# Quantum Mechanics

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**Q**

$$H = H_0 + \lambda\hbar\omega\left(a^2 + \bar{a}^2\right)$$

i)

$$E_n^{(1)} = \langle n|H'|n\rangle$$

$$= \lambda\hbar\omega\,\langle n|aa|n\rangle + \lambda\hbar\omega\,\langle n|\bar{a}\bar{a}|n\rangle$$

$$= \lambda\hbar\omega\left(\sqrt{(n+1)(n+2)}\,\langle n|n+2\rangle + \sqrt{n(n-1)}\,\langle n|0\rangle\right)$$

$$= 0.$$

ii)

$$E_n^{(2)} = \sum_{m\neq n} \frac{|\langle m|H'|n\rangle|^2}{E_n - E_m}$$

$$= \sum_{m\neq n} \frac{\left[\lambda\hbar\omega\langle m|a^2+\bar{a}^2|n\rangle\right]^2}{E_n - E_m}$$

$$= \lambda^2\hbar^2\omega^2 \sum_{m\neq n} \frac{\left[\sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + \sqrt{n(n-1)}\,\delta_{m,n-2}\right]^2}{(n+\frac{1}{2})\hbar\omega - (m+\frac{1}{2})\hbar\omega}$$

$$= \frac{(\lambda\hbar\omega)^2}{\hbar\omega} \sum_{m\neq n} \frac{\left((n+1)(n+2)\,\delta_{m,n+2} + n(n-1)\,\delta_{m,n-2}\right)}{n-m}$$

$$= \lambda^2\hbar\omega\left[\frac{(n+1)(n+2)}{n-(n+2)} + \frac{n(n-1)}{n-(n-2)}\right]$$

$$= \lambda^2\hbar\omega\left(-\frac{1}{2}(n+1)(n+2) + \frac{1}{2}n(n-1)\right)$$

$$= \frac{\lambda^2\hbar\omega}{2}\left(-n^2 - 3n - 2 + n^2 - n\right) = -2\lambda^2\hbar\omega\left(n+\frac{1}{2}\right).$$

$$|\psi_n^{(1)}\rangle = \sum_{m\neq n} \frac{\langle m|H'|n\rangle}{E_n-E_m}|m\rangle$$

$$= \sum_{m\neq n} \lambda\hbar\omega \frac{\langle m|\,a^2+\bar{a}^2\,|n\rangle}{E_n-E_m}|m\rangle$$

$$= \sum_{m\neq n} \lambda\hbar\omega \frac{\langle m|a^2|n\rangle + \langle m|\bar{a}^2|n\rangle}{E_n-E_m}|m\rangle$$

$$= \lambda\hbar\omega \sum_{m\neq n} \frac{\sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + \sqrt{n(n-1)}\,\delta_{m,n-2}}{(n+\frac{1}{2})\hbar\omega - (m+\frac{1}{2})\hbar\omega}|m\rangle$$

$$= \frac{\lambda\hbar\omega}{\hbar\omega} \sum_{m\neq n} \frac{\sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + \sqrt{n(n-1)}\,\delta_{m,n-2}}{n-m}|m\rangle$$

$$= \lambda\left[\frac{\sqrt{(n+1)(n+2}}{n-(n+2)} + \frac{\sqrt{n(n-1)}}{n-(n+2)}\right]|n+2\rangle$$

$$= -\frac{\lambda}{2}\left[\sqrt{(n+1)(n+2} + \sqrt{n(n-1)}\right]|n+2\rangle.$$

$$= -\frac{\lambda}{2}\left[\sqrt{(n+1)(n+2)}\,|n+2\rangle + \sqrt{n(n-1)}\,|n+2\rangle\right]$$