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Question Based On

"Moment of Inertia Tensor"

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Key Concept

- Moment of Inertia is a tensor of Rank 2,
- It is a 3x3 matrix with nine components
- It determines the magnitude of torque, for desired angular acceleration ($\tau = I\alpha$)

$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

Since $I_{xy} = I_{yx}$, $I_{xz} = I_{zx}$, $I_{yz} = I_{zy}$ $\therefore$ I = Symmetric Tensor

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- If we choose the axes of fixed in the body such that off diagonal elements are zero, then such axis is called Principle axes of body

- If $\mathbb{I}$ = Inertia tensor

$\hat{n}$ = unit vector on any axis $= \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} = (\alpha, \beta, \gamma)$

then moment of Inertia about the axes $\hat{n}$ $= n\,\mathbb{I}\,n^T$

where $n = (\alpha, \beta, \gamma)$

$n^T = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$

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Question: Gate 2008

The moment of inertia tensor of a rigid body is given by

$$I = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$$

The moment of Inertia about an axis $\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ is

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Solution

Given $$\underline{I} = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$$

and $$n = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$$

So $$n^T = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{pmatrix}$$

moment of Inertia about $\hat{n}$ is

$$\underline{I}'_{\hat{n}} = n \underline{I} n^T$$

putting the values

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(6)

$$I_n' = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} \underset{3\times3}{\begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}} \underset{3\times2}{\begin{pmatrix} 1/2 \\ \sqrt{3}/2 \\ 0 \end{pmatrix}}$$

$$= \begin{pmatrix} \frac{1}{2} & \boxed{\frac{\sqrt{3}}{2}} & 0 \end{pmatrix} \begin{pmatrix} 8\times\frac{1}{2} + 0\times\frac{\sqrt{3}}{2} + -4\times0 \\ 0\times\frac{1}{2} + \boxed{4\times\frac{\sqrt{3}}{2}} + 0\times0 \\ -4\times\frac{1}{2} + 0\times\frac{\sqrt{3}}{2} + 8\times0 \end{pmatrix}$$

$$= \underset{1\times3}{\begin{pmatrix} \frac{1}{2} & \boxed{\frac{\sqrt{3}}{2}} & 0 \end{pmatrix}} \underset{3\times1}{\begin{pmatrix} 4 \\ 2\sqrt{3} \\ -2 \end{pmatrix}}$$

$$= \frac{1}{2}\times4 + \frac{\sqrt{3}}{2}\times2\sqrt{3} + 0\times-2$$

$$= 2 + \sqrt{3}\times\sqrt{3} = 2 + 3^{\frac{1}{2}}3^{\frac{1}{2}} = 2 + 3^{\frac{1}{2}+\frac{1}{2}} = 2+3^1$$

$$= 2+3 = 5 \quad \text{(Ans)}$$

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