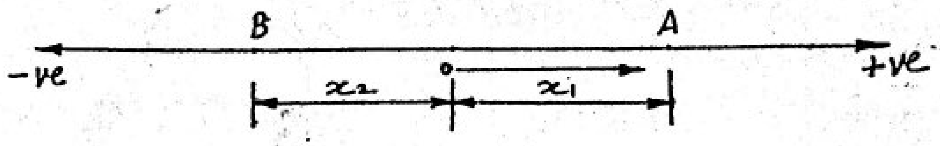
DISTANCE: The total distance covered by the particle or body along the path is known as distance.



Let a body move from reference point O to position A and then from A to position B.

Displacement of body $= -x_2$

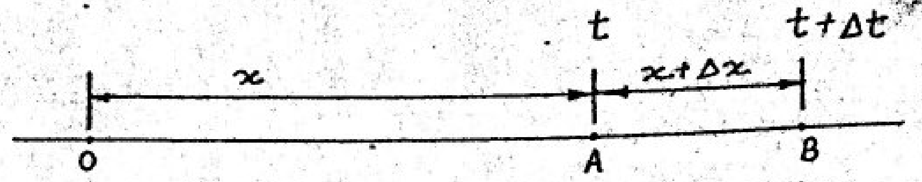
Distance of body $= x_1 + x_1 + x_2$

$= 2x_1 + x_2$

	DISPLACEMENT	DISTANCE
1.	Shortest distance covered by the particle from reference point.	Total distance covered by the particle along the path.
2.	It is taken - positive to right of reference point and negative to its left.	It is always taken as positive.
3.	It has both magnitude and direction.	It has magnitude only.
4.	It is a vector quantity.	It is a scalar quantity.

VELOCITY: The rate of change of position of a body with respect to time is called velocity.

The position A of a body at time t is at a distance x from reference point O. At time $(t+\Delta t)$, the body moves to position B at a distance $x+\Delta x$ from O.



Now, the average velocity of body over time interval Δt is

$$v_{av} = \frac{\Delta x}{\Delta t}$$

<u>INSTANTANEOUS VELOCITY</u> : It is defined as the velocity at a particular instant of time.

Taking time interval Δt and displacement Δx to be very very small.

$$\text{Instantaneous velocity}, v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$= \frac{dx}{dt}$$

1) Velocity is a measure of rate of change of position in a particular direction and hence is a vector quantity.

2) Velocity is positive if displacement is increasing and moving in positive direction.

ACCELERATION : The rate of change of velocity of a body with respect to time.

Let 'v' be the velocity at time t and later the velocity becomes $(v+dv)$ at time $(t+\Delta t)$.

$$\text{Average acceleration}, a = \frac{\Delta v}{\Delta t}$$

If time interval Δt and change in velocity Δv is taken to be very very small.

$$\text{Acceleration, } a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$= \frac{dv}{dt}$$

Acceleration is positive if velocity is increasing and body is moving in positive direction.

$$a = \frac{dv}{dt} \quad , \quad v = \frac{dx}{dt}$$

$$\therefore \quad a = \frac{d^2x}{dt^2}$$

Also $\quad a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt}$

$$\boxed{a = v \frac{dv}{dx}}$$

UNIFORM MOTION : When a body moves with constant velocity (i.e when acceleration is zero), the body is said to have uniform motion.

UNIFORMLY ACCELERATED MOTION : A body having a constant acceleration is referred to as uniformly accelerated motion.

GRAPHICAL REPRESENTATION

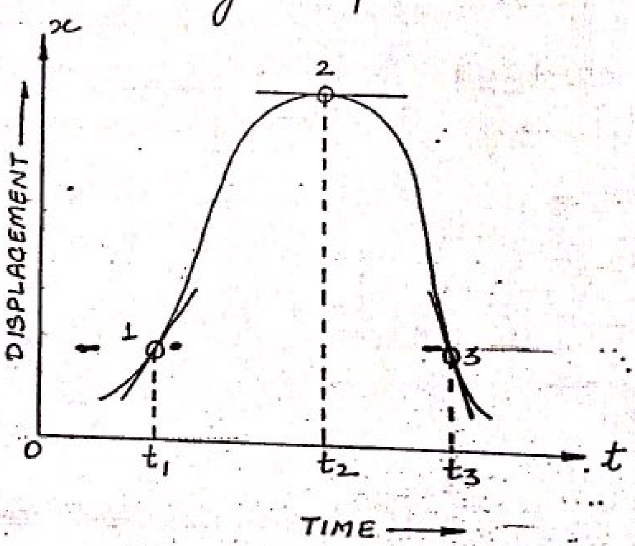
1.) DISPLACEMENT - TIME GRAPH ($x-t$ CURVE)

Displacement of the body is plotted with respect to time, displacement lies along y axis and time along x axis.

As $v = \frac{dx}{dt}$.

the slope of x-t curve at any instant gives the velocity of body at that instant.

a) At time t_1, the curve has a positive slope and hence the velocity is positive.



b) At time t_2, the slope is zero and hence the velocity of body is zero. The body is at rest.

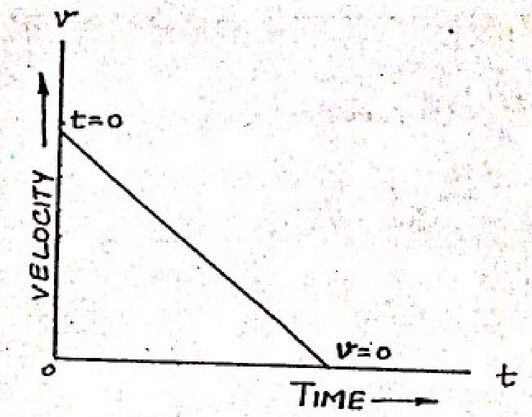
c) At time t_3, the curve has negative slope and hence the velocity is negative.

2.) VELOCITY - TIME GRAPH ($v-t$ CURVE)

The velocity is plotted as a function of time:

As $a = \frac{dv}{dt}$, the slope of v-t curve gives

b) The body has finite velocity and with passage of time velocity decreases linearly with time. Finally velocity becomes zero and body comes to rest. The body moves with constant retardation (negative acceleration).



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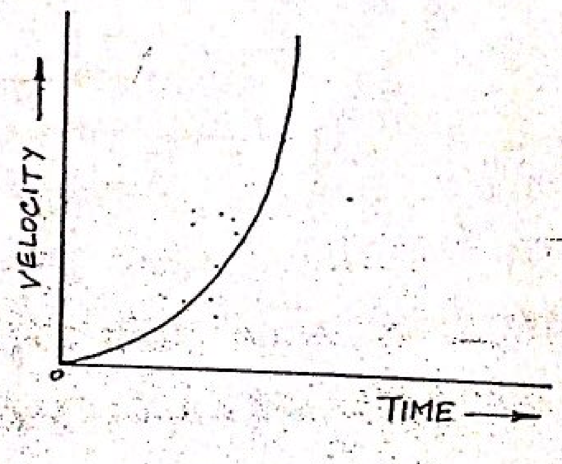
c) The slope of v-t curve is different at different times. No uniform acceleration is there as the velocity is not changing at constant interval of time.



3) ACCELERATION-TIME GRAPH (a-t CURVE)

The acceleration of the body is plotted as the function of time.

We know,

$$a = \frac{dv}{dt}$$

$$\int dv = \int a\,dt$$

$$v_2 - v_1 = \int a\,dt$$

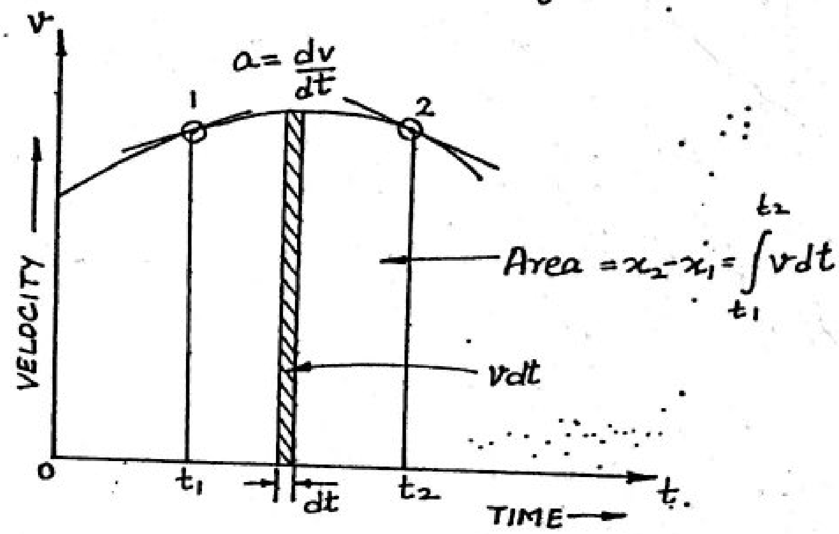
acceleration at any instant.

We know, $v = \frac{dx}{dt}$

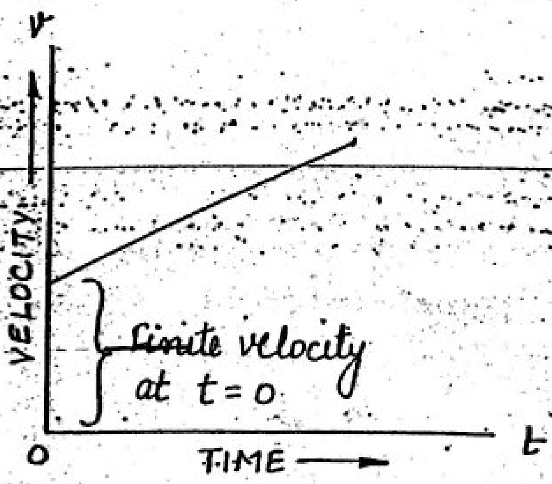
$$\int dx = \int v\,dt$$

$$x_2 - x_1 = \int v\,dt$$

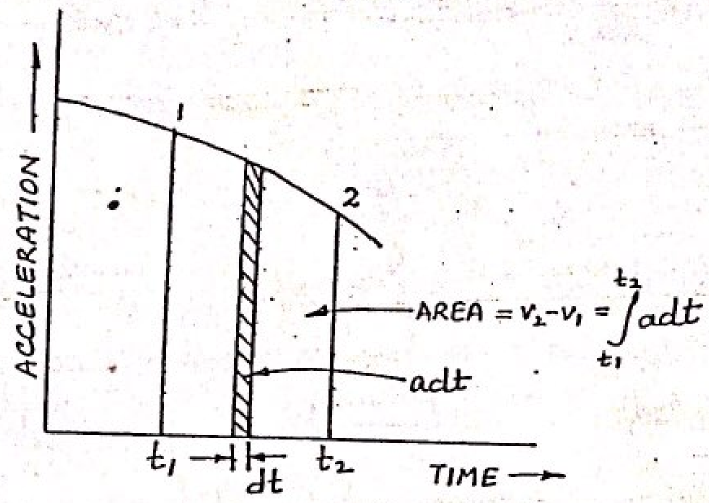
The above expression implies that area under v-t curve in a given time interval gives change in displacement or distance travelled during same interval



a) The velocity increases linearly with time.
As v-t curve has constant slope, the velocity increases by equal amounts at equal intervals of time. Hence the body moves with constant acceleration.



The above expression implies that the area under v-t curve in a given time interval gives change in velocity during same time interval.



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MOTION WITH UNIFORM ACCELERATION

1.) Acceleration, a = Constant

We know,
$$a = \frac{dv}{dt}$$
$$dv = a\,dt$$
$$\int_u^v dv = \int_0^t a\,dt$$
$$v - u = at$$
$$\boxed{v = u + at}$$

Also a = Rate of change of velocity
$$= \frac{\text{Change in velocity}}{\text{Time}}$$
$$= \frac{v-u}{t}$$
$$v - u = at$$
$$\boxed{v = u + at}$$

2.) Distance travelled = Average velocity × Time

$$s = \frac{u+v}{2} \times t$$

$$= \frac{u+u+at}{2} \times t \qquad [\because v = u+at]$$

$$= \left[u + \frac{at}{2}\right]t$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

Also

$$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$dx = (u+at)\,dt$$

$$\int_0^x dx = \int_0^t (u+at)\,dt$$

$$x = ut + \frac{at^2}{2}$$

$$\boxed{x = ut + \frac{1}{2}at^2}$$

3.) Distance = Average velocity × Time

$$s = \frac{u+v}{2} \times t \qquad \left[\because v = u+at, \; t = \frac{v-u}{a}\right]$$

$$= \frac{u+v}{2} \times \frac{v-u}{a}$$

$$= \frac{v^2-u^2}{2a}$$

$$\boxed{v^2-u^2 = 2as}$$

Q: A body is moving with uniform acceleration and covers 15 m in fifth second and 25 m in 10th second. Determine :

1) Initial velocity of the body
2) Acceleration of the body.

SOL^N: Distance covered in 5th second = 15m
Distance covered in 10th second = 25m

Let $u \to$ Initial velocity
$a \to$ Acceleration

Distance covered in 5th second

$$15 = u + \frac{a}{2}(2\times5-1)$$

$$15 = u + \frac{9a}{2} \quad \text{———} \ (1)$$

Distance covered in 10th second

$$25 = u + \frac{a}{2}(2\times10-1)$$

$$25 = u + \frac{19a}{2} \quad \text{———} \ (2)$$

Solving (1) and (2)

$$15 = u + \frac{9a}{2}$$

$$25 = u + \frac{19a}{2}$$

$$-10 = \left(\frac{9a}{2} - \frac{19a}{2}\right)$$

$$-10 = \frac{-10a}{2}$$

$$\boxed{a = 2\ m/sec^2}$$

$$15 = u + \frac{9\times2}{2}$$

$$u = 15 - 9$$

$$\boxed{u = 6\ m/sec}$$

6

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Also, $$a = v\frac{dv}{dx}$$

$$a\,dx = v\,dv$$

$$\int_0^x a\,dx = \int_u^v v\,dv$$

$$ax = \left[\frac{v^2}{2}\right]_u^v$$

$$ax = \frac{v^2-u^2}{2}$$

$$\boxed{v^2-u^2 = 2ax}$$

4.) Distance covered in n seconds

$$S_n = un + \frac{1}{2}an^2$$

Distance covered in (n-1) seconds

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$= u(n-1) + \frac{1}{2}a(n^2+1-2n)$$

$\therefore$ Distance covered in n^{th} second

$$S_{nth} = S_n - S_{n-1}$$

$$= un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n^2+1-2n)$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 - \frac{1}{2}a + an$$

$$= u - \frac{a}{2} + an$$

$$S_{nth} = u - \frac{a+2an}{2}$$

$$\boxed{S_{nth} = u + (2n-1)\frac{a}{2}}$$

MOTION WITH VARIABLE ACCELERATION

The motion under gravity is a special case of this motion type.

The four quantities displacement, velocity, acceleration and time are related as:

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$$

$$a = \frac{d^2x}{dt^2} \qquad a = v\frac{dv}{dx}$$

Equations of motions due to gravity in the downward and upward directions :

1) DOWNWARD MOTION

$$a = +g$$

$$v = u + gt$$

$$s = ut + \frac{1}{2}gt^2$$

$$v^2 - u^2 = 2gs$$

2) UPWARD MOTION

$$a = -g$$

$$v = u - gt$$

$$s = ut - \frac{1}{2}gt^2$$

$$v^2 - u^2 = -2gs$$

IMPORTANT POINTS.

1) If a body starts from rest, its initial velocity is zero.

$$u = 0$$

2) If a body comes to rest, its final velocity is zero.

$$v = 0$$

3) If a body is projected vertically upwards, the final velocity of the body at highest point is zero.

$$v = 0$$

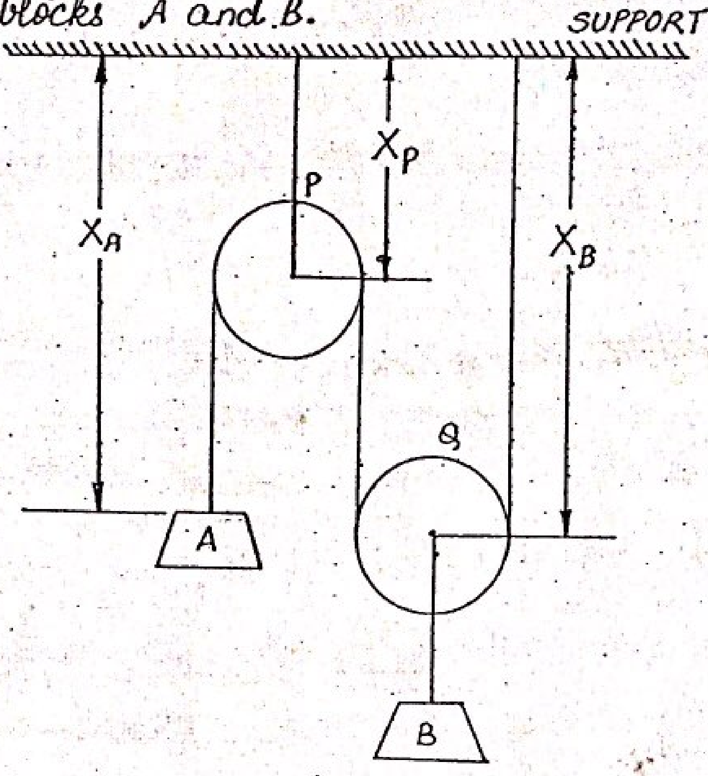
4) If a body starting to move vertically downwards, its initial velocity is zero

$$u = 0$$

5) Acceleration due to gravity is taken positive when a body is moving vertically downwards. But if the body is moving vertically upwards, the acceleration due to gravity is taken negative.

6) Displacement of the particle is always zero at time $t = 0$.

Q. A system of two pulleys is there. Find the relations connecting displacements, velocities and accelerations of the blocks A and B.



SOL^N: Measuring all the distances with respect to support.
Let length of rope be $= l$.
Effective length of rope

$$= (X_A - X_P) + (X_B - X_P) + X_B = l.$$

$$X_A + 2X_B - 2X_P = l$$

$$\boxed{X_A + 2X_B = l + 2X_P}$$

~~If the block A be lowered by distance Δx_A~~

$$\Delta x_A + 2\Delta x_B = 0 \qquad \left\{ 2\Delta x_B = -\Delta x_A \right.$$

$$\boxed{\Delta x_B = -\frac{\Delta x_A}{2}}$$

Differentiating the above equation w.r.t time

$$\frac{dx_A}{dt} + \frac{2dx_B}{dt} = 0$$

$$v_A + 2v_B = 0$$

$$\boxed{v_B = -\frac{v_A}{2}}$$

Differentiating again

$$\frac{d^2x_A}{dt^2} + \frac{2d^2x_B}{dt^2} = 0$$

$$a_A + 2a_B = 0$$

$$\boxed{a_B = -\frac{a_A}{2}}$$

Q. The rectilinear motion of a motor car starting from rest is prescribed by the relation $a = \frac{6}{1\cdot5V+2}$ where a is the acceleration in m/s^2 and V is the velocity in m/s. Calculate the time taken and distance covered by the motor car to attain the velocity of 6m/s.

Sol^N: Given $a = \frac{6}{1\cdot5V+2}$

We know,

$$a = \frac{dV}{dt} = \frac{6}{1\cdot5V+2}$$

$$(1\cdot5V+2)\,dV = 6\,dt$$

Given

$$u=0 \;,\; V=6\ m/s.$$

$\therefore$ Integrating with limits $u=0, t=0$ and $v=6, t=t$.

$$\int_0^6 (1.5v+2)\,dv = \int_0^t 6\,dt$$

$$\left[\frac{1.5v^2}{2} + 2v\right]_0^6 = \left[6t\right]$$

$$27 + 12 = 6t$$

9

$$6t = 39$$

$$t = \frac{39}{6}$$

$$\boxed{t = 6.5 \text{ sec}}$$

We know,

$$a = v\frac{dv}{dx}$$

$$v\frac{dv}{dx} = \frac{6}{1.5v+2}$$

$$(1.5v+2)v\,dv = 6\,dx$$

Integrating with limits $u=0, x=0$ and $v=6, x=x$

$$\int_0^6 (1.5v^2+2v)\,dv = \int_0^x 6\,dx$$

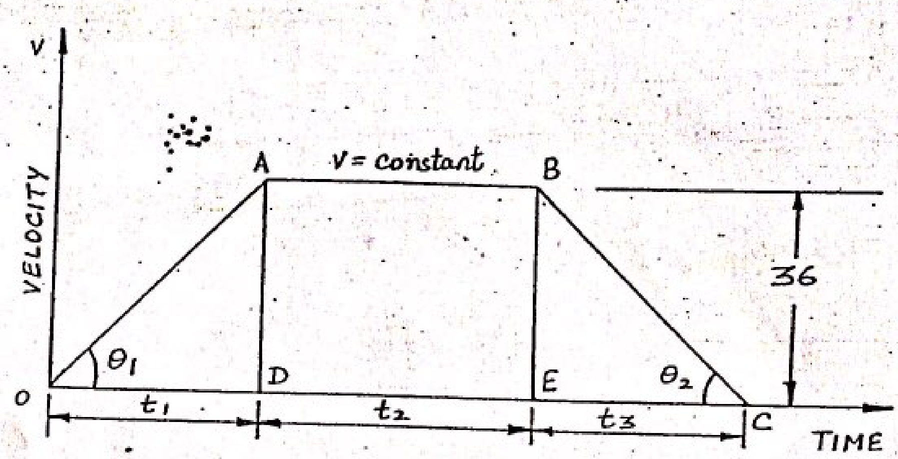
$$\left[\frac{1.5v^3}{3} + \frac{2v^2}{2}\right]_0^6 = \left[6x\right]$$

$$108 + 36 = 6x$$

$$6x = 144$$

$$\boxed{x = 24m}$$

Q. An electric train runs between two stations which are 2.5 Km apart and takes 6 minutes from start to stop. The train has a constant running speed of 36 Km/hr between the end of acceleration and beginning of retardation. If the acceleration and retardation are both uniform and numerically equal to each other, make calculations for their values.



SOL^N: Slope of velocity-time graph gives acceleration.

As acceleration = retardation [numerically, given]

$$\therefore \quad \tan\theta_1 = \tan\theta_2$$

$$\therefore \quad \theta_1 = \theta_2$$

We know $\quad \Delta OAD \simeq \Delta BEC$

$$\therefore \quad OD = EC$$

$$t_1 = t_2$$

Total travel time from start to stop = 6 minutes

$$6 \text{ minutes} = 0.1 \text{ hr}.$$

By graph

$$t_1 + t_2 + t_3 = 0.1$$

$$2t_1 + t_2 = 0.1 \qquad (t_1 = t_2)$$

The area of velocity-time graph gives distance travelled during any time interval.

$$S = S_1 + S_2 + S_3$$

$$= \text{Area of } \Delta AOD + \text{Area of rectangle } ABED + \text{Area of } \Delta BEC$$

$$2.5 = \frac{1}{2} \times 36 t_1 + t_2 \times 36 + \frac{1}{2} \times 36 t_3$$ 10

$$2.5 = 18t_1 + 36t_2 + 18t_3$$

$$2.5 = 18t_1 + 36t_2 + 18t_1$$

$$2.5 = 36t_1 + 36t_2$$

$$t_1 + t_2 = \frac{2.5}{36}$$

$$t_1 + t_2 = 0.0694$$

Using equation $2t_1 + t_2 = 0.1$

$$- \quad t_1 + t_2 = 0.0694$$

$$t_1 = 0.0306 \text{ hr}$$

$$\boxed{t_1 = 0.0306 \text{ hr}}$$

Consider ΔOAD

$$a = \frac{V_a - V_o}{t_1}$$

$$= \frac{36 - 0}{0.0306}$$

$$\boxed{a = 1176.47 \text{ Km/hr}^2}$$

As acceleration = retardation numerically

$$\therefore \quad a' = 1176.47 \text{ Km/hr}^2$$

retardation →

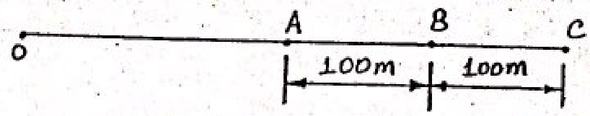
Q Three marks A, B and C at a distance of 100 m each are made along a straight road. A car starting from rest and with uniform acceleration passes the mark A and takes 10 seconds to reach B and further 8 second to reach the mark C. Calculate

(i) the magnitude of the acceleration of the car.

(ii) the velocity of the car at A.

(iii) the velocity of car at B.

(iv) the distance of the mark A from the starting point.



SOLN : Distance AB = 100m

Distance BC = 100m

Initial velocity of car, $u = 0$

Time taken from A to B

$t_1 = 10$ sec.

Time taken from B to C

$t_2 = 8$ sec.

Let a = Acceleration of car.

Velocity of car at A = v_A

Velocity of car at B = v_B

Motion of car from A to B

$$s = ut + \frac{1}{2}at^2$$

$$100 = v_A 10 + \frac{1}{2} a \times 10^2$$

$$100 = 10v_A + 50a \quad \text{--- (1)}$$

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∴ Motion of car from A to C.

$$s = ut + \frac{1}{2}at^2$$

$$100 + 100 = v_A \times 18 + \frac{1}{2}a(18)^2$$

[Total time = 10+8 = 18 sec.

$$200 = 18v_A + 162a. \quad \text{—— (2)}$$

∴ Solving (1) and (2)

~~200 = 20v_A + 100a.~~
~~200 = 18v_A + 162a~~
~~0 = 2v_A − 62a.~~

∴ Multiply (1) by 9 and (2) by 5

$$900 = 90v_A + 450a.$$

$$1000 = 90v_A + 810a.$$

$$-100 = -360a$$

$$\therefore a = \frac{100}{360}$$

$$\boxed{a = 0.278 \text{ m/sec}^2}$$

$$200 = 18v_A + 162 \times 0.278$$

$$18v_A = 200 - 45.036$$

$$18v_A = 154.964$$

$$\boxed{v_A = 8.61 \text{ m/sec}}$$

Consider motion of car from B to C.

$$100 = v_B \times 8 + \frac{1}{2}(0.278) \times 8^2$$

$$v_B = \frac{100 - 8.896}{8}$$

$$\boxed{v_B = 11.388 \text{ m/sec}}$$

Let the distance from O to A be = s.

using

$$v^2 - u^2 = 2as.$$

$$v_A^2 - 0 = 2 \times 0.278 s$$

$$(8.61)^2 = 2 \times 0.278 s$$

$$S = \frac{(8.61)^2}{2 \times 0.278}$$

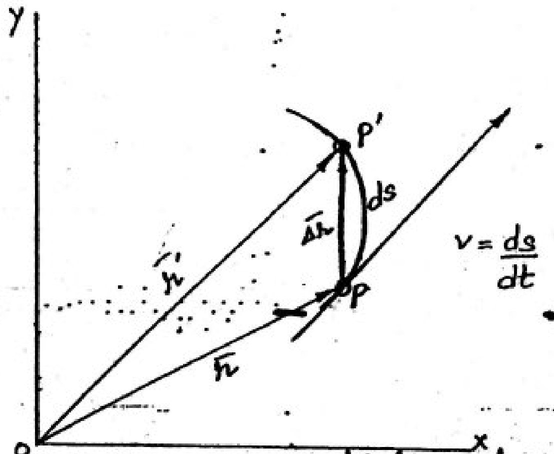
$$S = 133.33 \text{ m}$$

CURVILINEAR MOTION:

When a moving particle describes a curved path, it is said to have a curvilinear motion.

POSITION VECTOR, VELOCITY AND ACCELERATION

POSITION VECTOR: It defines the position of a particle moving along a curved path at any instant with respect to a reference axis.



Consider motion of a particle along a curved path and also choose a reference axes x-y. At time 't' let the position of particle be at P. Join O and P. The line OP is called position vector r of the point P.

At time 't+Δt', the position be P' for the particle. The position is defined by OP' i.e. r'.

The vector Δr joining P and P' gives change in the position vector r during time interval Δt.

Verification by triangle law :

$$r + \Delta r = r'$$

$$\Delta r = r' - r.$$

Change in position vector.

Gives both change in magnitude and change in direction.

VELOCITY : Instantaneous velocity is given by

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

Magnitude of Δr = Length of line segment PP'

As $\Delta t \to 0$, length of line segment approaches length Δs of the arc PP'.

$$\therefore \quad v = \lim_{\Delta t \to 0} \frac{PP'}{\Delta t}$$

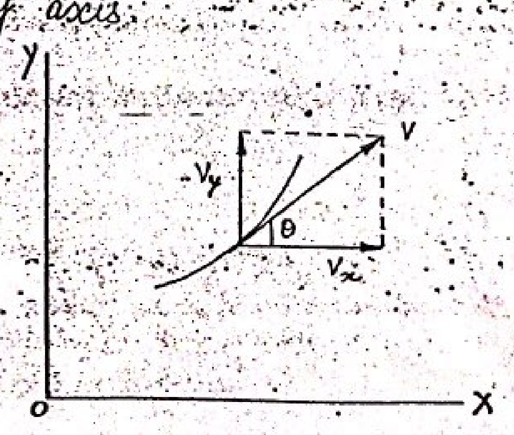
$$= \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}.$$

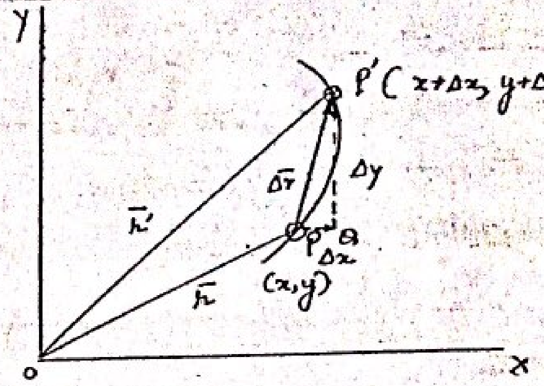
Length of arc described by the particle.

COMPONENTS OF MOTION : RECTANGULAR COMPONENTS

RECTANGULAR COMPONENTS OF VELOCITY :

As the direction of the velocity continuously changes therefore it is resolved in its components along x axis and y axis.





Calculation of v_x and v_y:

Resolve Δr into two components $\overline{PQ}$ and $\overline{QP'}$ parallel to x and y axes.

$$\therefore \quad \Delta r = \overline{PQ} + \overline{QP'}$$

We know,

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{(\overline{PQ} + \overline{QP'})}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\overline{PQ}}{\Delta t} + \lim_{\Delta t \to 0} \frac{\overline{QP'}}{\Delta t}$$

$\overline{QP'} \rightarrow$ Change in velocity along y axis, Δy.

$\overline{PQ} \rightarrow$ Change in velocity along x axis, Δx.

$$\therefore \quad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

$\boxed{v = v_x + v_y}$ $\longrightarrow$ VECTOR SUM

$$\therefore \quad v_x = \frac{dx}{dt} = \dot{x}$$

$$v_y = \frac{dy}{dt} = \dot{y}$$

Magnitude, $v = \sqrt{v_x^2 + v_y^2}$ Direction, $\theta = \tan^{-1} \frac{v_y}{v_x}$

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ACCELERATION :

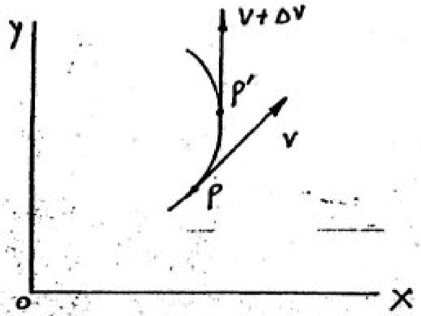
Consider a particle at a position P moving with velocity v at any time t.
After time Δt, position of particle changes to P' and velocity changes to $v+\Delta v$.

Instantaneous acceleration is given by

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

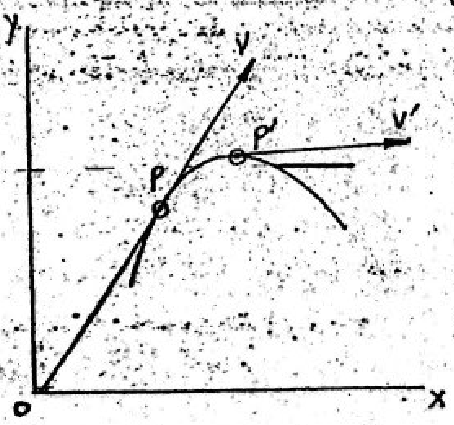
$$a = \frac{dv}{dt}$$

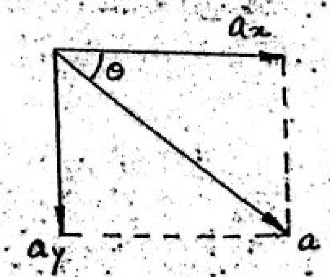
Acceleration of the particle at any instant is not tangential to path of particle. Hence direction of acceleration and velocity may not be same in a curvilinear motion.



RECTANGULAR COMPONENTS OF ACCELERATION :

Rectangular components of acceleration a are a_x and a_y parallel to x and y axes.





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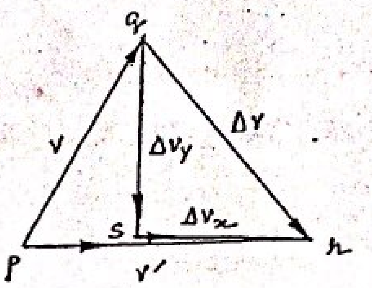


Calculations of a_x and a_y:

Vector $\overline{pq} \rightarrow v$

Vector $\overline{pr} \rightarrow v'$

Vector $\overline{qr} \rightarrow \Delta v$ (change in velocity of particle)

Resolve Δv into components Δv_x and Δv_y.

$$\Delta v = \overline{qr} = \overline{qs} + \overline{sr}$$

We know,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{(\overline{qs} + \overline{sr})}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\overline{qs}}{\Delta t} + \lim_{\Delta t \to 0} \frac{\overline{sr}}{\Delta t}$$

As $\Delta v = \Delta v_x + \Delta v_y$.

and $\Delta v_x = \overline{sr}$; $\Delta v_y = \overline{qs}$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}, \quad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$

$\boxed{a = a_x + a_y} \longrightarrow$ VECTOR SUM

Magnitude, $a = \sqrt{a_x^2 + a_y^2}$

Direction, $\theta = \tan^{-1}\dfrac{a_y}{a_x}$

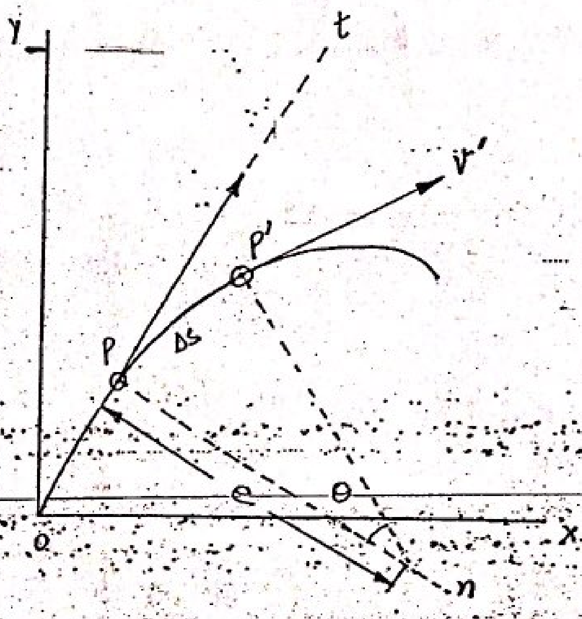
COMPONENTS OF ACCELERATION

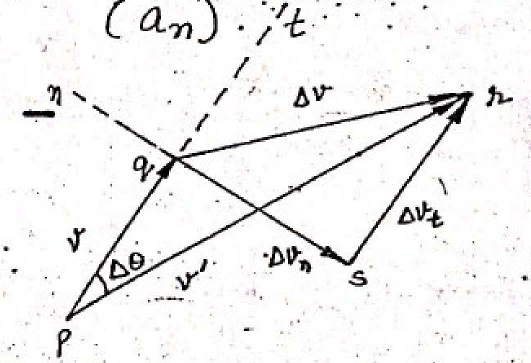
In a curvilinear path, the velocity of particle is a vector tangential to path at any instant. As acceleration may not be tangential to path, it is expressed as components:

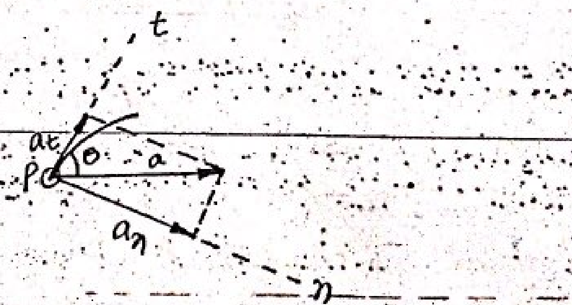
1) In the direction of the tangent to path.
2) In direction normal to path.

Tangential acceleration (a_t)

Normal acceleration (a_n)







Consider a particle having velocity v at time t and velocity v' at time $t + \Delta t$.

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Draw $\overline{pq}$ and $\overline{pr}$ representing v and v' respectively. Side $\overline{qr}$ of triangle represents change in velocity Δv in time Δt.

Resolving Δv into tangential and normal components i.e Δv_t and Δv_n.
Axis are denoted by t and n.

$$\Delta v = \overline{qr}$$
$$= \overline{qs} + \overline{sr}$$

$$\boxed{\Delta v = \Delta v_n + \Delta v_t} \longrightarrow \text{VECTOR SUM}$$

Acceleration, $$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta v_t}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta v_n}{\Delta t}$$

$$a_t = \lim_{\Delta t \to 0} \frac{\Delta v_t}{\Delta t}, \quad a_n = \lim_{\Delta t \to 0} \frac{\Delta v_n}{\Delta t}$$

$$\therefore \boxed{a = a_t + a_n} \longrightarrow \text{VECTOR SUM}$$

TANGENTIAL COMPONENT (a_t) :

$\overline{sr}$ represents change in magnitude of velocity in tangential direction.

$$a_t = \lim_{\Delta t \to 0} \frac{(v' - v)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$a_t = \frac{dv}{dt}$$

Tangential acceleration (a_t) is equal to rate of change of velocity of particle.

It is considered to be positive in direction of tangent coinciding with sense of motion.

NORMAL ACCELERATION (a_n) :

$\overline{qs}$ represents change in direction of velocity of particle in normal axis direction.

$$qs = v\Delta\theta$$

Also $\Delta v_n = v\Delta\theta$

$$\therefore \quad a_n = \lim_{\Delta t \to 0} \frac{v\Delta\theta}{\Delta t}$$

If R is the radius of curvature of curve then,

$$\Delta s = R\Delta\theta$$

$$\Delta\theta = \frac{\Delta s}{R} \longrightarrow \text{length of arc } PP'$$

$$\therefore \quad a_n = \lim_{\Delta t \to 0} \frac{v\Delta\theta}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{v}{R}\frac{ds}{dt}$$

As $\frac{ds}{dt} = v$

$$\boxed{a_n = \frac{v^2}{R}}$$

Direction of normal acceleration is such that it is always directed towards the centre of curvature of path. It is also known as centripetal acceleration (centre-seeking acceleration)

Total acceleration may be caused due to change in magnitude of velocity or change in direction of velocity or both.

$\therefore$

$$a = a_t + a_n$$

$$a_t = \frac{dv}{dt}, \quad a_n = \frac{v^2}{R}$$

Magnitude, $a = \sqrt{a_t^2 + a_n^2}$

Direction, $\theta = \tan^{-1} \frac{a_n}{a_t}$

Let R be equal to radius 'r' of a circular path. Let the constant speed be v.

$$a_n = \frac{v^2}{R}$$

$$= \frac{v^2}{r}$$

$$a_t = \frac{dv}{dt} = 0 \quad (v = \text{constant})$$

$$a = \sqrt{a_t^2 + a_n^2} = a_n$$

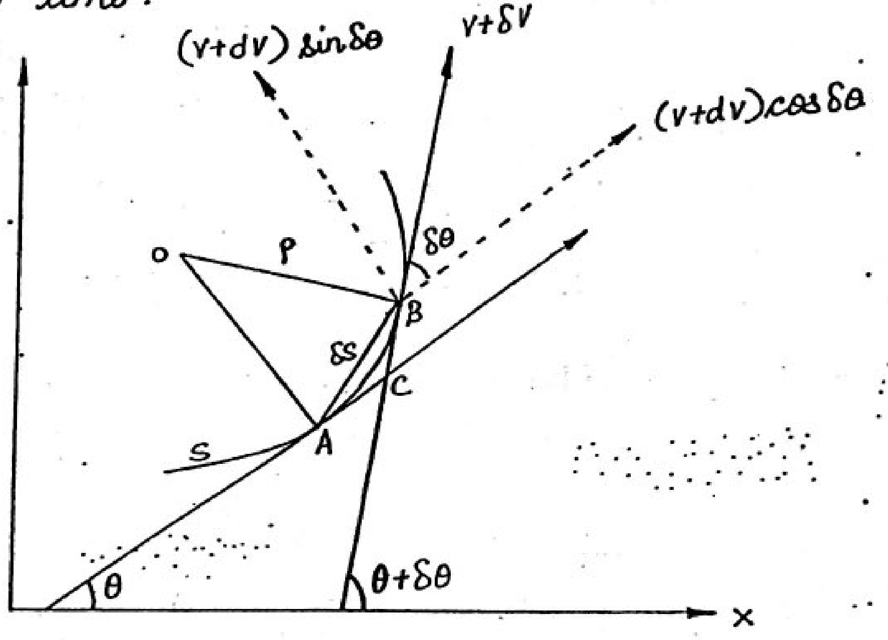
$$\theta = \tan^{-1} \frac{v^2/r}{0}$$

$$\boxed{\theta = 90}$$

Total acceleration 'a' is equal to its normal acceleration a_n and acts in the same direction as a_n.

TANGENTIAL AND NORMAL COORDINATES

Consider a particle that moves along a curved path from point A to point B and traverses an infinitely small distances δs in a small interval of time δt. When the particle moves, the distance S changes with time.



When time interval δt is very small

chord AB = arc AB = δs.

$\angle BAC = \delta\theta$ and hence the components of distance δs travelled along the tangential and normal directions are $\delta s \cos\delta\theta$ and $\delta s \sin\delta\theta$.

Velocity of particle at point A in tangential direction:

$$V_t = \frac{\text{Distance moved along tangential direction}}{\text{Time interval}}$$

$$= \lim_{\delta t \to 0} \frac{\delta s \cos\delta\theta}{\delta t} = \lim_{\delta t \to 0} \frac{\delta s}{\delta t} \cos\delta\theta$$

As $\delta\theta$ is very small, $\cos\delta\theta \to 1$

$$\therefore \quad V_t = \lim_{\delta t \to 0} \frac{\delta s}{\delta t}$$

$$\boxed{V_t = \frac{ds}{dt}}$$

Velocity of particle in normal direction

$$V_n = \frac{\text{Distance moved along normal direction}}{\text{Time interval}}$$

$$= \lim_{\delta t \to 0} \frac{\delta s \sin\delta\theta}{\delta t}$$

As $\delta\theta$ is very small, $\sin\delta\theta \to \delta\theta$

$$\therefore \quad V_n = \lim_{\delta t \to 0} \frac{\delta s \times \delta\theta}{\delta t}$$

$\delta s \times \delta\theta$ is very small, hence neglecting it.

$$\therefore \quad \boxed{V_n = 0}$$

Hence, the velocity of particle moving in curved path is equal to tangential velocity and the direction of velocity is tangential to the path at any instant.

CONCLUSION :

1) At point A, velocity V is directed along tangent that is inclined at an angle θ with horizontal. Velocity in a direction normal to tangent is zero.

2) At point B, velocity $(V+\delta V)$ is directed along tangent making an angle $\delta\theta$ with tangent at point B.

The components of velocity vector $(V+\delta V)$:

a) $(V+\delta V)\cos\delta\theta$ in direction of tangent at B.

b) $(V+\delta V)\sin\delta\theta$ in direction normal to tangent at B.

Tangential acceleration, $a_t = \dfrac{\text{Change in tangential velocity}}{\text{Time interval}}$

$$= \lim_{\delta t \to 0} \frac{(V+\delta V)\cos\delta\theta - V}{\delta t}$$

$$= \lim_{\delta t \to 0} \frac{\delta V \cdot \cos\delta\theta}{\delta t}$$

As δt is very small, arc AB is very small, hence $\delta\theta$ is very small, $\cos\delta\theta \to 1$.

$$\therefore a_t = \lim_{\delta t \to 0} \frac{\delta V}{\delta t}$$

$$\boxed{a_t = \frac{dV}{dt}}$$

Normal acceleration, $a_n = \dfrac{\text{Change in normal velocity}}{\text{Time interval}}$

$$a_n = \lim_{\delta t \to 0} \frac{(V+\delta V)\sin\delta\theta - 0}{\delta t}$$

For small $\delta\theta$, $\sin\delta\theta \to \delta\theta$

$$a_n = \lim_{\delta t \to 0} \frac{(V+\delta V)\times\delta\theta}{\delta t}$$

Neglecting $\delta v \times \delta\theta$ as it is very small.

$$\therefore \quad a_n = \lim_{\delta t \to 0} \frac{V\delta\theta}{\delta t}$$

$$a_n = \frac{V d\theta}{dt}$$

Multiply and divide by dS.

$$a_n = V \frac{d\theta}{dS} \times \frac{dS}{dt}$$

We know,

$$\frac{d\theta}{dS} = \rho \quad , \quad \frac{dS}{dt} = V$$

Radius of curvature of path

$$\therefore \quad \boxed{a_n = \frac{V^2}{\rho}}$$

Tangential acceleration is due to change in magnitude of velocity and normal acceleration is due to change in direction of velocity.

Resultant acceleration, $a = \sqrt{a_t^2 + a_n^2}$

Let resultant acceleration make angle ϕ with normal, then

$$\tan\phi = \frac{a_t}{a_n}$$

1) If magnitude of velocity is constant, then tangential acceleration, $a_t = 0$.

2) If direction does not change, the particle moves along straight line, then $\rho \to \infty$ and $a_n = 0$.

Q1. A motorist is driving at 80 km/hr on the curved position of a highway of 400m radius. He suddenly applies the brakes and that causes the speed to decrease to 45 Km/hr at a constant rate in 8 seconds. Determine the tangential and normal components of acceleration immediately after the application of brakes and 4 seconds later.

SOL^N:

Initial velocity, $u = 80 \text{ Km/hr}$

$$= \frac{80 \times 1000}{3600} = 22.22 \text{ m/s}.$$

Final velocity, $v = 45 \text{ Km/hr}$

$$= \frac{45 \times 1000}{3600} = 12.5 \text{ m/s}.$$

Tangential acceleration, $a_t = \frac{\text{Change in velocity}}{\text{Time}}$

$$= \frac{12.5 - 22.22}{8}$$

$$= -1.215 \text{ m/s}^2$$

Velocity when brakes are applied $= 22.22 \text{ m/s}$.

Normal acceleration, $a_n = \frac{V^2}{r} = \frac{(22.22)^2}{400}$

$$= 1.234 \text{ m/s}^2$$

Total acceleration $= \sqrt{a_n^2 + a_t^2} = \sqrt{(1.234)^2 + (-1.215)^2}$

$$= \sqrt{1.522756 + 1.476225}$$

$$= 1.731 \text{ m/s}^2$$

Velocity of vehicle 4 seconds after application of brakes,

$$v = u + at$$
$$= 22.22 + (-1.215 \times 4)$$
$$= 22.22 - 4.86$$
$$v = 17.36 \text{ m/s}.$$

$$a_t = -1.215 \text{ m/s}^2$$

$$a_n = \frac{v^2}{r} = \frac{(17.36)^2}{400}$$

$$a_n = 0.753 \text{ m/s}^2$$

$$a = \sqrt{a_n^2 + a_t^2}$$
$$= \sqrt{(0.753)^2 + (-1.215)^2}$$
$$= \sqrt{0.567009 + 1.476225}$$
$$a = 1.429 \text{ m/s}^2$$

Q2. The displacement-time equation for the oscillations of a simple pendulum is given by

$$s = S \cos\left(\sqrt{\frac{g}{L}}\right) t$$

where S is maximum displacement of oscillations.

Find i) the maximum velocity

ii) Maximum tangential and normal accelerations of the bob.

SOL^N:

$$s = S \cos\left(\sqrt{\frac{g}{L}}\right) t$$

Tangential velocity

$$v = \frac{ds}{dt}$$

$$v = \frac{d\left[s\cos\left(\sqrt{g/l}\right)t\right]}{dt}$$

$$v = -\sin\sqrt{\frac{g}{l}}\,t \times s\sqrt{\frac{g}{l}}$$

$$\boxed{v_{max} = \pm s\sqrt{\frac{g}{l}}} \qquad \text{when} \quad \sin\sqrt{\frac{g}{l}}\,t = 1.$$

$$a_t = \frac{dv}{dt}$$

$$= -s\sqrt{\frac{g}{l}}\cos\sqrt{\frac{g}{l}}\,t \times \sqrt{\frac{g}{l}}$$

$$a_t = -s\left(\frac{g}{l}\right)\cos\sqrt{\frac{g}{l}}\,t$$

$$\boxed{(a_t)_{max} = \pm\frac{sg}{l}}$$

$$a_n = \frac{v^2}{R} = \left[\frac{s^2 g}{l}\sin^2\sqrt{\frac{g}{l}}\,t\right] / l.$$

$$a_n = \frac{s^2 g}{l^2}\sin^2\sqrt{\frac{g}{l}}\,t$$

$$\boxed{(a_n)_{max} = \frac{s^2 g}{l^2}}$$

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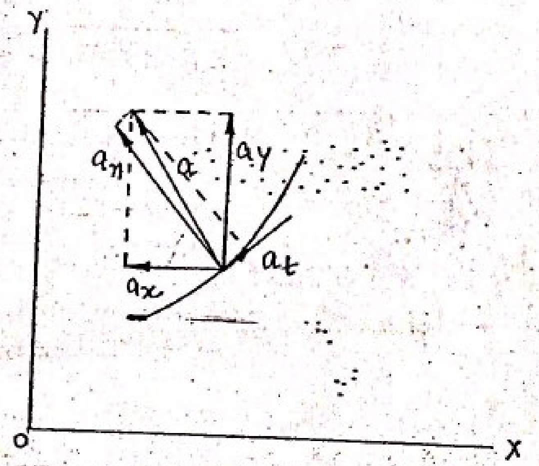
KINETICS

The equations of motion of a particle relate force, mass and acceleration of the particle.

Acceleration can be resolved into two components which are mutually perpendicular to each other.

1) a_x and a_y along directions of coordinate axes x and y.

2) a_t and a_n along directions of tangent and normal to curve.



EQUATIONS OF MOTION : IN RECTANGULAR COMPONENTS.

Let a system of forces act on a particle P.

Resolving the forces in x and y directions.

$\Sigma F_x \rightarrow$ Sum of components of forces in x direction

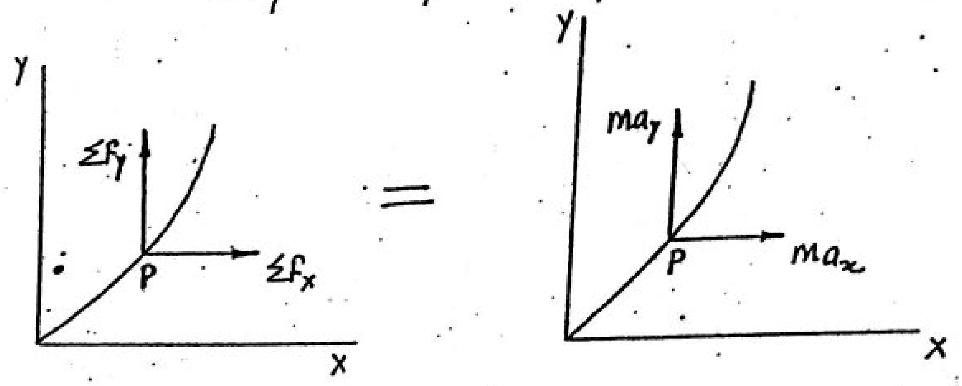
$\Sigma F_y \rightarrow$ Sum of components of forces in y direction.

Let a_x and a_y be components of acceleration in x and y directions.

According to Newton's second law

$$\Sigma F_x = ma_x$$

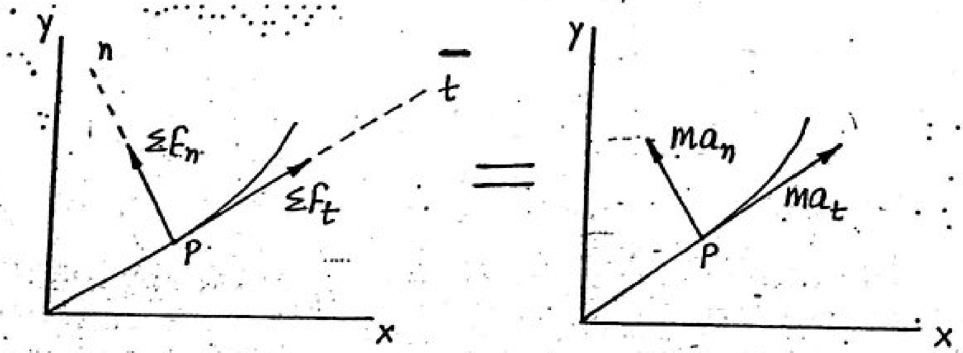
$$\Sigma F_y = ma_y.$$



EQUATIONS OF MOTIONS : IN TANGENTIAL AND NORMAL COMPONENTS

Let F_t be the component along the tangent to the path.

F_n be the component along normal to the path.



Applying Newton's second law

$$\Sigma F_t = ma_t$$

$$\Sigma F_n = ma_n$$

We know, acceleration during a curvilinear motion is given by

$$a_t = \frac{dv}{dt}, \quad a_n = \frac{v^2}{r}.$$

∴ Equations of motion becomes.

$$\Sigma F_t = m a_t$$

$$\boxed{\Sigma F_t = \frac{m\,dv}{dt}}$$

$$\Sigma F_n = m a_n$$

$$\boxed{\Sigma F_n = \frac{m v^2}{r}}$$

If velocity is constant

$$\boxed{a_t = 0}$$

EQUATIONS OF DYNAMIC EQULIBRIUM (D'ALEMBERT'S PRINCIPLE

1.) RECTANGULAR COMPONENTS :

$$\Sigma F_x = m a_x$$

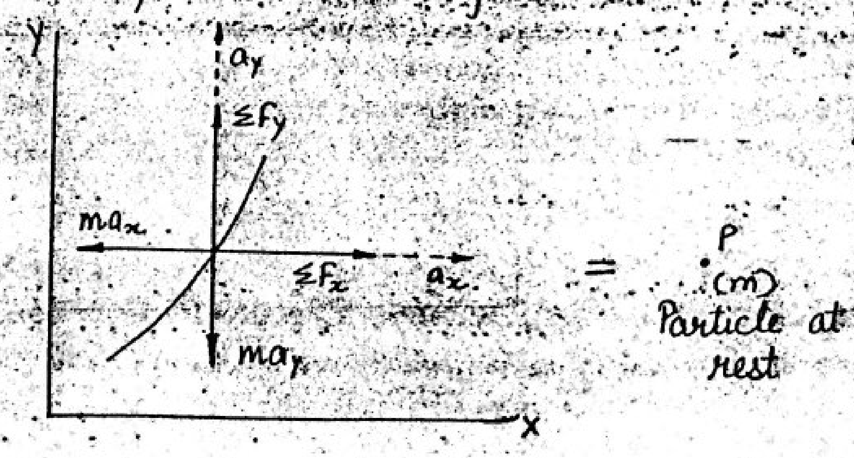
$$\Sigma F_x - m a_x = 0$$

$$\Sigma F_y = m a_y$$

$$\Sigma F_y - m a_y = 0$$

→ Equation of dynamic equilibrium

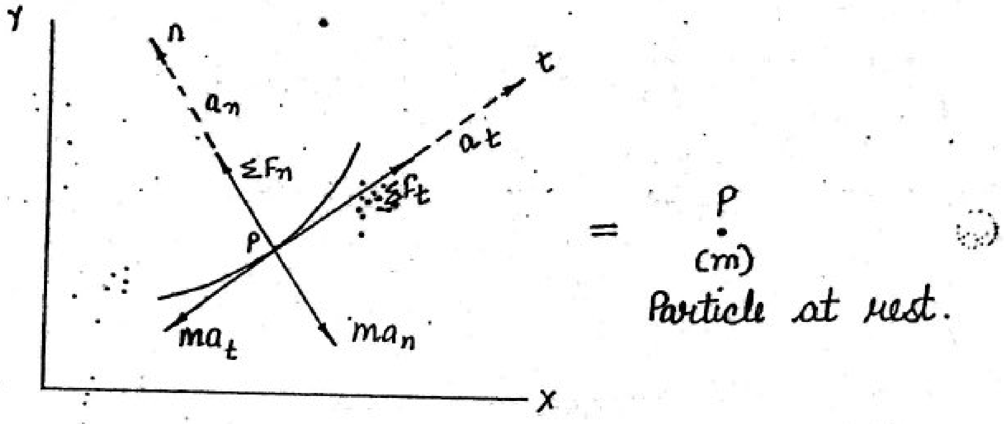
$-ma_x$ and $-ma_y$ are inertia forces.



2.) NORMAL AND TANGENTIAL COMPONENTS :

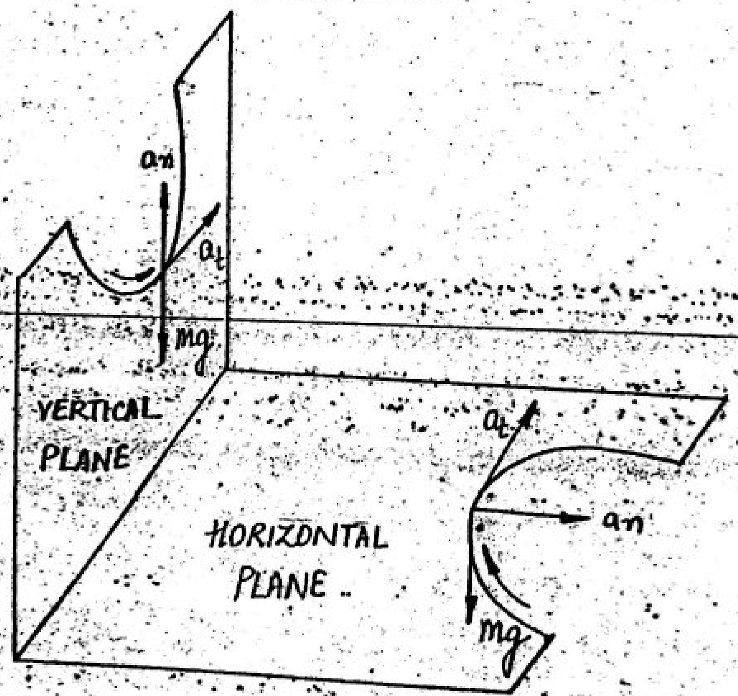
$$\Sigma F_t = ma_t$$

$$\left.\begin{aligned} \Sigma F_t - ma_t &= 0 \\ \Sigma F_n &= ma_n \\ \Sigma F_n - ma_n &= 0 \end{aligned}\right] \longrightarrow \text{Equations of dynamic equilibrium}$$



CONCEPTS OF CURVILINEAR MOTION

A particle may move along a curved path which may lie on a vertical plane ~~or~~ ~~in~~ a horizontal plane.



1.) Weight of particle 'mg' acts in plane of motion only when it moves in verticle plane.
When particle moves in horizontal plane, 'mg' acts in a plane normal to plane of motion.

2.) Normal acceleration is always directed towards centre of curvature of path.

$$a_n = \frac{v^2}{r}$$

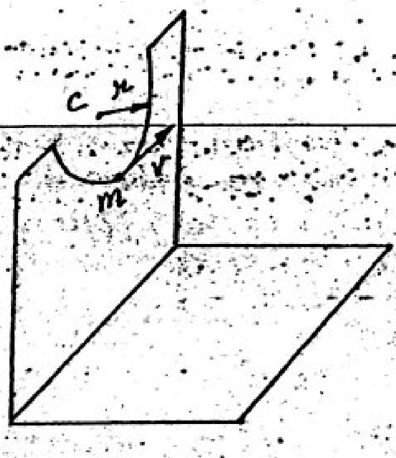
3.) Radius of curvature of path may or may not be same at all the points of the path.

$$\therefore \quad r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} \longrightarrow \text{General form.}$$

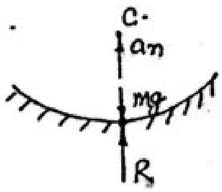
4.) Tangential acceleration, $a_t = \frac{dv}{dt}$ is taken positive (coincides with direction of motion of particle).

MOTION OF A PARTICLE WITH CONSTANT VELOCITY IN CIRCULAR PATH:

1.) Path in a Vertical Plane :



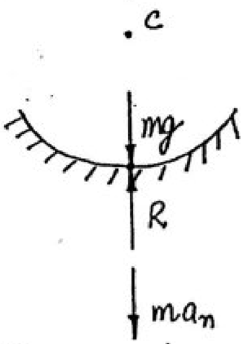
Equation of Motion



$\Sigma F_n = ma_n$

$R - mg = ma_n.$

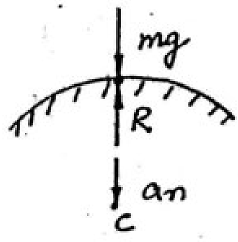
Equation of Dynamic Equilibrium



$\Sigma F_n = 0$

$R - mg - ma_n = 0$

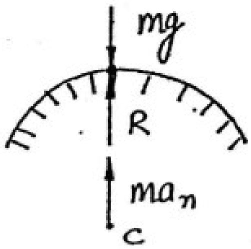
Equation of Motion



$\Sigma F_n = ma_n$

$mg - R = ma_n.$

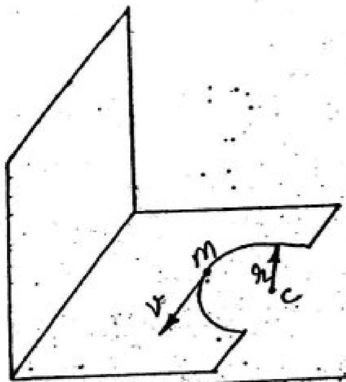
Equation of Dynamic Equilibrium



$\Sigma F_n = 0$

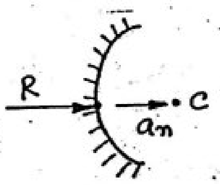
$mg - R - ma_n = 0$

2.) Path in a Horizontal Plane :



$a_n = \frac{v^2}{r}$

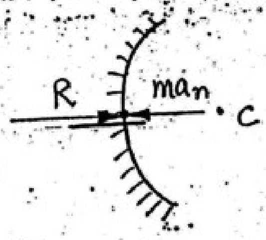
Equation of Motion



$\Sigma F_n = ma_n$

$R = ma_n$

Equation of Dynamic Equilibrium

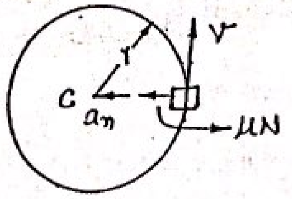


$\Sigma F_n = 0$

$R - ma_n = 0$

Q1. A small block of weight W rests on a horizontal turn table at a distance r=1m from the centre of the turn table. Find the maximum uniform speed the block can have without slipping off the table. Assume coefficient of friction between the block and turn table to be $\mu = 0.5$.

SOL^N: Block has a tendency to slip away from the centre.



Normal reaction = N

$$N = W.$$

Frictional force acting on block = μN.

Equation of motion of block

$$\Sigma F_n = a_n \times m.$$

$$\frac{W}{g} a_n = \mu W \qquad \left\{ m = \frac{W}{g} \; ; N = W \right.$$

Also $a_n = \frac{v^2}{r}$.

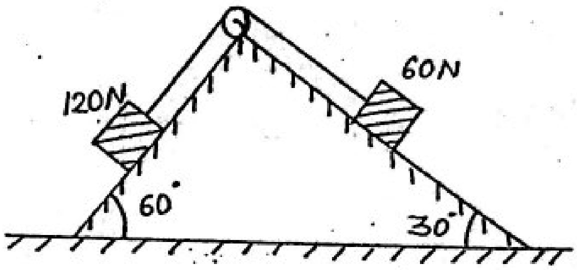
$$\frac{W}{g} \frac{v^2}{r} = \mu W$$

$$v^2 = \mu g r$$

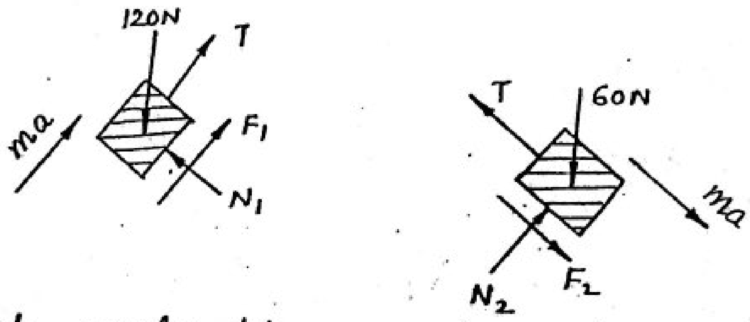
$$v = \sqrt{\mu g r} = \sqrt{0.5 \times 9.81 \times 1}$$

$$v = 2.215 \text{ m/s}.$$

Q2. Find the acceleration and tension in the string of the system. Coefficient of the friction $\mu = 0.3$ for all planes of the contact. Also determine the velocity of the system in 4 seconds after starting from rest.



SOL^N:



Let acceleration of the system be = 'a'

<u>For 120N block</u> :

$$N_1 = 120 \cos 60^\circ$$

$$\boxed{N_1 = 60N}$$

$$F_1 = \mu N_1 = 0.3 \times 60$$

$$\boxed{F_1 = 18N}$$

$$T - 120 \sin 60 + F_1 + ma = 0$$

$$T - 103.92 + 18 + \frac{120}{9.81} a = 0$$

$$\boxed{T + 12.23a = 85.92} \longrightarrow (1)$$

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For 60N block :

$$N_2 = 60 \cos 30^\circ$$

$$\boxed{N_2 = 51.96N}$$

$$F_2 = \mu N_2 \quad = 0.3 \times 51.96$$

$$\boxed{F_2 = 15.59N}$$

$$T - 60 \sin 30^\circ - F_2 - ma = 0$$

$$T - 30 - 15.59 - \frac{60}{9.81} a = 0$$

$$\boxed{T - 6.12a = 45.59} \longrightarrow (2)$$

Solving (1) and (2)

$$T + 12.23a = 85.92$$

$$T - 6.12a = 45.59$$

$$18.35a = 40.33$$

$$\boxed{a = 2.1\overline{98}\, m/s^2}$$

$$T = 85.92 - 12.23 \times 2.198$$

$$\boxed{T = 59N}$$

Velocity after 4 seconds:

$$v = u + at$$

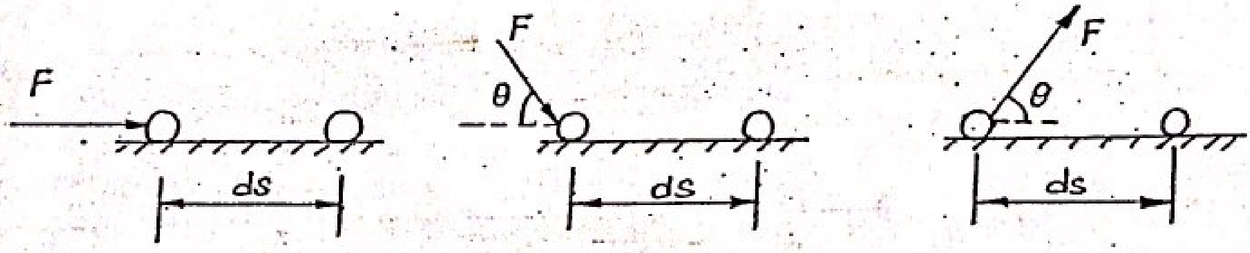
$$= 0 + 2.198 \times 4$$

$$\boxed{v = 8.792\, m/s}$$

KINETICS OF A PARTICLE : WORK AND ENERGY.

WORK : It is said to be done when point of application of force moves in direction of force.

Work done is equal to the product of displacement ds and component of force F in direction of the displacement.



Work, W = Force x Displacement

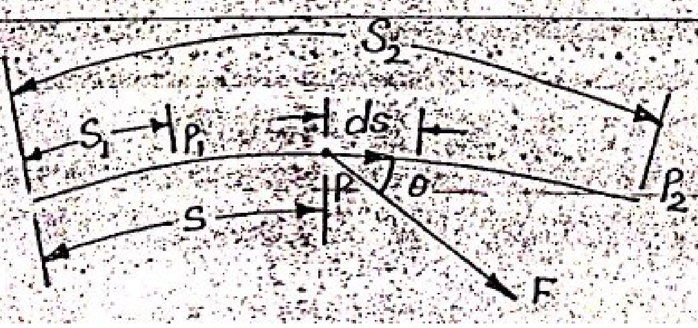
$$\boxed{W = F \times \vec{ds}}$$

$$\boxed{W = F\, ds \cos\theta}$$

where θ is angle between force and displacement vector.

Work done by a force during a finite displacement between two positions is given by :-

$$W_{1-2} = \int_{s_1}^{s_2} (F\cos\theta)\, ds$$

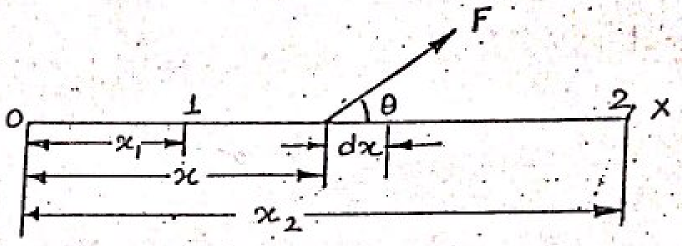


Work done by a constant force in rectilinear motion is given by

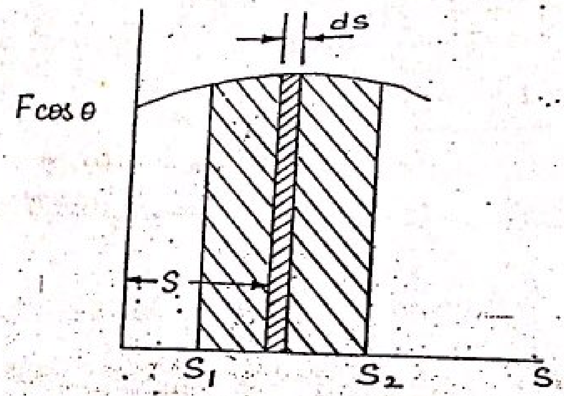
$$W_{1-2} = \int_{x_1}^{x_2} (F\cos\theta)\,dx$$

Work done for displacement x from origin

$$W_{1-2} = (F\cos\theta)\,x$$



Work done by a force can be represented by area under the curve.



IMPORTANT POINTS :

1) Work done by a force is zero if either displacement is zero or force acts normal to the displacement.

2) Work done by a force is positive if direction of force and direction of displacement are same.

Positive work $\longrightarrow$ Work done by a force.

Negative work $\longrightarrow$ Work done against a force.

3) Work is a scalar quantity. It has magnitude and sense/sign but no direction.

4) Work done by a force depends on path over which force moves except in case of conservative forces.

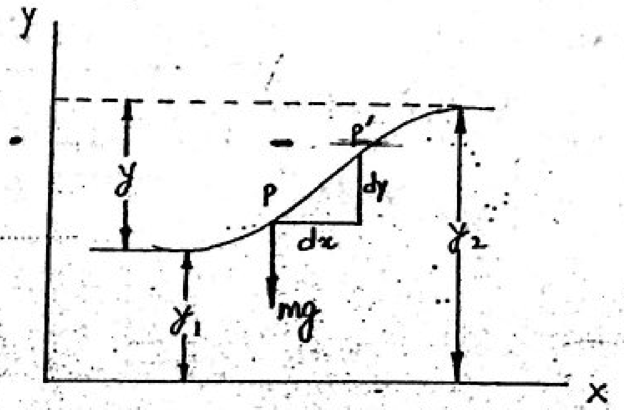
CONSERVATIVE FORCES → Force of gravity, spring force, elastic force.

NON-CONSERVATIVE FORCES → Friction force.

WORK OF VARIOUS FORCES

1.) WORK OF FORCE OF GRAVITY :

Let a particle of mass m move along a path in a vertical plane x-y.



Let particle P be displaced by dy vertically to new position P'.

$$dW = (-mg)\,dy$$

Work of gravity force

$$W_{1-2} = \int_{y_1}^{y_2} -mg\,dy$$

$$= -mg(y_2 - y_1)$$

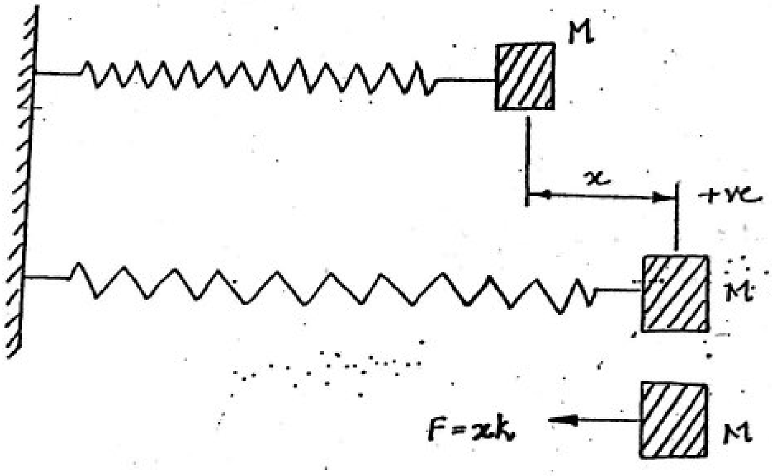
$$\boxed{W_{1-2} = -mgy}$$

Work of gravity force is negative because direction of force and direction of displacement are opposite.

Gravity force does no work when body moves horizontally.

2) WORK OF FORCE OF SPRING :

Let a spring of stiffness 'k' be stretched a distance x from undeformed position.



Force excited by spring on the body M

$$F = kx$$

Work done on spring during small displacement dx

$$dW = -F dx$$

Work done on spring when stretched

$$W_{1-2} = \int_1^2 dW$$

$$= \int_{x_1}^{x_2} -F dx$$

$$= \int_{x_1}^{x_2} -Kx dx$$

$$W_{1-2} = \frac{1}{2} K(x_2^2 - x_1^2)$$

If $Kx_1 = F_1$ → Force of spring when stretched through x_1 distance from initial position.

$Kx_2 = F_2$ → Force of spring when stretched through x_2 distance from initial position

Work done, $W_{1-2} = -\frac{1}{2}K(x_2^2 - x_1^2)$

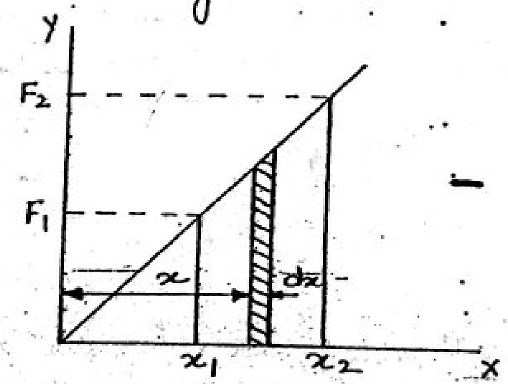
$$= -\frac{1}{2}K(x_2+x_1)(x_2-x_1)$$

$$= -\frac{1}{2}(Kx_2 + Kx_1)(x_2-x_1)$$

$$= -\left(\frac{F_2+F_1}{2}\right)(x_2-x_1)$$

$$W_{1-2} = -\frac{1}{2}\text{(Average Force)}\text{(Displacement)}$$

This is represented by area of trapezoid under force-displacement diagram.

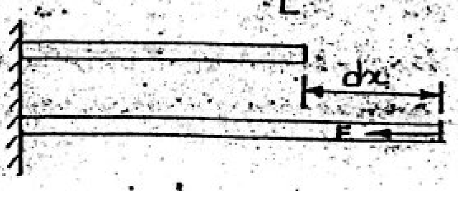


Work done in stretching a spring is negative.

3) WORK OF ELASTIC FORCE :

Consider a prismatic bar of area A and length L and elastic constant E is stretched, then work of elastic force is calculated as

$$K = \frac{AE}{L}$$



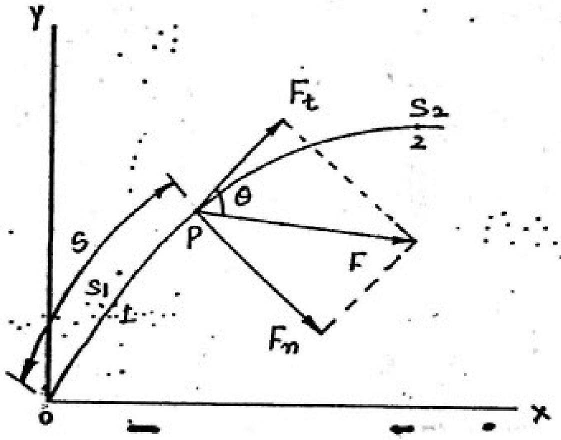
ENERGY : It is defined as the capacity to do work

KINETIC ENERGY : It is the energy possessed by a particle by virtue of its motion.

$$K.E. = \frac{1}{2} m v^2$$

WORK - ENERGY PRINCIPLE

Consider a particle P of mass m acted upon by a force F and moving with velocity v along a path.



Position P be at a distance of S from the origin along the path.

Let θ be the angle between force vector and the tangent to path at P. Resolve F along normal and tangential directions.

Tangential component ; $F_t = F\cos\theta$

Normal component ; $F_n = F\sin\theta$

Equation of motion in tangential direction

$$F_t = m a_t$$

$$F\cos\theta = m a_t$$

As $a_t = \frac{dv}{dt}$

$$F\cos\theta = m\frac{dv}{dt}$$

$$= m\frac{dv}{ds}\,\frac{ds}{dt}$$

$$\boxed{F\cos\theta = mv\frac{dv}{ds}.}$$

Velocity at point 1 $\rightarrow v_1$ Distance $\rightarrow s_1$
Velocity at point 2 $\rightarrow v_2$ Distance $\rightarrow s_2$

$$\int_{s_1}^{s_2} (F\cos\theta)\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$\underbrace{\int_{s_1}^{s_2} (F\cos\theta)\,ds}_{\text{Work done from position 1 to 2.}} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}.$$

$$\therefore \quad W_{1-2} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

$$\boxed{W_{1-2} = K.E_2 - K.E_1}$$

STATEMENT OF WORK-ENERGY PRINCIPLE :

The work done by a force acting on a particle during its displacement is equal to the change in the kinetic energy of the particle during that displacement.

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WORK AND ENERGY PRINCIPLE FOR A SYSTEM OF PARTICLES

For a single particle work-energy principle is given by

$$W_{1-2} = K.E_2 - K.E_1$$

For a system of particles, change of kinetic energy of all particles is added and equated to work of all the forces involved during the displacement of the system of particles.

$$\Sigma W_{1-2} = \Sigma (K.E_2 - K.E_1)$$

POTENTIAL ENERGY AND CONSERVATIVE FORCES

POTENTIAL ENERGY : It is the energy possessed by a particle by virtue of its position.

Consider a particle of mass m moving from position 1 to position 2 along the path 1.

Distance of position 1 from x axis $= y_1$

Distance of position 2 from x axis $= y_2$

Work of gravity force,

$$W_{1-2} = \int_{y_1}^{y_2} (-mg)\,dy$$

$$= -mg(y_2 - y_1)$$

$$\boxed{W_{1-2} = -mgy}$$

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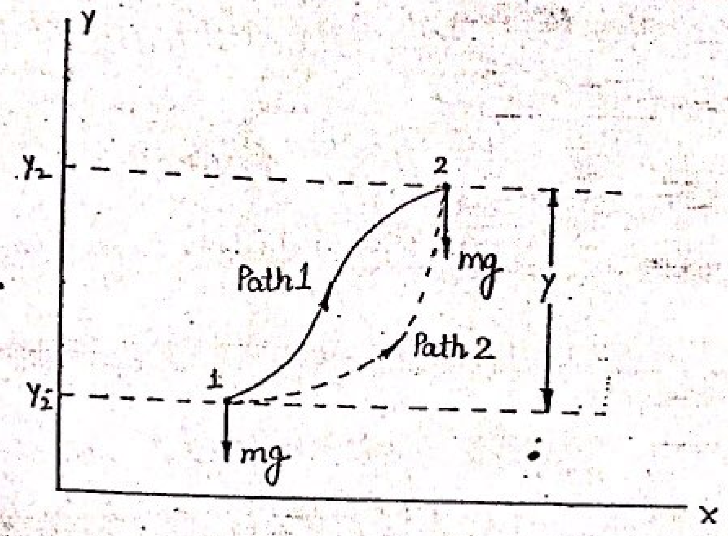


Negative sign indicates:

1) Work done against the force of gravity
2) Represents increase in potential energy of the particle.

Now if path 2 is followed the work done remains same and equal to $(-mgy)$.

Hence, work of gravity force is <u>independent</u> of the path followed and depends on the <u>initial and final values</u> of the function mgy.

<u>CONSERVATIVE FORCES.</u>

If the work of a force in moving a particle between two positions is independent of the path followed by the particle and can be expressed as change in potential energy, then such a force is called as a conservative force.

Example: Gravity force, spring force.

PRINCIPLE OF CONSERVATION OF ENERGY.

Using work-energy principle.

Work done = Change in kinetic energy.

$$W_{1-2} = K.E_2 - K.E_1$$

If particle moves under action of a conservative force, work done is stored as potential energy.

$$W_{1-2} = -(P.E_2 - P.E_1)$$

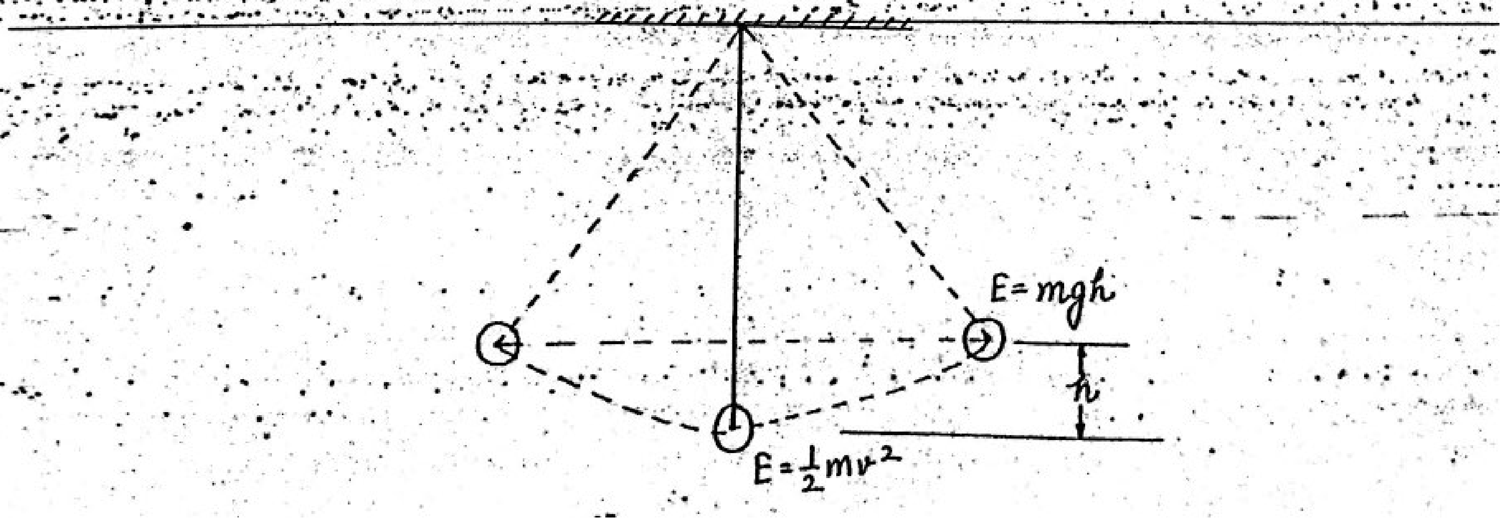
Work done = $-$(Change in potential energy)

$$K.E_2 - K.E_1 = -(P.E_2 - P.E_1)$$

$$\boxed{K.E_2 + P.E_2 = K.E_1 + P.E_1}$$

Sum of potential energy and kinetic energy of a particle remains constant during the motion under the action of conservative forces.

<u>EXAMPLE</u>: Sum of P.E. and K.E. remains constant in case of a simple pendulum in any position if frictional force at support and force due to air resistance are neglected.



POWER : It is defined as the time rate at which the work is done.

Average Power →

If ΔW is work done during an interval of time Δt, therefore

$$\text{Average Power} = \frac{\text{Work done}}{\text{Time}}$$

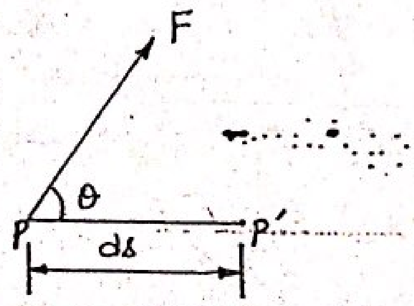
$$P_{avg} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power

$$P_i = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

$$\boxed{P_i = \frac{dW}{dt}}$$

We Know



$$dW = F\cos\theta \, ds.$$

$$v = \frac{ds}{dt}$$

$$P = \frac{dW}{dt} = \frac{F\cos\theta \, ds}{dt}$$

$$\boxed{P = (F\cos\theta)\, v}$$

$$\boxed{\text{Power} = \text{Force} \times \text{Velocity}}$$

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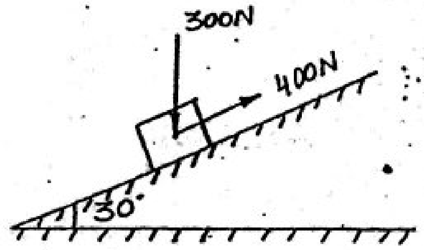
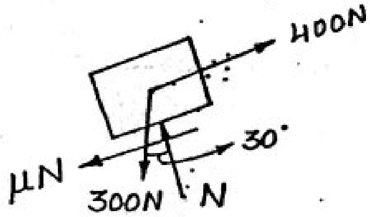
81. A body weighing 300N is pushed up a 30° plane by a force of 400 N acting parallel to the plane. If the initial velocity of the body is 1·5 m/sec and $\mu = 0.2$, what velocity will the body have after moving 6m?



<u>SOL^N</u>:



Work done = Change in K.E.

$\Sigma F_y = 0$

$N = 300 \cos 30$

$= 300 \times 0.866$

$\boxed{N = 259.8\,N}$

$F = \mu N = 0.2 \times 259.8$

$\boxed{F = 51.96 N}$

$\Sigma F_x : 400 - \mu N - 300 \sin 30$

$= 400 - 51.96 - 150$

$= 198.04\,N$

Work done by force along the plane

$= (\Sigma F_x) \times (\text{Distance})$

$= 198.04 \times 6$

$\boxed{W = 1188.24\,Nm}$

$$\text{Change in K.E.} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \frac{1}{2} \times \frac{300}{9.81}(v^2 - u^2)$$

$$= \frac{1}{2} \times \frac{300}{9.81}(v^2 - 1.5^2)$$

We know,

$$\text{Work done} = \text{Change in K.E.}$$

$$\therefore \quad 1188.24 = \frac{1}{2} \times \frac{300}{9.81}(v^2 - 1.5^2)$$

$$v^2 - 1.5^2 = 77.71$$

$$v^2 = 77.71 - 2.25$$

$$v^2 = 75.46$$

$$\boxed{v = 8.68\ m/sec}$$

32. An engine weighing 500 KN pulls the coaches weighing 2500 KN up a gradient 1 in 120, with a uniform velocity of 36 Kmph. Find the power transmitted by the engine, if tractive resistance is 6N/KN. Find also the power transmitted by the engine at the end of one second, if acceleration of the engine is $0.2\ m/sec^2$ and $g = 9.81\ m/sec^2$.

SOLN:

Weight of engine = 500KN

Weight of all the coaches = 2500KN

$\therefore$ Total weight = 2500 + 500

= 3000KN

Total tractive force = 3000×6

= 18000 N

= 18 KN

When velocity is uniform

$$V = 36 \text{ Kmph}$$
$$= 36 \times \frac{1000}{3600}$$
$$= 10 \text{ m/sec}.$$

Work done in one second

= Force × Distance

Force = $P - 3000 \sin\theta - F$ → Tractive force.

Distance = 10m.

As $\theta = \frac{1}{120}$ → Very small.

$\therefore \sin\theta = \frac{1}{120}$

$$\therefore \quad W = P - 3000 \times \frac{1}{120} - 18$$

Now change in K.E.

$$= \frac{1}{2}\frac{W}{g}\left[v^2 - u^2\right]$$

$v = u =$ Uniform.

$$= \frac{1}{2}\frac{W}{g}\left[10^2 - 10^2\right]$$

$$= 0$$

Now,

$$P - \frac{3000}{120} - 18 = 0$$

$$P = 18 + 25$$

$$\boxed{P = 43 \text{kN}}$$

Power = Force x Velocity

$= 43 \times 10$

$= 430 \text{ KNm/sec}.$

$\boxed{\text{Power} = 430 \text{ KW}.}$

(b) Acceleration $= 0.2 \text{ m/sec}^2$

$v = u + at$

Velocity of engine after 1 sec

$v = 10 + 0.2 \times 1$

$\boxed{v = 10.2 \text{ m/sec}}$

Work done/sec = Change in K.E. in 1 sec.

Distance travelled in 1 second.

$S = ut + \frac{1}{2}at^2$

$= 10 \times 1 + \frac{1}{2} \times 0.2 \times 1^2$

$= 10 + 0.1$

$\boxed{S = 10.1 \text{ m}.}$

$$\left[P - 18 - \frac{3000}{120}\right] \times 10.1 = \frac{1}{2} \times \frac{3000}{9.81}\left[(10.2)^2 - 10^2\right]$$

$$(P - 43)\, 10.1 = 152.9\,(104.04 - 100)$$

$$P - 43 = \frac{152.9 \times 4.04}{10.1}$$

$$P - 43 = 61.16$$

$$\boxed{P = 104.16 \text{ KN}}$$

Power = Force x Velocity

$= 104.16 \times 10.2$

Power $= 1062.4$ KW

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IMPULSE OF A FORCE:

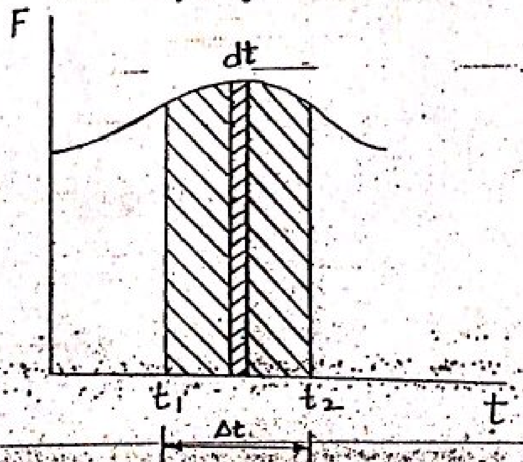
When a large force acts over a short period of time, that force is called impulsive force.

The impulse of force F acting over a time interval for t_1 to t_2 is given by

$$I = \int_{t_1}^{t_2} F\,dt$$

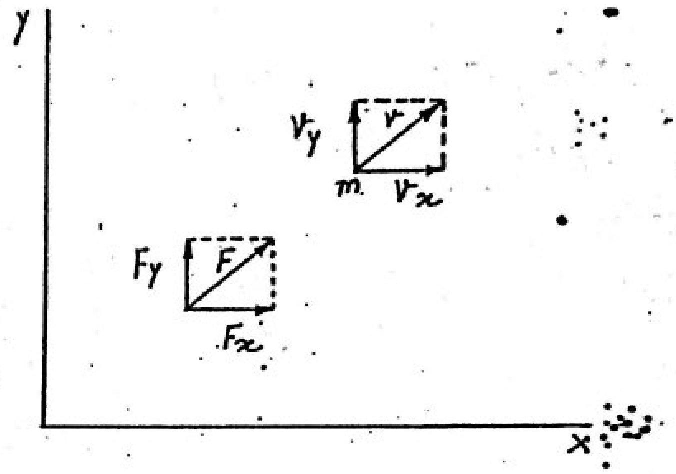
It is also given as area under the force vs time diagram. When variation of force with respect to time is unknown, the impulse is calculated as

$$I = F_{avg} \times \Delta t$$



MOMENTUM: It is the product of mass and velocity of a body and represents the energy of motion stored in a moving body.

Consider the motion of a particle of mass m acted upon by a force F. Let it move with velocity v.



Equation of motion in x and y directions:

$$F_x = m a_x \qquad F_y = m a_y$$

$$F_x = m \frac{dv_x}{dt} \qquad F_y = m \frac{dv_y}{dt}$$

$$F_x = \frac{d(mv_x)}{dt} \qquad F_y = \frac{d(mv_y)}{dt}$$

Equation in vector form.

$$F = \frac{d(mv)}{dt}$$

This implies

Force $\propto$ Rate of change of momentum.

~~Vector (mv) $\longrightarrow$ Momentum or Linear momentum~~

It has same direction as the velocity of the particle.

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PRINCIPLE OF IMPULSE AND MOMENTUM

Using the equation

$$F_x = \frac{d(mv_x)}{dt}$$

$$\underbrace{F_x\,dt}_{\text{Impulse of force } F_x} = \underbrace{d(mv_x)}_{\text{Differential change in momentum in } x\text{-direction.}}$$

Taking the limits t_1, v_{x_1} and t_2, v_{x_2}.

$$\int_{t_1}^{t_2} F_x\,dt = (mv_x)_2 - (mv_x)_1$$

$$\int_{t_1}^{t_2} F_y\,dt = (mv_y)_2 - (mv_y)_1$$

Combining above two equations

$$\int_{t_1}^{t_2} F\,dt = mv_2 - mv_1$$

If $t_1 = 0$, $t_2 = t$

$$mv_2 - mv_1 = \int_0^t F\,dt$$

$\therefore$ Final momentum − Initial momentum = Impulse of force

The principle of impulse and momentum states that the total change in momentum of a particle during a time interval is equal to impulse of force acting during the same interval of time.

SYSTEM OF PARTICLES :

All the momentums of all the particles and impulses of all the forces involved are added vectorially.

$$\Sigma m v_2 - \Sigma m v_1 = \Sigma \int_0^t F dt$$

IMPORTANT POINTS

1) When impulsive forces act on a system, the non-impulsive forces can be neglected. eg weight of the particle.

2) Internal forces between the particles need not be considered as sum of the impulses of internal forces is always zero. This is because internal forces exist in pairs of equal and opposite forces.

CONSERVATION OF MOMENTUM

When sum of impules due to external forces is zero, the momentum of system remains constant or is said to be conserved.

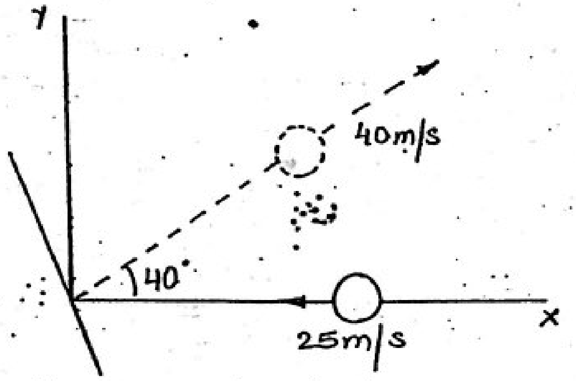
$$\Sigma \int_0^t F dt = \Sigma m v_2 - \Sigma m v_1$$

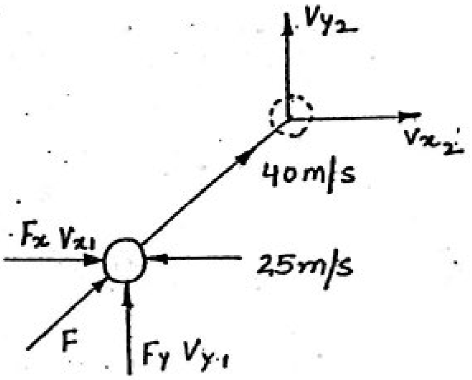
$$\Sigma \int_0^t F dt = 0$$

$$\therefore \Sigma m v_2 = \Sigma m v_1$$

Final momentum = Initial momentum

Q1. A ball of mass 100g is moving towards a bat with a velocity of 25m/s. When hit by a bat the ball attains a velocity of 40 m/s. If the bat and ball are in contact for a period of 0.015 s, Determine the average impulse force exerted by bat on the ball during the impact.





SOL^N: Applying principle of momentum to the ball.

Component equation in x-direction

$$m v_{x_2} - m v_{x_1} = \int_0^t F_x \, dt \quad \text{——} \quad (1)$$

We have,

$$m = \frac{100}{1000} = 0.1 \text{ Kg}, \quad \Delta t = 0.015 \text{ s}.$$

$$v_{x_1} = -25 \text{ m/s}$$

$$v_{x_2} = 40 \cos 40° = 40 \times 0.766 = 30.64 \text{ m/s}.$$

Also

$$\int_0^t F_x \, dt = F_{x(avg)} \times \Delta t \quad \text{——} \quad (2)$$

Equating (1) and (2)

$$F_{x(avg)} \Delta t = 0.1(30.64) - 0.1(-25)$$

$$F_{x(avg)} = \frac{5.564}{0.015}$$

$$\boxed{F_{x(avg)} = 370.9 \text{ N}}$$

Equation in y-direction

$$mv_{y_2} - mv_{y_1} = \int_0^t F_y dt$$

$$\int_0^t F_y dt = F_{x(avg)} \times \Delta t$$

∴ we have,

$$v_{y_1} = 0$$

$$v_{y_2} = 40 \sin 40^\circ = 40 \times 642$$

$$= 25.71 \text{ m/s}$$

$$F_{y(avg)} \times 0.015 = 0.1(25.71) - 0.1 \times 0$$

$$F_{y(avg)} = \frac{2.571}{0.015}$$

$$\boxed{F_{y(avg)} = 171.4 \text{ N}}$$

$$F_{avg} = \sqrt{F_{x(avg)}^2 + F_{y(avg)}^2}$$

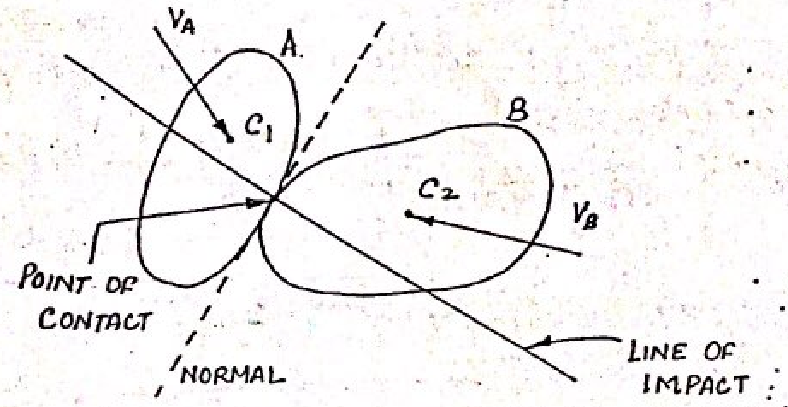
$$= \sqrt{(370.9)^2 + (171.4)^2}$$

$$= \sqrt{137566.81 + 29381.39}$$

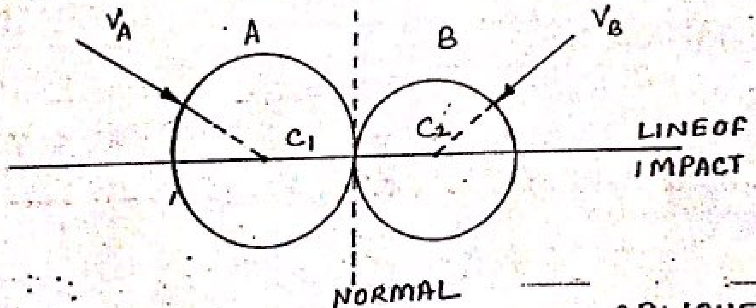
$$\boxed{F_{avg} = 408.6 \text{ N}}$$

IMPACT : COLLISION OF ELASTIC BODIES

The phenomenon of collision of two bodies which occurs in a very small interval of time and during which the two bodies exert very large force on each other is called an impact.



OBLIQUE NON CENTRAL IMPACT

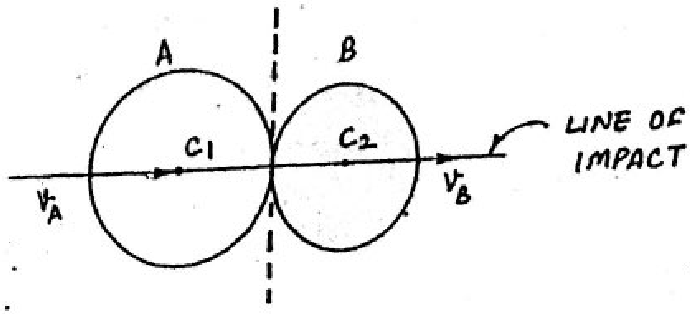


OBLIQUE CENTRAL IMPACT

LINE OF IMPACT : The common normal to the surfaces of two bodies in contact during the impact, is called line of impact.

CENTRAL/NON-CENTRAL IMPACT :

When the mass centers (C_1 and C_2) of the colliding bodies lie on the line of impact, it is called central Impact otherwise it is called non-central impact.

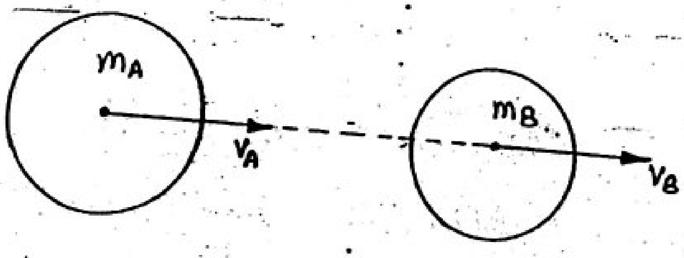


DIRECT IMPACT / INDIRECT (OBLIQUE) IMPACT :

If the velocities of the two bodies before collision are collinear with line of impact it is called direct impact.

DIRECT CENTRAL IMPACT

Consider the two spheres A and B of masses m_a and m_b moving in same direction and along the same straight line with the known velocities v_a and v_b respectively.



If $v_a > v_b$, the sphere A will strike the sphere B. As no external force is acting, the total momentum of the system of spheres A and B is conserved.

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

$$e = \frac{-(v_b' - v_a')}{(v_b - v_a)}$$

Coefficient of restitution

It depends upon nature of impact.

$$e = -\frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

Q1. Ball A of mass 1Kg moving with a velocity of 2m/s, impinges directly on a ball B of mass 2Kg at rest. Find the velocities of the two balls after the impact. Assume coefficient of restitution $e = 1/2$.

SOL^N:

$m_a = 1Kg$, $m_b = 2Kg$

$v_a = 2m/s$, $v_b = 0$

$e = 1/2$

Using principle of conservation of momentum

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

$$1 \times 2 + 2 \times 0 = 1 \times v_a' + 2 v_b'$$

$$\boxed{v_a' + 2v_b' = 2} \quad \text{--- (1)}$$

Coefficient of restitution relation

$$e = -\frac{(v_b' - v_a')}{(v_b - v_a)}$$

$$\frac{1}{2} = \frac{-(v_b' - v_a')}{(v_b - v_a)}$$

$$v_b - v_a = -(v_b' - v_a') \times 2$$

$$\frac{-2}{2} = -(v_b' - v_a')$$

$$\boxed{v_b' - v_a' = 1} \quad \text{--- (2)}$$

Solving ① and ②

$$v_a' + 2v_b' = 2$$
$$\underline{\overset{-}{2v_b'} \overset{+}{-2v_a'} = \overset{-}{2}}$$
$$3v_a' = 0$$
$$\boxed{v_a' = 0}$$

$$v_b' - v_a' = 1$$

$$\boxed{v_b' = 1 \text{ m/s}}$$

TYPE OF IMPACT :

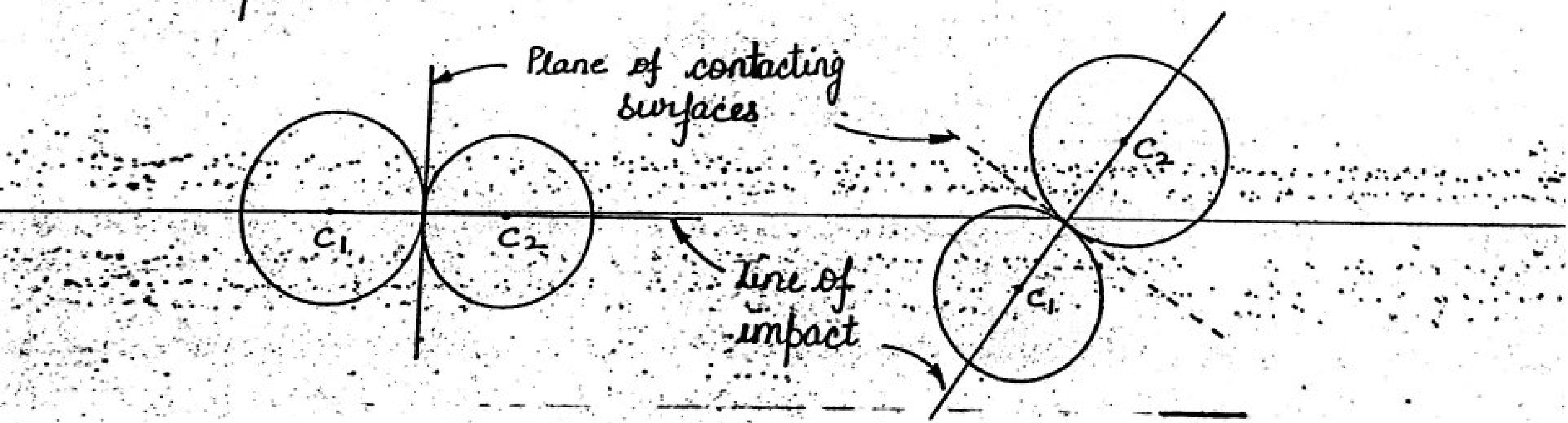
When the two bodies collide with one another, resulting impact may be:

(i) Central and eccentric impact

(ii) Direct and oblique (inclined) impact.

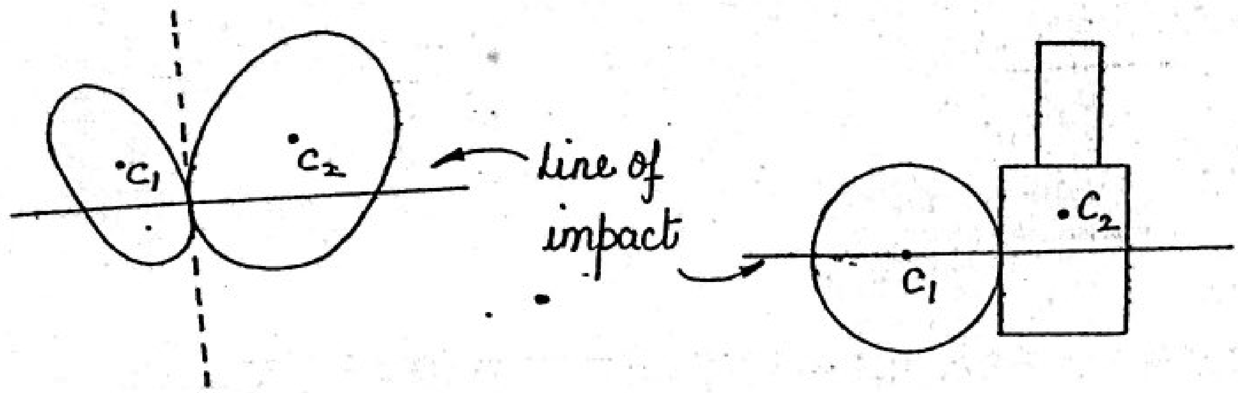
→ CENTRAL IMPACT :

The impact is central when mass centres of the colliding bodies are located on the line of impact.



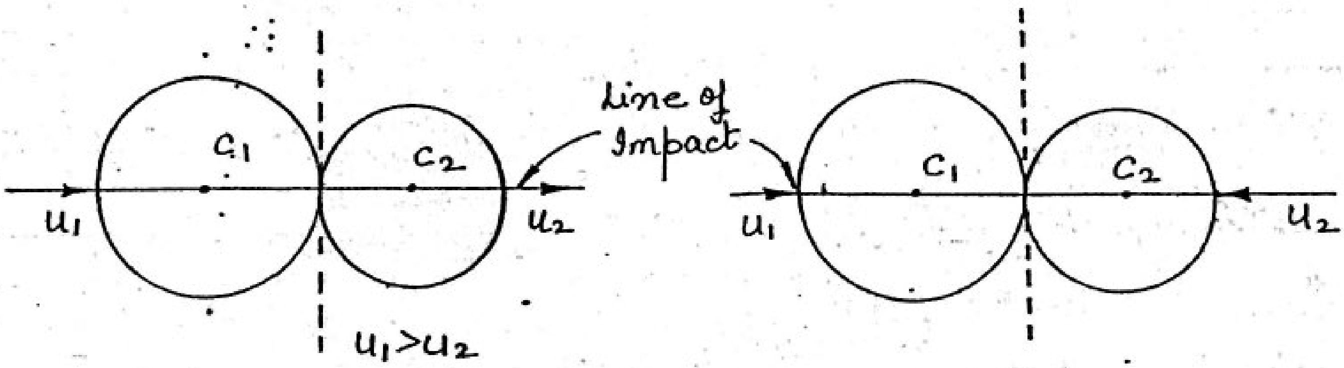
→ ECCENTRIC IMPACT :

When mass centres of colliding bodies are not located on the line of impact, the impact is referred as eccentric loading.



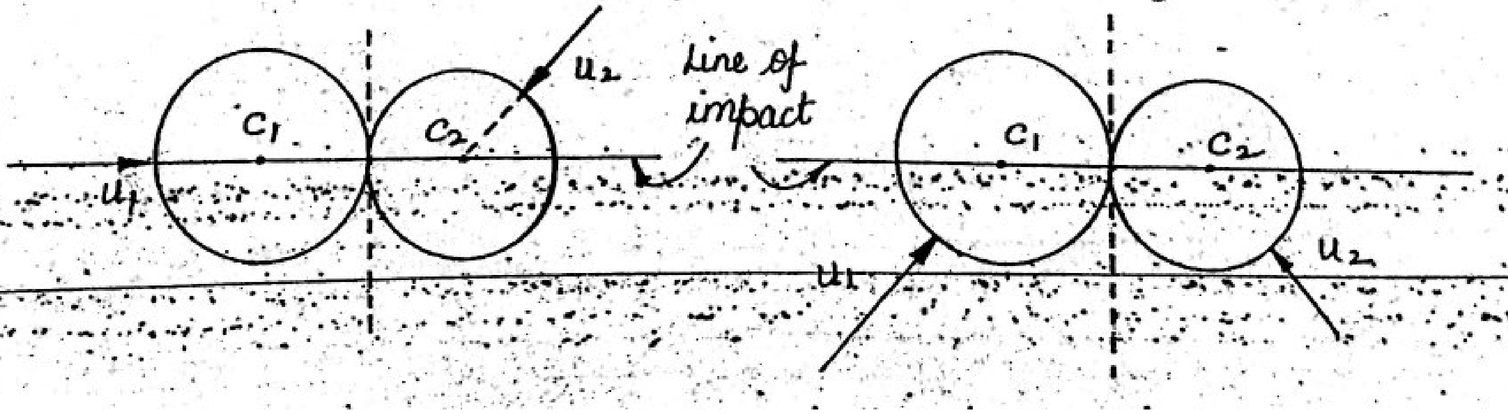
→ DIRECT IMPACT :

It is said to be direct if before impact the bodies are moving along the line of impact, i.e the motion of colliding bodies is directed along the line of impact.



→ INDIRECT / OBLIQUE IMPACT :

The impact is indirect if the motion of one or both of colliding bodies, before impact, is not directed along the line of impact.



NATURE OF IMPACT AND THE COEFFICIENT OF RESTITUTION

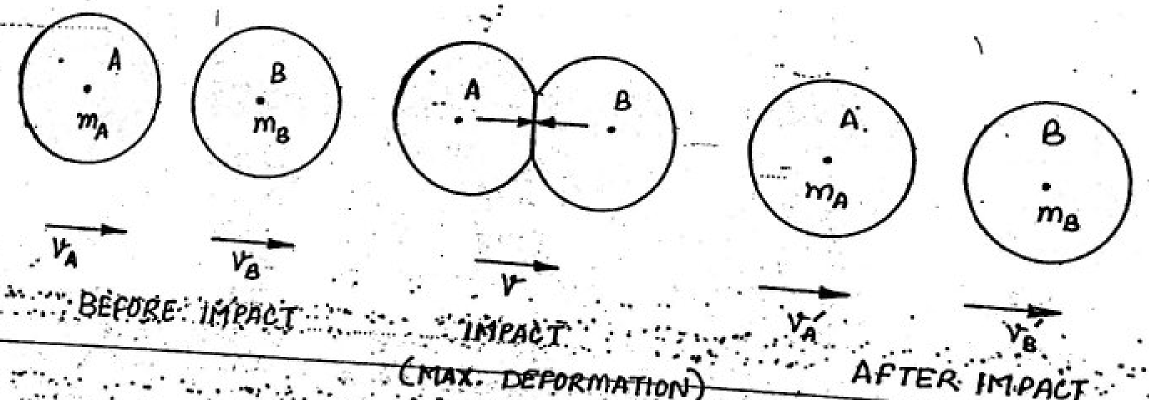
The phenomenon of impact consists of two phases:

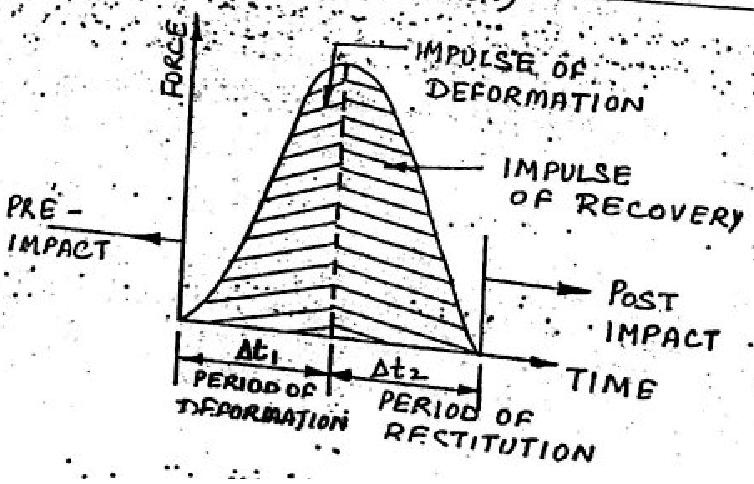
1). PERIOD OF DEFORMATION:

After the collision, the two bodies deform. The time interval from first contact to the maximum deformation is called period of deformation. At the end of this period, both bodies move with same velocity 'v'.

2.) PERIOD OF RESTITUTION:

Period of deformation is followed by the period of restitution (regain or recovery). At the end of resitution period either the two bodies regain their original shapes fully or partially or are permanently deformed. At the end, the two bodies separate and travel with different velocities.





Consider a body A moving with velocity v_a collides with body B. During deformation period, an impulsive force F_d is excerted by body B on body A and its velocity changes to v.

Using principle of impulse and momentum

$$m_a v_a - \int F_d\, dt = m_a v$$

Let F_r be impulsive force excerted by body B on body A and velocity changes from v to v_a'.

$$m_a v - \int F_r dt = m_a v_a'$$

<u>Coefficient of restitution:</u>

It is the ratio of magnitude of Impulses during the restitution period and deformation period.

$$e = \frac{\text{Impulse during restitution}}{\text{Impulse during deformation}}$$

$$e = \frac{\int F_r dt}{\int F_d dt}$$

$$= \frac{m_a (v_a - v)}{m_a (v - v_a')}$$

$$e = \frac{v_a - v}{v - v_a'} \longrightarrow \text{Analysis for body A}$$

$$e = \frac{v - v_b'}{v_b - v} = \frac{v_b' - v}{v - v_b} \longrightarrow \text{Analysis for body B.}$$

$$e = \frac{v - v_a'}{v_a - v}$$

$$e v_a - e v = v - v_a'$$

$$\boxed{v + ve = e v_a + v_a'}$$

$$e = \frac{v_b' - v}{v - v_b}$$

$$e v - e v_b = v_b' - v$$

$$\boxed{v + ev = v_b' + e v_b}$$

Equating both equations

$$e v_a + v_a' = v_b' + e v_b$$

$$e(v_a - v_b) = v_b' - v_a'$$

$$\boxed{e = \frac{v_b' - v_a'}{v_a - v_b}}$$

$$\Rightarrow \quad e = (-)\frac{v_b' - v_a'}{v_b - v_a}$$

$$= (-)\frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

Coefficient of restitution is a parameter indicating the energy loss during an impact.

IMPORTANT CASES OF IMPACT

1) Perfectly Elastic Impact ($e = 1$)

By coefficient of restitution

$$-\frac{(v_b' - v_a')}{(v_b - v_a)} = 1$$

$$\boxed{v_b' - v_a' = -(v_b - v_a)}$$

In perfectly elastic impact the energy of system is conserved.

Conservation of momentum gives

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

$$m_a (v_a - v_a') = m_b (v_b' - v_b) \quad \text{——— (1)}$$

Coefficient of restitution relation when $e = 1$

$$v_b' - v_a' = -(v_b - v_a)$$

$$v_a + v_a' = v_b + v_b' \quad \text{——— (2)}$$

Multiplying (1) and (2)

$$m_a (v_a - v_a')(v_a + v_a') = m_b (v_b' - v_b)(v_b + v_b')$$

$$m_a v_a^2 - m_a v_a'^2 = m_b v_b'^2 - m_b v_b^2$$

$$\frac{1}{2} m_a v_a^2 + \frac{1}{2} m_b v_b^2 = \frac{1}{2} m_a v_a'^2 + \frac{1}{2} m_b v_b'^2$$

K.E of two bodies before impact = K.E of two bodies after impact.

2) Perfectly Plastic Impact ($e = 0$)

By coefficient of restitution

$$e = 0 = (-)\frac{(v_b' - v_a')}{(v_b - v_a)}$$

$$v_b' = v_a' = v$$

After plastic impact, final velocities of both the bodies becomes equal and they move together as one body.

As the two bodies become permanently deformed, there is no period of restitution and kinetic energy of the system is not conserved.

But the total momentum of the system of bodies is conserved.

$$m_a v_a + m_b v_b = (m_a + m_b) v'$$

Common velocity,

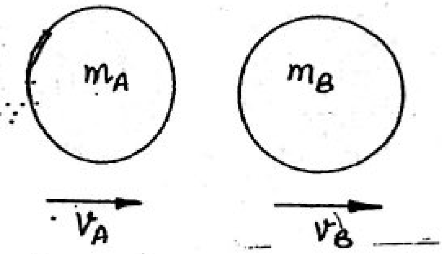
$$v' = \left[\frac{m_a v_a + m_b v_b}{m_a + m_b}\right]$$

IMPORTANT CASES : ELASTIC IMPACT

a) Impact of two Equal Masses

$$m_a = m_b = m$$

Conservation of momentum



$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

$$\boxed{v_a + v_b = v_a' + v_b'} \quad \text{--- (1)}$$

Coefficient of restitution

$$e = 1 = -\frac{v_b' - v_a'}{v_b - v_a}$$

$$\boxed{v_b - v_a = -(v_b' - v_a')} \quad \text{--- (2)}$$

Solving (1) and (2)

$$v_b + v_a = v_a' + v_b'$$
$$v_b - v_a = v_a' - v_b'$$
$$2v_b = 2v_a'$$

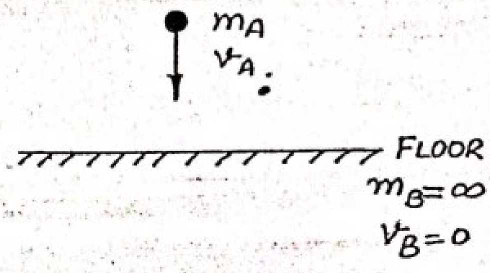
$$\boxed{v_b = v_a'} \quad , \quad \boxed{v_a = v_b'}$$

Hence after elastic impact, the two masses exchange velocities.

2) Impact of Two Bodies [Of the two bodies, one is immovable and of very large mass as compared to other body]

$m_b = \infty$

$v_b = 0$



Conservation of momentum

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

Dividing by m_b

$$\frac{m_a}{m_b} v_a + v_b = \frac{m_a}{m_b} v_a' + v_b'$$

As m_b is very large and $v_b = 0$

$$\boxed{v_b' = 0}$$

Coefficient of restitution

$$e = 1 = (-)\frac{(v_b' - v_a')}{(v_b - v_a)}$$

$$v_b - v_a = v_a' - v_b'$$

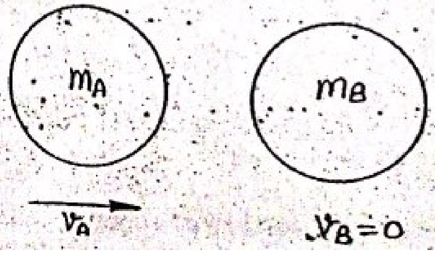
$$\boxed{v_a' = -v_a}$$

Body m_a rebound with same velocity.

3.) A Body strikes another body of Equal mass at Rest.

$m_a = m_b = m$

$v_b = 0$



Conservation of momentum

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

$$v_a + 0 = v_a' + v_b' \quad \text{——} \ (1)$$

Coefficient of restitution

$$e = 1 = (-)\frac{(v_b' - v_a')}{(v_b - v_a)}$$

$$v_b - v_a = v_a' - v_b' \quad \text{——} \ (2)$$

∴ Solving (1) and (2)

$$\begin{aligned} v_a &= v_a' + v_b' \\ -v_a &= v_a' - v_b' \\ \hline 0 &= 2v_a' \end{aligned}$$

$$\boxed{v_a' = 0}$$

$$\boxed{v_b' = v_a}$$

Striking mass m_a stops after imparting its entire velocity to the mass m_b.

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