = \frac{dy}{dx} \quad \& \quad \frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\frac{ds}{d\psi} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\left[\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right]}{\frac{d^2y}{dx^2}}$$

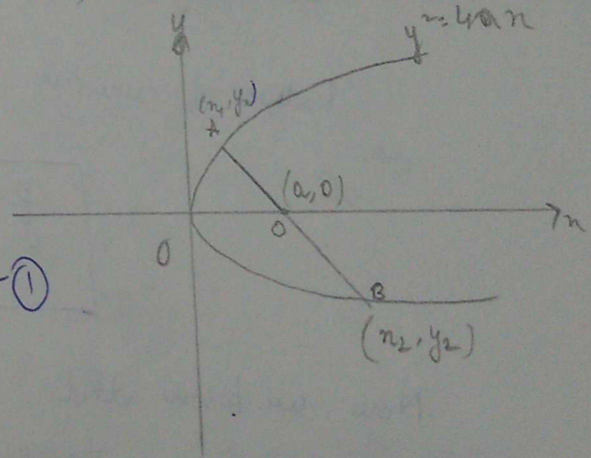
From (1)

$$\rho = \frac{\left[1+\left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2}.$$

$\boxed{\text{M.P}}$

Q5 If ρ_1 and ρ_2 are radii of curvatures at the extremeties of a focal chord of the parabola $y^2 = 4ax$, then prove that.

$$\rho_1^{-2/3} + \rho_2^{-2/3} = (2a)^{-2/3}.$$



$$y^2 = 4ax$$

$$2yy_1 = 4a$$

$$y_1 = \frac{2a}{y}$$

$$y_1' = \frac{2a}{\sqrt{4ax}} = \sqrt{\frac{a}{x}} \quad -(1)$$

Differentiating again

$$y'' = \frac{d}{dx}\left[\sqrt{a}\left(x^{-1/2}\right)\right]$$

$$y'' = -\frac{1}{2}\sqrt{a}\left[x^{-3/2}\right]$$

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Req surface Area

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{y''} = -\frac{\left[\frac{a+x}{x}\right]^{3/2}}{\frac{1}{2}\sqrt{a}\left[x^{-3/2}\right]}$$

$$= (-1)\frac{(a+x)^{3/2}}{x^{3/2}} \cdot \frac{x^{3/2}}{\sqrt{a}} \cdot 2$$

$$\rho = \frac{-2}{\sqrt{a}}(a+x)^{3/2} \quad - (2)$$

$$\rho^{2/3} = \frac{(-2)^{2/3}(a+x)}{a^{1/3}}$$

$$\rho^{2/3} = \left(\frac{4}{a}\right)^{1/3}(a+x)$$

$$\rho_1^{-2/3} = \left(\frac{a}{4}\right)^{1/3}\left(\frac{1}{a+x_1}\right) \qquad \rho_2^{-2/3} = \left(\frac{a}{4}\right)^{1/3}\left(\frac{1}{a+x_2}\right)$$

$$\therefore \rho_1^{-2/3} + \rho_2^{-2/3} = \left(\frac{a}{4}\right)^{1/3}\left[\frac{2a+x_1+x_2}{(a+x_1)(a+x_2)}\right].$$

We know that, for extremities of focal chords

$$x_1 x_2 = a^2$$

$$\therefore \rho^{-2/3} + \rho_2^{-2/3} = \left(\frac{a}{4}\right)^{1/3}\left[\frac{2a+x_1+x_2}{a^2+a(x_1+x_2)+x_1x_2}\right]$$

$$= \left(\frac{a}{4}\right)^{1/3}\frac{1}{a}\left[\frac{2a+\cancel{x_1+x_2}}{2a+\cancel{x_1+x_2}}\right] = \frac{1}{4^{1/3}(a^{2/3})}$$

$$= \frac{1}{(2a)^{2/3}} = 2a^{-2/3}$$

HP

[Proof for $x_1x_2 = a^2$: Slope of AO = OB.

$$\frac{y_2}{x_1-a} = \frac{y_1}{x_2-a} \quad \text{squaring both sides.}$$

$$\frac{y_1^2}{(x_1-a)^2} = \frac{y_2^2}{(x_2-a)^2}$$

$$\frac{4ax_1}{(x_1-a)^2} = \frac{4ax_2}{(x_2-a)^2}$$

$$x_1\left[x_2^2+a^2-2x_2a\right] = x_2\left[x_1^2+a^2-2x_1a\right]$$

$$x_1x_2^2 + x_1a^2 = x_2x_1^2 + x_2a^2$$

$\therefore \; x_1x_2(x_2-x_1) - a^2(x_2-x_1) = 0$

$(x_2-x_1)(x_1x_2 - a^2) = 0$

Since $x_2 - x_1 \neq 0$

$$\boxed{x_1x_2 = a^2}$$

Q6. Find the length of following curves :->

i) $x^2(a^2-x^2) = 8a^2y^2$

Tracing the curve

- Curve is symmetric about x, y axis.
- Curve passes through origin.
- intersection with axis

$\boxed{y=0} \Rightarrow x = \pm a, 0$

$\boxed{x=0} \Rightarrow y = 0$.

- Asymptotes.

No asymptotes exist parallel to x or y axis.

- Region of absence

$-a \leq x \leq a$

- $\frac{dy}{dx}$

$$2xa^2 - 4x^3 = 8a^2(2y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{xa^2 - 2x^3}{8a^2(y)}.$$

$\frac{dy}{dx} = 0$,

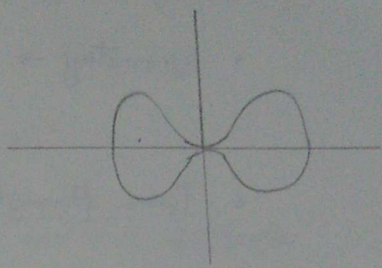
$xa^2 - 2x^3 = 0$

$x[a^2 - 2x^2] = 0$

$x = 0, \quad x = \pm a/\sqrt{2}$

$\frac{dy}{dx} \to \infty, \quad y = 0$

$x = \pm a$.



$$S = 4\int_0^a \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{x\left(a^2-2x^2\right)}{(8a^2)(y)}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{x^2\left(a^2-2x^2\right)^2}{(8a^2)^2\,(y^2)} = \frac{\cancel{x^2}\left(a^2-2x^2\right)^2}{(8a^2)^{\cancel{2}}\,\frac{(\cancel{x^2})(a^2-x^2)}{(\cancel{8a^2})}}$$

$$\therefore \quad 1+\left(\frac{dy}{dx}\right)^2 = \frac{8a^4-8a^2x^2+a^4+4x^4-4x^2a^2}{(8a^2)(a^2-x^2)}$$

$$= \frac{\left(3a^2-2x^2\right)^2}{8a^2\left(a^2-x^2\right)}$$

$$\therefore \quad \frac{S}{4} = \int_0^a \frac{3a^2-2x^2}{\sqrt{8}\,a\left(\sqrt{a^2-x^2}\right)}\, dx$$

Put $x = a\cos\theta$ when $x \to 0,\ \theta \to 90^\circ$

$\therefore\ dx = a\,-\sin\theta\, d\theta$ $\quad x \to a,\ \theta \to 0^\circ$

$$\frac{S}{4} = \int_{90^\circ}^{0^\circ} \frac{3a^2-2a^2\cos^2\theta}{\sqrt{8}\,a\left(\sqrt{a^2-a^2\cos^2\theta}\right)} \cdot a(-\sin\theta)\, d\theta$$

$$= \int_0^{\pi/2} \frac{a^2+2a^2\sin^2\theta}{\sqrt{8}\,a\,(\cancel{a\sin\theta})} (\cancel{a\sin\theta})\, d\theta$$

$$= \frac{1}{\sqrt{8}} \int_0^{\pi/2} \left(a+2a\sin^2\theta\right) d\theta$$

$$= \frac{a}{\sqrt{8}} \int_0^{\pi/2} \left(1+2\sin^2\theta\right) d\theta$$

$$= \frac{a}{\sqrt{8}} \int_0^{\pi/2} \left(2-\cos 2\theta\right) d\theta$$

$$\frac{S}{4} = \frac{a}{\sqrt{8}}\left[[2]\left[\tfrac{\pi}{2}-0\right] + [0-0]\right]$$

$$\boxed{S = \sqrt{2}\,\pi a}$$

ii) $r = a(1+\cos\theta)$

- Symmetry → $\theta \to -\theta$, no change

$\therefore$ symmetric about $\theta = 0°$

- Passes through origin →

$r = 0$

$1+\cos\theta = 0$

$\cos\theta = -1$

$\boxed{\theta = \pi}$ → tangent at origin.

- Asymptotes

$\frac{1}{r} = 0$

$a(1+\cos\theta) \to \infty$

Not possible, Hence asymptotes exist.

- Direction of tangent

$\frac{dr}{d\theta} = a(-\sin\theta)$

$\therefore \tan\phi = \frac{a(1+\cos\theta)}{a(-\sin\theta)}$.

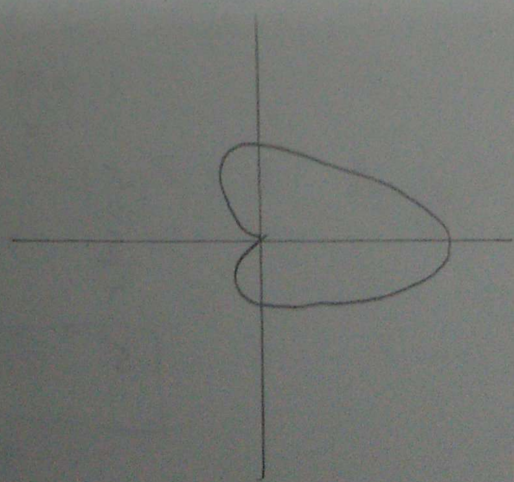
$\tan\phi = \frac{(1+\cos\theta)}{(-\sin\theta)}$

$\tan\phi \to \infty$

$\sin\theta = 0$

$\boxed{\theta = 0°}$ $\quad r = 2a$

$(2a, 0)$ → perpendicular



$$S = \int_{\alpha}^{\beta} \left(\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\right) d\theta$$

$$\frac{dr}{d\theta} = -a\sin\theta \qquad r^2 = a^2(1+\cos\theta)^2$$

$$\therefore \left(\frac{dr}{d\theta}\right)^2 + r^2 = a^2\sin^2\theta + a^2 + a^2\cos^2\theta + 2a^2\cos\theta$$

$$= 2a^2 + 2a^2\cos\theta$$

$$= 2a^2(1+\cos\theta)$$

$$= 4a^2\cos^2\theta/2 .$$

$$S = 2\int_0^{\pi} 2a\cos\theta/2 \; d\theta$$

$$S = \left[2 \times \frac{2a(\sin\theta/2)}{1/2}\right]_0^{\pi}$$

$$= 8a\sin\pi/2$$

$$\boxed{S = 8a}$$

iii) $x = a\cos^3 t \quad , \quad y = a\sin^3 t$

$$\frac{x}{a} = \cos^3 t \quad , \quad \frac{y}{a} = \sin^3 t .$$

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$$

$$x^{2/3} + y^{2/3} = a^{2/3} .$$

- Symmetric about x, y axis.
- Does not pass through origin.
- No // asymptotes to x or y axis.
- $-a \leq x \leq a$ because $\left(\frac{x}{a}\right)^{2/3} \leq 1$ & $\left(\frac{y}{a}\right)^{2/3} \leq 1$

  $-a \leq y \leq a$.
- $$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

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$\dfrac{dy}{dx} = 0$ $\qquad\qquad$ $y = 0$

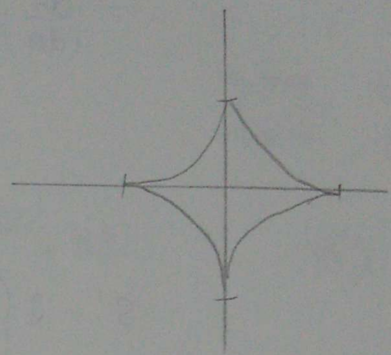
$x^{2/3} = a^{2/3}$

$\boxed{x = \pm a}$

$\dfrac{dy}{dx} \to \infty$ $\qquad\qquad$ $x = 0$

$y^{2/3} = a^{2/3}$

$y = \pm a$



π

$\pi/2$

In parametric form

$$S = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$S = 4\int_{\pi/2}^{0} \sqrt{[3a\cos^2 t(-\sin t)]^2 + [3a\sin^2 t \cos t]^2}\, dt$$

$$s = \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 4\int_{\pi/2}^{0} 3a\cos t \sin t \sqrt{\sin^2 t + \cos^2 t}\, dt$$

$x = 0,\ t = \pi/2$

$y = a,\ t = 0$

$$= 4 \times 3a \int_{\pi/2}^{0} \frac{\sin 2t}{2}\, dt$$

$$= 6a\left[\frac{-\cos 2t}{2}\right]_{\pi/2}^{0}$$

$$= 6a\left[\frac{-1-1}{2}\right]$$

$$s = |-6a| = \boxed{6a}$$

Q7 → Done at last

Q8 Show that surface area of the solid generated by the revolution of the tractrix

$$x = a\cos t + (a/2)\log\tan^2(t/2)$$

$y = a\sin t$, about x-axis is $4\pi a^2$.

Ans

Req surface Area is

$$A = 2\int_{\cancel{0}}^{\infty} 2\pi y \cdot d\cancel{x}s.$$

$$= 2\int_{\pi/2}^{\pi} 2\pi\, y \cdot \frac{ds}{dt} \cdot dt$$

$x \to \infty$, $t \to \pi$

$x \to \infty$, $t \to \pi/2$

$$A = 2\int_{\pi/2}^{\pi} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt \quad -(1)$$

$$x = a\cos t + \frac{a}{2}\log(\tan^2 t/2)$$

$$\frac{dx}{dt} = -a\sin t + \frac{a}{2}\frac{1}{\tan^2 t/2} \cdot 2\tan t/2 \cdot \sec^2 t/2 \cdot \frac{1}{2}$$

$$= -a\sin t + \frac{a}{2\sin t/2 \cos t/2}$$

$$= -a\sin t + \frac{a}{\sin t}.$$

$$y = a\sin t$$

$$\frac{dy}{dt} = a\cos t.$$

$$\therefore \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} = \sqrt{a^2\cos^2 t + a^2\sin t + \frac{a^2}{\sin^2 t} - 2a^2}$$

$$= a\left[\sqrt{\operatorname{cosec}^2 t - 1}\right.$$

$$= a\cot t.$$

$$\therefore A = 2\int_{\pi/2}^{\pi} 2\pi y \cdot a\,\sin t \cdot a\cot t \cdot dt$$

$$= 2\times 2\pi a^2 \int_{\pi/2}^{\pi} \cos t\, dt.$$

$$= 4\pi a^2 \left[\sin t\right]_{\pi/2}^{\pi}$$

$$S = 4\pi a^2[-1] \Rightarrow \boxed{|S| = 4\pi a^2}$$

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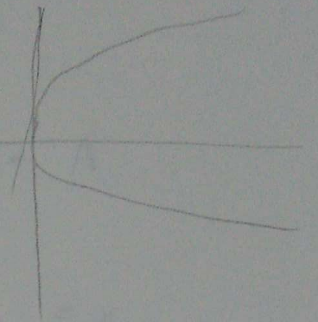
Q9 Find the volume of the reel shaped solid formed by the revolution about y-axis, of the part of parabola $y^2 = 4ax$ cut of by latus rectum.

∴ Reqd Volume = V $\qquad y^2 = 4ax$



$$V = \int_{-2a}^{2a} \pi (x^2)\, dy$$

$$= \int_{-2a}^{2a} \pi \left(\frac{y^2}{4a}\right)^2 . dy$$

$$= \frac{\pi}{16a^2} \int_{-2a}^{2a} y^4\, dy$$

$$= \frac{\pi}{16a^2} \cdot \left[\frac{y^5}{5}\right]_{-2a}^{2a}$$

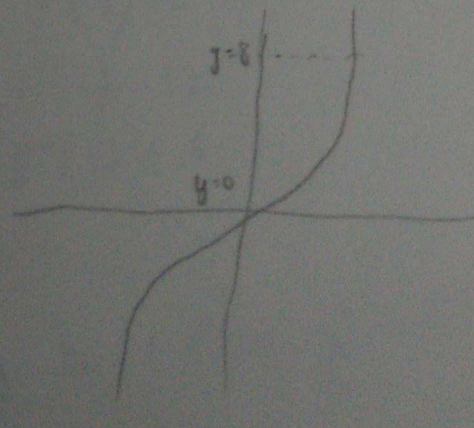
$$= \frac{\pi}{16a^2} \left[\frac{32a^5}{5} + \frac{32a^5}{5}\right]$$

$$= \frac{4a^3\pi}{5} \text{ cubic units.}$$

Q10 A basin formed by the revolution of the curve $x^3 = 64y$ $(y>0)$, about the axis of y. If the depth of the basin is 8 cm, how many cu. cm of water will it hold?

Volume of water hold

= volume of solid formed by revolution of curve about y-axis.



$$V = \int_0^8 \pi x^2\, dy$$

$\frac{h^3}{6}$

$$\therefore f = x^2 + \underline{2x_1^2 - xx_2}$$

$$= \int_0^8 \pi (64y)^{2/3} . dy$$

$$= \pi . 16 \int_0^8 y^{2/3} . dy$$

$$= 16\pi \left[\frac{y^{5/3}}{5/3} \right]_0^8$$

$$= \frac{16\pi}{5} \times 3 \times 2^5$$

$$= \frac{32 \times 16 \times 3}{5} \pi \ cm^3$$

$$= \frac{1536}{5} \pi \ cm^3 .$$

Basin can hold $\frac{1536\pi}{5}$ cubic centimeters of water, if depth of basin is 8 cm.

Qn Apply Taylor's series expansion to calculate the value of $f(2.001)$ if $f(x) = x^3 - 2x + 5$.

By Taylor's series expansion

$$f(x+h) = f(x) + f'(x) . \frac{h^1}{1!} + f''(x) . \frac{h^2}{2!} \ldots\ldots$$

here $x = 2$, $h = 10^{-3}$

$f'(x) = 3x^2 - 2$ $\qquad$ $f'(2) = 10$

$f''(x) = 6x$ $\qquad$ $f''(2) = 12$

$f'''(x) = 6$ $\qquad$ $f'''(2) = 6$.

$f''''(x) = 0$, $f'''''(x) = 0 \ldots$

$$f(2 + 10^{-3}) = f(2) + f'(2) . \underline{h} + f''(2) . \frac{h^2}{2} + f'''(2) . \frac{h^3}{6} + 0 .$$

$f(2) = 9$.

$\therefore \quad = 9 + 10^{-2} + 6\times10^{-6} + 10^{-9}.$

$f(2.001) \simeq 9.010006001$

$\simeq 9.01$

$\therefore \quad f(2.01) \simeq 9.01.$

Q12 Calculate the approximate value of $\sqrt{17}$ its four places of decimals using Taylor's expansion

$$f(x) = x^{1/2}.$$

$$x = 16$$

$$h = 1$$

$$\therefore f(17) = f(16) + \frac{(1)}{1!}\cdot f'(16) + \frac{1^2}{2!}f''(16) + \frac{1^3}{3!}f'''(16)\dots$$

$$f'(x) = \frac{1}{2}x^{-1/2} \qquad f'(16) = \frac{1}{2}\times\frac{1}{4} = \frac{1}{8}$$

$$f''(x) = -\frac{1}{4}x^{-3/2} \qquad f''(16) = -\frac{1}{4}\cdot\frac{1}{64} = -\frac{1}{256}.$$

$$f'''(x) = +\frac{3}{8}x^{-5/2} \qquad f'''(16) = \frac{3}{8}\times\frac{1}{1024} = \frac{3}{8192}.$$

$$\therefore f(17) = \sqrt{17} = 4 + \frac{1}{8} - \frac{1}{512} + \frac{1}{16384}\dots$$

$$= 4.123108$$

$$\boxed{\sqrt{17} = 4.1231}$$

Q13 Prove that the curvature of a circle is constant.

Eqn of circle in polar form

$$\boxed{r = a}$$

Differentiating wrt to θ

$$r_1 = 0$$

$$r_2 = 0$$

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$$\therefore \rho = \frac{r^2 + 2r_1^2 - r r_2}{(r_1^2 + r^2)^{3/2}}$$

Put value $r = a$

$r_1 = 0$, $r_2 = 0$

$$\rho = \frac{a^2 + 0 - 0}{(0 + a^2)^{3/2}}$$

$$= \frac{1}{a}$$, which is constant, since a is constant

Hence, proved

Q14 If ρ_1 and ρ_2 are the radii of curvature at the extremities of a focal chord of a parabola, whose semi latus rectum is l, prove that,

$$(\rho_1)^{-2/3} + (\rho_2)^{-2/3} = l^{-2/3}$$

Ans 14 → Same as Q5 in the assignment

$l = 2a$ [Semi latus rectum]

∴ To prove becomes same as Q5

Q15 Find the radius of curvature at the origin for the curve $y^2(a^2 - x^2) = a^3 x$.

Curves passes through origin with tangent

$$\boxed{x = 0}$$

By Newtonian method:

$$\rho(0,0) = \lim_{y \to 0} \left(\frac{y^2}{2x}\right)$$

$$\frac{y^2}{2x} = \frac{a^3 (\cancel{x})}{a^2 - x^2}\left(\frac{1}{2\cancel{x}}\right)$$

$$\frac{y^2}{2x} = \left(\frac{a^3}{a^2 - x^2}\right)\frac{1}{2}$$

Taking limit $y \to 0$, [$y \to 0$, then $x \to 0$.

$$\rho = \frac{a^3}{2a^2}$$

$$\boxed{\rho = \frac{a}{2}}$$

The radius of curvature at $(0,0)$ is $a/2$.

Q16 If ρ_1, ρ_2 be the radii of curvature at the extremeties of any chord of the cardioid $r = a(1+\cos\theta)$ which passes through the pole, show that $\rho_1^2 + \rho_2^2 = 16a^2/9$.

Ans → Pt at extremities of focal chord.

(r_1, θ_1) , (r_2, θ_2).

Calculating ρ

$$r = a(1+\cos\theta)$$

$$\frac{dr}{d\theta} = a(-\sin\theta)$$

$$\frac{d^2r}{d\theta^2} = a(-\cos\theta)$$

$$\therefore \rho = \frac{ds}{d\psi} = \frac{\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right)^{3/2}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 - r\left(\frac{d^2r}{d\theta^2}\right)}$$

$$= \frac{\left[a^2(1+\cos^2\theta + 2\cos\theta) + a^2\sin^2\theta\right]^{3/2}}{a^2(1+\cos^2\theta + 2\cos\theta) + 2a^2\sin^2\theta + a^2[\cos\theta + \cos^2\theta]}$$

$$= \frac{[2a^2 + 2a^2\cos\theta]^{3/2}}{[3a^2 + 3a^2\cos\theta]}$$

$$= \frac{2^{3/2} \cdot a^3}{3a^2} \frac{[1+\cos\theta]^{3/2}}{[1+\cos\theta]}$$

$$\rho = \frac{\sqrt{8}a}{3} \cdot [1+\cos\theta]^{1/2}$$

$$\therefore \rho_1^2 = \left[\frac{\sqrt{8}a}{3} \cdot [1+\cos\theta_1]^{1/2}\right]^2 \qquad \rho_2^2 = \left(\frac{\sqrt{8}a}{3}[1+\cos\theta_2]^{1/2}\right)^2$$

$$\rho_1^2 = \frac{8}{9}a^2[1+\cos\theta_1] \qquad \rho_2^2 = \frac{8}{9}a^2[1+\cos\theta_2]$$

We know that $\rightarrow$ $\boxed{\theta_2 = \pi + \theta_1}$ $\rightarrow$ [From figure.

LHS

$$\therefore = \rho_1^2 + \rho_2^2$$

$$= \frac{8a^2}{9}\left[2 + \cos\theta_1 + \cos(\pi+\theta_1)\right]$$

$$= \frac{8a^2}{9}\left[2 + \cancel{\cos\theta_1} - \cancel{\cos\theta_1}\right]$$

$$= \frac{16a^2}{9} = RHS$$