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QM Test-3

Q1)

$\beta_1 = 0 \qquad \beta_2 = 0$

$$\frac{1}{\hbar}\int_{x_1}^{x_2} \sqrt{2m(E-V_0)}\, dx = n\pi - (\beta_1 + \beta_2)$$

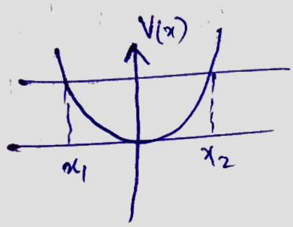
$$= \frac{1}{\hbar}\int_0^L \sqrt{2m(E-0)}\, dx = n\pi$$

$$\Rightarrow \sqrt{2mE}\int_0^L dx = n\pi\hbar$$

$$\sqrt{2mE}\, L = n\pi\hbar \Rightarrow 2mEL^2 = n^2\pi^2\hbar^2$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}.$$

Q3)



$$E = V(x_2) \Rightarrow \frac{1}{2}m\omega^2 x_2^2$$

$$x_1 = -x_2$$

$$\int_{x_1}^{x_2} p(x)dx = \left(n - \frac{1}{2}\right)\pi\hbar.$$

$$p(x) = \sqrt{2m(E-V)} = \sqrt{2m\left(E - \frac{1}{2}m\omega^2 x^2\right)}$$

$$E(V(x_2)) = \frac{1}{2}M\omega^2 x_2^2$$

$$p(x) = \sqrt{2m\left(\frac{1}{2}m\omega^2 x_2^2 - \frac{1}{2}M\omega^2 x^2\right)}$$

$$\Rightarrow \int_{x_1}^{x_2} m\omega\sqrt{x_2^2 - x^2}\, dx$$

$$= 2mw \int_0^{x_2} \sqrt{x_2^2 - x_1^2} \, dx$$

$x = x_2 \cos\theta$
$dx = -x_2 \sin\theta$.

$$\int_{\pi/2}^{0} \sin^2\theta \, d\theta = -\int_0^{\pi/2} (1 - \cos 2\theta) \, d\theta$$

$$\Rightarrow \frac{8 m w \pi x_2^2}{4 \cdot 2} = \left(n - \frac{1}{2}\right) \cancel{\pi} \hbar$$

$$\boxed{E = \frac{1}{2} m w^2 x_2^2} \qquad \frac{\cancel{\pi} m w x_2^2}{2} = \left(n - \frac{1}{2}\right) \cancel{\pi} \hbar$$

$$\Rightarrow \frac{2E}{mw^2} \frac{mw}{2} = \left(n - \frac{1}{2}\right) \pi \hbar$$

$$E = \hbar w \left(n - \frac{1}{2}\right), \quad n = 1, 2, 3 \ldots$$

Shifting $n \to n+1$,

$$\boxed{E = \hbar w \left(n + \frac{1}{2}\right)} \quad n = 0, 1, 2, 3 \ldots$$

Q4) Probability of transition from one energy eigen state of a quantum system to a group of states.

$$H_0 \psi_n = E_n \psi_n = \hbar w_n \psi_n.$$

↳ (corresponding energy eigen values).

$$\psi(t) = \sum_n b_n(t) \psi_n(r)$$

where $b(t) = C_n(t) - e^{-i w_n t}$

$$\psi(t) = \sum_n C_n(t) \, e^{i w_n t} \psi_n(r)$$

we have,

$$i\hbar \frac{\partial}{\partial t}\Psi(t) = (H_0+V)\Psi(t)$$

$$= (H_0+V)\sum_n C_n(t)\, e^{-i\omega_n t}\psi_n$$

$$= \sum_n (E_n+V)\, C_n(t)\,\psi_n(r)\, e^{-i\omega_n t}$$

also,

$$i\hbar\frac{\partial}{\partial t}\Psi(t) = i\hbar\sum_n [\dot{C}_n - i\omega_n C_n]\, e^{-i\omega_n t}\psi_n$$

$$i\hbar\sum_n [\cancel{\dot{C}_n} - i\omega_n C_n]\, e^{-i\omega_n t}\psi_n = \sum_n (E_n+V)\, \cancel{C_n(t)}\, e^{-i\omega_n t}\psi_n$$

orthonormality condition $\rightarrow$ $\int \psi_s^* \psi_n \, dr = \langle s|n\rangle = \delta_{sn}$

$$i\hbar\frac{d C_s}{dt} = \sum_n C_n(t)\, V_{sn}(t)\, e^{i\omega_{sn}t}$$

where, $\omega_{sn} = \omega_s - \omega_n = \frac{E_s - E_n}{\hbar}$

$$V_{sn}(t) = \int \psi_s^* V(t)\,\psi_n \, d\tau = \langle s|V|n\rangle$$

for obtaining exact solutions of time dependent equations,

$$i\hbar\,[C_s(t) - C_s(0)] = \sum_n C_n(0)\cdot\int_0^t V_{sn}\, e^{i\omega_{sn}t}\, dt$$

$C_s(0) = 1, \; s=i$
$\quad\;\; = 0, \; s\neq i$

$$C_f(t) = \frac{1}{i\hbar}\int_0^t V_{fi}(t)\, e^{i\omega_{fi}t}\, dt \;, \quad f\neq i$$

$$C_f(t) = \frac{1}{i\hbar} V_{fi}\int_0^t e^{i\omega_{fi}t}\, dt = -V_{fi}\frac{e^{i\omega_{fi}t}-1}{\hbar\omega_{fi}}$$

probability,

$$P_f(t) = |C_f(t)|^2 = |V_{fi}|^2 \frac{1}{\hbar^2}\frac{\sin^2(\omega_{fi}t/2)}{(\omega_{fi}/2)^2}$$

as perturbation theory is valid only when $P_f(t) << 1$,

$$\therefore (E_f - E_i)\frac{t}{2\hbar} << 1 .$$

$$\therefore P_f(t) \simeq |V_{fi}|^2 t^2/\hbar^2$$

to any state, density varying functn.

$$W = \int P_f(t)\, \rho_f(E_f)\, dE_f .$$

$$W \simeq \rho_f(E_i) \int_{\Delta E} P_f(t)\, dE_f .$$

$$W \simeq \rho_f(E_i) \frac{2t}{\hbar}|V|^2 \int_0^\infty \frac{\sin^2 \varepsilon^2}{\varepsilon^2}\, d\varepsilon .$$

$$\simeq \frac{2\pi}{\hbar} |V_{fi}|^2 \rho_f(E_i)\, t$$

then transitn probability per unit time is, $T = \frac{dw}{dt} = \underline{\frac{2\pi}{\hbar} |V_{fi}|^2 \rho_f(E_i)}$.

which is Fermi's Golden Rule.

It is also called the decay probability or decay constant and is related to mean liftime τ of the state by $\lambda = 1/\tau$. Fermi's Golden rule can apply to atomic transitions, nucleus decay, scattering. ~~etc~~.