

Gate: 2009

The Lagrangian of a free particle in spherical polar coordinate is given by

$$L = \frac{1}{2} m \left[\dot{r}^2 + r \dot{\theta}^2 + r^2 \dot{\phi}^2 \sin^2 \theta \right]$$

The quantity that is conserved is

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\phi}}$

Ans (b)

Explaination:

Given $L = \frac{1}{2} m \left[\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \dot{\phi}^2 \sin^2\theta \right]$

$$\Rightarrow \frac{\partial L}{\partial \dot{\phi}} = \frac{1}{2} m \frac{\partial}{\partial \dot{\phi}} \left[\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \dot{\phi}^2 \sin^2\theta \right]$$

$$= \frac{1}{2} m \left[\frac{\partial}{\partial \dot{\phi}} \left(\dot{r}^2 \right) + \frac{\partial}{\partial \dot{\phi}} \left(r^2 \dot{\theta}^2 \right) + \frac{\partial}{\partial \dot{\phi}} \left(r^2 \dot{\phi}^2 \sin^2\theta \right) \right]$$

$$= \frac{1}{2} m \left[0 + 0 + r^2 \sin^2\theta \frac{\partial}{\partial \dot{\phi}} \dot{\phi}^2 \right]$$

$$\frac{\partial L}{\partial \dot{\phi}} = \frac{1}{2m}\left[r^2 \sin^2\theta \cdot 2\dot{\phi}\right]$$

$$= \left(r^2 \sin^2\theta\right)\dot{\phi}$$

and $$\frac{\partial L}{\partial \phi} = \frac{\partial}{\partial \phi}\left[\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2 \sin^2\theta\right]$$

$$= \left[0 + 0 + 0\right]$$

$$= 0 \quad \text{—— (i)}$$

Using Lagrange's Equation.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = 0.$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - 0 = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = 0$$

$$\Rightarrow \frac{\partial L}{\partial \dot{\phi}} = \text{Constant}$$

Alternate method:

$$L = \frac{1}{2} m \left[\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \dot{\phi}^2 \sin^2\theta \right]$$

Since L does not contain ϕ

$\therefore$ ϕ is cyclic or ignorable coordinate

The generalised momentum corresponding to cyclic coordinate is constant with time i.e. p_ϕ = constant

But $p_\phi = \dfrac{\partial L}{\partial \dot{\phi}}$ (By formula.)

$\therefore$ $\dfrac{\partial L}{\partial \dot{\phi}}$ = constant.