Also, $V_{aa'} = -V_{a'a}$. The double-script notation is very useful in writing KVL equations without reference to the circuit diagram.

16.2 POLYPHASE SYSTEM

Systems with more than one phase are called *polyphase systems*: A polyphase system contains two or more a.c. voltage sources of the same frequency. These source voltages have a fixed phase angle difference between them. The most widely used polyphase system is the three-phase system. Three-phase systems are in common use for generation, transmission, distribution and utilisation of electric energy. For certain special applications, greater number of phases are also used. For example, systems of six or twelve phases are used in polyphase rectifiers.

A system is said to be *symmetrical* when the various voltages are equal in magnitude and are displaced from one another by equal angles. The system is *balanced* when the various voltages are equal in magnitude, the various currents are equal in magnitude and the phase angles are the same for each phase.

16.3 TWO-PHASE SYSTEM

When two identical coils are placed with their axes at right angles to one another and rotated in a uniform magnetic field, a sinusoidal voltage is generated across each coil. This is the basis of a simple two-phase generator (alternator). Fig. 16.2 shows a simple two-phase, two-pole alternator.

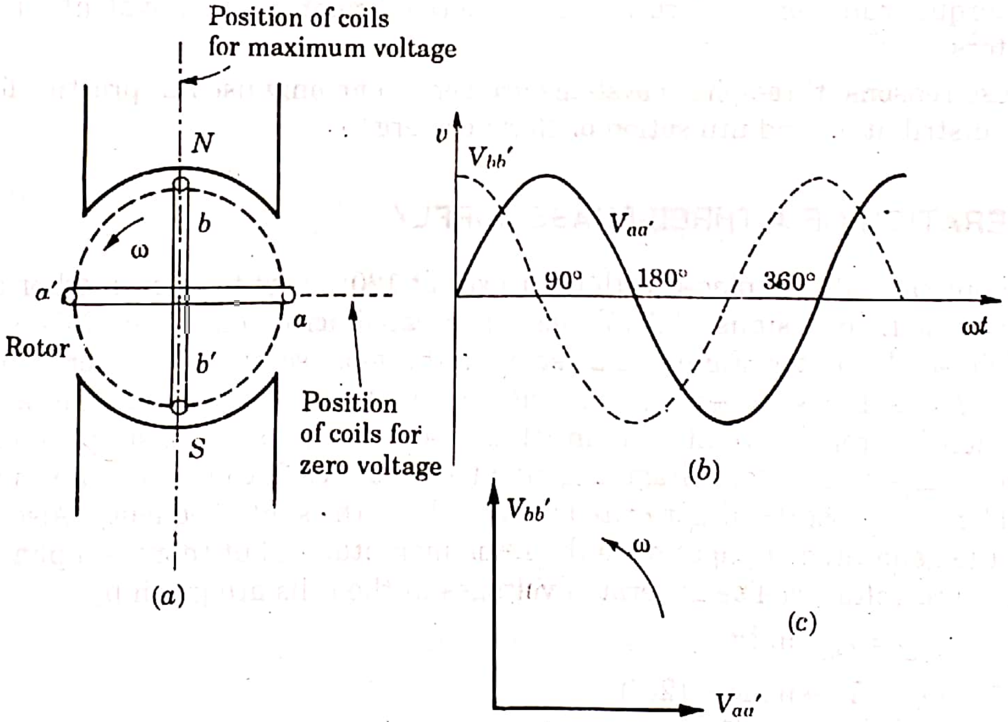

Fig. 16.2. Two-phase voltage generation : (*a*) two-phase, 2-pole alternator; (*b*) waveform of voltages; (*c*) phasor diagram.

It has two sets of coils aa' and bb' mounted on a rotor such that their axes are perpendicular to each other. When the rotor is rotated at a constant angular velocity ω radians per second, a sinusoidal voltage is generated across each coil. Since the two coils rotate at the same velocity ($\omega = 2\pi f$), the generated voltages have the same frequency. Also, since the coils are identical, the generated voltages have the same magnitudes, but there is a phase difference of 90° between these voltages. The generated voltages in the coils are given by

$$v_{aa'} = V_m \sin \omega t$$

$$v_{bb'} = V_m \sin(\omega t + 90°)$$

where V_m is the maximum voltage.

In polar form

$$\mathbf{V}_{aa'} = V \angle 0°$$

$$\mathbf{V}_{bb'} = V \angle 90°$$

where $\mathbf{V}$ is the r.m.s. value of the voltage and $\mathbf{V} = \mathbf{V}_m/\sqrt{2}$. The voltage waveforms are shown in Fig. 16.2*b* and the phasor diagram is shown in Fig. 16.2*c*. Two-phase system has limited applications in control and instrumentation.

16.4 ADVANTAGES OF THREE-PHASE SYSTEMS

The main advantages of three-phase system over single-phase system are

(1) A three-phase machine has a smaller size than a single-phase machine of the same power output.

(2) The conductor material required to transmit a given power at a given voltage over a given distance by a three-phase system is less than that by an equivalent single-phase system.

(3) Since three-phase supply produces a rotating magnetic field, three-phase motors are simpler in construction, smaller in size, start more conveniently, have uniform torque, run more smoothly and are more efficient than equivalent single-phase motors.

For these reasons, three-phase systems are very commonly used in practice for generation, transmission, distribution and utilisation of electric energy.

16.5 GENERATION OF A THREE-PHASE SUPPLY

When three identical coils are placed with their axes at 120° apart from each other and rotated in a uniform magnetic field, a sinusoidal voltage is generated across each coil. This is the basis of a simple three-phase alternator. Figure 16.3*a* shows a three-phase, two-pole alternator. It has three sets of coils *aa′*, *bb′* and *cc′* symmetrically mounted on a rotor such that their axes are at 120° from each other. When the rotor is rotated in anticlockwise direction at a constant angular velocity ω radians per second, a sinusoidal voltage is generated across each coil. Since the three coils rotate at the same velocity ($\omega = 2\pi f$), the generated voltages have the same frequency. Also, since the coils are identical, the generated voltages have the same magnitudes, but there is a phase difference of 120° between these voltages. The generated voltages in the coils are given by

$$v_{aa'} = V_m \sin \omega t$$

$$v_{bb'} = V_m \sin(\omega t - 120°)$$

$$v_{cc'} = V_m \sin(\omega t - 240°)$$

or

$$v_{cc}{}' = V_m \sin(\omega t + 120°)$$

It is to be noted that a phase angle of – 240° is the same as + 120°. In polar form,

$$\mathbf{V}_{aa'} = V \angle 0°$$

$$\mathbf{V}_{bb'} = V \angle -120°$$

$$\mathbf{V}_{cc'} = V \angle -240° = V \angle +120°$$

The voltage waveforms are shown in Fig. 16.3*b* and the phasor diagram is shown in Fig. 16.3*c*.

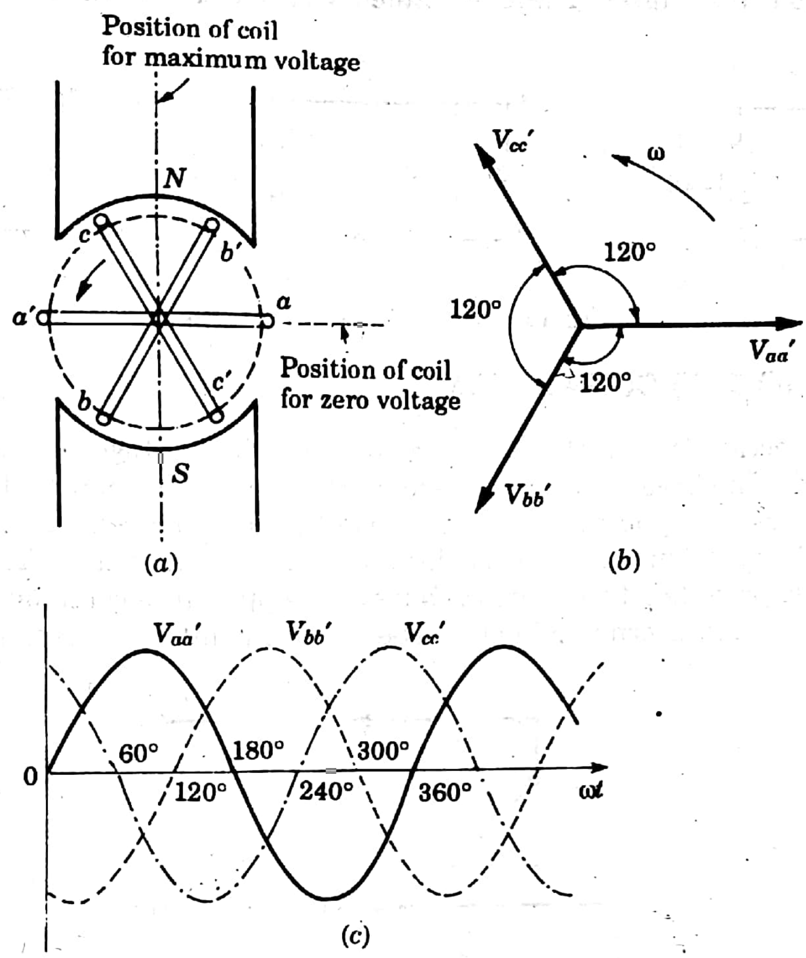

Fig. 16.3. Generation of three-phase supply : (*a*) three-phase two-pole alternator; (*b*) waveforms of voltages; (*c*) phasor diagram.

It is seen that $\mathbf{V}_{aa'}$ leads $\mathbf{V}_{bb'}$ by 120°, and $\mathbf{V}_{bb'}$ leads $\mathbf{V}_{cc'}$ by 120°. Also, the three voltages reach their positive maximum values in the order $\mathbf{V}_{aa'}$, $\mathbf{V}_{bb'}$, $\mathbf{V}_{cc'}$. The order in which the phase voltages reach their maximum values is called the *phase sequence.* For the arrangement shown in Fig. 16.3*a* in which the coils are rotating in anticlockwise direction, the phase sequence is *abc*. If the rotor is rotated in clockwise direction the voltages reach their positive maximum values in the order $\mathbf{V}_{cc'}$, $\mathbf{V}_{bb'}$, $\mathbf{V}_{aa'}$. In this case the phase sequence is *cba* or *acb*. Thus, the phase sequence determines the direction of rotation.

16.6 METHODS OF CONNECTION OF THREE-PHASE SUPPLIES

Since a voltage is generated in each coil, it may be considered as a source of voltage. The three coils together constitute a three-phase system and each coil is a phase. Let a load be connected across each phase. The arrangement given in Fig. 16.4 shows three loads supplied separately from the three phases. The end of a coil where the current leaves may be called the *starting end* or simply the *start*. The other end where the current enters the coil is called the *finishing end* or simply the *finish*. In Fig. 16.4, the ends *a*, *b*, *c* are the *starting ends* while the ends *a′*, *b′*, *c′* are the finishing ends. The arrangement shown in this figure requires six wires to carry energy from the coils (sources) to the loads. This is equivalent to three separate single-phase systems. Such a system is called a *three-phase, six-wire system*. The number of connecting wires may be reduced by the interconnection of the phases to form a single three-phase a.c. source. There are two methods of interconnecting the three phases. These are called star and delta connections.

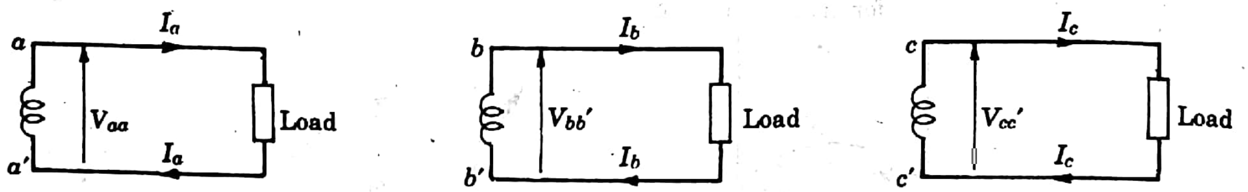

Fig. 16.4. Three-phase six-wire system.

16.7 STAR OF WYE (Y) CONNECTION

In this type of connection the finishing ends a', b', c' are joined together at a common point n as shown in Fig. 16.5. The free ends a, b, c are connected to the external circuit through three conductors called *lines*. The point n is called the *neutral point or neutral*. It is also called the *star point*. A wire brought out from the neutral point is called the *neutral wire*. The three line conductors and the neutral wire provide a three-phase, four-wire supply. It is common practice to connect the windings of a three-phase alternator in star. The neutral point is connected to ground.

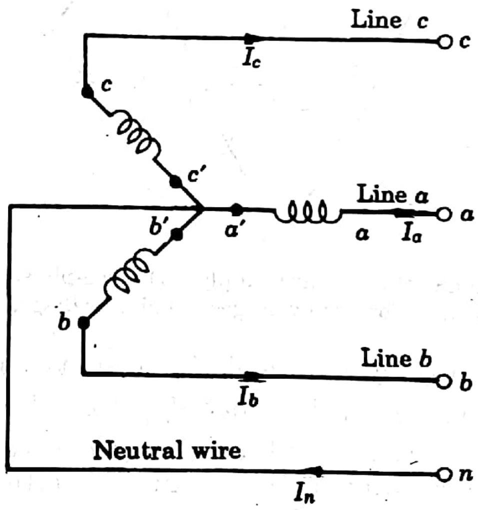

Fig. 16.5. Three-phase star connection.

Definition of a 'phase' : A phase is one of the three branches making up a $3-\phi$ circuit. In a Y connection, a phase consists of these circuit elements connected between one line and neutral.

In a Δ circuit, a phase consists of those circuit elements connected between two lines.

16.8 PHASE AND LINE VOLTAGES IN STAR CONNECTION

The voltage across each coil is called the *phase voltage*. This is also the voltage between each line and neutral. Therefore the phase voltage in star connection is also called the *line-to-neutral voltage*. The three phase voltages are $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$ and $\mathbf{V}_{cn}$. In a symmetrical system these voltages are equal in magnitude. The phase voltage is designated as V_p.

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| = V_p$$

If $\mathbf{V}_{an}$ is taken as reference

$$\mathbf{V}_{an} = V_p \angle 0° = V_p\,(1 + j0)$$

$$\mathbf{V}_{bn} = V_p \angle -120° = V_p\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$\mathbf{V}_{cn} = V_p \angle -240° = V_p \angle 120° = V_P\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

The voltage between two lines is called the *line-to-line voltage* or simply the *line voltage*. The line voltages are $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$ between the pairs of lines *a-b*, *b-c* and *c-a* respectively. In a symmetrical (balanced) system these voltages are equal in magnitude and each is designated as V_L.

$$|\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}| = V_L$$

The phase and line voltages in star connection are shown in Fig. 16.6. The relationship between the phase and line voltages in a balanced three-phase star connection can be found from the phasor diagram shown in Fig. 16.7.

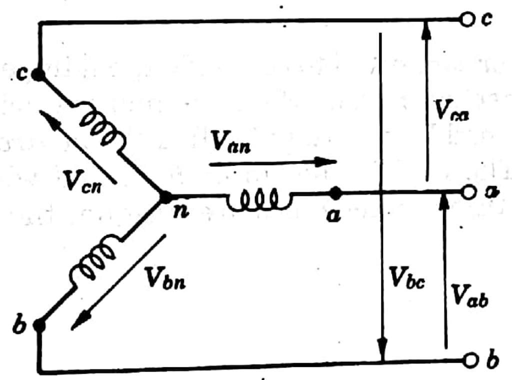

Fig. 16.6. Phase and line voltages in star connection.

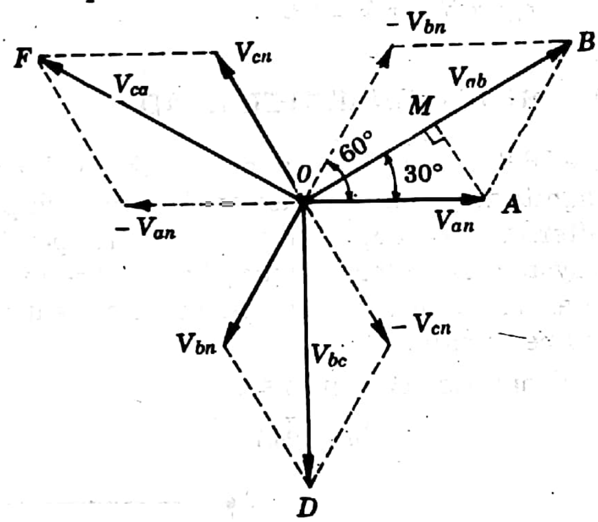

Fig. 16.7. Phasor diagram.

By KVL,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} \qquad (\because\ \bar{V}_{bn} = -\bar{V}_{nb})$$

$$= \mathbf{V}_{an} + (-\mathbf{V}_{bn}) = \text{phasor sum of } \mathbf{V}_{an} \text{ and } \mathbf{V}_{bn} \text{ reversed}$$

$$= \text{phasor } \mathbf{OB} = \text{OB} \angle 30°$$

Draw a perpendicular AM on OB. In triangle OAB,

$$\text{OA} = \text{AB} = V_p\ ;\ \text{OB} = \mathbf{V}_{ab} = V_L\ ;\ \angle\text{AOB} = 30°$$

Now $\text{OB} = 2\text{OM} = 2\text{OA} \cos 30° = 2V_p \dfrac{\sqrt{3}}{2}$

or $$V_L = \sqrt{3}\, V_P$$

$\therefore$ $$\mathbf{V}_{ab} = \sqrt{3}\, V_p \angle 30°$$

Thus, we see that in a star connected generator the magnitude of the line voltage is equal to $\sqrt{3}$ times the magnitude of the phase voltage. Again by KVL,

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} + \mathbf{V}_{nc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \mathbf{V}_b + (-\mathbf{V}_{cn})$$

$$= \text{phasor sum of } \mathbf{V}_{bn} \text{ and } \mathbf{V}_{cn} \text{ reversed}$$

$$= \text{phasor } \mathbf{OD} = \text{OD} \angle -90° = \sqrt{3}V_p \angle -90°$$

By KVL,

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} + \mathbf{V}_{na} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \mathbf{V}_{cn} + (-\mathbf{V}_{an})$$
$$= \text{phasor sum of } \mathbf{V}_{cn} \text{ and } \mathbf{V}_{an} \text{ reversed}$$
$$= \text{phasor } \mathbf{OF} = \text{OF} \angle -210° = \text{OF} \angle 150° = \sqrt{3}\, V_p \angle 150°$$

Thus, the line voltages are of equal magnitude and have phase angles of 120° between them.

$$|\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}| = V_L = \sqrt{3}V_P$$

The instantaneous values of line voltages may be written in the following forms

$$v_{ab} = \sqrt{3}V_p \sin(\omega t + 30°)$$
$$v_{bc} = \sqrt{3}V_p \sin(\omega t - 90°)$$
$$v_{ca} = \sqrt{3}\, V_p \sin(\omega t - 210°)$$

The phasor sum of the line voltages is zero, that is,

$$\mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{ca} = 0$$

16.9 STAR-CONNECTED LOAD

Figure 16.8 shows a star-connected, three-phase alternator connected to three identical impedances of magnitude $|\mathbf{Z}|$ and phase angle ϕ in each phase connected in star. Here the neutral point n of the alternator is connected to the neutral point n' of the load by a wire nn' called the *neutral wire*. This system is called a *three-phase, four-wire system*. The load is said to be *balanced* when the impedances in all the three phases are identical. If the three impedances are unequal the load is said to be *unbalanced.*

From Fig. 16.8, it is seen that

$$\mathbf{I}_{na} = \mathbf{I}_{aA} = \mathbf{I}_{An'}$$

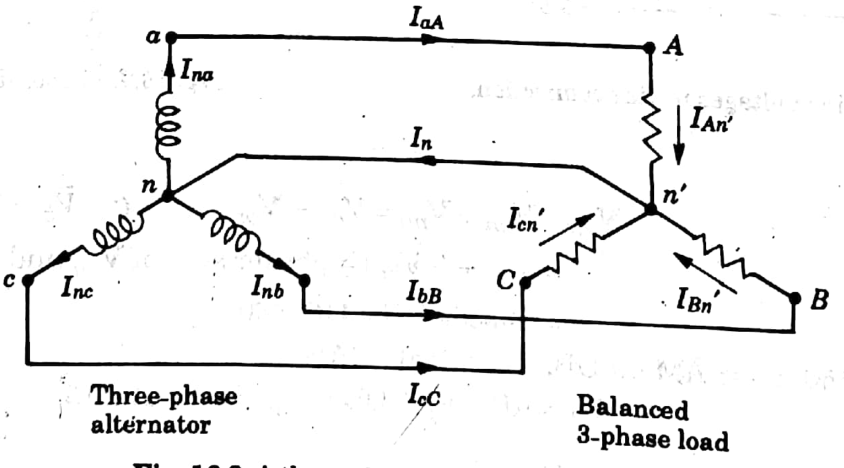

Fig. 16.8. A three-phase, four-wire supply system.

that is, current in phase a of the alternator = current in line a = current in phase A of the load

Also, $\mathbf{I}_{nb} = \mathbf{I}_{bB} = I_{Bn'}$

and $\mathbf{I}_{nc} = \mathbf{I}_{cC} = \mathbf{I}_{Cn'}$

Thus, the currents flowing through the voltage sources flow through the line conductors and through the load elements. Hence, the line current is the same as phase current both in magnitude and phase, that is, $\mathbf{I}_L = \mathbf{I}_p$ for a star connection.

$$= \text{phasor } \mathbf{OD} = \text{OD} \angle -90° = \sqrt{3} V_p \angle -90°$$

By KVL,

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} + \mathbf{V}_{na} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \mathbf{V}_{cn} + (-\mathbf{V}_{an})$$
$$= \text{phasor sum of } \mathbf{V}_{cn} \text{ and } \mathbf{V}_{an} \text{ reversed}$$
$$= \text{phasor } \mathbf{OF} = \text{OF} \angle -210° = \text{OF} \angle 150° = \sqrt{3}\, V_p \angle 150°$$

Thus, the line voltages are of equal magnitude and have phase angles of 120° between them.

$$|\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}| = V_L = \sqrt{3} V_P$$

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$$v_{ca} = \sqrt{3}\, V_p \sin(\omega t - 210°)$$

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From Fig. 16.8, it is seen that

$$\mathbf{I}_{na} = \mathbf{I}_{aA} = \mathbf{I}_{An'}$$

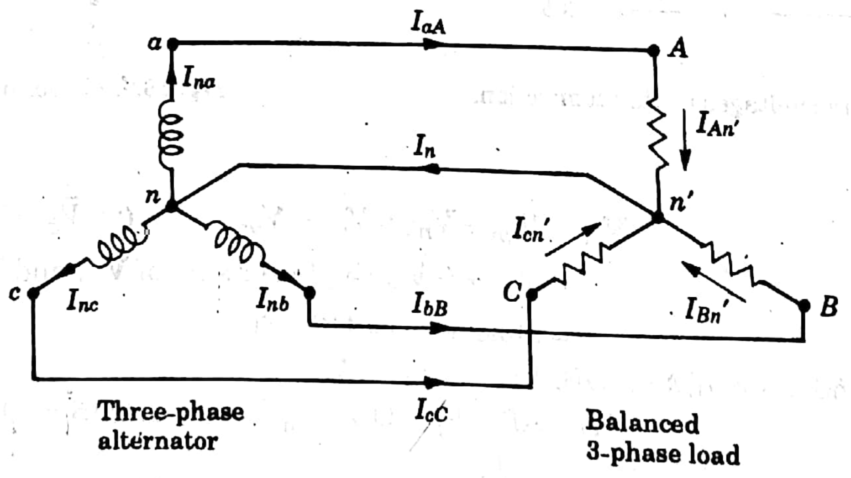

Fig. 16.8. A three-phase, four-wire supply system.

that is, current in phase a of the alternator = current in line a = current in phase A of the load

Also, $\mathbf{I}_{nb} = \mathbf{I}_{bB} = \mathbf{I}_{Bn'}$

and $\mathbf{I}_{nc} = \mathbf{I}_{cC} = \mathbf{I}_{Cn'}$

Thus, the currents flowing through the voltage sources flow through the line conductors and through the load elements. Hence, the line current is the same as phase current both in magnitude and phase, that is, $\mathbf{I}_L = \mathbf{I}_p$ for a star connection.

16.10 NEUTRAL CURRENT

In Fig. 16.8. each impedance is connected between a line and neutral. This is equivalent to three single-phase circuits. The line currents are given by

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_p}, \quad \mathbf{I}_{bB} = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_p}, \quad \mathbf{I}_{cC} = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_p}$$

Let us take an inductive load where

$$\mathbf{Z}_p = Z \angle\phi$$

We have

$$\mathbf{V}_{an} = V_p \angle 0°$$
$$\mathbf{V}_{bn} = V_p \angle -120°$$
$$\mathbf{V}_{cn} = V_p \angle 120°$$

Therefore,

$$\mathbf{I}_{aA} = \frac{V\angle 0°}{Z_p \angle\phi} = \frac{V_P}{Z_p} \angle -\phi$$

$$\mathbf{I}_{bB} = \frac{V_p \angle -120°}{Z_p \angle\phi} = \frac{V_p}{Z_p} \underline{/-120° - \phi}$$

$$\mathbf{I}_{cC} = \frac{V_p \angle 120°}{Z_p \angle\phi} = \frac{V_p}{Z_p} \underline{/120° - \phi}$$

It is seen that the magnitude of each line current is V_P/Z_P and the line currents are displaced by 120° phase angle from each other. The phasor diagram is shown in Fig. 16.9. It is also to be remembered that the line currents are the same as the phase currents. Since the line currents (or phase currents) form a balanced system, the phasor sum of these currents must be zero. This statement can be proved with the help of the current phasor diagram shown in Fig. 16.10.

$$\mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = \mathbf{OL} + \mathbf{MO} + \mathbf{LM}$$
$$= \mathbf{OL} + \mathbf{LM} + \mathbf{MO} = 0$$

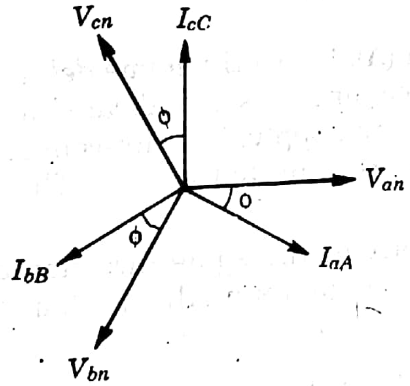

Fig. 16.9. Phasor diagram.

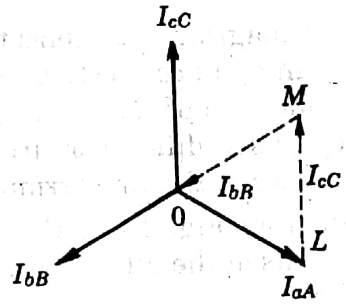

Fig. 16.10. Current phasor diagram.

Writing KCL at neutral point n',

$$\mathbf{I}_N = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 0$$

That is, in a *balanced* three-phase star-connected load the current in the neutral conductor is zero. Since there is no current in the neutral wire the term *neutral* is used. The neutral wire is unnecessary in a balanced system and therefore it may be omitted without affecting the system. This gives a three-phase, three-wire, star-connected supply system as shown in Fig. 16.11.

16.11 DELTA (Δ) OR MESH CONNECTION

If the three coils of the alternator shown in Fig. 16.3 are connected so that the *start* of one coil is connected to the *finish* of the next coil, the result is the delta connection. Figure 16.12 shows a three-phase delta connection. Here a is connected to b', b to c' and c to a'. Thus, a closed mesh is formed, hence the name *mesh* connection.

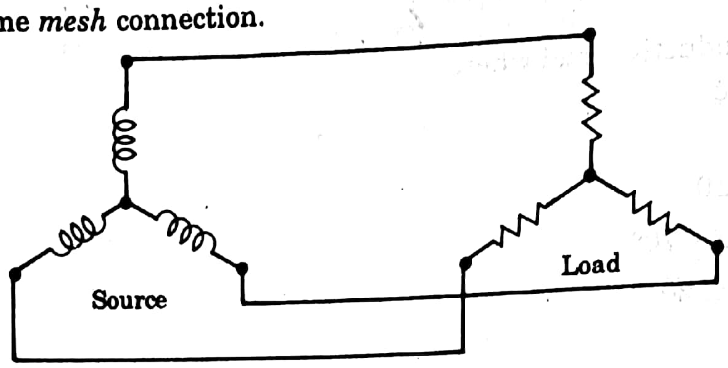

Fig. 16.11. A three-phase, three-wire, star-connected supply system.

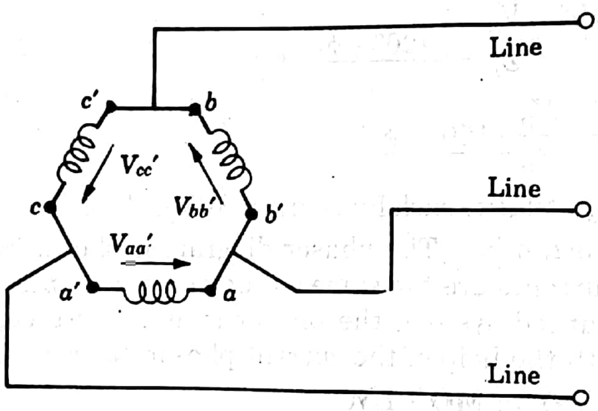

Fig. 16.12. Delta connection.

The circuit diagram resembles the Greek latter delta (Δ), hence the name *delta* connection. The supply lines in delta connection are taken from the three junctions as indicated in Fig. 16.12. Thus, a delta connection of coils gives a three-phase, three-wire supply. In delta-connected system the term phase is used differently from that used in the star-connected system. The phases are now connected between the line terminals.

It is seen from Fig. 16.12 that the phase voltages act in the same direction around the delta-connected coils of the alternator. The resultant voltage acting round the mesh at any instant

$$= v_{aa'} + v_{bb'} + v_{cc'}$$

$$= V_m \sin \omega t + V_m \sin (\omega t - 120°) + V_m \sin(\omega t + 120°)$$

$$= V_m \sin \omega t + V_m \sin \omega t \cos 120° - V_m \cos \omega t \sin 120° + V_m \sin \omega t \cos 120° + V_m \cos \omega t \sin 120°$$

$$= V_m \sin \omega t + 2V_m \sin \omega t \cos 120°$$

$$= V_m \sin \omega t + (2 V_m \sin \omega t)\left(-\frac{1}{2}\right) = V_m \sin \omega t - V_m \sin \omega t = 0$$

That is, there is no resultant voltage acting around the mesh. Hence there will be no circulating current in the mesh in the absence of an external load.

This result can also be proved with the help of the phasor diagram shown in Fig. 16.13.

Here, $\mathbf{V}_{aa'} + \mathbf{V}_{bb'} + \mathbf{V}_{cc'} = \mathbf{OL} + \mathbf{MO} + \mathbf{LM}$

$$= \mathbf{OL} + \mathbf{LM} + \mathbf{MO} = 0$$

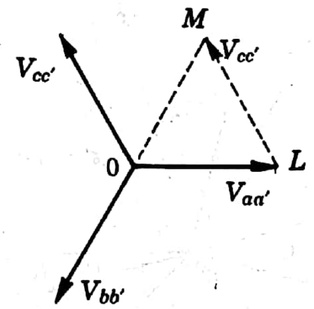

Fig. 16.13. Phasor diagram.

16.12 PHASE AND LINE VOLTAGES IN DELTA CONNECTION

The voltage between any two lines is the line voltage V_L. Figure 16.14 shows a delta-connected generator supplying a *balanced* delta-connected load. Since the phase coils are now connected between lines, the phase voltage is equal to the line voltage. Hence in a balanced delta-connected system,

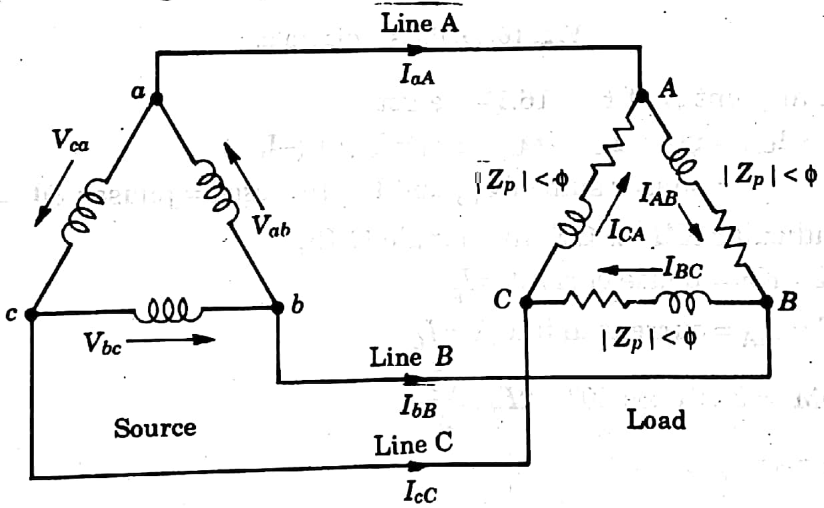

Fig. 16.14. Delta-connected generator supplying a delta-connected load.

magnitude of the line voltage = magnitude of the phase voltage

$$|\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}| = V_L = V_p$$

16.13 PHASE AND LINE CURRENTS IN DELTA CONNECTION

If we take $\mathbf{V}_{ab}$ as the reference, the phase currents can be found as follows :

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_p} = \frac{V_L \angle 0°}{Z_p \angle \phi} = \frac{V_L}{Z_p} \angle -\phi$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_p} = \frac{V_L \angle -120°}{Z_p \angle \phi} = \frac{V_L}{Z_p} \angle -120° - \phi$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_p} = \frac{V_L \angle 120°}{Z_p \angle \phi} = \frac{V_L}{Z_p} \angle 120° - \phi$$

The three phase currents, therefore, have equal magnitudes $I_P (= [V_L/Z_p])$ and are separated by 120° phase angles as shown in Fig. 16.15. The line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$ and $\mathbf{I}_{cC}$ can be found as follows :

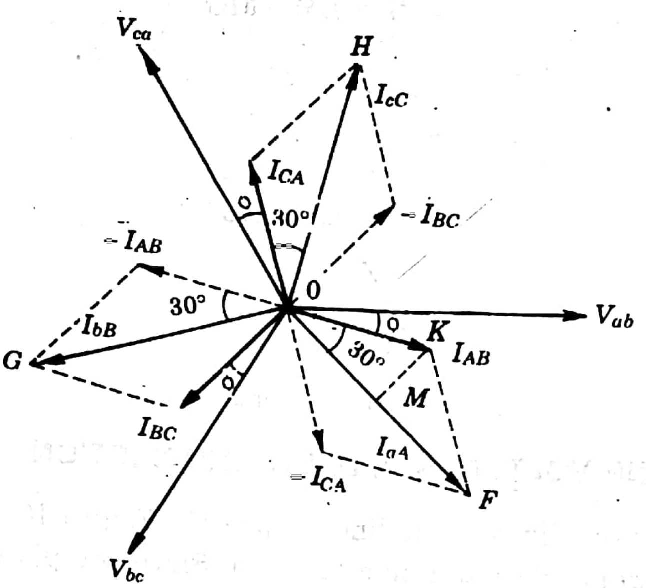

Fig. 16.15. Phasor diagram.

Applying KCL at point A of Fig. 16.14 we get

$$\mathbf{I}_{aA} + \mathbf{I}_{CA} = \mathbf{I}_{AB}\ ;\ \mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB} + (-\mathbf{I}_{CA})$$

$$= \text{phasor sum of } \mathbf{I}_{AB} \text{ and } \mathbf{I}_{CA} \text{ reversed} = \text{phasor } \mathbf{OF} = \text{OF} \underline{/-\phi - 30°}$$

Draw a perpendicular KM on OF. In triangle OKF,

$$\text{OK} = \text{KF} = \text{phase current} = I_p$$

$$\text{OF} = I_{aA} = \text{current in line A} = I_L$$

Now, $\text{OF} = 2\ \text{OM} = 2\ \text{OK} \cos 30° = 2I_p \cdot \dfrac{\sqrt{3}}{2}$

$$I_L = \sqrt{3}\, I_P$$

Thus, we see that in a balanced delta-connected system the magnitude of the line current is equal to $\sqrt{3}$ times the magnitude of the phase current.

Currents in lines B and C can also be found. By KCL at point B,

$$\mathbf{I}_{bB} + \mathbf{I}_{AB} = \mathbf{I}_{BC}\ ;\ \ \mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_{BC} + (-\mathbf{I}_{AB})$$

$$= \text{phasor sum of } \mathbf{I}_{BC} \text{ and } \mathbf{I}_{AB} \text{ reversed} = \text{phasor } \mathbf{OG} = \text{OG} \underline{/-150° - \phi}$$

$$= \sqrt{3}\, I_p \underline{/-150° - \phi}$$

By KCL at C,

$$\mathbf{I}_{cC} + \mathbf{I}_{BC} = \mathbf{I}_{CA}\ ;\ \mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

$$= \mathbf{I}_{CA} + (-\mathbf{I}_{BC}) = \text{phasor sum of } \mathbf{I}_{CA} \text{ and } \mathbf{I}_{BC} \text{ reversed}$$

$$= \text{phasor } \mathbf{OH} = \text{OH} \underline{/90° - \phi} = \sqrt{3}\, I_p \underline{/90° - \phi}$$

It is found that the line currents are of equal magnitude and have phase angles of 120° between them.

16.14 VOLTAMPERES, POWER AND REACTIVE VOLTAMPERES IN A THREE-PHASE SYSTEM

Voltamperes *S*

Voltamperes in a single-phase circuit is given by the expression

$$S_{1\phi} = VI \text{ VA (voltamperes)}$$

The voltamperes consumed by a three-phase load, whether balanced or unbalanced, is given by the sum of the voltamperes in each phase. If the load is balanced, the total voltamperes will be three times the voltamperes per phase, that is,

$$S_{3\phi} = 3\, S_{1\phi} = 3\, V_p I_p \text{ VA}$$

where V_p and I_p are the r.m.s. phase voltage and r.m.s. phase current.

The total voltamperes can also be calculated in terms of line values.

For a star-connected system,

$$V_p = \frac{V_L}{\sqrt{3}} \text{ and } I_P = I_L$$

$$\therefore \quad S_{3\phi} = 3V_p I_P = 3 \cdot \frac{V_L}{\sqrt{3}} \cdot I_L = \sqrt{3} V_L I_L \text{ VA}$$

For a delta-connected system,

$$V_p = V_L \text{ and } I_p = \frac{I_L}{\sqrt{3}}$$

$$\therefore \quad S_{3\phi} = 3\, V_P I_P = 3V_L \frac{I_L}{\sqrt{3}} = \sqrt{3} V_L I_L \text{ VA}$$

Hence, for both star and delta connections

$$S_{3\phi} = \sqrt{3} V_L I_L \text{ VA}$$

Power *P*

Power in a single-phase circuit is given by

$$P_{1\phi} = VI \cos\phi \quad \text{W}$$

Power consumed by a three-phase load, whether balanced or unbalanced, is equal to the sum of the power of each phase. For a balanced load, the total power is equal to three times the power per phase, that is,

$$P_{3\phi} = 3P_{1\phi} = 3V_P I_P \cos\phi \quad \text{W}$$

where ϕ is the phase angle between V_P and I_P. It is also the phase angle of the load. If R is the resistance, χ the reactance and Z the impedance of the load

$$Z^2 = R^2 + \chi^2$$

$$\cos\phi = \frac{R}{Z}$$

In terms of line values, for both star and delta connections.

$$P_{3\phi} = \sqrt{3}\, V_L I_L \cos\phi \text{ W}$$

This relation applies to balanced loads irrespective of their configuration.

Reactive Voltamperes *Q*

Reactive voltamperes in a single-phase circuit are given by

$$Q_{1\phi} = VI \sin\phi \quad \text{VAr}$$

Reactive voltamperes of a three-phase load, either balanced or unbalanced, are equal to the sum of the reactive voltamperes of each phase. For the balanced load, the total voltamperes are equal to three times the voltamperes per phase, that is,

$$Q_{3\phi} = 3Q_{1\phi} = 3V_P I_P \sin\phi \quad \text{VAr}$$

In terms of line values, for both star and delta connections,

$$Q_{3\phi} = \sqrt{3}\, V_L I_L \sin\phi \quad \text{VAr}$$

16.15 INTERCONNECTION OF STAR-AND DELTA-CONNECTED SYSTEMS

The windings of a three-phase alternator are always connected in star for the following reasons :

(1) For star connection,

phase voltage $= \frac{1}{\sqrt{3}}$ line voltage and for a delta connection, phase voltage = line voltage

Thus, for a given phase voltage the star-connected alternator gives a greater line voltage. Hence the star connection is economical.

(2) A star point is available in a star-connected alternator. The star point is connected to ground. This is advantageous from the protection point of view.

The *load* on a three-phase system may either be star or delta connected. For distribution purposes, usually three-phase, four-wire system is used. Here two voltages are available. One voltage is obtained between a line and the neutral, and the other voltage between any two lines. For example, if the voltage between any two lines is 400 V, the voltage between any line and the neutral is $400/\sqrt{3} = 230$ V. In practice, single-phase supplies are obtained from three-phase, four-wire systems. Singe-phase alternators are not used for obtaining single single-phase supplies. Lighting and heating loads are usually connected between any one of the lines and the neutral. Domestic consumers are supplied with single-phase voltage obtained from a three-phase, four-wire system. Industrial motors are connected to three-phase system directly. The load is normally distributed equally among the phases in order to keep the system as balanced as possible. A typical arrangement showing the connections of a single-phase and three-phase loads from a three-phase system is given in Fig. 16.16.

16.16 SOLUTION OF THREE-PHASE CIRCUITS

It is always better to solve balanced three-phase circuits on *per phase basis*. When a three-phase supply voltage is given without reference to the line or phase value then it is the line voltage that is given. The following steps are adopted for solving three-phase balanced circuits :

(1) Draw the circuit diagram.

(2) Draw one phase separately.

(3) Determine $X_{LP} = 2\pi fL$

(4) Determine $X_{CP} = \frac{1}{2\pi fC}$

(5) Determine $X_P = X_{LP} - X_{CP}$

(6) Determine $Z_P = \sqrt{R_P^2 + X_P^2}$

(7) Determine $\cos\phi = \frac{R_P}{Z_P}$

The power factor is lagging when $X_{LP} > X_{CP}$ and it is leading when $X_{CP} > X_{LP}$.

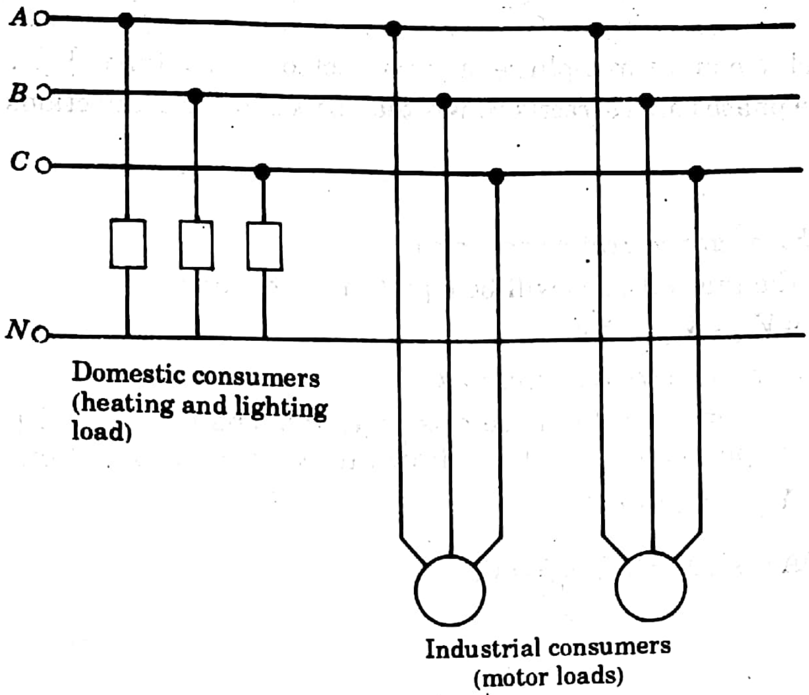

Fig. 16.16. Connections of one-phase and three-phase loads from a three-phase system.

(8) Determine V_P. For star connection $V_P = (V_L/\sqrt{3})$ and for delta connection $V_P = V_L$

(9) Determine $I_P = V_P/Z_P$

(10) Determine the line current I_L. For star connection $I_L = I_P$ and for delta connection $I_L = \sqrt{3}\, I_p$.

(11) Determine $P = 3I_P^2 R_P$

or $P = 3\, V_P I_P \cos\phi$

(12) Determine $Q = 3\, V_P I_P \sin\phi$

(13) Determine $S = 3\, V_P I_P$.

EXAMPLE 16.1

Calculate the line current of a three-phase alternator delivering 5 MW at 33 kV and working at 0.8 power factor.

SOLUTION

$P = 5\text{MW} = 5 \times 10^6$ W

$V_L = 33\text{kV} = 33 \times 10^3$ V

Since the alternator is star connected

$$V_P = V_L/\sqrt{3} = \frac{33 \times 10^3}{\sqrt{3}} = 19.052 \times 10^3 \text{ V}$$

$$3V_P I_P \cos\phi = P$$

$$I_P = \frac{P}{3V_p \cos\phi} = \frac{5 \times 10^6}{3 \times 19.052 \times 10^3 \times 0.8} = 109.4 \text{ A}$$

Line current $I_L = I_P = 109.4$ A

EXAMPLE 16.2

If the phase voltage of a three-phase star-connected alternator is V_p, what will be the line voltages : (*a*) when the phases are correctly connected; (*b*) when the connections to one of the phases are reversed ?

SOLUTION

(*a*) *When the phases are correctly connected*

In this case all the line voltages will be equal in magnitude

$$V_{ab} = V_{bc} = V_{ca} = V_L = \sqrt{3}V_P$$

(*b*) *When one phase connection is reversed*

Suppose that the connection to phase *a* is reversed. The circuit and phasor diagrams are shown in Fig. 16.17. The phasor V_{an} has been drawn in the reverse direction.

$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \text{phasor } \mathbf{OC}$$

$$\text{OC} = 2\text{OM} = 2\text{OA} \cos 60° = 2V_P \times \frac{1}{2} = V_P$$

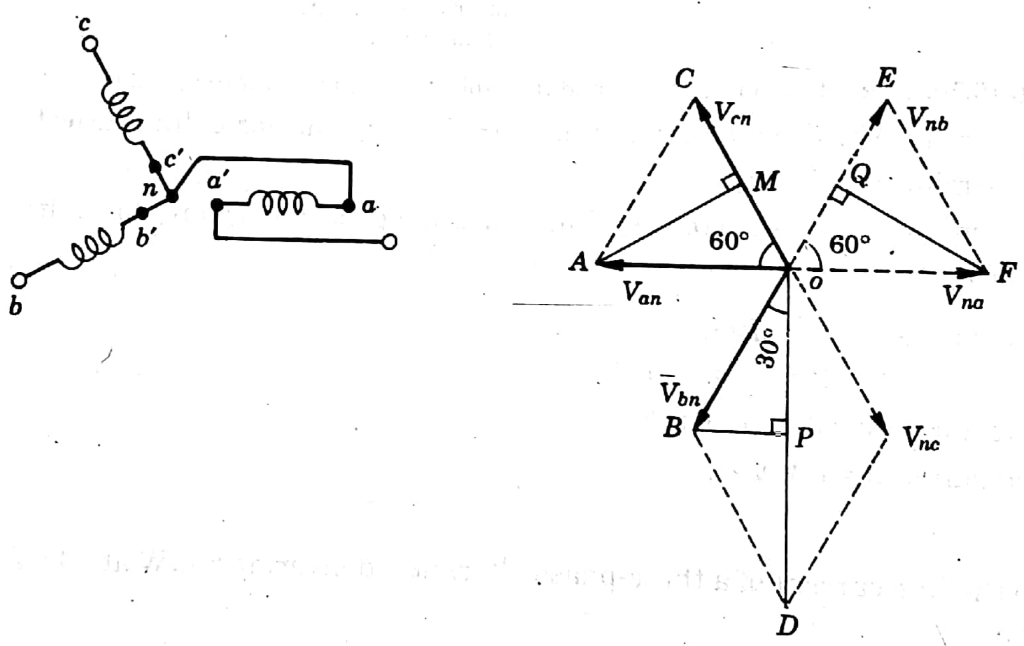

Fig. 16.17. (*a*) Circuit diagram; (*b*) phasor diagram.

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} + \mathbf{V}_{nc} = \text{phasor } \mathbf{OD}$$

$$\text{OD} = 2\text{OP} = 2\text{OB} \cos 30° = 2V_P \times \frac{\sqrt{3}}{2} = \sqrt{3}V_P$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} + \mathbf{V}_{na} = \text{phasor } \mathbf{OE}$$

$$\text{OE} = 2\text{OQ} = 2\text{OF} \cos 60° = 2V_P \times \frac{1}{2} = V_P$$

Thus, $|\mathbf{V}_{ab}| = V_P$; $|\mathbf{V}_{bc}| = \sqrt{3}\,V_p$; $|\mathbf{V}_{ca}| = V_P$

EXAMPLE 16.3

Show that the power consumed by three identical phase loads connected in delta is equal to three times the power consumed when the phase loads are connected in star.

SOLUTION

Let V_L = line voltage

R = resistance of each phase load

Z = impedance of each phase load

Star connection

$$V_P = V_L/\sqrt{3}$$

$$P_Y = 3\,V_P I_P \cos\phi = 3\,V_P \cdot \frac{V_P}{Z} \cdot \frac{R}{Z} = 3V_P^2\frac{R}{Z^2}$$

$$= 3\left(\frac{V_L}{\sqrt{3}}\right)^2 \frac{R}{Z^2} = V_L^2\frac{R}{Z^2}$$

Delta connection

$$V_P = V_L$$

$$P_\Delta = 3\,V_P I_P \cos\phi$$

$$= 3\,V_P \cdot \frac{V_P}{Z} \cdot \frac{R}{Z} = 3\,V_P^2\frac{R}{Z^2} = 3V_L^2\frac{R}{Z^2}$$

By comparison, $P_\Delta = 3\,P_Y$

EXAMPLE 16.4

Three identical resistors of 20 Ω are connected in star to a 415 V, three-phase, 50 Hz supply. (*a*) Calculate the total power taken by the load. (*b*) Also calculate the power consumed in the resistors if they are connected in delta to the same supply. (*c*) If one of the resistors is open-circuited in each case, calculate the power consumed.

SOLUTION

(*a*) *Star connection*

$$V_P = V_L/\sqrt{3} = 415/\sqrt{3} = 240\text{ V}$$

$$I_P = \frac{V_P}{R} = \frac{240}{20} = 12\text{ A}$$

$$P = 3I_P^2 R = 3 \times 12^2 \times 20 = 8640\text{ W}$$

(*b*) *Delta connection*

$$V_P = V_L = 415\text{ V}$$

$$I_P = \frac{V_P}{R} = \frac{415}{20} = 20.75\text{ A}$$

$$P = 3I_P^2 R_P = 3(20.75)^2 \times 20 = 25833\text{ W}$$

(*c*) *One resistor open-circuited in star connection*

From Fig. 16.18

$$\text{current } I_1 = \frac{V_L}{2R} = \frac{415}{2 \times 20} = 10.375\text{ A}$$

Power consumed

$$P = 2I_1^2 R = 2(10.375)^2 \times 20 = 4305\text{ W}$$

(d) One resistor open-circuited in delta connection

This arrangement is shown in Fig. 16.19.

Power consumed

$$P = \frac{V_L^2}{R} + \frac{V_L^2}{R} = 2\frac{V_L^2}{R} = 2 \times \frac{(415)^2}{20} = 17222 \text{ W}$$

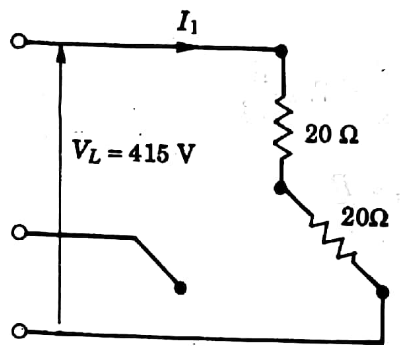

Fig. 16.18

EXAMPLE 16.5

A three-phase delta-connected 415 V induction motor has a power factor of 0.8 when supplying a load of 3.73 kW. Calculate the line current taken by the motor if the efficiency of the motor is 0.86.

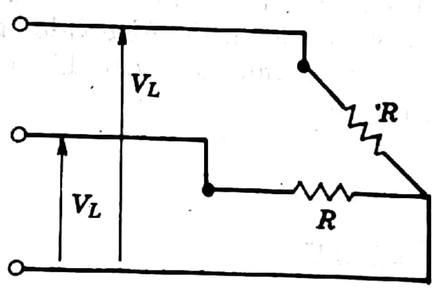

Fig. 16.19

SOLUTION

Motor output = 3.73 kW = 3730 W

Input power to motor,

$$P_i = \frac{\text{output}}{\text{efficiency}} = \frac{3730}{0.86} = 4337 \text{ W}$$

For delta connection,

$$V_P = V_L = 415 \text{ V}$$

$$P_i = 3V_P I_P \cos\phi$$

$$I_P = \frac{P_i}{3V_P \cos\phi} = \frac{4337}{3 \times 415 \times 0.8} = 4.355 \text{ A}$$

Line current

$$I_L = \sqrt{3} I_P = \sqrt{3} \times 4.355 = 7.55 \text{ A}$$

EXAMPLE 16.6

A delta-connected motor is fed from a transformer with a star-connected secondary. If the line voltage is 400 V and the motor develops 150 kW when operating at an efficiency of 0.86 and 0.8 p.f. lagging, calculate the line current, the current in each phase of the motor and the current in each phase of the transformer.

SOLUTION

Motor output = 150 kW = 150×10^3 W

Motor input,

$$P_i = \frac{\text{motor output}}{\text{efficiency}} = \frac{150 \times 10^3}{0.86} = 174.4 \times 10^3 \text{ W}$$

Let I_P be the motor phase current.

$$3V_P I_P \cos\phi = P_i$$

$$I_P = \frac{P_i}{3V_P \cos\phi} = \frac{174.4 \times 10^3}{3 \times 400 \times 0.8} = 181.7 \text{ A}$$

Line current,

$$I_L = \sqrt{3} I_P = \sqrt{3} \times 181.7 = 314.7 \text{ A}$$

Current in each phase of the transformer = line current = 314.7 A