

Comment If the temperature remains less than $T = 550$ K, then the intrinsic carrier concentration will contribute less than 5 percent of the total electron concentration for this donor impurity concentration.

Equations (3.65) and (3.67) are used to calculate the majority carrier electron concentration in an n -type semiconductor and majority carrier hole concentration in a p -type semiconductor, respectively. The minority carrier hole concentration in an n -type semiconductor could, theoretically, be calculated from Eq. (3.67). However, we would be subtracting two numbers on the order of 10^{16} cm^{-3} , for example, to obtain a number on the order of 10^4 cm^{-3} , which from a practical point of view is not possible. The minority carrier concentrations are calculated from $n_0 p_0 = n_i^2$ once the majority carrier concentration has been determined.

POSITION OF FERMI ENERGY LEVEL

3.7

We discussed qualitatively in Section 3.4.1 how the electron and hole concentrations change as the Fermi energy level moves through the bandgap energy. Then, in Section 3.6, we calculated the electron and hole concentrations as a function of donor and acceptor impurity concentrations. We can now determine the position of the Fermi energy level as a function of the doping concentrations and as a function of temperature. The relevance of the Fermi energy level will be further discussed after the mathematical derivations.

3.7.1 Mathematical Derivation

The position of the Fermi energy level within the bandgap can be determined by using the equations already developed for the thermal-equilibrium electron and hole concentrations. If we assume the Boltzmann approximation to be valid, then from Eq. (3.16) we have $n_0 = N_c \exp [-(E_c - E_F)/kT]$. We can solve for $E_c - E_F$ from this equation and obtain

$$E_c - E_F = kT \ln \left(\frac{N_c}{n_0} \right) \quad (3.68)$$

where n_0 is given by Eq. (3.65). If we consider an n -type semiconductor in which $N_d \gg n_i$, then $n_0 \approx N_d$, so that

$$E_c - E_F = kT \ln \left(\frac{N_c}{N_d} \right) \quad (3.69)$$

The distance between the bottom of the conduction band and the Fermi energy is a logarithmic function of the donor concentration. As the donor concentration increases, the Fermi level moves closer to the conduction band. Conversely, if the Fermi level moves closer to the conduction band, then the electron concentration in the conduction band is increasing. We may note that if we have a compensated semiconductor, then the N_d term in Eq. (3.69) is simply replaced by $N_d - N_a$, or the net effective donor concentration.

Example 3.18

Objective To determine the required donor impurity concentration to obtain a specified Fermi energy.

Silicon at $T = 300$ K contains an acceptor impurity concentration of $N_a = 10^{16} \text{ cm}^{-3}$. Determine the concentration of donor impurity atoms that must be added so that the silicon is n type and the Fermi energy is 0.20 eV below the conduction band edge.

From Eq. (3.69), we have

$$E_c - E_F = kT \ln \left(\frac{N_c}{N_d - N_a} \right)$$

which can be rewritten as

$$N_d - N_a = N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right]$$

Then

$$N_d - N_a = 2.8 \times 10^{19} \exp \left[\frac{-0.20}{0.0259} \right] = 1.24 \times 10^{16} \text{ cm}^{-3}$$

$$N_d = 1.24 \times 10^{16} + N_a = 2.24 \times 10^{16} \text{ cm}^{-3}$$

Comment A compensated semiconductor can be fabricated to provide a specific Fermi energy level.

We may develop a slightly different expression for the position of the Fermi level. We had from Eq. (3.44) that $n_0 = n_i \exp [(E_F - E_{Fi})/kT]$. We can solve for $E_F - E_{Fi}$ as

$$E_F - E_{Fi} = kT \ln \left(\frac{n_0}{n_i} \right) \quad (3.70)$$

Equation (3.70) can be used specifically for an *n*-type semiconductor, where n_0 is given by Eq. (3.65), to find the difference between the Fermi level and the intrinsic Fermi level as a function of the donor concentration. We may note that, if the net effective donor concentration is zero, that is, $N_d - N_a = 0$, then $n_0 = n_i$ and $E_F = E_{Fi}$. A completely compensated semiconductor has the characteristics of an intrinsic material in terms of carrier concentration and Fermi level position.

We can derive the same types of equations for a *p*-type semiconductor. From Eq. (3.19), we have $p_0 = N_v \exp [-(E_F - E_v)/kT]$, so that

$$E_F - E_v = kT \ln \left(\frac{N_v}{p_0} \right) \quad (3.71)$$

If we assume that $N_a \gg n_i$, then Eq. (3.71) can be written as

$$E_F - E_v = kT \ln \left(\frac{N_v}{N_a} \right) \quad (3.72)$$

The distance between the Fermi level and the top of the valence-band energy for a *p*-type semiconductor is a logarithmic function of the acceptor concentration: as the acceptor concentration increases, the Fermi level moves closer to the valence band. Equation (3.72) still assumes that the Boltzmann approximation is valid. Again, if we have a compensated *p*-type semiconductor, then the N_a term in Eq. (3.72) is replaced by $N_a - N_d$, or the net effective acceptor concentration.

We can also derive an expression for the relationship between the Fermi level and the intrinsic Fermi level in terms of the hole concentration. We have from Eq. (3.45) that $p_0 = n_i \exp [-(E_F - E_{Fi})/kT]$, which yields

$$E_{Fi} - E_F = kT \ln \left(\frac{p_0}{n_i} \right) \quad (3.73)$$

Equation (3.73) can be used to find the difference between the intrinsic Fermi level and the Fermi energy in terms of the acceptor concentration. The hole concentration p_0 in Eq. (3.73) is given by Eq. (3.67).

We may again note from Eq. (3.70) that, for an *n*-type semiconductor, $n_0 > n_i$ and $E_F > E_{Fi}$. The Fermi level for an *n*-type semiconductor is above E_{Fi} . For a *p*-type semiconductor, $p_0 > n_i$, and from Eq. (3.73) we see that $E_{Fi} > E_F$. The Fermi level for a *p*-type semiconductor is below E_{Fi} . These results are shown in Fig. 3.25.

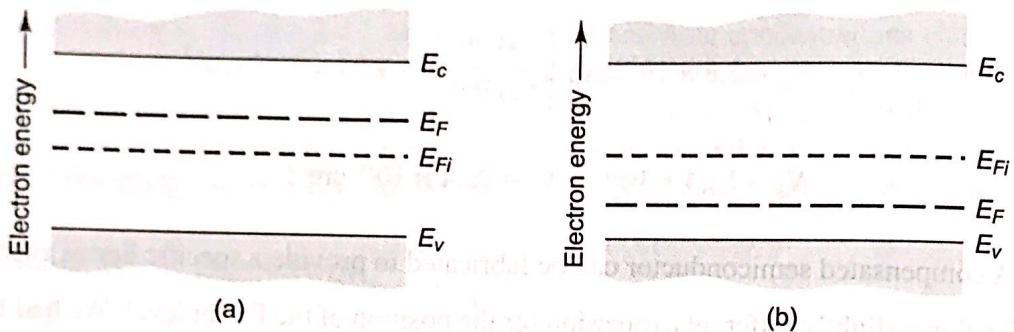


Fig. 3.25 Position of Fermi level for an (a) *n*-type ($N_d > N_a$), and (b) *p*-type ($N_d > N_a$) semiconductor

3.7.2 Variation of E_F with Doping Concentration and Temperature

We may plot the position of the Fermi energy level as a function of the doping concentration. Figure 3.26 shows the Fermi energy level as a function of donor concentration (*n* type) and as a function of acceptor concentration (*p* type) for silicon at $T = 300$ K. As the doping levels increase, the Fermi energy level moves closer to the conduction band for the *n*-type material and closer to the valence band for the *p*-type material. Keep in mind that the equations for the Fermi energy level that we have derived assume that the Boltzmann approximation is valid.

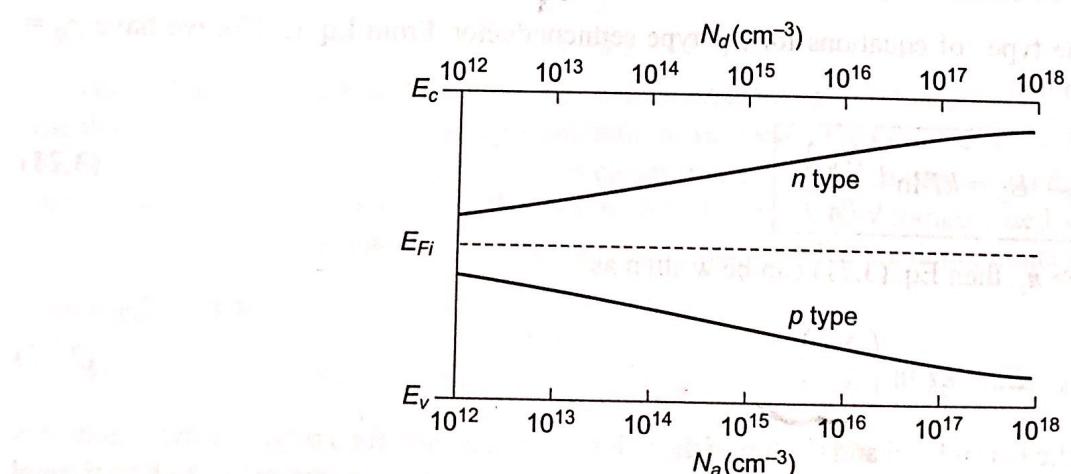


Fig. 3.26 Position of Fermi level as a function of donor concentration (*n* type) and acceptor concentration (*p* type)

Example 3.19

Objective To determine the Fermi-level position and the maximum doping at which the Boltzmann approximation is still valid.

Consider *p*-type silicon, at $T = 300$ K, doped with boron. We may assume that the limit of the Boltzmann approximation occurs when $E_F - E_a = 3kT$. (See Section 3.2.2.) From Table 3.3, we find the ionization energy is $E_a - E_v = 0.045$ eV for boron in silicon. If we assume that $E_{Fi} \approx E_{\text{midgap}}$, then from Eq. (3.73), the position of the Fermi level at the maximum doping is given by

or

$$E_{Fi} - E_F = \frac{E_g}{2} - (E_a - E_v) - (E_F - E_a) = kT \ln \left(\frac{N_a}{n_i} \right)$$

$$0.56 - 0.045 - 3(0.0259) = 0.437 = (0.0259) \ln \left(\frac{N_a}{n_i} \right)$$

$$N_a = n_i \exp \left(\frac{0.437}{0.0259} \right) = 3.2 \times 10^{17} \text{ cm}^{-3}$$

Comment If the acceptor (or donor) concentration in silicon is greater than approximately $3 \times 10^{17} \text{ cm}^{-3}$, then the Boltzmann approximation of the distribution function becomes less valid and the equations for the Fermi-level position are no longer quite as accurate.

The intrinsic carrier concentration n_i , in Eqs (3.70) and (3.73), is a strong function of temperature, so that E_F is a function of temperature also. Figure 3.27 shows the variation of the Fermi energy level in silicon with temperature for several donor and acceptor concentrations. As the temperature increases, n_i increases, and E_F moves closer to the intrinsic Fermi level. At high temperature, the semiconductor material begins to lose its extrinsic characteristics and begins to behave more like an intrinsic semiconductor. At the very low temperature, freeze-out occurs; the Boltzmann approximation is no longer valid and the equations we derived for the Fermi-level position no longer apply. At the low temperature where freeze-out occurs, the Fermi level goes above E_d for the *n*-type material and below E_a for the *p*-type material. At absolute zero degrees, all energy states below E_F are full and all energy states above E_F are empty.

3.7.3 Relevance of the Fermi Energy

We have been calculating the position of the Fermi energy level as a function of doping concentrations and temperature. This analysis may seem somewhat arbitrary and fictitious. However, these relations do become

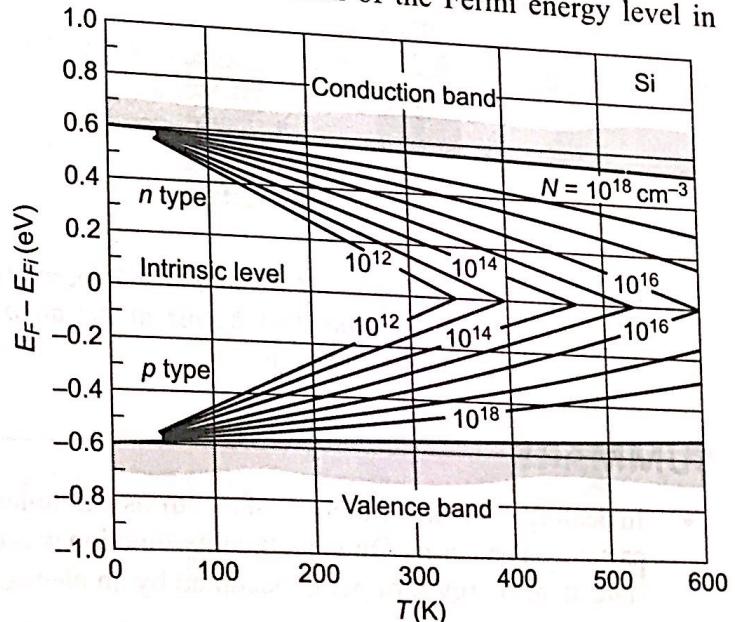


Fig. 3.27 Position of Fermi level as a function of temperature for various doping concentrations (From Sze [13].)

CARRIER DRIFT

An electric field applied to a semiconductor will produce a force on electrons and holes so that they experience a net acceleration and net movement, provided there are available energy states in the conduction and valence bands. This net movement of charge due to an electric field is called *drift*. The net drift of charge gives rise to a *drift current*.

4.1.1 Drift Current Density

If we have a positive volume charge density ρ , moving at an average drift velocity v_d , the drift current density is given by

$$J_{drf} = \rho v_d$$

where J is in units of $\text{C}/\text{cm}^2\text{-s}$ or amp/cm^2 . If the volume charge density is due to positively charged holes, then

$$J_{p,drf} = (ep)v_{dp}$$

where $J_{p,drf}$ is the drift current density due to holes and v_{dp} is the average drift velocity of the holes.

The equation of motion of a positively charged hole in the presence of an electric field is

$$F = m_p^* a = eE$$

where e is the magnitude of the electronic charge, a is the acceleration, E is the electric field, and m_p^* is the effective mass of the hole. If the electric field is constant, then we expect the velocity to increase linearly with time. However, charged particles in a semiconductor are involved in collisions with ionized impurity atoms and with thermally vibrating lattice atoms. These collisions, or scattering events, alter the velocity characteristics of the particle.

As the hole accelerates in a crystal due to the electric field, the velocity increases. When the charged particle collides with an atom in the crystal, for example, the particle loses most, or all, of its energy. The particle will again begin to accelerate and gain energy until it is again involved in a scattering process. This continues over and over again. Throughout this process, the particle will gain an average drift velocity which, for low electric fields, is directly proportional to the electric field. We may then write

$$v_{dp} = \mu_p E$$

where μ_p is the proportionality factor and is called the *hole mobility*. The mobility is an important parameter of the semiconductor since it describes how well a particle will move due to an electric field. The unit of mobility is usually expressed in terms of $\text{cm}^2/\text{V}\cdot\text{s}$.

By combining Eqs (4.2) and (4.4), we may write the drift current density due to holes as

$$J_{p,drf} = (ep)v_{dp} = e\mu_p p E$$

The drift current due to holes is in the same direction as the applied electric field.

The same discussion of drift applies to electrons. We may write

$$J_{n,drf} = \rho v_{dn} = (-en)v_{dn}$$

where $J_{n,drf}$ is the drift current density due to electrons and v_{dn} is the average drift velocity of electrons. The net charge density of electrons is negative. Values of mobilities of Si, Ge, and GaAs are given in Table 4.1.