Question Based On

"Velocity Dependent Potential"

and

"Cyclic Coordinates"

Key Concepts:

- If the ~~vecto~~ Potential is velocity dependent

  i.e. $U = U(q, \dot{q})$

  then force are dissipative in nature

  and Total Energy is not Conserved

- If Lagrangian 'L' does not contain a coordinate q_k

  then q_k is Cyclic Coordinate

  and corresponding momenta $p_k = \frac{\partial L}{\partial \dot{q}_k}$ = constant of motion.

Question: Gate 2007

The Lagrangian of particle of mass m is

$$L = \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] - \frac{V}{2}(x^2+y^2) + W \sin \omega t$$

where V, W, ω are constant

then conserved quantity are

Ans Energy and z- component of linear momentum.

Solution

Given: $L = \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] - \frac{V}{2}(x^2+y^2) - W \sin\omega t$

$$L = \underbrace{\frac{m}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2)}_{T} - \underbrace{\frac{V}{2}(x^2+y^2) - W \sin\omega t}_{V}$$

$$T = \frac{m}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2)$$

$$V = \frac{V}{2}(x^2+y^2) + W \sin\omega t$$

Since Potential V is Independent of velocity

$\therefore$ Energy will be conserved.

and the Lagrangian does not contain coordinate z

$\therefore$ 'z' = cyclic coordinate

$\therefore$ p_z = z-component of linear momentum will be conserved.