UNIT - IV

Shear Force & Bending Moment

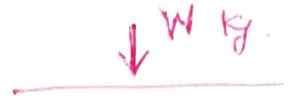
Learning Objectives

- Type of loads / Reaction
- Type of Beams
- Concept of Shear Force & Bending Moment
- Relation between Load, Shear-force & Bending Moment.
- Concept of drawing Shear Force (SF) & Bending Moment (BM) diagram

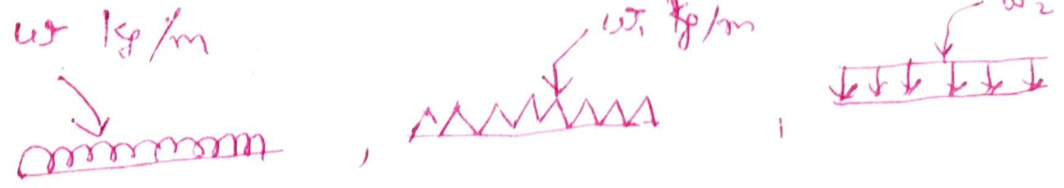
Types of Loads and Reactions

There ~~are~~ are basically three type of dead loads which can be applied on a beam.

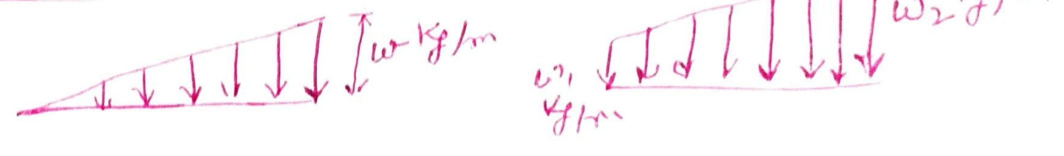
1. Point Load - (in Kg), represented by W



2. Uniformly Distributed Load (UDL) (Unit - Kg/m or Kg/cm), represented by



3. Varying Load



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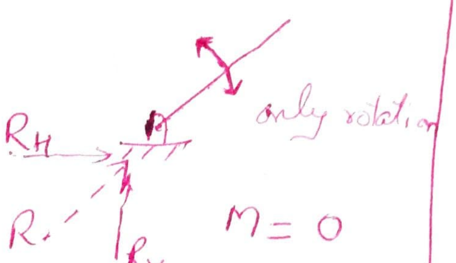
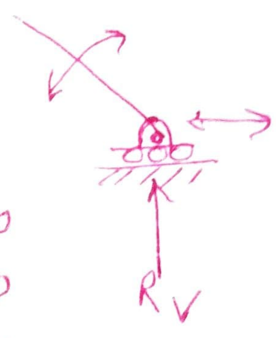
Type of Reaction / Support

1. **Hinge Support** – There is only rotatory motion about a axisal hinge support. ~~There is a~~ Reaction at hinge support may have both horizontal as well as vertical component, but moment will be zero.

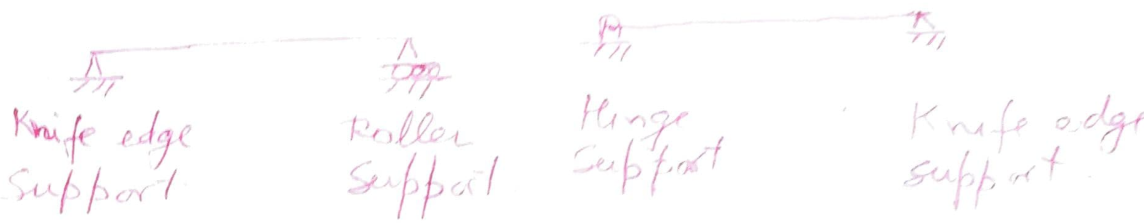
2. **Roller support** – There are rotatory motion as well as translatory motion at roller support. Due to which, reaction has only vertical component. Moment is zero

3. **Fixed Support** – No movement. Reaction has both horizontal & vertical component and there is some moment also at fixed support.

Hinge support	Roller Support	Fixed Support
		
$M = 0$ Depends on load → $R_H \neq 0$, $R_V \neq 0$	$R_V \neq 0$ $R_H = 0$ $M = 0$	$R_H \neq 0$, $R_V \neq 0$ (depends on load) $M \neq 0$

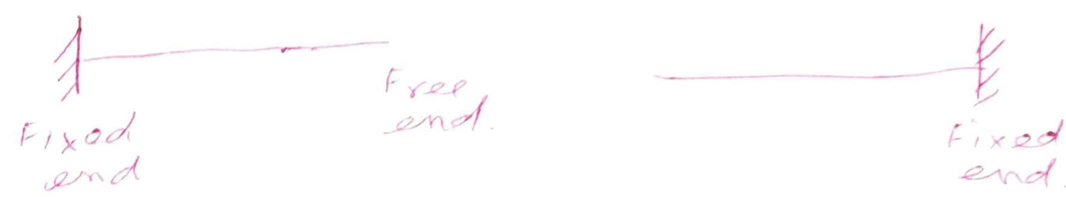
Type of Beams:

1. Simply Supported beam - A beam resting on two supports which may be knife edge or hinge or roller support. These support will not exert any moment on the beam

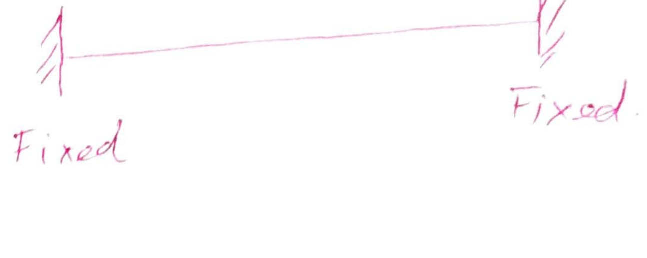


2. Overhanging beam - A beam having one or both ends extended over the supports.

3. Cantilever beam - A beam with one end fixed and other end free. There is no deflection or rotation at the fixed end.



4. Fixed Beam - A beam with both end fixed

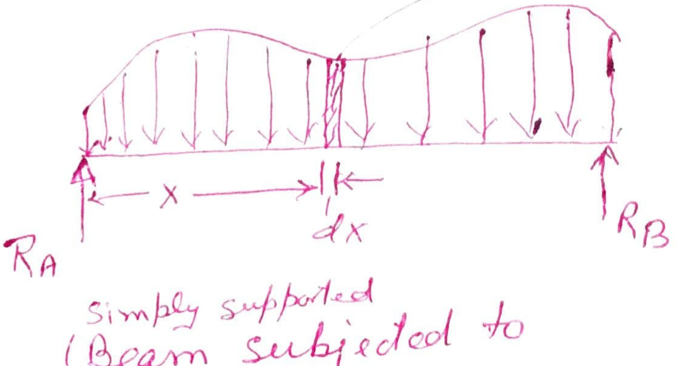


5. <u>Continuous beam</u> - A beam with more than two supports. Such beam may or may not have overhang.

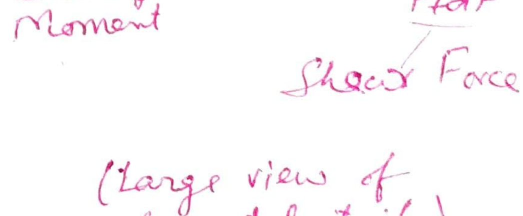
Sign Convention for SF & BM diagram

	Moving from Left side	Moving from Right side
Shear Force Diagram	Reaction +ve Load −ve	Reaction −ve Load +ve
Bending Moment Diagram	Reaction +ve Load −ve	Reaction +ve Load −ve

Relation between Load, Shear force & Bending moment -



Simply supported (Beam subjected to varying load)



(Large view of elemental strip)

Refer the figure of large view of elemental strip, the strip with length dx is supposed to be uniformly loaded with w KN/m, Shear force varying from F to F+dF and Bending moment varying from M to M+dM.

Taking moment at C (right side of elemental strip)

$\Sigma M = 0$

i.e. $$M - (M+dM) + F * dx - (F+dF) * 0 - \underbrace{(w.dx)}_{\text{load due to U.D.L}} \cdot \underbrace{\frac{dx}{2}}_{\text{Distance of C from C.G. of load}} = 0$$

Negligible.

$$-dM + F.dx - w \frac{(dx)^2}{2} = 0$$

$$dM = F.dx \quad \text{or} \quad \boxed{F = \frac{dM}{dx}}$$

[Shear force is rate of change of bending moment with respect to x]

For equilibrium.

$\Sigma F_V = 0$

$$F - w\,dx - (F+dF) = 0$$

$$\text{or} \quad \boxed{w = \frac{dF}{dx}}$$

So intensity of load is rate of change of shear force with respect to x.

Definition of SF & BM

Shear Force at a section in a beam is the force that is trying to shear off the section. It is obtained as algebric sum of all the forces acting normal to axis of beam: on either side of beam (left or right)

Bending Moment at a section in a beam is the moment that tends to bend the beam and is obtained as algebric sum of moment of all forces about the section, acting either to the left or right side of section.

Points to remember:

1. The shear force
   a) changes suddenly at a section where there is point load or reaction (Vertical line in SF diagram)
   b) remain constant between two point load, if there is no ~~Uniformly distributed~~ other load in between. (horizontal line in SF diagram)
   c) has a straight line variation for Uniformly distributed load (inclined line in SF diagram)

2. The bending moment (BM)
   a) is maximum at a section where S.F is zero and changes the sign.

   b) has a parabolic variation for UDL

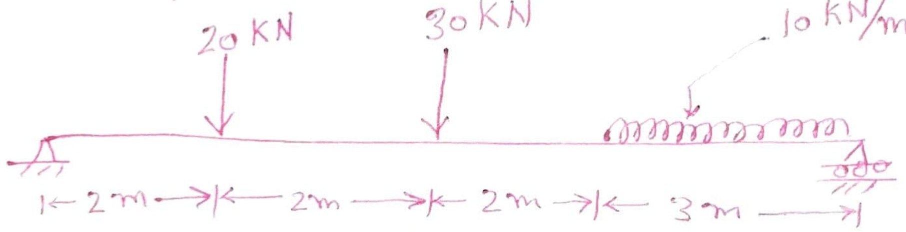
3. The point where B.M changes its sign and is zero, is called point of contraflexure or point of inflexion.

4. At a point, where couple is applied, the shear force remains unchanged, but there is sudden change in bending moment. (vertical line in BM diagram)

5. For calculation purpose, Uniformly distributed Load (UDL) or Varying Distributed Load (VDL) can be taken as point load acting at C.G. (center of gravity) or centroid of the area at which it is acting.

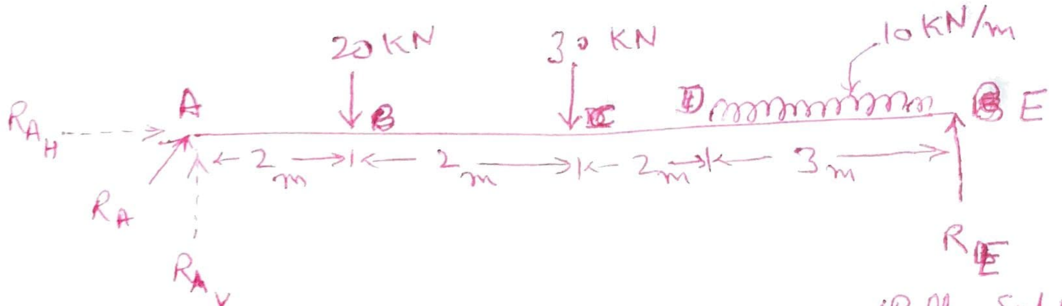
Some examples of S.F. & BM diagram

Ex 1 Draw S.F. & B.M. diagram for the given Load diagram.



(Load Diagram)

Ans. Step 1 Finding reactions



(Hinge Support)
Inclined reaction

(Roller Support)
only vertical reaction

For equilibrium

(i) $\Sigma F_H = 0$ (Sum of all horizontal forces must be zero)

$$R_{A_H} = 0$$

(ii) $\Sigma F_V = 0$ (Some of all vertical forces must be zero)

$$R_{A_V} + R_E = 20 + 30 + 10 * 3$$
$$= 80$$

(iii) $\Sigma M = 0$ (Sum of moment of all forces w.r.t. a point must be zero)

Taking moment w.r.t. A

$$R_E * 9 = 20 * 2 + 30 * 4 + (10 * 3) * 7.5$$

$$R_E = \frac{40 + 120 + 225}{9} = \frac{385}{9}$$
$$= 42.78 \text{ KN}$$

UDL will act at mid point (i.e. at C.G.)

$R_{A_V} = 80 - R_E$

$= 80 - 42.78 = 37.22$ KN

<u>Step 2</u> Drawing S.F. diagram.

Let us start <u>from lef side</u>
(Reaction will be taken +ve and load as -ve)

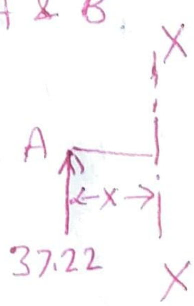
(i) Shear force at A.

As there is reaction at A, so there will be sudden change of S.F. at A from 0 to 37.22 (i.e. vertical line at A)

$S.F._{at\ A} = +37.22$ KN

(ii) Shear force between A & B.

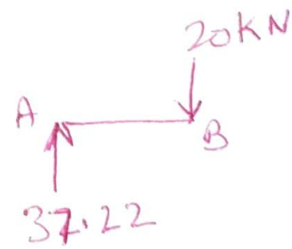
SF_{AB} = Sum of all forces left to X-X



$= +37.22$ KN

(Horizontal line between A & B)

(iii) Shear force at B

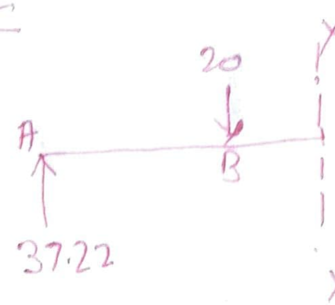


$SF_B = +37.22 - 20$

$= 17.22$ KN

As there is point load at B, so there will be sudden change of S.F, at B from 37.22 to 17.22 KN.
(i.e. Vertical line at B)

(iv) Shear force ~~at~~ between B & C



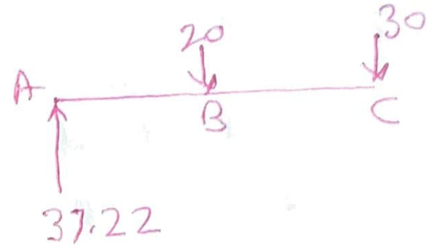
$SF_{BC} = +37.22 - 20$

$= 17.22$ KN

(Horizontal line)
between B & C

(Left side of cross section between B & C)

(V) Shear force at C



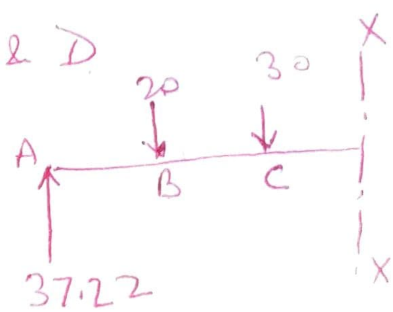
$SF_C = +37.22 - 20 - 30$

$= -12.78$ KN

(Left side of beam at ~~section~~ C)

As there is point load at C, so there will be sudden change of S.F. at C from 17.22 KN to −12.78 KN.
(Vertical line at C)

(vi) Shear force between C & D



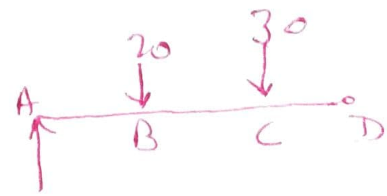
$SF_{CD} = +37.22 - 20 - 30$

$= -12.78$ KN

(Horizontal line between C & D)

(Left side of cross-section between C & D)

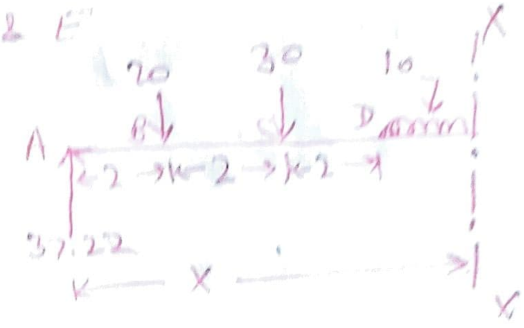
(vii) Shear force ~~between~~ at D



$SF_D = +37.22 - 20 - 30$

$= -12.78$ KN.

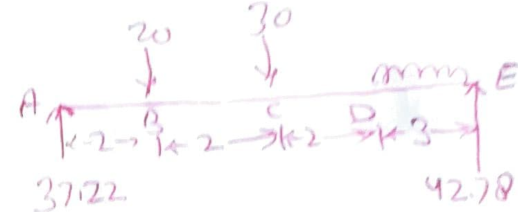
(viii) Shear force between D & E



$S.F_{DE} = +37.22 - 20 - 30$
$- 10\,(x-6)$
$= -12.78 - 10x + 60$
$= -10x + 47.22$

(Inclined line between D & E with downward slope)

(ix) Shear force at E



$SF_E = +37.22 - 20 - 30$
$- 10 \times 3 + 42.78 = 0$

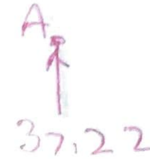
As there is vertical reaction at E, so there will be sudden change of SF at E from -42.78 to 0 (Vertical line at E)

Step 3: Drawing B.M. diagram.

- (Bending moment at the section will be sum of moment of all forces about the section, acting on left side of section)
  (as we are moving from left)
- Moment of Reaction are taken +ve and moment of load are taken −ve (as per sign convention, discussed earlier)

(i) Bending moment at A

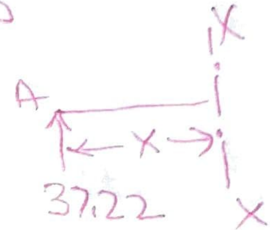
$M_A = \underbrace{37.22}_{\text{Reaction}} * \underbrace{0}_{\text{distance}} = 0$



(ii) Bending moment between A & B

$M_{AB} = 37.22\,X$

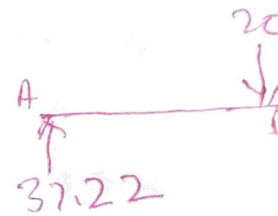
(Inclined line with upward slope)



(iii) Bending moment at B

$M_B = 37.22 * 2 - 20 \times 0$

$= 74.44$ KN-m



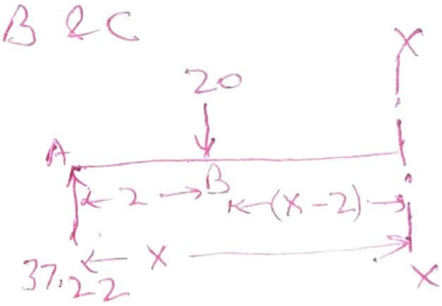
(iv) Bending moment between B & C

$M_{BC} = 37.22\,X - 20 * (X-2)$

$= 37.22\,X - 20\,X + 40$

$= 17.22\,X + 40$

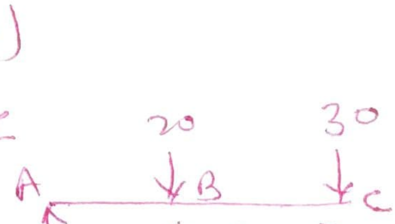
(Inclined line with upward slope)



(v) Bending moment ~~between~~ at C

$M_C = 37.22 * 4 - 20 * 2 - 30 \times 0$

$= 108.88$ KN-m

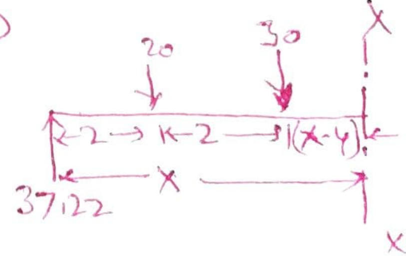


(vi) Bending moment between C & D

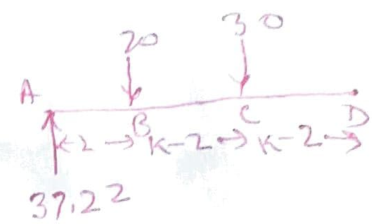
$M_{CD} = 37.22 * X - 20(X-2) - 30(X-4)$

$= -12.78\,X + 160$

(Inclined line with downward slope)



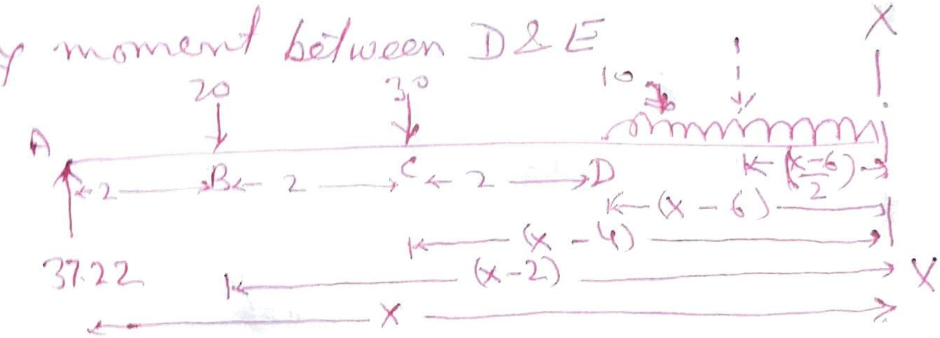
(vii) Bending moment at D



$M_D = 37.22 * 6 - 20 * 4 - 30 * 2$
$= 83.32$ KN-m

(viii) Bending moment between D & E

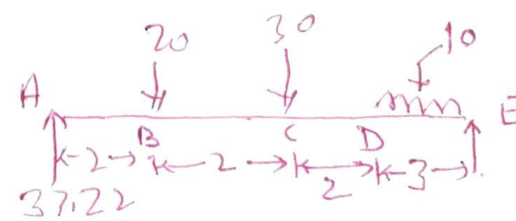
$\underline{M_{DE}}$



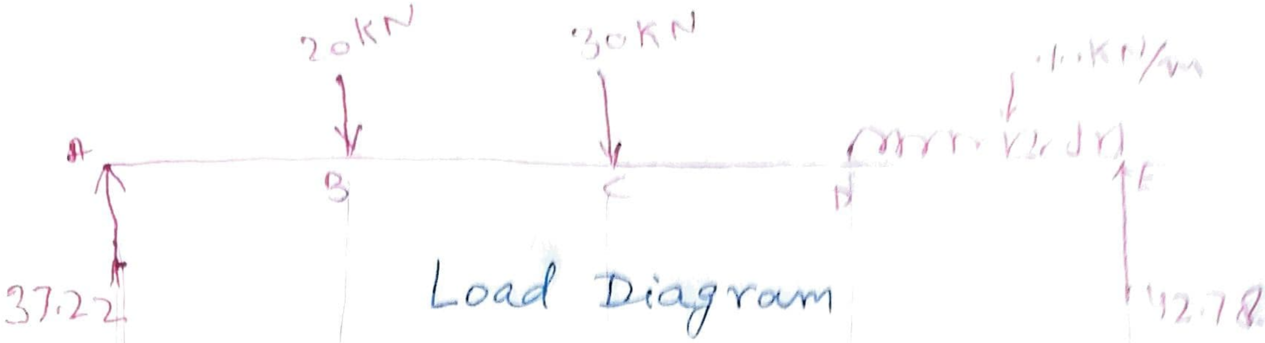
$$M_{DE} = 37.22 * X - 20 * (X-2) - 30 * (X-4)$$
$$- \underbrace{10 * (X-6)}_{\text{Load}} * \underbrace{\left(\frac{X-6}{2}\right)}_{\text{Distance}}$$
$$= -12.78\,X + 160 - 5(X-6)^2$$

(A parabola)

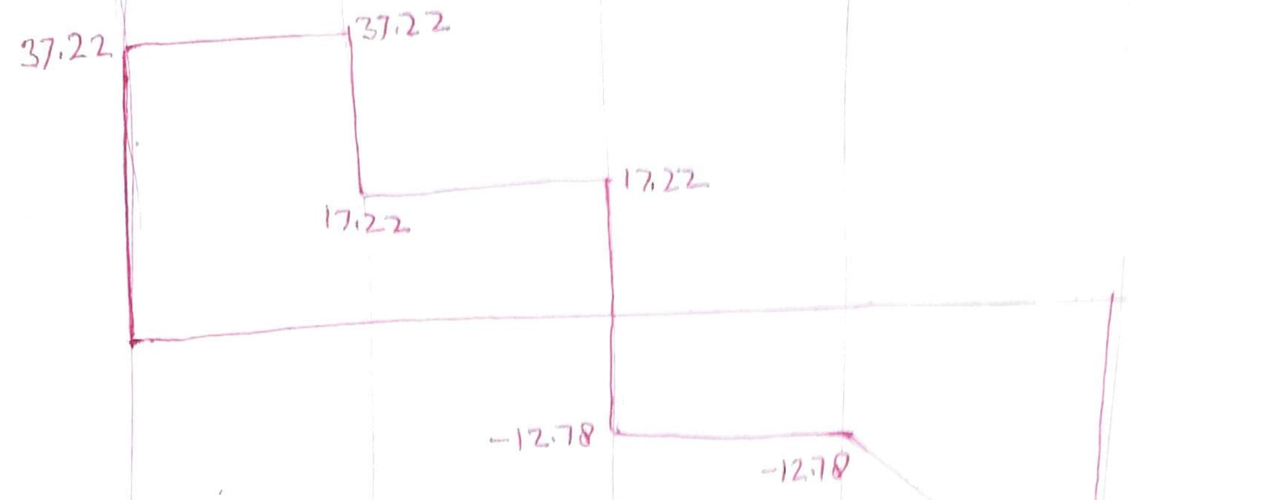
(ix) Bending moment at E



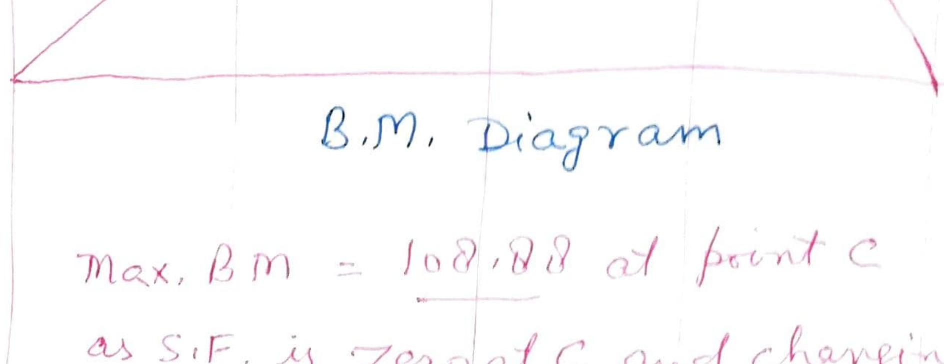
$M_E = 37.22 * 9 - 20 * 7$
$- 30 * 5 - (10 * 3) * 3/2$
$+ 42.78 * 0$
$= 0$



Load Diagram



S.F. Diagram



B.M. Diagram

Max. BM = 108.88 at point C as S.F. is Zero at C and changing its sign. ~~for~~

Q. Draw S.F, & B.M. diagram for the following. (write each step of SF & BM)