CLASSICAL MECHANICS

ASSIGNMENT I

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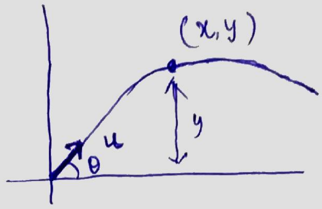
2K19 / EP / 005

**Q1**

Kinetic Energy,

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$




Potential Energy,

$$V = mgy$$

Lagrangian, $L = T - V = \frac{1}{2}m(\dot{x}^2 + \dot{y}) - mgy$

EOM, in x,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$m\ddot{x} = 0 \qquad \boxed{\ddot{x} = 0}$$

$\dot{x} = A$, for $t = 0$, $A = u_x \rightarrow$ initial velocity in x-directn.

$\dot{x} = u_x$

$$\boxed{x = u_x t}$$

EOM in y,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = 0$$

$$m\ddot{y} + mg = 0, \qquad \boxed{\ddot{y} = -g}$$

$\dot{y} = -gt + A$, for $t = 0 \rightarrow A = u_y$. (initial velocity in y-directn)

$$\int \dot{y} = \int -gt \cdot dt + \int u_y \, dt$$

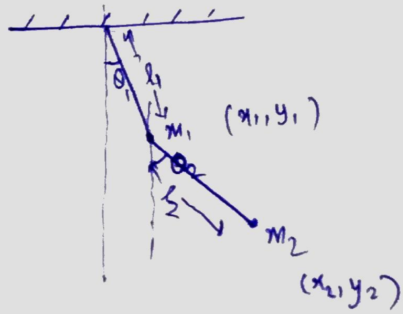
$$\boxed{y = u_y t - \frac{1}{2}gt^2}$$

Q3

$x_1 = l_1 \sin\theta_1 , \quad y_1 = l_1 \cos\theta_1 ,$

$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$

$y_2 = l_1 \cos\theta_1 + l_2 \cos\theta_2 .$




Now,

$\dot{x}_1 = l_1\dot{\theta}_1 \cos\theta_1 , \quad \dot{y}_1 = -l_1\dot{\theta}_1 \sin\theta_1 ,$

$\boxed{DOF = 2}$.

$\dot{x}_2 = l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2$

$\dot{y}_2 = -l_1\dot{\theta}_1 \sin\theta_1 - l_2\dot{\theta}_2 \sin\theta_2 .$

K.E of system,

$T = T_1 + T_2 = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2)$

$= \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\left[l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)\right]$

P.E of system

$V = -m_1 g l_1\cos\theta_1 - m_2 g(l_1\cos\theta_1 + l_2\cos\theta_2)$

$L = T - V$

eqn of Motions

① $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$ and ② $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0 .$

① $m_1 l_1^2\ddot{\theta}_1 + m_2 l_1^2\ddot{\theta}_1 + m_2 l_1 l_2\ddot{\theta}_2\cos(\theta_1-\theta_2) - m_2 l_1 l_2\dot{\theta}_2(\dot{\theta}_1-\dot{\theta}_2)\sin(\theta_1-\theta_2)$

$= -m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2) - m_1 g l_1\sin\theta_1 - m_2 g l_1\sin\theta_1 .$

② $m_2 l_2^2\ddot{\theta}_2 + m_2 l_1 l_2\ddot{\theta}_1\cos(\theta_1-\theta_2) - m_2 l_1 l_2\dot{\theta}_1(\dot{\theta}_1-\dot{\theta}_2)\sin(\theta_1-\theta_2)$

$= m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2) - m_2 g l_2\sin\theta_2$

if $m_1 = m_2$, and $l_1 = l_2$,

then — —

① $2l\ddot{\theta}_1 + l\ddot{\theta}_2 \cos(\theta_1-\theta_2) + l\dot{\theta}_2^2 \sin(\theta_1-\theta_2) = 2g\sin\theta_1$

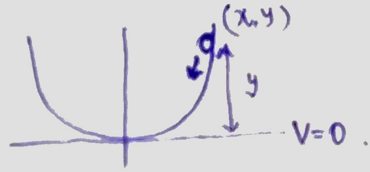
② $l\ddot{\theta}_1 \cos(\theta_1-\theta_2) + l\ddot{\theta}_2 - l\dot{\theta}_1^2 \sin(\theta_1-\theta_2) = -g\sin\theta_2$

if $\theta$ is small, $\sin\theta \approx \theta$ and $\cos\theta = 1$,

$$\boxed{\begin{array}{ll} ① & 2l\ddot{\theta}_1 + l\ddot{\theta}_2 = -2g\theta_1 \\ ② & l\ddot{\theta}_1 + l\ddot{\theta}_2 = -g\theta_2 \end{array}}$$

**Q4**

$y = ax^2$




KE, $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

as $y = ax^2$, $\dot{y} = 2ax\dot{x}$.

$T = \frac{1}{2}m(x^2 + (2ax\dot{x})^2)$.

P.E, $V = mgy = mgax^2$

$L = T - V = \frac{1}{2}m(\dot{x}^2 + 4a^2x^2\dot{x}^2) - mgax^2$

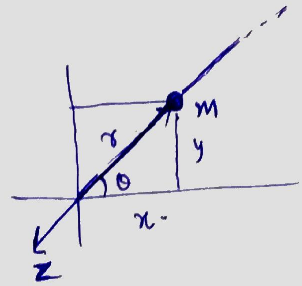
EOM: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$\frac{d}{dt}(m\dot{x} + m4a^2x^2\dot{x}) - (m4a^2x\dot{x}^2 + 2mgax) = 0$

$\Rightarrow \ddot{x}(1 + 4a^2x^2) - 4ax\dot{x}^2 - 2agx = 0$.

Q5

as constraint force is a functn of time → rheonomic.




$x = r\cos\theta \quad , \quad y = r\sin\theta$ .

$\dot{x} = -r\sin\theta\,\dot{\theta} + \dot{r}\cos\theta , \quad \dot{y} = r\cos\theta\,\dot{\theta} + \dot{r}\sin\theta$ .

K.E, $T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2)$

$= \frac{1}{2}m(r^2\dot{\theta}^2+\dot{r}^2)$

$V = 0$ ,

as it is a force free space.

$L = T = \frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)$

EOM: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$ . as $\dot{\theta} = \frac{d\theta}{dt} = \underline{\omega}$ .

$\frac{d}{dt}(m\dot{r}) - mr\dot{\theta}^2 = 0$ $\boxed{\ddot{r} = m\omega^2 r}$

where $r = Ae^{-\omega t} + Be^{\omega t}$ .

Q2

$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2 V(x) - [V(x)]^2$ .

EOM: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$\frac{d}{dt}\left(\frac{m^2\dot{x}^3}{3} + 2m\dot{x}V(x)\right) - \left(m\dot{x}^2\frac{dV(x)}{dx} - \frac{2dV(x)}{dx}\right)$

$\cancel{m^2\ddot{x}^3}$

$\cancel{\frac{m^2}{3}\ddot{x}^3 + 2m\ddot{x}V(x) + \dot{V}2m\dot{x} - m\dot{x}^2V'(x)}$

$- 2V'(x)$

$$m^2 \dot{x}^2 \ddot{x} + 2m\ddot{x}V + m\dot{x}^2 \frac{dV}{dx} + 2V\frac{dV}{dx} = 0.$$

factoring

$$\left(m\dot{x}^2 + 2V\right)\left(m\ddot{x} + \frac{dV}{dx}\right) = 0$$

true if,

$$\frac{1}{2}m\dot{x}^2 + V = 0$$

$$m\ddot{x} + \frac{dV}{dx} = 0$$

$$E = \frac{1}{2}m\dot{x}^2 + V \quad , \quad \dot{x}\dot{E}E = 0.$$

$$\dot{E}E = 0. \qquad \frac{d}{dt}(E^2) = 2E\dot{E}$$

$$\dot{E} = \dot{x}\left(m\ddot{x} + \frac{dV}{dx}\right).$$

$$\boxed{m\ddot{x} + \frac{dV}{dx} = 0}$$

$$\frac{1}{2}m\dot{x}^2 + V(x) = E$$

where

$$\dot{x} = \pm\sqrt{\frac{2E - V(x)}{m}}.$$