

Question Based On

"Equilibrium"

Basic Concept:

1) Equilibrium

A body is in Equilibrium if, net force acting on body is zero

$$\vec{F} = 0$$

if forces are conservative than $\vec{F} = -\frac{dU}{dr}$

for Equilibrium $F=0$

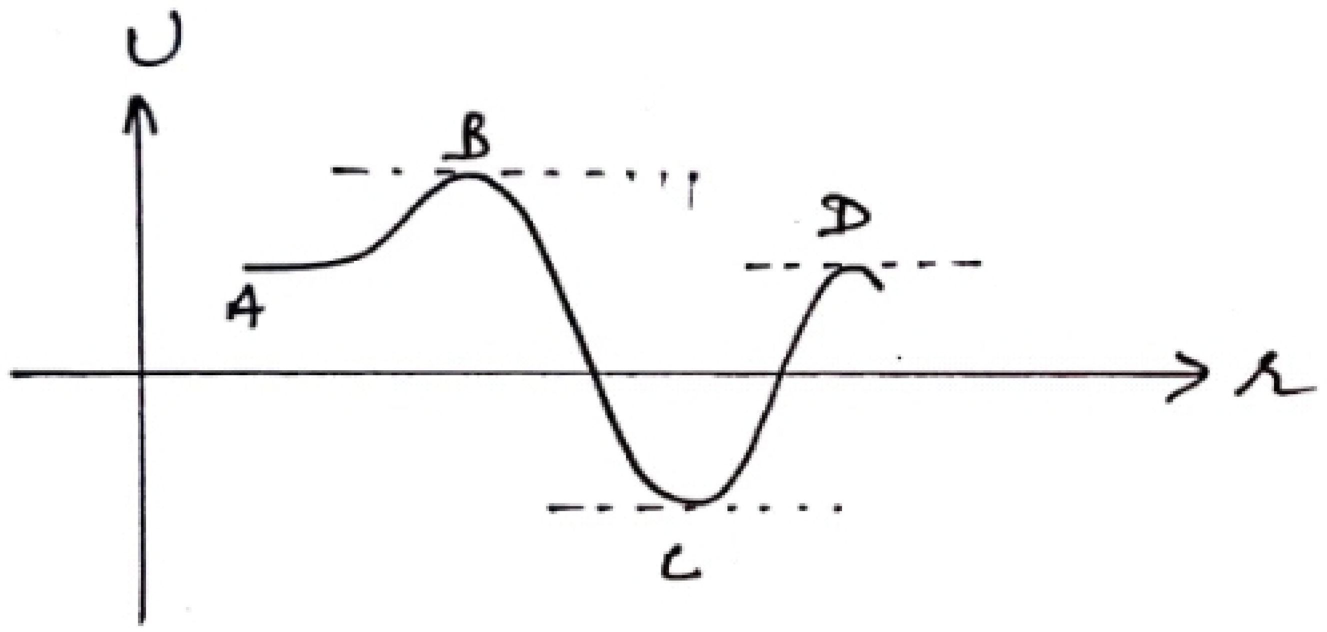
$$\Rightarrow -\frac{dU}{dr} = 0$$

$$\Rightarrow \frac{dU}{dr} = 0$$

i.e. at Equilibrium

slope of U-r graph = zero

This mean P.E. is optimum (max. or minimum)



A, B, C, D all are Equilibrium position, at these points $\boxed{\frac{dU}{dr} = \text{slope} = 0}$

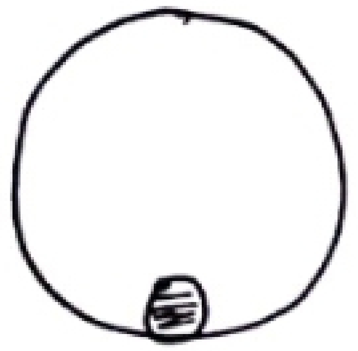
i.e. $\boxed{F = -\frac{\partial U}{\partial r} = 0}$

There are three types of Equilibrium

Equilibrium

- Stable Equilibrium
- unstable Equilibrium
- Neutral Equilibrium

Stable Equilibrium

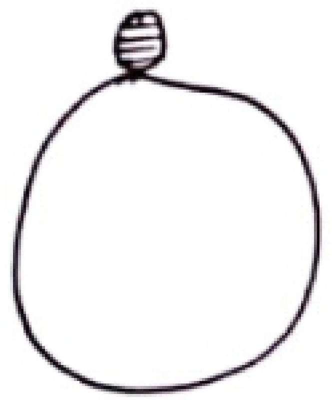


$f = 0$

$\frac{dU}{dr} = 0$

$\frac{d^2U}{dr^2} = +ve$

unstable Equilibrium



$f = 0$

$\frac{dU}{dr} = 0$

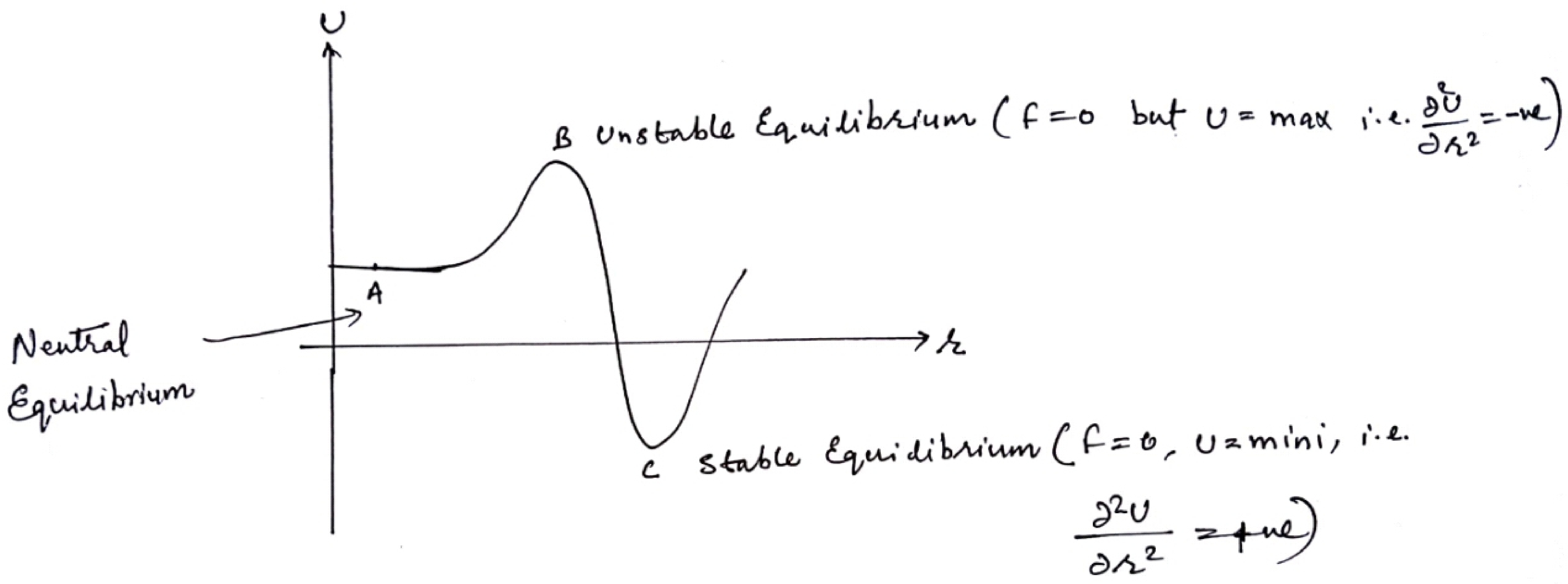
$\frac{d^2U}{dr^2} = -ve$

Neutral Equilibrium

$f = 0$

$\frac{dU}{dr} = 0$

$\frac{d^2U}{dr^2} = 0$


U
r
A
B Unstable Equilibrium (f = 0 but U = max i.e. $\frac{\partial^2 U}{\partial r^2} = -ve$)
Neutral Equilibrium
C Stable Equilibrium (f = 0, U = mini, i.e. $\frac{\partial^2 U}{\partial r^2} = +ve$)

Question: (CSIR- UGC NET DEC 2011)

The potential of a diatomic molecule as a function of distance 'r' between the atoms is given by $V(r) = \frac{-a}{r^6} + \frac{b}{r^{12}}$

The value to the potential at Equilibrium separation between the atoms is. ____

Solution :

Given: potential $V(r) = \frac{-a}{r^6} + \frac{b}{r^{12}}$

To find : The potential at equilibrium separation

for Equilibrium $\frac{\partial V}{\partial r} = 0$

$$\frac{\partial}{\partial r}\left(\frac{-a}{r^6} + \frac{b}{r^{12}}\right) = 0$$

$$-a \frac{\partial}{\partial r}\left(\frac{1}{r^6}\right) + b \frac{\partial}{\partial r} \frac{1}{r^{12}} = 0$$

$$-a \frac{\partial}{\partial r} r^{-6} + b \frac{\partial}{\partial r} r^{-12} = 0$$

$$-a(-6)r^{-6-1} + b(-12)r^{-12-1} = 0$$

$$+6a\,r^{-7} - 12b\,r^{-13} = 0$$

$$\frac{6a}{r^7} - \frac{12b}{r^{13}} = 0$$

$$\frac{6a}{r^7} - \frac{12b}{r^7 r^6} = 0$$

$$\frac{1}{r^7}\left[\frac{6a}{1} - \frac{12b}{r^6}\right] = 0$$

$\Rightarrow \frac{1}{r^7} = 0$ is not useful.

$$\therefore \quad \frac{6a}{1} - \frac{12b}{r^6} = 0$$

$$6a = \frac{12b}{r^6}$$

$$r^6 = \frac{12b}{6a}$$

$$r^6 = \frac{2b}{a}$$

$$\left(r^6\right)^{1/6} = \left(\frac{2b}{a}\right)^{1/6}$$

$$r = \left(\frac{2b}{a}\right)^{1/6}$$

This is the equilibrium position.

potential at this Equilibrium position

$$V\left(r=\left(\frac{2b}{a}\right)^{1/6}\right) = \frac{-a}{\left\{\left(\frac{2b}{a}\right)^{\frac{1}{6}}\right\}^{6}} + \frac{b}{\left\{\left(\frac{2b}{a}\right)^{\frac{1}{6}}\right\}^{12}}$$

$$= \frac{-a}{2b/a} + \frac{b}{(2b/a)^2}$$

$$= \frac{-a^2}{2b} + \frac{a^2 b}{2^2 b^2}$$

$$= \frac{-a^2}{2b} + \frac{a^2}{4b}$$

$$= \left(-\frac{1}{2}+\frac{1}{4}\right)\frac{a^2}{b}$$

$$V = \left(\frac{-2+1}{4}\right)\frac{a^2}{b}$$

$$= \frac{-1}{4}\,\frac{a^2}{b}$$

$$= -\frac{a^2}{4b} \quad \text{(Ans)}$$

This is the required potential at Equilibrium position.