Gate 2009

If q and p are position and momentum variables, which of the following is not canonical transformation.

(a) $Q = \alpha q$ and $P = \frac{p}{\alpha}$ for $\alpha = 0$

(b) $Q = p$ and $P = q$

Ans (b)

Explaination:

if q, p are old position and momentum

and Q, P are new ones.

Then these coordinates are related by the following transformation

$$P = P(p, q, t)$$

$$Q = Q(p, q, t)$$

Now, Transformation are canonical only if

$$[Q,P]_{q,p} = \left(\frac{\partial Q}{\partial q}\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\frac{\partial P}{\partial q}\right) = 1$$

for objective (a) $Q = \alpha q,\ P = p/\alpha$

$$\frac{\partial Q}{\partial q} = \frac{\partial}{\partial q}(\alpha q) = \alpha \ ; \ \frac{\partial Q}{\partial p} = \frac{\partial}{\partial p}(\alpha q) = 0$$

$$\frac{\partial P}{\partial q} = \frac{\partial}{\partial q}\left(\frac{p}{\alpha}\right) = 0 \ ; \ \frac{\partial P}{\partial p} = \frac{\partial}{\partial p}\left(\frac{p}{\alpha}\right) = \frac{1}{\alpha}$$

$$\therefore [Q,P] = \left(\alpha \cdot \frac{1}{\alpha} - 0 \cdot 0\right) = 1$$

For objective (b)

$$Q = p \quad , \quad P = q.$$

$$\frac{\partial Q}{\partial q} = \frac{\partial p}{\partial q} = 0 \quad ; \quad \frac{\partial Q}{\partial p} = \frac{\partial p}{\partial p} = 1$$

$$\frac{\partial P}{\partial q} = \frac{\partial q}{\partial q} = 1 \quad ; \quad \frac{\partial P}{\partial p} = \frac{\partial q}{\partial p} = 0.$$

$$\therefore \; [Q, P] = [0 \cdot 0 - 1 \cdot 1] = -1$$

So this is not Canonical Transformation.