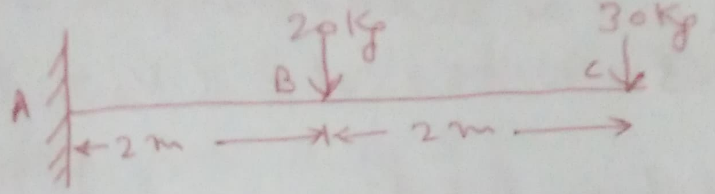
Shear Force & Bending Moment

Ex. 2 Draw S.F & BM diagram for



Soln

Note - 1. In case of cantilever, calculation of SF & BM is started from free end.

2. SF & BM diagrams are closed by drawing a vertical line at fixed end

3. In S.F. diagram, vertical line at fixed end will show the reaction.

4. In BM diagram, vertical line at fixed end will show the Resisting moment at fixed end

S.F. diagram calculations

(i) S.F at C = +30

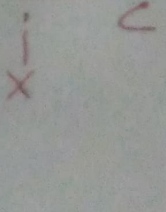
From right side
Load +ve
Reaction -ve

There will be a sudden change in shear force from 0 to 30 (Vertical line) at C)

(ii) SF between B & C

$SF_{BC} = +30$

(Horizontal line between B & C)

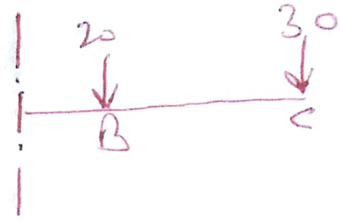


(iii) SF at B

$SF_B = +30 + 20$
$= +50$

There will be sudden change in Shear force from +30 to +50 at B (Vertical line at B)

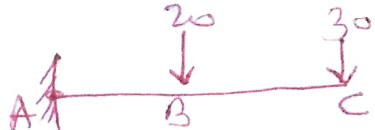
(iv) SF between B & A



$SF_{AB} = +30 + 20 = +50$

(Horizontal line between A & B)

(V) SF at A



$SF_A = 0$

(SF diagram should close and should be zero at end)

There will be sudden change in shear force from +50 to 0
(Vertical line at A)

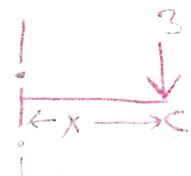
B.M. diagram calculation

From Right Side
Reaction +ve
load -ve

(i) BM at C

$BM_c = -30 * 0 = 0$

(ii) BM between B & C



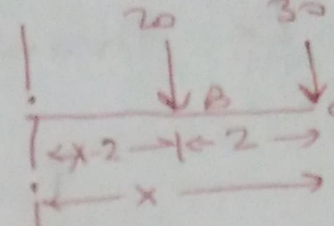
$BM_{BC} = -30 * X = -30X$

(Inclined line with downward slope from C to B)

(iii) BM at B

$BM_B = -30 * 2 - 20 * 0$
$= -60$ Kg.m

(iv) BM between A & B



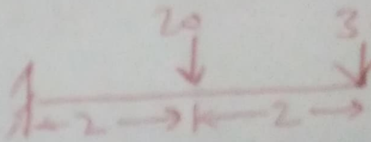
$$BM_{AB} = -30 \times x - 20(x-2)$$

$$= -50x + 40$$

(Inclined line with downward slope from B to A)

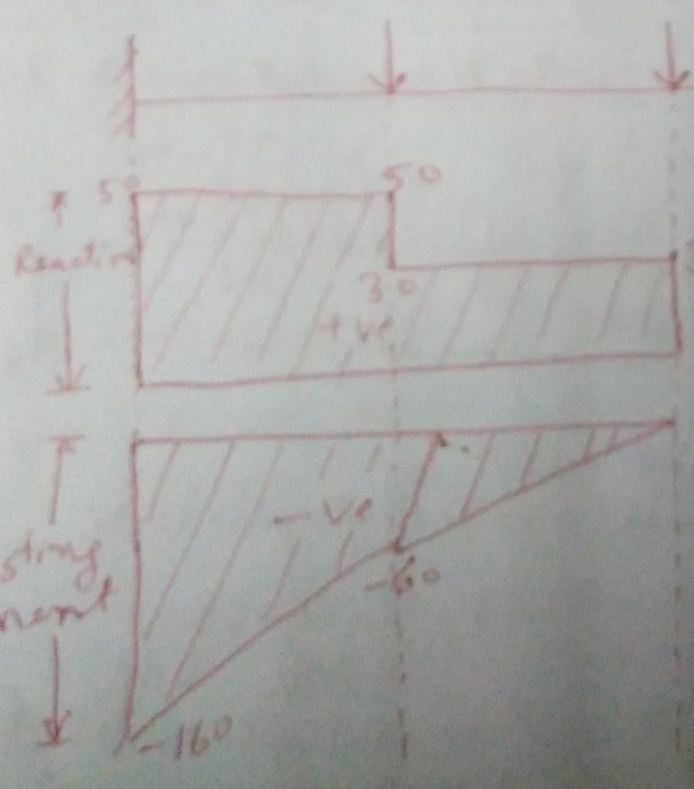
$$BM_{AB \text{ at } x=4} = -50 \times 4 + 40 = -160 \text{ kg-m}$$

(v) BM at A



$$BM_A = 0$$

- As BM diagram should close, BM at A will be zero.
- There will be sudden change in BM diagram from -160 to 0 due to resisting moment at fixed end (Vertical line at A)



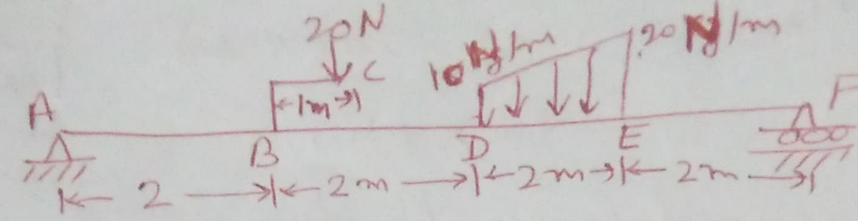
Load Diagram

SF Diagram

BM diagram

Reaction = 50 kg
Resisting moment = 160 kg-m (CCW)

Ex. Draw a S.F. & B.M. diagram for



Soln

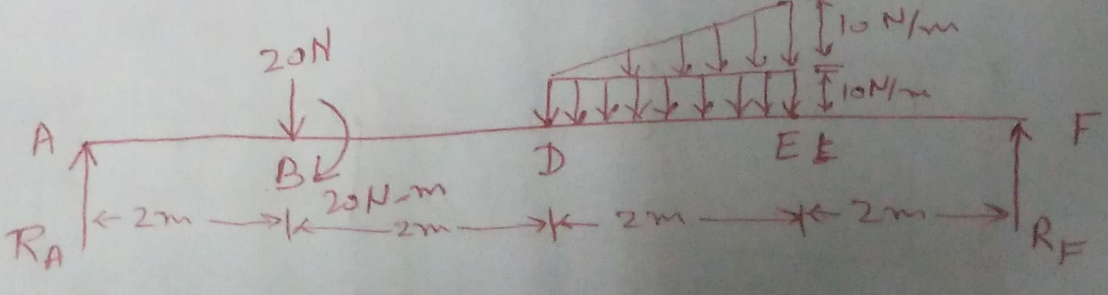
Note - There are two load on beam

① - A load of 20 N at the end of L section of 1m * 1 m welded at 2 m from left side.

- This load will be considered as a load of 20 N at B and a moment (clockwise) ~~at B~~ of 20 * 1 = 20 N-m at B

② - A uniformly varying load from 10 N/m to 20 N/m from D to E.

- This load may be considered as a UDL of 10 N/m and a varying load from 0 to 10 N/m between D to E



- At B - Load - 20 N
  Moment - 20 N (clockwise) CW

- Between D & E - Load - 10 N/m - UDL
  0 - 10 N/m - VDL

- Reaction at A will be vertical as there is no Horizontal Load / force

Reaction Calculation

$\Sigma F_y = 0$

$$R_A + R_F = 20 + \underbrace{10 * 2}_{UDL} + \underbrace{\tfrac{1}{2} * 10 * 2}_{VDL}$$

$$= 20 + 20 + 10 = 50\ N$$

$\Sigma M = 0$

Taking moment at A.

$$R_F \times 8 = 20 \times 2 + \overset{\text{Moment}}{20} + (10 * 2) \times 5$$

UDL will act at centre of D&E

$$+ \left(\tfrac{1}{2} \times 10 \times 2\right) \times \left(4 + 2 \times \tfrac{2}{3}\right)$$

VDL will act at CG of triangle i.e. 2/3 from vertex point D.

$$= 40 + 20 + 100 + \frac{160}{3}$$

$$R_F = 80/3\ N$$

$$R_A = 50 - R_F = \frac{70}{3}\ N$$

S.F. diagram Calculation

Start from left side

Reaction will be +ve

Load as −ve

(i) SF at A

A ↑ 70/3

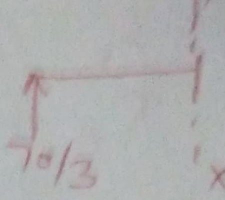
$$SF_A = \frac{70}{3}$$

There will be sudden change in SF from 0 to 70/3 N (vertical line at A)

(ii) SF between A&B

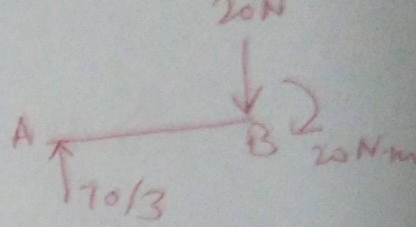
$SF_{AB} = \frac{70}{3}$ N

(Horizontal line between A & B)



(iii) SF at B

$SF_B = \frac{70}{3} - 20 = \frac{10}{3}$ N



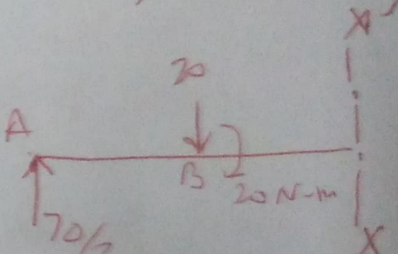
There is sudden change in SF from $\frac{70}{3}$ to $\frac{10}{3}$ at B

(Note – There will be no impact of 20N-m moment acting at point B on Shear force at B)

(iv) SF between B & D

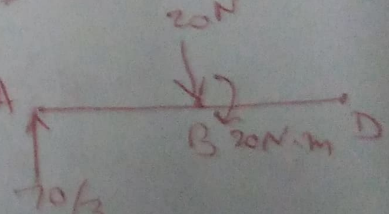
$SF_{BD} = \frac{70}{3} - 20 = \frac{10}{3}$ N

(Horizontal line between B & D)



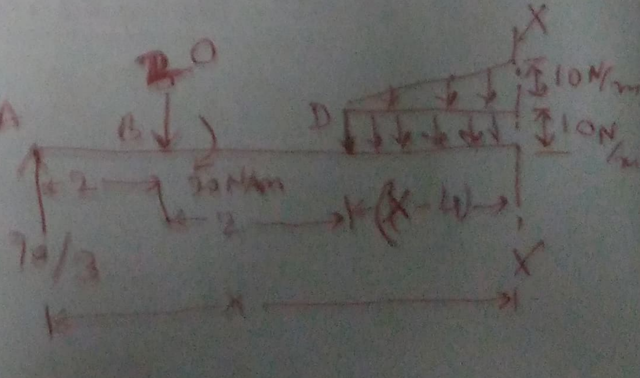
(v) SF at D

$SF_D = \frac{70}{3} - 20 = \frac{10}{3}$ N



(vi) SF between D & E

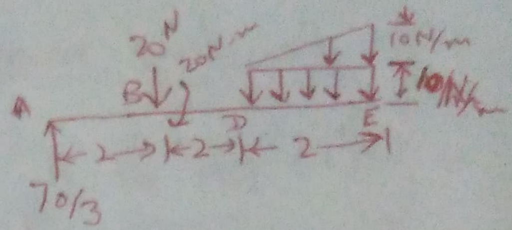
$$SF_{DE} = \frac{70}{3} - 20 - [10(x-4)] - \frac{1}{2}[10\times(x-4)]$$



$$SF_{DE} = \frac{10}{3} - 15x + 60$$
$$= -15x + \frac{190}{3}$$

(Inclined line with downward slope)

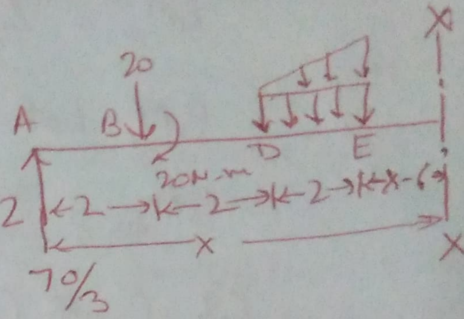
(vii) SF at E



$SF_E = +\frac{70}{3} - 20 - 10\times2 - \frac{1}{2}\times10\times2$

$= -\frac{80}{3}$ N

(viii) SF between E & F

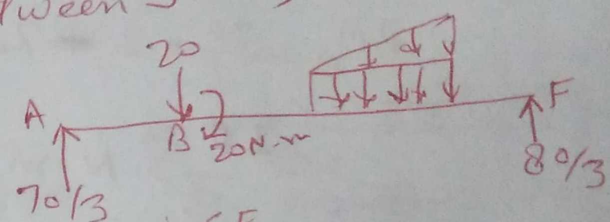


$SF_{EF} = +\frac{70}{3} - 20 - 10\times2 - \frac{1}{2}\times10\times2$

$= -\frac{80}{3}$ N

(Horizontal line between D & E)

(ix) SF at F



SF at F = 0

There will be sudden change in S.F. at F ~~due to~~ ~~reaction~~ from $-\frac{80}{3}$ to 0 due to reaction.

(Vertical line at F)

(x) As $SF_D = 10/3$ and $SF_E = -80/3$, there will be a point between D & E where shear force will be zero.

$SF_{DE} = -15x + \frac{190}{3} = 0$

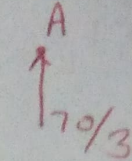
$\therefore x = \frac{190}{45} = \frac{38}{9}$

$\therefore SF_{x=38/9} = 0$

(Shear force will be zero and change its sign at a distance $\frac{38}{9}$ from point A)

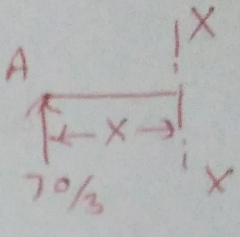
Bending moment calculation

(i) BM at A



$$BM_A = \frac{70}{3} \times 0 = 0$$

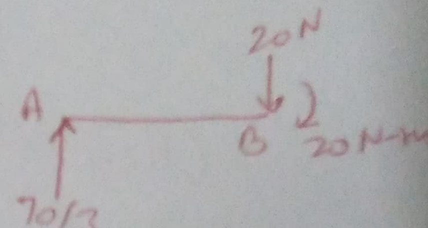
(ii) BM between A & B



$$BM_{AB} = \frac{70}{3} \cdot x = \frac{70}{3} x$$

(Inclined line with upward slope betwe A & B)

(iii) BM at B



$$BM_B = \frac{70}{3} \times 2 - 20 \times 0 + 20$$

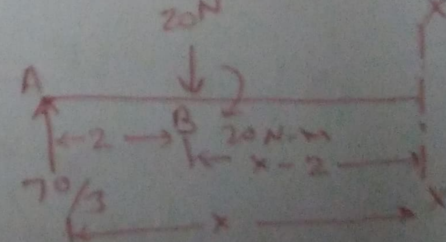
Moment

$$= \frac{200}{3} \text{ N-m}$$

There will be a sudden change in moment from $\frac{70}{3} \times 2 = \frac{140}{3}$ N-m to $\frac{80}{3}$ N-m at B due to moment available at B.

(Vertical line at B)

(iv) BM between B & D



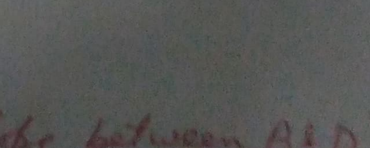
$$BM_{BD} = \frac{70}{3} x - 20(x-2) + 20$$

$$= \frac{70}{3} x - 20x + 40 + 20$$

$$= \frac{10}{3} x + 60$$

(Inclined line with upward slope between B & D)

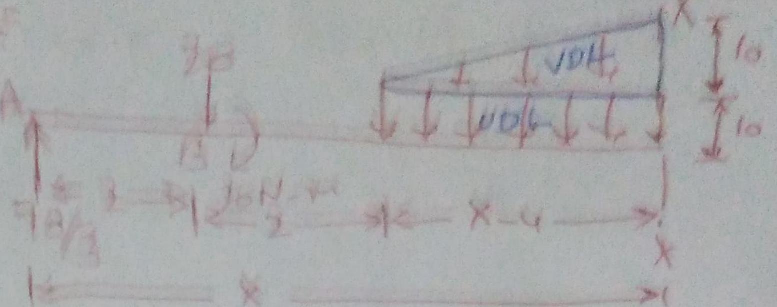
(v) BM at D



$$BM_D = \frac{70}{3} \times 4 - 20 \times 2 + 20$$

$$= \frac{220}{3} \text{ N-m}$$

(vi) BM between D & E



Moment at X X

$BM_{DE} = \frac{70}{3}x$

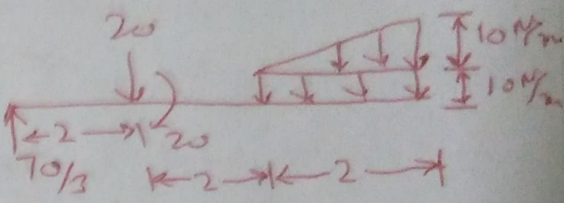
$- 20(x-2) + 20$

$- [10 * (x-4)] * \left(\frac{x-4}{2}\right)$ ← Distance of CG from XX for UDL

$- \frac{1}{2}[10 * (x-4)] * \left(\frac{x-4}{3}\right)$ ← Distance of CG from XX for VDL

(There will be parabolic curve between D & E)

(vii) BM ~~between~~ at E

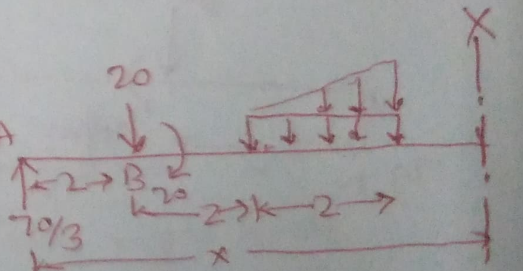


$BM_E = \frac{70}{3} * 6 - 20 * 4 + 20$

$- (10*2) * \frac{2}{2} - \frac{1}{2} * (10*2) * \frac{2}{3}$

$= \frac{160}{3}$ N-m

(viii) BM between E & F



$BM_{EF} = \frac{70}{3}x - 20(x-2) + 20$

$- (10*2)(x-5) - \frac{1}{2}(10*2)\left(x - \frac{16}{3}\right)$

$(2+2+2/2)$ $(2+2+2*2/3)$

$= \frac{70}{3}x - 20x + 40 + 20 - 20x + 100 - 10x + \frac{160}{3}$

$= -\frac{80}{3}x + 160 * \frac{4}{3}$

(ix) BM at F

$BM_F = \frac{70}{3} * 8 - 20 * 6 + 20 - (10*2) * 3$

$- \frac{1}{2}(10*2) * \frac{8}{3} + \frac{80}{3} * 0$

$= 0$

(X) B. M. will be maximum where SF changes its sign and is zero.

As SF is zero at $x = \frac{38}{9}$ between D & E

$$BM_{DE\,(x=38/9)} = \frac{70}{3}x - 20(x-2) + 20 - 10(x-4)\left(\frac{x-4}{2}\right) - \frac{1}{2} \times 10 \times (x-4) \cdot \frac{x-4}{3}$$

Putting value of $x = 38/9$

$$BM = \frac{70}{3} * \frac{38}{9} - 20\left(\frac{38}{9} - 2\right) + 20 - 5\left(\frac{38}{9} - 4\right)^2 - \frac{5}{3}\left(\frac{38}{9} - 4\right)^2$$

$$= \frac{70}{3} * \frac{38}{9} - 20 * \frac{20}{9} + 20 - \frac{20}{3} * \left(\frac{2}{9}\right)^2$$

$$= \quad N\text{-}m$$