$$m\ddot{r} - mr\dot{\theta}^2 + \frac{dV}{dr} = 0 \quad \text{— (A)}$$

$$mr^2\dot{\theta} = \text{constant} = L \quad \text{— (B)}$$

Expressions for $r(t)$ and $\theta(t)$

eqⁿ (A)

$$m\ddot{r} - mr\dot{\theta}^2 = -\frac{dV}{dr}$$

$$m\ddot{r} - \frac{L^2}{mr^3} = -\frac{dV}{dr}$$

$$m\ddot{r} = -\frac{dV}{dr} + \frac{L^2}{mr^3} = -\frac{d}{dr}\left(V + \frac{L^2}{2mr^2}\right).$$

$* \dot{r}$,

$$m\ddot{r}\dot{r} = -\frac{d}{dr}\left(V + \frac{L^2}{2mr^2}\right)\dot{r}$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{r}^2\right) = -\frac{d}{dr}\left(V + \frac{L^2}{2mr^2}\right)\frac{dr}{dt}$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{r}^2\right) = -\frac{d}{dt}\left(V + \frac{L^2}{2mr^2}\right)$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V\right) = 0$$

$$\underbrace{\frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2}}_{T} + \underset{V}{V} = \text{constant} = E$$

$$\dot{r}^2 = \frac{2}{m}\left(E - \frac{L^2}{2mr^2} - V\right)$$

$$\dot{r} = \pm\sqrt{\frac{2}{m}}\left(E - \frac{L^2}{2mr^2} - V\right)^{1/2}$$

$$\frac{dr}{dt} = \sqrt{\frac{2}{m}}\left(E - \frac{L^2}{2mr^2} - V\right)^{1/2}$$

$$\text{or } dt = \frac{dr}{\sqrt{\frac{2}{m}}\left(E - \frac{L^2}{2mr^2} - V\right)^{1/2}}$$

$$\boxed{E, L, r_0, \theta_0}$$

$$\boxed{t = \int_{r_0}^{r} \frac{dr}{\sqrt{\frac{2}{m}}\left(E - \frac{L^2}{2mr^2} - V\right)^{1/2}}}$$

at $t = 0$, $r = r_0$ (say)

$$\boxed{\theta = L\int_0^t \frac{dt}{mr^2(t)} + \theta_0}$$

General features of orbits

$$m\ddot{r} = -\frac{d}{dr}\left(V + \frac{L^2}{2mr^2}\right)$$

$$= -\frac{dV}{dr} - \frac{d}{dr}\left(\frac{L^2}{2mr^2}\right)$$

$$= -\frac{dV}{dr} + \frac{L^2}{mr^3}$$

$$f' = -\frac{dV}{dr} + \frac{L^2}{mr^3}$$

$$= f(r) + \frac{m(r\dot{\theta})^2}{r}$$

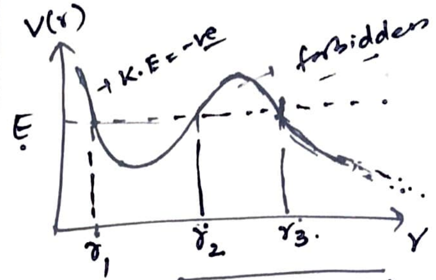
$$= f(r) + \frac{mv_\theta^2}{r} \rightarrow \text{centrifugal force.}$$

$f(r)$ ↓ central force

false or pseudo force

$$V' = V + \frac{L^2}{2mr^2}$$

Motion of particle in arbitrary Potential field :-



$E = T + V' = 0$

$T = -V'$

$\boxed{\text{Escape velocity}}$

$$\dot{r} = \sqrt{\frac{2}{m}\left(E - V'(r)\right)}$$

$\dot{r} = 0$ at $r = r_1, r_2, r_3$ Turning points

① $r \cancel{<} r_1$

② between r_1 and r_2 → $E > V'$ Apsidal distances

③ betn r_2 and r_3

④ $r \geq r_3$