Unit vector in the direction of $\vec{Q} = \dfrac{i + 5j}{5.1} = 0.196\, i + 0.98\, j$

(b) $\vec{S} = 3\vec{P} + 2\vec{Q} = 3(5i - j) + 2(i + 5j)$

$= (15i - 3j) + (2i + 10j) = 17i + 7j$

Magnitude of vector $\vec{S} = \sqrt{(17)^2 + (7)^2} = 18.38$ unit

Unit vector in the direction of $\vec{S} = \dfrac{17i + 7j}{18.38} = 0.925\, i + 0.381\, j$

4.2. DOT (SCALAR) PRODUCT

The dot product of vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity, and is defined as the product of the magnitude of the vectors and cosine of their included angle

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta = AB\cos\theta$$

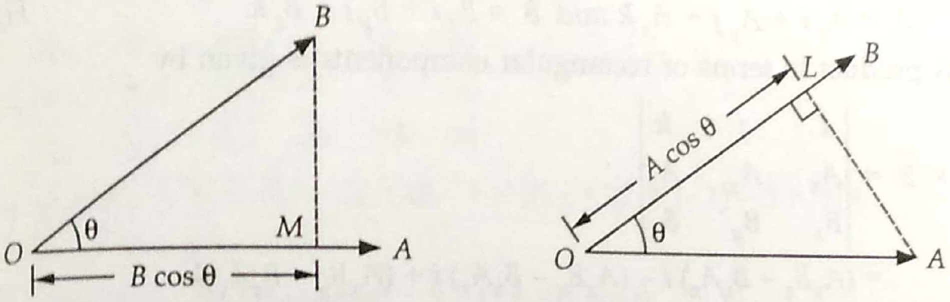

Fig. 4.6

It is to be noted that :

(i) When $\theta = 0°$, i.e., when the vectors $\vec{A}$ and $\vec{B}$ are along the same direction

$$\vec{A} \cdot \vec{B} = AB\cos 0° = AB$$

(ii) When $\theta = 90°$, i.e., when the vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other

$$\vec{A} \cdot \vec{B} = AB\cos 90° = 0$$

(iii) $\vec{A} \cdot \vec{B} = A$ times projection of $\vec{B}$ on $\vec{A}$

$= B$ times projection of $\vec{A}$ on $\vec{B}$

(iv) In terms of components,

$$\vec{A} \cdot \vec{B} = (A_x i + A_y j + A_z k) \times (B_x i + B_y j + B_z k)$$

$$= A_x B_x + A_y B_y + A_z B_z$$

i, j and k are the unit vectors along x, y and z directions respectively, and

$i \cdot i = j \cdot j = k \cdot k = 1$ $\quad$ ($\because \theta = 0°$ and $\cos 0° = 1$)

$i \cdot j = j \cdot k = k \cdot i = 0$ $\quad$ ($\because \theta = 90°$ and $\cos 90° = 1$)

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(*v*) The angle between the vectors $\vec{A}$ and $\vec{B}$ is

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{\vec{A}\cdot\vec{B}}{AB}$$

(*vi*) The dot product defines the term work in the field of dynamics.

4.3. CROSS (VECTOR) PRODUCT

The cross product of vectors $\vec{A}$ and $\vec{B}$ is a vector quantity and is defined as the product of the magnitude of the vectors and the sine of the smaller angle between the vectors. Its direction is perpendicular to the plane containing the vectors.

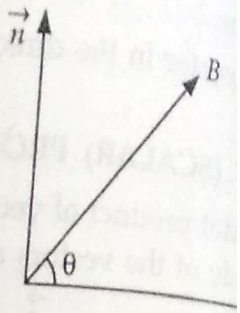

Fig. 4.7

If $\vec{n}$ is the unit vector which gives the direction of the resultant vector, then

$$\vec{R} = \vec{A}\times\vec{B} = |A||B|\sin\theta\,\vec{n} = AB\sin\theta\,\vec{n}$$

If $\vec{A} = A_x i + A_y j + A_z k$ and $\vec{B} = B_x i + B_y j + B_z k$

then the cross product in terms of rectangular components is given by

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - B_yA_z)\,i - (A_xB_z - B_xA_z)\,j + (A_xB_y - B_xA_y)k$$

Further it is to be noted that :

(*i*) $\vec{A}\times\vec{B} = 0$ if $\theta = \theta$ or π

(*ii*) $i\times i = j\times j = k\times k = 0$

$i\times j = k;\;\; j\times k = i$ and $k\times i = j$

(*iii*) The cross product defines the term moment of force.

Example 4.2. **(*a*) Check whether the following set of vectors are perpendicular to each othe not**

$$\vec{A} = 6i + 10j - 5k \quad\text{and}\quad \vec{B} = 5i + 2j + 10k$$

(*b*) Check whether the following set of vectors are parallel to each other or not

$$\vec{A} = 2i - 3j - k \quad\text{and}\quad \vec{B} = -6i + 9j + 3k$$

Solution :

$$\vec{A}\cdot\vec{B} = (6j + 10j - 5k)\cdot(5i + 2j + 10k)$$

$$= 6\times 5 + 10\times 2 + (-5\times 10) = 0$$

Since the dot product of the given vectors is zero, the vectors are perpendicular to each other.

(*b*) $\vec{A}\times\vec{B} = (2i - 3j - k)\times(-6i + 9j + 3k)$

$$= \begin{vmatrix} i & j & k \\ 2 & -3 & -1 \\ -6 & 9 & 3 \end{vmatrix}$$

$= i[-3 \times 3 - 9 \times (-1)] - j[2 \times 3 - (-6) \times (-1)] + k[2 \times 9 - (-6 \times -3)]$

$= i(-9 + 9) - j(6 - 6) + k(18 - 18) = 0$

Since the cross product of the given vectors is zero, the vectors are parallel to each other.

Example 4.3. Two vectors $\vec{A}$ and $\vec{B}$ are given as

$$\vec{A} = 2i + 3j \quad \text{and} \quad \vec{B} = 3i - j$$

Determine : (*a*) the dot product and cross product of the vectors, (*b*) the angle between vectors $\vec{A}$ and $\vec{B}$ (*c*) the included angle between vector $\vec{A}$ and the vector resulting from the cross product.

Solution : The dot product is given as

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

$$= 2 \times 3 + 3 \times (-1) = 6 - 3 = 3 \text{ units}$$

The cross product is expressed as

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 3 & -1 & 0 \end{vmatrix}$$

$$= [3 \times 0 - (-1) \times 0]\, i - (2 \times 0 - 3 \times 0)\, j + [2 \times (-1) - 3 \times 3]\, k$$

$$= 0 - 0 - 11k = -11k$$

(*b*) From the definition of dot product : $\vec{A} \cdot \vec{B} = AB \cos\theta$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{3}{\sqrt{2^2 + 3^2} \times \sqrt{3^2 + (-1)^2}}$$

$$= \frac{3}{\sqrt{13} \times \sqrt{10}} = 0.263$$

$$\theta = \cos^{-1}(0.263) = 74.75^\circ$$

Check : From the definition of cross product : $|\vec{A} \times \vec{B}| = AB \sin\theta$

$$\sin\theta = \frac{\vec{A} \times \vec{B}}{AB} = \frac{11}{\sqrt{13} \times \sqrt{10}} = 0.965$$

$$\theta = \sin^{-1}(0.965) = 74.79^\circ$$

$$\text{(c) } \vec{A} \times (\vec{A} \times \vec{B}) = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 0 & 0 & -11 \end{vmatrix} = -33i + 22j$$

The above vector has a magnitude of $\sqrt{(-33)^2 + (-22)^2} = 39.66$

$$\sin\theta = \frac{\vec{A} \times (\vec{A} \times \vec{B})|}{|\vec{A}| \times |\vec{A} \times \vec{B}|} = \frac{39.6}{\sqrt{13} \times 11} = 1.00$$

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Thus the angle between $\vec{A}$ and $(\vec{A} \times \vec{B})$ is 90°. The included angle between vector $\vec{B}$ and the vector $(\vec{A} \times \vec{B})$ would also workout to be 90°.

Remarks : The included angle between the vector resulting from the cross product and either of the constituent vectors must be 90°.

Example 4.4. Determine the work done when a force $\vec{F} = 2i + 3j - k$ moves form point P (3, 1, – 1) to point Q (2, – 1, 1).

Solution : Distance moved = position vector of point P – position vector of point Q

$$\vec{S} = (3i + j - k) - (2i - j + k) = i + 2j - 2k$$

Work done by the force is given by the dot product of force applied and distance moved

$$\text{Work done} = \vec{F} \cdot \vec{S} = (2i + 3j - k) \cdot (i + 2j - 2k)$$

$$= 2 \times 1 + 3 \times 2 + (-1) \times (-2) = \mathbf{10\ units}$$

4.4. COMPONENT OF A FORCE

Force is a vector quantity having both magnitude and direction

Let a force $\vec{F}$ be designated by vector OP. If through point P, planes are drawn parallel to the coordinate planes, then a rectangular box or parallelopiped gets formed (Fig. 4.8). The force $\vec{F}$ is then represented by the diagonal OP of the box, and the three components F_x, F_y and F_z are represented by the edges of the box.

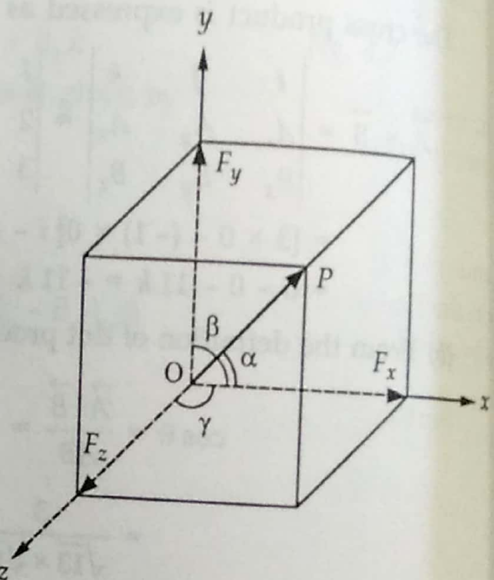

Fig. 4.8

Salient aspects :

(*i*) If i, j and k are the unit vectors in the positive x, y and z directions respectively, then force $\vec{F}$ is designated as

$$\vec{F} = F_x i + F_y j + F_z k$$

$$\text{Magnitude } F = |\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

The components F_x, F_y and F_z are called the scalar components of the force vector $\vec{F}$.

(*ii*) If α, β and γ are the angles which the force vector $\vec{F}$ makes with the coordinate axes x, y and z axes respectively, then

$$F_x = F \cos \alpha \;; \quad F_y = F \cos \beta \quad \text{and} \quad F_z = F \cos \gamma$$

That gives

$$\vec{F} = (F \cos \alpha)\, i + (F \cos \beta)\, j + (F \cos \gamma)\, k$$

(*iii*) The cosines of angles α, β and γ are called direction cosines and are denoted as

$$l = \cos \alpha \;; \quad m = \cos \beta \quad \text{and} \quad n = \cos \gamma$$

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The direction cosines are not independent and are related by the expression

$$l^2 + m^2 + n^2 = 1$$

4.5. MOMENT OF A FORCE

Moment of force refers to the turning effect produced by the force $\vec{F}$ about a point. It is a vector quantity and it equals the product of force and the perpendicular distance of the line of action of force from the point.

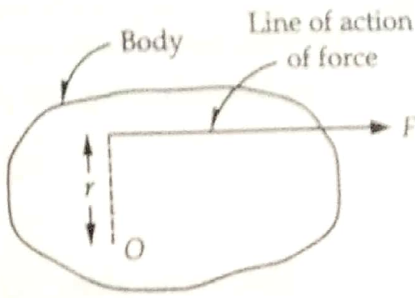

Fig. 4.9

Let $\vec{F}$ = force acting on the body

$\vec{r}$ = perpendicular distance from the point O to the line of action of force

Moment of force is then given by

$$\vec{M} = \vec{r} \times \vec{F}$$

In terms of the components of the force vector along the coordinate axes,

$$\vec{F} = F_x i + F_y j + F_z k$$

and $\vec{r} = xi + yj + zk$ is the position vector of any point on the line of action of forces with respect to O (the point about which turning takes place).

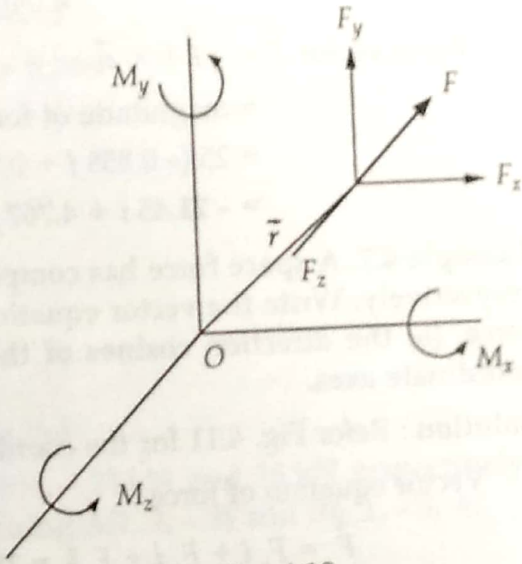

Fig. 4.10

Further $\vec{M} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (y\,F_z - z\,F_y)\,i - (x\,F_z - z\,F_x)\,j + (x\,F_y - y\,F_x)\,k$$

Example 4.5. A point is acted upon by a set of three forces given by

$$\mathbf{(3\,i + 5\,j + 2\,k),\ (2\,i + 7\,j + 3\,k)\ and\ (i + 2\,j + 5\,k).}$$

Find the magnitude of the resultant force and its direction cosines.

Solution: Resultant force $\vec{R}$

$$= (3\,i + 5\,j + 2\,k) + (2\,i + 7\,j + 3\,k) + (i + 2\,j + 5\,k)$$

$$= 6\,i + 14\,j + 10\,k$$

Magnitude of resultant $= \sqrt{6^2 + 14^2 + 10^2}$ = **18.22 units**

Direction cosines of the resultant are:

$$l = \cos\alpha = \frac{F_x}{F} = \frac{6}{18.22} = 0.329; \qquad \alpha = 70.79^\circ$$

$$m = \cos\beta = \frac{F_y}{F} = \frac{14}{18.22} = 0.768; \qquad \beta = 39.82^\circ$$

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$$n = \cos\gamma = \frac{F_z}{F} = \frac{10}{18.22} = 0.5488; \quad \gamma = 56.71^\circ$$

Check : $l^2 + m^2 + n^2 = (0.329)^2 + (0.768)^2 + (0.5488)^2 \simeq 1$

Example 4.6. A force 25 kN starts from a point P(1, 3, – 2) and passes through the point Q(- 5, 3). Express the force in terms of unit vectors *i*, *j* and *k*.

Solution : Vector joining the points $P(1, 3, -2)$ and $Q(-8, 5, 3)$ is

$$\overrightarrow{PQ} = \vec{r} = (-8-1)\,i + (5-3)\,j + [3-(-2)]\,k$$
$$= -9i + 2j + 5k$$

$$\text{Unit vector along } PQ = \frac{-9i+2j+5k}{\sqrt{(-9)^2+(2)^2+(5)^2}} = \frac{-9i+2j+5k}{\sqrt{110}}$$
$$= -0.858\,i + 0.1907\,j + 0.4767\,k$$

$$\text{Force vector } \vec{F} = |\vec{F}| \times \vec{n}$$
$$= \text{magnitude of force} \times \text{unit vector in the direction of force}$$
$$= 25(-0.858\,i + 0.1907\,j + 0.4767\,k)$$
$$= \mathbf{-21.45\,i + 4.767\,j + 11.917\,k}$$

Example 4.7. A space force has components of 10 N, 20 N and – 50 N along the *x*, *y* and *z* axes respectively. Write the vector equation for the force and calculate : (*a*) the magnitude of the force, (*b*) the direction cosines of the force and (*c*) the angles made by the force along the coordinate axes.

Solution : Refer Fig. 4.11 for the coordinate axes and the orientation of the given forces.

Vector equation of force

$$\vec{F} = F_x i + F_y j + F_z k = 10\,i + 20\,j - 50\,k$$

(*a*) Magnitude of force

$$F = |\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{10^2 + 20^2 + (-50)^2}$$
$$= 54.77 \text{ N}$$

(*b*) Direction cosines of the force

$$l = \cos\alpha = \frac{F_x}{F} = \frac{10}{54.77} = 0.1826$$

$$m = \cos\beta = \frac{F_y}{F} = \frac{20}{54.77} = 0.3652$$

$$n = \cos\gamma = \frac{F_z}{F} = \frac{-50}{54.77} = -0.9129$$

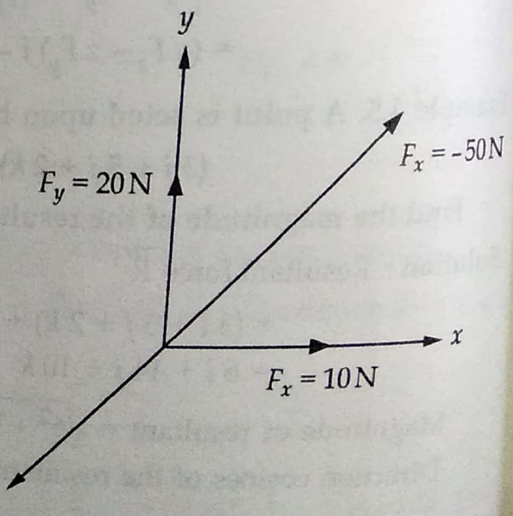

Fig. 4.11

Check : $l^2 + m^2 + n^2 = (0.1826)^2 + (0.3652)^2 + (-0.9129)^2$
$= 0.0333 + 0.1334 + 0.8334 \simeq 1.0$

(c) Angles made by the forces along the coordinate axes

$$\cos\alpha = 0.1826\,; \quad \alpha = 79.48°$$
$$\cos\beta = 0.3652\,; \quad \beta = 68.58°$$
$$\cos\gamma = -0.929\,; \quad \gamma = 155.91°$$

Example 4.8. A space force of magnitude 125 N makes angles of 30° and 65° with the x and y axes respectively.

Make calculations for the scalar components of the force along the x, y and z axes and write the vector equation for the force.

Solution :

$$l = \cos\alpha = \cos 30° = 0.866$$
$$m = \cos\beta = \cos 65° = 0.423$$

From the relation; $l^2 + m^2 + n^2 = 1$, we have

$$n = \cos\gamma = \sqrt{1 - l^2 - m^2} = \sqrt{1-(0.866)^2-(0.423)^2}$$
$$= \sqrt{1-0.75-0.179} = 0.266$$

That gives

$$\gamma = \cos^{-1}(0.266) = 74.57°$$

Scalar components of the force are :

$$F_x = F\cos\alpha = 125\cos 30° = 108.25 \text{ N}$$
$$F_y = F\cos\beta = 125\cos 65° = 52.83 \text{ N}$$
$$F_z = F\cos\gamma = 125\cos 74.57° = 33.26 \text{ N}$$

Vector equation of the force

$$= F_x i + F_y j + F_z k = 108.25\, i + 52.83\, j + 33.26\, k$$

Example 4.9. The x, y, z components of a force are 30 kN, – 20 kN and 16 kN respectively. Determine the component of this force along the line joining $A(1, 2, -3)$ and $B(-1, -3, 4)$.

Solution : Force $\vec{F} = 30i - 20j + 16k$

$$|\vec{F}| = F = \sqrt{30^2 + (-20)^2 + 16^2} = \sqrt{1556} = 39.446 \text{ kN}$$

$$\text{Unit vector of force} = \frac{30i - 20j + 16k}{39.446} = 0.760\, i - 0.507\, j + 0.406\, k$$

Then in terms of unit vector $\vec{F} = 39.446\,(0.760\, i - 0.507\, j + 0.406\, k)$.

Vector joining the points $A(1, 2, -3)$ and $B(-1, -3, 4)$ is

$$\overrightarrow{AB} = \vec{r} = (-1-1)\, i + (-3-2)\, j + [4-(-3)]\, k$$
$$= -2i - 5j + 7k$$

$$\text{Unit vector along } AB = \frac{-2i - 5j + 7k}{\sqrt{(-2)^2 + (-5)^2 + (7)^2}} = \frac{-2i - 5j + 7k}{\sqrt{78}}$$
$$= -0.226\, i - 0.566\, j + 0.792\, k$$

The component of force F along AB is

$$= (\vec{F} \text{ in terms of unit vectors}) \cdot (\text{unit vector along } AB)$$
$$= 39.446\,(0.760\, i - 0.507\, j + 0.406\, k) \cdot (-0.226\, i - 0.566\, j - 0.792\, k)$$
$$= 39.446\,[0.760 \times (-0.226) - 0.507 \times (-0.566) + 0.406 \times 0.792]$$
$$= 39.446\,(-0.1718 + 0.2869 + 0.3215) = \mathbf{17.224 \text{ kN}}$$

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Example 4.10. A force $\vec{F}$ acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 68°$ and $\theta_z = 55°$. The component of force $\vec{F}$ along y-direction is -125 N. Determine;

(*a*) angle θ_y (*b*) magnitude of force $\vec{F}$

(*c*) components of force along x and z directions, and

(*d*) component of force on a line that passes through the origin and point (1, 1, 1)

Solution : The direction cosines along the x and z directions are :

$$l = \cos\theta_x = \cos 68° = 0.3746$$
$$n = \cos\theta_z = \cos 55° = 0.5736$$

From the relation : $l^2 + m^2 + n^2 = 1$, we have

$$m = \cos\theta_y = \sqrt{1 - l^2 - n^2} = \sqrt{1-(0.3746)^2-(0.5736)^2} = \pm 0.7332$$

That gives : $\theta_y = 42.85°$ and $137.15°$

(*b*) From the relation : $F_y = F\cos\theta_y$ we have

$$\text{Magnitude of force } F = \frac{F_y}{\cos\theta_y} = \frac{-125}{\cos 137.15} = 170.53 \text{ N}$$

(*c*) Then scalar components of force in the x and z directions are :

$$F_x = F\cos\theta_x = 170.53 \times \cos 68° = 63.88 \text{ N}$$
$$F_z = F\cos\theta_z = 170.53 \times \cos 45° = 97.81 \text{ N}$$

Vector equation of force

$$\vec{F} = F_x i + F_y j + F_z k = 63.88\, i - 125\, j + 97.81\, k$$

(*d*) Coordinates of the given point $A(1, 1, 1)$

$$\text{Vector } \overrightarrow{OA} = (1-0)\, i + (1-0)\, j + (1-0)\, k = i + j + k$$

$$\text{Unit vector along } OA = \frac{i+j+k}{\sqrt{1^2+1^2+1^2}} = \frac{i+j+k}{1.732}$$

Force vector $\vec{F}$ = magnitude of force × unit vector along the direction OA

$$= 170.53 \times \frac{i+j+k}{1.732} = 98.46\,(i + j + k)$$

Thus the components of force on a line that passes through the origin and point (1, 1, 1) are

$$F_x = 98.46 \text{ N}; \quad F_y = 98.46 \text{ N} \quad \text{and} \quad F_z = 98.96 \text{ N}$$

Example 4.11 Forces 30 kN, 20 kN, 25 kN and 100 kN are respectively directed through the points whose coordinates are $A(2, 1, 5)$, $B(3, -1, 4)$, $C(-3, -2, 1)$ and $D(4, 1 - 2)$. If these forces are concurrent at the origin O, make calculations for the resultant of the system.

Solution : Distance $OA = \sqrt{(2-0)^2+(1-0)^2+(5-0)^2} = 5.478$

$$OB = \sqrt{(3)^2+(-1)^2+(4)^2} = 5.1$$
$$OC = \sqrt{(-3)^2+(-2)^2+(1)^2} = 3.742$$
$$OD = \sqrt{(4)^2+(1)^2+(-2)^2} = 4.582$$

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Unit vectors along these directions are :

$$n_1 = \frac{1}{5.478}(2\,i + j + 5\,k)$$

$$n_2 = \frac{1}{5.1}(3\,i - j + 4\,k)$$

$$n_3 = \frac{1}{3.742}(-3\,i - 2\,j + k)$$

$$n_4 = \frac{1}{4.582}(4\,i + j - 2\,k)$$

Now force vector $\vec{F} = |\vec{F}| \times \vec{n}$

= magnitude of force × unit vector in the of direction force

$$\vec{F_1} = 30 \times \frac{(2\,i + j + 5\,k)}{5.478} = 10.95\,i + 5.48\,j + 27.38\,k$$

$$\vec{F_2} = 20 \times \frac{(3\,i - j + 4\,k)}{5.1} = 11.76\,i - 3.92\,j + 15.69\,k$$

$$\vec{F_3} = 25 \times \frac{(-3\,i - 2\,j + k)}{3.742} = -20.04\,i - 13.36\,j + 6.68\,k$$

$$\vec{F_4} = 100 \times \frac{(4\,i + j - 2\,k)}{4.582} = 87.30\,i + 21.82\,j - 43.65\,k$$

Resultant $\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = 89.97\,i + 10.02\,j + 6.1\,k$

Magnitude of resultant force

$$|\vec{R}| = R = \sqrt{(89.97)^2 + (10.02)^2 + (6.1)^2} = 90.73 \text{ N}$$

The scalar components of the resultant along the x, y and z axes are 89.97 N, 10.02 N and 6.1 N respectively.

Direction cosines of the resultant are,

$$l = \cos\alpha = \frac{89.97}{90.73} = 0.9916\,; \qquad \alpha = 7.43^\circ$$

$$m = \cos\beta = \frac{10.02}{90.73} = 0.1104\,; \qquad \beta = 83.66^\circ$$

$$n = \cos\gamma = \frac{6.1}{90.73} = 0.0672\,; \qquad \gamma = 86.15^\circ$$

Check : $l^2 + m^2 + n^2 = (0.9916)^2 + (0.1104)^2 + (0.067)^2 = 1.0$

Example 4.12. A tower guy wire shown in Fig. 4.12 is anchored by means of a bolt at *A* and the tension in the wire is estimated to be 2000 kN. Determine

(a) the components F_x, F_y and F_z of the force acting on the bolt, and

(b) the angles α, β and γ which define the direction of force along the *x*, *y* and *z* directions respectively.

Solution : The coordinates of points *A* and *B* are :

$$A(30,\ 0,\ -20) \text{ and } B(0,\ 60,\ 0)$$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= (0 - 30)\, i + (60 - 0)\, j$
$+ [0 - (-20)]\, k$
$= -30\, i + 60\, j + 20\, k$

$AB = |\overrightarrow{AB}| = \sqrt{(-30)^2 + (60)^2 + (20)^2} = 70$ m

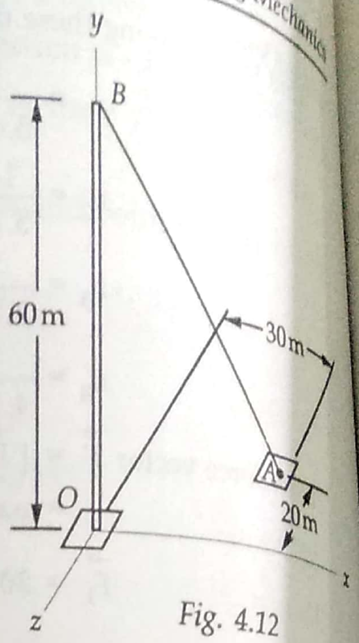

Fig. 4.12

Unit vector along $AB = \dfrac{-30\,i + 60\,j + 20\,k}{70}$

Force vector $\vec{F} = |\vec{F}| \times \vec{n}$
= magnitude of force × unit vector in the direction of force
$= 2000 \times \dfrac{-30i + 60j + 20k}{70}$
$= -857.1\, i + 1714.2\, j + 571.4\, k$

Comparing it with the general expression $\vec{F} = F_x i + F_y j + F_z k$, we have

$F_x = -857.1$ N ; $F_y = 1714.2$ N and $F_z = 571.4$ N

These are called the scalar components of the force vector.

(*b*) The direction cosines of the force vector are :

$$l = \cos\alpha = \frac{-857.1}{2000} = -0.4285\,; \qquad \alpha = 115.37^\circ$$

$$m = \cos\beta = \frac{1714.2}{2000} = 0.8571\,; \qquad \beta = 31.0^\circ$$

$$n = \cos\gamma = \frac{571.4}{2000} = 0.2857\,; \qquad \gamma = 73.4^\circ$$

Check : $l^2 + m^2 + n^2 = (-0.4285)^2 + (0.8571)^2 + (0.2857)^2 \simeq 1$

Example 4.13. A force $\vec{F} = 3\,i - 4\,j + 12\,k$ acts at a point *A* whose coordinates are (1, – 2, 3) m. Compute:

(*a*) moment of force about origin
(*b*) moments of force about the point *B* (2, 1, 2) m
(*c*) vector component of force F along line *AB* and the moment of this force about the origin.

Solution : Position vector $\overrightarrow{OA}$ = Position of O with respect to *A*
$= (x_a - x_0)\, i + (y_a - y_0)\, j + (z_a - z_0)\, k$
$= (1 - 0)\, i + (-2 - 0)\, j + (3 - 0)\, k = i - 2j + 3k$

Moment of force about the origin
$= \overrightarrow{OA} \times \vec{F} = (i - 2j + 3k) \times (3i - 4j + 12k)$

$$= \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 3 & -4 & 12 \end{vmatrix}$$

$= [(-2 \times 12) - (-4 \times 3)]\, i - [(1 \times 12) - (3 \times 3)\, j$
$+ [(-4 \times 1) - (-2 \times 3)]\, k$
$= -12\,i - 3\,j + 2\,k$

(b) Position vector $\overrightarrow{BA}$ = Position of B with recpect to A

$$= (x_a - x_b)\,i + (y_a - y_b)\,j + (z_a - z_b)\,k$$
$$= (1 - 2)\,i + (-2 - 1)\,j + (3 - 2)\,k = -i - 3j + k$$

Moment of force about point B

$$= \overrightarrow{BA} \times \vec{F} = (-i - 3j + k) \times (3i - 4j + 12k)$$

$$= \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 3 & -4 & 12 \end{vmatrix}$$

$$= [-3 \times 12 - (-4 \times 1)]\,i - [(-1 \times 12) - (3 \times 1)]\,j + [-1 \times (-4) - 3 \times (-3)]\,k$$

$$= -32\,i + 15\,j + 13\,k$$

(c) Position vector $\overrightarrow{AB} = (2 - 1)\,i + [1 - (-2)]\,j + (2 - 3)\,k = i + 3j - k$

Unit vector along $\overrightarrow{AB}$ (along the direction of force)

$$= \frac{i + 3j - k}{\sqrt{1^2 + 3^2 + (-1)^2}} = \frac{i + 3j - k}{\sqrt{11}} = 0.3\,i + 0.9\,j - 0.3\,k$$

Projection of $\vec{F}$ on AB = $\vec{F}$ · unit vector along AB

$$= (3i - 4j + 12k) \cdot (0.3\,i + 0.9\,j - 0.3\,k)$$
$$= 3 \times 0.3 - 4 \times 0.9 + 12 \times (0.3) = -6.3 \text{ m}$$

Example 4.14. A force of 100 N is directed along the line drawn from point $A(5, 2, 3)$ to the point $B(2, -5, 8)$. Determine the moment of this force about a point $C(4, 3, 1)$. The distances are in metres.

Solution: Position vector $\overrightarrow{AB} = (2 - 5)\,i + (-5 - 2)\,j + (8 - 3)\,k$
$$= -3\,i - 7\,j + 5\,k$$

Unit vector along $\overrightarrow{AB}$ (along the direction of force)

$$= \frac{-3i - 7j + 5k}{\sqrt{(-3)^2 + (-7)^2 + (5)^2}} = \frac{-3i - 7j + 5k}{9.11}$$

∴ Force vector $\vec{F}$ = magnitude of force × unit vector along the direction of force

$$= 100 \times \frac{-3i - 7j + 5k}{9.11} = -32.93i - 76.84j + 54.88k$$

Position vector $\overrightarrow{CA} = (4 - 5)\,i + (3 - 2)\,j + (1 - 3)\,k = -i + j - 2k$

∴ Moment of the force vector about point C

$$= \begin{vmatrix} i & j & k \\ -1 & 1 & -2 \\ -32.93 & -76.84 & 54.88 \end{vmatrix}$$

$$= [1 \times 54.88 - (-76.84)(-2)]\,i - [(-1) \times 54.88 - (-32.93)(-2)]\,j + [(-1) \times (-76.84) - (-32.93) \times 1]\,k$$

$$= [-98.80\,i + 120.74\,j + 109.77k] \text{ Nm}$$

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Example 4.15 A cable supporting a 20 m high vertical post is anchored to the ground as shown in Fig. 4.13. If the tensile force in the cable is 10 kN, find its moment about the z-axis passing through base of the post.

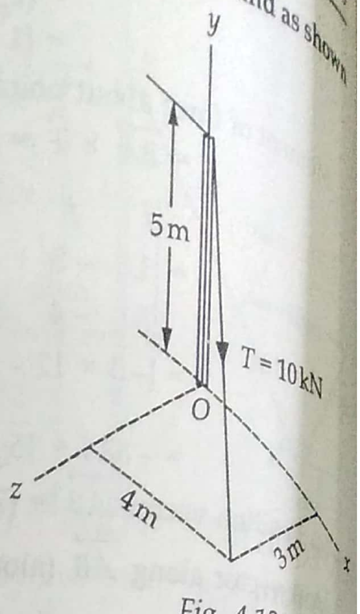

Fig. 4.13

Solution : With point O as origin, the coordinates of points A and B are :

$$A\,(0, 5, 0) \quad \text{and} \quad Q\,(4, 0, 3)$$

Position vector $\overrightarrow{AB}$

$$= (4 - 0)\,i + (0 - 5)\,j + (3 - 0)\,z$$

$$= 4i + 5j + 3z$$

$$\text{Unit vector along } AB = \frac{4i - 5j + 3z}{\sqrt{4^2 + (-5)^2 + 3^2}}$$

$$= \frac{4i - 5j + 3z}{7.07}$$

Force vector $\vec{F}$ = magnitude of force × unit vector along the direction AB

$$= 10 \times \frac{4i - 5j + 3z}{7.07} = 5.66i - 7.07j + 4.24k$$

Moment of force $\vec{F}$ about origin $O = \vec{r} \times \vec{F}$

$$\vec{r} = \text{position vector } \overrightarrow{OA} = (0 - 0)\,i + (5 - 0)\,j + (0 - 0)\,k = 5\,j$$

$\therefore$ Moment of force $\vec{M}_o = 5\,j \times (5.66\,i - 7.07\,j + 4.24\,k)$

$$= -\,28.3\,k + 0 + 21.2\,i = 21.2\,i - 28.3\,k$$

The desired moment of $\vec{F}$ about z axis

$$\vec{M}_z = -\,28.3 \text{ kNm (clockwise)}$$

The minus sign is indicative of the fact that the moment is clockwise when viewed from positive z-direction.

4.6. RESULTANT OF SPACE CONCURRENT FORCES

The resultant of any number of space concurrent forces can be determined by writing the vector expression for each force and adding the vector expressions of all forces.

Consider the forces identified as

$$\vec{F_1} = F_{1x}\,i + F_{1y}\,j + F_{1z}\,k$$

$$\vec{F_2} = F_{2x}\,i + F_{2y}\,j + F_{2z}\,k$$

$$\vec{F_3} = F_{3x}\,i + F_{3y}\,j + F_{3z}\,k$$

....

....

$$\vec{F_n} = F_{nx}\,i + F_{ny}\,j + F_{nz}\,k$$

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The resultant of these forces is

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots \vec{F_n}\, k$$
$$= (F_{1x} + F_{2x} + F_{3x} + \dots F_{nx})\, i + (F_{1y} + F_{2y} + F_{3y} + \dots F_{ny})\, j + (F_{1z} + F_{2z} + F_{3z} + \dots F_{nz})\, k$$
$$= \Sigma F_x i + \Sigma F_y j + \Sigma F_z k$$

The magnitude of the resultant force is

$$R = |\vec{R}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

and the direction cosines are

$$l = \frac{\Sigma F_x}{R}; \quad m = \frac{\Sigma F_y}{R}; \quad n = \frac{\Sigma F_z}{R}$$

4.7. EQUILIBRIUM OF A PARTICLE IN SPACE

A particle in space will be in equilibrium when the resultant of all the forces acting on it is zero.

Mathematically :

$$\vec{R} = \Sigma F_x i + \Sigma F_y j + \Sigma F_z k = 0$$

This also implies that for a particle in space to be in equilibrium, the algebraic sum of the forces acting on the particle along x-direction, y-direction and z-direction are individually equal to zero. Hence

$$\Sigma F_x = 0; \quad \Sigma F_y = 0; \quad \Sigma F_z = 0$$

Example 4.16. A vertical pole is guyed by three cables PA, PB and PC tied at a common point P, 10 m above the ground. The base points of the cable are

A (– 3, 0, – 4), B (1, – 1, 5)

and C (5, 0, – 1)

If the tensile forces in the cables are adjusted to be 15, 18 and 20 kN, make calculations for the resultant force on the pole at P.

Solution : Refer Fig. 4.14 for the arrange-ment of the three cables and the pole.

The pole P is 10 m above the ground at O, and as such the forces in the cables would be directed along PA, PB and PC.

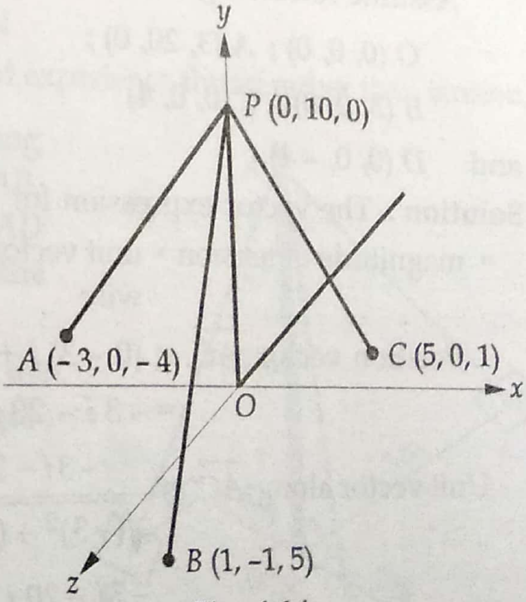


Fig. 4.14

$$\text{Distance } PA = \sqrt{(x_a - x_p)^2 + (y_a - y_b)^2 + (z_a - z_p)^2}$$
$$= \sqrt{(-3-0)^2 + (0-10)^2 + (-4-0)^2}$$
$$= \sqrt{125} = 11.18$$

$$PB = \sqrt{(1-0)^2 + (-1-10)^2 + (5-0)^2} = \sqrt{147} = 12.12$$

$$PC = \sqrt{(5-0)^2 + (0-10)^2 + (-1-0)^2} = \sqrt{126} = 11.22$$

Unit vectors along these directions are :

$$n_1 = \frac{1}{11.18}(-3\,i - 10\,j - 4\,k)$$

$$n_2 = \frac{1}{12.12}(i - 11\,j + 5\,k)$$

$$n_3 = \frac{1}{11.22}(5\,i - 10\,j - 1\,k)$$

Force vector $\vec{F} = |\vec{F}| \times \vec{n}$

= magnitude of force × unit vector in the direction of force

$$\vec{F_1} = 15 \times \frac{(-3i - 10j - 4k)}{11.18} = -4.02\,i - 13.4\,j - 5.36\,k$$

$$\vec{F_2} = 18 \times \frac{(i - 11j + 5k)}{12.12} = 1.48\,i - 16.34\,j + 7.42\,k$$

$$\vec{F_3} = 20 \times \frac{(5i - 10j + k)}{11.22} = 8.91\,i - 17.8\,j - 1.78\,k$$

Resultant $\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 6.37\,i - 47.54\,j + 0.28\,k$

Magnitude of resultant force $= \sqrt{(6.37)^2 + (-47.54)^2 + (0.28)^2} = 47.96$ kN

The resultant force on the pole is 47.96 kN in magnitude and it acts predominantly downwards to hold the pole in position.

Example 4.17. A vertical tower *AB* shown in Fig. 4.15 is subjected to a horizontal force F = 50 kN at its top and it is anchored by two guy wires *AC* and *AD*. Compute the tensions in the guy wires and thrust in the pole.

Assume following coordinates for the points :

O (0, 0, 0) ; A (3, 20, 0) ;

B (3, 0, 0) ; C (0, 0, 4)

and D (0, 0, – 4).

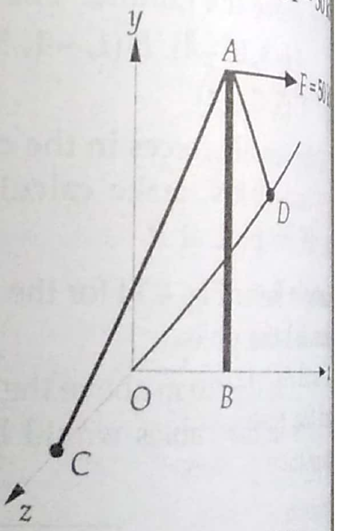


Fig. 4.15

Solution : The vector expression for tension in a guy wire is

= magnitude of tension × unit vector along the direction of that wire

Position vector $\overrightarrow{AC} = (0 - 3)\,i + (0 - 20)\,j + (4 - 0)\,k$

$= -3\,i - 20\,j + 4\,k$

Unit vector along $\overrightarrow{AC} = \dfrac{-3i - 20j + 4k}{\sqrt{(-3)^2 + (-20)^2 + 4^2}}$

$= \dfrac{-3i - 20j + 4k}{20.61}$

∴ $\vec{T}_{ac} = T_{ac} \times \dfrac{-3i - 20j + 4k}{20.61}$

$= T_{ac}[-0.146\,i - 0.97\,j + 0.194\,k]$

Similarly

$$\vec{T}_{ad} = T_{ad} \times \frac{-3i - 20j - 4k}{\sqrt{(-3)^2 + (-20)^2 + (-4)^2}}$$

$$= T_{ad}[-0.146\,i - 0.097\,j - 0.195\,k]$$

$$\vec{T}_{ab} = T_{ab} \times \frac{0i - 20j - 0k}{\sqrt{0^2 + (-20)^2 + (0)^2}} = T_{ab}(-j)$$

The force $F = 50$ kN can be expressed as a force vector

$$\vec{F} = 50\,i$$

Resultant of all forces meeting at point A

$$= \vec{T}_{ac} + \vec{T}_{ad} + \vec{T}_{ab} + \vec{F}$$

$$= (-0.146\,T_{ac} - 0.146\,T_{ad} + 50)\,i + (-0.97\,T_{ac} - 0.97\,T_{ad} - T_{ab})\,j + (0.194\,T_{ac} - 0.194\,T_{ad})\,k$$

Applying the equilibrium conditions to point A : $\vec{R} = 0$

Then equating the coefficients of i, j and k to zero, we get

$$-0.146\,T_{ac} - 0.146\,T_{ad} + 50 = 0 \quad ...(i)$$

$$-0.97\,T_{ac} - 0.97\,T_{ad} - T_{ab} = 0 \quad ...(ii)$$

$$0.194\,T_{ac} - 0.194\,T_{ad} = 0\,;\ T_{ac} = T_{ad} \quad ...(iii)$$

From identities (*i*) and (*iii*) we obtain

$$-0.146\,T_{ac} - 0.146\,T_{ac} + 50 = 0$$

$$\therefore \quad T_{ac} = \frac{50}{0.146 + 0.146} = \mathbf{171.23\ N}$$

$$T_{ad} = T_{ac} = \mathbf{171.23\ N}$$

Substituting these values of T_{ad} and T_{ac} in identify (*ii*), we get

$$-0.97 \times 171.23 - 0.97 \times 171.23 - T_{ab} = 0$$

$$\therefore \quad T_{ab} = -2 \times 0.97 \times 171.23 = \mathbf{-332.18\ N}$$

The negative value for T_{ad} indicates that the tower would experience thrust rather than tension.

Example 4.18. In the system shown in Fig. 4.16, a 5 m long beam is held in vertical position *AO* by three guy wires *AB*, *AC* and *AD*. If a tension equivalent to 600 N is induced in *AD* and the resultant force at *A* is to be vertical, compute the magnitude of forces in *AC* and *AB*.

Solution : Taking y and z axis along A and B respectively, the coordinates of various points with respect to O as origin are :

$A(0, 5, 0)$;

$B(0, 0, -3)$;

$C(-1.5, 0, 1)$

and $D(2, 0, 1.5)$

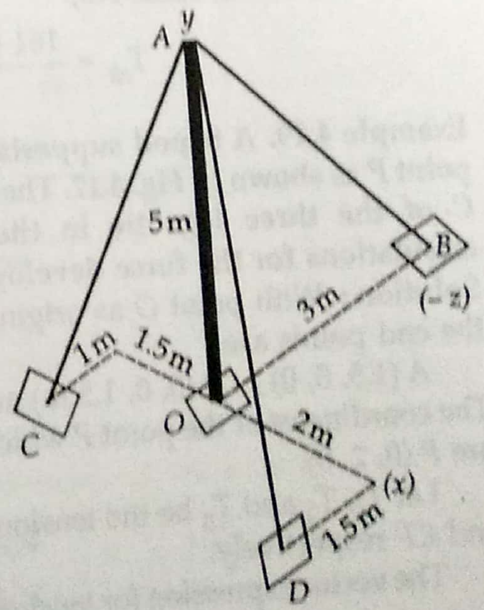


Fig. 4.16

The vector expression for tension induced in guy wire is

= magnitude of tension × unit vector along the direction of force

Position vector $\overrightarrow{AD} = (2-0)\,i + (0-5)\,j + (1.5-0)\,k = 2i - 5j + 1.5k$

Unit vector along $\overrightarrow{AD} = \dfrac{2i-5j+1.5k}{\sqrt{1^2+(-5)^2+1.5^2}} = \dfrac{2i-5j+1.5k}{5.59}$

$$\therefore \quad \overrightarrow{T_{ad}} = T_{ad} \times \frac{2i-5j+1.5k}{5.59} = 600 \times \frac{2i-5j+1.5k}{5.59}$$

$$= 214.67i - 536.67j + 161k$$

Similarly :

$$\overrightarrow{T_{ab}} = T_{ab} \times \frac{(-5j-3k)}{\sqrt{(-5)^2+(-3)^2}} = T_{ab} \times \frac{(-5j-3k)}{5.83}$$

$$= -T_{ab}(0.857j + 0.514k)$$

$$\overrightarrow{T_{ac}} = T_{ac} \times \frac{-1.5i-5j+k}{\sqrt{(-1.5)^2+(-5)^2+1^2}} = T_{ac} \times \frac{(-1.5i-5j+k)}{5.31}$$

$$= -T_{ac}(0.282\,i + 0.942\,j - 0.188\,k)$$

Resultant of all forces meeting at point A

$$= \overrightarrow{T_{ad}} + \overrightarrow{T_{ab}} + \overrightarrow{T_{ac}}$$

$$= (214.67 - 0.282\,T_{ac})\,i - (536.67 + 0.857\,T_{ab} + 0.942\,T_{ac})\,j + (161 - 0.514\,T_{ab} + 0.188\,T_{ac})$$

As resultant force at A is to be vertical

$$\Sigma F_x = 0 \text{ and } \Sigma F_z = 0$$

That gives :

$$214.67 - 0.282\,T_{ac} = 0$$

$\therefore$ Tension in cable AC,

$$T_{ac} = \frac{214.67}{0.282} = 761.24 \text{ N}$$

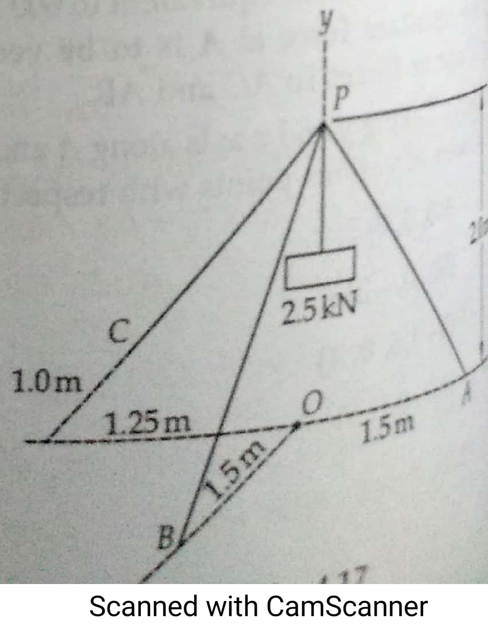
Also

$$161 - 0.514\,T_{ab} + 0.188\,T_{ac} = 0$$

$\therefore$ Tension in cable AB,

$$T_{ab} = \frac{161 + 0.188 \times 761.24}{0.514} = 591.66 \text{ N}$$

Example 4.19. A tripod supports a load of 2.5 kN at point P as shown in Fig. 4.17. The end points A, B and C of the three legs lie in the x-z plane. Make calculations for the force developed in each leg.



Solution : With point O as origin, the coordinates of the end points are :

A (1.5, 0, 0) ; B (0, 0, 1.5 m) and C (− 1.25, 0, − 1).

The coordinates of the point P which supports the load are P (0, 2, 0).

Let T_1, T_2 and T_3 be the tension in the legs AP, BP and CP respectively.

The vector expression for tension induced in a leg is

= magnitude of tension × unit vector along the direction of that leg

$$\overrightarrow{AP} = (0 - 1.5)\, i + (2 - 0)\, j + (0 - 0)\, k = -1.5\, i + 2\, j$$

$$\therefore \quad \vec{T_1} = T_1 \times \frac{-1.5i + 2j}{\sqrt{(-1.5)^2 + (2)^2}} = T_1(-0.6\, i + 0.8\, j)$$

Similarly :

$$\vec{T_2} = T_2 \times \frac{2j - 1.5k}{\sqrt{(2)^2 + (-1.5)^2}} = T_2(0.8\, j - 0.6\, k)$$

$$\vec{T_3} = T_3 \times \frac{1.25i + 2j - k}{\sqrt{(1.25)^2 + 2^2 + 1^2}} = T_3(0.49\, i + 0.78\, j + 0.39\, k)$$

Also :

$$\vec{W} = -2.5\, j$$

Resultant of all the forces meeting at point P

$$= \vec{T_1} + \vec{T_2} + \vec{T_3} + \vec{W}$$

$$= (-0.6T_1 + 0.49T_3)\, i + (0.8T_1 + 0.8T_2 + 0.78T_3 - 2.5)\, j + (-0.6T_2 + 0.39T_3)\, k$$

Applying the equilibrium conditions to point P

$$\vec{R} = \Sigma\vec{F} = 0$$

Then equating the coefficients of i, j and k to zero, we get

$$-0.6T_1 + 0.49T_3 = 0 \quad \text{...(i)}$$

$$0.8T_1 + 0.8T_2 + 0.78T_3 - 2.5 = 0 \quad \text{...(ii)}$$

$$-0.6T_2 + 0.39T_3 = 0 \quad \text{...(iii)}$$

From identities (*i*) and (*iii*) :

$$T_1 = \frac{0.49}{0.6} T_3 = 0.816T_3$$

and

$$T_2 = \frac{0.39}{0.6} T_3 = 0.65\, T_3$$

Substituting these values for T_1 and T_2 in identity (*ii*), we get

$$0.8 \times 0.816\, T_3 + 0.8 \times 0.65\, T_3 + 0.78T_3 = 2.5$$

or

$$(0.653 + 0.52 + 0.78)\, T_3 = 2.5$$

$\therefore$ Force developed in leg CP : $\quad T_3 = \dfrac{2.5}{1.953} = 1.28$ kN

Force developed in leg BP : $\quad T_2 = 0.65 \times 1.28 = 0.832$ kN

Force developed in leg AP : $\quad T_1 = 0.816 \times 1.28 = 1.04$ kN

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$= 42.4$ ...

Alternatively : The cosine of angle θ between ...

$$\cos\theta = \frac{\sqrt{3^2+4^2}}{\sqrt{3^2+4^2+5^2}} = 0.707; \qquad \theta = 45^\circ$$

$$F_{xy} = F\cos\theta = 100 \times \cos 45^\circ = 70.7\ N$$

(c) Vector $\overrightarrow{OB} = 6i + 6j + 2k$

Unit vector along $\overrightarrow{OB} = \dfrac{6i+6j+2k}{\sqrt{6^2+6^2+2^2}} = 0.688\,i + 0.688\,j + 0.229\,k$

Then

$$\begin{aligned} F_{OB} &= \vec{F} \times \text{unit vector along } OB \\ &= (42.4\,i + 56.6\,j + 70.7\,k) \cdot (0.688\,i + 0.688\,j + 0.229\,k) \\ &= 42.4 \times 0.688 + 56.6 \times 0.688 + 70.7 \times 0.229 \\ &= 29.17 + 38.94 + 16.19 = 84.3\ N \end{aligned}$$

REVIEW QUESTIONS

1. What is meant by concurrent forces in space?
2. Define the terms unit vector and position vector.
3. Explain the concept of dot and cross products in the context of vectors.
4. Express vectorially the resultant of concurrent forces in space.
5. State the conditions of equilibrium for a particle in space.
6. Define moment of force about a point in space.
7. A line originates at the points $(4, 1, -2)$ and passes through the point $(2, 2, 6)$. What will be the unit vector along this line?
8. Two vectors $\vec{A}$ and $\vec{B}$ are defined by the relations.

   $$\vec{A} = 3i + 4j + 6k \quad \text{and} \quad \vec{B} = 4i + 5j - 9k$$

   Determine the sum, difference and dot product of these vectors. Also find the angle between $\vec{A}$ and $\vec{B}$.

   $(7i + 9j - 3k;\ -i - j + 15k;\ -22;\ 109.2^\circ)$

9. Two vectors $\vec{A}$ and $\vec{B}$ with magnitude 10 units and 15 units respectively have a dot product equivalent to 50 units. Find the magnitude of their cross product.

   (144.4 units)

10. The coordinates of the initial and terminal points of a vector are $(3, 1, -2)$ and $(4, -7, 10)$. Specify the vector, evaluate its magnitude and direction cosines.

    $(i - 8j + 12k,\ 14.46,\ 0.069,\ -0.553,\ 0.830)$

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11. The line of action of a 100 N force $\vec{F}$ passes through points A (2, 5, 8) and B (7, 2, 6). The direction of force is from A to B. Express $\vec{F}$ in terms of the unit vectors i, j and k. (81.11 i - 48.67 j - 32.44 k)
12. A constant force 4 i + 3 j moves a particle along straight line from position (6, 9, 5) to position (4, 6, 8). What work will be done by the force? Take force in newtons and distances in metres. (– 17 Nm)
13. A force $\vec{F} = (10\,i + 8\,j - 5\,k)$ N acts at point A (2, 5, 6) m. What is the moment of the force about the point B (3, 1, 4) m. (– 36 i + 15 j – 48 k) Nm
14. A point P is acted upon by the following set of forces as shown in Fig. 4.20.

    F_1 = 900 N acting along line PA

    F_2 = 1200 N acting along line PB

    F_3 = 1500 N

    If the resultant of these forces is parallel to y-axis, what should be the direction of force F_3? Proceed to calculate the magnitude of the resultant force.

    (– 3020 N downwards)

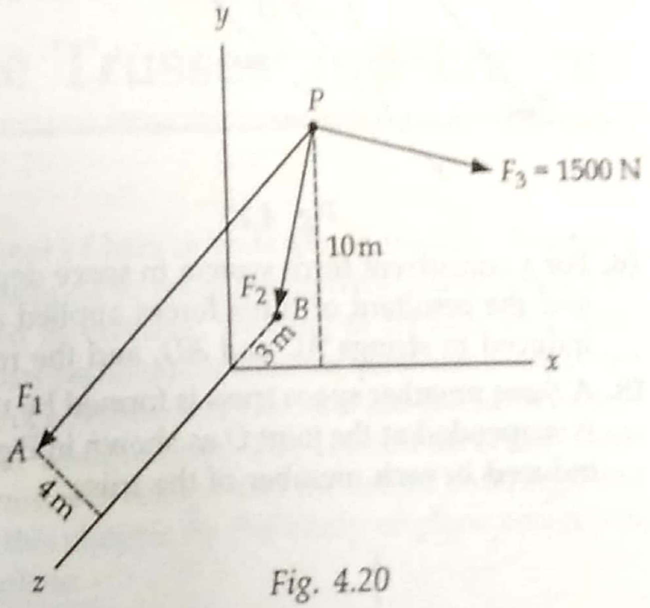

Fig. 4.20

15. A vertical load of 1000 N is suppor-ted by three bars as shown in Fig. 4.21 Points C, O and D are in x-y plane while B is 1.5 m above this plane. The base points of the bars are:

    A (0, 6, 0), B (– 3, 1.5, 0)

    C (0, 0, 3) and D (3, 0, – 3)

    Compute the forces in each member.
16. A tripod carrying a load of 50 kN has its supports A, B and C which are coplanar in x-z plane. Assuming all points to be of ball and socket type, make calculations for the forces in members AD, BD and CD (Refer Fig. 4.22)

    (– 13 kN, 58 kN, 50.4 kN)
17. An electric post is kept in equilibrium by a metal guy rope tied to a peg P as shown in Fig. 4.23. If the force in guy rope is estimated to be 1250 N, calculate the components of the force at the peg and the angles of inclination with the rectangular axes.
    [– 603 N, 1060 N, 210 N, 120.3°, 31.8°, 80.3°]

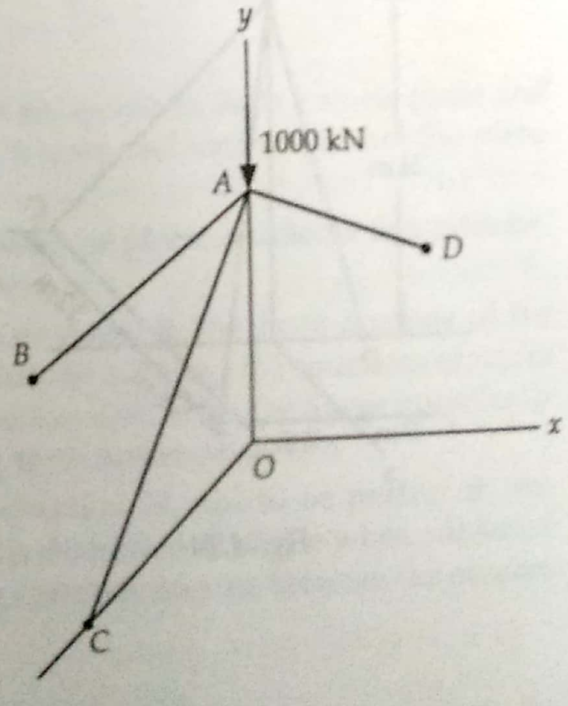

Fig. 4.21

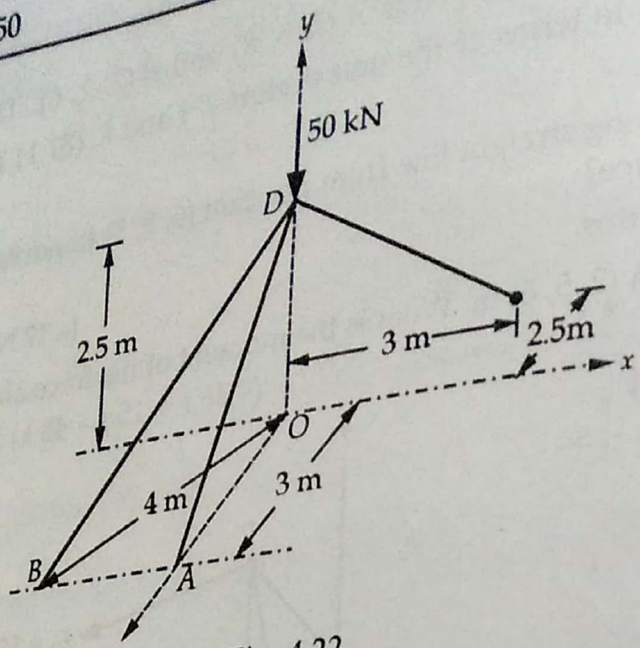

Fig. 4.22

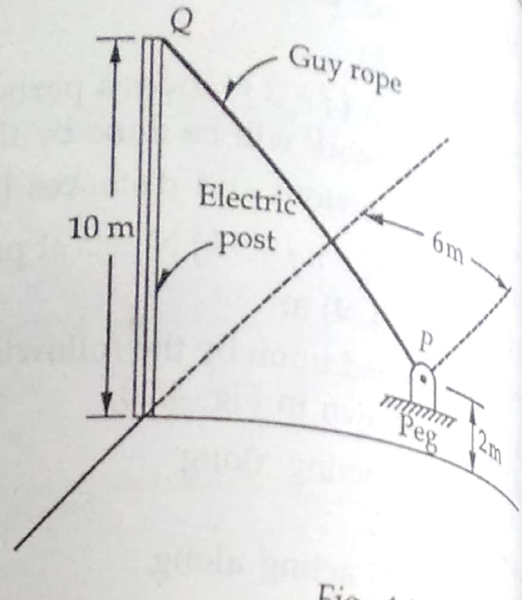

Fig. 4.23

18. For a concurrent force system in space depicted in Fig. 4.24, the tension in string AB is 75 and the resultant of three forces applied at A is vertical. Make calculations for the tensi induced in strings AC and AD, and the resultant force.(40.5 kN, 123.7 kN, 222.6 kN)
19. A three member space truss is formed by the member OA, OB and OC, and a weight of 10 is suspended at the joint O as shown in Fig. 4.25. Calculate the magnitude and nature of fo induced in each member of the truss.

(– 12.5 kN, 18 kN, – 12.5

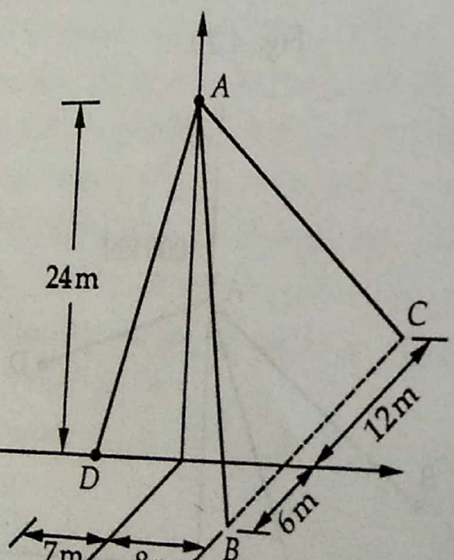

Fig. 4.24

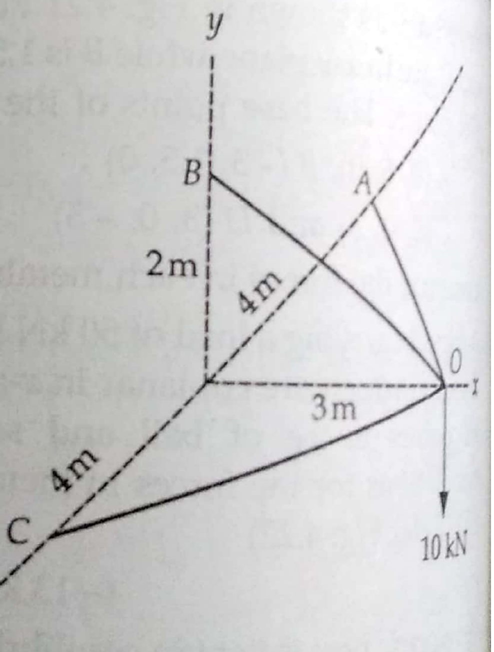

Fig. 4.25