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Assignment: Central forces

2nd Nov. 2020

1. Use Hamilton's equations to find the differential eqⁿ for planetary motion and prove that the areal velocity is constant.
2. A particle of mass m moves under the action of central force whose potential is $V(r) = Kmr^3$ ($K > 0$), then
 - (i) For what kinetic energy and angular momentum will the orbit be a circle of radius R about the origin?
 - (ii) Calculate the period of circular motion.
3. The eccentricity of the earth's orbit is 0.0167. Calculate the ratio of ~~momentum~~ maximum and minimum speeds of the earth in its orbit.
4. The maximum and minimum velocities of a satellite are v_{\max} and v_{\min} respectively. Prove that the eccentricity of the orbit of the satellite is,

$$e = \frac{v_{\max} - v_{\min}}{v_{\max} + v_{\min}}$$
5. A particle of mass ' m ' is observed to move in a spiral orbit given by the eqⁿ $r = c\theta$ where c is a constant. Is it moving in a central force field? If it is so, find the force law. [Hint: Eqⁿ of orbit: $\frac{d^2U}{d\theta^2} + U = -\frac{mf(\frac{1}{r})}{L^2U^2}$]
6. A particle describes a circular motion under the influence of an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of the distance.