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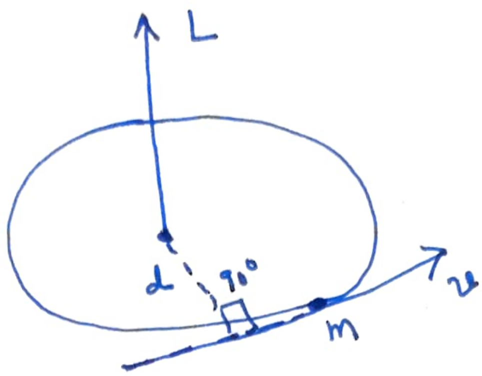
Orbital Angular Momentum

Central force



$L = mvd$, Direction: Right hand rule

Torque on particle $\tau = \frac{dL}{dt}$.

Since the force on particle is a Central force torque exherted will be Zero

$\Rightarrow$ L has constant magnitude and direction at every point along with its trajectory

$\Rightarrow$ Angular Momentum is CONSERVED ✓

Total Energy = KE + PE

$$\boxed{KE = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{2r}}$$

$PE = qV = (-e)V = -e \frac{1}{4\pi\epsilon_0} \frac{Ze}{r}$

$PE = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r}$

$TE = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{2r} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{2a}$

$$\boxed{TE = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{2a}}$$

$\cancel{KE = \frac{1}{2} mv^2}$

$\cancel{\frac{K(e)(Ze)}{r^2} = \frac{mv^2}{r}}$, $K = \frac{1}{4\pi\epsilon_0}$

$\cancel{\Rightarrow v^2 = \frac{KZe^2}{mr}}$

$\Rightarrow v^2 = \frac{KZe^2}{mr}$

$$\boxed{KE = \frac{KZe^2}{2r}}$$

Magnetic Dipole momentum

$$\mu = IA = ef \cdot \pi r^2 \quad \text{frequency}$$

Orbital Angular Moment $|L| = mvr = m(2\pi r f)r$

$$|L| = \frac{2m}{e}|\mu| \quad \text{Magnetic Dipole mom}$$

$$\boxed{\mu = -\frac{e}{2m}L}$$

Magnetic Dipole Moment in an external Magnetic Field B

$$\tau = \mu \times B$$

~~They~~ ~~B tries~~ μ tries to allign with μ

E_B = Potential Energy of μ in B. $= -\mu \cdot B = \frac{e}{2m} L \cdot B$

$$\boxed{E_B = \frac{e}{2m} L_z B}$$

$$= \frac{e}{2m} m_\ell B = \frac{e}{2m} \times m_\ell B$$

$m_\ell \to \ell, \ell-1, \ldots$

ZEEMAN EXPERIMENT

Dutch Physicist Pieter Zeeman in 1896
Prior to the advent of Quantum Mechanics

When an atom is placed in external magnetic field, its excitation spectrum is measured and compared with the spectrum when there is no magnetic field. ~~Ext~~ Additional Spectrum lines.

Each spectral line split into number of additional Discrete energy levels

Explain ?

'Quantization of the magnitude of L'

$$|L| = \sqrt{l(l+1)}\,\hbar$$

where l is an integer i.e. $l = 0, 1, 2 \ldots (n-1)$

n number of level

Bohr's theory $E_n = \frac{E_1^0}{n^2}$

For $n=1$, the lowest value for l is zero

$\Rightarrow L = 0$ for $l = 0,\ n = 1$

Quantization of the direction of L

$\boxed{l=1 \quad 2l+1 = 3}$

$$L_z = m_l \hbar$$

$$m_l = l,\ l-1,\ l-2,\ \ldots 0 \ldots -(l-1),\ -l.$$

$(2l+1)$ values

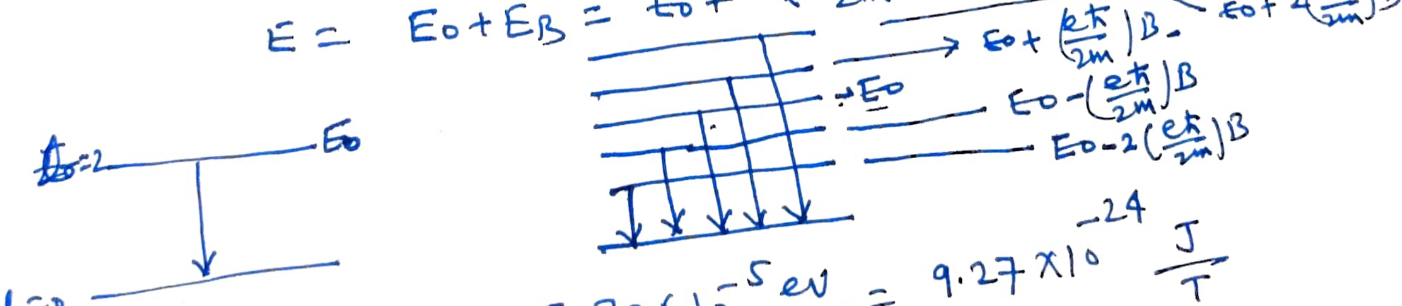
For a given 'l', the maximum value of $L_z = l\hbar$

The maximum value of $L = \sqrt{l(l+1)}\,\hbar$

$$\Rightarrow L_z < L$$

Explanation Zeeman Effect

$$E = E_0 + E_B = E_0 + m_l \frac{e\hbar}{2m} B = E_0 \pm l \frac{e\hbar}{2m} B$$



$$\frac{e\hbar}{2m} = 5.79 \times 10^{-5} \frac{eV}{T} = 9.27 \times 10^{-24} \frac{J}{T}$$

$\Delta l = \pm 1,\ \Delta m_l = \pm 1$ or 0

$$\Delta E = \Delta E_0 + \frac{e\hbar}{2m} B$$

$$\boxed{\Delta E_{zee} = \frac{e\hbar}{2m}}$$

S.A. Goudsmit & G E Uhlenbeck – Electron possesses an intrinsic Angular Momentum called SPIN

Magnetic moment $\mu_s \Rightarrow$ Spin Angular Momentum S

Quantum No of Spin Angular momentum S is specified as ~~$s = \frac{1}{2}$~~ s,

Two orientations are possible $2 = 2s+1$

$$\Rightarrow s = \frac{1}{2}$$

$$|\mathbf{S}| = \sqrt{s(s+1)}\,\hbar = \sqrt{\tfrac{1}{2}(\tfrac{1}{2}+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar$$

$S_z = m_s\hbar$, where $m_s = s, s-1 = \frac{1}{2}, -\frac{1}{2}$

$m_s = +\frac{1}{2}$ - Spin up, $m_s = -\frac{1}{2}$ = Spin down

Intrinsic ~~Angular~~ magnetic ~~momentum~~ moment μ_s and intrinsic Angular momentum S are proportion to each other

$$\mu_s = -g_s \frac{e}{2m} S$$

g_s = Gyromagnetic Ratio = $\dfrac{|\mu_s|/|S|}{|\mu|/|L|}$ Spin Angular Mom / Orbital

Proton, Neutron also possess an intrinsic Angular momentum

Spin orbit Coupling.

$$E_B = -\mu \cdot B$$

Total Angular Momentum (Vector model)

$$J = L + S$$

$$|J| = \sqrt{J(J+1)}\,\hbar$$

$$J = L+S,\ L+S-1,\ \ldots (L-S)$$

where L and S are orbital and spin Quantum numbers.

$$J_z = M_j \hbar \quad \text{where } M_J = J, J-1, J-2, \ldots -J$$

For Hydrogen ~~atom~~ like atom

$$S = \tfrac{1}{2}$$

$$\therefore J = \begin{cases} L+S,\ L-S & \text{for } L>0 \\ S & \text{for } L=0 \end{cases}$$

Calculate $L \cdot S$ for $L = 1$ and $S = \frac{1}{2}$

$$J = L+S = 1 + \tfrac{1}{2} = \tfrac{3}{2}.$$

$$J = L+S-1 = 1+\tfrac{1}{2}-1 = \tfrac{1}{2}$$

$$L \cdot S = \tfrac{1}{2}\left[J(J+1) - L(L+1) - S(S+1)\right]\hbar^2$$

$$J \cdot J = |J|^2 = (L+S)(L+S) = L \cdot L + 2L \cdot S + S \cdot S$$

$$= |L|^2 + 2L\cdot S + |S|^2$$

$$\Rightarrow \quad L \cdot S = \cancel{\tfrac{1}{2}\left[|L|^2\right.} = \tfrac{1}{2}\left[|J|^2 - |L|^2 - |S|^2\right]$$

$$\tfrac{1}{2}\left[J(J+1)\hbar^2 - l(l+1)\hbar^2 - S(S+1)\hbar^2\right]$$

$$L \cdot S = \tfrac{1}{2}\left[J(J+1)\hbar^2 - \right.$$

$$\boxed{\begin{array}{l} J = 3/2 \Rightarrow L \cdot S = \tfrac{1}{2}\hbar^2 \\ \text{For } J = \tfrac{1}{2} \Rightarrow -\hbar^2 \end{array}}$$