Question Based On

"Generalised force"

Key Concept:

1) if Generalised Potential $V = V(q)$

then Generalised force $F = -\frac{\partial V}{\partial q}$

2) if Generalised Potential $V = V(q, \dot{q})$

then Generalised force $F = \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}}\right) - \frac{\partial V}{\partial q}$.

Question: Gate 2011

A particle is moving under the action of Generalised potential

$$V(q, \dot{q}) = \frac{(1+\dot{q})}{q^2}$$

The magnitude of Generalised force is

Solution. we will use

$$f = \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}}\right) - \frac{\partial V}{\partial q}$$

$$= \frac{d}{dt}\left\{\frac{\partial}{\partial \dot{q}}\left(\frac{1+\dot{q}}{q^2}\right)\right\} - \frac{\partial}{\partial q}\left(\frac{1+\dot{q}}{q^2}\right)$$

$$= \frac{d}{dt}\left\{\frac{1}{q^2}\frac{\partial}{\partial \dot{q}}(1+\dot{q})\right\} - (1+\dot{q})\frac{\partial}{\partial q}\left(\frac{1}{q^2}\right)$$

$$= \frac{d}{dt}\left\{\frac{1}{q^2}(0+1)\right\} - (1+\dot{q})\frac{\partial}{\partial q}(q^{-2})$$

$$= \frac{d}{dt}\left(\frac{1}{q^2}\right) - (1+\dot{q})\{(-2)\,q^{-2-1}\}$$

(P-5)

$$f = \frac{d}{dt}\left(\frac{1}{q^2}\right) - (1+\dot{q})(-2)\,q^{-3}$$

$$f = \frac{d}{dq}\,q^{-2}\cdot\frac{dq}{dt} + 2\frac{(1+\dot{q})}{q^3}$$

$$f = \{-2\,q^{-2-1}\}\,\dot{q} + \frac{2(1+\dot{q})}{q^3}$$

$$f = \boxed{-\frac{2\dot{q}}{q^3} + \frac{2(1+\dot{q})}{q^3}}$$

$$f = -\frac{2\dot{q}}{q^3} + \frac{2}{q^3} + \frac{2\dot{q}}{q^3}$$

$$f = \frac{2}{q^3} \qquad \text{(Ans)}$$