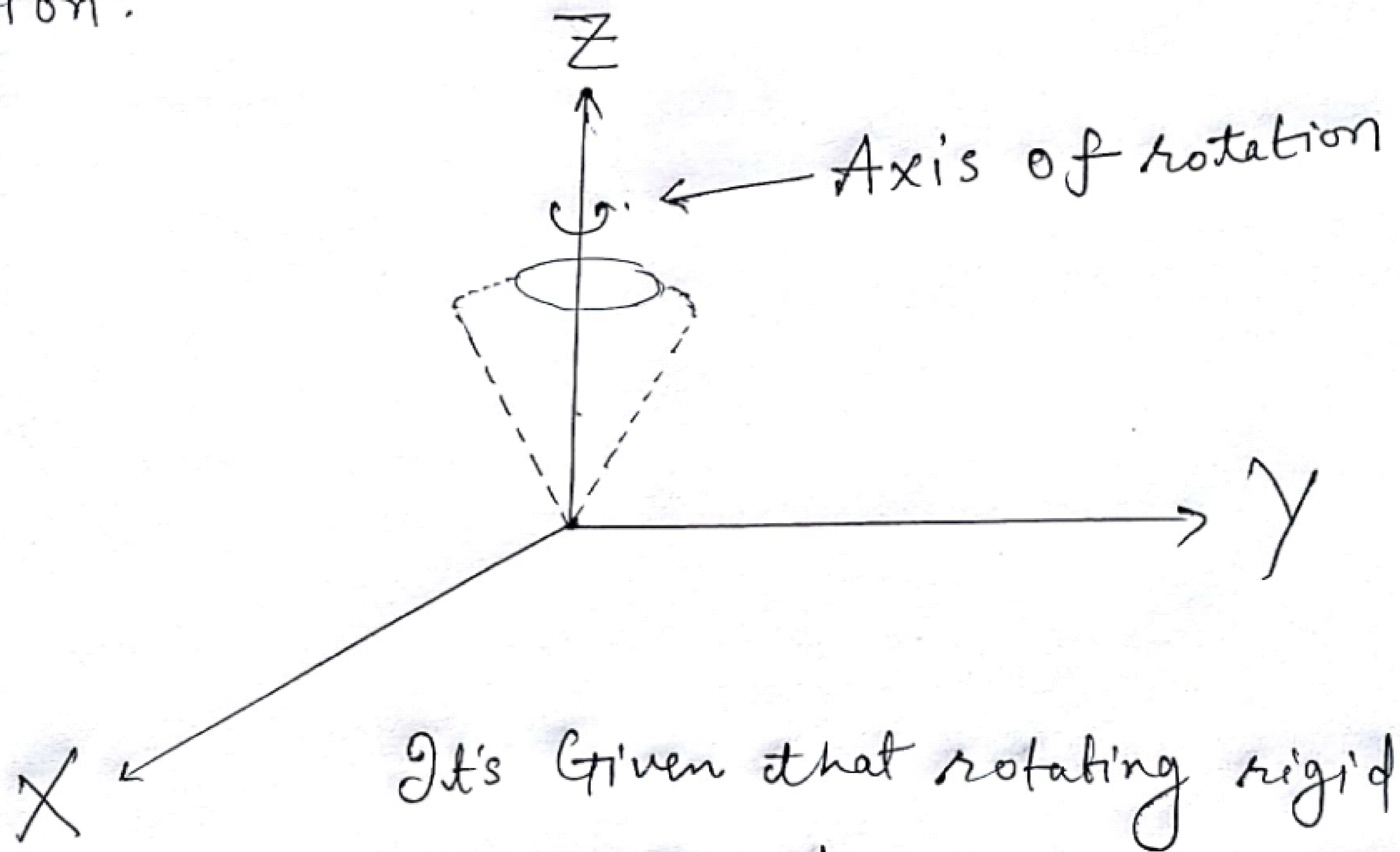
Gate 2011:

A heavy symmetrical top is rotating about its own axis of symmetry (z-axis). If I_1, I_2 and I_3 are principle moment of Interia along x, y, z axes respectively then

(a) $I_2 = I_3$; $I_1 \neq I_2$ (b) $I_1 = I_2$, $I_1 \neq I_3$

Ans. (b) $I_1 = I_2$, $I_1 \neq I_3$

Explaination:



It's Given that rotating rigid body is symmetrical.

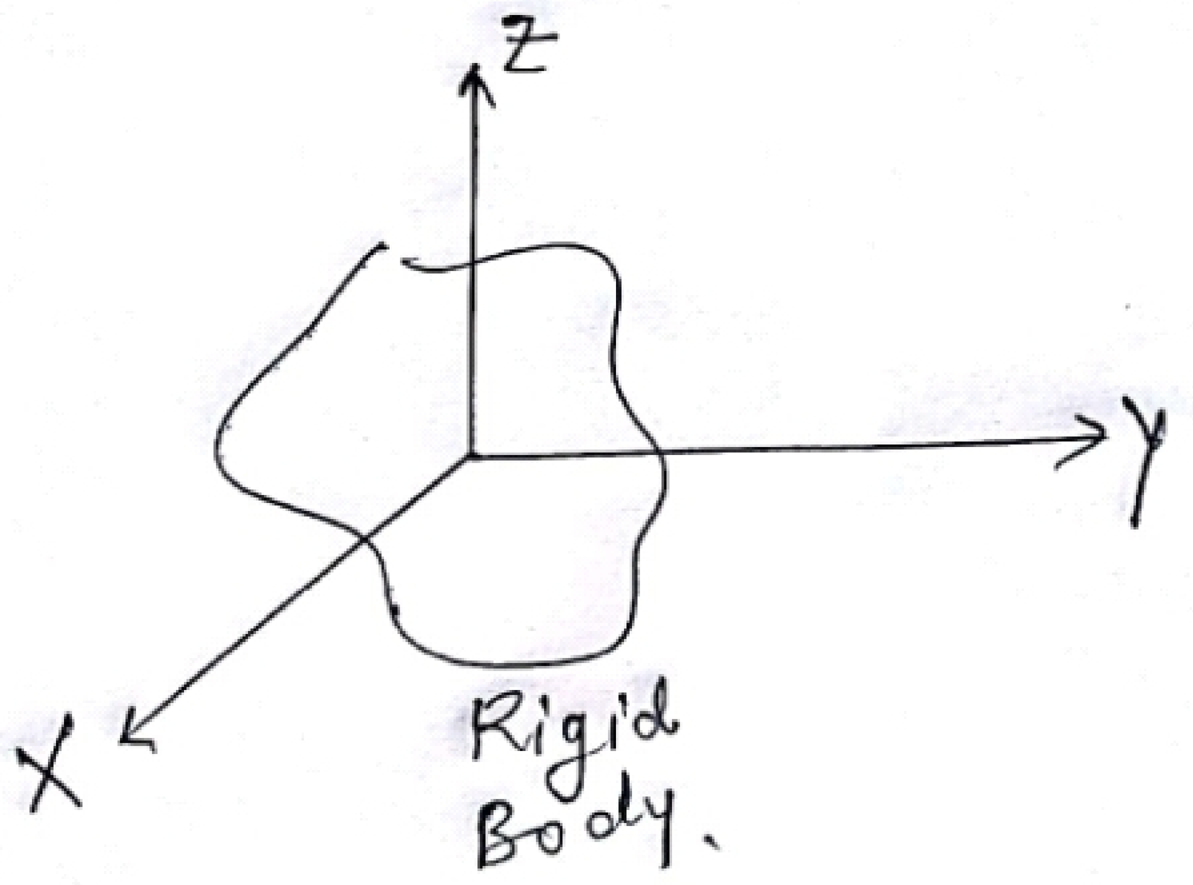
Fact: In general, a rigid body is Symmetrical, if two of its principle moment of interia are equal $I_1 = I_2$

(note, we do not select I_3 because $I_Z = I_3$ is axis of rotation).

Now, By Theorem of perpendicular axes.

$$I_Z = I_x + I_y$$

or $$I_3 = I_1 + I_2$$



But $I_1 = I_2$

$\therefore \quad I_3 = I_1 + I_1$

$\Rightarrow \quad I_3 = 2I_1$

$\therefore \quad I_1 \neq I_3$

So we have $I_1 = I_2$ and $I_1 \neq I_3$

is the required answer.

Gate 2006 :

A particle is moving in an inverse square field. If the total energy of the particle is positive. Than trajectory of particle is

(a) Circular

(b) elliptical

(c) parabolic

(d) Hyperbolic

Correct Ans (d)

Explaination:

Important table:

Total Energy 'E'	eccentricity	Trajectory.
$E > 0$	$e > 1$	Hyperbola
$E = 0$	$e = 1$	Parabola
$E < 0$	$e < 1$	$\boxed{\text{Ellipse}}$
$E = \frac{-mk^2}{2J}$	$e = 0$	Circle.

here $k =$ Constant $\left[f(r) = \dfrac{-k}{r^2} \right]$

$J =$ angular momentum magnitude $= rmv = rm\, r\omega$

$= r^2 m \omega = r^2 m \dfrac{d\theta}{dt}$

$= r^2 m \dot{\theta}$