Class test - 02

Total Marks = 20
Time: 50 Minutes.

Answer all questions.
Each question carry 05 Marks

(1) The normalized eigenstates of a particle in one dimensional potential well $V(x) = 0$ if $0 \le x \le a$ and ∞ otherwise, are given by $\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$ where $n = 0, 1, 2 \cdots$

The particle is subjected to a perturbation

$$V'(x) = V_0 \cos \frac{\pi x}{a} \quad \text{for } 0 \le x \le a/2$$
$$= 0 \quad \text{otherwise}$$

Calculate the shift in the ground state energy due to the perturbation in the first-order correction.

(2) Consider a particle that has the Hamiltonian $H = H_0 + \lambda \hbar\omega (a^2 + \bar{a}^2)$, where H_0 is the Hamiltonian of a simple one dimensional Harmonic Oscillator, and a and $\bar{a}$ are the usual annihilation (lowering) and creation (raising) operators which obey $[a, \bar{a}] = 1$ and λ is a very small real number.

(i) Calculate the ground state energy to second order in λ

(ii) Find the energy of the n^{th} excited state E_n to second order in λ and the corresponding $|\psi_n\rangle$ to the first order in λ.

(3) What do you mean by Time-Independent Perturbation theory. Derive an expression for the first and second order correction in energy for non-degenerate Eigenstate.

(4) Estimate the ground state energy of a particle confined in one-dimensional potential $V(x) = g|x|$ using trial wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{c}{a^5}} (a^2 - x^2) & \text{for } x < |a| \\ 0 & \text{for } x \ge |a| \end{cases}$$

where g and c are Constants.

xx All the Best xx