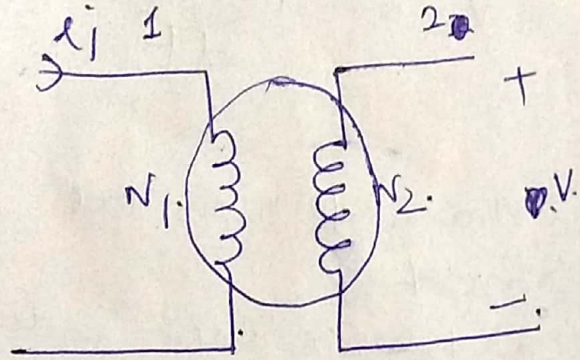
Dot Convention in magnetically Coupled ckts



whenever the current i flows in one coil, it ~~induces~~ produces a voltage in the nearby coil

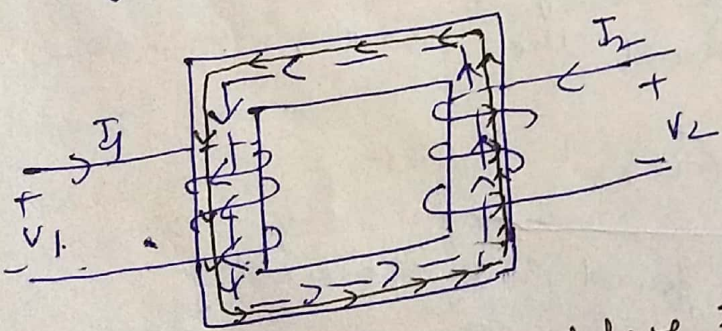
when the current flows through one coil, ~~it~~ the flux of the one coil gets coupled with the other coil.

This kind of ckts are magnetically coupled ckts -

The voltage induced in the nearby coil is given by

$V = M\frac{di_1}{dt}$ M is the mutual inductance b/w the two coils:

the polarity of the induced voltage induced in the nearby coil depends on the way the windings are wound around the core



$$V_{21} = \frac{M di_1}{dt}$$

voltage induced in 2 due to current flowing in 1.

$$V_{12} = \frac{M di_2}{dt}.$$

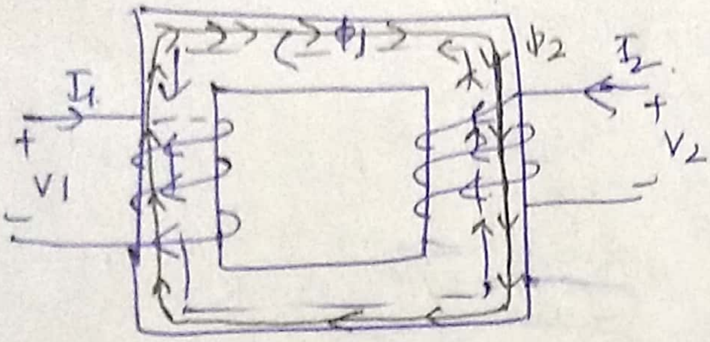
These two fluxes are adding to each other. The current I_1 & I_2 are flowing simultaneously.

The voltage which will be generated due to mutual coupling will be +ve:

$$\boxed{V_1 = \underbrace{L_1\frac{di_1}{dt}}_{\text{Self induced emf}} + \underbrace{\frac{M di_2}{dt}}_{\text{emf induced due to mutual coupling.}}}$$

$$\boxed{V_2 = L_2\frac{di_2}{dt} + \frac{M di_1}{dt}}$$

...d voltage ...



$$V_{21}' = -M\frac{di_1}{dt}$$

$$V_{12} = -M\frac{di_2}{dt}$$

The Direction of winding has changed then the direction of flux has been changed and hence the induced The net flux will be opposite and hence the induced voltage in the coil due to mutual coupling will be negative

$$v_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$v_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$

Hence the polarity of the induced voltage depended on the ~~direction of the~~ (direction of the windings) way in which the windings are wound around the core.

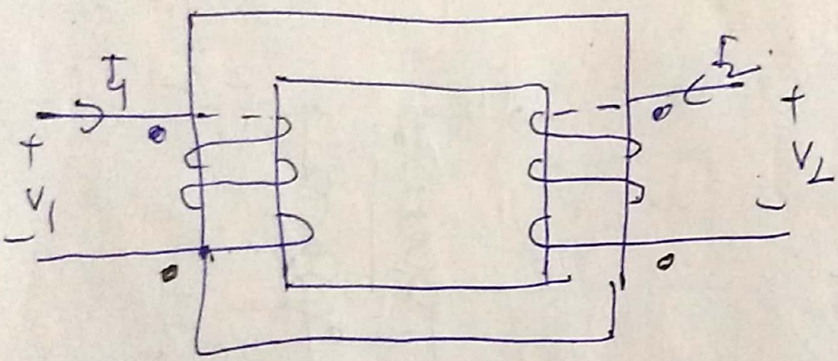
This is a lengthy process

Everything we have to find the polarity of the voltage we have to see the direction of the windings & the direction of the flux.
This is very lengthy process
We put dots on the either side of the coil & we put the dot on all the coils which are coupled together.
Depending on the current which are entering

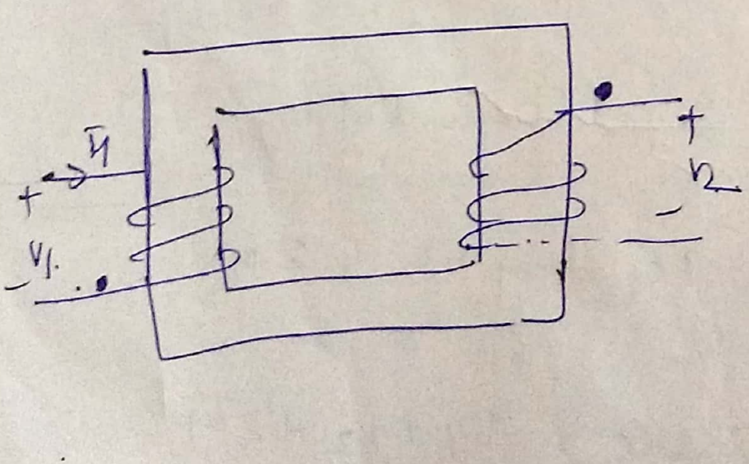
of leaving the dot, we determine the polarity of the induced voltage due to mutual coupling.

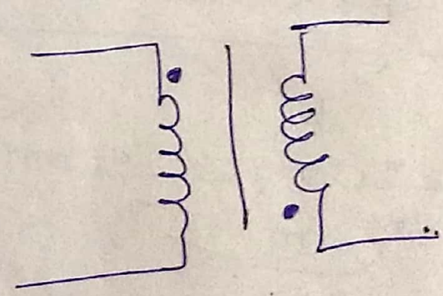
Dot Convention

If two currents are entering (leaving) into the dot at the same time then voltage generated due to mutual inductance will be positive

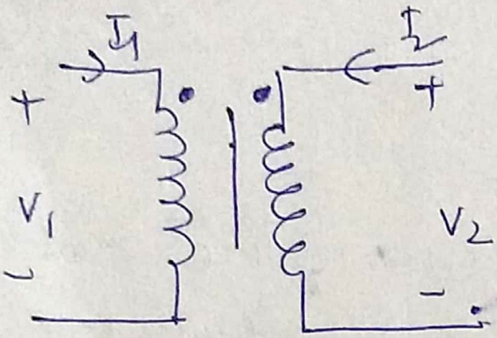


If one current is entering (leaving) the dot and another current is leaving (entering) the dot then voltage generated due to mutual inductance will be negative.

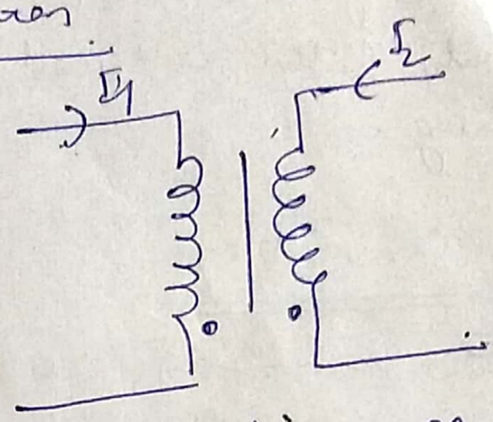




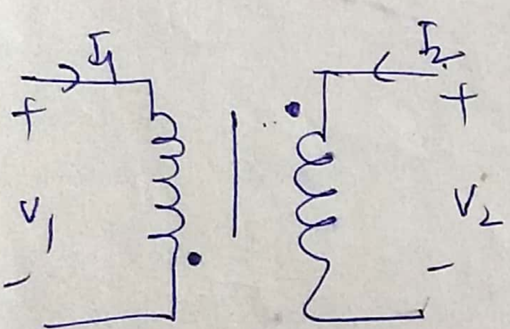
Dot Convention



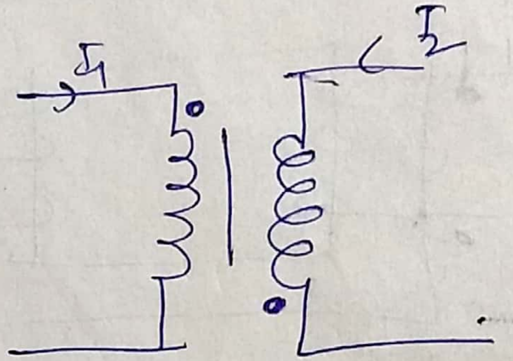
$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$



$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

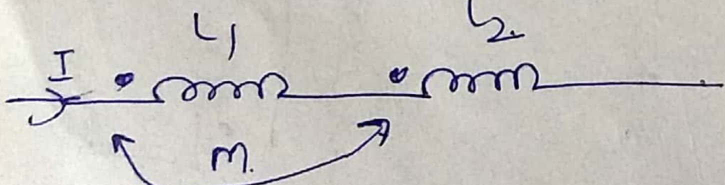


$$V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$



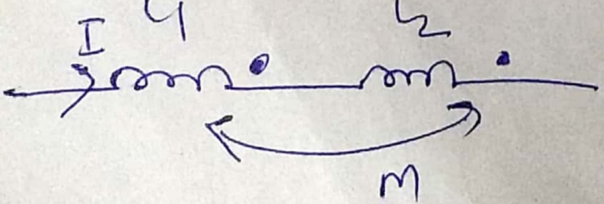
$$V_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}.$$

Coupled Coil in Series



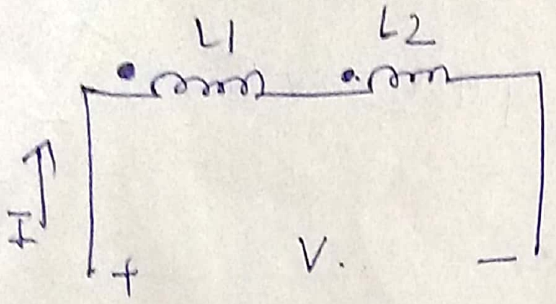
Series aiding Connection

$$L_{eq} = L_1 + L_2 + 2M$$



$$L_{eq} = L_1 + L_2 + 2M$$

Series aiding

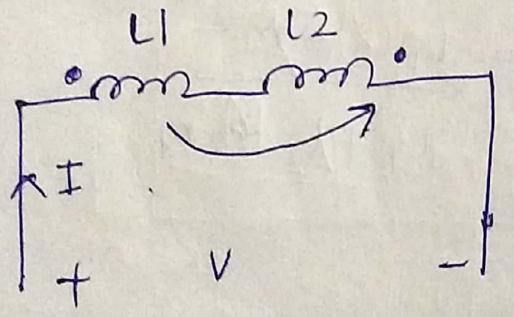


$V = j\omega L_1 I + j\omega L_2 I + j\omega M I + j\omega M I$

$\frac{V}{I} = Z_{eq}$

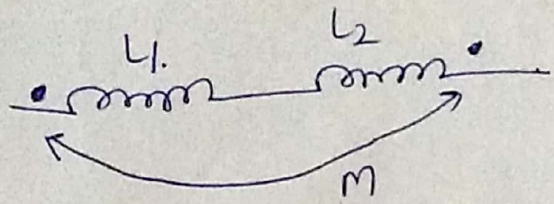
$j\omega [L_1 + L_2 + 2M] = Z_{eq}$

$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$



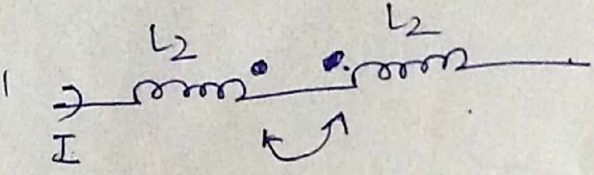
$V = j\omega L_1 I + j\omega L_2 I - j\omega M I - j\omega M I$

$$\boxed{V = L_1 + L_2 - 2M}$$



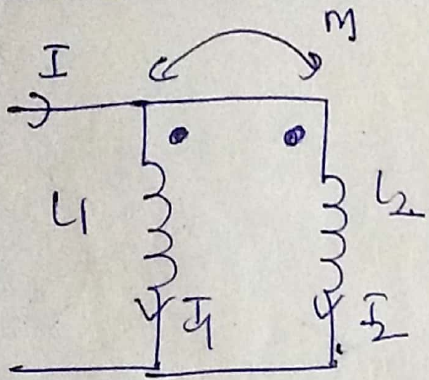
Series opposing connection

$$L = L_1 + L_2 - 2M$$

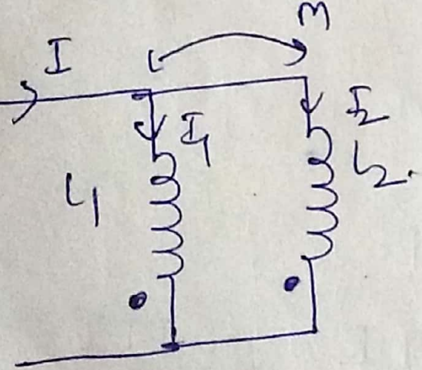


SOC

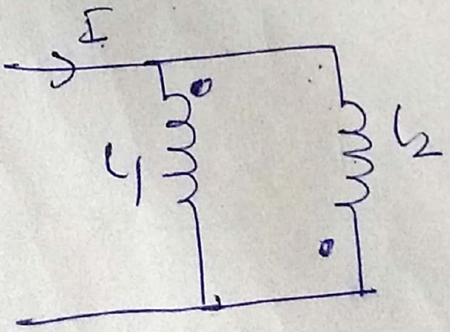
Coupled Coils in Parallel



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$



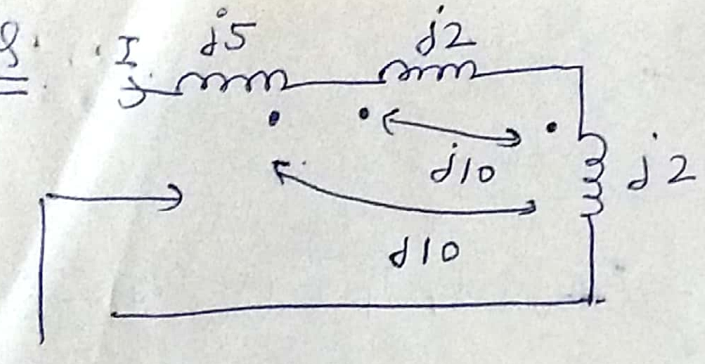
$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Effect cause

M_{21} = Effect on 2 due to current flowing in 1.

M_{12} = Effect on 1 due to current flowing in 2.

Q.



Z:

$$V = j\omega L_1 I + j\omega L_2 I + j\omega L_3 I - j\omega M_{21} I + j\omega M_{31} I + j\omega M_{12} I + j\omega M_{32} I - j\omega M_{13} I + j\omega M_{23} I$$

$$= j5 + j2 + j2 + j10 - j10 - j10 + j10 + j10 - j10 + j10$$

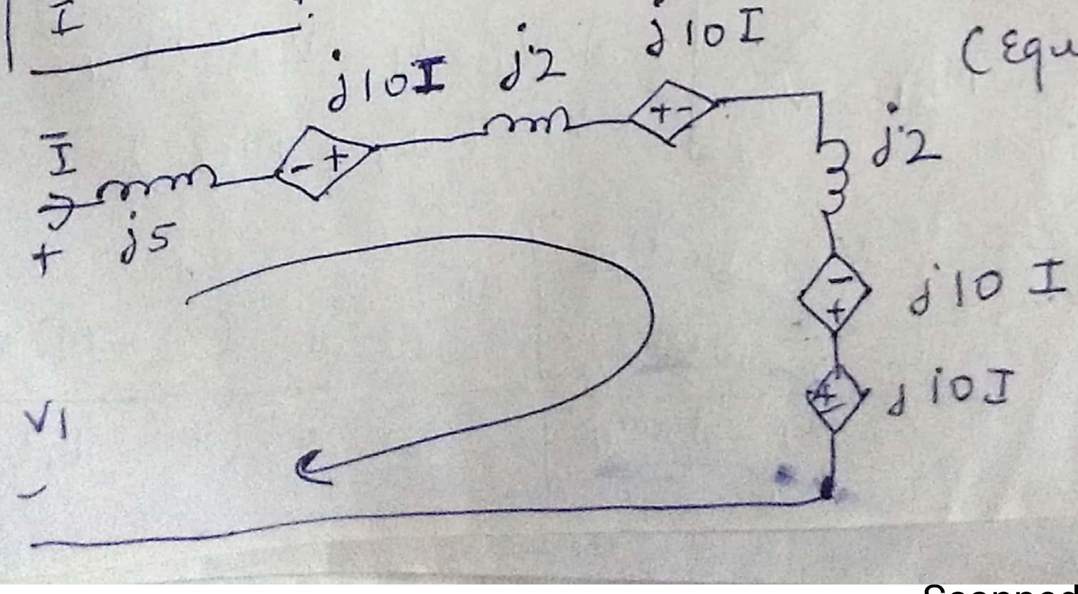
$$j9$$

$$V = j\omega L_1 I + j\omega L_2 I + j\omega L_3 I - j\omega M_{31} I + j\omega M_{23} I + j\omega M_{32} I - j\omega M_{13} I$$

13: effect on 1, cause due to 3

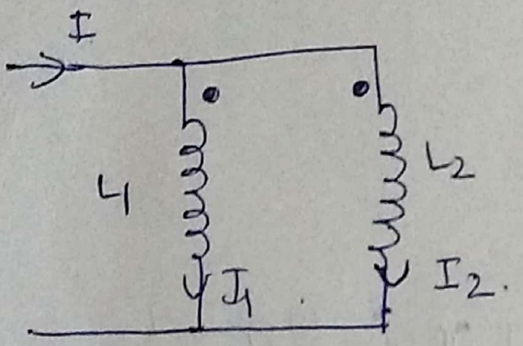
$$V = j9 I$$

$$\boxed{\frac{V}{I} = j9}$$



(Equivalent ckt:)

Coupled Coils in Parallel



In KVL 1

$-V + j\omega L_1 I_1 + j\omega M_{12} I_2 = 0$

In KVL 2

$-V + j\omega L_2 I_2 + j\omega M_{21} I_1 = 0.$

$$\begin{bmatrix} V \\ V \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M_{12} \\ j\omega M_{21} & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

$$\Delta = \begin{bmatrix} j\omega L_1 & j\omega M_{12} \\ j\omega M_{21} & j\omega L_2 \end{bmatrix} = j^2\omega^2 L_1 L_2 - j^2\omega^2 M^2$$

$$= j^2\omega^2 (L_1 L_2 - M^2)$$

$$\Delta_1 = \begin{bmatrix} V & j\omega M_{12} \\ V & j\omega L_2 \end{bmatrix} = V(j\omega L_2) - V(j\omega M_{12})$$

$$= V j\omega (L_2 - M_{12})$$

$$\Delta_2 = \begin{bmatrix} j\omega L_1 & V \\ j\omega M_{21} & V \end{bmatrix} = V j\omega L_1 - V j\omega M_{21}$$

$$= V j\omega (L_1 - M_{21})$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} V & j\omega M_{12} \\ V & j\omega L_2 \end{bmatrix}}{\begin{bmatrix} j\omega L_1 & j\omega M_{12} \\ j\omega M_{21} & j\omega L_2 \end{bmatrix}} = \frac{j\omega (L_2 - M) V}{\omega^2 (M^2 - L_1 L_2)}$$

$$I_2 = \frac{\begin{bmatrix} j\omega L_1 & V \\ j\omega M & V \end{bmatrix}}{\begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix}} = \frac{j\omega (L_1 - M) V}{\omega^2 (M^2 - L_1 L_2)}$$

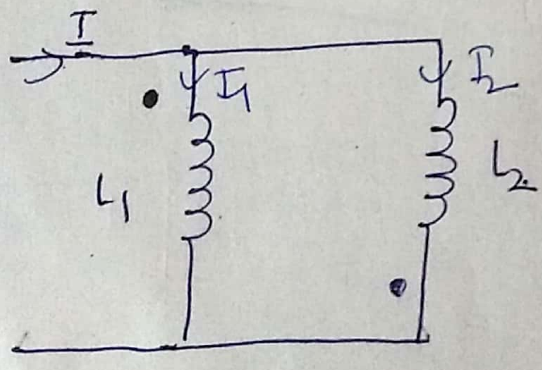
$$I = I_1 + I_2$$

$$= \frac{j\omega (L_1 + L_2 - 2M) V}{\omega^2 (M^2 - L_1 L_2)}$$

$$Z = \frac{V}{I} = j\omega \left(\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \right)$$

$$\boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

Q2.



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

KVL 1

$$-V_1 + j\omega L_1 I_1 - j\omega M_{12} I_2 = 0 \quad \text{--- (1)}$$

KVL 2

$$-V_1 + j\omega L_2 I_2 - j\omega M_{21} I_1 = 0 \quad \text{--- (2)}$$

$$\begin{bmatrix} V \\ V \end{bmatrix} = \begin{bmatrix} j\omega L_1 & -j\omega M_{12} \\ -j\omega M_{21} & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I \end{bmatrix}$$

$$\Delta = \begin{bmatrix} j\omega L_1 & -j\omega M_{12} \\ -j\omega M_{21} & j\omega L_2 \end{bmatrix}$$

$$= j\omega L_1 (j\omega L_2) - (j\omega M_{21})(j\omega M_{12})$$

$$= j^2 \omega^2 L_1 L_2 - j^2 \omega^2 M^2$$

$$\Delta = j^2 \omega^2 (L_1 L_2 - M^2)$$

$$\Delta_1 = \begin{bmatrix} V & -j\omega M_{12} \\ V & j\omega L_2 \end{bmatrix}$$

$$\Delta_1 = V(j\omega L_2) + j\omega M_{12}(V)$$

$$\Delta_2 = \begin{bmatrix} j\omega L_1 & V \\ -j\omega M_{21} & V \end{bmatrix}$$

$$\Delta_2 = V(j\omega L_1) + V(j\omega M_{21})$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{V(j\omega L_2) + j\omega M_{12}(V)}{j^2 \omega^2 (L_1 L_2 - M^2)} = \frac{V j\omega (L_2 + M)}{\omega^2 (M^2 - L_1 L_2)}$$

$$I_1 = \frac{V(L_2 + M)}{j\omega (L_1 L_2 - M^2)}$$

$$\cancel{I_1 = \frac{V j\omega (L_2 + M)}{\omega^2 (M^2 - L_1 L_2)}}$$

$$\boxed{j\omega \left(\frac{L_1 L_2 - M^2}{L_2 + M} \right) = \frac{V}{I_1}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} j\omega L_1 & V \\ -j\omega M_{21} & V \end{bmatrix}}{\begin{bmatrix} j\omega L_1 & -j\omega M_{12} \\ -j\omega M_{21} & j\omega L_2 \end{bmatrix}}$$

$$I_2 = \frac{V(j\omega L_1) + V(j\omega M_{21})}{j^2 \omega^2 (L_1 L_2 - M^2)}$$

$$I_2 = \frac{V j\omega (L_1 + M_{21})}{j^2 \omega^2 (L_1 L_2 - M^2)} = \frac{j\omega (L_1 + M) V}{\omega^2 (M^2 - L_1 L_2)}$$

$$I = I_1 + I_2$$

$$= \frac{V(L_2 + M)}{j\omega (L_1 L_2 - M^2)} + \frac{V(L_1 + M)}{j\omega (L_1 L_2 - M^2)}$$

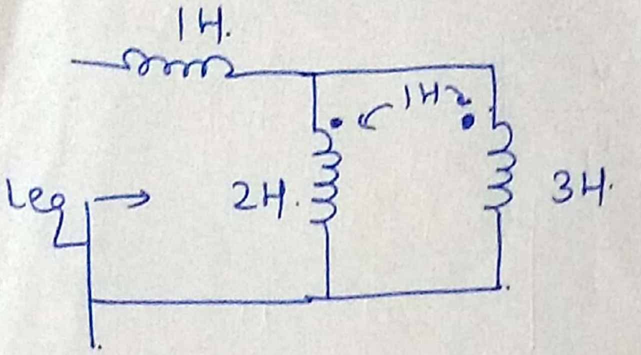
$$I = \frac{V (L_1 + L_2 + 2M)}{j\omega (L_1 L_2 - M^2)}$$

$$j\omega \left(\frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \right) = \frac{V}{I}$$

$$\boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

Scanned with CamScanner

The Equivalent Inductance L_{eq} of the ckt :

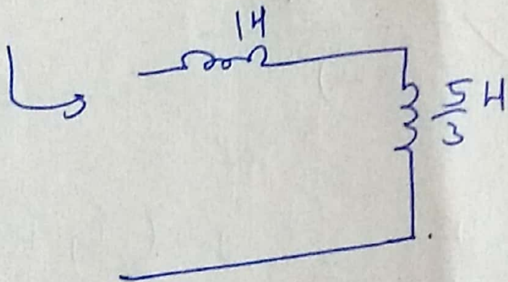


$L_1 = 2H$

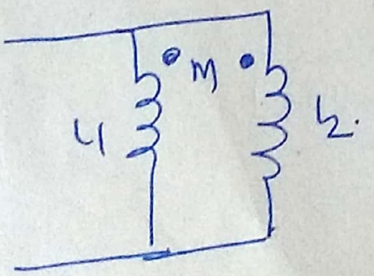
$L_2 = 3H$

$$L_0 = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{2 \times 3 - (1)^2}{2 + 3 - 2} = \frac{5}{3} H$$

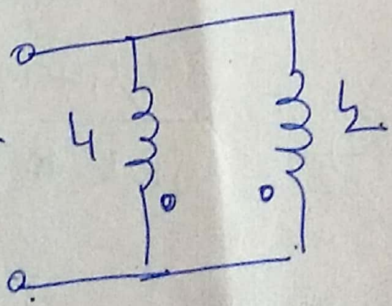
$$L_{eq} = L + L_0 = 1 + \frac{5}{3} = \frac{8}{3} H$$



If no magnetic coupling



or



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

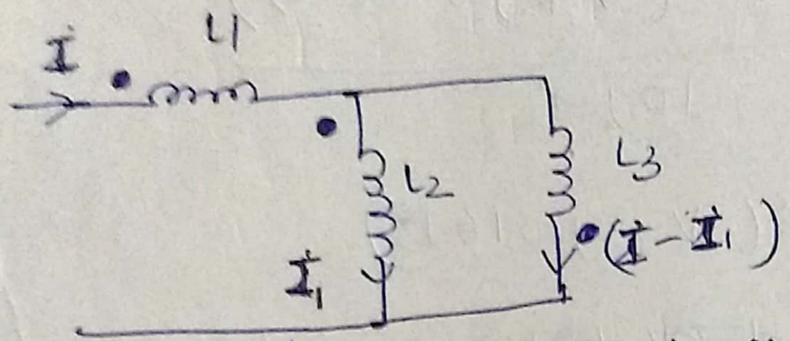
If no magnetic coupling $\boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$ or $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

8) $L_1 = L_2 = L_3 = 10\,H$

$M_{12} = M_{21} = 1\,H.$

$M_{13} = M_{31} = 2\,H$

$M_{23} = M_{32} = 3\,H$



KVL 1

$$V = j\omega L_1 I + j\omega L_2 I_1 + j\omega M_{12} I_1 - j\omega M_{13} (I - I_1) - j\omega M_{23} (I - I_1) + j\omega M_{21} I \quad \text{— (1)}$$

$$V = j\omega (L_1 + M_{21} - M_{23} - M_{13}) I + j\omega (L_2 + M_{12} + M_{13} + M_{23}) I_1$$

$$V = j\omega (10 + 1 - 3 - 2) I + j\omega (10 + 1 + 2 + 3) I_1$$

$$V = j\omega (11 - 5) I + j\omega (16) I_1$$

$$\boxed{V = 6 j\omega I + 16 j\omega I_1} \quad \text{— (1)}$$

KVL 2

$$-V_1 + j\omega L_1 I + j\omega L_3 (I - I_1) + j\omega M_{12} I_1 - j\omega M_{13} (I - I_1) - j\omega M_{31} I - j\omega M_{32} I_1 = 0$$

$$-V + j\omega I (L_1 + L_3 - M_{13} - M_{31}) - j\omega (L_3 - M_{13} - M_{12} + M_{32}) I_1 = 0$$

$$-V_1 + j\omega I(10+10-2-2) - j\omega I_1(10) = 0$$

$$\boxed{V = -10j\omega I_1 + j\omega I(16)} \quad (2)$$

$$V = j\omega 6I - \frac{j\omega 16(V - j\omega I(16))}{10j\omega}$$

$$V = j\omega 6I - 1.6(V - j\omega 16 I)$$

$$2.6V = j\omega 6I + j\omega 25.6 I$$

$$2.6V = 31.6 I$$

$$\boxed{\frac{V}{I} = 12.5\,\Omega}$$

$$= j\omega(12.5)$$

L_{eq}