Assignment – I

Chapter – I : Review of vector Analysis.

1. State and prove Gauss's divergence theorem,
2. state and prove Stoke's theorem in vector analysis.
3. state and prove Green's theorem in vector analyses.
4. Evaluate $\iint_S (x^3 - yz)\,dy\,dz - 2x^2y\,dz\,dx + z\,dx\,dy$ over the surface bounded by the coordinate planes and the planes $x = y = z = a$.
5. Verify Green's theorem in the plane for $\int_C [(xy + y^2)\,dx + x^2\,dy]$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$
6. Using ~~green~~ Green's theorem in space, evaluate $\iint_S (4xz\,dy\,dz - y^2\,dz\,dx + yz\,dx\,dy)$, where S is the surface of a cube bounded by the planes $x=0$, $y=0$, $z=0$, $x=1$, $y=1$, and $z=1$.
7. Evaluate $\int_C \vec{f}\cdot d\vec{r}$ by Stoke's theorem, where $f = y^2 i + x^2 j - (x+z)k$ and C is the boundary of the triangle with vertices at (0,0,0), (1,0,0) and (1,1,0).
8. Find curl $\vec{F}$, where $\vec{F} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$
9. Find div $\vec{F}$, where $F = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$
10. Show that the vector $\vec{V} = (yz)\hat{i} + (zx)\hat{j} + (xy)\hat{k}$ is irrotational

Tutorials

Chapter – I : Review of Vector Analysis

1. If $\vec{V} = \dfrac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$, find the value of curl $\vec{V}$ & div $\vec{V}$

2. prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.

3. show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} - 3\vec{b} + 4\vec{c}$, $-\vec{b} + 2\vec{c}$ are coplanar.

4. Given vectors $a = 2\hat{i} - 3\hat{j}$, $b = \hat{i} + \hat{j} - \hat{k}$, $c = 3\hat{i} - \hat{k}$. Construct a vector 'v' orthogonal to 'a' and 'b' and having unit scalar product with 'c'.

5. (a) Prove that (i) div curl $A = 0$ and (ii) curl grad $\phi = 0$

Here ϕ, ψ are scalar point functions and A is vector point function in a certain region.

(b) prove that curl $(\phi \text{ grad } \psi) = \nabla\phi \times \nabla\psi = -$ curl $(\psi \text{ grad } \phi)$

6. Using divergence theorem evaluate

$$\iint_S (x^3 dy\, dz + y^3 dz\, dx + z^3 dx\, dy)$$

7. verify stoke's theorem for the vector $A = (2x-y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ over the upper half of the surface of the sphere $x^2+y^2+z^2=1$.

8. Using Green's theorem, evaluate $\int_C (x^2 y\, dx + x^2 dy)$, where C is the boundary described counter-clockwise of triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$.

9. From the equation of continuity, show that for the flow of an incompressible fluid, velocity can be written as curl of another vector.

10. If ρ denotes the charge density and J the current density due to the charges, show that the equation $\dfrac{\partial \rho}{\partial t} + \text{div } J = 0$ expresses conservation of the total energy.