① Assignment : Central forces 2nd Nov. 2020

1. Use Hamilton's equations to find the differential eq^n for planetary motion and prove that the areal velocity is constant.

2. A particle of mass m moves under the action of central force whose potential is $V(r) = Kmr^3$ $(K>0)$, then
(i) for what kinetic energy and angular momentum will the orbit be a circle of radius R about the origin ?
(ii) Calculate the period of circular motion.

3. The eccentricity of the earth's orbit is 0.0167. Calculate the ratio of ~~momentum~~ maximum and minimum speeds of the earth in its orbit.

4. The maximum and minimum velocities of a satellite are v_{max} and v_{min} respectively. Prove that the eccentricity of the orbit of the satellite is,

$$e = \frac{v_{max} - v_{min}}{v_{max} + v_{min}}.$$

5. A particle of mass 'm' is observed to move in a spiral orbit given by the eq^n $r = c\theta$ where c is a constant. Is it moving in a central force field ? If it is so, find the force law. [Hint : Eq^n of orbit : $\frac{d^2u}{d\theta^2} + u = -\frac{m f(\frac{1}{u})}{L^2 u^2}$]

6. A particle describes a circular motion under the influence of an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of the distance.

7. The potential energy function betⁿ two atoms of a diatomic molecule is given by

$$V = \frac{a}{x^{12}} - \frac{b}{x^6}$$ where a, b are +ve constants. x is separation betⁿ atoms

find the equilibrium point and check its stability.

8. A particle of mass 'm' moves along the x-axis under the influence of PE $V(x) = -k\,x\,e^{-\beta x}$
find the equilibrium position and its stability.

9. A mass m, moves in a circular orbit of radius r_0 under the influence of a central force whose potential is $-k/r^n$. Show that the circular orbit is stable under small oscillations.

10. Define Poisson bracket of two dynamical variables. Show that for any three dynamical variables u, v, w the Jacobi identity

$$[u, [v, w]] + [v, [w, u]] + [w [u, v]] = 0$$

11. Prove that Poisson's bracket donot obey commutative law of algebra but obeys distributive law of algebra.

12. Prove that $[X, YZ] = Y[X, Z] + [X, Y]Z$ for Poisson's brackets.

13. Using Poisson's brackets, show that total time derivative for a function $f(q, p, t)$ is given as,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H]$$ H = Hamiltonian