CENGAGE Learning

MECHANICS of MATERIALS

SECOND EDITION

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MECHANICS OF SOLIDS

ENGINEERING MECHANICS

STATICS　　DYNAMICS　　MECHANICS OF MATERIALS

As expected that you have gone through STATICS & DYNAMICS

STATICS
DYNAMICS } Study of External effects on <u>Rigid bodies</u>

MECHANICS OF MATERIALS } Study of <u>internal effects</u> and <u>deformations</u> that are caused by applied loads.

We should understand the design. If we design a particular size of member you need to how much deform and what stress acts on member.

MECHANICS OF MATERIALS

STRENGTH OF MATERIALS

MECHANICS OF DEFORMABLE BODIES

MECHANICS OF SOLIDS

All provides same informations.

Building structure : Different types of loads, Earthquake, Wind loadings, moving persons etc subjected to external loads.

SECTION - B.

Mechanics and Strength of Materials.

Unit -3 : Force System and Analysis

i) Basic concept

ii) Friction

unit 4 ; Structure Analysis

i) Beam's

ii) Trusses

unit 5 ; Stress and Strain Analysis

i) Simple stress and strain

ii) compound stress and strain

(iii) Pure Bending of Beam

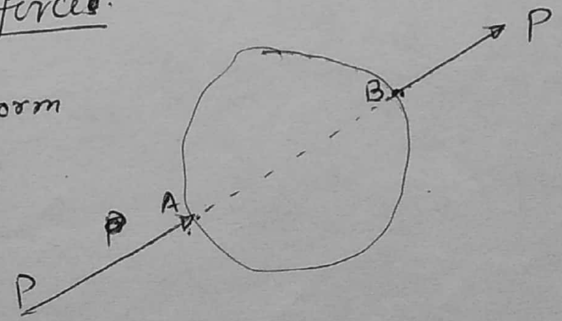
(iv) Torsion

Unit -3- BASIC CONCEPT.

Fundamental Laws in applied Mechanics

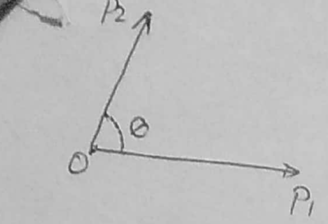
1. Newton's first law (Also known as law of inertia)
2) Newton's Second law ($F \propto \frac{dp}{dt}$) $\Rightarrow F = ma$
3) Newton's third law (For each action there is equal & opposite reaction)
4) Law of transmissibility of forces and
5) parallelogram law of forces

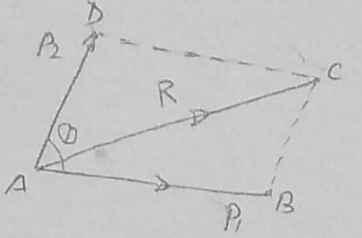
Law of transmissibility of forces.

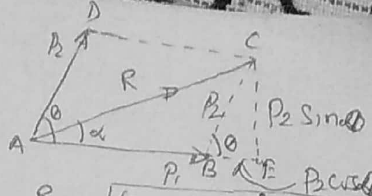


" The state of rest or uniform motion of a body (rigid body) is unaltered if a force acting on the body is replaced by another force of same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.

... forces







$$R = \sqrt{(P_1 + P_2\cos\theta)^2 + (P_2\sin\theta)^2}$$

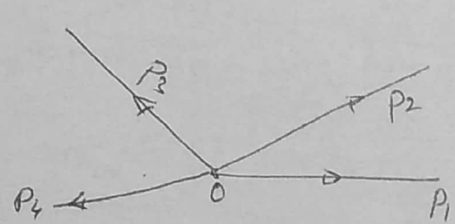
$$= \sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\theta}$$

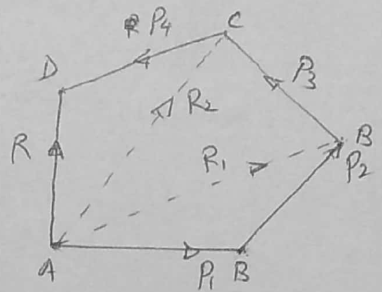
$$\tan\alpha = \frac{P_2\sin\theta}{P_1 + P_2\cos\theta}$$

Coplanar Concurrent forces.

If all the forces in a system lie in a single plane and pass through a single point.

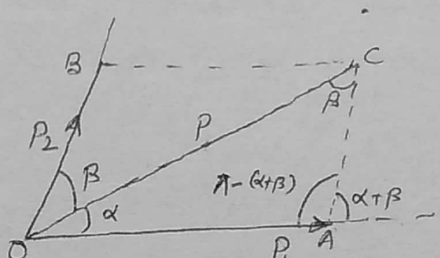
Triangle Law of forces.





Coplanner and concurrent forces

RESOLUTION OF A FORCE.

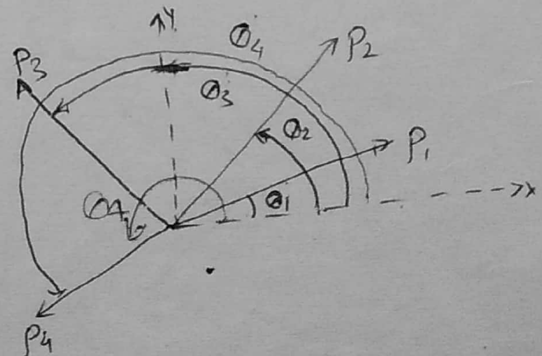


$$\frac{P_1}{\sin\beta} = \frac{P_2}{\sin\alpha} = \frac{P}{\sin[\pi - (\alpha+\beta)]}$$

$$P_1 = \frac{P\sin\beta}{\sin(\alpha+\beta)}$$

$$P_2 = \frac{P\sin\alpha}{\sin(\alpha+\beta)}$$

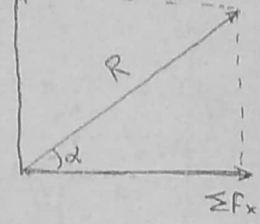
RESULTANT OF A NUMBER OF FORCES



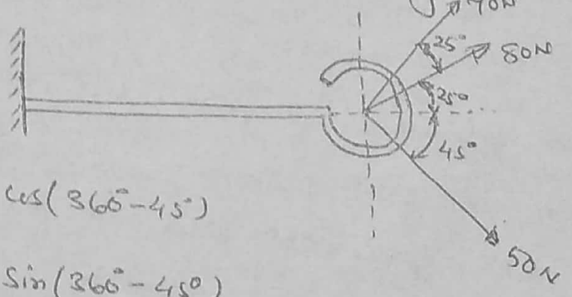
$$\Sigma F_x = P_1\cos\theta_1 + P_2\cos\theta_2 + P_3\cos\theta_3 + P_4\cos\theta_4$$

$$\Sigma F_y = P_1\sin\theta_1 + P_2\sin\theta_2 + P_3\sin\theta_3 + P_4\sin\theta_4$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$



Ex 1 Determine the resultant of the three forces acting on a hook as shown in the fig.



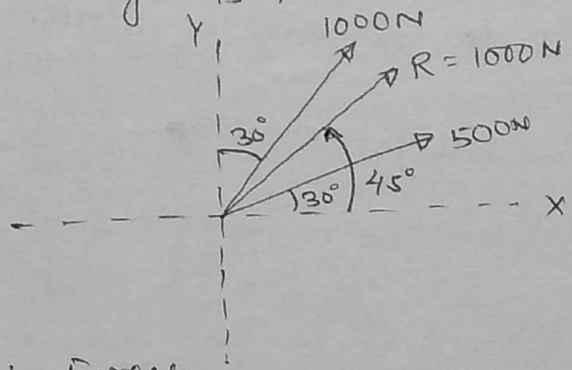
$\Sigma F_x = 80\cos 25^\circ + 70\cos 50^\circ + 50\cos(360^\circ - 45^\circ)$
$= 152.85N$

$\Sigma F_y = 80\sin 25^\circ + 70\sin 50^\circ + 50\sin(360^\circ - 45^\circ)$
$= 52.07N$

$\therefore R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(152.85)^2 + (52.07)^2} = 161.48N$

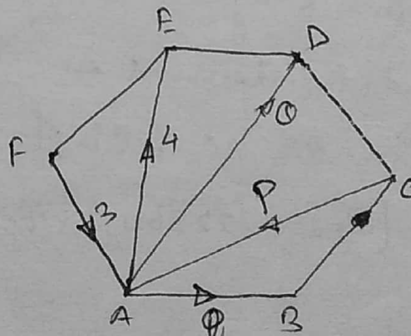
$\alpha = \tan^{-1}\left(\frac{52.07}{152.85}\right) = 18.81^\circ$

Q. Two forces acting on a body are 500N and 1000N as shown in the fig. Determine the third force F such that resultant of all three forces is 1000N directed as angle 45° to x-axis



Ans $F = 467.2N$
$\theta = 61.08^\circ$

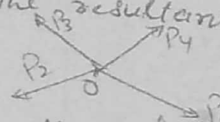
Q. ABCDEF is a reguler hexagen. Forces 2, P, Q, 4 and 3KN act along AB, CA, AD, AE and FA respectively are in equilibrium. Determine the value of P and Q.

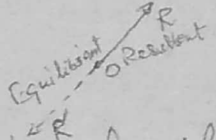


Ans. $P = 4.65KN$
$Q = 1.044KN$

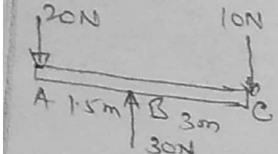
Q. Forces 2, $\sqrt{3}$, 5, $\sqrt{3}$ and 2KN respectively act at one of the angular points of regular hexagon towards five other angular points. Determine the magnitude and direction of resultant force.

... given force in equilibrium. Evidently, equilibriant is equal and opposite to the resultant.





EQUILIBRIUM OF BODY.



$\Sigma F_x = 0$

$\Sigma F_Y = 0$

$\Sigma M = 0$

Net moment about B = ~~....~~ $20 \times 1.5 + 10 \times 3 = 0$

Equilibrium

Condition of Equilibrium of coplaner forces

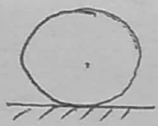
1. Algebric sum of the resolved components of all the forces in any direction in the plane of forces equals zero ie $\Sigma FX = 0$ & $\Sigma FY = 0$

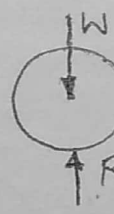
11) The algebric sum of all the forces about any point in the plane of forces equals zero.
$\Sigma M = 0$

FREE BODY DIAGRAMS

In many problem's it is essensial to isolate the body under consideration from the other bodies in contact and draw all the forces acting on the body. For this " first the body is drawn and then applied forces, self weight and reaction's at the point of contact with other bodies are drawn.

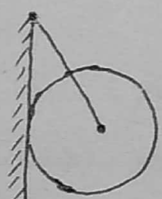
Examples. (1)

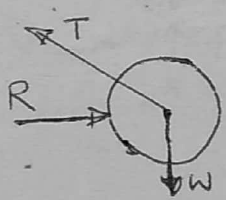




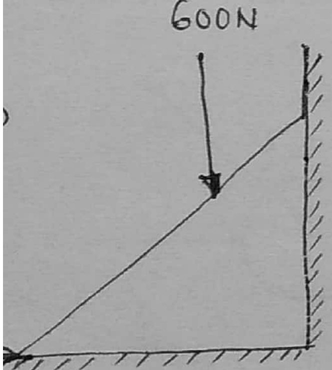
FBD for ball

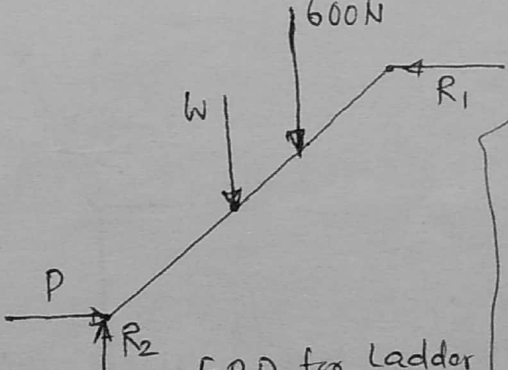
(11)



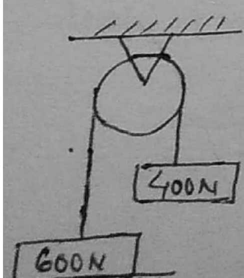


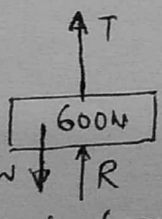
FBD for ball





FBD for Ladder





FBD for 600N Block.

Some particular Cases

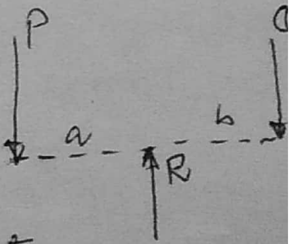
(a) Single force cannot remain in equilibrium.

(b) Two forces ~~|~~ can be in equilibrium only when they act ~~only~~ along same straight line. Further the force must be equal in magnitude and act in opposite direction.

(c) when Three forces are parallel fig. 1, the resultant of two like forces should balance the ~~f~~ third force

$P + Q = R$

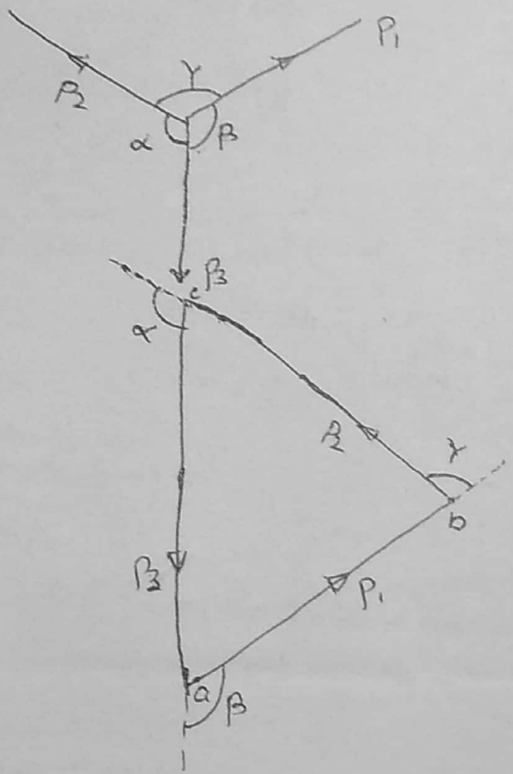
and $Pa = Qb$



When they are concurrent, Lami's theorem and principle of triangle of forces hold good.

LAMI'S THEOREM

If a body is in equilibrium under the action of three forces, each force is proportional to the sine of the angle between the other two forces.



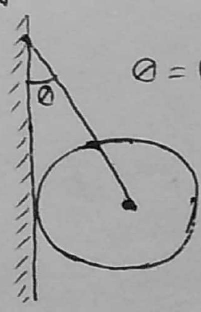
$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$

$$\frac{P_1}{\sin(180-\alpha)} = \frac{P_2}{\sin(180-\beta)} = \frac{P_3}{\sin(180^\circ-\gamma)}$$

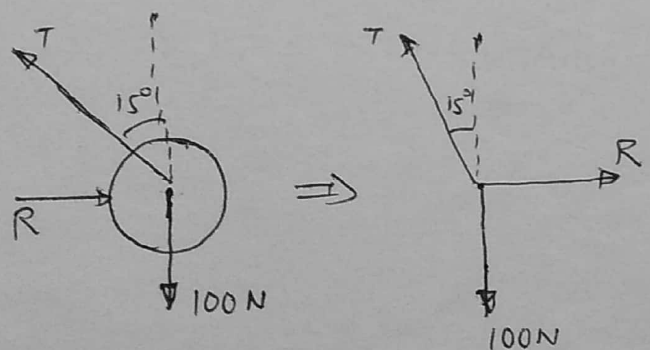
$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$

Note: (i) P_1, P_2, P_3 are Coplanar
(ii) P_1, P_2, P_3 are Concurrent

Q. A sphere of weight 100 N is tied to a smooth wall by a string as shown in fig.(1). Find the tension T in the string and reaction R on the wall.



Solution: Draw the FBD of sphere.



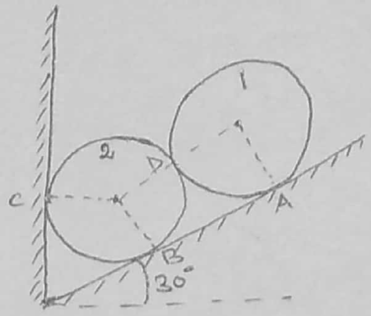
Note (1) String can only pull a body it can't push a body.
(2) Wall can only give pushing reaction.

Apply Lami's Theorem

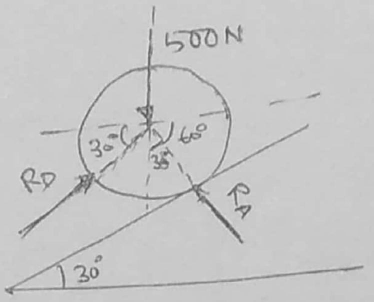
$$\frac{100}{\sin(90+15)} = \frac{R}{\sin(180^\circ-15^\circ)} = \frac{T}{\sin 90^\circ}$$

$\Rightarrow T = 103.51\ N$
$\Rightarrow R = 26.788\ N$ } Ans.

Q. Two identical rollers each of weight Q = 500 N are supported by an inclined plane and vertical wall as shown in fig. Assuming smooth surfaces find the reactions induced at points of supports A, B and C.



Sol^n FBD for roller (1)

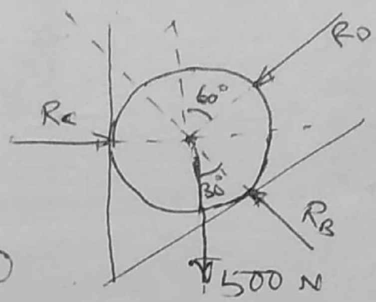


$$\frac{R_A}{\sin(90+30)} = \frac{R_D}{\sin(90+60)} = \frac{500}{\sin 90^\circ}$$

$$R_A = 500 \times 0.866 = 433.012\ N$$

$$R_D = 500 \times 0.5 = 250\ N$$

FOR ROLLER 2



$\Sigma F_x = 0$

$$\Rightarrow R_c - R_D \sin 60^\circ - R_B \sin 30^\circ = 0$$

$$R_c - \frac{R_B}{2} = \frac{250 \times \sqrt{3}}{2} \quad \text{---(1)}$$

$\Sigma F_y = 0$

$$\Rightarrow -500 - R_D \cos 60^\circ + R_B \cos 30^\circ = 0$$

$$R_B \cos 30^\circ = 500 + 250 \times \tfrac{1}{2} = 625$$

$$R_B = \frac{625 \times 2}{\sqrt{3}} = 721.7\ N$$

from eq (1)

$$R_c - 721.7 \times \tfrac{1}{2} = \frac{250 \times \sqrt{3}}{2}$$

$$\Rightarrow R_c = 577.4\ N \text{ Ans.}$$

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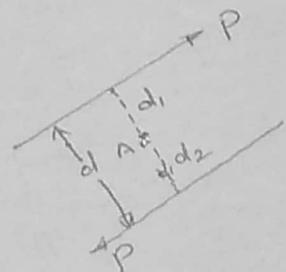
COUPLE.

"Two parellal forces equal in magnitude and opposite and separated by a definite distance are said to be form a couple"



$M_A = p d_1 + p d_2$
$= p(d_1 + d_2)$
$= pd$
$=$ force $\times$ $\perp^r$ distance between forces.

COPLANAR NON CONCURRENT FORCES

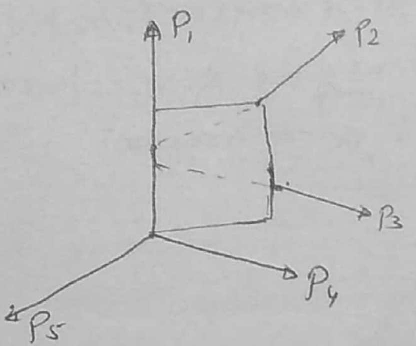


fig (a)

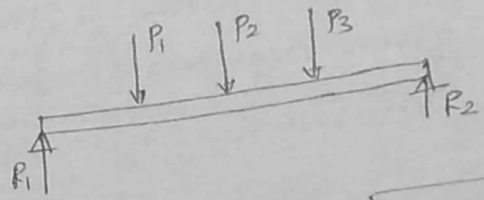
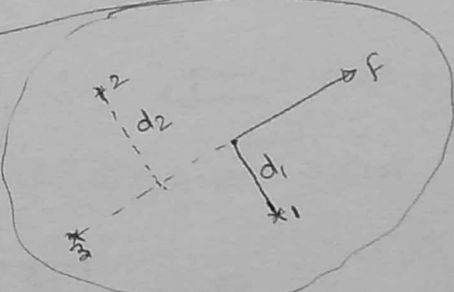


fig (b)

Forces and moment two important agents responsible of motion of a body. Force will cause a linear motion while moment will cause a angular displacement.

MOMENT OF A FORCE.



Moment of a force about any point is the measure of its rotational effect. Moment is defined as the product of force and perpendicular distance of point from the line of action of the force.

The point about ~~at~~ which the moment is considerd is called moment centre" and perpendicular distance of the point from the line of action of the force is called "moment arm".

Moment of force about point '1' ie $M_1 = Fd_1$

" " " " 2 ie $M_2 = Fd_2$

" " " " 3 ie $M_3 = F \times 0 = 0$

Note: If point lie on the line of action of force F, the moment of force about that point is zero.

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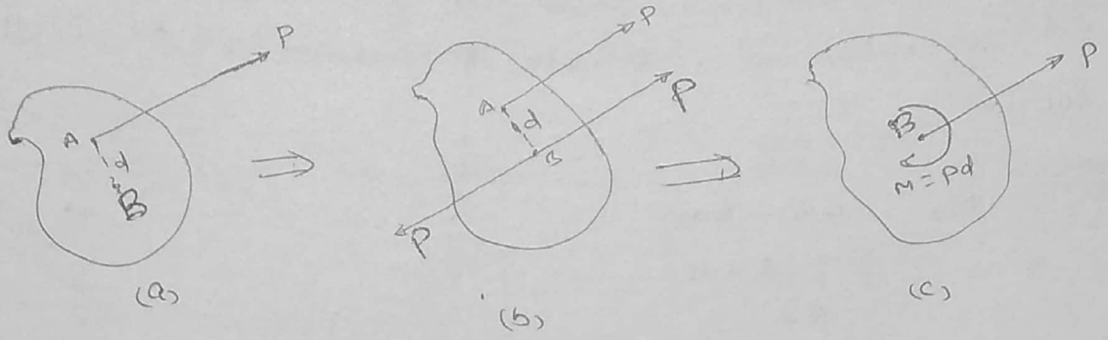
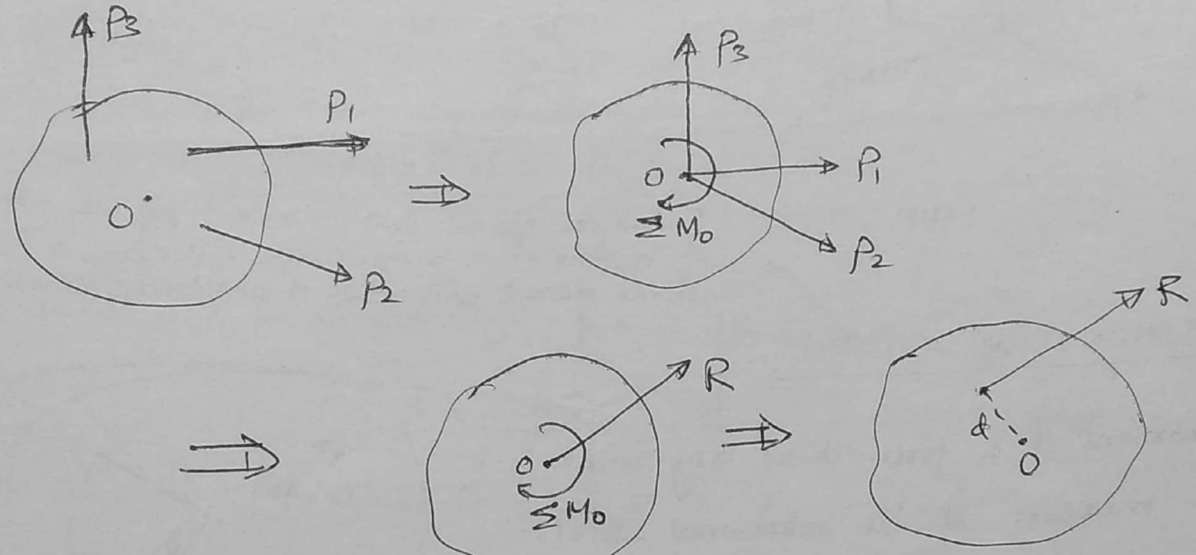
RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE.



RESULTANT OF FORCE SYSTEM'S

The resultant of force system is the one which will have the same rotational and translational effect as the given system of forces. It may be single force, a pure moment or a force and a moment.



$$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$$

$$\tan\alpha = \frac{\Sigma f_y}{\Sigma f_x}$$

$$\boxed{d = \frac{\Sigma M_o}{R}}$$

Q11 A system of load acting on 'a' beam is shown in fig.1
Determine the resultant of loads

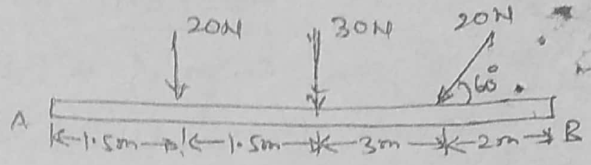


fig 1

$\Sigma f_x = -20\cos 60^\circ = -10$ N

$\Sigma f_y = -20 - 30 - 20\sin 60^\circ = -67.32$ N

$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2} = \sqrt{(-10)^2 + (-67.32)^2}$

$= 68.1$ N

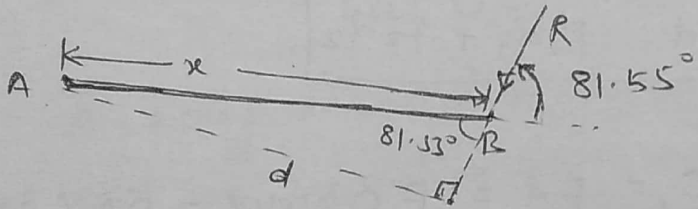
$\tan\alpha = \dfrac{\Sigma f_y}{\Sigma f_x} \Rightarrow \alpha = 81.55^\circ$

Now taking moment about point A.

$\Sigma M_A = 20 \times 1.5 + 30 \times 3 + 20\sin 60^\circ \times 6 = 223.92$ N

The distance of resultant from point A.

$d = \dfrac{\Sigma M_A}{R} = \dfrac{223.92}{68.0592} = 3.29$ m



$x \sin\alpha = d$

$x = \dfrac{d}{\sin\alpha} = \dfrac{3.29}{\sin 81.55} = 3.326$ m

The value of x intercept can also be calculated by using

$x = \dfrac{\Sigma M_A}{\Sigma R_y} = \dfrac{223.9231}{67.3205} = 3.326$ m.

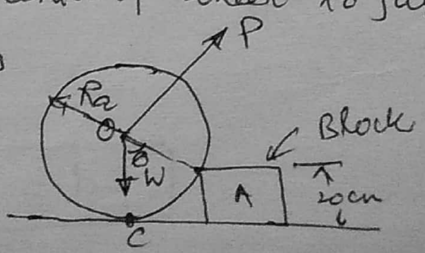
Q. A uniform wheel of 50 cm diameter and 1 kN weight rest against a rigid rectangular block of thickness 20 cm (fig.1) Considering all surfaces smooth, determine

(a) least pull to be applied through the centre of wheel to just turn it over the corner of the block,

(b) Reaction of the block

Ans $P = 0.8$ kN

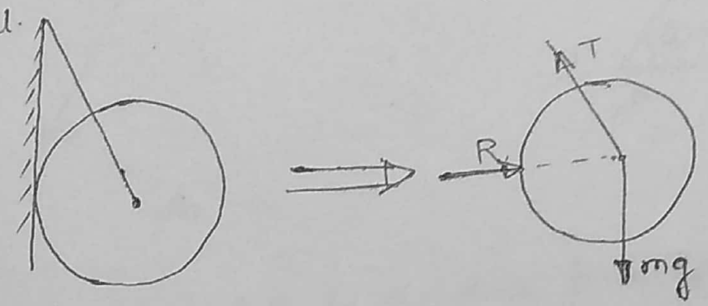
$R_a = 0.6$ kN



FREE BODY DIAGRAM

A free body diagram of a body is a diagram of the body in which the body under consideration is freed from all contact surfaces with reaction forces and the diagram of the body is shown with applied forces and reaction forces at points where the body makes contact with other surfaces.

1) A circular roller of weight W hangs by string and rests against a smooth vertical wall.

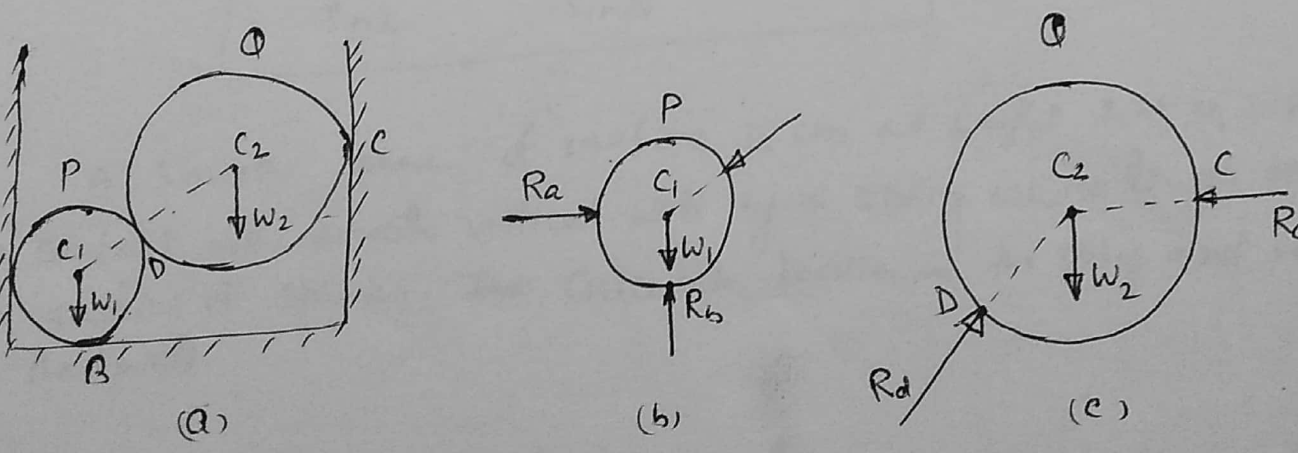


Free body diagram

A body is said to be in equilibrium when it is at rest or in ~~motion~~ uniform motion. It means that the resultant of all forces acting is zero. Mathematically

$$\Sigma F_x = 0 , \quad \Sigma F_Y = 0 , \quad \Sigma M = 0$$

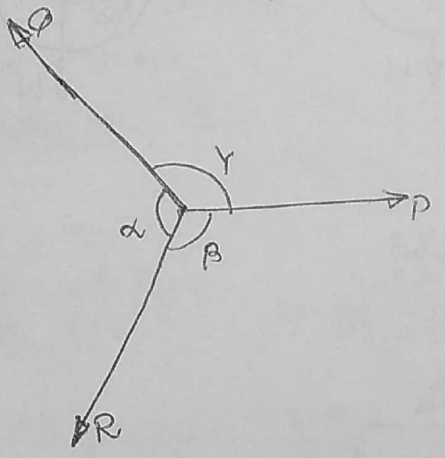
ii) Two sphere P and Q placed in a vessel.



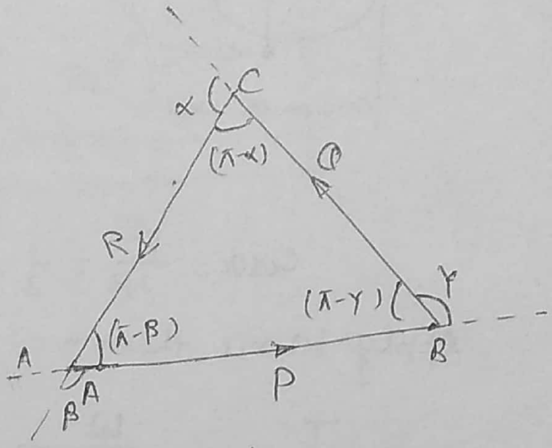
Free body diagram

LAMI'S THEOREM

"If a body is in equilibrium under the action of three forces, then each force is proportional to the sine of the angle between the other two force."



(a)



(b)

Let P, Q and R be the three forces acting on a body along the directions as indicated in fig (a). Since these forces are in equilibrium, they can be represented by the sides of ABC

Applying the Sine rule for triangle ABC

$$\frac{P}{\sin(\pi-\alpha)} = \frac{Q}{\sin(\pi-\beta)} = \frac{R}{\sin(\pi-\gamma)}$$

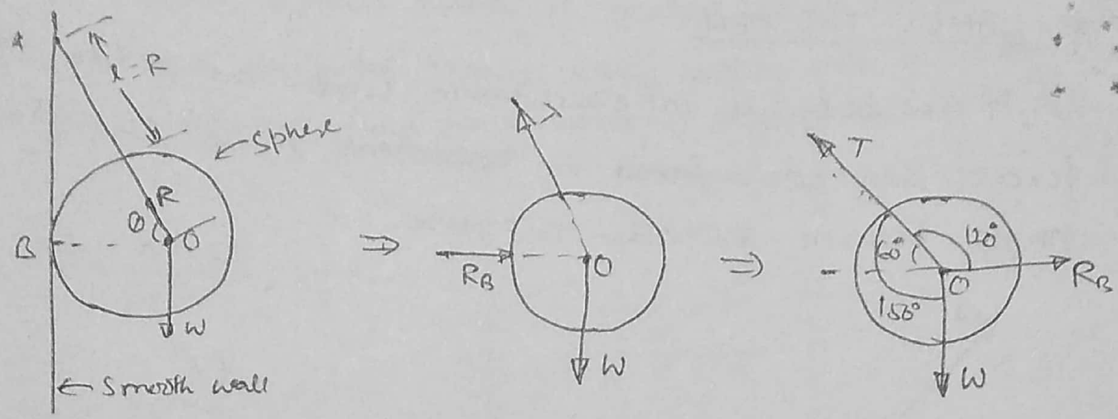
$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

Condition:

(I) Three forces should be co-planner

(II) Three forces should be concurrent.

Q A smooth sphere of radius 15 cm and weight 2 N is supported in contact with smooth vertical wall by a string whose length equals to radius of sphere. ~~The~~ Calculate tension in the string and reaction of the wall.



$$\cos\theta = \frac{R}{2R} = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

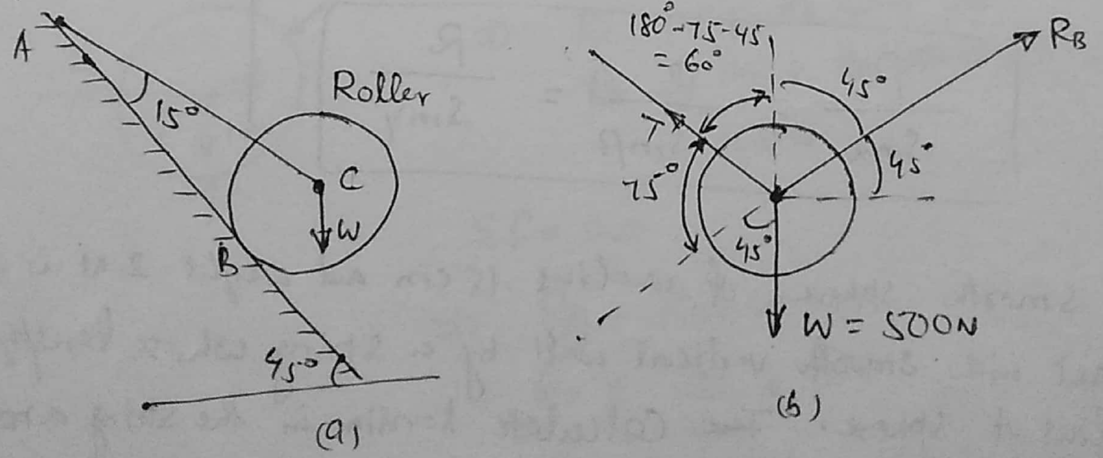
Apply Lamis theorem

$$\frac{T}{\sin 90^\circ} = \frac{W}{\sin 120^\circ} = \frac{R_B}{\sin 150^\circ}$$

$$T = W\frac{\sin 90^\circ}{\sin 120^\circ} = 2 \times \frac{1}{0.866} = 2.31N$$

$$R_B = W\frac{\sin 150^\circ}{\sin 120^\circ} = 2 \times \frac{0.5}{0.866} = 1.15N$$

Q. A roller of weight 500N rests on a smooth inclined plane and (fig). find tension in the string and reaction at the point of contact.



Apply LAMIS Theorem

$$\frac{W}{\sin(60^\circ+45^\circ)} = \frac{R_B}{\sin(75^\circ+45)} = \frac{T}{\sin(90+45^\circ)}$$

$$R_B = 500 \times \frac{\sin 120^\circ}{\sin 105^\circ} = 500 \times \frac{0.866}{0.966} = 448.24N$$

$$T = 500\frac{\sin 135^\circ}{\sin 105^\circ} = 365.94N$$

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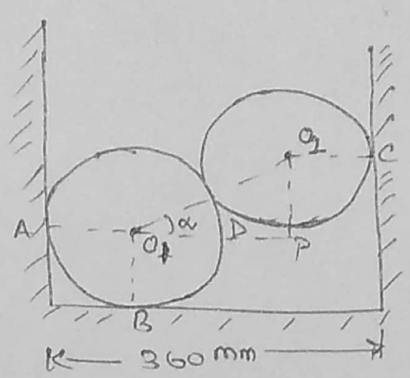
Q. Two smooth spheres each of radius 100 mm and weight 100N, rests in a horizontal channel having vertical walls, the distance between which is 360 mm. Find the reactions at point of contact A, B, C and D.

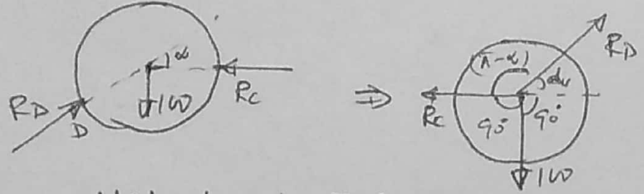


(a)

$$\cos\alpha = \frac{360-100-100}{100+100} = 0.8$$

$$\sin\alpha = 0.6$$

Consider sphere 2



Apply LAMI'S THEOREM

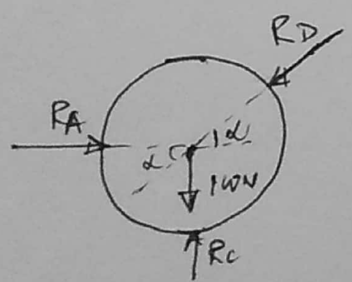
$$\frac{R_C}{\sin(90°+\alpha)} = \frac{R_D}{\sin 90°} = \frac{100}{\sin(\pi-\alpha)}$$

$$\frac{R_C}{\cos\alpha} = \frac{R_D}{1} = \frac{100}{\sin\alpha}$$

$$R_C = 100\,\frac{\cos\alpha}{\sin\alpha} = 100 \times \frac{0.8}{0.6} = 133.33N$$

$$R_D = \frac{100}{\sin\alpha} = \frac{100}{0.6} = 166.67N.$$

Consider sphere 1



$$\Sigma F_y = 0$$

$$\Rightarrow R_C - 100 - R_D \sin\alpha = 0$$

$$R_C = 100 + \cancel{166.67}\ \frac{100}{0.6} \times 0.6$$

$$= 200N$$

$$\Sigma F_x = 0$$

$$\Rightarrow R_A - R_D\cos\alpha = 0$$

$$R_A = \frac{100}{0.6} \times 0.8$$

$$= 100 \times \frac{4}{3} = 133.33N \text{ Ans.}$$

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