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Gate 2003

The lagrangian of a particle of mass m moving in a plane is given by

$$L = \frac{1}{2}\left[m\left(V_x^2 + V_y^2\right)\right] + a\left(x V_y - y V_x\right)$$

where V_x and V_y are velocity component and a is constant. The canonical momenta is

(a) $P_x = mV_x$ and $P_y = mV_y$

(b) $P_x = mV_x - ay$ and $P_y = mV_y + ax$

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Correct answer (b)

Explaination:

Canonical Momenta or Generalized Momenta

$$p = \frac{\partial L}{\partial \dot{q}}$$

if q has dimension of length then $\dot{q} = v$

$$\therefore \quad p = \frac{\partial L}{\partial v}$$

For x- component $p_x = \frac{\partial L}{\partial v_x}$ — (1)

For y- component $p_y = \frac{\partial L}{\partial v_y}$ — (2)

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Solving eq (1)

$$p_x = \frac{\partial L}{\partial v_x} = \frac{\partial}{\partial v_x}\left[\frac{1}{2}m(v_x^2+v_y^2)\right] + a(xv_y - yv_x)$$

$$p_x = \frac{1}{2}m\frac{\partial}{\partial v_x}(v_x^2+v_y^2) + a\frac{\partial}{\partial v_x}(xv_y - yv_x)$$

$$p_x = \frac{1}{2}m\left(\frac{\partial}{\partial v_x}v_x^2 + 0\right) + a\left(0 - y\frac{\partial v_x}{\partial v_x}\right)$$

$$p_x = \frac{1}{2}m(2v_x) + (a - y(1))$$

$$p_x = mv_x - ay \quad \text{——— (3)}$$

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Solving eq (2) (p-4)

$$p_y = \frac{\partial L}{\partial v_y} = \frac{\partial}{\partial v_y}\left[\frac{1}{2}m\left(v_x^2 + v_y^2\right) + a\left(x v_y - y v_x\right)\right]$$

$$p_y = \frac{1}{2}m\left(\frac{\partial}{\partial v_y}v_x^2 + \frac{\partial}{\partial v_y}v_y^2\right) + a\left(\frac{\partial}{\partial v_y}(x v_y) - y\frac{\partial}{\partial v_y}v_x\right)$$

$$p_y = \frac{1}{2}m\left(\boxed{0} + 2v_y\right) + a\left(x\frac{\partial v_y}{\partial v_y} - 0\right)$$

$$p_y = m v_y + a\left(x(1)\right)$$

$$p_y = m v_y + a x \quad \text{——— (4)}$$

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Hence from eq (3) & (4)

$$p_x = mv_x - ay$$

$$p_y = mv_y + ax$$

These are the required canonical momenta.

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