

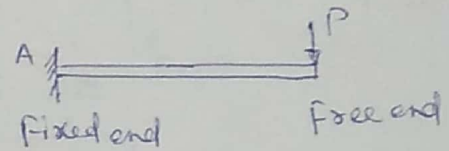
# SHEAR FORCE AND BENDING MOMENT

BEAM: A bar subject to force or couples that lie in a plane containing the longitudinal axis of bar is called beam. The forces act on the beam are perpendicular to the longitudinal axis.

## TYPES OF BEAM.

### (a) CANTILEVER BEAM:

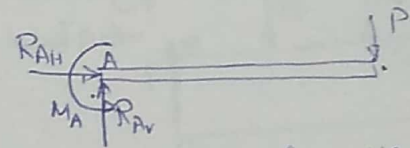
One end of cantilever beam is fixed and other end is free. The beam cannot rotate about point A. The fixed end is also called restrained.



(a) Cantilever Beam

### (b) SIMPLY SUPPORTED BEAM

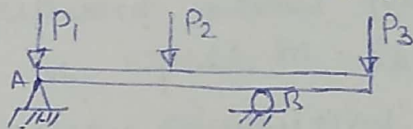
If one end of beam is hinged supported and other end is roller supported, then beam is called simply supported beam. Here beam is freely supported i.e. it can only exert the force but not capable to exert moments.



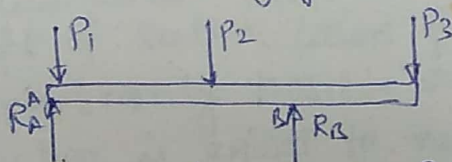
(b) FBD of Cantilever Beam

### (c) OVER HANGING BEAM

A beam freely supported at two ~~ends~~ points and having one or both ends extending beyond these supports is called an overhanging beam.



(a) Over hanging Beam



(b) FBD of Overhanging Beam

### (d) STATICALLY DETERMINATE BEAMS:

All <sup>the</sup> beams discussed above, i.e. the cantilever beams, simply supported beams and overhanging beam are such type in which the reactions of support may be determined by use of equation of static equilibrium. The values of these reactions are independent of the deformation of beam. Such beams are called to be statically determinate beams.

### (e) STATICALLY INDETERMINATE BEAMS:



# SHEARING FORCES AND BENDING MOMENTS IN BEAMS THEIR SIGN CONVENTIONS

## 1. (a) Shear forces and its Sign Conventions:

For equilibrium of beam fig(a)

$$R_A + R_B = P_1 + P_2 + P_3 + P_4$$

$$R_A - P_1 - P_2 = P_3 + P_4 - R_B \quad \text{--- (1)}$$

Consider the beam shown in fig(a). Cut the beam at C and consider left hand portion of free body i.e. AC.

Here  $R_A - P_1 - P_2$  (i.e. force acting on left hand portion AC)

$$\text{let } R_A - P_1 - P_2 > 0$$

Net force acting on free body AC

in upward direction i.e.  $R_A - P_1 - P_2$

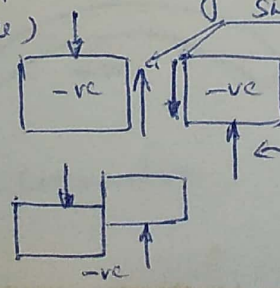
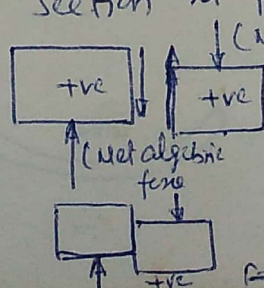
In order to maintain the free body in equilibrium an internal force  $F$  of the magnitude  $R_A - P_1 - P_2$  acting in downward direction is called an internal force or resisting shear force or simply shear force. Similarly

Similarly for free body CB, ~~an a downward internal~~ an upward internal force  $P_3 + P_4 - R_B$  acting on free body. The upward internal force  $P_3 + P_4 - R_B$  acting on free body CB must be equal to  $R_A - P_1 - P_2$  from eqn (1).

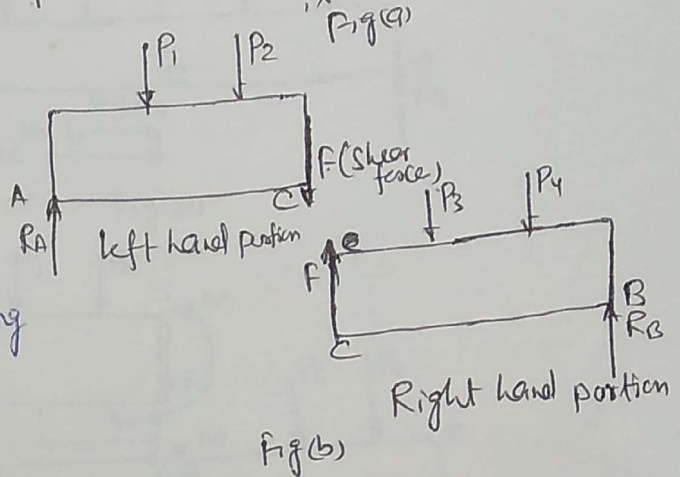
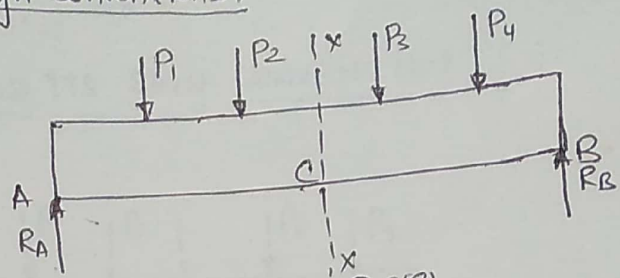
## (b) SIGN CONVENTION OF SHEAR FORCE:

A downward internal shear force at a section when forces to the left of section are considered is arbitrarily positive value. Since downward internal force at the section is equal to net algebraic sum of external force to the left of section, it follows that net algebraic sum of the external forces to the left of the section acts, upward, the shear force at that section is positive otherwise negative.

Fig(a) Shear force Sign Convention.



Fig(a) Shear force Sign Convention.



Fig(b)



Thus the shearing forces within a beam can be calculated at all section by just finding the algebraic sum of forces to left or right of the particular section.

## 2.2 (a) BENDING MOMENTS AND ITS SIGN CONVENTION

### (a) BENDING MOMENTS.

Consider the beam shown in fig. Cut the beam at c and consider the left hand free body AC, the algebraic sum of moments of all the external forces must be zero. Moments of all the external force about c is equal to

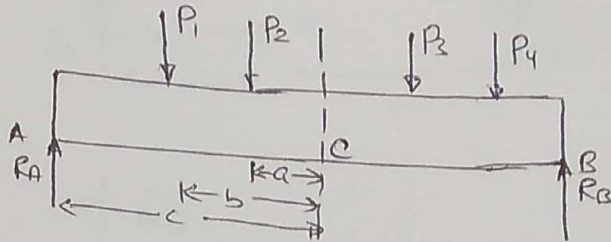


Fig (a)

$(R_1c - P_1b - P_2a)$  for the free body AC. To satisfy the condition of equilibrium, an internal (bending) moment  $M$  equal in magnitude to the external

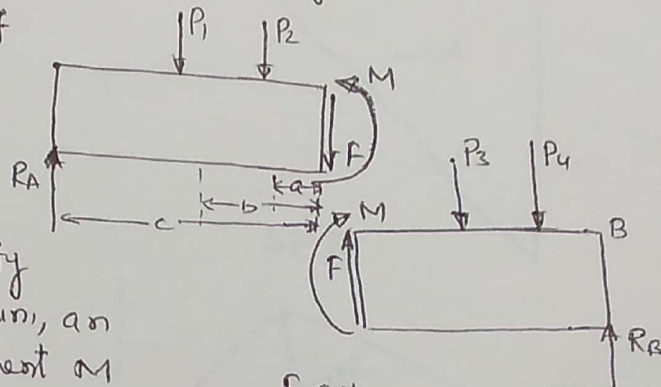
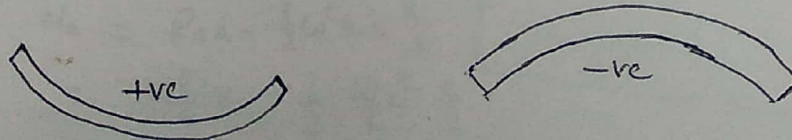


Fig (b)

(or imposed) moments (ie  $R_1c - P_1b - P_2a$ ) must exist at section c of the free body AC in the direction shown. Such external moments existing in the beam section are called bending moments of all the external forces to the left of the section (or to the right of section). Note that the net internal bending (also called resisting moment) at a section is equal to the external bending moment (ie bending due to external forces) but they have opposite signs.

### (b) SIGN CONVENTION OF BENDING MOMENTS : A bending moment

that produces compression on the top and tension at the bottom of beam is arbitrarily assigned a positive value. Bending moment (B.M) at a section if produces tension at top and compression at the bottom is negative. It will be noted that if the algebraic sum of moments of all external forces to the left of a section is directed clockwise, the B.M will be positive, otherwise negative.



B.M Sign Convention

## SFD AND BMD FOR IMPORTANT CASES

(a) Simply Supported Beam with a concentrated Load at the middle span.  
For equilibrium

$$R_A + R_B = W \quad (1)$$

$$\sum M_A = 0$$

$$\Rightarrow L R_B - W \times \frac{L}{2} = 0$$

$$R_B = \frac{W}{2}, R_A = \frac{W}{2}$$

§ For portion AC ( $0 \leq x < \frac{L}{2}$ )

$$SF = +R_A = +\frac{W}{2}$$

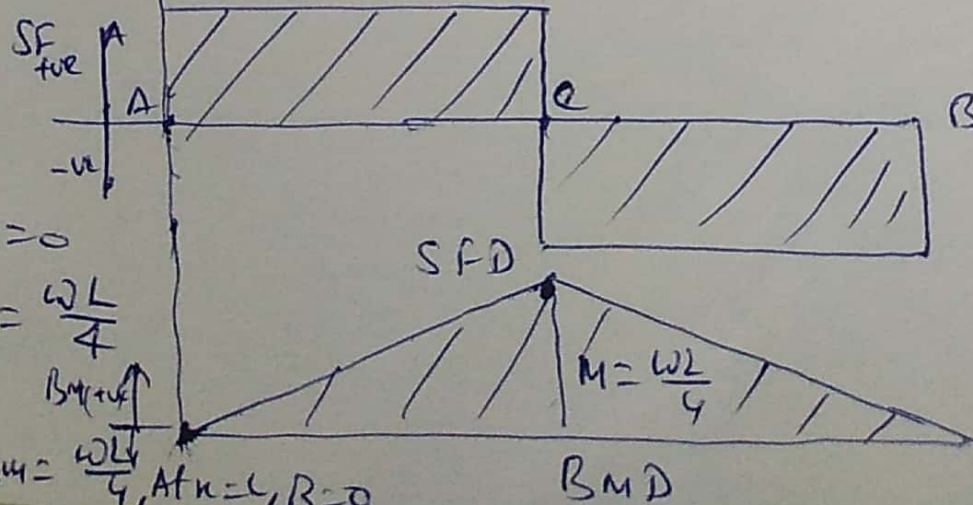
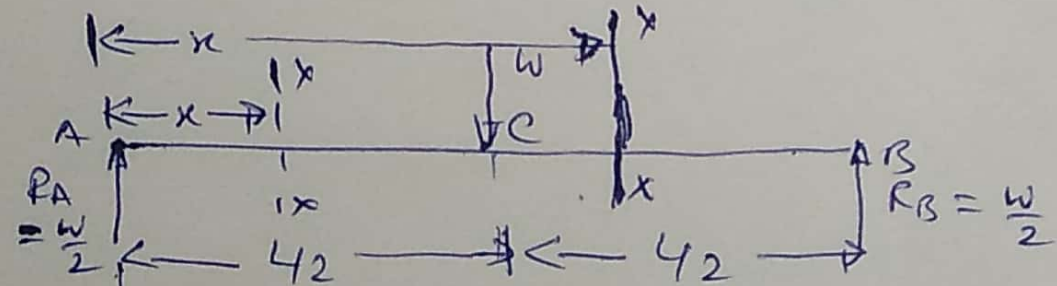
$$BM = R_A x \quad \text{At } x=0, BM=0$$

$$\text{At } x = \frac{L}{2}, BM = \frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$$

For portion CB ( $\frac{L}{2} \leq x < L$ )

$$SF = R_A - W = \frac{W}{2} - W = -\frac{W}{2}$$

$$BM = R_A x - W(x - \frac{L}{2}) \quad \text{At } x = \frac{L}{2}, BM = \frac{WL}{4}, \text{ At } x = L, BM = 0$$

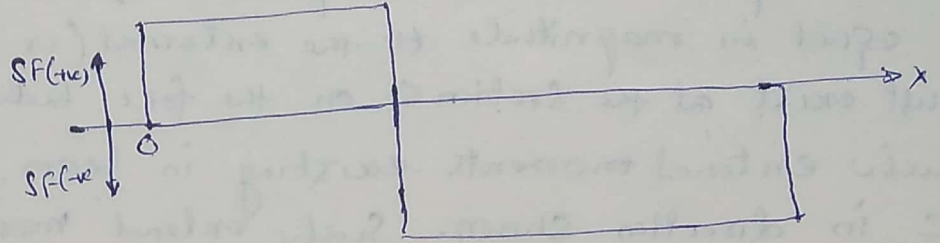
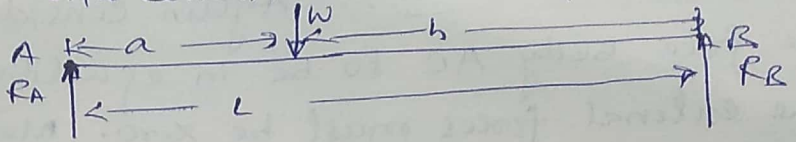




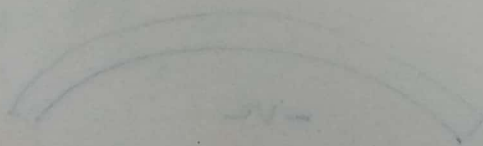
(b) Simply Supported Beam with Concentrated Load not at mid span

$$R_A = \frac{Wb}{L}$$

$$R_B = \frac{Wa}{L}$$



Sign convention for B.M. : A bending moment that produces compression at the top and tension at the bottom of beam is considered a positive value. Bending moment at a section if produces tension at top and compression at bottom is negative. If the algebraic sum of the moments of all the external forces to the left of the section is directed clockwise, the B.M. will be positive otherwise negative.



B.M sign convention.

SFD AND BMD FOR IMPORTANT CASES

(i) Simply supported beam with a concentrated load at the middle