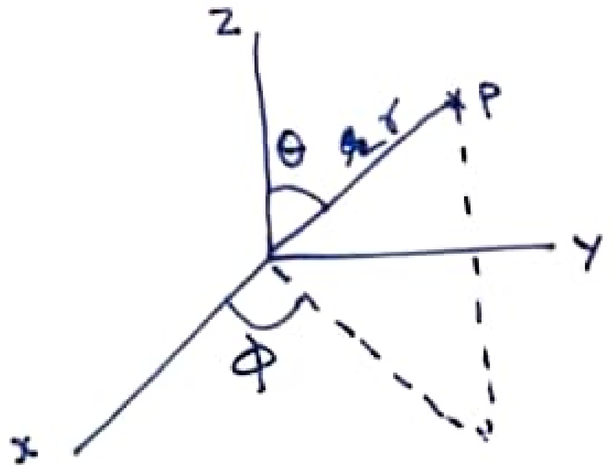
Spherical Polar Coordinate (L_z, L)



$$L_z = -i\hbar \frac{\partial}{\partial \phi}$$

$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\cdot\frac{\partial}{\partial\theta}\sin\theta\cdot\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\ldots\right.$$

$$L^2 Y(\theta,\phi) = \lambda\hbar^2 Y(\theta,\phi)$$

$$\boxed{\begin{aligned} L_z Y_{lm}(\theta,\phi) &= m\hbar Y_{lm}(\theta,\phi) \\ L^2 Y_{lm}(\theta,\phi) &= l(l+1)\hbar^2 \ldots \end{aligned}}$$

Spherical Harmonics

$$x = r\sin\theta.\cos\phi$$
$$y = r\sin\theta.\sin\phi$$
$$z = r\cos\theta$$

$$r^2 = x^2+y^2+z^2$$

$$\tan\phi = \frac{y}{x}$$

$$\tan^2\theta = \frac{x^2+y^2}{z^2}$$

$$\psi(r,\theta,\phi)$$

$$L_z\psi = -i\hbar\left[x\frac{\partial\psi}{\partial y} - y\frac{\partial\psi}{\partial x}\right]$$

$$x\frac{\partial\psi}{\partial y} = r\sin\theta.\cos\phi\left[\frac{\partial\psi}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial\psi}{\partial\theta}\cdot\frac{\partial\theta}{\partial y} + \frac{\partial\psi}{\partial\phi}\cdot\frac{\partial\phi}{\partial y}\right]$$

$$y\frac{\partial\psi}{\partial x} = r\sin\theta.\sin\phi\left[\frac{\partial\psi}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial\psi}{\partial\theta}\cdot\frac{\partial\theta}{\partial x} + \frac{\partial\psi}{\partial\phi}\cdot\frac{\partial\phi}{\partial x}\right]$$

$$r^2 = x^2+y^2+z^2 \Rightarrow 2r\frac{\partial r}{\partial x} = 2x \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}$$

$$\Rightarrow \frac{\partial r}{\partial x} = \sin\theta.\sin\phi$$

$$\Rightarrow \frac{\partial r}{\partial y} = \sin\theta.\cos\phi$$

$$\tan\phi = \frac{y}{x} \Rightarrow \sec^2\phi.\frac{\partial\phi}{\partial x} = \frac{-y}{x^2}$$

$$\Rightarrow \frac{\partial\phi}{\partial x} = -\frac{1}{r}\cdot\frac{\sin\phi}{\sin\theta}$$

$$[x\ p_x] \equiv [x p_x - p_x x] = i\hbar$$

Doesn't commute : Measurments ?

$$[2, 3] = [2 \cdot 3 - 3 \cdot 2] = 0$$

Commute.

$$L = \vec{r} \times \vec{p}$$

$$r = (x, y, z)$$

$$p = (p_x, p_y, p_z)$$

$$L = \begin{bmatrix} \hat{i}, & \hat{j}, & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{bmatrix} = i(y p_z - z p_y) + j(z p_x - x$$

$$= \hat{i} L_x + \hat{j} L_y + \hat{k} L_z$$

$$p_x = -i\hbar \frac{\partial}{\partial x}, \quad p_x = -i\hbar \frac{\partial}{\partial y}, \quad p_z = -i$$

$$L_x = -i\hbar\left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y}\right)$$

$$L_y = -i\hbar\left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z}\right)$$

$$L_z = -i\hbar\left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}\right)$$

$$\boxed{L^2 = L_x^2 + L_y^2 + L_z^2}$$

$$[L_x, L_y]\psi = i\hbar L_z \psi$$