EP-205

Assignment-4

Classical Mechanics

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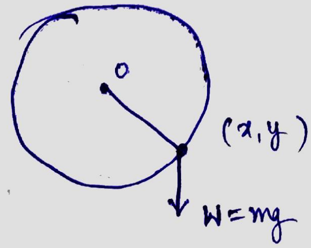
1

Consider a particle moving along circle in xy plane. its equatn of contraint is



$$x^2+y^2=r^2$$

$$2x\,\delta x + 2y\,\delta y = 0 \Rightarrow \delta x = -\frac{y}{x}\,\delta y \quad -①$$

Acc. to D'Alembert's principle,

$$(F - m\ddot{r})\,\delta r = 0$$

as $F_x = 0$

$F_y = -mg$

$$\therefore \quad (F_x - m\ddot{r})\,\delta x + (F_y - m\ddot{r})\,\delta y = 0$$

$$\Rightarrow \quad -m\ddot{x}\,\delta x - (mg + m\ddot{y})\,\delta y = 0$$

using ①

$$m(-\ddot{x}y + \ddot{y}x + gx)\,\delta x = 0$$

for $\delta x \neq 0$, $m \neq 0$,

we get $\boxed{\ddot{x}y - \ddot{y}x - gx = 0}$.

2

$$\phi = x^2 + y^2 - z$$

$$2x\,\delta x + 2y\,\delta y - \delta z = 0$$

$$\delta z = 2(x\delta x + y\delta y).$$

Acc to D'alembert's principle,

$$(F - m\ddot{r})\,\delta r = 0 .$$

$$\therefore \quad (F_x - m\ddot{x})\,\delta x + (F_y - m\ddot{y})\,\delta y + (F_z - m\ddot{z})\,\delta z = 0 .$$

As, $F_x = 0$, $F_y = 0$, $F_z = -mg$.

$$\Rightarrow \quad -m\ddot{x}\,\delta x - m\ddot{y}\,\delta y - (mg + m\ddot{z})\,\delta z = 0$$

$$\Rightarrow \quad -m\ddot{x}\,\delta x - m\ddot{y}\,\delta y - (mg + m\ddot{z})\,2(x\delta x + y\delta y) = 0$$

$$\Rightarrow \quad -m\ddot{x}\,\delta x - m\ddot{y}\,\delta y - 2mgx\,\delta x - 2mgy\,\delta y - 2m\ddot{z}x\,\delta x - 2m\ddot{z}y\,\delta y = 0$$

$$\Rightarrow \quad (-m\ddot{x} - 2mgx - 2mx\ddot{z})\,\delta x + (-m\ddot{y} - 2mgy - 2my\ddot{z})\,\delta y = 0$$

EOM: $\quad \ddot{z}x + gx + \dfrac{\ddot{x}}{2} = 0,$

$\quad \ddot{z}y + gy + \dfrac{\ddot{y}}{2} = 0 .$

$\boxed{3}$

$$p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

$$p_y = \frac{\partial L}{\partial \dot{y}} = m\dot{y} \qquad\qquad H = \sum_i p_i \dot{q}_i - L$$

$$p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z} +$$

$$H = m\dot{x}^2 + m\dot{y}^2 + m\dot{z}^2 - \left(\frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{\omega}{2}L_z\right)$$

$$= \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{\omega}{2}L_z .$$

$\boxed{4}$

$$L = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - e\phi$$
$$+ \frac{e}{c}\left(A_x\dot{x} + A_y\dot{y} + A_z\dot{z}\right)$$

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$
$$\vec{v} = \dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z}.$$

$$p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x} + \frac{e}{c}A_x$$
$$p_y = \partial L/\partial \dot{y} = m\dot{y} + \frac{e}{c}A_y$$
$$p_z = \partial L/\partial \dot{z} = m\dot{z} + \frac{e}{c}A_z$$

$$H = \sum_i \left(\dot{q}_i p_i\right) - L$$
$$= m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + \frac{e}{c}\left(A_x\dot{x} + \cancel{A_y\dot{y}} + A_z\dot{z}\right)$$
$$- \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + e\phi$$
$$- \frac{e}{c}\left(A_x\dot{x} + \cancel{A_y\dot{y}} + A_z\dot{z}\right)$$

$$H = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + e\phi$$

$$\boxed{H = T + e\phi}$$

$\boxed{5}$

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{1}{2}K(r-r_0)^2$$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r} \qquad \frac{\partial L}{\partial r} = mr\dot{\theta}^2 - K(r-r_0)$$

$$m\ddot{r} - mr\dot{\theta}^2 + K(r-r_0) = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}, \qquad \frac{\partial L}{\partial \theta} = 0$$

$$\because \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \qquad \frac{d}{dt}\left(mr^2\dot{\theta}\right) = 0$$

$$\rightarrow mr^2\dot{\theta} = L = \text{conserved.}$$

$\boxed{6}$

$$\delta t \Rightarrow \delta\int_{t_1}^{t_2} L\,dt = \int_{t_1}^{t_2} \delta L\,dt = 0.$$

$$\delta L = \frac{\partial L}{\partial q_i}\delta q_i + \frac{\partial L}{\partial \dot{q}_i}\delta \dot{q}_i + \frac{\partial L}{\partial t}\delta t \rightarrow 0$$

$$\delta L = \frac{\partial L}{\partial q_i}\delta q_i + \left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\delta q_i\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)\delta q_i\right]$$

$$\delta L = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\delta q_i\right) - \left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}\right]\delta q_i$$

$$\int_{t_1}^{t_2}\delta L\,dt = \cancel{\frac{\partial L}{\partial q}}\int_{t_1}^{t_2}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\delta q_i\right)dt - \int_{t_1}\left(\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}\right)\delta q_i\,dt = 0$$

$$= \text{As } \delta q_i = 0\Big|_{t_1}^{t_2}$$

$$\therefore \frac{\partial L}{\partial \dot{q}_i}\delta q_i\Big]_{t_1}^{t_2} = 0.$$

$$= -\int_{t_1}^{t_2}\left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}\right]\delta q_i \, dt = 0$$

as each term of δq_i is independent,

$$\therefore \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 .$$

$\boxed{7}$ consider a two-particle system,

$$m_1\ddot{r}_1 = F_{21} , \quad m_2\ddot{r}_2 = F_{12}$$

$$V_{12}(r_1 - r_2) = V_{21}(r_2 - r_1)$$

Total momentum of this system will be conserved,
so considering center of mass,

$$R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \frac{m_1 r_1 + m_2 r_2}{M}$$

$$V_{CM} = \frac{m_1 V_1 + m_2 V_2}{M}$$

$$R(t) = V_{cm}^{(0)} \times t$$

$$\therefore \quad F_{ext} = M a_{cm} .$$

$$m_1 m_2 \ddot{r}_1 = m_2 F_{21} \quad ; \quad m_1 m_2 \ddot{r}_2 = m_1 F_{12}$$

subtracting,

$$m_1 m_2(\ddot{r}_1 - \ddot{r}_2) = m_2 F_{21} - m_1 F_{12}$$

$$\Rightarrow \quad \frac{m_1 m_2}{m_1 + m_2}\ddot{r} = F_{21} = -\frac{\partial}{\partial r_1} V_{12}(r_1 - r_2)$$

$$\Rightarrow \quad m_* \ddot{r} = -\frac{\partial}{\partial r} V(r) \equiv F(r) \quad \rightarrow \text{ one body problem.}$$

where $m_* = \dfrac{m_1 m_2}{m_1 + m_2}$