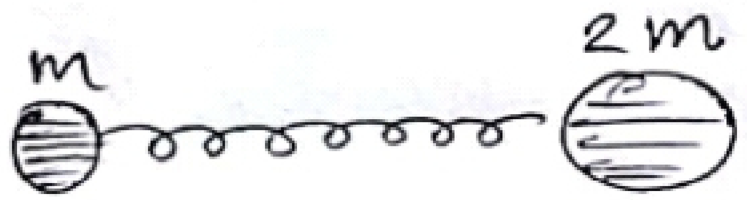
unacademy

Alternate Method :



In classical Mechanics, two body problem can be reduced to one body problem by the concept of reduced mass.

$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{2m} \quad ; \ \mu = \text{reduced mass of system}$$

$$\frac{1}{\mu} = \frac{2+1}{2m}$$

$$\frac{1}{\mu} = \frac{3}{2m} \implies \mu = \frac{2m}{2}$$

Scanned by CamScanner

Lagrangian $L = T - V$

$$L = \frac{1}{2}\mu\dot{x}^2 - \frac{1}{2}k x^2$$ [x = Coordinate for μ]

$$L = \frac{1}{2}\left(\frac{2m}{3}\right)\dot{x}^2 - \frac{1}{2}kx^2$$

$$L = \frac{m}{3}\dot{x}^2 - \frac{1}{2}kx^2$$

Now $$\frac{\partial L}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}}\left(\frac{m}{3}\dot{x}^2 - \frac{1}{2}kx^2\right)$$

$$= \frac{\partial}{\partial \dot{x}}\frac{m}{3}\dot{x}^2 - 0$$

$$= \frac{m}{3}\cdot 2\dot{x} = \frac{2m}{3}\dot{x}$$

Scanned by CamScanner

unacademy

also $\frac{\partial L}{\partial x} = \frac{\partial}{\partial x}\left(\frac{m}{3}\dot{x}^2 - \frac{1}{2}kx^2\right)$

$= 0 - \frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)$

$= -\frac{1}{2}k\frac{\partial}{\partial x}x^2$

$= -\frac{1}{2}k \cdot 2x$

$= -kx$

using the Lagrange Equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

Scanned by CamScanner

unacademy

$$\frac{d}{dt}\left(\frac{2m}{3}\dot{x}\right) - (-kx) = 0$$

$$\frac{2m}{3}\frac{d}{dt}\dot{x} + kx = 0$$

$$\frac{2m}{3}\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{3}{2}\frac{k}{m}x = 0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{3}{2}\frac{k}{m}x = 0$$

Comparing this eq. with Standard Equation of SHO

Scanned by CamScanner

unacademy

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

we get $\omega^2 = \frac{3k}{2m}$

$$\Rightarrow \quad \omega = \sqrt{\frac{3k}{2m}}$$

$=$ Normal freq. (Ans)

Scanned by CamScanner