Assignment

Quantum Mechanics

Aditya Singh

2K19/EP/005

[1]

$$[x, p_x^2]\psi = (xp_x^2 - p_x^2 x)\psi$$

$$= -i\hbar\left(xp_x \frac{\partial \psi}{\partial x} - p_x \frac{\partial (x\psi)}{\partial x}\right)$$

$$= -i\hbar\left(xp_x \frac{\partial \psi}{\partial x} - p_x\psi - p_x\left(x\frac{\partial \psi}{\partial x}\right)\right)$$

$$= -i\hbar\left[xp_x \frac{\partial \psi}{\partial x} - p_x\psi - \left[i\hbar\left(\frac{d\psi}{dx} - x\frac{\partial^2 \psi}{\partial x^2}\right)\right]\right)$$

$$= -i\hbar\left(xp_x \cancel{\frac{\partial \psi}{\partial x}} - p_x\psi - \left(p_x\psi - xp_x \cancel{\frac{\partial \psi}{\partial x}}\right)\right)$$

$$= i\hbar(-2p_x\psi) = \boxed{-2i\hbar p_x}$$

[2]

infinite potential well, $E \geq 0$

$$\lambda_I \geq \frac{2\pi\hbar}{\sqrt{2mE}}, \quad \lambda_F \geq \frac{2\pi\hbar}{\sqrt{2m(V_0 - E)}}$$

where $V_0 > E$.

there is a probability of particle to be found outside the well boundaries in finite potential well but not in infinite well.

$$\therefore \boxed{\lambda_F > \lambda_I}$$

3 $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

Probability, $P = \int_0^{a/3} \psi(x)\psi^*(x)\,dx = \int_0^{a/3} \frac{2}{a} \sin^2\left(\frac{\pi x}{a}\right) dx$

$= \frac{2}{a} \int_0^{a/3} 1 - \cos^2\left(\frac{\pi x}{a}\right) dx.$

$= \frac{2}{a} \int 1 - \left(\frac{1}{2} + \frac{\cos 2\pi x}{2}\right) dx.$

$= \frac{1}{a}\left[\frac{a}{3} - \frac{a}{2\pi} \sin \frac{2\pi a}{3a} - 0\right]$

$= \frac{1}{a}\left[\frac{a}{3} - \frac{a}{2\pi}\frac{\sqrt{3}}{2}\right] = \boxed{\frac{1}{3} - \frac{\sqrt{3}}{4\pi}}.$

4 $|\psi_1\rangle = 4i\,|\phi_1\rangle - 12i\,|\phi_2\rangle$

$|\psi_2\rangle = |\phi_1\rangle - 6i\,|\phi_2\rangle$ $\quad \langle\phi_1|\phi_2\rangle = 0$

$\langle \psi_1 + \psi_2| = \underline{(1-4i)\langle\phi_1| + 16i\langle\phi_2|}$.

5 $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \lambda x.$

$E_n = 0, \quad E_n^2 = \frac{-\lambda^2}{2m\omega^2}$

$\underline{T \cdot E} = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{\lambda^2}{2m\omega^2}.$

$\underline{\text{oscillator}}$, $V(x) = \frac{1}{2} m\omega^2 x^2 + bx$

first & second order correction,

$$E_n^1 = 0, \quad E_n^2 = \frac{b^2\cancel{\hbar}}{2m\omega}\left(\frac{\cancel{n}}{\cancel{\hbar}\omega} - \frac{n\lambda^2}{\cancel{\hbar}\omega}\right)$$

for $n=0$,

$$E_0 = \frac{1}{2}\hbar\omega - \frac{\lambda^2}{2m\omega^2}$$

for $n=1$,

$$E_1 = \frac{3}{2}\hbar\omega - \frac{\lambda^2}{2m\omega^2}$$

$$\underline{E_1 - E_0} = \frac{3}{2}\hbar\omega - \frac{1}{2}\hbar\omega = \boxed{\hbar\omega}$$

$\boxed{6}$

a) $[L_x, L_y] = i\hbar L_z$.

$$[L_x, L_y]\psi = (L_xL_y - L_yL_x)\psi$$

$$= -\hbar\left[\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\left(z\frac{\partial\psi}{\partial x} - x\frac{\partial\psi}{\partial z}\right) - \left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)\left(y\frac{\partial\psi}{\partial z} - z\frac{\partial\psi}{\partial y}\right)\right]$$

$$= \hbar^2\left[y\frac{\partial}{\partial z}\left(z\frac{\partial\psi}{\partial x}\right) - y\frac{\partial}{\partial z}\left(x\frac{\partial\psi}{\partial z}\right) - z\frac{\partial}{\partial y}\left(z\frac{\partial\psi}{\partial x}\right) + z\frac{\partial}{\partial y}\left(x\frac{\partial\psi}{\partial z}\right) - z\frac{\partial}{\partial x}\left(y\frac{\partial\psi}{\partial z}\right) + z\frac{\partial}{\partial x}\left(z\frac{\partial\psi}{\partial y}\right) + x\frac{\partial}{\partial z}\left(y\frac{\partial\psi}{\partial z}\right) - x\frac{\partial}{\partial z}\left(z\frac{\partial\psi}{\partial y}\right)\right]$$

$$= -\hbar^2\left(y\frac{\partial\psi}{\partial x} - x\frac{\partial\psi}{\partial y}\right)$$

$$= -\hbar\frac{L_z}{i\hbar}\psi = \underline{i\hbar L_z\psi}.$$

(b) $[L_z, x] = i\hbar y$

$[L_z, x]\psi = (L_z x - x L_z)\psi$

$= i\hbar \left(x\frac{\partial}{\partial y}(x\psi) - y\frac{\partial}{\partial x}(x\psi) \right)$

$= -i\hbar \left(\cancel{x\frac{\partial \psi}{\partial y}} - xy\cancel{\frac{\partial \psi}{\partial x}} - y\psi - \cancel{x\frac{\partial \psi}{\partial y}} + xy\cancel{\frac{d\psi}{\partial x}} \right)$

$= \underline{i\hbar y\,\psi}$.

(c) $[L_z, p_z] = i\hbar x$

$\left(L_z = i\hbar \left[x\frac{d}{\partial y} - y\frac{\partial}{\partial x} \right] \right)$

$\left(p_x = -i\hbar \frac{\partial}{\partial z} \right)$

$[L_z, p_z]\psi = [L_z p_z - p_z L_z]\psi$

$= -i\hbar^2 \left[x\frac{\partial}{\partial y}\cancel{\left(\frac{\partial \psi}{\partial z}\right)} - y\frac{\partial}{\partial x}\cancel{\left(\frac{\partial \psi}{\partial z}\right)} \right.$

$\left. - x\frac{\partial}{\partial z}\cancel{\left(\frac{\partial \psi}{\partial y}\right)} - y\frac{\partial}{\partial z}\cancel{\left(\frac{\partial \psi}{\partial x}\right)} + \frac{\partial \psi}{\partial y}\frac{\partial x}{\partial z} - \frac{\partial \psi}{\partial x}\frac{\partial y}{\partial z} \right]$

$= -\hbar^2 \left[\frac{\partial \psi}{\partial x}\frac{\partial y}{\partial z} - \frac{\partial \psi}{\partial y}\frac{\partial x}{\partial z} \right]$

as LHS $\neq$ RHS , this option is incorrect .

(d) $[L_x, r^2] = 0$

$[L_x, r^2] = [L_x, x^2] + [L_x, y^2] + [L_x, z^2]$

$= [L_x, x]x + [L_x, y]y + [L_x, z]z$.

$= 0 + 0 + 0 = \underline{0}$.

(e)

$$L^2 = \frac{\hbar}{2}\left(L_+L_- + L_-L_+ \right) + L_z^2$$

$$L_+ = L_x + iL_y$$
$$L_- = L_x - iL_y$$

$$L_x = \frac{L_+ + L_-}{2} \quad , \quad L_y = \frac{L_+ - L_-}{2}.$$

$$\rightarrow \quad L^2 = L_x^2 + L_y^2 + L_z^2$$

$$L_+L_- = (L_x + iL_y)(L_x - iL_y)$$
$$= L_x^2 + L_y^2$$
$$L_-L_+ = (L_x - iL_y)(L_x + iL_y)$$
$$= L_x^2 + L_y^2$$

$$\Rightarrow \quad L^2 = (L_{+-}) + L_z^2 = \frac{1}{2}(L_{+-}) + L_z^2$$

$$\Rightarrow \quad L^2 = \frac{1}{2}\left(L_+L_- + L_-L_+ \right) + L_z^2$$

correct.

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END.