2K19/EP/005

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(EP-203) MATHEMATICAL PHYSICS

CLASS TEST-1

① Let us consider the temp. of crystal increases uniformly by a small amount ΔT and this produces strain components as:

$$\varepsilon_{ij} = \alpha_{ij} \Delta T$$

↳ component of strain rise.

$\varepsilon_1 = \alpha_1 \Delta T$
$\varepsilon_2 = \alpha_2 \Delta T$
$\varepsilon_3 = \alpha_3 \Delta T$

where $\alpha_1, \alpha_2, \alpha_3$ are the principal expansion coefficients.

$$\alpha_1 x_1^2 + \alpha_2 x^2 + \alpha_3 x_3^2 = 1 .$$

The change in overall volume of a cube of side L for a 1K temp change is given by;

$$\delta V = L^3 (1+\alpha_1)(1+\alpha_2)(1+\alpha_3) - L^3$$
$$= V(\alpha_1 + \alpha_2 + \alpha_3)$$

hence volume expansivity $\alpha_V = \delta V / V$

$$\Rightarrow \alpha_V = (\alpha_1 + \alpha_2 + \alpha_3) .$$

② $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$; $h = 0.1$.

$$K_1 = hf(x_0, y_0) = 0.1 f(1, 1.5) = (0.1)(3.25) = 0.325$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = 0.1 f(1.05, 1.6625)$$

$$= (0.1)(3.96983) = 0.39698 .$$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.1)f(1.05, 1.69332)$

$= (0.1)(3.96983) = 0.39698$.

$K_4 = hf(x_0 + h, y_0 + k_3) = 0.1 f(1.1, 1.89698)$

$= (0.1)(4.80855) = 0.48085$.

$y_1 = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$y_1 = 1.5 + \frac{1}{6}\left[0.325 + 2(0.38664) + 2(0.39698) + (0.48085)\right]$

$y(1.1) = \underline{1.89552}$ ..

(3) Aij is a tensor of rank 2, it is said to be skew-symmetric if,

$$\underline{A^{ij} = -A^{ji}}$$

Total elements of n tensor $= n^2$

diagonal elements $= A^{ii}$

skew-symmetric ; $A^{ij} = -A^{ji}$

$\Rightarrow A^{ii} = -A^{ii}$ $\therefore \boxed{A^{ii} = 0}$

$\therefore$ diagonal elements are zero.

different components $= n(n-1)$

as $A^{12} = -A^{21}$, $A^{13} = A^{31}$, ... so on ..

$\therefore$ Total different components $= \frac{n(n-1)}{2}$.

(4) Dextoral.

Any index which is repeated in a given term is called dummy / dextoral index

Consider the expression $a_i x^i$ where i, is dextral index,

then $a_i x^i = a_1 x^1 + a_2 x^2 + \ldots a \ldots a_n x^n$.

and $a_j x^j = a_1 x^1 + x_2^2 + \ldots \quad a_n x^n$

i.e. $\underline{a_i x^i = a_j x^j}$.

so it can be replaced by any other index.

Real

Any index which appears only once is the real index.

ex. A_k^{ij} all the indexes appears only once and A is tensor of rank 3.

(5) The shift operator E is the operath of increasing the argument x by h, i.e

$$E f(x) = f(x+h), \quad E^2 f(x) = f(x+2h) \ldots \text{etc.}$$

inverse $E^{-1} f(x) = f(x-h)$.

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central difference operator is given by:

$$y_1 - y_0 = \Delta y_0 = \delta y_{1/2}.$$

and mean operator $\mu f(x) = \frac{1}{2}\left[f(x+\frac{1}{2}h) + f(x-\frac{1}{2}h)\right]$

Relatn b/w them

$$\mu f(x) = \frac{1}{2}\left(f\left(x+\frac{1}{2}h\right) + f\left(x-\frac{1}{2}h\right)\right)$$

$$\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$$ where E is shift operator.

$$= \left[\sqrt{\frac{1}{4}\left(E^{1/2} + E^{-1/2}\right)^2}\right]$$

$$= \left[\sqrt{\frac{1}{4}\left(E + E^{-1} + 2E^{1/2}E^{-1/2}\right)}\right]$$

$$= \left[\sqrt{\frac{1}{4}\left(E^{1/2} - E^{-1/2}\right)^2 + 4}\right]$$

as $\delta = E^{1/2} - E^{-1/2}$

$$= \sqrt{\frac{1}{4}\left(\delta^2 + 4\right)}$$

$$= \sqrt{\frac{\delta^2}{4} + 1}$$

$$\therefore \boxed{\mu^2 = 1 + \frac{\delta^2}{4}}$$