Perturbation Theory of Non degenerate State

$$H_0 = -\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + \frac{1}{2}kx^2$$

$$H' = \frac{1}{2}bx^2 \text{ — perturbation in } H_0$$

The Eigenvalue and Eigenfunctions of H_0 are Well Known

$$H_0 U_n = E_n U_n$$

where $E_n = (n+\frac{1}{2})\hbar\omega = (n+\frac{1}{2})\hbar\sqrt{\frac{k}{\mu}}$

$$U_n = N_n H_n(\xi)\exp(-\tfrac{1}{2}\xi^2)$$

where $\xi = \gamma x$, $\gamma = \sqrt{\frac{\mu\omega}{\hbar}} = \left[\frac{\mu k}{\hbar^2}\right]^{1/4}$

$$H = H_0 + H'$$

$$H\psi_n = W_n\psi_n$$

$$W_n = (n+\tfrac{1}{2})\hbar\sqrt{\frac{k+b}{\mu}} = (n+\tfrac{1}{2})\hbar\sqrt{\frac{k}{\mu}\left(1+\frac{b}{k}\right)}$$

$$= (n+\tfrac{1}{2})\hbar\sqrt{\frac{k}{\mu}}\left(1+\frac{b}{k}\right)^{1/2}$$

$$= (n+\tfrac{1}{2})\hbar\sqrt{\frac{k}{\mu}}\left[1+\frac{1}{2}\frac{b}{k}-\frac{1}{8}\frac{b^2}{k^2}+\dots\right]$$

for $\frac{b}{k} < 1$

Since $W_n = W_n^{(0)} + gW_n^{(1)} + g^2 W_n^{(2)} + \dots$

$\Rightarrow W_n^{(0)} = (n+\frac{1}{2})\hbar\sqrt{\frac{k}{\mu}}$ — unperturbed value

Linear in b: $W_n^{(1)} = \frac{1}{2}(n+\frac{1}{2})\hbar\sqrt{\frac{k}{\mu}}\,\frac{b}{k}$ — First order Correction

Quadratic in b: $W_n^{(2)} = -\frac{1}{8}(n+\frac{1}{2})\hbar\sqrt{\frac{k}{\mu}}\,\frac{b^2}{k^2}$ etc. — Second Order Correction

(5) The motion of a particle of mass 'm' in one dimension is described by the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x$

what is the difference between the quantized energies of the first two levels

(A) $\hbar\omega - \lambda x$ (B) $\hbar\omega + \lambda x$ (C) $\hbar\omega + \frac{\lambda^2}{2m\omega^2}$ (D) $\hbar\omega$

(E) None of the above

<u>Here</u>

$$V = \frac{1}{2}m\omega^2 x^2 + \lambda x$$

$$= \frac{1}{2}m\omega^2\left(x^2 + 2x\frac{\lambda}{m\omega^2} + \frac{\lambda^2}{m^2\omega^4} - \frac{\lambda^2}{m^2\omega^4}\right)$$

$$= \frac{1}{2}m\omega^2\left(x^2 + \frac{\lambda}{m\omega^2}\right)^2 - \frac{\lambda^2}{m^2\omega^4}$$

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{\lambda^2}{m^2\omega^4}$$

$$E_1 - E_0 = \frac{3}{2}\hbar\omega \cancel{-\frac{\lambda^2}{m^2\omega^4}} - \frac{1}{2}\hbar\omega \cancel{+ \frac{\lambda^2}{m^2\omega^4}}$$

$$\boxed{E_1 - E_0 = \hbar\omega}$$