B2 physics Notes

(3)

Chapter 5

Nuclear Shell Model

5.1 Magic Numbers

The binding energies predicted by the Liquid Drop Model underestimate the actual binding energies of "magic nuclei" for which either the number of neutrons $N = (A - Z)$ or the number of protons, Z is equal to one of the following "magic numbers"

$$2,\ 8,\ 20,\ 28,\ 50,\ 82,\ 126.$$

This is particularly the case for "doubly magic" nuclei in which *both* the number of neutrons and the number of protons are equal to magic numbers.

For example for $^{56}_{28}$Ni (nickel) the Liquid Drop Model predicts a binding energy of 477.7 MeV, whereas the measured value is 484.0 MeV. Likewise for $^{132}_{50}$Sn (tin) the Liquid Drop model predicts a binding energy of 1084 MeV, whereas the measured value is 1110 MeV.

There are other special features of magic nuclei:

- The neutron (proton) separation energies (the energy required to remove the last neutron (proton)) peaks if N (Z) is equal to a magic number.

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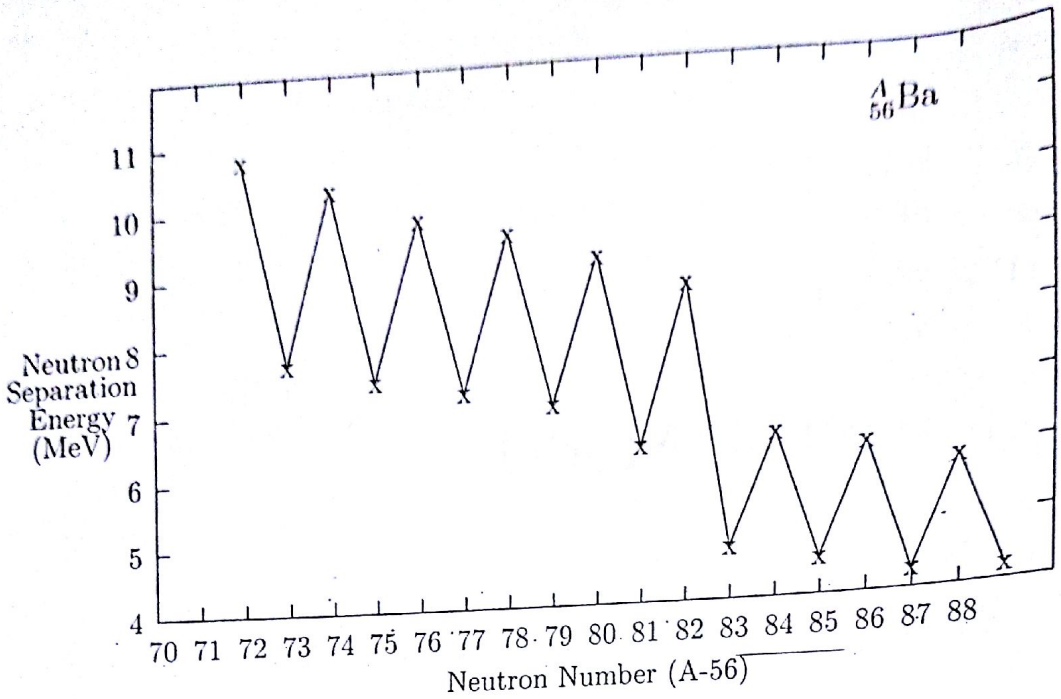
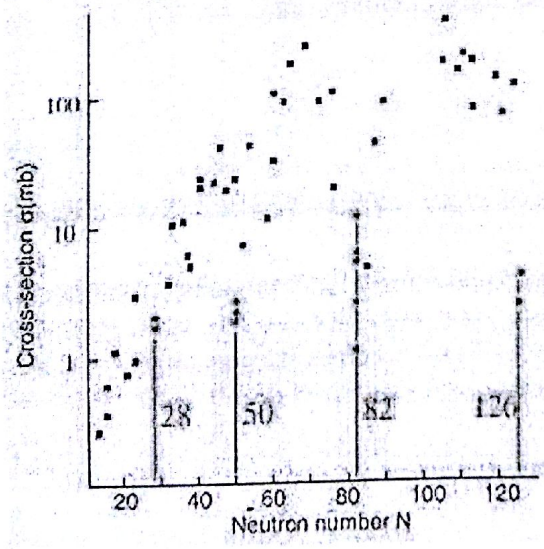


- There are more stable isotopes if Z is a magic number, and more stable isotones if N is a magic number.

- If N is magic number then the cross-section for neutron absorption is much lower than for other nuclides.



Nuclei with magic numbers of neutrons have neutron absorption cross-sections up to two orders of magnitude less than other nuclei with similar masses

- The energies of the excited states are much higher than the ground state if either N or Z or both are magic numbers.

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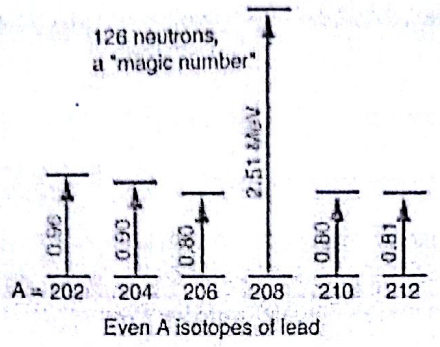

- Elements with Z equal to a magic number have a larger natural abundance than those of nearby elements.

5.2 Shell Model

These magic numbers can be explained in terms of the Shell Model of the nucleus, which considers each nucleon to be moving in some potential and classifies the energy levels in terms of quantum numbers $n\,l\,j$, in the same way as the wavefunctions of individual electrons are classified in Atomic Physics.

For a spherically symmetric potential the wavefunction (neglecting its spin for the moment) for any nucleon whose coordinates from the centre of the nucleus are given by polar coordinates (r, θ, ϕ) is of the form

$$\Psi_{nlm} = R_{nl}(r) Y_l^m(\theta, \phi).$$

The energy eigenvalues will depend on the principle quantum number, n, and the orbital angular momentum, l, but are degenerate in the magnetic quantum number m. These energy levels come in 'bunches' called "shells" with a large energy gap just above each shell.

In their ground state the nucleons fill up the available energy levels from the bottom upwards with two protons (neutrons) in each available proton (neutron) energy level.

Unlike Atomic Physics we do not even understand in principle what the properties of this potential are - so we need to take a guess.

A simple harmonic potential (i.e. $V(r) \propto r^2$) would yield equally spaced energy levels and we would not see the shell structure and hence the magic numbers.

It turns out that once again the Saxon-Woods model is a reasonable guess, i.e.

$$V(r) = -\frac{V_0}{1 + \exp\left(((r - R)/\delta)\right)}$$

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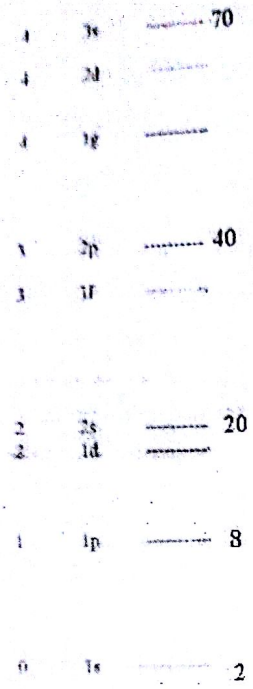

For such a potential it turns out that the lowest level is $1s$ (i.e. $n = 1,\ l = 0$) which can contain up to 2 protons or neutrons. Then comes $1p$ which can contain up to a further 6 protons (neutrons). This explains the first 2 magic numbers (2 and 8). Then there is the level $1d$, but this is quite close in energy to $2s$ so that they form the same shell. This allows a further 2+10 protons (neutrons) giving us the next magic number of 20.

The next two levels are $1f$ and $2p$ which are also quite close together and allow a further 6+14 protons (neutrons). This would suggest that the next magic number was 40 - but experimentally it is known to be 50.

The solution to this puzzle lies in the spin-orbit coupling. Spin-orbit coupling - the interaction between the orbital angular momentum and spin angular momentum occurs in Atomic Physics. In Atomic Physics, the origin is magnetic and the effect is a small correction. In the case of nuclear binding the effect is about 20 times larger, and it comes from a term in the nuclear potential itself which is proportional to $\mathbf{L} \cdot \mathbf{S}$, i.e.

$$V(r) \rightarrow V(r) + W(r)\mathbf{L} \cdot \mathbf{S}.$$

As in the case of Atomic Physics (j-j coupling scheme) the orbital and spin angular momenta of the nucleons combine to give a total angular momentum j which can take the values $j = l + \frac{1}{2}$ or $j = l - \frac{1}{2}$. The spin-orbit coupling term leads to an energy shift proportional to

$$j(j+1) - l(l+1) - s(s+1), \quad (s = 1/2).$$

A further feature of this spin-orbit coupling in nuclei is that the energy split is in the opposite sense from its effect in Atomic Physics, namely that states with higher j have *lower energy*.

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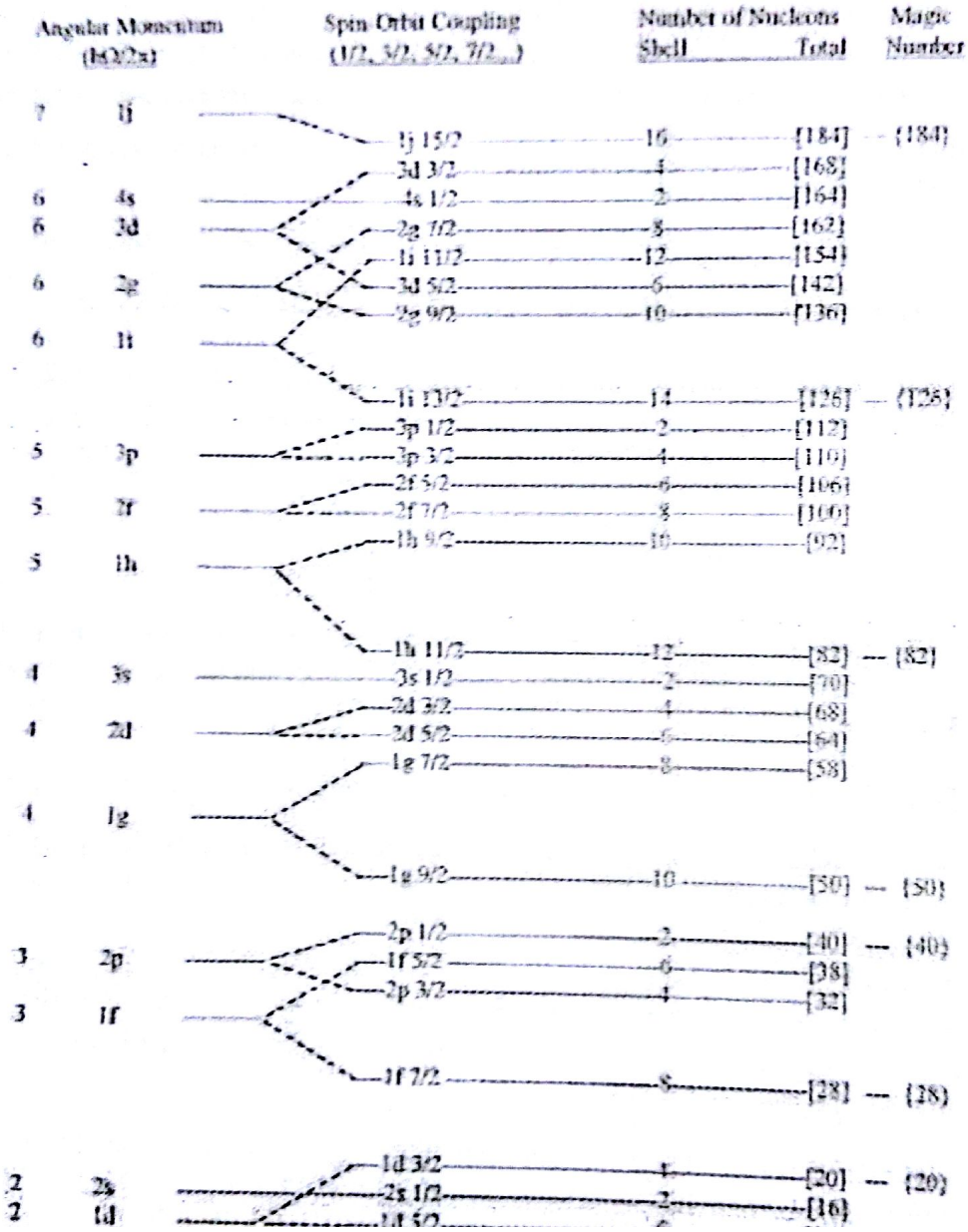
Nuclear Shell Structure



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We see that this large spin-orbit effect leads to crossing over of energy levels into different shells. For example the state above the $2p$ state is $1g$ (l=4), which splits into $1g_{\frac{9}{2}}$, $(j = \frac{9}{2})$ and $1g_{\frac{7}{2}}$, $(j = \frac{7}{2})$. The energy of the $1g_{\frac{9}{2}}$ state is sufficiently low that it joins the shell below, so that this fourth shell now consists of $1f_{\frac{7}{2}}, 2p_{\frac{3}{2}}$, $1f_{\frac{5}{2}}$, $2p_{\frac{1}{2}}$ and $1g_{\frac{9}{2}}$. The maximum occupancy of this state ($(2j+1)$ protons (neutrons) for each j) is now 8+4+6+2+10=30, which added to the previous magic number, 20, gives the next observed magic number of 50.

Further up, it is the $1h$ state that undergoes a large splitting into $1h_{\frac{11}{2}}$ and $1h_{\frac{9}{2}}$, with the $1h_{\frac{11}{2}}$ state joining the lower shell.

5.3 Spin and Parity of Nuclear Ground States.

Nuclear states have an intrinsic spin and a well defined parity, $\eta = \pm 1$, defined by the behaviour of the wavefunction for all the nucleons under reversal of their coordinates with the centre of the nucleus at the origin.

$$\Psi(-\mathrm{r}_1, -\mathrm{r}_2 \cdots - \mathrm{r}_A) = \eta\Psi(\mathrm{r}_1, \mathrm{r}_2 \cdots \mathrm{r}_A)$$

The spin and parity of nuclear ground states can usually be determined from the shell model. Protons and neutrons tend to pair up so that the spin of each pair is zero and each pair has even parity ($\eta = 1$). Thus we have

- Even-even nuclides (both Z and A even) have zero intrinsic spin and even parity.
- Odd A nuclei have one unpaired nucleon. The spin of the nucleus is equal to the j-value of that unpaired nucleon and the parity is $(-1)^l$, where l is the orbital angular momentum of the unpaired nucleon.

  For example $^{47}_{22}$Ti (titanium) has an even number of protons and 25 neutrons. 20 of the neutrons fill the shells up to magic number 20 and there are 5 in the $1f_{\frac{7}{2}}$ state ($l = 3$, $j = \frac{7}{2}$) Four of these form pairs and the remaining one leads to a nuclear spin of $\frac{7}{2}$ and parity $(-1)^3 = -1$.
- Odd-odd nuclei. In this case there is an unpaired proton whose total angular momentum is j_1 and an unpaired neutron whose total angular momentum is j_2. The total spin of the nucleus is the (vector) sum of these angular momenta and can take values between $|j_1 - j_2|$ and $|j_1 + j_2|$ (in unit steps). The parity is given by $(-1)^{(l_1+l_2)}$, where l_1 and l_2 are the orbital angular momenta of the unpaired proton and neutron respectively.

  For example $^6_3$Li (lithium) has 3 neutrons and 3 protons. The first two of each fill the $1s$ level and the thrid is in the $1p_{\frac{3}{2}}$ level. The orbital angular mometum of each is $l = 1$ so the parity is $(-1) \times (-1) = +1$ (even), but the spin can be anywhere between 0 and 3.

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5.4 Magnetic Dipole Moments

Since nuclei with an odd number of protons and/or neutrons have intrinsic spin they also in general possess a magnetic dipole moment.

The unit of magnetic dipole moment for a nucleus is the "nuclear magneton" defined as

$$\mu_N = \frac{eh}{2m_p},$$

which is analogous to the Bohr magneton but with the electron mass replaced by the proton mass. It is defined such that the magnetic moment due to a proton with orbital angular momentum $\mathbf{l}$ is $\mu_N \mathbf{l}$.

Experimentally it is found that the magnetic moment of the proton (due to its spin) is

$$\mu_p = 2.79\mu_N = 5.58\mu_N s, \quad \left(s = \frac{1}{2}\right)$$

and that of the neutron is

$$\mu_n = -1.91\mu_N = -3.82\mu_N s, \quad \left(s = \frac{1}{2}\right)$$

If we apply a magnetic field in the z-direction to a nucleus then the unpaired proton with orbital angular momentum $\mathbf{l}$, spin $\mathbf{s}$ and total angular momentum $\mathbf{j}$ will give a contribution to the $z-$ component of the magnetic moment

$$\mu^z = (5.58s^z + l^z)\,\mu_N.$$

As in the case of the Zeeman effect, the vector model may be used to express this as

$$\mu^z = \frac{(5.58 < \mathbf{s}\cdot\mathbf{j} > + < \mathbf{l}\cdot\mathbf{j} >)}{< \mathbf{j}^2 >} j^z\,\mu_N$$

using

$$\begin{aligned}
< \mathbf{j}^2 > &= j(j+1)\hbar^2 \\
< \mathbf{s}\cdot\mathbf{j} > &= \frac{1}{2}\left(< \mathbf{j}^2 > + < \mathbf{s}^2 > - < \mathbf{l}^2 >\right) \\
&= \frac{\hbar^2}{2}\left(j(j+1) + s(s+1) - l(l+1)\right) \\
< \mathbf{l}\cdot\mathbf{j} > &= \frac{1}{2}\left(< \mathbf{j}^2 > + < \mathbf{l}^2 > - < \mathbf{s}^2 >\right) \\
&= \frac{\hbar^2}{2}\left(j(j+1) + l(l+1) - s(s+1)\right)
\end{aligned} \tag{5.4.1}$$

We end up with expression for the contribution to the magnetic moment

$$\mu = \frac{5.58\,(j(j+1) + s(s+1) - l(l+1)) + (j(j+1) + l(l+1) - s(s+1))}{2j(j+1)}\,j\,\mu_N$$

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and for a neutron with orbital angular momentum l' and total angular momentum j' we get (not contribution from the orbital angular momentum because the neutron is uncharged)

$$\mu = -\frac{3.82\,(j'(j'+1) + s(s+1) - l'(l'+1))}{2j'(j'+1)}\,j'\,\mu_N$$

Thus, for example if we consider the nuclide ${}^{7}_{3}\mathrm{Li}$ for which there is an unpaired proton in the $2p_{\frac{3}{2}}$ state ($l = 1,\ j = \frac{3}{2}$ then the estimate of the magnetic moment is

$$\mu = \frac{5.58\left(\frac{3}{2}\times\frac{5}{2} + \frac{1}{2}\times\frac{3}{2} - 1\times 2\right) + \left(\frac{3}{2}\times\frac{5}{2} + 1\times 2 - \frac{1}{2}\times\frac{3}{2}\right)\frac{3}{2}}{2\times\frac{3}{2}\times\frac{5}{2}} = 3.79\mu_N$$

The measured value is $3.26\mu_N$ so the estimate is not too good. For heavier nuclei the estimate from the shell model gets much worse.

The precise origin of the magnetic dipole moment is not understood, but in general they cannot be predicted from the shell model. For example for the nuclide ${}^{17}_{9}\mathrm{F}$ (fluorine), the measured value of the magnetic moment is $4.72\mu_N$ whereas the value predicted form the above model is $-0.26\mu_N$. !! There are contributions to the magnetic moments from the nuclear potential that is not well-understood.

5.5 Excited States

As in the case of Atomic Physics, nuclei can be in excited states, which decay via the emission of a photon (γ-ray) back to their ground state (either directly ore indirectly).

Some of these excited states are states in which one of the neutrons or protons in the outer shell is promoted to a higher energy level.

However, unlike Atomic Physics, it is also possible that sometimes it is energetically cheaper to promote a nucleon from an inner closed shell, rather than a nucleon form an outer shell into a high energy state. Moreover, excited states in which more than one nucleon is promoted above its ground state is much more common in Nuclear Physics than in Atomic Physics.

Thus the nuclear spectrum of states is very rich indeed, but very complicated and cannot be easily understood in terms of the shell model.

Most of the excited states decay so rapidly that their lifetimes cannot be measured. There are some excited states, however, which are metastable because they cannot decay without violating the selection rules. These excited states are known as "isomers", and their lifetimes can be measured.

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5.6 The Collective Model

The Shell Model has its shortcomings. This is particularly true for heavier nuclei. We have already seen that the Shell Model does not predict magnetic dipole moments or the spectra of excited states very well.

One further failing of the Shell Model are the predictions of electric quadrupole moments, which in the Shell Model are predicted to be very small. However, heavier nuclei with A in the range 150 - 190 and for A > 220, these electric quadrupole moments are found to be rather large.

The failure of the Shell Model to correctly predict electric quadrupole moments arises from the assumption that the nucleons move in a spherically symmetric potential.

The Collective Model generalises the result of the Shell Model by considering the effect of a non-spherically symmetric potential, which leads to substantial deformations for large nuclei and consequently large electric quadrupole moments.

One of the most striking consequences of the Collective Model is the explanation of low-lying excited states of heavy nuclei. These are of two types

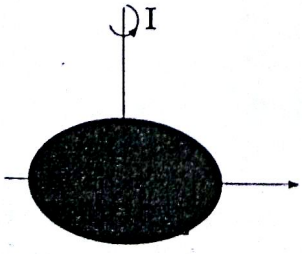
- **Rotational States:** A nucleus whose nucleon density distributions are spherically symmetric (zero quadrupole moment) cannot have rotational excitations (this is analogous to the application of the principle of equipartition of energy to monatomic molecules for which there are no degrees of freedom associated with rotation).

  On the other hand a nucleus with a non-zero quadrupole moment can have excited levels due to rotational perpendicular to the axis of symmetry.

  For an even-even nucleus whose ground state has zero spin, these states have energies

$$E_{\text{rot}} = \frac{I(I+1)\,\hbar^2}{2\mathcal{I}}, \tag{5.6.2}$$

  where $\mathcal{I}$ is the moment of inertia of the nucleus about an axis through the centre perpendicular to the axis of symmetry.



It turns out that the rotational energy levels of an even-even nucleus can only take even values of I. For example the nuclide $^{170}_{72}$Hf (hafnium) has a series of rotational states with excitation energies

E (KeV) : 100, 321, 641

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These are almost exactly in the ratio $2 \times 3 : 4 \times 5 : 6 \times 7$, meaning that these are states with rotational spin equal to 2, 4, 6 respectively. The relation is not exact because the moment of inertia changes as the spin increases.

We can extract the moment of inertia for each of these rotational states from eq.(5.6.2). We could express this in SI units, but more conveniently nuclear moments of inertia are quoted in $\mathrm{MeV}/c^2\,\mathrm{fm}^2$, with the help of the relation

$$\hbar c = 197.3\,\mathrm{MeV\,fm}.$$

Therefore the moment inertia of the $I = 2$ state, whose excitation energy is 0.1 MeV, is given (inseting $I = 2$ into eq.(5.6.2) by

$$\mathcal{I} = 2 \times 3 \times \frac{\hbar^2 c^2}{2c^2 E_{\mathrm{rot}}} = \frac{6}{2}\frac{197.3^2}{0.1} = 1.17 \times 10^6\,\mathrm{MeV}/c^2\,\mathrm{fm}^2$$

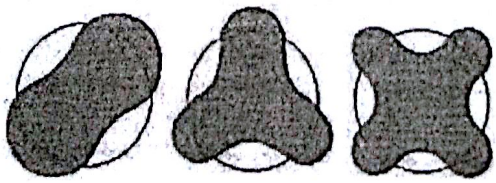
For odd-A nuclides for which the spin of the ground state I_0 is non-zero, the rotational levels have excitation levels of

$$E_{\mathrm{rot}} = \frac{1}{2\mathcal{I}}\left(I(I+1) - I_0(I_0+1)\right)\hbar^2,$$

where I can take the values $I_0 + 1$, $I_0 + 2$ etc. For example the first two rotational excitation energies of $^{143}_{60}$Nd (neodynium), whose ground state has spin $\frac{7}{2}$, have energies 128 KeV and 290 KeV. They correspond to rotational levels with nuclear spin $\frac{9}{2}$ and $\frac{11}{2}$ respectively. The ratio of these two excitation energies (2.27) is almost exactly equal to

$$\frac{\frac{11}{2} \times \frac{13}{2} - \frac{7}{2} \times \frac{9}{2}}{\frac{9}{2} \times \frac{11}{2} - \frac{7}{2} \times \frac{9}{2}} = 2.22$$

- **Shape oscillations:** These are modes of vibration in which the deformation of the nucleus oscillates - the electric quadrupole moment oscillates about its mean value. It could be that this mean value is very small, in which case the nucleus is oscillating between an oblate and a prolate spheroidal shape. It is also possible to have shape oscillations with different shapes



The small oscillations about the equilibrium shape perform simple harmonic motion. The energy levels of such modes are equally spaced. Thus an observed sequence of equally spaced energy levels within the spectrum of a nuclide is interpreted as a manifestation of such shape oscillations.

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Concepts & Skills to Review

- Rutherford scattering experiment; discovery of the nucleus (Section 27.6)
- fundamental forces (Section 4.12)
- mass and rest energy (Section 26.7)
- exclusion principle (Section 28.7)
- exponential functions (Appendix A.3, Section 18.10)
- tunneling (Section 28.10)

Mass of proton & nutron.
m = 1.672 x 10^-27 kg
me = 9.1 x 10^-31 kg

29.1 NUCLEAR STRUCTURE

In an atom, electrons are bound electrically to a positively charged nucleus. In Chapters 27 and 28, we generally treated the nucleus as a point charge so massive that it is not affected by electrical forces on it due to the electrons. In reality, the atomic nucleus is several thousand times more massive than the electrons in an atom and occupies only a tiny fraction of the atom's volume (about 1 part in 10^{12} or less). The finite mass and volume of the nucleus have subtle effects on the electronic configuration and thus on the chemical properties of atoms. However, the nucleus has a complex structure of its own that manifests itself in radioactive decay and nuclear reactions.

The nucleus is a bound collection of protons and neutrons. Together, protons and neutrons are referred to as **nucleons** (particles found inside the nucleus). The **atomic number** Z is the number of protons in the nucleus. Each proton has a charge of $+e$ and the neutron is uncharged, so the electrical charge of a nucleus is $+Ze$. The number of electrons in a neutral atom is also equal to Z. The number of protons determines to which element, or chemical species, an atom belongs.

Once it was thought that all atoms of a given element were identical. However, we now know that there exist different isotopes of a given element. The isotopes of an element all have the same number of protons in the nucleus, but they have different masses because the number of neutrons (N) differs. The total number of nucleons therefore also differs from one isotope to another. The **nucleon number** A is the total number of protons and neutrons:

$$A = Z + N \tag{29-1}$$

Any particular species of nucleus, called a **nuclide**, is characterized by the values of A and Z. The nucleon number A is also called the **mass number**. Since almost all of the mass of an atom is found in the nucleus, and since protons and neutrons have *approximately* the same mass, the mass of an atom is roughly proportional to the number of nucleons.

Since their masses differ, the isotopes of an element can be separated using a mass spectrometer (Section 19.3). Sometimes the differing masses of isotopes have an effect on chemical reaction rates, but on the whole, the chemical properties of different isotopes are virtually identical. On the other hand, different nuclides have *very* different nuclear properties. The number of neutrons present affects how strongly the nucleus is held together, so that some are stable while others are unstable (**radioactive**). Nuclear energy levels, radioactive half-lives, and radioactive decay modes are all particular to a specific nuclide; they are very different for two isotopes of the same element.

Several notations are used to distinguish nuclides. The chemical symbol O stands for the element oxygen. To specify a particular isotope of oxygen, the mass number must also be specified. Oxygen-18, O-18, O^{18}, and ^{18}O all stand for the isotope of oxygen with $A = 18$. Sometimes it is helpful to include the atomic number as well, even though it is redundant; oxygen by definition has 8 protons. When including the atomic number, the preferred form is $^{18}_{8}O$, although $_{8}O^{18}$ is found in some older texts.

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Example 29.1

Finding the Number of Neutrons

How many neutrons are present in an ^{18}O nucleus?

Strategy The superscript gives the number of nucleons (A). We consult the Periodic Table to find the atomic number (Z) for oxygen. The number of neutrons is $N = A - Z$.

Solution An ^{18}O nucleus has 18 nucleons. Oxygen has atomic number 8, so there are 8 protons in the nucleus. That leaves $18 - 8 = 10$ neutrons.

Discussion Different isotopes of oxygen have different numbers of neutrons but the same number of protons.

Practice Problem 29.1 Identifying the Element

Write the symbol (in the form $^{A}_{Z}X$) for the nuclide with 44 protons and 60 neutrons and identify the element.

It is usually more convenient to write the mass of a nucleus in **atomic mass units** instead of kg. The modern symbol for the atomic mass unit is "u"; in older literature it is often written "amu." The atomic mass unit is defined as exactly $\frac{1}{12}$ the mass of a neutral ^{12}C atom. The conversion factor between u and kg is

$$1\ \text{u} = 1.660\,539 \times 10^{-27}\ \text{kg} \qquad (29\text{-}2)$$

Nucleons have masses of *approximately* 1 u, while the electron is much less massive (Table 29.1). Therefore, the mass of a nucleus (or an atom) is *approximately* A atomic mass units—which is why A is called the mass number.

Table 29.1

Masses and Charges of the Proton, Neutron, and Electron

Particle	Mass (u)	Charge
Proton	1.007 276 5	$+e$
Neutron	1.008 664 9	0
Electron	0.000 548 6	$-e$

$\frac{m_p}{m_e} = 1836$

$m_n = 1838$

Example 29.2

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Sizes of Nuclei

How do we know the *sizes* of nuclei? The first experimental evidence came from the Rutherford scattering of alpha particles from gold nuclei (Section 27.6). From analysis of the number of alpha particles observed at different scattering angles, we can estimate the size of the gold nucleus. Similar experiments were performed on other nuclei. More recently, electron diffraction has been used to probe the structure of the nucleus. Using electrons of very short wavelength, we can determine not only the size of the nucleus but learn about its internal structure as well.

These and other experiments show that the mass density of all nuclei is approximately the same—the volume of a nucleus is proportional to its mass. Imagine a nucleus to be like a spherical container full of marbles (Fig. 29.1); each marble represents a nucleon. The nucleons are tightly packed together, as if touching each other. Both the mass and volume of the nucleus are proportional to the number of nucleons, so the mass per unit volume (density ρ) is approximately independent of the number of nucleons. If m is the mass of a nucleus, V is its volume, and A is its mass number, then

$$m \propto A \quad \text{and} \quad V \propto A$$

$$\Rightarrow \rho = \frac{m}{V} \text{ is independent of } A$$

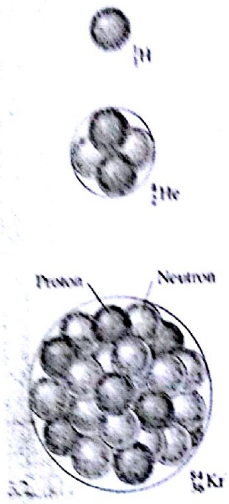

Figure 29.1 Simplified model of the nucleus as a set of hard spheres (representing the nucleons) packed together into a sphere.

Most nuclei are approximately spherical in shape, so

$$V = \frac{4}{3}\pi r^3 \propto A$$

$$\Rightarrow r^3 \propto A \quad \text{or} \quad r \propto A^{1/3} \tag{29-3}$$

The radius of a nucleus is proportional to the cube root of its mass number. Experiment shows that the constant of proportionality is approximately 1.2×10^{-15} m:

Radius of a nucleus:

$$r = r_0 A^{1/3} \tag{29-4}$$

$$r_0 = 1.2 \times 10^{-15}\ \text{m} = 1.2\ \text{fm} \tag{29-5}$$

The SI prefix "f-" stands for *femto*; the fm is properly called a *femtometer* but is also called a *fermi*, after Enrico Fermi (1901–1954). The nuclear radius ranges from 1.2 fm (for $A = 1$) to 7.7 fm (for $A \approx 260$).

Although nuclei all have about the same mass density, *atoms* do not. More massive atoms are generally more dense than lighter atoms. The increase in volume of an atom does not keep pace with the increase in mass. Although larger atoms have more electrons, these electrons are on average more tightly bound, due to the increased charge of the nucleus. Thus, some solids and liquids (in which the atoms are tightly packed) are more dense than others.

Example 29.3
Radius and Volume of Barium Nucleus

What are the radius and volume of the barium-138 nucleus?

Strategy To find the radius of a nucleus, all we need to know is the mass number A, which in this case is 138. To find the volume, we assume that the nucleus is approximately spherical.

Solution To find the radius we apply Eq. (29-4), substituting $A = 138$:

$$r = r_0 A^{1/3}$$

$$r = 1.2\ \text{fm} \times 138^{1/3} = 6.2\ \text{fm}$$

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Mass defect and binding energy:

$$\Delta m = (\text{mass of } Z \text{ protons and } N \text{ neutrons}) - (\text{mass of nucleus}) \quad (29\text{-}7)$$

$$E_B = (\Delta m)c^2 \quad (29\text{-}8)$$

The energy unit most commonly used in nuclear physics is the MeV. When using MeV for energy and atomic mass units for mass in Eq. (29-8), it is convenient to know the value of c^2 in units of MeV/u. It can be shown (Problem 16) that

$$c^2 = 931.494 \text{ MeV/u} \quad (29\text{-}9)$$

Mass tables such as Appendix B give the masses of *neutral atoms*, which include the masses of the electrons as well as the mass of the nucleus. To find the mass of a nucleus with atomic number Z, subtract the mass of Z electrons from the mass of the neutral atom. The binding energy of the electrons to the nucleus is much smaller than the rest energy of the electrons and can be ignored.

Example 29.4

Binding Energy of a Nitrogen-14 Nucleus

Find the binding energy of the ^{14}N nucleus.

Strategy From Appendix B, the mass of the ^{14}N *atom* is 14.003 074 0 u. The mass of the N atom includes the mass of seven electrons. Subtracting $7m_e$ from the mass of the atom gives the mass of the nucleus. Then we can find the mass defect and the binding energy.

Solution

$$\text{mass of } ^{14}\text{N nucleus} = 14.003\,074\,0 \text{ u} - 7m_e$$
$$= 14.003\,074\,0 \text{ u} - 7 \times 0.000\,548\,6 \text{ u}$$
$$= 13.999\,233\,8 \text{ u}$$

The ^{14}N nucleus has seven protons and seven neutrons. The mass defect is

$$\Delta m = (\text{mass of 7 protons and 7 neutrons}) - (\text{mass of nucleus})$$
$$= 7 \times 1.007\,276\,5 \text{ u} + 7 \times 1.008\,664\,9 \text{ u} - 13.999\,233\,8 \text{ u}$$
$$= 0.112\,356\,0 \text{ u}$$

The binding energy is therefore,

$$E_B = (\Delta m)c^2 = 0.112\,356\,0 \text{ u} \times 931.494 \text{ MeV/u}$$
$$= 104.659 \text{ MeV}$$

Discussion Since the binding energy of the electrons in an atom is so small, we can assume that the mass of an atom is equal to the mass of its nucleus plus the mass of the electrons. As a shortcut, we can calculate the mass defect using the mass of the nitrogen *atom* instead of the nitrogen nucleus and the mass of the *hydrogen atom* instead of the proton. Since each term contains the extra mass of seven electrons, the masses of the electrons subtract out:

$$\Delta m = (\text{mass of 7 } ^1\text{H atoms and 7 neutrons}) - (\text{mass of } ^{14}\text{N atom})$$
$$= 7 \times 1.007\,825\,0 \text{ u} + 7 \times 1.008\,664\,9 \text{ u} - 14.003\,074\,0 \text{ u}$$
$$= 0.112\,355\,3 \text{ u}$$

Practice Problem 29.4 Binding Energy of Nitrogen-15

Calculate the binding energy of the ^{15}N nucleus. The mass of the ^{15}N *nucleus* is 14.996 269 u. [*Hint:* This time you have been given the mass of the *nucleus*, not the mass of the atom.]

Binding Energy Curve

Figure 29.2 shows a graph of the binding energy *per nucleon* as a function of mass number. Recall that the strong force binds nucleons only to their nearest neighbors. In small nuclides there are not enough nucleons for all to fully bind since the average number of nearest neighbors is small. Increasing the number of nucleons leads to a larger binding energy per nucleon, up to a point, because the average number of nearest neighbors is increasing. Thus, we see a steep increase in the binding energy per nucleon as A increases.

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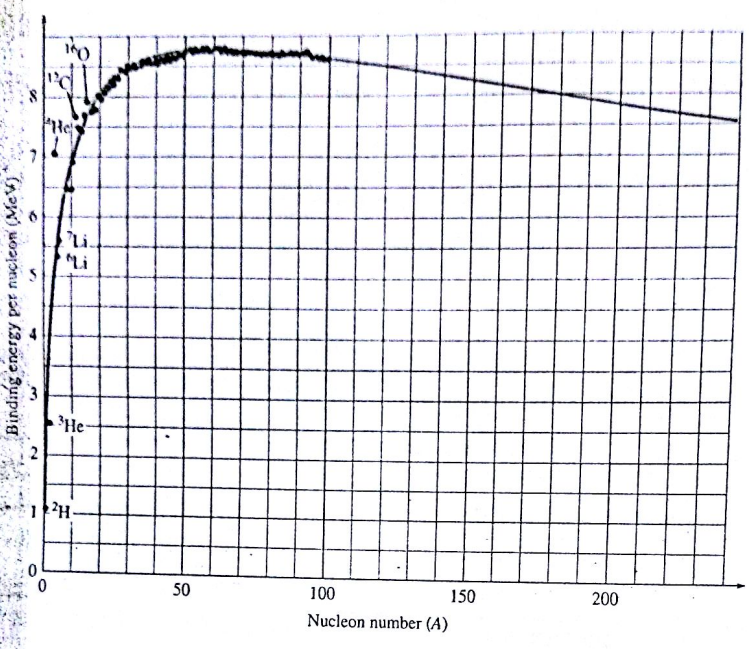

Figure 29.2 Binding energy per nucleon (E_B/A) for the most stable nuclide with nucleon number A. Individual data points are shown for $A < 100$; a smooth curve showing the general trend is shown in red. (Data points are omitted for $A \geq 100$ since they differ little from the values given by the red curve.) $^{62}_{28}$Ni has the largest binding energy per nucleon of all the nuclides (8.795 MeV), followed by $^{58}_{26}$Fe and $^{56}_{26}$Fe (8.792 MeV and 8.790 MeV, respectively). Data points for $^{4}_{2}$He, $^{12}_{6}$C, and $^{16}_{8}$O lie significantly above the red curve—these nuclides are particularly stable compared to nuclides with similar values of A.

Once nuclei reach a certain size, all nucleons except those on the surface have as many nearest neighbors as possible. Adding more nucleons doesn't increase the average binding energy per nucleon due to the strong force very much, but the Coulomb repulsion keeps adding up since it is long range. Thus, above $A \approx 60$, adding more nucleons *decreases* the average binding energy per nucleon. The decrease is relatively gentle, compared to the steep increase for small A, since the Coulomb repulsion is weak compared to the strong force.

The binding energy per nucleon is within the range 7–9 MeV for all but the smallest nuclides. For example, in Example 29.4 we found that the binding energy of ^{14}N is 104.659 MeV. The binding energy *per nucleon* for ^{14}N is

$$\frac{104.659 \text{ MeV}}{14 \text{ nucleons}} = 7.475\,64 \text{ MeV/nucleon}$$

The most tightly bound nuclides are around $A \approx 60$, where the binding energy is about 8.8 MeV/nucleon.

Nuclear Energy Levels

Neutrons and protons obey the Pauli exclusion principle: no two identical nucleons in the same nucleus can be in the same quantum state. As for atomic energy levels, a group of closely spaced nuclear energy levels is called a *shell*. We can describe the quantum state of the nucleus by specifying how the proton and neutron states are occupied, much as we did for electron states in the atom (Fig. 29.3). Two protons can occupy each proton energy level (one spin up, one spin down) and two neutrons can occupy each neutron energy level. The energy levels for the proton and neutron are similar; as far as the nuclear force is concerned, protons and neutrons are pretty much the same. The main difference is that the protons are affected by the Coulomb repulsion in addition to the strong force.

In Problem 20, an order of magnitude calculation shows that the energy level spacings in nuclei are expected to be in the MeV range. The structure of the nucleus is complex; energy level spacings range from tens of keV to several MeV. A nucleus in an

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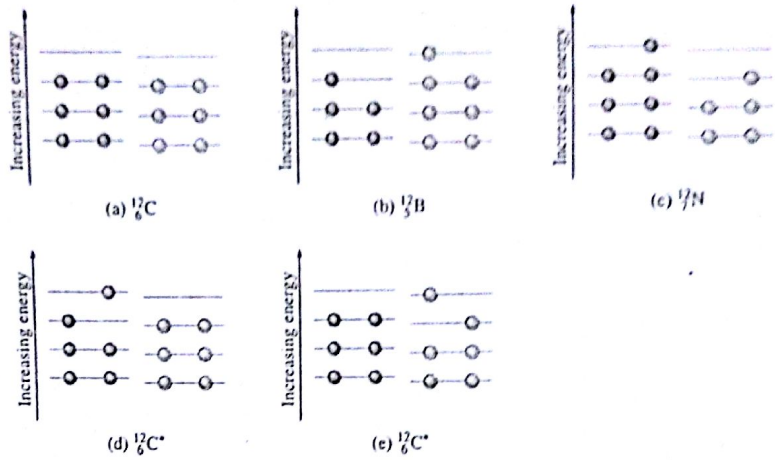

Figure 29.3 Qualitative energy level diagrams for some nuclides with $A = 12$. Red spheres represent protons and gray spheres represent neutrons. Compare the *atomic* energy level diagram in Figure 28.19. $^{12}_{6}C$ is stable, while $^{12}_{5}B$ and $^{12}_{7}N$ are unstable. The asterisks in (d) and (e) indicate that $^{12}_{6}C^*$ is in an excited state. $^{12}_{6}C^*$ can return to the ground state ($^{12}_{6}C$) by emitting a photon whose energy equals the difference in the energy levels. $^{12}_{5}B$ and $^{12}_{7}N$ can emit an electron or positron, respectively, to change into $^{12}_{6}C$ (see Beta decay, Section 29.3).

excited state can return to the ground state by emitting one or more *gamma-ray* photons. [The distinction between gamma rays and x-rays is based more on the source than the energy. A photon emitted by an excited nucleus or in pair annihilation (Section 27.8) is called a gamma ray; a high-energy photon emitted by an excited *atom*, by an electron slowing down on striking a target (Section 27.4), or by a circulating charged particle in a synchrotron is usually called an x-ray.] Just as the energy levels of atoms can be deduced by measuring the wavelengths of photons radiated by excited atoms, measurement of the gamma-ray energies emitted by excited nuclei enables us to deduce the nuclear energy levels. Each nuclide has its own characteristic gamma-ray spectrum, which can be used to identify it. A gamma-ray spectrum usually identifies the energy of the photons, in contrast to a visible spectrum where the wavelength is usually specified. In both cases, the quantity used is the one that is easier to measure.

Energy level diagrams help explain why, in stable light nuclides, the number of neutrons and protons tends to be approximately equal. Figure 29.3 shows energy level diagrams for three different nuclides, each of which has 12 nucleons. The energy levels are *not* quantitatively correct, but serve to illustrate the general idea. A maximum of two protons can be in any proton energy level and a maximum of two neutrons can be in any neutron energy level. The proton and neutron energy levels are similar; the proton levels are slightly higher in energy than the neutron levels to account for the Coulomb repulsion between the protons. The energy is lower with 6 protons and 6 neutrons than is possible with 5 of one and 7 of the other.

The story is more complicated for heavier nuclides. The Coulomb repulsion between protons favors more neutrons ($N > Z$) since the neutrons are immune to the Coulomb repulsion. For larger nuclides, the Coulomb repulsion becomes more and more important since it is long range: each proton repels *every other proton* in the nucleus. The proton energy levels get higher and higher with respect to the neutron energy levels as the electric potential energy of all those repelling protons adds up. Thus, large nuclides tend to have an excess of neutrons ($N > Z$). On the other hand, there is a limit to the neutron excess: neutrons are slightly more massive than protons, so if there is too much of a neutron excess, the mass (and therefore the energy) of the nucleus

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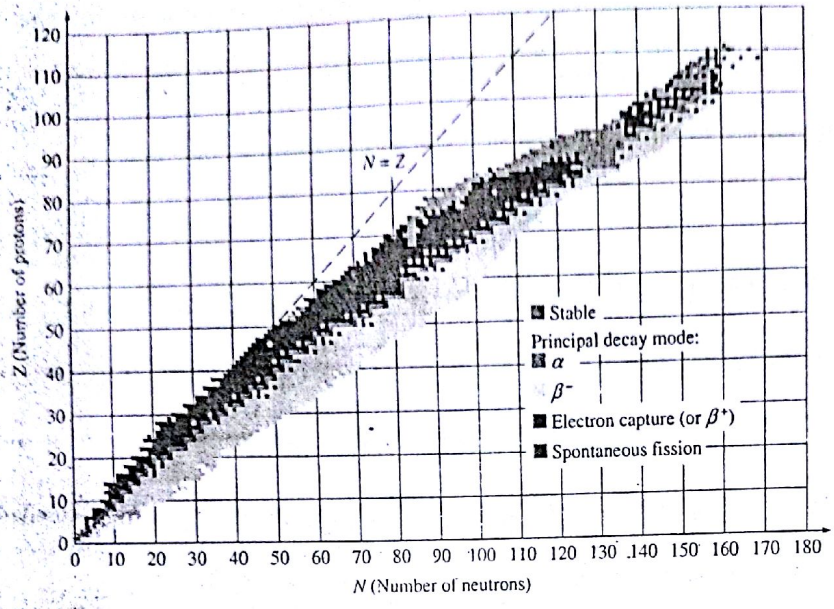

Figure 29.4 Chart of the most common nuclides. Stable nuclides are shown as green points. Note the general trend of increasing N/Z ratio for stable nuclides.

29.3 RADIOACTIVITY

Henri Becquerel (1852–1908) discovered radioactivity in 1896 when, quite by accident, he found that a uranium salt spontaneously emitted radiation in the absence of an external source of energy, such as sunlight. The radiation exposed a photographic plate even though the plate was wrapped in black paper to keep light out.

Nuclides can be divided into two broad categories. Some are stable; others are unstable or **radioactive.** An unstable nuclide **decays**—takes part in a spontaneous nuclear reaction—by emitting radiation. (The radiation may include but is not limited to *electromagnetic* radiation.) Depending on the kind of radiation emitted, the reaction may change the nucleus into a different nuclide, with a different charge or nucleon number or both.

Scientists studying radioactivity soon identified three different kinds of radiation emitted by radioactive nuclei; they were named alpha rays, beta rays, and gamma rays after the first three letters of the Greek alphabet. The initial distinction between the three was their differing abilities to penetrate matter (Fig. 29.5). Alpha rays are the least penetrating; they can only make it through a few centimeters of air and are completely blocked by human skin, thin paper, and other solids. Beta rays can travel farther in air—about a meter typically—and can penetrate a hand or a thin metal foil. Gamma rays are much more penetrating than either alpha or beta rays. Later, the electrical charge and mass were determined and used to distinguish the three types of radiation—and ultimately to identify them.

Of the approximately 1500 known nuclides, only about 20% are stable. All of the largest nuclides (those with $Z > 83$) are radioactive. As far as we know, stable nuclei last

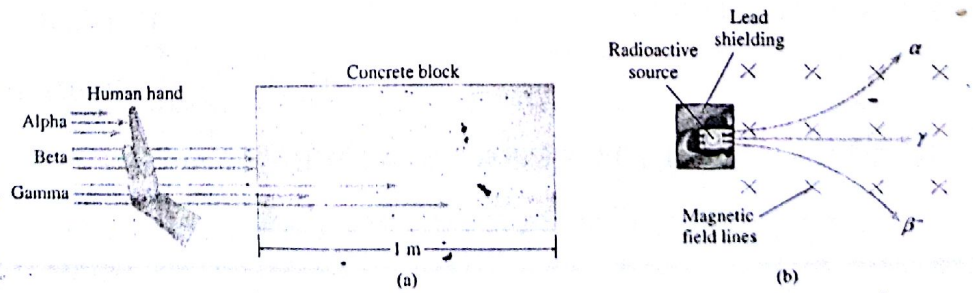

Figure 29.5 Alpha, beta, and gamma rays differ (a) in their abilities to penetrate matter as well as (b) in their electrical charges.

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forever without decaying spontaneously. Each radioactive nuclide decays with an average lifetime characteristic of that nuclide. The lifetimes cover an enormous range, from about 10^{-22} s (roughly the time it takes light to travel a distance equal to the diameter of a nucleus) to 10^{+28} s (10^{10} times the age of the universe).

Conservation Laws in Radioactive Decay

In a nuclear reaction, whether spontaneous or not, the total electric charge is conserved. Another conservation law says that the total number of nucleons must stay the same. We *balance* a nuclear reaction by applying these two conservation laws. It is helpful to write symbols for electrons, positrons, and neutrons as if they were nuclei, with a superscript for the number of nucleons and a subscript for the electrical charge in units of e (see Table 29.2). Then the reaction is balanced with regard to nucleon number if the sum of the superscripts is the same on both sides; it is balanced with regard to charge if the sum of the subscripts is the same on both sides.

Another conservation law is important in radioactive decay: all nuclear reactions also conserve energy. How can a nucleus with little or no kinetic energy decay, leaving products with significant kinetic energies? Where did this energy come from? In a spontaneous nuclear reaction, some of the rest energy of the radioactive nucleus is converted into kinetic energy of the products. The amount of rest energy that is converted into other forms of energy is called the **disintegration energy**. In order for kinetic energy to increase, there must be a corresponding decrease in rest energy. The total mass of the products must be less than the mass of the original radioactive nucleus in order for that nucleus to decay spontaneously. In other words, the products must be more tightly bound than is the original nucleus; the disintegration energy is the difference between the binding energy of the radioactive nucleus and the total binding energy of the products.

Disintegration energy = binding energy of radioactive nucleus − total binding energy of products

Alpha Decay

Alpha "rays" are now known to be ^{4}He nuclei. The helium nucleus is a group of two protons and two neutrons and it is very tightly bound. The mass of an alpha particle is 4.001 506 u and its charge is $+2e$.

In alpha decay, the original (*parent*) nuclide is converted to a "*daughter*" by the emission of an alpha particle. Balancing the reaction shows that the daughter nuclide has a nucleon number reduced by four and a charge reduced by two. Using P for the parent nuclide and D for the daughter nuclide, the spontaneous reaction in which an alpha particle is emitted is

Alpha decay:

$$^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D + {}^{4}_{2}\alpha \qquad \textbf{(29-10)}$$

Table 29.2

Particles Commonly Involved in Radioactive Decay and Other Nuclear Reactions

Particle Name	Symbols	Charge (in units of e)	Nucleon Number
Electron	$e^-, \beta^-, {}^{0}_{-1}e$	−1	0
Positron	$e^+, \beta^+, {}^{0}_{1}e$	+1	0
Proton	$p, {}^{1}_{1}p, {}^{1}_{1}H$	+1	1
Neutron	$n, {}^{1}_{0}n$	0	1
Alpha particle	$\alpha, {}^{4}_{2}\alpha, {}^{4}_{2}He$	+2	4
Photon	$\gamma, {}^{0}_{0}\gamma$	0	0
Neutrino	$\nu, {}^{0}_{0}\nu$	0	0
Antineutrino	$\bar{\nu}, {}^{0}_{0}\nu$	0	0

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Polonium-210 decays via alpha decay. Identify the daughter nuclide.

Strategy First we look up the atomic number of polonium in the Periodic Table (inside back cover). Next we write the nuclear reaction with an unknown nuclide and an alpha particle as the products. Balancing the reaction gives us the values of Z and A of the daughter nucleus.

Solution Polonium is atomic number 84. Then the reaction is

$$^{210}_{84}\text{Po} \rightarrow {}^{A}_{Z}(?) + {}^{4}_{2}\alpha$$

where A and Z are the nucleon number and atomic number of the daughter nucleus. To conserve charge,

$$84 = Z + 2$$

Thus, $Z = 82$. To conserve nucleon number,

$$210 = A + 4$$

and $A = 206$. Looking up atomic number 82 in the Periodic Table reveals that the element is lead. Thus, the daughter nucleus is lead-206 ($^{206}_{82}\text{Pb}$).

Discussion Writing out the reaction makes it easy to check that the total number of nucleons and the total electric charge are both conserved by the reaction:

$$^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\alpha$$

Practice Problem 29.5 Finding the Parent Given the Daughter

Radon-222, the radioactive gas that poses a significant health risk in some areas, is itself produced by the alpha decay of another nuclide. Identify its parent nuclide.

In alpha decay, the disintegration energy released is shared between the daughter nucleus and the alpha particle. Momentum conservation determines exactly how the energy is shared. Therefore, the alpha particles released in a particular radioactive decay have a characteristic energy (assuming that the initial kinetic energy of the parent is insignificant and can be taken to be zero).

Emission of an alpha particle is the most common type of radioactive decay for large nuclides ($Z > 83$). Since no nuclide with $Z > 83$ is stable, emitting an alpha particle moves toward stability most directly by decreasing both Z and N by 2. Emission of an alpha particle increases the ratio of neutrons to protons. For example, $^{238}_{92}\text{U}$ has a neutron-to-proton ratio of $(238 - 92)/92 = 1.587$. By emitting an alpha particle, $^{238}_{92}\text{U}$ becomes $^{234}_{90}\text{Th}$ with a higher neutron-to-proton ratio: $(234 - 90)/90 = 1.600$. Thus, large nuclides with a smaller neutron-to-proton ratio are more likely to alpha decay than are similar nuclides with a greater neutron-to-proton ratio.

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Example 29.6 Continued

Solution (a) The total mass of the products is

$$234.043\,595\,5\text{ u} + 4.002\,603\,2\text{ u} = 238.046\,198\,7\text{ u}$$

which is less than the mass of the parent nucleus. The change in mass is

$$\Delta m = 238.046\,198\,7\text{ u} - 238.050\,782\,6\text{ u} = -0.004\,583\,9\text{ u}$$

Δm stands for the *change* in mass: final mass minus initial mass. (When we write the mass defect of a nucleus as Δm, we imagine a reaction that separates the nucleus into its constituent protons and neutrons.) The decrease in mass for this reaction means that the rest energy decreases. According to Einstein's mass-energy relation, the change in rest energy is

$$E = (\Delta m)c^2 = -0.004\,583\,9\text{ u} \times 931.494\text{ MeV/u}$$
$$= -4.2699\text{ MeV}$$

By conservation of energy, the kinetic energy of the products is 4.2699 MeV more than the kinetic energy of the parent. The disintegration energy is 4.2699 MeV.

(b) Assuming for the moment that the daughter nucleus and the alpha particle can be treated nonrelativistically, their kinetic energies are related to their momenta by

$$K = \frac{p^2}{2m}$$

Momentum conservation says that their momenta must be equal in magnitude and opposite in direction. Therefore, the ratio of the kinetic energies is

$$\frac{K_\alpha}{K_{\text{Th}}} = \frac{p^2/(2m_\alpha)}{p^2/(2m_{\text{Th}})} = \frac{m_{\text{Th}}}{m_\alpha} = \frac{234.043\,595\,5}{4.002\,603\,2} = 58.4728$$

The two kinetic energies must add up to 4.2699 MeV.

$$K_\alpha + K_{\text{Th}} = 4.2699\text{ MeV}$$

Substituting for K_{Th} from the kinetic energy ratio,

$$K_\alpha + \frac{K_\alpha}{58.4728} = 4.2699\text{ MeV}$$

Solving yields $K_\alpha = 4.1981$ MeV.

Discussion The change in mass is *negative*: the total mass after the decay is less than the mass before. Some of the mass (or, more accurately, rest energy) of the U nucleus is converted into the kinetic energy of the products. The disintegration energy is positive because it is the quantity of energy *released*.

Since the alpha particle's kinetic energy is much smaller than its rest energy (about 4 u × 931.494 MeV/u ≈ 3700 MeV), the nonrelativistic expression for kinetic energy was appropriate. A relativistic calculation shows that our answer is correct to three significant figures.

Practice Problem 29.6 Alpha Energy in the Decay of Polonium-210

Find the kinetic energy of the alpha particle emitted by the decay of ^{210}Po:

$$^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + \alpha$$

Beta Decay

Beta particles are electrons or positrons (sometimes still called beta-minus and beta-plus particles). In beta-minus decay, an electron is emitted and a neutron in the nucleus is converted into a proton. Thus, the mass number does not change, but the charge of the nucleus increases by one:

Beta-minus decay:

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} \qquad \textbf{(29-11)}$$

The positron is the antiparticle of the electron (see Section 27.8); it has the same mass as the electron but a positive charge of $+e$.

The symbol $\bar{\nu}$ represents an antineutrino, an uncharged particle with negligible mass. In beta-plus decay, a positron is emitted and a proton in the nucleus is converted into a neutron. This time the charge of the nucleus decreases by one:

Beta-plus decay:

$$^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + {}^{0}_{+1}e + {}^{0}_{0}\nu \qquad \textbf{(29-12)}$$

The symbol $^{0}_{+1}e$ represents the emitted positron and ν is a neutrino with no charge and negligible mass. Before long, the positron will run into one electron and the pair will be annihilated producing a pair of photons (Section 27.8).

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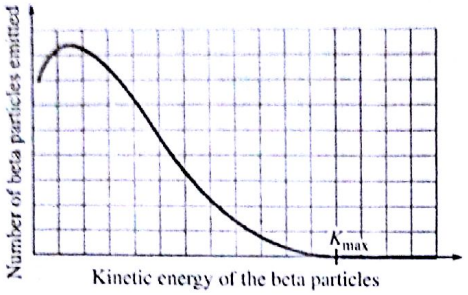

Figure 29.6 Typical continuous energy spectrum of electrons emitted in beta decay from a particular nuclide.

Unlike alpha decay, beta decay of a radionuclide does not change the number of nucleons. In essence, beta decay changes a neutron into a proton or vice versa. Since the mass of the neutron is greater than the combined mass of a proton plus an electron, free neutrons decay spontaneously by beta-minus emission. The half-life of this process is 10.2 min. A free proton cannot spontaneously decay into a neutron plus a positron; that would violate energy conservation. But within a nucleus, a proton can change into a neutron by emitting a positron; the energy required to make this happen comes from the change in the binding energy of the nucleus. Thus, the basic beta decay reactions that take place inside the nucleus are

$$\beta^-: \quad {}^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$$

$$\beta^+: \quad {}^{1}_{1}\text{p} \rightarrow {}^{1}_{0}\text{n} + {}^{0}_{+1}\text{e} + {}^{0}_{0}\nu$$

Beta decay does not change the mass number, but it does change the ratio of neutrons to protons. A nuclide that has too many neutrons to be stable is likely to decay via β^-. By emitting an electron, a neutron is changed into a proton inside the nucleus. A nuclide that has too few neutrons is likely to decay by β^+, emitting a positron and turning a proton into a neutron. In either case, total electric charge is conserved.

Beta decay was a puzzle at first because a *continuous spectrum* of electron (or positron) energies was observed. In alpha decay, the definite kinetic energy of the alpha particles emitted in a given decay reaction is understood to come from conservation of both energy and linear momentum. For the same reasons, scientists thought that beta particles emitted in a given decay reaction should also be monoenergetic. However, when the kinetic energies were measured, the emitted beta particles had a continuous range of kinetic energies up to a maximum value (Fig. 29.6). The *maximum* kinetic energy was consistent with what scientists thought the beta particle's kinetic energy should have been.

Why did many of the beta particles have lower energies than expected? Had scientists found an exception to one of the conservation laws (energy or momentum)? Although some quite respectable scientists—including Niels Bohr—started to think that energy conservation had been violated, Wolfgang Pauli finally suggested another explanation, which turned out to be correct. Pauli speculated that not one, but two particles were being emitted, the beta particle and another, as yet undetected, particle. If a nucleus emits two particles instead of one, then they can conserve both energy and momentum while splitting up the kinetic energy in every possible way. Two momentum vectors that add to zero must be equal in magnitude and opposite in direction, but *three* momentum vectors can add to zero in an infinite number of ways and still share the same total kinetic energy.

Enrico Fermi named this hypothetical particle the **neutrino**. The symbol for the neutrino is the Greek letter "nu" (ν). An **antineutrino** is written with a bar over it ($\bar{\nu}$). For reasons that we study in Chapter 30, an antineutrino ($\bar{\nu}$) is emitted in β^- decay, while a neutrino (ν) is emitted in β^+ decay. Neutrinos are famously hard to detect because they do not interact via the electromagnetic or strong interactions. It took 25 years after Pauli predicted their existence before one was actually observed. A neutrino can pass through the Earth with only about a 1 in 10^{12} chance of interacting. Enormous numbers of neutrinos, streaming toward us from the Sun, pass through your body every second but cause no ill effects.

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Example 29.7

Beta Decay of Nitrogen-13

The isotope of nitrogen with mass number 13 (^{13}N) is unstable to beta decay. (a) ^{14}N and ^{15}N are the stable isotopes of nitrogen. Do you expect ^{13}N to decay via β^- or β^+? Explain. (b) Write the decay reaction. (c) Calculate the maximum kinetic energy of the emitted beta particle.

Strategy The key in deciding between β^- and β^+ is whether the nucleus has too many or too few neutrons to be stable.

Solution (a) The stable isotopes of nitrogen have more neutrons than ^{13}N, so ^{13}N has too few neutrons to be stable. The beta decay should convert a proton into a neutron to increase the neutron-to-proton ratio. That means the charge of the nucleus decreases by e, so a positron (charge $= +e$) must be created to conserve charge. We expect the isotope ^{13}N to undergo β^+ decay.

(b) Since a positron is emitted, it must be accompanied by a neutrino (not an antineutrino). Z decreases by 1, from 7 (for nitrogen) to 6 (which is carbon). A is unchanged. The reaction is

$$^{13}_{7}N \rightarrow {}^{13}_{6}C + {}^{0}_{+1}e + {}^{0}_{0}\nu$$

Both charge and nucleon number are conserved: 13 = 13 + 0 and 7 = 6 + 1.

(c) From Appendix B, the atomic masses of $^{13}_{7}N$ and $^{13}_{6}C$ are 13.005 738 6 u and 13.003 354 8 u. To get the masses of the nuclei, we subtract Zm_e from each. The mass of the positron is the same as that of the electron: $m_e = 0.000\,548\,6$ u. The neutrino mass is negligibly small. If M_N and M_C represent atomic masses, then

$$\begin{aligned}\Delta m &= [(M_C - 6m_e) + m_e] - (M_N - 7m_e)\\ &= M_C - M_N + 2m_e\\ &= 13.003\,354\,8\text{ u} - 13.005\,738\,6\text{ u} + 2\times 0.000\,548\,6\text{ u}\\ &= -0.001\,286\,6\text{ u}\end{aligned}$$

The mass decreases, as it must for a spontaneous decay. The disintegration energy is

$$E = |\Delta m|c^2 = 0.001\,286\,6\text{ u}\times 931.494\text{ MeV/u} = 1.1985\text{ MeV}$$

This *is* the maximum kinetic energy of the positron, since it can get virtually all of the energy and leave the neutrino and daughter nucleus with a negligibly small amount.

Discussion It is *usually* possible to determine whether a radioactive nuclide decays via β^+ or β^-, but there are exceptions. For example, $^{40}_{19}K$ can decay by *either* β^+ or β^-. The only way to be sure is to compare the masses of the products to the mass of the radionuclide to see whether the spontaneous decay is energetically possible.

Note that in β^+ decay the electron masses (which are included in the atomic masses) do not automatically "cancel out" as they did for alpha decay.

Practice Problem 29.7 Maximum Electron Energy in the Decay of Potassium-40

Find the maximum energy of the electron emitted in the β^- decay of $^{40}_{19}K$.

Electron Capture

Any nuclide that can decay via β^+ can also decay by **electron capture**. Both processes convert a proton into a neutron. In electron capture, instead of emitting a positron, the nucleus absorbs one of the atom's electrons. The basic reaction is

$$^{0}_{-1}e + {}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{0}\nu \qquad \textbf{(29-13)}$$

When a nucleus captures an electron, the only reaction products are the daughter nucleus and the neutrino. With only two particles, conservation of momentum and energy determine what fraction of the energy released is taken by each particle. The neutrino, with its tiny mass, takes almost all of the kinetic energy, leaving the daughter to recoil with only a few eV of kinetic energy. Some nuclides can decay by electron capture but *not* by β^+ because the difference in mass between the parent and daughter is less than the mass of a positron.

Gamma Decay

Gamma rays are high-energy photons. Emission of a gamma ray does not change the nucleus into a different nuclide, since neither the charge nor the number of nucleons is

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Example 29.7

Beta Decay of Nitrogen-13

The isotope of nitrogen with mass number 13 (^{13}N) is unstable to beta decay. (a) ^{14}N and ^{15}N are the stable isotopes of nitrogen. Do you expect ^{13}N to decay via β^- or β^+? Explain. (b) Write the decay reaction. (c) Calculate the maximum kinetic energy of the emitted beta particle.

Strategy The key in deciding between β^- and β^+ is whether the nucleus has too many or too few neutrons to be stable.

Solution (a) The stable isotopes of nitrogen have more neutrons than ^{13}N, so ^{13}N has too few neutrons to be stable. The beta decay should convert a proton into a neutron to increase the neutron-to-proton ratio. That means the charge of the nucleus decreases by e, so a positron (charge $= +e$) must be created to conserve charge. We expect the isotope ^{13}N to undergo β^+ decay.

(b) Since a positron is emitted, it must be accompanied by a neutrino (not an antineutrino). Z decreases by 1, from 7 (for nitrogen) to 6 (which is carbon). A is unchanged. The reaction is

$$^{13}_{7}N \rightarrow {}^{13}_{6}C + {}^{0}_{+1}e + {}^{0}_{0}\nu$$

Both charge and nucleon number are conserved: $13 = 13 + 0$ and $7 = 6 + 1$.

(c) From Appendix B, the atomic masses of $^{13}_{7}N$ and $^{13}_{6}C$ are 13.005 738 6 u and 13.003 354 8 u. To get the masses of the nuclei, we subtract Zm_e from each. The mass of the positron is the same as that of the electron: $m_e = 0.000\,548\,6$ u. The neutrino mass is negligibly small. If M_N and M_C represent atomic masses, then

$$\begin{aligned}\Delta m &= [(M_C - 6m_e) + m_e] - (M_N - 7m_e)\\ &= M_C - M_N + 2m_e\\ &= 13.003\,354\,8\text{ u} - 13.005\,738\,6\text{ u} + 2 \times 0.000\,548\,6\text{ u}\\ &= -0.001\,286\,6\text{ u}\end{aligned}$$

The mass decreases, as it must for a spontaneous decay. The disintegration energy is

$$E = |\Delta m|c^2 = 0.001\,286\,6\text{ u} \times 931.494\text{ MeV/u} = 1.1985\text{ MeV}$$

This *is* the maximum kinetic energy of the positron, since it can get virtually all of the energy and leave the neutrino and daughter nucleus with a negligibly small amount.

Discussion It is *usually* possible to determine whether a radioactive nuclide decays via β^+ or β^-, but there are exceptions. For example, $^{40}_{19}K$ can decay by *either* β^+ or β^-. The only way to be sure is to compare the masses of the products to the mass of the radionuclide to see whether the spontaneous decay is energetically possible.

Note that in β^+ decay the electron masses (which are included in the atomic masses) do not automatically "cancel out" as they did for alpha decay.

Practice Problem 29.7 Maximum Electron Energy in the Decay of Potassium-40

Find the maximum energy of the electron emitted in the β^- decay of $^{40}_{19}K$.

Electron Capture

Any nuclide that can decay via β^+ can also decay by **electron capture**. Both processes convert a proton into a neutron. In electron capture, instead of emitting a positron, the nucleus absorbs one of the atom's electrons. The basic reaction is

$$^{0}_{-1}e + {}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{0}\nu \qquad \textbf{(29-13)}$$

When a nucleus captures an electron, the only reaction products are the daughter nucleus and the neutrino. With only two particles, conservation of momentum and energy determine what fraction of the energy released is taken by each particle. The neutrino, with its tiny mass, takes almost all of the kinetic energy, leaving the daughter to recoil with only a few eV of kinetic energy. Some nuclides can decay by electron capture but *not* by β^+ because the difference in mass between the parent and daughter is less than the mass of a positron.

Gamma Decay

Gamma rays are high-energy photons. Emission of a gamma ray does not change the nucleus into a different nuclide, since neither the charge nor the number of nucleons is

changed. A gamma-ray photon is emitted when a nucleus in an excited state makes a transition to a lower energy state, just as photons are emitted when electrons in atoms make transitions between energy levels.

Figure 29.7 shows some of the energy levels of the thallium-208 ($^{208}_{81}$Tl) nucleus. The nucleus in an excited state can radiate a photon to jump to a state of lower energy. For instance, the third arrow from the right, from 492 keV to 40 keV, shows a transition that results in the emission of a 452-keV photon.

To emphasize that a nucleus is in an excited state, we put an asterisk as a superscript after the symbol: $^{208}_{81}\text{Tl}^*$. The gamma decay of an excited Tl-208 nucleus by emitting one photon is written as

$$^{208}_{81}\text{Tl}^* \rightarrow {}^{208}_{81}\text{Tl} + \gamma$$

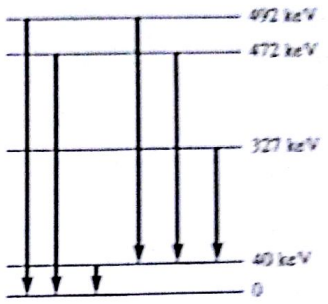

Figure 29.7 An energy level diagram for $^{208}_{81}$Tl. Downward arrows show the allowed transitions for gamma decay.

Alpha and beta decay do not always leave the daughter nucleus in its ground state. Sometimes the daughter nucleus is left in an excited state that then emits one or more gamma-ray photons until it reaches the ground state. In alpha decay, therefore, there may be different possible kinetic energies of the alpha particles emitted, depending on which excited state of the daughter nucleus is produced by the decay. For example, $^{212}_{83}$Bi can alpha decay to form any of the five energy states of $^{208}_{81}$Tl shown in Fig. 29.7 (the ground state and four excited states). The alpha-particle spectrum in the decay of $^{212}_{83}$Bi is still discrete, but there are five discrete values instead of one (see Problem 71). In beta decay, if the daughter nucleus can be left in an excited state, then the amount of kinetic energy shared by the electron (or positron), the antineutrino (or neutrino), and the daughter nucleus is smaller. The spectrum of electron (or positron) kinetic energies is still continuous.

29.4 RADIOACTIVE DECAY RATES AND HALF-LIVES

What determines when an unstable nucleus decays? Radioactive decay is a quantum-mechanical process that can only be described in terms of *probability*. Given a collection of identical nuclides, they do not all decay at the same time, and there is no way to predict which one decays when. The decay probability for one nucleus is independent of its past history and of the other nuclei. Each radioactive nuclide has a certain decay probability per unit time, written λ (no relation to wavelength). The decay probability per unit time is also called the **decay constant**. Since probability is a pure number, the decay constant has SI units s^{-1} (probability *per second*).

$$\text{decay constant } \lambda = \frac{\text{probability of decay}}{\text{unit time}} \qquad \textbf{(29-14)}$$

The probability that a nucleus decays *during a short time interval* Δt is $\lambda\,\Delta t$.

In a collection of a large number N of identical radioactive nuclei, each one has the same decay probability per unit time. The nuclei are independent—the decay of one has nothing to do with the decay of another. Since the decays are independent, the average number that decay during a short time interval Δt is just N times the probability that any one decays:

$$\Delta N = -N\lambda\,\Delta t \qquad \textbf{(29-15)}$$

The negative sign is necessary because as nuclei decay, the number of nuclei that *remain* is *decreasing*, so the change in N is *negative*. Equation (29-15) gives the *average* number that are expected to decay during Δt. Since radioactive decay is a statistical process, we may not observe exactly that number of decays. If N is sufficiently large, then we expect Eq. (29-15) to be very close to what we observe; for small N, however, deviations from the expected number can be significant. Statistical fluctuations in the number of decays $|\Delta N|$ actually measured are of order $\sqrt{|\Delta N|}$; that is, if the average number of decays expected is 10,000, the actual number of decays that occur in a particular experiment can vary by about $\sqrt{10{,}000} = 100$ above or below the average number.

Equation (29-15) is only valid for a *short* time interval $\Delta t \ll 1/\lambda$ because it assumes that the number of nuclei is a constant N. If the time interval is long enough that N changes

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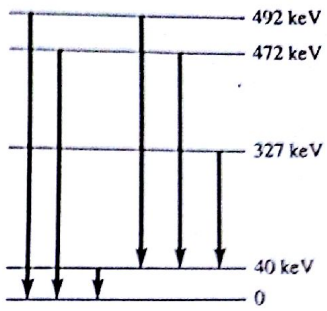

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significantly, then what N should we use? The initial number? The final number? Some sort of average? As long as $\Delta t \ll 1/\lambda$, then we are sure that $|\Delta N| \ll N$, which means that N has not changed significantly.

The number of radioactive decays from a sample per unit time is called the **decay rate** or **activity** (symbol R). The SI unit of activity is the becquerel (Bq), which is one decay per second. These three ways of writing the SI unit of activity are equivalent:

$$1\ \text{Bq} = 1\ \frac{\text{decay}}{\text{s}} = 1\ \text{s}^{-1} \tag{29-16}$$

Another unit of activity in common use is the curie (Ci):

$$1\ \text{Ci} = 3.7 \times 10^{10}\ \text{Bq} \tag{29-17}$$

If the number of decays during a short interval Δt is $|\Delta N|$, then the activity is

$$R = \frac{\text{number of decays}}{\text{unit time}} = \frac{-\Delta N}{\Delta t} = \lambda N \tag{29-18}$$

In Eq. (29-18), the rate of change of N ($\Delta N/\Delta t$) is a negative constant ($-\lambda$) times N. Whenever the rate of change of a quantity is a negative constant times the quantity, the quantity is an *exponential* function of time. The number of remaining nuclei N in radioactive decay (the number that have *not* decayed) is

$$N(t) = N_0 e^{-t/\tau} \tag{29-19}$$

A graph of N versus t is shown in Fig. 29.8. For radioactive decay, the time constant is

$$\tau = \frac{1}{\lambda} \tag{29-20}$$

and N_0 is the number of nuclei at $t = 0$. The time constant is also called the **mean lifetime** since it is the *average* time that a nucleus survives before decaying. However, it would be a misconception to think that nuclei "get old." A uranium-238 nucleus that has been sitting in rock for millions of years has the *same* probability per second of decay as one that has just been created seconds ago in a nuclear reaction; no more, no less. Equations such as (29-18) and (29-19) tell us *how many* nuclei are expected to decay, but not *which ones*.

Since the decay rate is proportional to the number of nuclei, the rate also decays exponentially:

$$R(t) = R_0 e^{-t/\tau} \tag{29-21}$$

As for exponential decay in any other context, the time constant τ is the time for the quantity to decrease to $1/e \approx 36.8\%$ of its initial value. During a time interval τ, 63.2% of the nuclei decay, leaving 36.8%. After a time interval of 2τ, $1/e^2 \approx 13.5\%$ of the nuclei still have not decayed, while $1 - 1/e^2 \approx 86.5\%$ have decayed.

Radioactive decay is often described in terms of the **half-life** $T_{1/2}$ instead of the time constant τ. The half-life is the time during which half of the nuclei decay. After

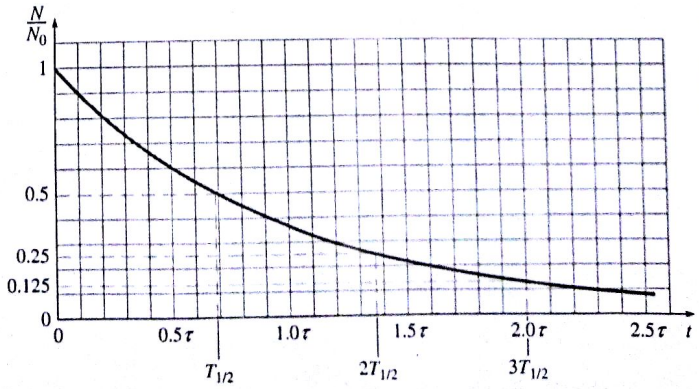

Figure 29.8 Fraction of radioactive nuclei remaining (N/N_0) as a function of time.

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two half-lives, $\frac{1}{4}$ of the nuclei remain; after m half-lives, $\left(\frac{1}{2}\right)^m$ remain. It can be shown (Problem 43) that

$$T_{1/2} = \tau \ln 2 \approx 0.693\,\tau \tag{29-22}$$

where ln 2 is the natural (base-e) logarithm of 2. Then

$$N(t) = N_0(2^{-t/T_{1/2}}) = N_0\left(\tfrac{1}{2}\right)^{t/T_{1/2}} \tag{29-23}$$

Example 29.8

Radioactive Decay of Nitrogen-13

The half-life of ^{13}N is 9.965 min. (a) If a sample contains 3.20×10^{12} ^{13}N atoms at $t = 0$, how many ^{13}N nuclei are present 40.0 min later? (b) What is the ^{13}N activity at $t = 0$ and $t = 40.0$ min? Express the activities in Bq. (c) What is the probability that any one ^{13}N nucleus decays during a one-second time interval?

Strategy (a, b) The number of nuclei at $t = 0$ is $N_0 = 3.20 \times 10^{12}$ and the half-life is $T_{1/2} = 9.965$ min. The data for N at $t = 40.0$ min and for R at both $t = 0$ and 40.0 min. Since the time interval of 40.0 min is approximately four times the half-life, we can first estimate the solution: both N and R are multiplied by $\frac{1}{2}$ during each half-life.

(c) The probability of decay during a 1-s interval is λ *only* if it can be considered a short time interval. Since the half-life is 9.965 min = 597.9 s, 1 s is a tiny fraction of the half-life and therefore *can* be considered a short time interval.

Solution (a) Half of the nuclei are left after one half-life, $\left(\frac{1}{2}\right)^2$ after two half-lives, and $\left(\frac{1}{2}\right)^4$ after four half-lives. Therefore, the number remaining after four half-lives is

$$N = \left(\tfrac{1}{2}\right)^4 \times 3.20 \times 10^{12} = 2.00 \times 10^{11}$$

Eq. (29-23) gives the precise result:

$$N = N_0\left(\tfrac{1}{2}\right)^{t/T_{1/2}} = N_0\left(\tfrac{1}{2}\right)^{40.0/9.965} = 1.98 \times 10^{11}$$

(b) Activity and number of nuclei are related by Eq.

$$R = \lambda N = \frac{N}{\tau}$$

Now τ is related to the half-life by Eq. (29-22):

$$\tau = \frac{9.965\ \text{min} \times 60\ \text{s/min}}{0.693} = 862.6\ \text{s}$$

Next we substitute the number of nuclei N at $t = 0$ and at $t = 40.0$ min to determine the rate of decay at those two times. The time constant does not change.

At $t = 0$,

$$R_0 = \frac{N_0}{\tau} = \frac{3.20 \times 10^{12}}{862.6\ \text{s}} = 3.71 \times 10^9\ \text{Bq}$$

At $t = 40.0$ min,

$$R = \frac{N}{\tau} = \frac{1.98 \times 10^{11}}{862.6\ \text{s}} = 2.30 \times 10^8\ \text{Bq}$$

(c) The probability per second is

$$\lambda = \frac{1}{\tau} = 1.1593 \times 10^{-3}\ \text{s}^{-1}$$

A nucleus has a 0.001 159 3 probability of decaying during a 1-s interval.

Discussion As a check, R after four half-lives should be $\frac{1}{16}$ of R_0:

$$\tfrac{1}{16} \times 3.71 \times 10^9\ \text{Bq} = 2.32 \times 10^8\ \text{Bq}$$

Since 40.0 min is slightly more than four half-lives, the activity at $t = 40.0$ min is slightly less than 2.32×10^8 Bq.

The probability of decay in one second would *not* be equal to λ if the half-life were not large compared to 1 s. For a longer time interval, we find the decay probability as follows:

$$\text{probability of decay} = \frac{\text{number expected to decay}}{\text{original number}} = \frac{|\Delta N|}{N_0} = \frac{N_0 - N}{N_0} = 1 - e^{-t/\tau}$$

Practice Problem 29.8 Number Remaining After One Half of a Half-Life

How many ^{13}N atoms are present at $t = 5.0$ min?

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Exa 29.12 Continued

In cases, the competing reaction might include alpha, beta decay, or fission.

Practice Problem 29.12 The Reaction That Produces Carbon-14

Determine whether the reaction $n + {}^{14}N \rightarrow {}^{14}C + p$ is exoergic or endoergic. How much energy is released or absorbed?

Neutron Activation Analysis

Making the Connection: neutron activation analysis

Neutron activation analysis (NAA) is a technique used to study precious works of art, rare biological specimens, geological objects, and the like. It is used to determine which elements are present in the sample being studied—even those present in only trace amounts. The great advantage of NAA over other methods of analysis is that it is some of the most invasive. An entire painting can be analyzed without the need to scrape off paint, as would have to be done to use a mass spectrometer. Art historians know different paint pigments were used in different historical periods. Determination of the pigments used can help establish the date of a painting; it can also detect forgeries, repairs, and canvasses that have been painted over.

The elements present in a sample are identified by the characteristic gamma-ray energies emitted by the activated nuclei when they decay. By taking gamma-ray spectra at different times, the half-lives can also be used for identification purposes. Quantitative analysis of the gamma-ray spectrum yields the concentrations of various elements in the sample studied. Neutron activation analysis of this type has been used to study lunar samples from the Apollo missions, bullets and gunshot residue swabs used as forensic evidence in criminal investigations, oceanographic fossils and sediments, textiles, and artifacts from archaeological excavations, just to name a few examples.

NAA allows the art historian to determine which pigments have been used on which parts of the painting, even in the underlying layers, without damaging the painting. In *Jousting with a Bust of Homer*, NAA helped reveal the extent of the damage to the painting, even to the hat. Art historians also drew some conclusions about how the composition was altered as he worked, such as changes in Aristotle's costume, the positions of the arms and shoulders, a change in the position of the medallion, and a change in the height of the bust of Homer. Historians knew that the canvas had shrunk from its original height; the early position of the bust helped them conclude that the missing canvas was at the bottom.

29.7 FISSION

As shown in Figure 29.2, very large nuclei have a smaller binding energy per nucleon than those of intermediate mass. The binding energy of large nuclides is reduced by the Coulomb repulsion of the protons. Each proton in the nucleus repels every other proton. The strong force, which holds the nucleons together in a nucleus, is short-range; a nucleon is bound only to its nearest neighbors. In large nuclides the number of nearest neighbors is approximately constant, so the strong force adds the binding energy per nucleon to compensate for the Coulomb repulsion, which reduces the binding energy per nucleon.

A heavy nucleus can therefore increase energy by splitting into two smaller, more tightly bound nuclei in the process called **fission**. The term is borrowed from biology; a cell that splits into two. Nuclear fission was first observed by Otto Hahn and Fritz Strassmann in 1938.

Very large nuclei can fission spontaneously. Radioactive uranium-238, for instance, breaks apart into two fission products, though it is much more likely to decay by emitting an alpha particle. Fission can also be induced by an incident neutron, a ^2H nucleus, alpha particle, or photon. Fission due to the capture of

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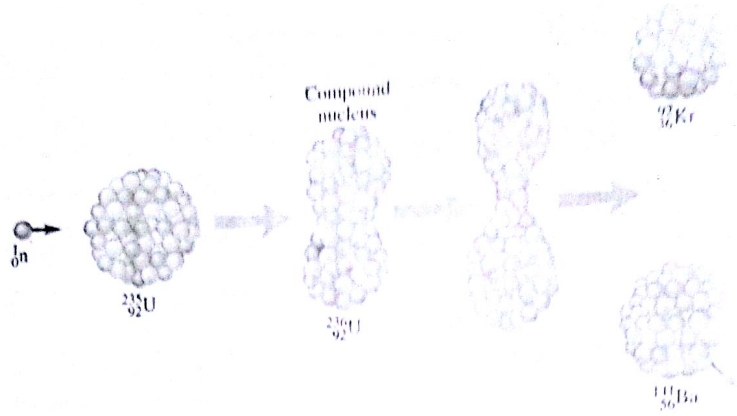

Figure 29.12 Fission of ^{235}U induced by the capture of a slow neutron. In addition to the two daughter nuclei, some neutrons are released.

a slow neutron allows the possibility of a chain reaction. Uranium-235 is natu-rally occurring nuclide that can be induced to fission by slow neutrons.

Suppose that a slow neutron is captured by a ^{235}U nucleus. The com ucleus formed, ^{236}U, is in an excited state since the neutron gives up energy wh comes bound to the nucleus (Problem 54). The excited nucleus is elonga shape (Fig. 29.12). The attractive force between nucleons tends to pull the nucle into a sphere, while the Coulomb repulsion between protons tends to push the art. If the excitation energy is sufficient, a neck forms and the nucleus splits i parts. The Coulomb repulsion then pushes the two fragments apart so they do n mbine into a single nucleus.

Figure 29.13 shows the potential energy of a nucleus as it elongates a ts into two. To form an elongated shape, the potential energy of the nucleus n crease about 6 MeV. In the absence of an incident particle to supply this energy, *neous* fission can occur only by quantum-mechanical tunneling through the 6 energy barrier (Section 28.10). The tunneling probability is much lower than the p ility of alpha decay. If ^{238}U decayed only by spontaneous fission, its half-life wo e about 10^{16} yr (instead of 4×10^{9} yr).

Many different fission reactions are possible for a given parent nuclid n fission products are not always the same, and there is no way to predict which of t y pos-sibilities actually occur. Here are two examples of the induced fission o after it captures a slow neutron:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^{*} \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n \qquad (29\text{-}30)$$

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^{*} \rightarrow ^{139}_{54}Xe + ^{95}_{38}Sr + 2^{1}_{0}n \qquad (29\text{-}31)$$

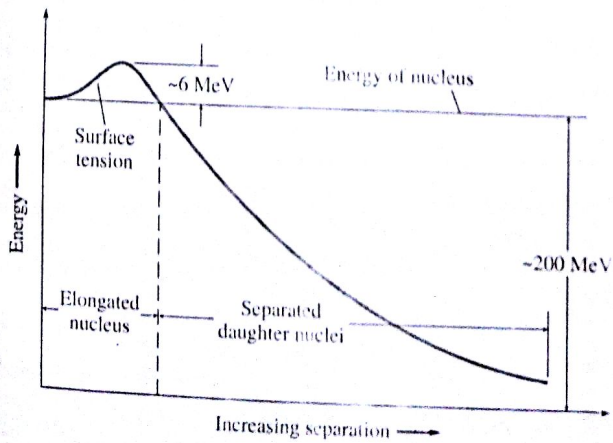

Figure 29.13 Potential energy as a function of separation of the two daughter nuclei in spontaneous fission.

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stable as was originally thought. The Nevada state government and some other groups oppose the planned facility; as this book goes to press, the legal battles continue and high-level nuclear waste continues to be stored on site at the reactors.

29.8 FUSION

The energy radiated by the Sun and other stars is produced by nuclear **fusion**. Fusion is essentially the opposite of fission. Instead of a large nucleus splitting into two smaller pieces, fusion combines two small nuclei to form a larger nucleus. Both fission and fusion release energy, since they move toward larger binding energies per nucleon (see Fig. 29.2). Due to the steep slope of the binding energy per nucleon curve at low mass numbers, fusion can be expected to release significantly more energy per nucleon than fission.

Here is an example of a fusion reaction:

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + \mathrm{n} \qquad \textbf{(29-32)}$$

Two hydrogen nuclei fuse to form a helium nucleus. The reaction releases 17.6 MeV of energy, which is 3.52 MeV per nucleon—much more than the 0.75–1 MeV per nucleon typical of fission reactions. Although this reaction produces a tremendous amount of energy, it cannot occur at room temperature. The deuterium (^{2}H) and tritium (^{3}H) nuclei must get close enough to react. At room temperature, the two positively charged nuclei have kinetic energies much too small to overcome their mutual Coulomb repulsion. However, in the Sun's interior the temperature is about 2×10^{7} K and the average kinetic energy of the nuclei is $\frac{3}{2}k_BT = 2.52$ keV. This *average* kinetic energy is still far too small to allow a fusion reaction (see Example 29.14), but some of the more energetic nuclei do have enough kinetic energy to overcome the Coulomb repulsion. Fusion reactions are also called thermonuclear reactions because they depend on the large kinetic energies available at high temperatures.

Two fusion cycles to explain energy production in stars were proposed by Hans Bethe (1906–2005). One cycle is called the **proton-proton cycle**:

$$\mathrm{p} + \mathrm{p} \rightarrow {}^{2}\mathrm{H} + \mathrm{e}^{+} + \nu \qquad \textbf{(29-33a)}$$

$$\mathrm{p} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He} \qquad \textbf{(29-33b)}$$

$${}^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + 2\mathrm{p} \qquad \textbf{(29-33c)}$$

The net effect of the proton-proton cycle is to fuse four protons into a ^{4}He nucleus. (The first two reactions must each take place twice to form the two ^{3}He nuclei needed for the third reaction.) The three steps can be summarized as

$$4\mathrm{p} \rightarrow {}^{4}\mathrm{He} + 2\mathrm{e}^{+} + 2\nu$$

Each positron annihilates with an electron, so the overall reaction due to the proton-proton cycle is

$$4\mathrm{p} + 2\mathrm{e}^{-} \rightarrow {}^{4}\mathrm{He} + 2\nu \qquad \textbf{(29-34)}$$

Another cycle of fusion reactions that occurs in some stars is the **carbon cycle**:

$$\mathrm{p} + {}^{12}\mathrm{C} \rightarrow {}^{13}\mathrm{N} \qquad \textbf{(29-35a)}$$

$${}^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu \qquad \textbf{(29-35b)}$$

$$\mathrm{p} + {}^{13}\mathrm{C} \rightarrow {}^{14}\mathrm{N} \qquad \textbf{(29-35c)}$$

$$\mathrm{p} + {}^{14}\mathrm{N} \rightarrow {}^{15}\mathrm{O} \qquad \textbf{(29-35d)}$$

$${}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu \qquad \textbf{(29-35e)}$$

$$\mathrm{p} + {}^{15}\mathrm{N} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} \qquad \textbf{(29-35f)}$$

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Here the carbon-12 nucleus acts as a catalyst. It is present in the beginning and at the end. After the annihilation of the two positrons, the net effect is the same as the proton-proton cycle:

$$4p + 2e^- \rightarrow {}^4He + 2\nu$$

The *total* energy released by the carbon cycle is the same as the total energy released by the proton-proton cycle. By "total energy released" we mean the total energy of all the photons [not shown in Eqs. (29-33) through (29-35)] and neutrinos produced plus the kinetic energy of the ^{4}He nucleus minus the initial kinetic energies of the protons and electrons.

Example 29.14

First Step of the Carbon Cycle

(a) Calculate the energy released in the first step of the carbon cycle. (b) Estimate the minimum kinetic energy of the proton and ^{12}C nucleus required for this reaction to take place.

Strategy To calculate the energy released, we must determine the mass difference between reactants and products. For the minimum initial kinetic energy, we know that the two positively charged particles repel each other. We can find the distance between the two when they just "touch," and find the electric potential energy in that position.

Solution (a) The reaction in question is

$$p + {}^{12}C \rightarrow {}^{13}N$$

We use atomic masses in the calculations since the extra mass of seven electrons is equally present in the atomic masses of the reactants and products. The initial mass is then

$$1.007\,825\,0\text{ u} + 12.000\,000\,0\text{ u} = 13.007\,825\,0\text{ u}$$

The mass change is

$$\Delta m = 13.005\,738\,6\text{ u} - 13.007\,825\,0\text{ u} = -0.002\,086\,4\text{ u}$$

The energy released is

$$E = 0.002\,086\,4\text{ u} \times 931.494\text{ MeV/u} = 1.9435\text{ MeV}$$

(b) From Eq. (29-4), the radii of the [illegible] are 1.2 fm and

$$1.2\text{ fm} \times 12^{1/3} = 2.[illegible]$$

For an *estimate* of the electric pot[illegible] proton and ^{12}C nucleus are just "[illegible] electric potential energy of two [illegible] $+6e$, at a separation of 3.95 fm.

$$U_E = \frac{6ke^2}{r} = \frac{6 \times (9 \times 10^9\text{ N·m}^2/\text{C}^2)[illegible]}{3.95 \times 1[illegible]}$$

$$= 3.50 \times 10^{-13}\text{ J} = 2\text{ Me[illegible]}$$

The minimum total kinetic energy [illegible] nucleus that [illegible]lows the reaction t[illegible] 2 MeV.

Discussion The energy released, [illegible] both the increase in kinetic energy [illegible] photon.

Practice Problem 29.14 Se[illegible] Carbon Cycle

Calculate the energy released in th[illegible] carbon cycle.

Stars act as factories to form heavier nuclides from lighter nuclides. In a star like our Sun, most of the fusion reactions produce helium from hydrogen. At higher core temperatures, heavier nuclides can take part in fusion reactions. Nuclides all the way up to the peak of the binding energy curve (Fig. 29.2), around $A = 60$, are formed by fusion reactions in the interiors of stars. Once a star core is rich in iron and nickel, elements near the top of the binding energy curve, fusion reactions die out. Heavier nuclides are *less* tightly bound than iron and nickel, so fusion reactions no longer release energy. Eventually the large star implodes under its own gravity; the implosion provides the energy for the fusion of heavier nuclides. Ultimately the star may explode, an event called a supernova. Additional fusion and neutron-capture reactions occur in the shock waves caused by the explosion, forming the heaviest nuclides. The nuclides formed in the supernova, plus the ones that had already been formed in the star's core, are dispersed by the explosion into space. The atoms that make up all of us and our surroundings were distributed into space by one or more supernovae before making their way to

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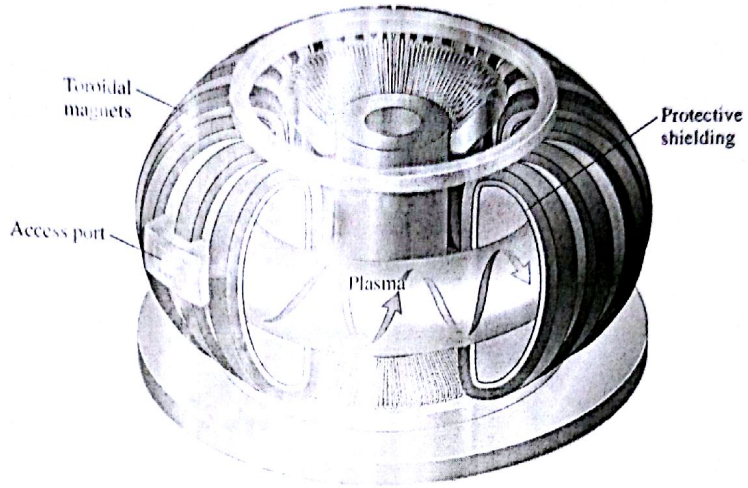
Figure 2 ... tokamak is one of th... using methods fo... a controlled ... on. At such high t... the atoms in th... apart to form a *pla*... ure of electrons a... charged nuclei. Ma... confine the charge... interior of a dough... evacuated chamber. ... al around ma... es and are confine... iding with the w... um chamber.



Earth. Other than hydrogen (and a small fraction of some of the other light elements), all of the elements found on Earth were either made in the core of a star or in a supernova (or are radioactive decay products of these elements).

Ma... connection:

Controlled Fusion

In a thermonuclear bomb (or hydrogen bomb), a fission bomb creates the high temperatures that enable an uncontrolled fusion reaction to take place. For decades, researchers have been trying to make possible a sustained, *controlled* fusion reaction. Fusion as an energy source would have several advantages over fission. The fuel for fusion is more easily obtained than the fuel for fission. The most promising reactions for controlled fusion are deuteron-deuteron fusion ($^2H + {}^2H$) or deuteron-triton fusion [$^2H + {}^3H$, as in Eq. (29-3...)]. Deuterium is readily available in seawater; about 0.015% of the water molecules in the ocean contain a deuterium atom. Tritium's natural abundance is very small, but is not difficult to produce.

One of the biggest problems associated with *fission* reactors is the radioactive waste that must be safely stored for thousands of years. A fusion reactor would produce less radioactive waste and the waste would not have to be stored for as long.

However, a sustained, controlled fusion reaction has not yet been achieved. The main problem is containing the fuel at extremely high temperatures (estimated to be about 10^8 K, which is higher than the temperature of the Sun's interior) needed for fusion to take place while maintaining a high density of nuclei so that they collide into each other. An ordinary container cannot be used; the nuclei would lose too much kinetic energy when they collide with the walls of such a container and the container would be vaporized by the high temperatures. Two principal confinement schemes are being tried. One is *magnetic confinement* (see Fig. 29.18). The other is *inertial confinement*, in which a small fuel pellet is heated rapidly by intense laser beams from all sides, causing the fuel pellet to implode and the fusion reactions to take place before the pellet is vaporized.

MASTER THE CONCEPTS

- A ... clide is characterized by its atomic ... number of protons) and its nucleon ... total number of protons and neu... topes of an element have the same ... but different numbers of neutrons.

- The mass density of all nuclei is approximately the same. The radius of a nucleus is

$$r = r_0 A^{1/3} \tag{29-4}$$

where

$$r_0 = 1.2 \times 10^{-15}\ \text{m} = 1.2\ \text{fm} \tag{29-5}$$

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